Multidimensional poverty among children in Uruguay 2004-2006. Evidence from panel data

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Abstract

Multidimensional poverty measurement is an expanding field and a consensus on which are the best practices yielding to reasonable composite indicators has not yet emerged. In this paper, we try to address these issues by examining the usefulness of some of the more extended approaches to multidimensional poverty measurement. Specifically, the aim of this paper is to compare three existing methodologies: generalized Foster Greer and Thorbecke indexes (Chakravarty and Bourguignon, 2003), fuzzy sets (Lemmi, 2005; Chiappero-Martinetti, 2001) and stochastic dominance (Duclos and Sahn, 2006).
We assess these three approaches in terms of their advantages and disadvantages to build multidimensional poverty measures, concerning aggregation methods, flexibility of substitution among dimensions and weights.
After that, we present an empirical illustration of the convergence and divergence of poverty profiles using the three methodologies and of their time path considering the evolution of each of the dimensions included. This exercise is based on panel data information gathered for the years 2004 and 2006 from a representative survey of 1600 children attending public schools in Montevideo and the metropolitan area. The questionnaire of this survey was specially designed in order to include dimensions that reflect a wide set of capabilities. In each wave, field work included gathering information for each child at schools and then interviewing an adult in charge of the child.
The dimensions we chose aim at reflecting several aspects of capability deprivation that are commonly included in most lists of dimensions of well-being (Alkire, 2002). These are nutritional status of children, participation in social life, educational attainment of household adults and housing conditions. We also consider the monetary dimension and analyze its instrumental role in the achievement of well-being.
Our results show that the three methodologies present advantages and disadvantages in terms of their flexibility. The three of them reflect a reduction in poverty in the period 2004-2006 when income is included among the dimensions assessed. Once income is removed, stochastic dominance yields to different results than the other two methodologies, suggesting that poverty remained constant over the period.

Keywords: multidimensional poverty, Uruguay, children, FGT indexes, stochastic dominance

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I. Introduction

In 2002 Uruguay experienced one of the most severe economic crises since the beginning of the XX century. This economic breakdown was originated in the regional situation, mainly fostered by the Argentinean finacial crisis, and by internal macroeconomic problems. As a result, that year GDP fell 11%, unemployment grew from 10 to 18% and income poverty incidence was twice the pre-crisis level. By the end of 2003, the Uruguayan economy started to recover but most income based household welfare indicators remained almost unchanged until 2006. Only that year poverty incidence began to decrease significantly, but it still is considerably above the pre-crisis level.
One of the demographic groups most affected by the crisis were children, who also were the group that previously showed the highest income poverty rates (Table 1). This strong association between age and poverty is a result of various factors, among which the labor market performance, the social protection system and differences in fertility rates are the main explanatory factors.

Table 1. Poverty incidence by age group. 2001-2006

	Total	Less than 6 years old	6-12 years old	13-17 years old	18-64 years old	Elder than 64
2001	18,8	38,5	35,6	27,8	15,3	3,9
2002	24,3	47,7	42,3	35,7	21,0	5,6
2003	31,3	56,8	51,0	43,2	28,2	9,9
2004	31,9	56,5	54,0	44,8	28,4	10,7
2005	29,2	53,6	50,7	42,3	25,6	9,3
2006	24,1	42,5	39,9	35,8	20,0	6,5

Most poverty analyses in Uruguay have been done on the basis of income unidimensional measures. Multidimensional poverty measurement has been mainly focused on basic needs following the ECLAC tradition.[1] The only previous work on multidimensional poverty using composite indexes for Uruguay, carried out by Arim and Vigorito (2007), computed cross-sectional multidimensional FGT following Bourguignon and Chakravarty (2003), and was based on household survey data. The indexes turned out to be very sensitive to the time variation of their different components and their values were sometimes difficult to interpret. We consider that paper as a first approach to the issue of multidimensional poverty, and in this paper we aim at advancing on that line of reseach.
Following Sen, we understand poverty as capability deprivation. This means considering the well-being of a person in terms of the quality of the person's being. Living is then seen as a set of interrelated funcionings, consisting of beings and doings. A person's

[1] This approach has been subject to important criticisms due to the lack of a conceptual framework.

achievement can be seen as the vector of his or her functionings, and poverty is the failure to achieve certain minimal or basic capabilities (Sen, 1992). So SCA emphasises people's ability to enjoy various sets of alternative beings and doings, and this implies considering human freedom. In strict terms, poverty is capability deprivation, and not functioning deprivation. But it is not possible to capture this freedom component in a poverty measure, because it is not possible to extrapolate from an achieved functioning set to the capablity set associated with this achieved functionings. For this reason, most of empirical applications of SCA approach have argued that capability measurement is empirically very difficult or impossible to consider with existing data, and have been based on functionings. This research undertakes the same road, focusing on functionings or achievements.

The need to concentrate on functionings and not capabilities is not the only problem when trying to implement SCA. To make the approach operational, decisions about which dimensions to consider and how to measure and aggregate them in order to approximate the concept of functioning have to be made. In this paper we consider four dimensions that are mentioned in Sen´s writings and that can be traced in most of the lists proposed in the capabilities literature. These dimensions are health, participation in social life, housing conditions and adult education. We also consider the role of income, conceived as an instrument to achieve well-being in SCA approach. Our first objective is then to provide multidimensional measures of child poverty based on panel data for the years 2004 and 2006/2007 and to compare its evolution with that of income poverty data.

The choice of the multidimensional index or measure to be computed is also a relevant issue as there is an increasing supply of multidimensional poverty and well-being indicators, sometimes yielding to different results. As argued by Atkinson (2003), the definition of an aggregation procedure, the weight assigned to each component and the rates of substitution among dimensions are key issues in order to create multidimensional poverty measures. Hence, the second purpose of this paper is to assess the advantages and disadvantages of building multidimensional poverty measures by comparing three existing methodologies: generalized Foster Greer and Thorbecke indexes (Chakravarty and Bourguignon, 2003), fuzzy sets (Lemmi, 2005; Chiappero-Martinetti, 2001) and stochastic dominance (Duclos and Sahn, 2006). Each methodological approach is analyzed in terms of aggregation methods, flexibility of substitution among dimensions and weights.

Finally, we aim at having an empirical illustration of the convergence and divergence of poverty profiles using the three methodologies and of their time path compared to the evolution of each of the dimensions separately. We use panel data gathered for the years 2004 and 2006 from a representative survey of 1600 children attending public schools in Montevideo and the metropolitan area. This survey was specially designed in order to include dimensions that reflect a wide set of capabilities. In each wave, field work included gathering information for each child at schools and then interviewing an adult in charge of the child.

The paper is organized as follows. Section II contains information on the datasets we used, the variables we constructed and the different methodologies selected for computing multidimensional poverty indexes. Section III presents our main results and section IV gathers some final remarks.

II. Methodology

In the process of measuring multidimensional poverty, important decisions have to be taken about many aspects: the selections of the dimensions to be considered and the indicators that reflect these dimensions, the weighting scheme attached to the selected dimensions, the setting of a threshold to identify deprived population, and the way of aggregating the results in each dimension. In this section we present our decisions related to these aspects: first we discuss the main dimensions, variables and thresholds of our analysis (II.1), and then we discuss the methods selected to compute multidimensional indexes (II.2). These methods may imply different weighting schemes and aggregation criteria, aspects discussed in the following paragraphs.

