# Calculating Poverty Measures from the Beta Income Distribution

(Preliminary)

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26 Feb 09

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**Abstract**

Data for measuring poverty and income inequality are frequently available in a summary form that describes the proportion of income or mean income for each of a number of population proportions, ordered according to increasing income. While various discrete measures can be directly applied to data in this limited form, these discrete measures typically ignore inequality within each group. To overcome this problem Chotikapanich, Griffiths and Rao (2007a) proposed a method of moments estimator for fitting a generalized beta distribution to limited data. They examined shifts in the income distributions and Lorenz curves for the period 1988 to 1993. In a subsequent paper [Chotikapanich *et al* (2007b)], the authors estimated generalized beta income distributions for 91 countries and used these estimates to examine changes in global inequality over the period 1993 to 2000.

In this paper we extend this work to the estimation of poverty measures. We show how values of poverty measures (the head-count ratio, the poverty-gap ratio, and measures suggested by Foster, Greer and Thorbecke, Watt, Atkinson, and Sen) can be computed from the parameters of beta distributions. The methodology is illustrated using World Bank data for Bangladesh, Thailand, urban and rural India, and urban and rural China for two periods around 1993 and 2000. The sensitivity of poverty assessments and their changes to the poverty threshold is examined.

## 1. Introduction

Given the prominence of research into global poverty and major efforts to reduce that poverty, measuring the incidence of poverty and how it has changed over time is of vital importance. The purpose of this paper is to illustrate how poverty measures can be computed from the parameters of beta income distribution. The World Bank has a large ongoing research project documented on its web page, http://iresearch.worldbank.org/PovcalNet/jsp/index.jsp. On this page it is possible to calculate the incidence of poverty for numerous countries and several years. These calculations were done by the World Bank and are based on Lorenz curves estimated from income or expenditure and population proportions which in turn are compiled from data on a number of household surveys. Overviews of the World Bank's findings on the extent of poverty and how it has changed over time can be found in Chen and Ravallion (2004, 2007). In this paper we examine an alternative approach to estimating poverty. Instead of using estimated Lorenz curves, we fit beta distributions to population and income/expenditure share data, following the method suggested by Chotikapanich, Griffiths and Rao (2007), and then estimate a variety of poverty measures from the beta distributions for income/expenditure. Both approaches – the direct estimation of Lorenz curves or the direct estimation of income distributions – have been used widely in the literature for measuring income inequality and other characteristics of income distributions. Kleiber and Katz (2003) review the various income distributions that have been estimated and their characteristics, and also provide a short summary of parametric Lorenz curves that have been estimated (p.26-29). The Lorenz curves employed by the World Bank are the general quadratic (Villasenor and Arnold, 1989) and the beta Lorenz curve (Kakwani, 1980). We have chosen the beta distribution as an alternative to Lorenz curve estimation for calculating poverty measures because (a) use of a distribution instead of a Lorenz curve is an obvious gap in the literature, (b) in our earlier work (Chotikapanich et al 2007a, 2007b) the beta distribution provided an excellent fit to available share data, and (c) estimated income distributions can be used to compute a variety of characteristics of those distributions, including the Lorenz curve, but it is not always possible to retrieve an underlying income distribution and its characteristics from a Lorenz curve.

In Section 2 we describe the beta distribution and how various poverty measures can be expressed in terms of the parameters of that distribution. The countries and years selected to illustrate the techniques are described in Section 3 along with a discussion of the data. The results are presented in Section 4 and concluding remarks made in Section 5.

**2. Poverty Measures and the Beta Distribution**

In this Section we show how various poverty measures can be computed under the assumption that the underling distribution for income is a beta distribution. We begin by introducing the beta distribution and notation related to it. The beta distribution that we use is more specifically described as the beta-2 distribution. Denoting its parameters as *b*, *p* and *q*, its probability density function (pdf) is given by

$$f(y) = \frac{y^{p-1}}{b^p B(p,q)\left(1+\dfrac{y}{b}\right)^{p+q}} \qquad y > 0 \tag{1}$$

where $b > 0, p > 0$ and $q > 0$ and

$$B(p,q) = \frac{\Gamma(p)\,\Gamma(q)}{\Gamma(p+q)} = \int_0^1 t^{p-1}(1-t)^{q-1}dt$$

For the mode of $f(y)$ to be nonzero we require $p > 1$; for the mean to exist $q > 1$ is required. For the variance to exist we require $q > 2$. The corresponding cumulative distribution function (cdf) is given by

$$F(y) = \frac{1}{B(p,q)} \int_0^{[y/(b+y)]} t^{p-1}(1-t)^{q-1}dt = B_{y/(b+y)}\left(p,\ q\right) \tag{2}$$

The function $B_t(p,q)$ is the cdf for the normalized beta distribution defined on the (0,1) interval. It is a convenient representation because it is commonly included as a readily-computed function in statistical software. If $T$ is a standard beta random variable defined on the interval (0, 1), then the relationship between $T$ and $Y$ is

$$T = \frac{Y}{b+Y} \qquad\qquad Y = \frac{bT}{1-T}$$

When they exist, the mean, mode and variance of $Y$ are given by

$$\mu = \frac{bp}{q-1} \qquad\qquad m = \frac{(p-1)b}{q+1}$$

$$\sigma^2 = \mu\left[\frac{b(p+1)}{q-2} - \mu\right] = \frac{b^2 p(p+q-1)}{(q-1)^2(q-2)} \tag{3}$$

The Gini coefficient is given by

$$G = \frac{2B\left(2p, 2q-1\right)}{pB^2\left(p,q\right)} \tag{4}$$

All poverty measures are defined as the integral of a function over the interval $(0,z)$ where $z$ is called the poverty line or poverty threshold. Where that integral is an expectation of a function with the expectation taken with respect to the income density, it can be estimated using a sample average of that function. Thus, in such instances one way to compute (estimate) a poverty measure is to generate a large number of observations from the beta distribution, compute a value of the function for each observation, and then average those values. Where possible, we prefer to derive exact expressions that are readily computable by software, but, in those instances where derivations are not possible, we can rely on a sample average from generated observations. Also, for instances where derivations of exact expressions are lengthy, computing the corresponding sample average provides a check on the validity of the derivations.

The most common poverty measure is the <u>head-count ratio</u> or proportion of poor which is given by

$$H_z = F(z) = B_{z/(b+z)}(p,q) \tag{5}$$

The head-count measure provides useful information in its own right but it is also a component of other measures. It is commonly used to describe the number of poor in a country, but because it fails to provide additional information such as the intensity of poverty, other measures are also used. Before describing the other measures, it is convenient to introduce another quantity which is also a component of other measures. Specifically, the aggregate income-gap ratio is given by

$$g_z = \frac{z-\mu_z}{z} \tag{6}$$

where $\mu_z$ is mean income of the poor. Poverty is greater the larger the difference between the poverty line and the mean income of the poor. For the beta distribution $\mu_z$ is given by

$$\mu_z = \frac{\int_0^z y\,f(y)dy}{F(z)} = \frac{\mu B_{z/(b+z)}(p+1,\,q-1)}{B_{z/(b+z)}(p,q)} \tag{7}$$

The second equality in (7) holds because $y\,f(y)$ is equal to $\mu$ multiplied by a beta *pdf* with parameters $\left[b,(p+1),(q-1)\right]$. Kakwani (1999) notes that most poverty measures can be written as some function of $H_z, g_z$ and a measure of inequality among the poor. The different measures differ in the way they combine these three components and in the measure of inequality that is used or implied. In our empirical work we compute $H_z, g_z$ and the Gini coefficient for the poor, $G_z$, as well as the poverty measures described below. The Gini coefficient for the poor is not the inequality measure implied by all poverty measures, but, to avoid adding to an already large number of graphs and results, we do not explicitly compute the other measures of inequality among the poor. In what follows we draw heavily from Kakwani (1999). Proofs of results appear in Appendix A.

