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Consistent multidimensional poverty comparisons*

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Abstract

We study multidimensional poverty comparisons. It is assumed that overall poverty is measured by the sum of individual poverty levels. We do not from the outset impose restrictions on how to identify the poor or on how to compare individual poverty levels. Instead, we require only that the individual poverty ranking of bundles is consistent with the overall poverty ranking of distributions of bundles. Aside from the conventional "transferable" attributes (typical examples are income and wealth), we allow for "non-transferable" attributes (typical examples are educational attainment and housing conditions). A new distributional priority axiom is shown to imply that the individual poverty function is quasi-linear in the transferable attributes.

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1 Introduction

It is broadly agreed by now that income alone provides too narrow a basis for the assessment of poverty. It is therefore necessary to adopt a multidimensional perspective. We consider three among the several difficulties involved in multidimensional poverty measurement.

The first difficulty is that one may want to treat certain attributes such as self-reported health, educational attainment or housing conditions in a different way than attributes such as income or goods. That is, for certain attributes of an ordinal or even nominal nature, it may not be meaningful or desirable to think of these attributes as transferable across individuals. Those attributes should then be left out of the domain of the conventional transfer and monotonicity axioms. Instead of ignoring such attributes, as is the common approach in the literature,[1] we explicitly allow for them in our analysis by distinguishing between "transferable" and "non-transferable" attributes. In this way it is possible to assign to non-transferable attributes a special (and more passive) role.

The second difficulty is that of identifying the poor. In the multidimensional context there is a poverty bundle, i.e., a poverty line for every dimension, and this complicates the identification. Several approaches have been proposed and all have different advantages and disadvantages. The union approach classifies an individual as poor if she is poor in at least one of the dimensions, while the intersection approach classifies an individual as poor if she is poor in all dimensions. Approaches intermediate between these two extremes are possible as well. We will not impose one particular identification approach from the outset. What we ask instead is that the identification criterion used is *consistent* with the poverty ranking itself. With this we mean that an individual with bundle x is classified as poor if the distribution in which everyone has bundle x is according to the poverty ranking worse than the distribution in which everyone has the poverty bundle.

The third difficulty we consider is that of how to give priority to worse off poor individuals over better off poor individuals. Distributional concerns in poverty measurement are probably best seen as concerns for priority to the worse off: if the policymaker has an amount of an attribute available and can give it to any two poor individuals, then we want this amount to go to the worst off among these two. But many proposed multidimensional poverty measures fail to satisfy this condition. As an illustration, consider the following multidimensional version of the (absolute) Foster-Greer-Thorbecke (1984) poverty index: poverty equals $\sum_i \prod_j (z_j - x_j^i)^{\alpha_j}$

[1] See Alkire and Foster (2008) for a notable exception.

(the sum over poor individuals only) with x^i_j individual i's amount of attribute j, z_j the poverty line for attribute j and α_j the weight chosen for attribute j. Assume there are two attributes and two individuals. The poverty bundle is $(z_1, z_2) = (1, 1)$ and the bundles of individuals 1 and 2 are $x^1 = (0.4, 0.6)$ and $x^2 = (0.65, 0.4)$, respectively. Letting $\alpha_1 = \alpha_2 = 1$, poverty equals $0.6 \times 0.4 + 0.35 \times 0.6$. Note that individual 1 is the worst off poor according to our *consistency* requirement stated above: giving everyone $(0.4, 0.6)$ yields a worse distribution according to the poverty index than giving everyone $(0.65, 0.4)$. Now suppose an indivisible amount of 0.05 of the first attribute becomes available. If we give the amount to the worst off, then poverty decreases with $(0.6 - 0.55) \times 0.4$, while if we give it to the best off, then poverty decreases with $(0.35 - 0.30) \times 0.6$. So we arrive at the undesirable conclusion that according to this index poverty is minimized by giving the amount to the best off individual.[2] We will impose a condition of priority which avoids this undesirable conclusion and study its implications on poverty rankings using a reinterpretation of Bosmans, Lauwers and Ooghe (2009, Theorem 1).

