"Tell me what you need": Signaling with limited resources

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Abstract

We study the project allocation mechanisms trade-off between minimizing the waste of resources in the application process and maximizing the match of needs and projects when the recipient's needs and resources are private information.

We propose a signaling mechanism where the set of signals available to each agent is constrained by his capacity and by his truthful need of the project. The principal can control, at a given cost, the agent's application cost and the utility of receiving the project by non-needy agents.

Our findings suggest that there exists a threshold in the principal's budget such that for smaller budgets, all instruments are used in the optimal mechanism, while for bigger budgets the optimal application complexity is independent of the budget and waste of resources is a decreasing share of the resources available.

1 Introduction

In many real life situations, like Foreign Aid and Research Grants, both principals and agents play an active role in the information disclosure. On the one hand, the principals, willing to obtain information on the quality/type of the pool of applicants for funds, screen them with the design of application mechanisms and disbursement schemes. On the other hand, applicants are not passive players in the game. Applicants compete among them to obtain the funding and hence see the application process as a signaling game to let the principals know their type: *"the better is my application, the more likely I am to be awarded the funds"*.

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The design of application mechanisms, the evaluation of these applications, and the design and monitoring of funds disbursements are costly activities. Principals have a limited amount of funds to award projects and to cover all these expenditures, therefore they face a trade-off between more refined application mechanisms, that increase the probability of good matches of recipient's type and projects but "waste" more resources on design, evaluation and implementation, or less complex application mechanisms, that may decrease the average quality of the agents being awarded a project.

Our objective is to study how to design a mechanism (application) to obtain information on the agents' needs when the agents have limited, and maybe unobservable, capacity to signal their needs[1]. The principal can control, at a given cost, the number of projects to be awarded, the difficulty for each type of agent to fill in the application or his probability of being caught cheating, and the utility of receiving the project by non-needy recipients, through in-kind awards. The principal's objective is to award the projects to the high need agents, and among them give preference to the ones with lowest capacity, maximizing sum of agent's utility from the projects awarded minus application expenditure, subject to his budget constraint.

The choice of the instruments available to the principal is motivated by two existing mechanisms. On the one hand, Private Foundations, like the Bill and Melinda Gates Foundation, are well known for complex application procedures and specialization in a few topics, fact likely to reduce their application review costs. On the other hand, institutional aid agencies, like the World Bank, have traditionally had less complex application procedures and have linked the project's budget to in-kind transfers, decreasing the value of the project for non-needy agents. Our objective is to study the appropriate mix of both technologies for different sizes of projects, donor's budgets, and needy agents and capacity distributions.

We model the application process as a signaling game with unobservable capacity and needs, where the set of signals available to each agent is constrained by his capacity and by his true need of the project. Our novelty is to introduce in a lobbying game à la Esteban and Ray (2000, 2003, 2006) need dependent signaling-lobbying costs (and hence type and wealth dependent feasible signals) and to study the optimal mix of principal's technologies that ensures the existence of a separating equilibrium on needs subject to the principal's budget constraint. Moreover, we study how the distribution of (unobservable) capacity and needs affect the cost of the optimal application mechanism.

We consider two levels of capacity and two levels of needs, which leave us with four types of agents. For simplicity of language and to avoid confusion with needs, we call rich the high capacity agents and poor the low capacity agents. Non-needy poor and needy rich agents will not be problematic, since they are able to separate themselves when desired, not sending any signal (i.e. not applying) or with the largest possible signal respectively (i.e. with the more

[1] It is kwnoledged that capacity to fill application procedures can be an important constraint for some potential candidates. For example, electricity availability to keep computers on, sintesizing capacity,... are difficult to measure.

complete application feasible). The problematic types are non-needy rich and needy pour, and are the first type the ones whose binding incentive compatibility constraint determines the separating equilibrium for a given combination of parameters. For a separating equilibrium to exist, we need to make sure that (1) if incentive compatibility constraint binds for the non-needy rich agents, the needy poor have enough capacity to send the separating signal, and (2) if non-needy rich resources constraint binds, the needy pour can signal themselves without being imitated by non-needy rich.