To obtain a measure that accommodates not just the proportion of poor, but also the magnitude of poverty among those defined as poor, a number of alternatives to the head-count ratio have been suggested. The *poverty gap ratio* is the simplest extension. It is given by

$$PG_z = \int_0^z \left( \frac{z-y}{z} \right) f(y)dy = H_z g_z \tag{8}$$

A generalization of this measure is that suggested by Foster, Greer and Thorbecke (1984)

$$FGT_z(\alpha) = \int_0^z \left( \frac{z-y}{z} \right)^{\alpha} f(y)dy \qquad \text{for } \alpha \geq 1 \tag{9}$$

The poverty gap ratio is given by $\alpha = 1$. That is, $FGT_z(1) = PG_z$. Obtaining an exact expression for $FGT_z(\alpha)$ in terms of beta distribution integrals is difficult for the general case. However, note that

$$FGT_z(\alpha) = E_f \left[ \left( \frac{z-y}{z} \right)^{\alpha} I(y \leq z) \right] \tag{10}$$

where *I*(.) is an indicator function equal to unity when its argument is true and zero otherwise. Thus, we can estimate $FGT_z(\alpha)$ accurately by drawing a large number of observations from $f(y)$, say *M*, and computing the average

$$\widehat{FGT}_z(\alpha) = \frac{1}{M} \sum_{i=1}^{M} \left( \left( \frac{z-y_i}{z} \right)^{\alpha} I(y_i \leq z) \right) \tag{11}$$

For the case $\alpha = 2$, a case commonly used in practice, more progress towards an exact expression can be made. Using the results in the Appendix we have

$$\begin{aligned} FGT_z(2) &= \int_0^z \left( \frac{z-y}{z} \right)^2 f(y)dy \\ &= H_z \left[ g_z^2 + (1-g_z)^2 \frac{\sigma_z^2}{\mu_z^2} \right] \end{aligned} \tag{12}$$

where $\sigma_z^2$ is the variance of the incomes of the poor. It can be computed from

$$\sigma_z^2 = \mu_z^{(2)} - \mu_z^2$$

where the second moment $\mu_z^{(2)}$ is

$$\mu_z^{(2)} = \frac{1}{F(z)} \int_0^z y^2 f(y) dy = \frac{1}{F(z)} \frac{\mu b(p+1)}{q-2} B_{z/(b+z)}(p+2, q-2) \tag{13}$$

The next poverty measure that we consider is the Atkinson measure

$$\begin{aligned} A_z(e) &= \frac{1}{e} \int_0^z \left[ 1 - \frac{y}{z}^e \right] f(y) dy \\ &= \frac{H_z}{e} \left[ 1 - (1 - g_z)^e \frac{\mu_z^{(e)}}{\mu_z^e} \right] \qquad o < e \le 1 \end{aligned} \tag{14}$$

where $\mu_z^{(e)}$ is the '$e$-th moment' for the income distribution of the poor. It can be calculated from

$$\mu_z^{(e)} = \frac{b^e B(p+e, q-e) B_{z/(b+z)}(p+e, q-e)}{H_z B(p,q)} \tag{15}$$

As $e \to 0$, the Atkinson measure approaches what is known as the Watt's poverty measure.

$$W_z = \int_0^z \left( \ln(z) - \ln(y) \right) f(y) dy \tag{16}$$

Expressing this measure in terms of beta functions is not straightforward so we estimate it from

$$\widehat{W_z} = H_z \ln(z) - \frac{1}{M} \sum_{i=1}^{M} \ln(y_i) I(y_i \le z) \tag{17}$$

where $(y_1, y_2, \ldots, y_M)$ are a set of draws from the beta distribution with parameters $b$, $p$ and $q$. Kakwani (1999) suggest a closely related measure given by $K_z^* = 1 - \exp(-W_z)$.

Another popular poverty measure is that proposed by Sen. It is given by

$$S_z = 2\int_0^z \left(\frac{z-y}{z}\right)\left(\frac{F(z)-F(y)}{F(z)}\right) f(y)dy$$
$$= H_z\left(g_z + (1-g_z)G_z\right) \tag{18}$$

where $G_z$ is the Gini coefficient for the poor given by

$$G_z = -1 + \frac{2}{\mu_z F^2(z)}\int_0^z yF(y)f(y)dy$$
$$= -1 + \frac{\mu}{\mu_z F^2(z)}\left[GB_{z/(b+z)}(2p+1,2q-2) + B^2_{z/(b+z)}(p+1,q-1)\right] \tag{19}$$

The necessary steps for computing the above measures are summarized in Appendix B. In the next Section we describe the data used to illustrate our computations.

**3. Data and Examples**

The formulae described in the previous Section were used to estimate and compare poverty in four Asian countries for two years around 1992 and 2000. The four countries chosen were China, India, Bangladesh and Thailand, with estimates for both rural and urban regions being obtained for India and China. China, India, and Bangladesh were chosen because they are three of the world's most populous countries; Thailand was chosen as an example of a transitional economy that has grown rapidly over the past few decades.

Ideally distribution data should refer either to income or expenditure of persons or households. In the current data set, there is a mix of countries with income or expenditure data. Most data used in this paper are for the distribution of expenditure; the exceptions where the distribution of income is used are those for rural and urban China in 1992. These differences could influence the estimates of the parameters of the respective "income" distributions.

Currently China is the world's most populous country. It has an estimated population of 1.30 billion (in 2005) with an average annual growth rate of 0.65 during the period of 2001-05. India is the second most populated country with 1.08 billion people in 2005. Its average population growth rate of 1.49 over the period of 2001-05 is significantly higher than that of

China. It has been forecast that India will replace China as the most populous country in the world in about 30 years (US Bureau of Statistics). Although the urban population in India has increased over recent years due migration to larger cities, around 70% still lives in rural areas.

China and India are among the five largest economies in the world (in PPP terms in 2005) with an estimated GDP (PPP) of 8.81 and 3.77 trillion dollars, respectively (World Bank, 2006). The average annual GDP growth rate in China for the period of 2001-05 was 9.5% while India grew at a rate of 7% during the same period. However, GDP per capita in these countries is relatively low, being 6,757 in China and 3,452 in India, compared to the other larger economies (41,890 in United Sates for example).

With 144.82 million people, Bangladesh is ranked as the 8th most populous country in the world (World Bank, 2006). Bangladesh's average GDP growth was 5.4% over the period of 2001-05. However, Bangladesh still remains as one of the lowest income countries with per capita income of just $2,053 (ppp terms).

To estimate the beta distributions for income/expenditure for each of the countries in each of the years we used the grouped data on the distribution of either income or expenditure provided on the World's Bank web pages (used there for estimating Lorenz curves). The estimation technique used is that described in Chotikapanich et al (2007a). Mean monthly income/expenditure for each income classes in PPP terms was calculated from the income/expenditure proportions in the raw data and from the World Bank's calculation of country mean monthly income/expenditure in PPP terms. Table 1 contains the estimates of the beta distribution parameters for the different countries for each of the two years. Also included are mean monthly income/expenditure and the Gini coefficients. Two estimates of the Gini coefficients areg given, one from the Lorenz curves estimated by the World Bank and one from the expression in equation (4). In general the two sets of Gini estimates are very similar. A check of the mean incomes implied by the beta distribution estimates $\mu = bp/(q-1)$ also yields mean incomes close to those used by the World Bank.

## 4. Results

In this paper the poverty lines used follow the conventional poverty lines of \$1.08 and \$2.16 per day. The \$1.08 poverty line is known as “a dollar a day” poverty line and it is used to measure extreme poverty. We present the results in a variety of tables and graphs. Tables 2 and 3 present the measures $H$ = head-count ratio, $G$ = aggregate income-gap ratio, $PG$ = poverty gap ratio, $FGT2$ = Foster-Greer-Thorbecke measure with parameter $\alpha = 2$, $S$ = Sen measure, $A5$ = Atkinson measure with parameter $e = 0.5$, and $W$ = Watts measure. Although less conventional, we also report values of the measures for a poverty line of \$5.40 in Table 4. Doing so gives a more complete picture of the left side of the income/expenditure distribution and the extent of poverty, although it may include parts of the population not regarded as experiencing extreme poverty. In Figures 1 to 7 the values of each of the measures is graphed against alternative values for the poverty line varying from \$30 per month to \$160 per month for each of the countries/areas and the two years. From the tables and graphs we can assess which countries/areas exhibit the greatest extent of poverty and we can examine whether the ranking of countries is sensitive to the choice of poverty measure or the choice of poverty line. In addition we can examine whether poverty has declined in each country over time and where this decline, if it exists, has been greatest.

In 1992 the ranking of countries/areas according to highest incidence of poverty is generally India-rural, China-rural, Bangladesh, India-urban, Thailand and China-urban. For the head-count ratio this ordering depends on the poverty line. Bangladesh exhibits a higher proportion of poor than China-rural at larger values of the poverty line, although, in general, the proportions from these countries are similar; also, for income/expenditure greater than approximately \$130 per month the proportion of poor in China-urban is greater than that for Thailand. For all other poverty measures the ranking is consistent for all values of the poverty line.

In 2000 India-rural continues to exhibit the highest incidence of poverty and China-urban the least, but Bangladesh has taken over from China-rural as the country/area with the second greatest incidence of poverty. Also, there is not a great deal of difference in the measures for India-rural and Bangladesh with Bangladesh being the poorest country/area for lower values of the poverty line. Otherwise, the ranking of countries according to the highest incidence of poverty is the same as in 1992.

