Measuring Multidimensional Poverty: the Generalized Counting Approach*

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Abstract. We examine the measurement of multidimensional poverty following the counting approach. In contrast to earlier contributions, dimensions of human well-being are not forced to be equally important but different weights can be assigned to different dimensions. We characterize a class of individual multidimensional poverty measures reflecting this feature. In addition, we axiomatize an aggregation procedure to obtain a class of multidimensional poverty measures for entire societies allowing for different degrees of inequality aversion in poverty. *Journal of Economic Literature* Classification No.: D63.

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1 Introduction

An important development in the study of inequality and poverty in the recent past is the shift of emphasis from a single dimension to a multidimensional framework. This is because the well-being of a population and hence its inequality and poverty are dependent on many dimensions of human life, such as housing, education, life expectancy, and income is just one such dimension. In such a structure poverty is defined as a situation that reflects failures in different dimensions of human well-being.

In this framework each person possesses a vector of several attributes that represent different dimensions of well-being. For measuring multidimensional poverty, it then becomes necessary to check whether a person has "minimally acceptable levels" (Sen, 1992, p.139) of these attributes. These minimally acceptable quantities of the attributes represent their threshold limits or cut-offs that are necessary for a subsistence standard of living. Therefore, a person is treated as deprived or poor in a dimension if its consumption level of the dimension falls below its cut-off. In this case we say that the individual is experiencing a functioning failure. Poverty at the individual level is an increasing function of these failures.

We are concerned with what Atkinson (2003) referred to as the "counting" approach to deprivation. A counting measure of individual poverty is simply the number dimensions in which a person is poor, that is the number of the individual functioning failures. But this measure is rather ad hoc. It also treats all the dimensions symmetrically in the sense that in the aggregation of individual's failures the same weight (one) is assigned to each dimension. Since some of the dimensions may be more important than others, a more appropriate counting measure can be obtained by assigning different weights to different dimensions and then summing up these weights. These weights may be assumed to reflect the importance a policy maker attaches to alternative dimensions in a poverty alleviation proposal.

Identification of the poor in a multivariate framework is still a debatable issue. One obvious way of regarding a person as poor is if it is deprived in all dimensions and this enables us to identify the number of poor as the total number of persons who are deprived in all dimensions. This is known as the intersection method of identification of the poor. But if a person is deprived in one dimension and non-deprived in another, then trading off the two dimensions may not be possible. Bad health status, say, cannot be compensated by housing. Clearly, such a person cannot be regarded as rich. In view of this, a person may be treated as poor if it is poor in at least one dimension. This is the union method of identifying the poor (see Tsui, 2002, and Bourguignon and Chakravarty, 2003). In between these two extremes there lies the intermediate identification method which regards a person as poor if it is deprived in at least m dimensions, where $1 \leq m \leq K$, with K being the number of dimensions (or weighted sum of dimensions) on which human well-being depends (see Mack and Lindsay, 1985, Gordon, Nandy, Pantazis, Pemberton and Townsend, 2003, and Alkire and Foster, 2007). Evidently, the intermediate method contains the union and the intersection methods as special cases for $m = 1$ and $m = K$. It should be clear that the calculation of the individual counting measure does not depend on any specific method of identification of the poor. More precisely, whatever the method of identification of the poor, the general counting measure can be calculated.

The first aim of this paper is to characterize the generalized individual counting measure of poverty. We proceed further by axiomatizing a class of aggregate poverty measures that permit us to compare different societies with respect to the poverty suffered by their members. We wish to take into account inequality in the distribution of individual poverty. The resulting distribution-sensitive measures are the extended symmetric means of order $r > 1$ applied to the individual multidimensional poverty values. The restriction on the possible values of the parameter r is a consequence of requiring strict inequality aversion with respect to individual poverty.