We find that there exists a threshold in the donor's budget, increasing in percentage of needy agents and in the unit cost of providing the project, such that separating signal and application requirements are increasing in number of projects for budgets below and constant afterwards. For smaller budgets, all instruments are used in the optimal mechanism whenever technology available is cheap enough. Separating signal and complexity of the application process, together with number of projects awarded, increase with budget until threshold is reached. This implies that resources spend on the design of the application process increase with the budget. For budgets over the threshold, separating application is independent of number of projects awarded, and marginal application cost for needy agents is decreasing with the share of needy agents and independent of the number of projects awarded. Share of resources spend in the mechanism decreases with budget.

Our results suggest that for expensive projects and large shares of needy agents, both instruments are used in the welfare maximizing mechanism and hence World Bank's technology is necessary, together with Application Evaluation. For small percentages of needy agents and cheap projects, separating signal only depends on rich agent's wealth and a complex application mechanism is the only instrument used. Hence, Bill Gate's technology is an asset in this situation.

The structure of the paper is as follows. We complete this section with a brief review of related literature. Section 2 presents the structure of the game, and Sections 3 presents the signaling game and the characteristics of the separating equilibriums in needs. Section 4 presents the Social Welfare Function and describes the Optimal Application Mechanism. And Section 5 concludes.

1.1 Related Literature

In the lobbying literature, Esteban and Ray (2000) present a license assignment problem between productive and non-productive agents where the number of licenses is smaller than the number of high types with constant wealth. Esteban and Ray (2003) expands this setting to account for different wealth distributions, and Esteban and Ray (2006) accounts for a lobby cost that depends on the amount of lobby expenditures and on the wealth, but assumes no correlation between productivity and wealth. We introduce need dependent signaling-lobbying costs (and hence type and wealth/capacity dependent feasible signals) and we study the optimal mix of principal's technologies to ensure the existence of a separating equilibrium on needs.

Fernandez and Gali (1999) shows in a matching-assignment problem that tournament allocations provide better matches than market and that welfare is larger for a good enough tournament mechanism when there exist borrowing constraints. We show that a tournament-application mechanism is always necessary (with different degrees of complexity) when agents have limited resources, but we allow for an additional instrument, the distortion on the non-needy's value of the projects[2], and show that a mix is optimal when resources constraints are not binding for all types. Moreover, we approach the question from a budget allocation perspective that accounts for the separability constraints that a tournament-application requires.

We differ from income maintenance problems (Besley and Coase (1992, 1995), Shapiro (2001)) in that our two unobservables are independent on the agent's decisions, as opposite to income maintenance where the problem is to avoid a decrease in labor supply to qualify for the program by high ability agents. In this sense, our setting is closer to the direct and indirect taxation literature with unobservable endowments and productivity, as in Cremer, Pestieau and Rochet (2001), but we only have one instrument, the application, to handle both unobservables.

Our signaling mechanism reminds of litigation procedures, where interested parties can choose the information to disclose[3] and the regulator can have different abilities to interpret this information. In our case, agents decide signal to send in a mechanism designed by the principal according to his costs and budget and distribution of the characteristics of the agents applying. Milgrom and Roberts (1986) study how the regulator's sophistication and the number of applicants affect the information disclosure; Maggi and Rodriguez-Clare (1995) show how falsification can be optimal since it helps to reduce information rents; our results are aligned with theirs in the ambiguous effect of the falsification technology, and we introduce an additional source of waste, number of projects over needed, as an instrument for the principal to extract information from the agents.

2 Design of the Application Mechanism

We present an application game where the principal's objective is to award identical projects to a continuum of agents that differ in their capacity to complete applications and in their need of the project, and where both these characteristics are unobservable by the principal. The principal can choose, subject to his budget, the number of projects to award and the resources to spend in the design and evaluation of the application procedure.

The application procedure has two characteristics to be chosen by the principal: (1) the complexity of the application to be submitted and the subsequent

[2] The parallelism in their school allocation setting would be low ability students feeling inferior in a high quality school and performing even under their ability.