It is also of interest to examine whether there has been a decline in poverty in each country/area over the period 1992-2000. Figure 8 is useful for this purpose, as are Tables 2, 3 and 4. The incidence of poverty has fallen unambiguously in Thailand, China-rural and India-rural. With the exception of the head-count ratio, all poverty measures, evaluated at all poverty lines, indicate that the incidence of poverty in Bangladesh has increased. For a poverty line of $50 per month or greater the head-count ratio shows a slightly lower proportion of people living in poverty in Bangladesh in 2000, but for poverty lines less than approximately $50, the proportion of poor has increased. For China-urban there has generally been a reduction in poverty. Strictly speaking, for low values of the poverty line the calculated values of the poverty measures suggest an increase in poverty. However, these values are so close to zero one can conclude that, for low values of the poverty line, poverty is negligible in both 1992 and 2000. For larger poverty lines the incidence of poor is no longer negligible, but there has been a dramatic decline in poverty over the period. In the remaining country/area, India-urban, there is some evidence of increased poverty for the lower poverty lines, but generally the difference is small at these levels and for poverty lines greater than say $50 per day, there has been a marked decline in the incidence of poverty.

## 5. Concluding Remarks

We have derived expressions for several poverty measures in terms of the parameters of the beta-2 distribution and used those expressions to compute measures of poverty in six country/areas in South-East Asia. Hitherto, such measures have mainly been computed using discrete grouped data or estimated Lorenz curves. Thus, estimating poverty via an estimated income or expenditure distribution fills an existing gap in the literature. The beta-2 distribution has proved to be one that fits well when applied to grouped data. Future work will extend these results to include measures of the pro-poorness of growth.

## 6. Appendices

### Appendix A  Some Derivations

Poverty Gap Ratio

The result in equation (8) follows because

$$\int_0^z \left(\frac{z-y}{z}\right) f(y)dy = H_z - \frac{1}{z}\int_0^z yf(y)dy$$

$$= H_z - \frac{H_z \mu_z}{z}$$

$$= H_z\left(1 - \frac{\mu_z}{z}\right)$$

$$= H_z g_z$$

Foster , Greer and Thorbecke (2) Measure

From equation (12)

$$FGT_z(2) = \int_0^z \left(1 - \frac{2y}{z} + \frac{y^2}{z^2}\right) f(y)dy$$

$$= H_z - \frac{2}{z} H_z \mu_z + \frac{1}{z^2} H_Z \mu_z^{(2)}$$

$$= H_z \left[\left(1 - \frac{\mu}{z}\right)^2 + \frac{\sigma_z^2}{z^2}\right]$$

$$= H_z \left[g_z^2 + \left(1 - g_z\right)^2 \frac{\sigma_z^2}{\mu_z^2}\right]$$

To obtain the expression for $\mu_z^{(2)}$ in (13) note that

$$\int_0^z y^2 f(y)dy = \int_0^z \frac{y^2 y^{p-1}}{b^p B\left(p,q\right)\left(1 + \frac{y}{b}\right)^{p+q}} dy$$

$$= \frac{b^2 B\left(p+2, q-2\right)}{B\left(p,q\right)} \int_0^z \frac{y^{p+1}}{b^{(P+2)} B(p+2, q-2)\left(1 + \frac{y}{b}\right)^{p+q}} dy$$

$$= \frac{b^2 p\left(p+1\right)}{\left(q-1\right)\left(q-2\right)} B_{z/(b+z)}\left(p+2, q-2\right)$$

$$= \frac{\mu b\left(p+1\right)}{q-2} B_{z/(b+z)}\left(p+2, q-2\right)$$

<u>Atkinson Measure</u>

The result in (14) follows because

$$\frac{1}{e}\int_0^z\left[1-\left(\frac{y}{z}\right)^e\right]f(y)dy=\frac{1}{e}\left[H_z-\frac{1}{z^e}\right]\int_0^z y^e f(y)dy$$

$$=\frac{1}{e}\left[H_z-\frac{1}{z^e}H_z\mu_z^{(e)}\right]$$

$$=\frac{H_z}{e}\left[1-\frac{\mu_e^z}{z^e}\frac{\mu_z^{(e)}}{\mu_z^e}\right]$$

$$=\frac{H_z}{e}\left[1-\left(1-g_z\right)^e\frac{\mu_z^{(e)}}{\mu_z^e}\right]$$

where

$$\mu_z^{(e)}=\frac{1}{F(z)}\int_0^z\frac{y^e y^{p-1}}{b^p B(p,q)\left(1+\frac{y}{b}\right)^{p+q}}dy$$

$$=\frac{b^e B(p+e,q-e)}{F(z)B(p,q)}\int_0^z\frac{y^{p+e-1}}{b^{p+e}B(p+e,q-e)\left(1+\frac{y}{b}\right)^{p+q}}dy$$

$$=\frac{b^e B(p+e,q-e)B_{z/(b+z)}(p+e,q-e)}{H_z B(p,q)}$$

<u>Sen Measure</u>

To derive the result in (18) we write

$$S_z=2\int_0^z\left(\frac{z-y}{z}\right)\left(\frac{F(z)-F(y)}{F(z)}\right)f(y)dy$$

$$=2\int_0^z\left(1-\frac{y}{z}-\frac{F(y)}{F(z)}+\frac{yF(y)}{zF(z)}\right)f(y)dy$$

$$=2\left[\int_0^z\left(1-\frac{y}{z}\right)f(y)dy-\frac{1}{F(z)}\int_0^z F(y)f(y)dy+\frac{1}{zF(z)}\int_0^z yF(y)f(y)dy\right]$$

The first integral is the poverty gap ratio. To evaluate the second integral we make the transformation $R=F(y)$ which gives

$$\int_0^z F(y) f(y) dy = \int_0^{F(z)} R dR = \frac{F^2(z)}{2} = \frac{H_z^2}{2}$$

Also recognizing that

$$\frac{1}{z} = \frac{1-g_z}{\mu_z}$$

we have

$$S_z = 2\left[H_z g_z - \frac{H_z}{2} + \frac{1-g_z}{\mu_z H_z}\int_o^z yF(y) f(y) dy\right]$$

$$= H_z g_z + H_z g_z - H_z + \frac{2(1-g_z)}{\mu_z H_z}\int_0^z yF(y) f(y) dy$$

$$= H_z\left[g_z + (1-g_z)\left(-1 + \frac{2}{\mu_z H_z^2}\int_0^z yF(y) f(y) dy\right)\right]$$

$$= H_z\left[g_z + (1-g_z) G_z\right]$$

To obtain an expression for the Gini coefficient for the poor in terms of parameters of the beta distribution and beta distribution functions, we begin by considering the distribution function

$$F(y) = \frac{1}{B(p,q)}\int_0^{y/(b+y)} t^{p-1}(1-t)^{q-1} dt$$

Using integration by parts, we have

$$F(y) = \frac{1}{B(p,q)}\left[\frac{1}{p}\left(\frac{y}{b+y}\right)^p \left(1-\frac{y}{b+y}\right)^{q-1} + \left(\frac{q-1}{p}\right)\int_0^{y/(b+y)} t^p (1-t)^{q-2} dt\right]$$

The beauty of this expression is that it allow us to express $F(y)$ in terms of the distribution function $F^*(y)$ for a beta distribution with parameters $[b,(p+1),(q-1)]$. Specifically, let,

$$F^*(y) = \frac{1}{B(p+1,q-1)}\int_0^{y/(b+y)} t^p (1-t)^{q-2} dt$$

and note that

$$\frac{q-1}{p}=\frac{B(p,q)}{B(p+1,q-1)}$$

implying that

$$F(y)=\frac{1}{pB(p,q)}\left(\frac{y}{b+y}\right)^{p}\left(1-\frac{y}{b+y}\right)^{q-1}+F^{*}(y)$$

Armed with this expression we can consider the integral

$$\begin{aligned}
I_z &= \int_0^z yF(y)f(y)dy \\
&= \int_o^z \frac{1}{pB(p,q)}\left(\frac{y}{b+y}\right)^{p}\left(1-\frac{y}{b+y}\right)^{q-1} yf(y)dy+\int_0^z F^{*}(y)yf(y)dy \\
&= \frac{1}{pb^{p}B^{2}(p,q)}\int_0^z \frac{y^{p}}{(1+y/b)^{p+q}}\left(\frac{y}{b+y}\right)^{p}\left(1-\frac{y}{b+y}\right)^{q-1} dy+\mu\int_0^z F^{*}(y)f^{*}(y)dy
\end{aligned}$$

where $yf(y)=\mu f^{*}(y)$ with $f^{*}(y)$ being the beta density function with parameters $\left[b,(p+1),(q-1)\right]$. Setting $y=bt/(1-t)$ and simplifying yields

$$\begin{aligned}
I_z &= \frac{b}{pB^{2}(p,q)}\int_0^{z/(b+z)} t^{2p}(1-t)^{2q-3}\,dt+\frac{\mu\left[F^{*}(z)\right]^{2}}{2} \\
&= \frac{bB(2p+1,2q-2)B_{z/(b+z)}(2p+1,2q-2)}{pB^{2}(p,q)}+\frac{\mu}{2}B^{2}_{z/(b+z)}(p+1,q-1) \\
&= \frac{\mu}{2}\left[\frac{2B(2p,2q-1)B_{z/(b+z)}(2p+1,2q-2)}{pB^{2}(p,q)}\right]+B^{2}_{z/(b+z)}(p+1,q-1) \\
&= \frac{\mu}{2}\left[GB_{z/(b+z)}(2p+1,2q-2)+B^{2}_{z/(b+z)}(p+1,q-1)\right]
\end{aligned}$$