2 Result

The set of individuals is I and the set of attributes is J. A multidimensional distribution is a list $X = (x^i)_{i \in I}$, where x^i in $B = \mathbb{R}^{|J|}_+$ is the attribute bundle of individual i. The domain of all distributions is $D = B^{|I|}$.[3] We partition the set J into the set of transferable attributes T and the set of the non-transferable attributes N. Accordingly, every bundle x (or any other vector in $\mathbb{R}^{|J|}_+$) can be decomposed into a transferable part $x_T = (x_j)_{j \in T}$ and a non-transferable part $x_N = (x_j)_{j \in N}$. Let z be a poverty bundle in B, containing a poverty line for each dimension. We assume z to be fixed.

Poverty comparisons of distributions are made using a poverty ordering $\succcurlyeq_z$, a reflexive, transitive and complete binary relation on D. The expression $X \succcurlyeq_z Y$ means that X is at least as good as Y, i.e., that poverty in X is at least as low as in Y. The asymmetric and symmetric components of $\succcurlyeq_z$ are denoted by $\succ_z$ and $\sim_z$, respectively.[4] We next define four axioms. The first three—representation, focus and monotonicity—are standard.

[2] Note that increasing the weights $\alpha_1 = \alpha_2$ to 2 helps avoid the conclusion. However, one can find examples to obtain the undesirable conclusion for any finite values of α_1 and α_2.

[3] It is possible to replace B by a cartesian product of closed non-degenerate intervals, one interval for each attribute j in J.

[4] We have $X \succ Y$ if $X \succcurlyeq Y$ and not $Y \succcurlyeq X$ and we have $X \sim Y$ if $X \succcurlyeq Y$ and $Y \succcurlyeq X$.

Representation requires that overall poverty can be written as a sum of individual poverty levels.

Representation. There exists a C^1-function $\pi_z : B \to \mathbb{R}$ such that, for all X and Y in D, we have

$$X \succcurlyeq_z Y \quad \Leftrightarrow \quad \sum_{i \in I} \pi_z(x^i) \leqslant \sum_{i \in I} \pi_z(y^i).$$

Representation forces the poverty ordering to be continuous, separable (the ranking of two distributions is independent of any individual who has the same bundle in both distributions) and anonymous (switching individuals' bundles does not affect the ranking). The axiom also imposes the technical condition of differentiability. The assumption of additive representability is common in the literature (see, e.g., Tsui, 2002, Bourguignon and Chakravarty, 2003, and Duclos, Sahn and Younger, 2006).

The focus axiom requires poverty to be unaffected by a change of the bundle of a non-poor individual (if it leaves the individual non-poor). In order to define this axiom, we need to be able to distinguish the individuals who are poor from those who are not. We base the criterion to identify the poor on the poverty ordering $\succcurlyeq_z$ itself. More precisely, we say that an individual with bundle x is poor if the distribution $(x, x, \ldots, x)$, in which every individual receives bundle x, is considered worse according to $\succcurlyeq_z$ than the distribution $(z, z, \ldots, z)$. Thus, the set of poor individuals in distribution X is $P_{\succcurlyeq_z}(X) = \{i \in I \,|\, (x^i, x^i, \ldots, x^i) \prec_z (z, z, \ldots, z)\}$. Given representation, the poor have bundles x such that $\pi_z(x) < \pi_z(z)$, while the non-poor have bundles x such that $\pi_z(x) \geqslant \pi_z(z)$.

Focus. For all X and Y in D and for each i in I that is neither in $P_{\succcurlyeq_z}(X)$ nor in $P_{\succcurlyeq_z}(Y)$, we have that if $x^k = y^k$ for all k in $I \setminus \{i\}$, then $X \sim_z Y$.

Given representation, focus requires that the individual poverty function π_z is constant for all bundles x such that $\pi_z(x) \geqslant \pi_z(z)$.

The monotonicity axiom requires poverty to decrease if a poor individual receives more of at least one of the *transferable* attributes, ceteris paribus. For vectors x and y, we write $x > y$ if the inequality $x_k \geqslant y_k$ holds for each component k with at least one inequality holding strictly.

Monotonicity. For all X and Y in D and for each i in $P_{\succcurlyeq_z}(Y)$, we have that if $x^i_T > y^i_T$ and $x^i_N = y^i_N$, while $x^k = y^k$ for all k in $I \setminus \{i\}$, then $X \succ_z Y$.