[3] Sanchirico and Triantis (2008) studies the fabrication of evidence in mergers litigations. Che and Severinof (2007) study how lawyer advice affects information disclosure.

evaluation and monitoring of this information, and (2) the share of the total value of the project to be awarded in-kind. Both this instruments determine, on the one hand, the agent's costs and benefits of submitting a request depending on their valuation of the project and their capacity, and hence they determine the agent's willingness to participate in the process. On the other hand, these instruments determine the cost for the principal of design and evaluation of the applications, and the cost of awarding and managing the in-kind shares of the projects. Figure 1 presents the timing of events: principal announces application rules, agents decide to send their application and projects are awarded according to the announced rules.

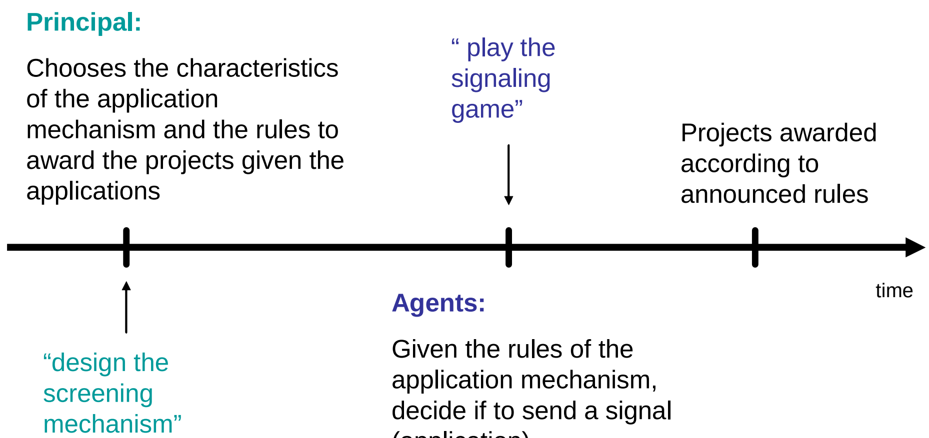

Figure 1: Timing of events

Next sub-section presents the structure and notation of each step. We proceed backwards: In Section 3 we present the signaling game the agents face when deciding if to apply for the project, and given their behavior we proceed in Section 4 to analyze how the principal chooses the welfare maximizing application mechanism given the instruments available and his budget.

2.1 Step 1: "Signaling" Application Game

We assume that each agent i gets a utility of being awarded the project V_i that for the agents that need the project comes from its use, and for the non-needy agents comes from the flow of funds the project involves. Therefore, as the in-kind share of the project increases, the value of the project for non-needy agents decreases as they can not use funds for alternative utility generating activities.

The principal announces the application procedure to be used to award the projects. We model this application process as a *signaling game*, where as usual to send a given signal is cheaper for the agents that value the project, but feasible for all of them as long as they have capacity to send them.

Let w_i be the capacity of agent i, $p(a)$ the probability of being awarded the project when an application of quality a is send and $\phi_i a$ the cost for agent i to send an application of quality a. The expected utility of agent i to send a signal

a is given by

$$U_i(a) = p(a)V_i + (w_i - \phi_i a)$$

and the feasible signals are subject to the capacity constraint

$$\phi_i a \leqslant w_i$$

i.e. the set of feasible signals for each agent is constrained by his capacity and by his need of the project.

We assume that there are two types of agents with respect to need: the ones that need the project (high types), that have an application cost ϕ_h, and the ones that do not need it (low types) with application cost $\phi_l > \phi_h$. Let β be the proportion of high type agents. Rich/ high capacity agents are able to send high signals since they have the resources/capacity to make them up, while poor agents are only able to send high signals when they really need the projects. In other words, if the agent does not need the project he faces a higher marginal cost of producing a given quality signal since he has to 'fake' the need[4]. This difference in marginal application cost can also be interpreted as probability of being caught lying and hence being disclassified from the application procedure in case of a random audit. Cost of this threat would be bigger for needy agents.