II.1 Data and variables

The majority of poverty studies are based on available data coming from household surveys. Although these surveys are extremely useful to consider the more traditional dimensions usually included in poverty profiles, they may not be the best alternative when trying to operationalize Sen's capability approach .The analysis presented in this paper is based on a data set that was specially designed in order to include dimensions that reflect a wide set of capabilities. Our survey is based on a representative sample of children attending the first year of primary school in public institutions in Montevideo and the metropolitan area.[2] We gathered panel data information for the years 2004 and 2006. In each wave, field work included gathering information for each child at schools and then interviewing an adult in charge of the child. The sample consists of 1660 children in 2004, 85% of them were included in the second wave in 2006.[3]

The identification of the dimensions and variables to include in a multidimensional analysis of poverty is a crucial step. Although the discussion of these issues by Nussbaum, Alkire and others has been very extense, there is not a consensus on the list of capabilities that should be considered. This has been named as the problem of horizontal vagueness in multidimensional measurement of poverty, meaning vagueness about the dimensions of well being which are relevant in a poverty analysis (Qizilbash, 2003).At the empirical level, the problem can be expressed as a trade off between redundancies because of overlapping variables and the risk of not including relevant variables. Despite these problems, there is a "hard core" of dimensions that can be found in most SCA lists as well as in empirical applications of the approach. Considering this, we identified the relevant dimensions based on the existing literature, and chose to include the following: health, participation in social life, educational attainment of household adults, housing conditions and income. For each dimension, we specified a threshold level. This is another step in poverty measurement that implies arbitrariness, and has been named as vertical vagueness, implying vagueness about the "bottom line" in each dimension.

For the health dimension, we reflect nutritional status of children through the height for age index. Specifically, mild and moderate malnutrition are detected through the z-scores.

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[2] The sample is based on a Height Census that was undertaken in 2002 in all public schools in Uruguay.

[3] The first wave of this survey was financed by Comisión Sectorial de Invesigación Científica (Universidad de la República), UNDP and UNICEF. The second wave of the survey was financed by Fondo Clemente Estable (Ministerio de Educación y Cultura).

Following the National Center for Health Statistics (NCHS) criteria, we consider that a child suffers malnutrition (including moderate and mild malnutrition) when his z-score (for height for age) is higher than –1 standard deviations from the mean.

To consider participation in social life, we constructed an index based on the methodology of principal components, proposed by Filmer y Prittchet (2001). The index considers if adult members of the household take part in a wide range of community, political and social activities. Among other, participation at parental associations at school, trade unions, political parties, civil associations were considered.[4] A household is considered not deprived if at least one of its members participates in one or more activities.

Education is measured considering average years of schooling of the adults of the household, and the threshold is set at nine years of schooling.

The dimension of housing conditions is reflected by a crowding variable that considers that a household is deprived if the number of people sleeping in one room is higher than three.

Our dimensions try to reflect achievements, in order to be consistent with Sen's capability approach. Nevertheless, we decided to include income that reflects a mean and not an achievement, and so is not strictly comparable with the other dimensions. Our decision is based on the fact that income is a central variable, particularly in a developing country emerging from a deep economic crisis. In other words, income is a mean but it provides information about the evolution of household welfare in the short run. In this case, the threshold is given by the national poverty line (INE, 2002). Throughout the analysis we make sensitivity analysis including and excluding this variable. The following table describes the dimensions and variables, as well as their respective threshold.

Table 2. Dimensions, variables and thresholds for measuring poverty

Dimension	Variable	Threshold
Health	Height for age (z score)	Moderate and mild malnutrition: more than –1 zscores
Participation	Index of social participation	No member of the household participates in at least one activity
Education	Educational attainment of household adults	Nine years of schooling
Housing	Crowding	More than 3 persons sleeping in the same room
Income	Per capita household income	National poverty line

II.2 Methods

[4] The questionnaire can be downloaded from http://www.fhuce.edu.uy/academica/filosofia/filPractica/InvEJE/Textos/Cuest_prefadap.pdf

In order to analyze poverty within the SCA, the poverty measure must capture the multidimensional nature of deprivation. This can be done by aggregating various attributes into a single measure or index. The advantage of this kind of multidimensional approach is that it tends to uncover the subjective nature of poverty assessments, because it requires a choice process that is not clear in income based analysis.

The literature on the measurement of multidimensional poverty is relatively new, and two approaches can be identified. Some methodologies are based on cardinal comparison; this means a precise quantification of how much difference there is in the magnitude of poverty (poverty measurement). Others use an ordinal approach, and in this case we get rankings of distributions according to multiple poverty criteria (poverty orderings).

In both cases, shortfalls are considered in terms of thresholds levels of the different dimensions. These shortfalls are then aggregated under different rules in order to obtain a single indicator. So poverty is defined as a shortfall from a threshold on each dimension of an individual's well being.

In this paper we want to assess the advantages and disadvantages of building multidimensional poverty measures by comparing three existing methodologies: generalized Foster Greer and Thorbecke indexes (Chakravarty and Bourguignon, 2003), fuzzy sets (Lemmi, 2005; Chiappero-Martinetti, 2001) and stochastic dominance (Duclos and Sahn, 2006). The first two are based on cardinal comparisons, while the later provides poverty orderings. These different measurement proposals are put together and compared in order to sort out their properties, theoretical frameworks and limitations. In the following paragraphs, each methodological approach is briefly presented and analyzed in terms of aggregation methods, flexibility of substitution among dimensions and weights.

a) Generalized FGT

Bourguignon and Chakravarty (2003) propose a multidimensional poverty measure that considers a specific threshold or poverty line for each of the multiple dimensions of poverty. A person is poor if she falls beneath at least one of the poverty lines.

Following their proposal, we can consider a vector $X_i=(x_{i1},x_{i2},....,x_{im})$ that gives the quantity of attributes j (with $j=1, ...,m$) that a person i possess, and a vector $Z=(z_1,z_2,...,z_m)$ that reflects the thresholds or minimally acceptable levels for each of these attributes. A simple way of defining and measuring poverty is to account for the possibility of being poor in any poverty dimension, so person i is poor in terms of attribute j if $x_{ij} < z_j$..

In this context, a multidimensional poverty index can be defined as a non constant function $P(X; Z)$ that gives the extent of poverty associated to attributes X and thresholds Z. The P index satisfies a set of postulates, including strong focus, weak focus, symmetry, monotonicity, and continuity, principle of population, scale invariance and subgroup decomposability.