2 Individual Multidimensional Poverty Measures

Suppose there are $K \in \mathbb{N} \setminus \{1\}$ attributes that are relevant for the degree of well-being of an individual, such as health status, housing conditions, access to certain goods and services, employment status, ability to satisfy basic necessities. These characteristics are the same across societies and represented by binary variables: a value of one indicates that the individual is poor with respect to this attribute, a value of zero identifies a characteristic with respect to which the individual is not poor. Thus, an *individual characteristics vector* is an element of $\mathcal{P} = \{0, 1\}^K$ and an *individual multidimensional poverty measure for individual* i is a function $P_i \colon \mathcal{P} \to \mathbb{R}$. This paper is concerned with the aggregation of individual poverty over characteristics and the across-society aggregation of these individual measures into a social measure of multidimensional poverty. We begin with a discussion of individual multidimensional poverty.

Let $\mathbf{0}$ be the vector consisting of K zeroes and, for all $j \in \{1, \ldots, K\}$, let $\mathbf{1}^j$ be the

K-dimensional vector defined by

$$\mathbf{1}^j_k = \begin{cases} 1 & \text{if } k = j \\ 0 & \text{if } k \in \{1, \ldots, K\} \setminus \{j\}. \end{cases}$$

We require P_i to possess the following properties.

Zero normalization. For all $j \in \{1, \ldots, K\}$,

$$P_i(\mathbf{1}^j) > P_i(\mathbf{0}) = 0.$$

Additivity. For all $x, y \in \mathcal{P}$ such that $(x + y) \in \mathcal{P}$,

$$P_i(x + y) = P_i(x) + P_i(y).$$

The normalization assumption is very standard: when the individual is not poor in any attribute we require the value of the index to be zero. The additivity property we use is very straighforward as well. Many social index numbers have an additive structure.

Additivity entails a separability property: the contribution of any variable to the overall index value can be examined in isolation, without having to know the values of the other variables. Thus, additivity properties are often linked to independence conditions of various forms.

These two properties characterize the class of measures identified in the following theorem.

Theorem 1 *An individual multidimensional poverty measure P_i satisfies zero normalization and additivity if and only if there exists $\alpha \in \mathbb{R}^K_{++}$ such that, for all $x \in \mathcal{P}$,*

$$P_i(x) = \begin{cases} 0 & \textit{if } x = \mathbf{0} \\ \sum_{j \in \{1,\ldots,K\}: x_j = 1} \alpha_j & \textit{if } x \neq \mathbf{0}. \end{cases} \tag{1}$$

Proof. 'If.' Clearly, the measures defined in (**??**) satisfy the required axioms.

'Only if.' Suppose P_i satisfies zero normalization and additivity. That $P_i(\mathbf{0}) = 0$ follows immediately from the equality in normalization. Define, for all $j \in \{1, \ldots, K\}$, $\alpha_j = P_i(\mathbf{1}^j)$. By the inequality in the definition of zero normalization, it follows that $\alpha_j > 0$ for all $j \in \{1, \ldots, K\}$. Finally, consider the case in which $x \neq \mathbf{0}$. Writing x as

$$x = \sum_{\substack{j \in \{1,\ldots,K\}: \\ x_j = 1}} \mathbf{1}^j,$$

additivity requires

$$P_i(x) = \sum_{\substack{j \in \{1,\ldots,K\}: \\ x_j = 1}} P_i(\mathbf{1}^j) = \sum_{\substack{j \in \{1,\ldots,K\}: \\ x_j = 1}} \alpha_j$$