The **call for applications** the principal publishes consists of minimal application quality to be considered for a project, and the rules for the distribution of the projects to the applications received. The equilibrium of the game consists of three objects:

1. The quality of the submitted application, that maps (w_i, ϕ_i, V_i) into $a(w_i, \phi_i, V_i)$ given the application process announced.
2. A function μ that maps application quality into posterior beliefs about the need of the agent, consistent with Bayes' rule when applicable.
3. A probability function $p(a)$ that maps application quality to probability of being awarded the project.

The *equilibrium* of the application game depends on the distribution of the unobservables of the model (capacity and need) and on the quantity of projects to be awarded (α), together with each agent's application costs and valuation of the project.

2.2 Step 2: "Screening" Design of the Application Procedure Characteristics

Our objective is to design the application mechanism, given by $(\alpha^*, \phi^*, C_l^*)$, that maximizes social welfare given the budget of the principal, his cost of

[4]An example of this resources constraint would be the energy (w_i) students need to complete a term paper: it is function of how long the paper has to be (a, equal for every student) and the difficulty of the subject (ϕ_i, different for each student given his ability), both manipulable by the professor when setting the conditions for the paper. Students with a lot of energy can complete a long exam, but students with low energy can only complete it if they have high ability.

evaluating applications and his ability to alter the utility the project provides to the non-needy agents. Moreover, the principal needs to ensure that the contract proposed is such that a separating equilibrium in needs exists and hence the signals of the agents contain information about needs. We present in Sections 3 and 4 the equilibrium of the signaling game for any level of the parameters (α, ϕ, C_l), and in Section 5 we choose from the separating equilibrium candidates the optimal as to maximize social welfare given the donor's budget constraint.

3 Application_ Signaling Game

In the first step of the game, we study how the agents play the signaling game given the announced rules. We start with the situation with two levels of need and equal capacity for all agents, and we continue with the case of two unobservables, namely need and capacity. For each of this information frameworks, we look at the existence of pooling and separating equilibriums with respect to need. We analyze the conditions for the existence of a separating equilibrium in needs, and the existence of partially separating equilibria with more than two signals.

3.1 Two levels of needs and equal capacity for all agents

Let us start by presenting the equilibrium of the application game when all the candidates have the same capacity w. Let β be the proportion of agents that value the project. In this case, needy agents can always separate themselves from the not needy by sending an application a such that

$$\frac{w}{\phi_h} > a > \frac{w}{\phi_l}$$

i.e. all applications that are better than w/ϕ_l come from an agent that needs the project since low type agents are not able to send them. We consider both pooling and separating equilibriums and see how their existence depends on the number of projects available α, the capacity level w and the difficulty to fill the application ϕ for needy and non-needy agents. Let V_h and V_l be the benefit from being awarded the project for needy and non-needy agents respectively. We assume $V_h \geqslant V_l$.

Proposition 1 (Pooling Equilibrium Constant Capacity) *The exists a pooling equilibrium where all players send the same application $a^* = 0$ when*

$$w > \frac{V_h(1-\alpha)\phi_l}{\phi_h}$$

Proof is provided in Appendix A. Intuitively, no needy agent deviates and signals himself with an application that non-needy can not fill when capacity is so big that the cost of resources to separate is greater than the expected benefit.

Let us look for a **separating equilibrium**. We need to take into account how the number of projects available compares with the number of needy agents: if $\alpha > \beta$, agents that not send a signal still have a positive probability of receiving the project, and if $\alpha < \beta$, even if a separating equilibrium in needs exists, there will be needy agents that will not receive the project.

Let (a^*_H, a^*_L) be the candidate separating applications for high and low type respectively, and let the application procedure be

$$\begin{aligned} \mu(a) &= 0 \text{ for } a < a^*_H \\ &\rightarrow \quad \tilde{p}(a) = 0 \text{ if } \alpha < \beta \text{ and } p(a) = \frac{\alpha - \beta}{1 - \beta} \text{ if } \alpha > \beta \end{aligned} \tag{1}$$

$$\begin{aligned} \mu(a) &= 1 \text{ for } a \geqslant a^*_H \\ &\rightarrow \quad \tilde{p}(a) = \frac{\alpha}{\beta} \text{ if } \alpha < \beta \text{ and } p(a) = 1 \text{ if } \alpha > \beta \end{aligned} \tag{2}$$