They also consider two transfer properties that deal with the redistributive criterion involving two attributes. To illustrate these postulates, they assume that there are two persons, i and t, and a two dimensional poverty space associated with attributes j and k. Person i has more of k but less of j. If the two persons interchange an amount of attribute j and after that person i, who had more of k, has now more of j too, there is an increase in the correlation of the attributes within the population. It is reasonable to expect that such a switch will not decrease poverty if the two attributes correspond to similar aspects of poverty. The poorer person cannot compensate the lower quantity of one attribute by a higher quantity of the other. On the same token, it is reasonable to expect that such a switch will not increase poverty if the two attributes correspond to different aspects of poverty. The non decreasing poverty under correlation increasing switch (NDCIS) postulate indicates that poverty can not decrease with such correlation increasing switches. The converse property is denoted NICIS, non increasing poverty under correlation increasing switches. The similarity or difference of attributes can be expressed in terms of substituibility or complementarities. If attributes are considered substitutes, then the marginal utility of one attribute decreases when the quantity of other increases. So a decrease in poverty due to the increase in one attribute is less when attributes are substitutes than when they are complements. The first postulate (NDCIS) holds for attributes that are substitutes, whereas the second one (NICIS) holds for attributes that are complements in the individual poverty function

The authors propose a full specification of a poverty multidimensional measure based on the FGT index and derived from a CES (constant elasticity substitution) function. For the two dimensions case, the $P(X;Z)$ index becomes:

$$P_{\alpha}^{\theta}(X;Z)=\frac{1}{n}\sum_{i=1}^{n}\left[a_1\left[\max\left(1-\frac{x_{i1}}{z_1};0\right)\right]^{\theta}+a_2\left[\max\left(1-\frac{x_{i2}}{z_2};0\right)\right]^{\theta}\right]^{\alpha/\theta}$$

Where a_1 and a_2 are positive weights attached to the attributes, $\alpha>0$ is a parameter that reflects “poverty aversion” and θ is the parameter of substitution between the shortfalls of the attributes. In this way, multidimensional poverty is defined as the average of aggregate shortfalls, raised to the power α, over the whole population, and reflects a generalization of the FGT index for the multidimensional case. When $\alpha=0$, the index becomes the multidimensional headcount. When $\alpha=1$, the index becomes a multidimensional poverty gap, obtained by some particular averaging of the poverty gaps of the included dimensions. As in the one dimensional case, higher values of α reflect more aversion towards extreme poverty.

As stated by the authors, this measure has the property that it satisfies NDCIS or NICIS depending on the relation between α and θ. If attributes are substitutes ($\alpha>\theta$), then a transfer that increases correlation of attributes among individuals does not decrease poverty, whereas if attributes are complements ($\alpha<=\theta$) such a transfer does not increase poverty. This implies that the drop in poverty due to a unit decrease in income is less important for people who have an educational level close to the education poverty threshold than for persons with very low education, if income and education are considered substitutes. On the contrary, the drop in poverty is larger for persons with higher education if these two attributes are supposed to be complements.

One of the limitations of the generalization of this index for more than two dimensions implies assuming the same elasticity of substitution between attributes.

b) Fuzzy sets

Both at the unidimensional or multidimensional level, poverty measurement face the problem of vertical vagueness that is the arbitrariness involved in the specification of the threshold level, for one or many dimensions. The set up of threshold levels and so the dichotomisation of the population in two excluding groups hides the fact that deprivation is a matter of the degree, not a clear condition free of ambiguity (Betti *et al*, 2005). Diverse authors have proposed to use the fuzzy set theory to capture simultaneously the vagueness of deprivation boundaries and the multidimensional character of poverty.

The fuzzy set theory replaces the traditional approach to the demarcation of poverty through of a binary function that assigns people to two non-superposed sets (poor and non-poor) by a generalized function, which varies between zero and one. This function is named as *membership function* and larger values indicate higher degrees of membership (Martinet, 2000). In more formal terms, we can denote as *X* the population, *A* the fuzzy set, and μ_A the membership function. Then, μ_A is defined as:

$$\mu_A : X \rightarrow [0,1]$$

If $\mu_A(x)=1$, the person *x* belongs completely to the set *A* whereas if $\mu_A(x)=0$ the person does not belong to *A* at all. In the intermediate cases, a person may belong partially to *A*.
Therefore, the application of this approach to analyse well-being requires specifying three aspects: 1) define indicators with an appropriate ordering of its values that reflect different degrees of well-being in each dimension, 2) identify the extreme conditions that allow considering that either the person belongs completely to the dimension- poverty set or she does not belong at all to that set, 3) specify the membership function.
Cheli and Lemmi (1995) propose a membership function directly derived from the distribution function, which presents the following form:

$$\mu(x^j) = \begin{cases} 0 \quad if \;\; j = 1 \\ \mu(x^{j-1}) + \dfrac{F(x_j) - F(x_{j-1})}{1 - F(x_1)} \end{cases}$$

Where *F(x)* is the sampling distribution function of the *x* and *j* indicates the rank of the observation in an increasing ordering of *x*. However, this membership function is a totally relative approach, since it assigns extreme values (zero and one) only to the lowest and highest positions in the rank. If the researcher thinks that there are both an upper threshold (z_u) over which a given functioning is fully achieved by a person and a lower threshold (z_l) below which she does not achieve an acceptable functioning at all, then the membership function can be reformulated in order to capture this situation: [5]

[5] This approach has been named Totally Fuzzy and Relative (TFR).

$$\mu(x^j)=\begin{cases}1 & if\ x_j < z_l \\ \mu(x^{j-1})+\dfrac{F(x_j)-F(x_{j-1})}{1-F(x_1)} & x_j \in (z_l, z_u) \\ 0 & if\ x_j > z_l\end{cases}$$

The last expression combines a relative fuzzy approach in the cases that belong to the interval (z_l,z_u) with absolute thresholds to identify the situations of extreme deprivation or complete fulfilment of basic functionings. In this paper, we use a TFP approach presented previously to compute the membership function for the social participation dimension. In the remaining dimensions, the mixed approach represented by h_α is used. The following table shows the upper and lower thresholds for each dimension.

Table 3. Upper and lower threshold by dimension		
Dimension	Lower threshold	Upper threshold
Housing	1 person per room	3 persons per room
Income	Extreme poverty line	1.2 poverty lines
Education	6 years of education (equivalent to complete primary school)	12 years of education (equivalent to complete high school)
Nutrition	-2 (more than to two standard deviations of the distribution of height for age z-score indicator)	+2 (more than two standard deviation of the distribution of height for age z-score indicator)
Participation	Totally relative approach	

The fuzzy theory allows overcoming the rigidity of the traditional one-dimensional poverty approach. However, in order to build multidimensional and fuzzy poverty indexes, it is necessary to define aggregation criteria by the membership functions of the different dimensions. So, a fuzzy multidimensional index (μ) can be expressed as:

$$\mu = h(\mu_1, \mu_2 ... \mu n)\ with\quad h:[0,1]^n \rightarrow [0,1]$$

Where $(\mu_1, \mu_2 ... \mu n)$ are the membership functions of the *n* dimensions. The aggregation operator *h* can reflect relations of either substitution or complementarities among the functionings. The polar cases are represented by the union and intersection operators.
Thus, the *fuzzy intersection* requires the simultaneous satisfaction of all the basic functionings. In this context, the reduction of poverty is only possible if advances in the less developed dimensions are achieved. Analogously, the *fuzzy union* implies that the well-being is determined for the dimension with a better performance. Thus, a person is not considered poor at least one of her membership functions equals zero.
There are different functions *h* that capture the union or intersection postulates. Chiappero Marinetti (2000) proposes to use strong and weak intersection and union operators, defined respectively as:

Table 4. Functions for fuzzy intersection or union	
Fuzzy intersection	Fuzzy union
Strong intersection: $h = \min[\mu 1, \mu 2, ..., \mu n]$	Strong union $h = \max[\mu 1, \mu 2, ..., \mu n]$
Weak intersection: $h = \mu 1 . \mu 2 .... \mu n$	Weak intersection $h = \max[\mu 1 + \mu 2 + .. + \mu n - \mu 1 . \mu 2 .. \mu n]$

In more general terms, the function *h* can be expressed as a generalised mean:

$$h_\alpha = \left[\left(w_1 v_1^\alpha + w_2 v_2^\alpha + ... + w_n v_n^\alpha\right)\right]^{\frac{1}{\alpha}}$$

Where the vector $(w_1 ..... w_2)$ specifies the relative importance assigned to each dimension. When α is equal to one the expression h_α is similar to the weak intersection operator, whereas if the parameter is zero h_α represents the weak union operator.

c) Multidimensional Stochastic Dominance

Duclos *et al* (2006) depart from the diagnosis that multidimensional measures show an important degree of dependence on aggregation rules. Hence, their purpose is to develop measures that are valid for a wide range of aggregation rules and that are independent from ethical considerations, extending previous work on univariate stochastic dominance (Davidson and Duclos, 2000). But this method can also be used in order to analyze whether multidimensional comparisons are robust to alternative ethical assumptions.

The authors cast some doubts on the validity of estimating multidimensional indexes, as long as these indexes tend to reduce poverty to one dimension again and are sensitive to aggregation rules, as discussed before. At the same time, they recall on the difficulties of estimating multidimensional poverty lines. Hence, they propose a method to undertake multidimensional poverty comparisons that they claim is more general, creating orderings that are robust despite the poverty measure and poverty line chosen. This method also allows for union and intersection definitions of multidimensional poverty, considering different poverty frontiers. The authors conclude that multidimensional rankings can yield to different orderings than univariate ones due to the interactions and correlations among dimensions.

In the bidimensional space, consider x_1 and x_2 to be two individual well-being indicators and λ a summary indicator of individual well-being such that:

$$\lambda(x_1, x_2) : R^2 \rightarrow R\left[\frac{\delta\lambda(x_1, x_2)}{\delta x_1} \geq 0, \frac{\delta\lambda(x_1, x_2)}{\delta x_2} \geq 0\right.$$

Assuming the existence of an unknown poverty frontier that separates the poor from the rich ($\lambda(x_1, x_2)$ =0), the set of the poor can be obtained as follows:

$$\Lambda(\lambda) = [(x_1, x_2) | (\lambda(x_1, x_2) \leq 0]$$

Then, a bidimensional additive poverty index can be defined as:

$$P(\lambda) = \iint_{\Lambda(\lambda)} \pi(x_1, x_2; \lambda) dF(x_1, x_2)$$

Being $\pi(x_1, x_2; \lambda)$ the weight that P(λ) attaches to someone under the poverty threshold and (z_{x1}, z_{x2}) the contribution to poverty of an individual with well-being indicators *x* and y which is positive if $\lambda(x_1, x_2)$ is lower than zero and zero otherwise. Thus, instead of creating multidimensional composite indexes, they study poverty rankings over classes of aggregation procedures defined in terms of the reaction of $\pi(x_1, x_2; \lambda)$ to changes in x_1, x_2 and over domains of poverty frontiers.
In regard to poverty comparisons, in the unidimensional framework the stochastic dominance curve for *x* can be defined as an FGT index:

$$P^{\alpha}(z) = \int_0^z (z - x)^{\alpha} dF(x)$$

for $\alpha \geq 0$.
In the bidimensional case, a stochastic dominance surface can be defined as:

$$P^{\alpha_{x1}\alpha_{x2}}(z_{x1}, z_{x2}) = \int_0^{z_{x1}} \int_0^{z_{x2}} (z_{x1} - x_1)^{\alpha_{x1}} (z_{x2} - x_2)^{\alpha_{x2}} dF(x_1, x_2)$$

For inequality aversion parameters $\alpha_{x1} \geq 0; \alpha_{x2} \geq 0$. This expression can be generalized for *n* dimensions. The dominance surface is generated by moving the respective poverty lines over a chosen domain. This expression can be understood as a generalized FGT where the gap in one dimension is weighted by the gap in the other, but it holds for union and intersection definitions of poverty. Being the integrand multiplicative, the dominance surface is influenced by the covariance between the dimensions. The height of the dominance surface is the product of two unidimensional curves plus the covariance in the poverty gaps in the two dimensions.

As in the unidimensional case, poverty comparisons use orders of dominance s_{x1} and s_{x2}, corresponding to αx_1+1 and αx_1+1, respectively. The properties derived for poverty comparisons to hold, rule out the headcount ratio. At the same time, they require substitutability among dimensions (π^{x1x2}>0,): "The more some has of x, the less is overall poverty deemed to be reduced if his value of y is increased" (pp.8).
Consider two moments or regions, *A* and *B*. The difference between the two can be written as $\Delta F = F_A - F_B$. The poverty dominance theorem or order one derived by Duclos *et al* (2006) requires that the bi-dimensional dominance surface be higher for *A* than for *B* for all intersection poverty frontiers which lie in $\Lambda(\lambda^*)$, regardless the poverty lines. The theorem requires checking dominance in the intersection set, although it holds also for union and intermediate options. Tests can be done on higher dominance orders

in all or some dimensions by imposing more restrictive assumptions that rule out more poverty measures. These assumptions are analogous to the requirements of unidimensional dominance tests.

d) Summary: comparison among the methodologies used in this study

Dimension	Generalized FGT	Stochastic dominance	Fuzzy sets
Theoretical referent	Vaguely SCA	Explicitly tries to avoid an ethical referent	SCA
Type of variables admitted	continuous	Continuous and discrete	Continuous and discrete
Relation among variables	Substitution or complementation	Substitution	Substitution or complementation
Substitution relation	Constant (CES)		
Weights	Defined by the researcher		
Type of poverty ranking	Cardinal	Ordinal	Cardinal

III. Main results

As discussed before, we take as indicators of capability deprivation child nutritional status, failure to participate in social life, failure to reach nine years of schooling, and household crowding. We also considered income in order to assess the extent of its instrumental role and its potential overlapping with achievements. Our data set covers a panel of children attending public schools and poverty is measured at the child level.

Poverty incidence decreased between 2004 and 2006 in all dimensions considered (table 4). The major decrease takes place in the social participation dimension. This is probably related to the increase in union density during 2005, as the mechanism of centralised wage bargaining was established again after fifteen years. Income poverty shows a slight decrease in our sample, although it decreased significantly (almost 25%) for the whole population (see table 1).[6]

Table 4. Poverty incidence in each dimension. 2004 and 2006 (%).

	2004	2006	Perceptual change
Health	18,1	16,8	-7,1%
Participation in social life	66,0	59,3	-10,2%
Education	65,9	63,7	-3,4%
Housing	31,2	30,5	-2,3%
Income	75,2	72,2	-3,9%

The correlations between deprivations in the different dimensions are low. Both in 2004 and 2006 the highest correlation between different dimensions corresponds to income and education. In all cases, deprivation on a certain dimension in 2006 is highly correlated with previous deprivation in that dimension, showing persistence of poverty. This is especially important in the health dimension. The low correlation between the levels of poverty in the different dimensions justifies the multidimensional approach.