which completes the proof. ■

3 Aggregate Multidimensional Poverty Measures

Given the individual multidimensional poverty measures P_i for each individual in a society, we use an *aggregate multidimensional poverty index* to obtain an overall measure of poverty that allows us to compare multidimensional poverty across societies. In the comparison among societies we want to take into account inequality in the distribution of individual poverty. (For a discussion of distribution-sensitive multidimensional poverty indices in the case of continuous functionings see Tsui, 2002.) The more equally distributed the latter is, the lower aggregate poverty. For instance, consider two societies, A and B, where two attributes are equally relevant for the evaluation of individual well-being. Suppose that, while in society A only one individual is poor in both attributes, in society B there are two individuals poor in one attribute each. Is multidimensional poverty the same in A and B? This does not seem to be the case—poverty is more severe in society A than in B. We proceed by implicitly assuming that the individual aggregation across poverty dimensions is performed first and the second step consists of aggregating the resulting indicators across individuals in a society to arrive at an overall measure of multidimensional poverty. This choice is motivated primarily by our desire to keep the exposition simple. To describe the second part of the aggregation process, let $\mathcal{N} = \mathbb{N} \setminus \{1, 2\}$ and $\Omega = \cup_{n \in \mathcal{N}} \mathbb{R}^n_+$. Now consider a function $\mathbf{P}\colon \Omega \to \mathbb{R}_+$, to be interpreted as a measure that assigns an aggregate value of multidimensional poverty $\mathbf{P}(\mathbf{p})$ to each vector of individual poverty values $\mathbf{p} = (p_1, \ldots, p_n) \in \Omega$, where $n \in \mathcal{N}$ is the population size corresponding to $\mathbf{p}$. For all $n \in \mathcal{N}$, the restriction of $\mathbf{P}$ to $\mathbb{R}^n_+$ is denoted by $\mathbf{P}^n$.

The aggregate multidimensional poverty measures we propose are the *extended symmetric means of order* $r > 1$ of individual multidimensional poverty indices, that is, we employ the indices $\mathbf{P}_r$ defined by

$$\mathbf{P}_r(\mathbf{p}) = \left(\frac{1}{n} \sum_{i=1}^{n} p_i^r \right)^{1/r}$$

for all $n \in \mathcal{N}$ and for all $\mathbf{p} \in \mathbb{R}^n_+$. Note that we exclude all values of the parameter r that are less than or equal to one. This is the case because the corresponding means fail to be strictly S-convex. As r approaches one, the index approaches the arithmetic mean.

For $n \in \mathcal{N}$, let $\mathbf{1}_n$ denote the vector consisting of n ones. We employ the following axioms in our characterization of the extended symmetric means of order r.

Equality normalization. For all $n \in \mathcal{N}$ and for all $a \in \mathbb{R}_+$,

$$\mathbf{P}^n(a\mathbf{1}_n) = a.$$

Continuity. For all $n \in \mathcal{N}$, $\mathbf{P}^n$ is continuous.

Monotonicity. For all $n \in \mathcal{N}$, $\mathbf{P}^n$ is strictly increasing.

Strict S-convexity. For all $n \in \mathcal{N}$, for all $\mathbf{p} \in \mathbb{R}^n_+$ and for all bistochastic $n \times n$ matrices B,

(i) $\mathbf{P}^n(B\mathbf{p}) \leq \mathbf{P}^n(\mathbf{p})$;

(ii) if $B\mathbf{p}$ is not a permutation of $\mathbf{p}$, then $\mathbf{P}^n(B\mathbf{p}) < \mathbf{P}^n(\mathbf{p})$.

Linear homogeneity. For all $n \in \mathcal{N}$, for all $\mathbf{p} \in \mathbb{R}^n_+$ and for all $\lambda \in \mathbb{R}_{++}$,

$$\mathbf{P}^n(\lambda\mathbf{p}) = \lambda\mathbf{P}^n(\mathbf{p}).$$

Let, for any $n \in \mathcal{N}$, for any $\mathbf{p} \in \mathbb{R}^n_+$ and for any non-empty proper subset I^s of $\{1, \ldots, n\}$, $\mathbf{p}^s$ be the subvector of $\mathbf{p}$ corresponding to the elements of I^s and let $\mathbf{p}^c$ be the subvector of $\mathbf{p}$ indexed by the elements of the complement $I^c = \{1, \ldots, n\} \setminus I^s$ of I^s. A non-empty proper subset I^s of $\{1, \ldots, n\}$ is *strictly separable from its complement I^c in* $\mathbf{P}^n$ if and only if, for all $\mathbf{p}, \mathbf{q} \in \mathbb{R}^n_+$,

$$\mathbf{P}^n(\mathbf{p}^s, \mathbf{p}^c) \geq \mathbf{P}^n(\mathbf{q}^s, \mathbf{p}^c) \;\Leftrightarrow\; \mathbf{P}^n(\mathbf{p}^s, \mathbf{q}^c) \geq \mathbf{P}^n(\mathbf{q}^s, \mathbf{q}^c).$$

Complete strict separability. For all $n \in \mathcal{N}$, any non-empty proper subset of $\{1, \ldots, n\}$ is strictly separable from its complement in $\mathbf{P}^n$.