Given representation, monotonicity requires the individual poverty function π_z to be strictly decreasing in each transferable attribute for all bundles x such that $\pi_z(x) < \pi_z(z)$.

Finally, we consider a new axiom called priority, which requires that if there is an indivisible amount of transferable attributes available, then this amount should be given to the worse off among any two poor individuals. We again make individual comparisons of poverty on the basis of the poverty ordering $\succcurlyeq_z$ itself. More precisely, an individual with bundle x is considered less poor than an individual with bundle y if $(x, x, \ldots, x) \succcurlyeq_z (y, y, \ldots, y)$. Let 0 denote a vector of zeros (of variable length).

Priority. For all X and Y in D, for each vector α in $\mathbb{R}_+^{|J|}$ with $\alpha_T > 0$ and $\alpha_N = 0$ and for all individuals k and ℓ in $P_{\succcurlyeq_z}(Y)$, we have that if $(x^k, x^k, \ldots, x^k) \succ_z (x^\ell, x^\ell, \ldots, x^\ell)$ and if $X = (\ldots, x^k, \ldots, x^\ell + \alpha, \ldots)$ and $Y = (\ldots, x^k + \alpha, \ldots, x^\ell, \ldots)$ with X and Y coinciding except for individuals k and ℓ, then $X \succ_z Y$.

The following theorem shows that, simultaneously, the axioms representation, focus, monotonicity and priority imply that the individual poverty function is quasi-linear in the transferable attributes. The theorem is a reinterpretation of Bosmans, Lauwers and Ooghe (2009, Theorem 1) and we therefore omit the proof. For two vectors p and x in $\mathbb{R}^k$, we write $p \cdot x$ for the sum $p_1 x_1 + p_2 x_2 + \cdots + p_k x_k$.

Theorem. *Consider a poverty bundle z in B. A poverty ordering $\succcurlyeq_z$ satisfies representation, focus, monotonicity and priority if and only if there exist*

(a) *a vector p_T of positive weights, one weight for each attribute in T,*

(b) *a C^1-function $\psi : \mathbb{R}_+^{|N|} \to \mathbb{R}$, to aggregate the attributes in N, and*

(c) *a C^1-function $\varphi : \mathbb{R} \to \mathbb{R}$, strictly decreasing and strictly convex for all bundles in $\{x \in B \,|\, p_T \cdot x_T + \psi(x_N) < p_T \cdot z_T + \psi(z_N)\}$ and equal to a constant for all other bundles,*

such that, for all X and Y in D, we have

$$X \succcurlyeq_z Y \quad \Leftrightarrow \quad \sum_{i \in I} \varphi\big(p_T \cdot x_T^i + \psi(x_N^i)\big) \;\leqslant\; \sum_{i \in I} \varphi\big(p_T \cdot y_T^i + \psi(y_N^i)\big).$$

3 Discussion

The representation we obtained in the theorem is quite restrictive. We have linearity in the transferable attributes so that trade-offs between those attributes are constant irrespective of the attribute amounts. The function φ specifies the weights given to the worse off relative to the better off in the poverty aggregation. The higher the degree of convexity of φ, the higher the maximum leak that is allowed in transferring an amount of a transferable attribute from a better off to a worse off poor individual. The non-transferable attributes play a specific and rather passive role. They have no impact on the trade-offs between the transferable attributes (the level sets of the individual poverty function with respect to the transferable attributes are unaffected by the choice of the function ψ). However, they contribute to the level of individual poverty of one individual relative to the others and therefore help determine the weight the individual gets in the poverty aggregation (through the function φ).

The theorem demonstrates that the considered axioms lead to a criterion for the identification of the poor that is intermediate between the intersection and union criteria. That is, some individuals who are not poor in all dimensions will be classified as poor, and some individuals who are poor in some dimensions will not be classified as poor. All approaches that do not use this intermediate criterion (e.g., Tsui, 2002, and Bourguignon and Chakravarty, 2003) fail to satisfy priority. Consider however the following unanimity poverty quasi-ordering: X is better than Y if and only all poverty orderings satisfying the axioms in the theorem agree that X is better than Y. This poverty quasi-ordering belongs to the intersection approach because all possible weights on the transferable attributes have to be considered.

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