The next to last equality uses the result

$$B(2p+1,2q-2)=\frac{\mu}{b}B(2p,2q-1)$$

while the last equality uses the expression for the Gini coefficient of the whole sample given in equation (4). For readers who have lasted the distance, the Gini coefficient for the poor can now be written as

$$G_z = -1 + \frac{2}{\mu_z F^2(z)} I_z$$

$$= -1 + \frac{\mu}{\mu_z H_z^2}\left(GB_{z/(b+z)}(2p+1, 2q-2) + B_{z/(b+z)}^2(p+1, q-1)\right)$$

**Appendix B   Steps for Calculation**

The objective is to compute various poverty measures given values for the beta distribution parameters *b*, *p* and *q*. The following steps are convenient ones for these calculations.

1. $\mu = \dfrac{bp}{q-1}$

2. $t = \dfrac{z}{b+z}$

3. $H_z = F(z) = B_t(p, q)$

4. $\mu_z = \dfrac{\mu}{H_Z} B_t(p+1, q-1)$

5. $g_z = \dfrac{z - \mu_z}{z}$

6. $PG_z = H_z g_z$

7. $\mu_z^{(2)} = \dfrac{\mu b(p+1)}{H_z(q-2)} B_t(p+2, q-2)$

8. $\sigma_z^2 = \mu_z^{(2)} - \mu_z^2$

9. $FGT_z(2) = H_z\left[g_z^2 + (1-g_z)^2 \dfrac{\sigma_z^2}{\mu_z^2}\right]$

10. $$\mu_z^{(e)} = \frac{b^e B\left(p+e, q-e\right) B_t\left(p+e, q-e\right)}{H_z B^2\left(p, q\right)}$$

11. $$A_z\left(e\right) = \frac{H_z}{e}\left[1-\left(1-g_z\right)^e \frac{\mu_z^{(e)}}{\mu_z^e}\right]$$

12. $$G = \frac{2B\left(2p, 2q-1\right)}{PB^2\left(p, q\right)}$$

13. $$G_z = -1 + \frac{\mu}{\mu_z H_Z^2}\left(GB_t\left(2p+1, 2q-2\right) + B_t^2\left(p+1, q-1\right)\right)$$

14. $$S_z = H_z\left(g_z + (1-g_z)G_z\right)$$

15. Generate $M$ value from $f(y)$. Compute $\widehat{W} = H_z \ln z - \frac{1}{M}\sum_{i=1}^{n} \ln(y_i) I(y_i \le z)$

## Appendix C EViews Program for Computing Poverty Measures

```
scalar b=.469226
scalar p= 368.8472
scalar q= 4.759436
scalar e=0.5
scalar mu=b*p/(q-1)
scalar bpq=@beta(p,q)
scalar bpeqe=@beta(p+e,q-e)
scalar gini=2*@beta(2*p,2*q-1)/(p*bpq^2)
smpl 1 51
genr z=30+@trend*2.4
genr pie=.01+@trend*.0196
genr tet=@qbeta(peep,p,q)
genr eta=@cbeta(tet,p+1,q-1)
vector(51) h
vector(51) g
vector(51) pg
vector(51) w_est
vector(51) fgt2
vector(51) a5
vector(51) giniz
vector(51) s
vector(51) s_est
vector(51) fgt2_est
vector(51) a5_est
smpl 1 50000
```

```
genr xxx=@rbeta(p,q)
genr x=b*xxx/(1-xxx)
genr cat=@cbeta(xxx,p,q)
for !j=1 to 51
scalar zed=z(!j)
scalar tee= zed/(b+zed)
h(!j)=@cbeta(tee,p,q)
scalar muz=mu*@cbeta(tee,p+1,q-1)/h(!j)
g(!j)=(zed-muz)/zed
pg(!j)= h(!j)*g(!j)
scalar muz2=(mu*b*(p+1)/(h(!j)*(q-2)))*@cbeta(tee,p+2,q-2)
scalar sig2z=muz2-muz^2
fgt2(!j)=h(!j)*(g(!j)^2+(1-g(!j))^2*sig2z/muz^2)
scalar muze=b^e*bpeqe*@cbeta(tee,p+e,q-e)/(h(!j)*bpq)
a5(!j)=(h(!j)/e)*(1-(1-g(!j))^e*muze/muz^e)
giniz(!j)=-1+(mu/(muz*h(!j)^2))*(gini*@cbeta(tee,2*p+1,2*q-2)+@cbeta(tee,p+1,q-1)^2)
s(!j)=h(!j)*(g(!j)+(1-g(!j))*giniz(!j))
genr y=(x<=zed)
genr temp=y*((zed-x)/zed)^2
fgt2_est(!j)=@mean(temp)
genr temp=y*log(x)
w_est(!j)= h(!j)*log(zed)-@mean(temp)
genr temp=y*(1-(x/zed)^e)
scalar c5=@mean(temp)
a5_est(!j)=c5/e
genr temp=2*y*((zed-x)/zed)*((h(!j)-cat)/h(!j))
s_est(!j)=@mean(temp)
next
```

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**Table 1 : Estimated Coefficients and Related Quantities from Beta Distributions**

| Country | Year | b | p | q | Mean Income (Monthly) | Gini Coefficient | WB Gini |
|---|---|---|---|---|---|---|---|
| Bangladesh | 1992 | 0.469 | 368.847 | 4.759 | 46.11 | 0.283 | 0.2827 |
| | 2000 | 0.106 | 1187.899 | 3.690 | 46.85 | 0.329 | 0.3342 |
| China_ Rural | 1992 | 35.192 | 6.210 | 5.922 | 44.00 | 0.329 | 0.3203 |
| | 1999 | 0.681 | 199.097 | 3.290 | 59.27 | 0.355 | 0.3539 |
| China_Urban | 1992 | 50.366 | 16.947 | 8.491 | 114.02 | 0.243 | 0.2417 |
| | 1999 | 81.701 | 9.152 | 5.433 | 168.71 | 0.316 | 0.3155 |
| India_ Rural | 1992 | 2.600 | 49.516 | 4.638 | 35.24 | 0.296 | 0.2988 |
| | 2000 | 0.468 | 348.555 | 4.861 | 42.33 | 0.280 | 0.2811 |
| India_Urban | 1992 | 4.390 | 49.516 | 4.638 | 59.51 | 0.296 | 0.3551 |
| | 2000 | 8.959 | 20.872 | 3.655 | 70.46 | 0.351 | 0.35 |
| Thailand | 1992 | 5.145 | 32.322 | 2.283 | 129.80 | 0.462 | 0.4622 |
| | 2000 | 8.124 | 25.333 | 2.542 | 133.75 | 0.432 | 0.4315 |

Note: p,p q values and gini coefficients are authors calculations. Mean monthly income from World Bank PovCal website http://iresearch.worldbank.org/PovcalNet/jsp/index.jsp

**Table 2: Poverty Measures for Selected Asian Countries**

**Poverty Line: $1.08 a day (1993 ppp)**

| Year | Country | H | G | PG | FGT2 | S | A5 | W |
|---|---|---|---|---|---|---|---|---|
| 1992 | Bangladesh | 35.17 | 22.85 | 8.04 | 2.62 | 0.11 | 0.09 | 0.09 |
| | China -Rural | 41.73 | 31.64 | 13.20 | 5.82 | 0.18 | 0.15 | 0.18 |
| | China -Urban | 0.26 | 10.88 | 0.03 | 0.01 | 0.00 | 0.00 | 0.00 |
| | India -Rural | 57.72 | 31.17 | 17.99 | 7.43 | 0.24 | 0.21 | 0.25 |
| | India -Urban | 18.29 | 19.23 | 3.52 | 1.02 | 0.05 | 0.04 | 0.04 |
| | Thailand | 6.78 | 19.17 | 1.30 | 0.38 | 0.02 | 0.01 | 0.01 |
| 2000 | Bangladesh | 40.29 | 26.63 | 10.73 | 3.95 | 0.15 | 0.12 | 0.14 |
| | China -Rural | 27.20 | 23.68 | 6.44 | 2.19 | 0.09 | 0.07 | 0.08 |
| | China -Urban | 0.34 | 14.72 | 0.05 | 0.01 | 0.00 | 0.00 | 0.00 |
| | India -Rural | 41.91 | 24.60 | 10.31 | 3.54 | 0.14 | 0.11 | 0.13 |
| | India -Urban | 17.09 | 21.71 | 3.71 | 1.20 | 0.05 | 0.04 | 0.04 |
| | Thailand | 4.35 | 17.48 | 0.76 | 0.21 | 0.01 | 0.01 | 0.01 |