[6] Nevertheless, it should be noticed that poverty incidence in our sample coincides with that of the households surveys if we consider the same population, that is children attending first year of primary in public schools.

Table 5. Correlations between dimensions. 2004 and 2006.										
	2004					2006				
	Housing	Malnutrition	Education	Income	Participation in social life	Housing	Malnutrition	Education	Income	Participation in social life
2004										
Housing	1									
Malnutrition	0,088	1								
Education	0,209	0,135	1							
Income	0,251	0,106	0,355	1						
Participation in social life	0,056	0,023	0,072	0,115	1					
2006										
Housing	0,423	0,119	0,245	0,258	0,073	1				
Malnutrition	0,077	0,798	0,122	0,110	0,033	0,102	1			
Education	0,244	0,127	0,598	0,415	0,085	0,283	0,107	1		
Income	0,248	0,132	0,383	0,546	0,109	0,257	0,100	0,461	1	
Participation in social life	0,088	0,046	0,208	0,100	0,090	0,100	0,038	0,158	0,160	1

We now turn to multidimensional poverty measures with the $P_{\alpha}^{\theta}(X;Z)$ proposed by Bourguignon and Chakravarty (2003). We considered values of θ=1 and θ=2, implying perfect substitutability in the first case. With respect to the weighting scheme, we considered two alternatives usually adopted in the literature: the same weight for each dimension, and the inverse of the poverty incidence of the corresponding dimension. In the first option, we are avoiding the need of assigning different relevance to each dimension. The last option implies giving a higher weight to dimensions in which fewer people are poor, in order to be able to bring out the sub-group of poorest people. In our case, this means that the dimension of health status has a higher weight, whereas income (in the case when it is included), has a lower weight.[7] In order to isolate the effect of income, that has an instrumental role and reflects a mean and not an achievement, we measured multidimensional poverty including and excluding income poverty.

When income is not considered, the headcount index shows a decrease between 2004 and 2006 (table 6). In this multidimensional case, the headcount index reflects individuals who are poor in any of the dimensions. So our results indicate that a very high percentage of children in our sample are deprived in at least one of the dimensions considered. The magnitude of deprivation is higher when the substitutability between attributes falls (higher θ). This responds to the fact that low substitutability between attributes gives more weight for each observation to attributes with the largest shortfalls. Nevertheless, changes in deprivation are similar under different assumptions of substitutability.

If we compare the two alternative weighting schemes, results based on the inverse of poverty incidence in each dimension show lower levels of deprivation. This reflects the fact that this weighting scheme gives more importance to dimensions that show less level of deprivation (in our case, health dimension). The weighting scheme has a considerable impact on the magnitude of deprivation, although the evolution is similar under both weighting schemes. Results for of α=1 and α=2 show a higher decrease of poverty indexes.

Table 6. Generalized FGT.Income dimension is excluded.

	α=0		α=1		α=2	
year	θ=1	θ=2	θ=1	θ=2	θ=1	θ=2
(aggregation weight =1/5 for each dimension)						
2004	0,906	0,906	0,262	0,411	0,092	0,213
2006	0,841	0,842	0,232	0,366	0,082	0,187
change	*-7,1%*	*-7,1%*	*-11,4%*	*-11,1%*	*-11,6%*	*-12,0%*
(aggregation weight=inverse of dimension poverty incidence)						
2004	0,905	0,906	0,165	0,313	0,039	0,123
2006	0,841	0,842	0,145	0,278	0,034	0,107
change	*-7,1%*	*-7,1%*	*-11,8%*	*-11,3%*	*-13,1%*	*-12,6%*

When income is included, results show a similar pattern. As expected, the indexes are higher than in the previous chart, because we are adding one dimension that shows a high level of deprivation.

Table 7. Generalized FGT.Income dimension is included.

[7] Table A.1 in the statistical appendix presents the weights assigned to each dimension in each case.

	α=0		α=1		α=2	
year	θ=1	θ=2	θ=1	θ=2	θ=1	θ=2
(aggregation weight =1/6 for each dimension)						
2004	0,940	0,940	0,299	0,447	0,115	0,234
2006	0,891	0,892	0,260	0,390	0,096	0,197
change	*-5,2%*	*-5,2%*	*-13,0%*	*-12,6%*	*-16,3%*	*-16,0%*
(aggregation weight=inverse of dimension poverty incidence)						
2004	0,940	0,940	0,192	0,347	0,051	0,143
2006	0,891	0,892	0,168	0,304	0,043	0,120
variation	*-5,2%*	*-5,2%*	*-12,7%*	*-12,4%*	*-15,9%*	*-15,6%*

Our second step in the empirical analysis was the application of the fuzzy set method. We included the six dimensions in the well-being assessment. Two results must be emphasized: there are relatively high degrees of achievement in income and housing dimensions, whereas performances in health, education and specially participation in social life are relatively low. The average values of individual membership functions fall moderately between both waves. The exception is participation in social life, whose indicator decreases more significantly (table 6). These results are consistent with the ones obtained from the traditional one-dimensional measurement (see table 4).

Table 8 : Membership degrees by dimension. Basic statistics

	2004			2006			Change	
	Mean (1)	Stand. Dev	Median (2)	Mean (3)	Stand. Dev	Median (4)	(3)/(1)	(4)/(2)
Housing	0,606	0,307	0,578	0,593	0,309	0,594	-2,1%	2,7%
Health	0,464	0,309	0,461	0,438	0,323	0,427	-5,6%	-7,4%
Income	0,604	0,374	0,634	0,597	0,343	0,616	-1,2%	-2,7%
Education	0,534	0,318	0,569	0,515	0,324	0,476	-3,5%	-16,3%
Participation in social life	0,179	0,297	0,000	0,243	0,327	0,000	35,8%	-.-

In order to analyze the association degree among one-dimensional fuzzy indicators we compute the correlation matrix (table 9). The absolute values of the coefficients are low. In particular, they are lower than ones obtained from traditional measures (see table 5). Therefore, when poverty is treated like a matter of degree rather than an attribute, the pertinence of a multidimensional approach for well-being assessment emerges still more clearly. The selected functionings seem to capture complementary dimensions of human well-being (Chiapparo Martinetti, 2000)

Table 9: Pearson correlation coefficients among membership individual function

year/dimension	2004					2006				
	Housing	Health	Education	Income	Participation in social life	Housing	Health	Education	Income	Participation in social life
2004										
Housing	1									
Health	0,061	1								
Education	-0,039	-0,023	1							
Income	-0,029	-0,155	0,208	1						
Participation in social life	-0,005	0,006	-0,127	-0,140	1					
2006										
Housing	0,189	0,059	0,008	0,008	-0,032	1				
Health	0,059	0,720	0,015	-0,140	-0,021	0,052	1			
Education	0,042	-0,108	0,442	0,143	-0,102	0,001	-0,055	1		
Income	-0,029	-0,109	0,151	0,552	-0,086	0,063	-0,106	0,188	1	
Participation in social life	-0,023	0,073	-0,150	-0,174	0,190	-0,029	0,066	-0,129	-0,157	1

We computed the average membership degrees to the composite fuzzy set. These indicators are overall poverty indexes obtained by interaction, union and average unweighted operators. It is important to recall that the union and intersection operators are associated to the presence of extreme complementation and substitution relations among dimensions, whereas the un-weighed average postulates a condition of symmetry among functionings.