Proposition 2 (Separating Equilibrium Constant Capacity) *There exist separating equilibrium when capacity is uniform among players where low types do not apply and the application of the high types a^* is such that:*

$$\begin{aligned} \textit{For } \alpha &< \beta\textit{: If } w \geqslant \frac{\alpha V_l}{\beta},\ \tilde{a}^* = \frac{\alpha V_l}{\beta \phi_l} \\ \textit{if } w &< \frac{\alpha V_l}{\beta},\ \tilde{a}^* = \frac{w}{\phi_l} \end{aligned}$$

$$\begin{aligned} \textit{For } \alpha &> \beta\textit{: If } w \geqslant \frac{(1-\alpha)V_l}{(1-\beta)},\ a^* = \frac{(1-\alpha)V_l}{(1-\beta)\phi_l} \\ \textit{if } w &< \frac{(1-\alpha)V_l}{(1-\beta)},\ a^* = \frac{w}{\phi_l} \end{aligned}$$

Equilibrium separating signal depends on the capacity and on the marginal cost of signaling for each type: either resources constraint binds so incentive compatibility for the low type can not bind, or incentive compatibility binds when capacity allows. Proof is provided at Appendix A.

It is interesting to look at the *allocative loss* (L, match and mismatch of projects and needs) and the *bureaucratic cost* (W, resources spend by the agents in the application process) as function of the number of projects awarded, the difficulty of the application process and the value of the project for the low type agents. We consider two types of allocative losses: needy agents not getting the project ($L1$), and non-needy agents getting the project ($L2$). Whenever $\alpha > \beta$ there is always $L2$, since there are more projects than needy agents, but we need to consider how this affects the equilibrium applications submitted and ultimately application costs W.

Corollary 3 *To award projects to non-needy agents decreases bureaucratic application costs in a separating equilibrium for high enough level of capacity.*

Proof. For $\alpha = \beta$, $a^* = \tilde{a}^*$. For a fixed β, and for $w \geqslant \max(\frac{\alpha V_l}{\beta}, \frac{(1-\alpha)V_l}{(1-\beta)})$, let $A(\alpha)$ represent equilibrium signal as function of α

$$A^*(\alpha) = \begin{cases} \frac{\alpha V_l}{\beta \phi_l} \text{ for } \alpha < \beta \\ \frac{V_l}{\phi_l} \text{ for } \alpha = \beta \\ \frac{(1-\alpha)V_l}{(1-\beta)\phi_l} \text{ for } \alpha > \beta \end{cases}$$

$A^*(\alpha)$ is an increasing function of α for $\alpha < \beta$ and decreasing for $\alpha > \beta$. Given the separating equilibrium signal, the total application cost is given by

$$W = \beta \phi_h A^*(\alpha)$$

that is decreasing with α for $\alpha > \beta$. Allocative loss $L1$ decreases with α until $\alpha = \beta$ and $L2$ increases with α after $\alpha = \beta$. So *waste of resources* $L2$ may be optimal to decrease bureaucratic application costs. ■

3.2 Two levels of needs and two levels of capacity

Let us consider the application procedure when both need and capacity differ among agents and are not observable by the principal. The capacity unobservability assumption aims to reflect the difficulty to verify the resources available for the agent to use in the application process: wealth, access to credit markets, etc., that together with the agent's marginal cost of the application ϕ_i determine the applications the agent is able to present. We assume that even the richest agents do not have enough resources to implement the project without aid.

We consider two capacity levels, $w_r > w_p$, and let σ be the proportion of rich agents. Thus, we have four types of agents: needy rich (HR), needy poor (HP), not needy rich (LR) and not needy poor (LP). Each of these agents faces a different constraint on the set of feasible applications: needy rich agents are the ones able to send the highest quality application, and the non needy poor are the more restricted ones. The problematic groups are the intermediate: needy poor and not needy rich. We need to consider three possibilities, $w_r/\phi_l > w_p/\phi_h$, $w_r = w_p/\phi_h$ and $w_r/\phi_l < w_p/\phi_h$, and discuss the existence of signaling equilibriums for each of the alternatives[5].