**Table 3: Poverty Measures for Selected Asian Countries**

**Poverty Line: $ 2.16 a day (1993 ppp)**

| Year | Country | H | G | PG | FGT2 | S | A5 | W |
|---|---|---|---|---|---|---|---|---|
| 1992 | Bangladesh | 84.28 | 43.42 | 36.59 | 18.95 | 0.46 | 0.44 | 0.53 |
| | China -Rural | 83.27 | 47.83 | 39.83 | 22.96 | 0.50 | 0.49 | 0.62 |
| | China -Urban | 14.08 | 17.54 | 2.47 | 0.67 | 3.49 | 2.68 | 0.03 |
| | India -Rural | 92.32 | 53.19 | 49.10 | 29.54 | 0.59 | 0.61 | 0.78 |
| | India -Urban | 11.53 | 36.08 | 25.16 | 69.75 | 0.33 | 0.29 | 0.34 |
| | Thailand | 36.09 | 31.21 | 11.26 | 4.80 | 0.15 | 0.13 | 0.16 |
| 2000 | Bangladesh | 82.96 | 46.43 | 38.52 | 21.19 | 0.48 | 0.47 | 0.58 |
| | China -Rural | 72.16 | 41.26 | 29.78 | 15.18 | 0.38 | 0.35 | 0.44 |
| | China -Urban | 7.75 | 19.88 | 1.54 | 0.48 | 0.02 | 0.02 | 0.02 |
| | India -Rural | 87.75 | 46.12 | 40.47 | 21.85 | 0.50 | 0.49 | 0.59 |
| | India -Urban | 61.13 | 36.46 | 22.29 | 10.51 | 0.29 | 0.26 | 0.30 |
| | Thailand | 52.39 | 36.68 | 19.22 | 9.20 | 0.25 | 0.23 | 0.27 |

**Table 4: Poverty Measures for Selected Asian Countries**

**Poverty Line: $ 5.4 a day (1993 ppp)**

| Year | Country | H | G | PG | FGT2 | S | A5 | W |
|---|---|---|---|---|---|---|---|---|
| 1992 | Bangladesh | 99.30 | 72.61 | 72.10 | 54.36 | 0.79 | 0.98 | 1.40 |
| | China -Rural | 99.23 | 73.67 | 73.10 | 56.31 | 0.81 | 1.01 | 1.48 |
| | China -Urban | 85.69 | 40.55 | 34.74 | 17.10 | 0.44 | 0.41 | 0.50 |
| | India -Rural | 99.70 | 78.71 | 78.48 | 63.26 | 0.85 | 0.32 | 1.67 |
| | India -Urban | 97.89 | 65.64 | 64.26 | 45.00 | 0.73 | 0.84 | 1.16 |
| | Thailand | 79.94 | 53.15 | 42.49 | 26.43 | 0.52 | 0.54 | 0.70 |
| 2000 | Bangladesh | 98.64 | 73.11 | 72.11 | 55.12 | 0.80 | 0.99 | 1.43 |
| | China -Rural | 96.67 | 67.87 | 65.61 | 47.66 | 0.75 | 0.88 | 1.23 |
| | China -Urban | 60.47 | 35.39 | 21.40 | 9.99 | 0.28 | 0.25 | 0.31 |
| | India -Rural | 99.53 | 74.67 | 74.32 | 57.25 | 0.81 | 1.02 | 1.48 |
| | India -Urban | 94.86 | 62.78 | 59.56 | 40.94 | 0.70 | 0.78 | 1.06 |
| | Thailand | 78.16 | 50.26 | 39.29 | 23.44 | 0.49 | 0.49 | 0.64 |

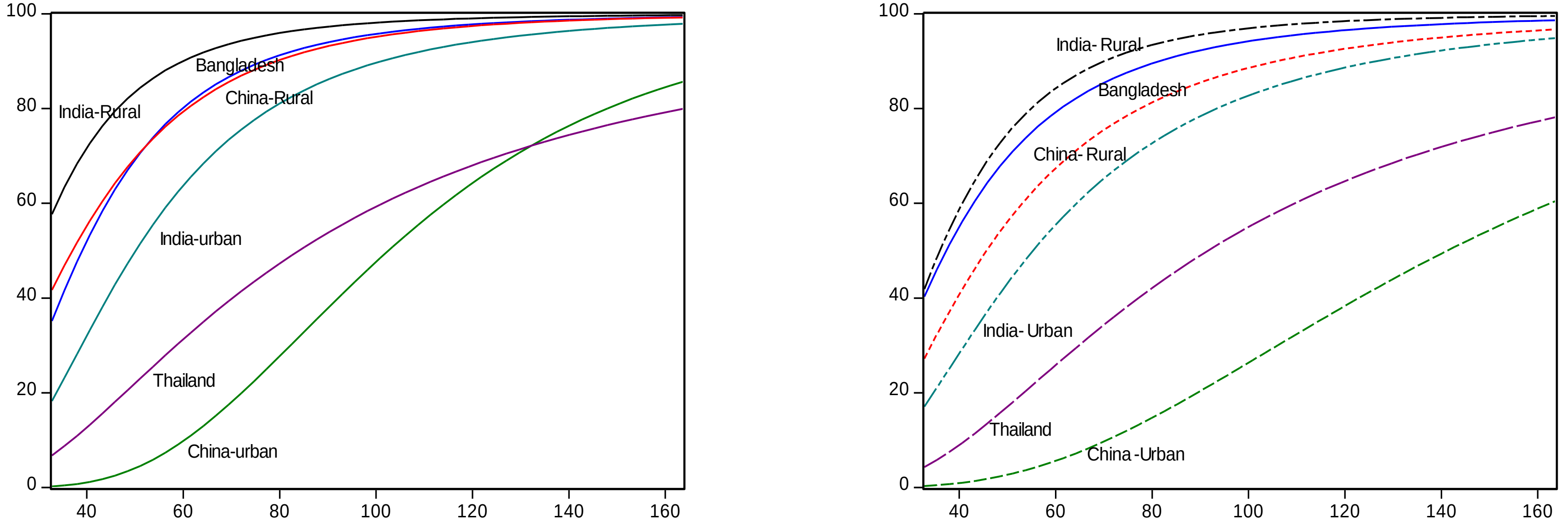
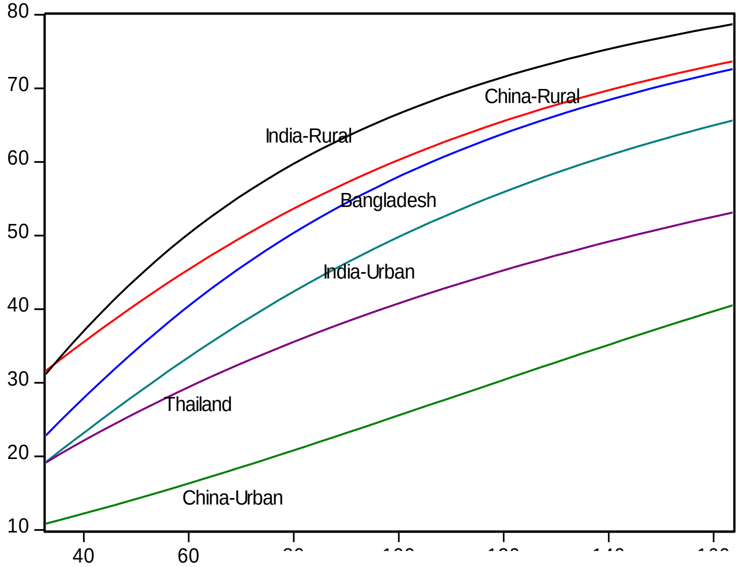