See Blackorby, Primont and Russell (1978) for a detailed discussion of complete strict separability and generalizations of this property.

Poverty Wicksell population principle. For all $n \in \mathcal{N}$ and for all $\mathbf{p} \in \mathbb{R}^n_+$,

$$\mathbf{P}^{n+1}\left(\mathbf{p}, \mathbf{P}^n(\mathbf{p})\right) = \mathbf{P}^n(\mathbf{p}).$$

See Blackorby and Donaldson (1984) for a discussion of this property and its link to general averaging principles.

We obtain

Theorem 2 *A function $\mathbf{P}\colon\Omega \to \mathbb{R}_+$ satisfies equality normalization, continuity, monotonicity, strict S-convexity, linear homogeneity, complete strict separability and the poverty Wicksell population principle if and only if there exists $r > 1$ such that $\mathbf{P} = \mathbf{P}_r$.*

Proof. The 'if' part of the theorem statement is straightforward to verify. To prove the 'only if' part, suppose $\mathbf{P}$ satisfies the required axioms.

Consider first the fixed-population-size case. It is well-known that, for any $n \in \mathcal{N}$, the class of symmetric means of order $r_n > 1$ is characterized by the fixed-population restrictions of the axioms equality normalization, continuity, monotonicity, strict S-convexity, linear homogeneity and complete strict separability; see, for instance, Hardy, Littlewood and Pólya (1934), Kolm (1976), among others. Note that the possible values of the parameter r_n are restricted due to our assumption of strict S-convexity. Furthermore, note that, without invoking additional properties, the parameter r_n can depend on the population size n and only vectors of dimension n can be compared according to $\mathbf{P}^n$. Thus, we have, for all $n \in \mathcal{N}$ and for all $\mathbf{p} \in \mathbb{R}^n_+$,

$$\mathbf{P}^n(\mathbf{p}) = \left(\frac{1}{n}\sum_{i=1}^{n} p_i^{r_n}\right)^{1/r_n}$$

where this function can be used to compare vectors of population size n.

We complete the proof of the theorem by using the poverty Wicksell population principle to establish that the r_n must be identical for all n and that the resulting function $\mathbf{P}$ can be employed in the comparison of any two vectors of different dimensions as well.

Let $n \in \mathcal{N}$ and define $r = r_{n+1}$. Thus,

$$\mathbf{P}^{n+1}(\mathbf{p}) = \left(\frac{1}{n+1}\sum_{i=1}^{n+1} p_i^{r}\right)^{1/r} \tag{2}$$

for all $\mathbf{p} \in \mathbb{R}^{n+1}_+$.

Now let $\mathbf{p} \in \mathbb{R}^n_+$. By the poverty Wicksell population principle and (**??**), we must have

$$\begin{aligned}\mathbf{P}^n(\mathbf{p}) &= \mathbf{P}^{n+1}\left(\mathbf{p}, \mathbf{P}^n(\mathbf{p})\right) \\ &= \left(\frac{1}{n+1}\left(\sum_{i=1}^{n} p_i^r + \left(\mathbf{P}^n(\mathbf{p})\right)^r\right)\right)^{1/r}\end{aligned}$$

and, solving for $\mathbf{P}^n(\mathbf{p})$, we obtain $\mathbf{P}^n(\mathbf{p}) = \mathbf{P}^{n+1}_r(\mathbf{p})$. Thus, the same parameter value r can be used for population size n and for population size $n+1$. Because this is true for all values of n, $\mathbf{P}$ is an extended symmetric mean of order $r > 1$, as was to be established. ■

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