This result also shows a decreasing pattern of deprivation, except when income is excluded and the strong union operator is used. Therefore, the pattern of poverty evolution is similar to that obtained with generalized FGT.

Table 10. An overall well-being assessment. Average membership degrees by union, intersection and average un-weighed operators

	2004	2005	Change
Income included			
strong union operator	0.919	0.918	-0.1%
strong intersection operator	0.155	0.140	-9.5%
Un-weighted average operators			
alfa=0 (geometric mean)	0.649	0.635	-2.2%
alfa=1 (arithmetic mean)	0.003	0.003	-1.6%
alfa=-1 (harmonic mean)	81.933	77.471	-5.4%
Income excluded			
strong union operator	0.880	0.888	0.9%
strong intersection operator	0.178	0.156	-12.1%
Un-weighted average operators			
alfa=0 (geometric mean)	0.729	0.688	-5.6%
alfa=1 (arithmetic mean)	0.002	0.002	-0.8%
alfa=-1 (harmonic mean)	94.103	90.076	-4.3%

Finally, we tested stochastic dominance. We defined 20 groups for each variable. This yielded a 20*20*20*20*20*20 matrix in which to test dominance for each dominance order and group of variables. [8] The tests were performed for the whole group of variables and removing income. At the same time we tested dominance for orders 2 and 3. In each case we used the same parameter α for all variables.

When income is included, 2004 dominates 2006 for all the groups considered suggesting a poverty reduction. This is consistent with the results obtained using generalized FGT and fuzzy sets. This dominance ordering holds for both orders 2 and 3.

Considering that this result could be led by income, we run a second round of tests removing income from the dimensions considered. In this case, we did not reject the null hypothesis, and hence, there is no dominance of one year over the other. This result can be interpreted considering SCA and the different time horizons of the variables used in the analysis.

It can be argued that being the rates of conversion of income into achievements parametrically different between households and individuals, increased household income

[8] Table A.2 presents 800 points but the whole results are available on request to the authors.

may not be translated into achievements into the other dimensions. This could reflect a problem of timing as long as income varies in shorter spans of time than the remaining variables. In order to observe achievements in the remaining variables, we need a longer period of sustained income growth. But if income is considered a mean and not an outcome, there is no theoretical reason to include it in a multidimensional index that tries to capture long run welfare. Recall that in this second result of no dominance when income is removed, our results are sensitive to the methodology we used.

IV. Final remarks

The translation of the richness of Sen's ideas into the empirical measurement of multidimensional poverty faces a lot of problems. The literature that addresses these problems proposes different methodologies that try to preserve the informative and conceptual contents of the approach. In this paper, we considered three of them: generalized FGTs, fuzzy sets and stochastic dominance.

The evolution of multidimensional poverty under the three approaches yields similar results when income is included among the dimensions. If income is not included, there is no dominance of one year over the other, so this methodology leads to somehow different results than the other two. This reflects the fact that dominance analysis is much more restrictive in order to allow for significant changes in poverty measures. Nevertheless, more sensitivity analysis about the robustness of the results of the evolution of multidimensional poverty under generalized FGT and fuzzy sets is needed.

References

Alkire, Sabina (2002). Dimensions of Human Development. *World Development*, 30: 191-205.

Arim R. and Vigorito A. (2007) *Un análisis multidimensional de la pobreza en Uruguay. 1991-2005*. Documento de Trabajo 10/06, Instituto de Economía, UDELAR.

Atkinson, A. B. (2003) "Multidimensional Deprivation: Contrasting Social Welfare and Counting Approaches", *Journal of Economic Inequality,* (1): 51.

Betti G., Cheli B., Lemmi A. y Verma V (2005). *The fuzzy approach to multidimensional poverty: the case of Italy in the 90´s.* Paper presented to the conference The Multiple Dimensions of Poverty. International Poverty Centre. Brazilia.

Bourguignon and Chakravarty (2003) "The measurement of multidimensional poverty". *Journal of Economic Inequality*. Nº1: 25-49.

Cheli B. and Lemmi A. (1995) "A 'totally' fuzzy and relative approach to the measurement of Poverty". *Economic Notes* 24(1): 115-134.

Chiappero Martinetti E. (2000), A multi-dimensional assessment of well-being based on Sen's functioning theory. *Rivista Internationale di Scienzie Sociali* CVIII: 207-231.

Duclos J.I., Sahn D. and Younger S. (2006). "Robust Multidimensional Poverty Comparisons" *Economic Journal*.

INE (2002) Evolución de la pobreza en el Uruguay por el método del ingreso. 1986-2001. http://www.ine.gub.uy/biblioteca/pobreza

Lemmi A. (2005). "The fuzzy approach to multidimensional poverty. The case of Italy in the 90s." Paper presented at the conference *The Many Dimensions of Poverty*. Centre for Poverty Analysis. Brazilia.

Qizilbash M. (2003). "Vague language and precise measurement: the case of poverty". *Journal of Economic Methodology,* 10(1) 41-58.

Sen A. 1992. *Inequality re-examined*, Cambridge University Press. Cambridge.

Appendix

Table A.1. Weights for generalized FGT.						
	Including income			Excluding income		
	Equal weights	2004	2006	Equal weights	2004	2006
Health	0,2	0,422	0,429	0,25	0,470	0,476
Participation in social life	0,2	0,116	0,122	0,25	0,129	0,135
Education	0,2	0,116	0,113	0,25	0,129	0,126
Housing	0,2	0,245	0,237	0,25	0,273	0,263
Income	0,2	0,102	0,100			
Total	1	1	1	1	1	1