**Figure 1 : Head Count Ratio in Selected Countries/ areas, 1992 and 2000**



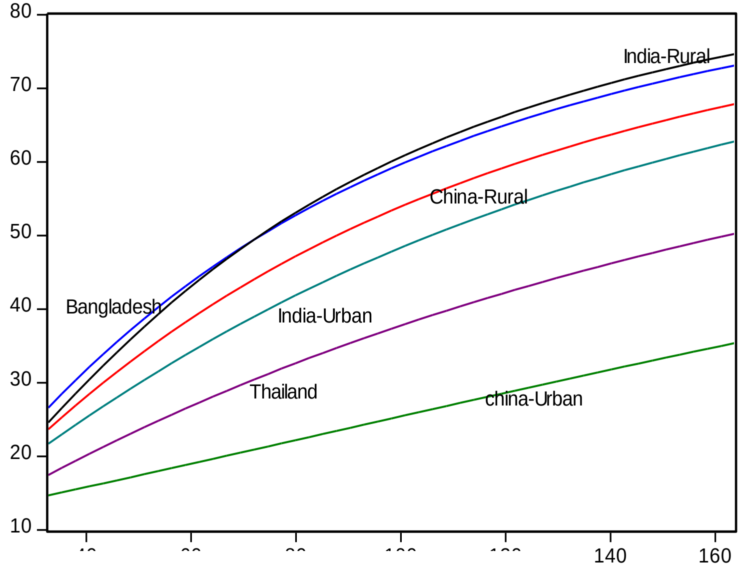
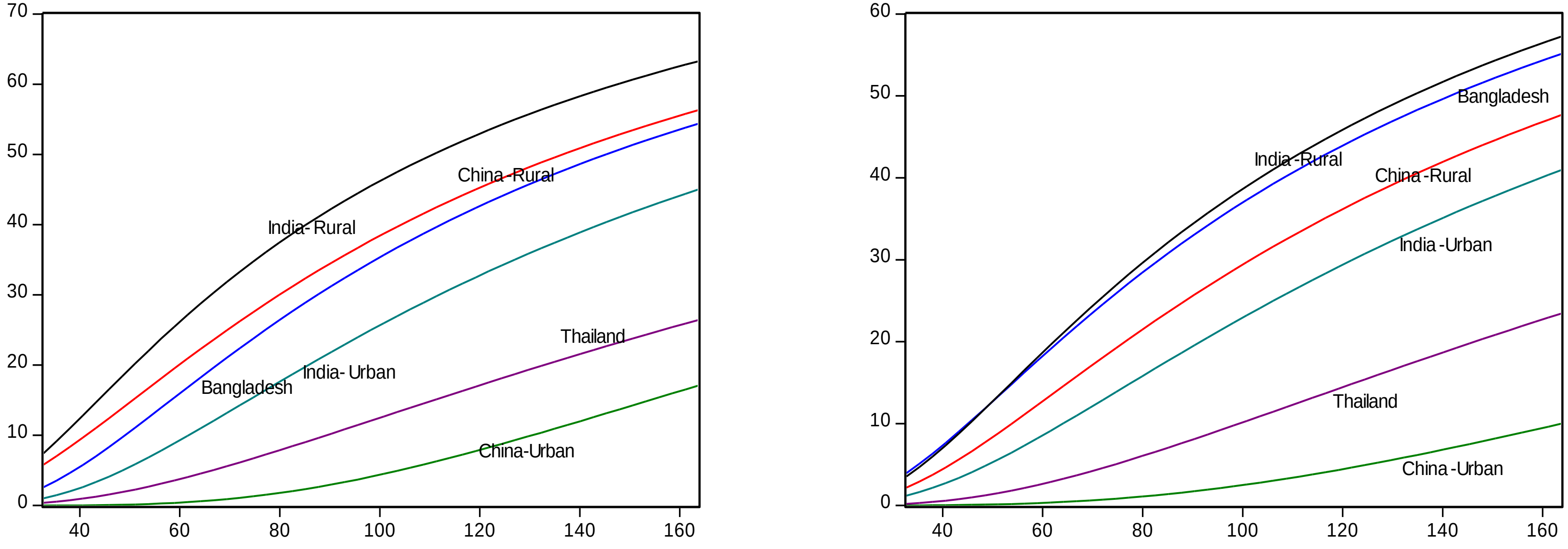




**Figure 2: Aggregate Poverty Gap Ratio in Selected Countries/ areas, 1992 and 2000**




**199** **Figure 3: FGT (2) in Selected Countries/ areas, 1992 and 2000**

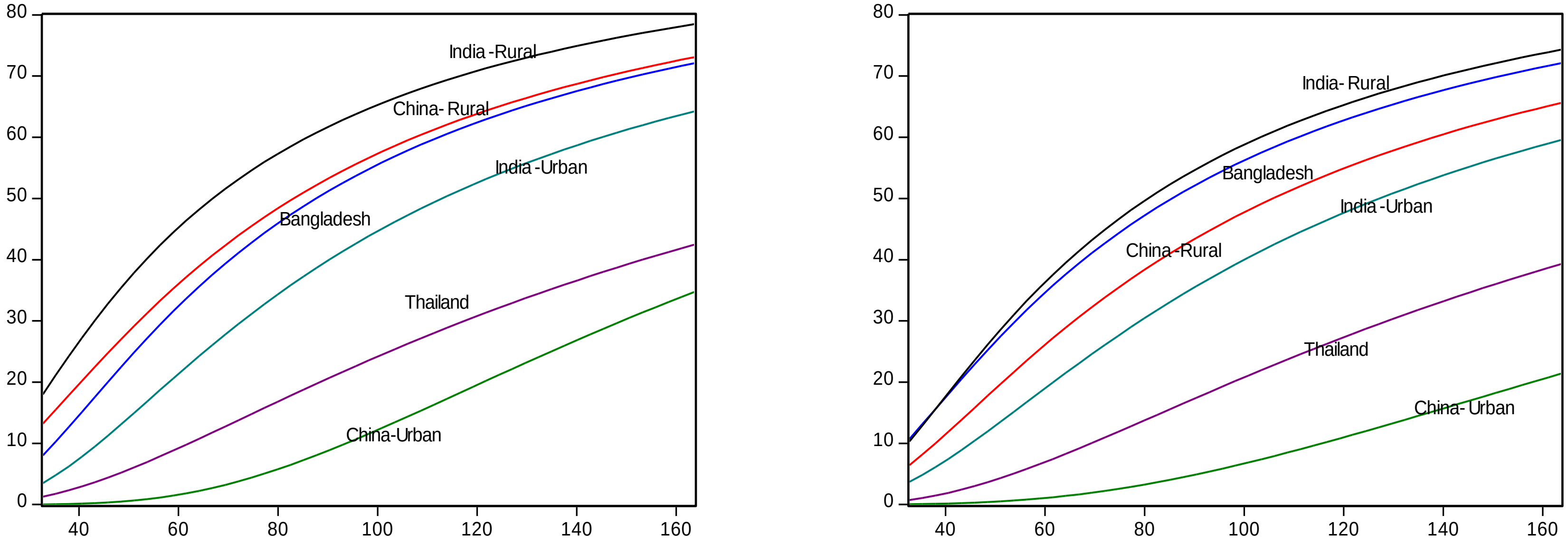



**Figure 4: Poverty Gap Ratio in Selected Countries/ areas, 1992 and 2000**

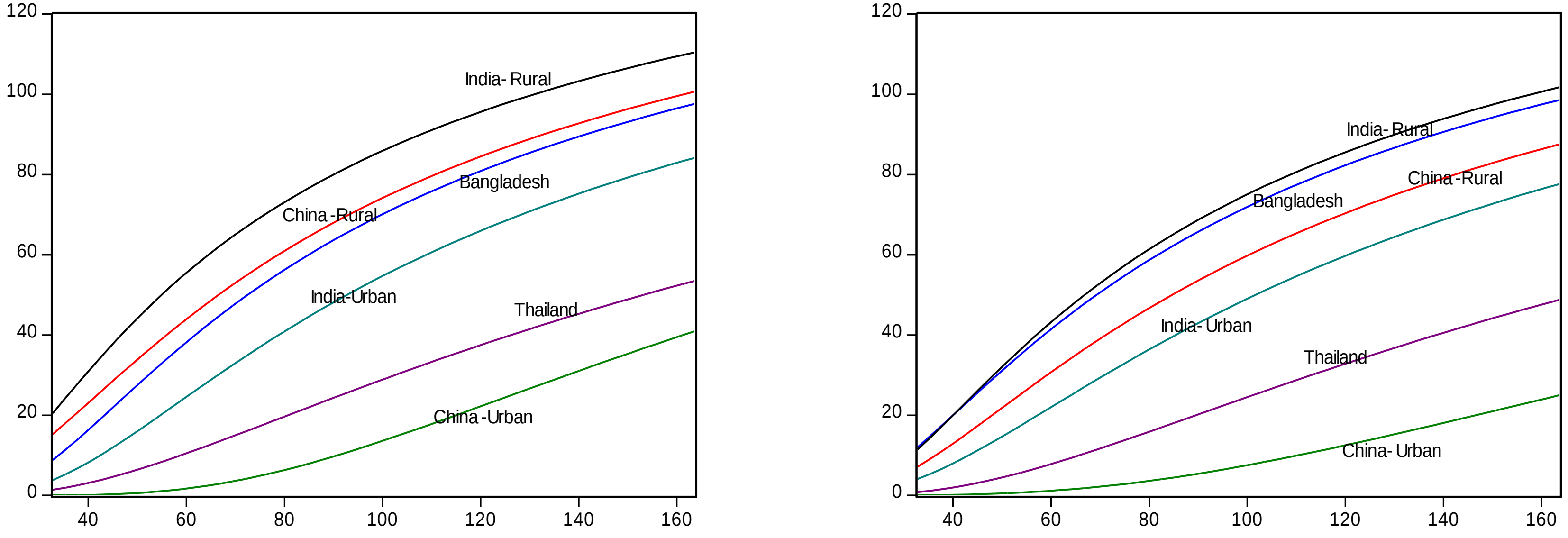
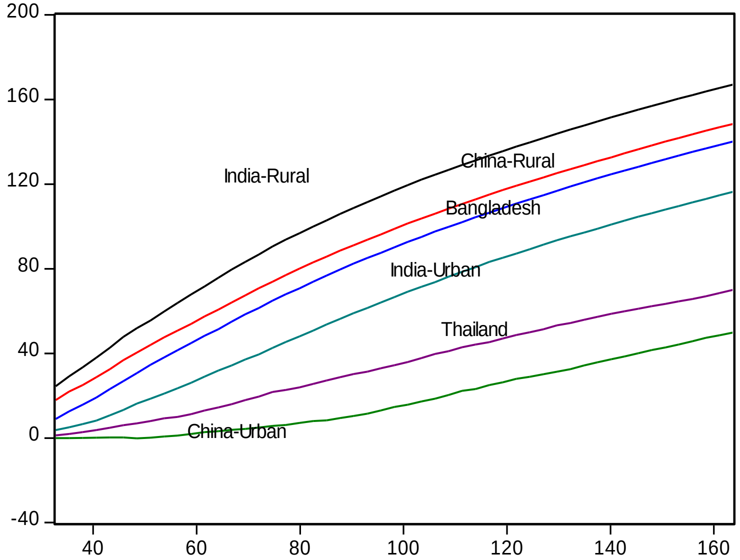



**Figure 5: Atkinson Measure in Selected Countries/ areas, 1992 and 2000**



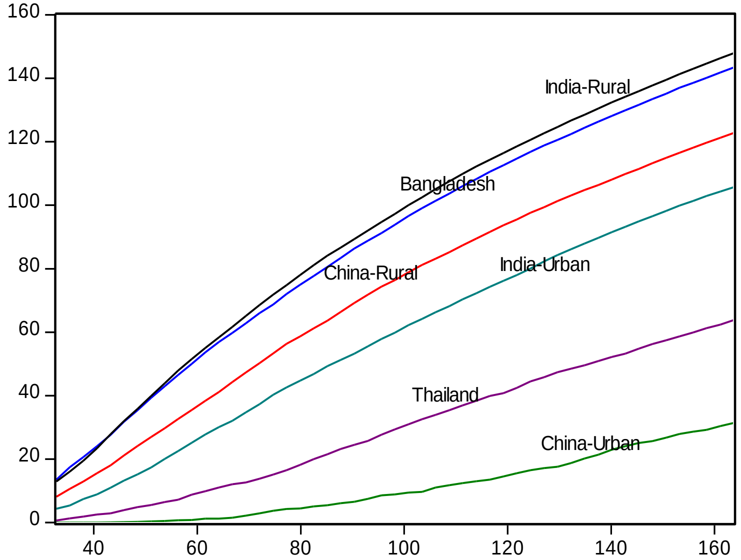




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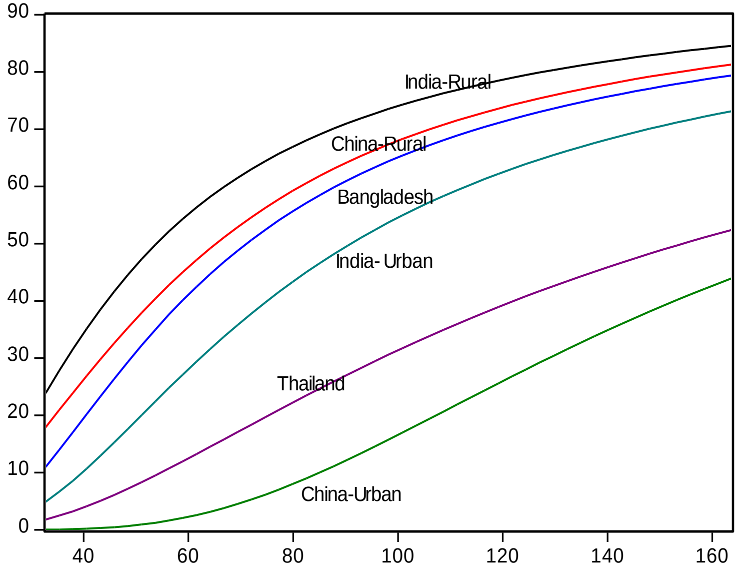
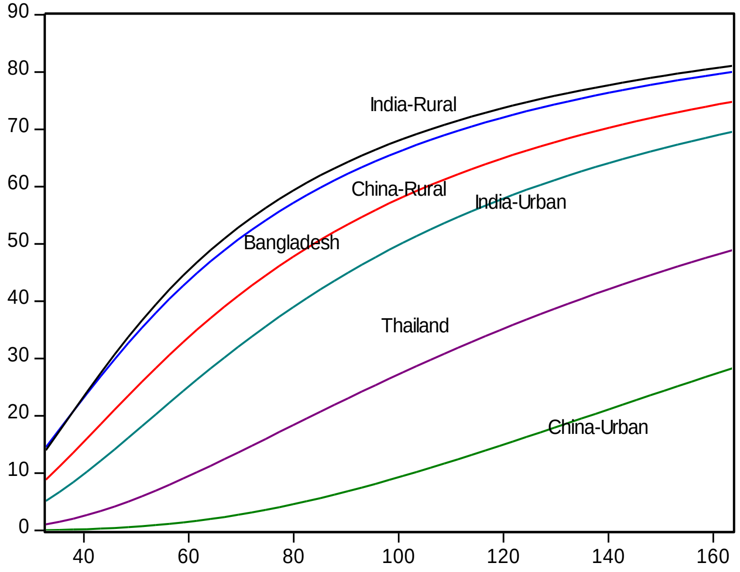
**Figure 6: Watt's Measure in Selected Countries/ areas, 1992 and 2000**