Table A.2 Stochastic Dominance Tests

Stochastic dominance results. Comparison 2004-2006 15625

		Sign of the variation													Value of T statistic										
									income excluded													income excluded			
		income							education					income							education				
	zhe	1	2	3	4	5		zhe	1	2	3	4	5	1	2	3	4	5		zhe	1,00	2,00	3,00	4,00	5,00
part=1	1	-	-	-	-	-	part=1	1	=	=	=	=	=	-14,08	-4,148	-11,89	-6,84	-12,74	part=1	1	1,76	0,59	2,97	0,76	1,42
educ=1	2	-	-	-	-	-	crowding=1	2	=	=	=	=	=	-18,24	-34,39	-8,659	-10,1	-12,19	crowding=	2	2,28	4,91	2,16	1,12	1,35
crowding=1	3	-	-	-	-	-	zwh=1	3	=	=	=	=	=	-11,86	-29,05	-1,045	-10,17	-11,53	zwh=1	3	1,48	4,15	0,26	1,13	1,28
zwh=1	4	-	-	-	-	-		4	=	=	=	=	=	-12,59	-12,95	-10,11	-13,29	-22,29		4	1,57	1,85	2,53	1,48	2,48
	5	-	-	-	-	-		5	=	=	=	=	=	-14,03	-11,92	-9,506	-13,7	-30,09		5	1,75	1,70	2,38	1,52	3,34
part=1	1	-	-	-	-	-	part=1	1	=	=	=	=	=	-12,45	-11,26	-4,872	-16,54	-10,19	part=1	1	1,56	1,61	1,22	1,84	1,13
educ=2	2	-	-	-	-	-	crowding=1	2	=	=	=	=	=	-7,948	-6,221	-7,555	-14,11	-8,453	crowding=	2	0,99	0,89	1,89	1,57	0,94
crowding=1	3	-	-	-	-	-	zwh=1	3	=	=	=	=	=	-7,499	-4,791	-0,664	-20,21	-21,28	zwh=1	3	0,94	0,68	0,17	2,25	2,36
zwh=1	4	-	-	-	-	-		4	=	=	=	=	=	-7,098	-2,79	-0,769	-22,26	-4,607		4	0,89	0,40	0,19	2,47	0,51
	5	-	-	-	-	-		5	=	=	=	=	=	-7,284	-7,478	-4,738	-7,197	-4,873		5	0,91	1,07	1,18	0,80	0,54
part=1	1	-	-	-	-	-	part=1	1	=	=	=	=	=	-4,833	-7,985	-0,521	-1,481	0	part=1	1	0,60	1,14	0,13	0,16	0,00
educ=3	2	-	-	-	-	-	crowding=1	2	=	=	=	=	=	-7,229	-9,276	-1,043	-1,384	-0,384	crowding=	2	0,90	1,33	0,26	0,15	0,04
crowding=1	3	-	-	-	-	-	zwh=1	3	=	=	=	=	=	-9,351	-6,508	-5,152	-20,04	-13,82	zwh=1	3	1,17	0,93	1,29	2,23	1,54
zwh=1	4	-	-	-	-	-		4	=	=	=	=	=	-6,241	-9,254	-3,642	-9,615	-10,66		4	0,78	1,32	0,91	1,07	1,18
	5	-	-	-	-	-		5	=	=	=	=	=	-6,397	-3,79	-5,637	-2,626	-3,345		5	0,80	0,54	1,41	0,29	0,37
part=1	1	-	-	-	-	-	part=1	1	=	=	=	=	=	-8,38	-5,598	-2,631	-10,63	-8,206	part=1	1	1,05	0,80	0,66	1,18	0,91
educ=1	2	-	-	-	-	-	crowding=1	2	=	=	=	=	=	-4,553	-4,422	-0,505	-8,139	-9,366	crowding=	2	0,57	0,63	0,13	0,90	1,04
crowding=2	3	-	-	-	-	-	zwh=1	3	=	=	=	=	=	-3,553	-6,402	-0,726	-0,422	0	zwh=1	3	0,44	0,91	0,18	0,05	0,00
zwh=1	4	-	-	-	-	-		4	=	=	=	=	=	-2,49	-5,958	-0,27	-0,755	-1,332		4	0,31	0,85	0,07	0,08	0,15
	5	-	-	-	-	-		5	=	=	=	=	=	-12,45	-11,26	-4,872	-16,54	-10,19		5	1,56	1,61	1,22	1,84	1,13
part=1	1	-	-	-	-	-	part=1	1	=	=	=	=	=	-13,33	-1,791	-2,131	-1,392	0	part=1	1	1,67	0,26	0,53	0,15	0,00
educ=1	2	-	-	-	-	-	crowding=1	2	=	=	=	=	=	-7,484	-4,023	-3,552	0	-1,139	crowding=	2	0,94	0,57	0,89	0,00	0,13
crowding=1	3	-	-	-	-	-	zwh=1	3	=	=	=	=	=	-7,499	-4,791	-0,664	0	0	zwh=1	3	0,94	0,68	0,17	0,00	0,00
zwh=1	4	-	-	-	-	-		4	=	=	=	=	=	-7,098	-2,79	-0,769	0	0		4	0,89	0,40	0,19	0,00	0,00
	5	-	-	-	-	-		5	=	=	=	=	=	-6,79	-7,861	-2,181	-4,76	-0,2		5	0,85	1,12	0,55	0,53	0,02
part=1	1	-	-	-	-	-	part=1	1	=	=	=	=	=	-7,841	-9,598	-6,239	-9,732	-0,334	part=1	1	0,98	1,37	1,56	1,08	0,04
educ=1	2	-	-	-	-	-	crowding=1	2	=	=	=	=	=	-7,438	-4,229	-4,563	-1,171	-1,481	crowding=	2	0,93	0,60	1,14	0,13	0,16
crowding=1	3	-	-	-	-	-	zwh=1	3	=	=	=	=	=	-10,58	-6,325	-5,3	-2,346	-1,384	zwh=1	3	1,32	0,90	1,33	0,26	0,15
zwh=1	4	-	-	-	-	-		4	=	=	=	=	=	-3,514	-4,09	-1,053	-2,185	0		4	0,44	0,58	0,26	0,24	0,00
	5	-	-	-	-	-		5	=	=	=	=	=	-4,332	-9,864	-1,167	-3,345	-0,278		5	0,54	1,41	0,29	0,37	0,03
part=1	1	-	-	-	-	-		1	=	=	=	=	=	-3,708	-3,321	-6,493	-4,069	-8,267		1	0,46	0,47	1,62	0,45	0,92
educ=1	2	-	-	-	-	-		2	=	=	=	=	=	-0,045	-1,457	-3,564	-4,531	-1,04		2	0,01	0,21	0,89	0,50	0,12
crowding=1	3	-	-	-	-	-		3	=	=	=	=	=	-7,463	-5,174	-3,862	-22,22	-6,463		3	0,93	0,74	0,97	2,47	0,72
zwh=1	4	-	-	-	-	-		4	=	=	=	=	=	-3,309	-5,407	-0,66	-19,77	-13,38		4	0,41	0,77	0,17	2,20	1,49
	5	-	-	-	-	-		5	=	=	=	=	=	-3,795	-11,36	-1,808	-8,267	-16,54		5	0,47	1,62	0,45	0,92	1,84
part=1	1	-	-	-	-	-	part=1	1	=	=	=	=	=	-3,775	-10,08	-7,301	-0,304	-2,112	part=1	1	0,47	1,44	1,83	0,03	0,23
educ=1	2	-	-	-	-	-	crowding=1	2	=	=	=	=	=	-0,146	-5,951	-3,497	-20,43	-0,239	crowding=	2	0,02	0,85	0,87	2,27	0,03
crowding=1	3	-	-	-	-	-	zwh=1	3	=	=	=	=	=	-3,368	-7,875	-2,832	-16,77	-14,77	zwh=1	3	0,42	1,12	0,71	1,86	1,64