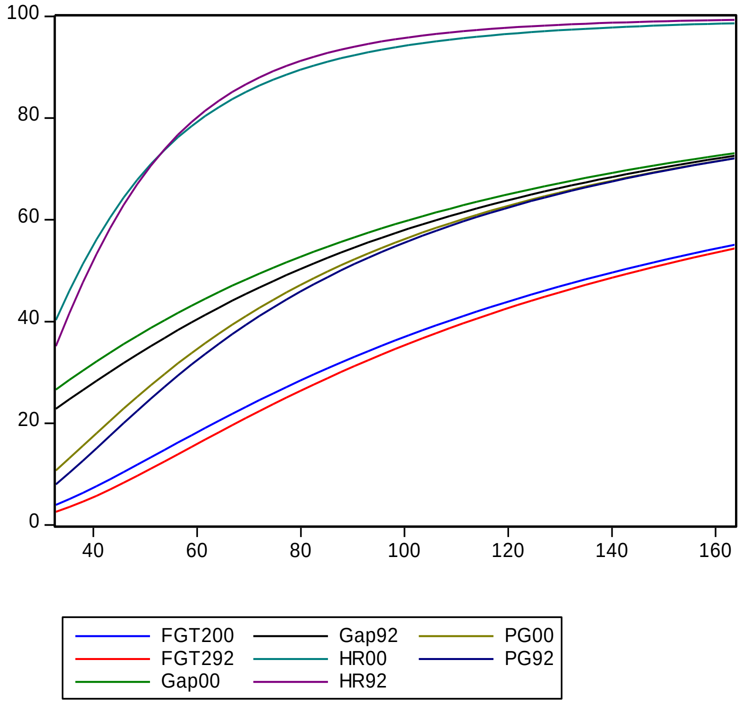



**1992**

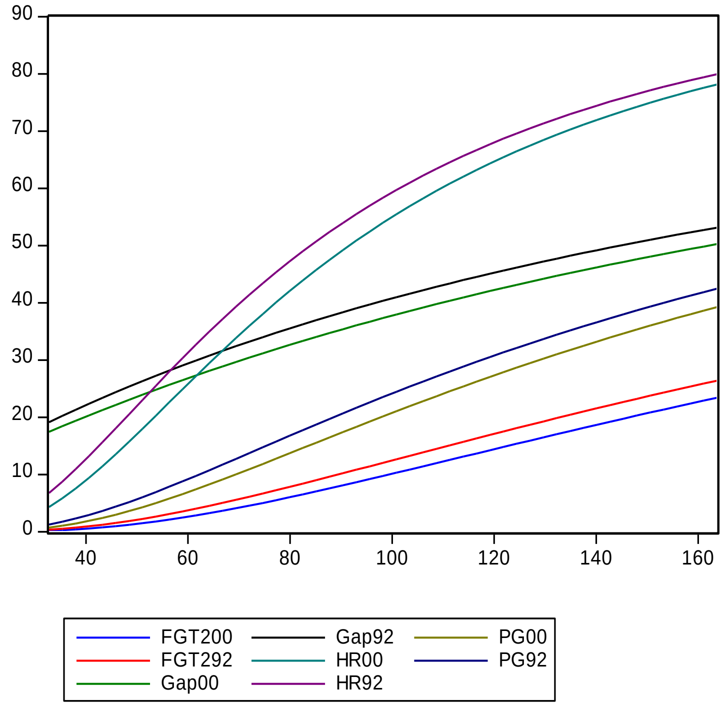



**2000**

**Figure 7: Sen's Measure in Selected Countries/ areas, 1992 and 2000**

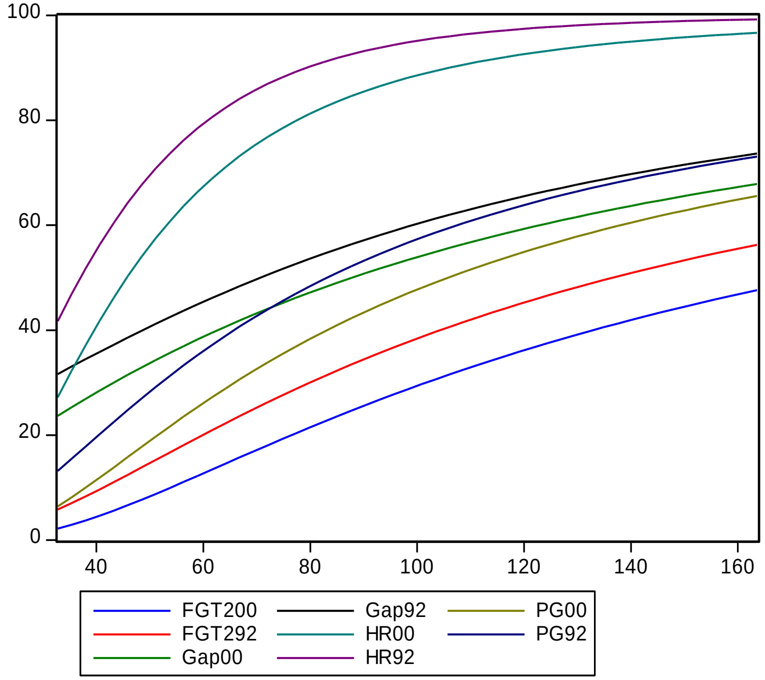



**Bangladesh**

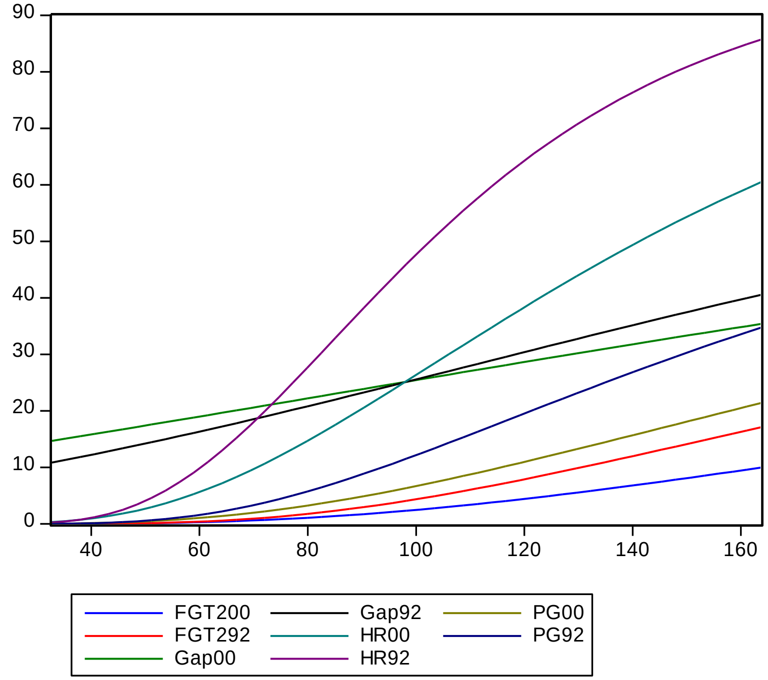



**Thailand**

**Figure 8: Selected Poverty Measures for Different Countries/areas in 1992 and 2000**

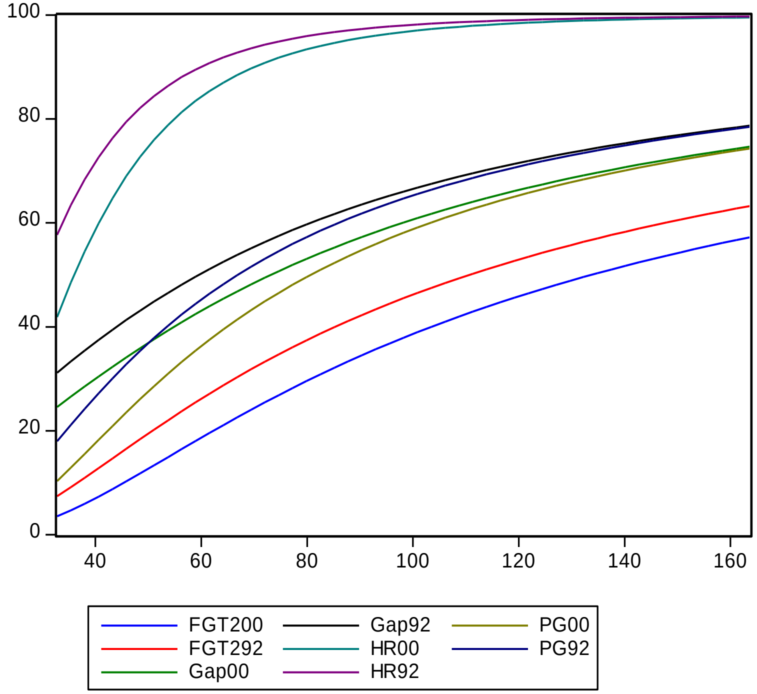



**China-Rural**

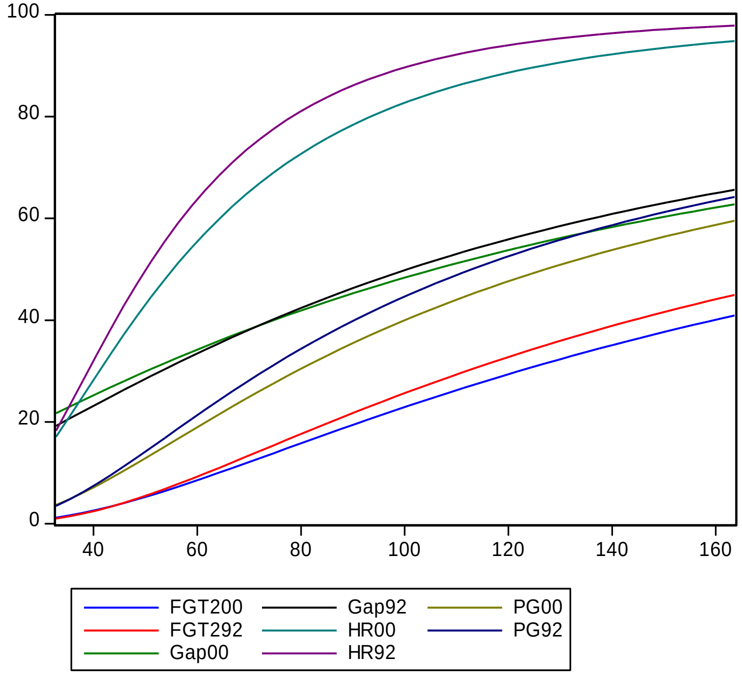



**China-Urban**

**Figure 8 cont.: Selected Poverty Measures for Different Countries/areas in 1992 and 2000**




**India-Rural**




**India-Urban**

**Figure 8 cont.: Selected Poverty Measures for Different Countries/areas in 1992 and 2000**