zwh=1	4	-	-	-	-	-		4	=	=	=	=	=	-0,75	-3,849	-2,66	-1,509	-19,31		4	0,09	0,55	0,66	0,17	2,15
	5	-	-	-	-	-		5	=	=	=	=	=	-0,146	-5,951	-3,497	-20,43	-0,239		5	0,02	0,85	0,87	2,27	0,03
part=1	1	-	-	-	-	-	part=1	1	=	=	=	=	=	-3,368	-7,875	-2,832	-16,77	-14,77	part=1	1	0,42	1,12	0,71	1,86	1,64
educ=1	2	-	-	-	-	-	crowding=1	2	=	=	=	=	=	-0,75	-3,849	-2,66	-1,509	-19,31	crowding=	2	0,09	0,55	0,66	0,17	2,15
crowding=1	3	-	-	-	-	-	zwh=1	3	=	=	=	=	=	-3,795	-11,36	-1,808	-8,267	-16,54	zwh=1	3	0,47	1,62	0,45	0,92	1,84
zwh=1	4	-	-	-	-	-		4	=	=	=	=	=	-1,665	-6,238	-2,014	-1,04	-12,46		4	0,21	0,89	0,50	0,12	1,38
	5	-	-	-	-	-		5	=	=	=	=	=	-3,111	-1,402	-3,732	-6,652	-8,69		5	0,39	0,20	0,93	0,74	0,97
part=1	1	-	-	-	-	-	part=1	1	=	=	=	=	=	-3,603	-6,392	-6,605	-15,26	-10,63	part=1	1	0,45	0,91	1,65	1,70	1,18
educ=1	2	-	-	-	-	-	crowding=1	2	=	=	=	=	=	-0,049	-2,519	-2,056	-2,829	-12,14	crowding=	2	0,01	0,36	0,51	0,31	1,35
crowding=1	3	-	-	-	-	-	zwh=1	3	=	=	=	=	=	-1,667	-2,278	-1,12	-6,778	-15,79	zwh=1	3	0,21	0,33	0,28	0,75	1,75
zwh=1	4	-	-	-	-	-		4	=	=	=	=	=	-3,082	-1,003	-2,925	-8,717	-1,395		4	0,39	0,14	0,73	0,97	0,15
	5	-	-	-	-	-		5	=	=	=	=	=	-0,024	-2,82	-1,887	-16,6	-5,562		5	0,00	0,40	0,47	1,84	0,62
part=1	1	-	-	-	-	-	part=1	1	=	=	=	=	=	-10,11	-13,26	-7,331	-23,2	-9,55	part=1	1	1,26	1,89	1,83	2,58	1,06
educ=1	2	-	-	-	-	-	crowding=1	2	=	=	=	=	=	-3,289	-0,824	-0,807	-3,694	-8,943	crowding=	2	0,41	0,12	0,20	0,41	0,99
crowding=1	3	-	-	-	-	-	zwh=1	3	=	=	=	=	=	0	0	0	-1,461	-6,41	zwh=1	3	0,00	0,00	0,00	0,16	0,71
zwh=1	4	-	-	-	-	-		4	=	=	=	=	=	-0,071	0	-0,678	-3,789	-10,12		4	0,01	0,00	0,17	0,42	1,12
	5	-	-	-	-	-		5	=	=	=	=	=	0	0	0	-0,844	-4,948		5	0,00	0,00	0,00	0,09	0,55
part=1	1	-	-	-	-	-	part=1	1	=	=	=	=	=	-3,993	-1,172	-0,21	-4,247	-12,96	part=1	1	0,50	0,17	0,05	0,47	1,44
educ=1	2	-	-	-	-	-	crowding=1	2	=	=	=	=	=	0	0	0	-0,164	-7,651	crowding=	2	0,00	0,00	0,00	0,02	0,85
crowding=1	3	-	-	-	-	-	zwh=1	3	=	=	=	=	=	-5,664	-13,04	-6,566	-20,78	-21,43	zwh=1	3	0,71	1,86	1,64	2,31	2,38
zwh=1	4	-	-	-	-	-		4	=	=	=	=	=	-5,32	-1,174	-8,58	-32,65	-27,61		4	0,66	0,17	2,15	3,63	3,07
	5	-	-	-	-	-		5	=	=	=	=	=	0	-2,697	-0,573	-6,581	-8,717		5	0,00	0,39	0,14	0,73	0,97
part=1	1	-	-	-	-	-	part=1	1	=	=	=	=	=	0	0	-0,012	-3,626	-4,247	part=1	1	0,00	0,00	0,00	0,40	0,47
educ=1	2	-	-	-	-	-	crowding=1	2	=	=	=	=	=	-3,289	-0,824	-0,807	-3,694	-8,943	crowding=	2	0,41	0,12	0,20	0,41	0,99
crowding=1	3	-	-	-	-	-	zwh=1	3	=	=	=	=	=	0	0	0	-1,461	-6,41	zwh=1	3	0,00	0,00	0,00	0,16	0,71
zwh=1	4	-	-	-	-	-		4	=	=	=	=	=	-6,385	-6,249	-1,912	-7,005	-8,475		4	0,80	0,89	0,48	0,78	0,94
	5	-	-	-	-	-		5	=	=	=	=	=	-2,712	-5,426	-2,061	-8,448	-13,84		5	0,34	0,78	0,52	0,94	1,54
part=1	1	-	-	-	-	-	part=1	1	=	=	=	=	=	-4,112	-2,2	-5,396	-5,487	-4,164	part=1	1	0,51	0,31	1,35	0,61	0,46
educ=1	2	-	-	-	-	-	crowding=1	2	=	=	=	=	=	0	-0,988	-2,237	-7,603	-4,968	crowding=	2	0,00	0,14	0,56	0,84	0,55
crowding=1	3	-	-	-	-	-	zwh=1	3	=	=	=	=	=	-3,603	-6,392	-6,605	-15,26	-10,63	zwh=1	3	0,45	0,91	1,65	1,70	1,18
zwh=1	4	-	-	-	-	-		4	=	=	=	=	=	0	-2,964	-1,998	-12,81	-9,567		4	0,00	0,42	0,50	1,42	1,06
	5	-	-	-	-	-		5	=	=	=	=	=	-13,56	-8,264	-12,9	-13,78	-6,68		5	1,70	1,18	3,23	1,53	0,74
part=1	1	-	-	-	-	-	part=1	1	=	=	=	=	=	-11,38	-7,441	-10,19	-16,6	-11,69	part=1	1	1,42	1,06	2,55	1,84	1,30
educ=1	2	-	-	-	-	-	crowding=1	2	=	=	=	=	=	-3,111	-1,402	-3,732	-6,652	-8,69	crowding=	2	0,39	0,20	0,93	0,74	0,97
crowding=1	3	-	-	-	-	-	zwh=1	3	=	=	=	=	=	0	0	-1,655	-6,952	-1,485	zwh=1	3	0,00	0,00	0,41	0,77	0,17
zwh=1	4	-	-	-	-	-		4	=	=	=	=	=	-3,708	-3,321	-6,493	-4,069	-8,267		4	0,46	0,47	1,62	0,45	0,92
	5	-	-	-	-	-		5	=	=	=	=	=	-0,045	-1,457	-3,564	-4,531	-1,04		5	0,01	0,21	0,89	0,50	0,12
part=1	1	-	-	-	-	-	part=1	1	=	=	=	=	=	-6,79	-7,861	-2,181	-4,76	-0,2	part=1	1	0,85	1,12	0,55	0,53	0,02
educ=1	2	-	-	-	-	-	crowding=1	2	=	=	=	=	=	-7,841	-9,598	-6,239	-9,732	-0,334	crowding=	2	0,98	1,37	1,56	1,08	0,04
crowding=1	3	-	-	-	-	-	zwh=1	3	=	=	=	=	=	-1,667	-2,278	-1,12	-6,778	-15,79	zwh=1	3	0,21	0,33	0,28	0,75	1,75
zwh=1	4	-	-	-	-	-		4	=	=	=	=	=	0	0	-1,076	-3,5	-11,38		4	0,00	0,00	0,27	0,39	1,26