Let us look for a **separating equilibrium**. Let (a_H^*, a_L^*) be the candidate applications for high and low type respectively, and let the application procedure be

$$\begin{aligned} \mu(a) &= 0 \text{ for } a < a_H^* \\ &\rightarrow \quad \tilde{p}(a) = 0 \text{ if } \alpha < \beta \text{ and } p(a) = \frac{\alpha - \beta}{1 - \beta} \text{ if } \alpha > \beta \qquad (3) \\ \mu(a) &= 1 \text{ for } a \geqslant a_H^* \\ &\rightarrow \quad \tilde{p}(a) = \frac{\alpha}{\beta} \text{ if } \alpha < \beta \text{ and } p(a) = 1 \text{ if } \alpha > \beta \qquad (4) \end{aligned}$$

[5]Following the education example, the professor's objective is to separate high ability students, not the high energy ones that may be able to write the same term paper.

Proposition 4 (Separating Equilibrium for Needy and Not needy) *There exists a separating equilibrium where all high types apply and low types do not apply that is given by a pair $(0, a^*)$ such that*

$$\begin{aligned} \textit{For } \alpha &< \beta\text{: } \textit{If } \frac{\alpha V_l}{\beta\phi_l} \leqslant \min(\frac{w_r}{\phi_l}, \frac{w_p}{\phi_h}),\ \tilde{a}^* = \frac{\alpha V_l}{\beta\phi_l} \\ \textit{if } \frac{w_p}{\phi_h} &> \frac{\alpha V_l}{\beta\phi_l} > \frac{w_r}{\phi_l},\ \tilde{a}^* = \frac{w_r}{\phi_l} < \frac{w_p}{\phi_h} \\ \textit{if } \frac{w_p}{\phi_h} &< \frac{w_r}{\phi_l} < \frac{\alpha V_l}{\beta\phi_l},\ \textit{or } \frac{w_p}{\phi_h} < \frac{\alpha V_l}{\beta\phi_l} < \frac{w_r}{\phi_l} \textit{ no separating equilibrium} \end{aligned}$$

$$\begin{aligned} \textit{For } \alpha &> \beta\text{: } \textit{If } \frac{(1-\alpha)V_l}{(1-\beta)\phi_l} \leqslant \min(\frac{w_r}{\phi_l}, \frac{w_p}{\phi_h}),\ a^* = \frac{(1-\alpha)V_l}{(1-\beta)\phi_l} \\ \textit{if } \frac{w_p}{\phi_h} &> \frac{(1-\alpha)V_l}{(1-\beta)\phi_l} > \frac{w_r}{\phi_l},\ a^* = \frac{w_r}{\phi_l} < \frac{w_p}{\phi_h} \\ \textit{if } \frac{w_p}{\phi_h} &< \frac{w_r}{\phi_l} < \frac{(1-\alpha)V_l}{(1-\beta)\phi_l},\ \textit{or } \frac{w_p}{\phi_h} < \frac{(1-\alpha)V_l}{(1-\beta)\phi_l} < \frac{w_r}{\phi_l} \textit{no separating equilibrium} \end{aligned}$$

Proof is provided in Appendix B. Structure of the proof is as follows:

1. Agents that send the low signal are identified as low types and are not awarded the project. It is in their best interest to send the lowest possible signal that is zero.

2. When Incentive Compatibility for LR does not bind, for $\frac{\alpha V_l}{\beta\phi_l} > \frac{w_r}{\phi_l} > \frac{w_p}{\phi_h}$, no separating equilibrium exists since the LR would always deviate and mimic the HP type. For these wealth levels, only an investment of the principal to increase ϕ_l and/or decrease ϕ_h and/or V_l could make a separating equilibrium feasible.

3. Whenever incentive compatibility for LR does bind, $\frac{w_p}{\phi_h} < \frac{\alpha V_l}{\beta\phi_l} < \frac{w_r}{\phi_l}$, we need the low type to have enough capacity to send the application.

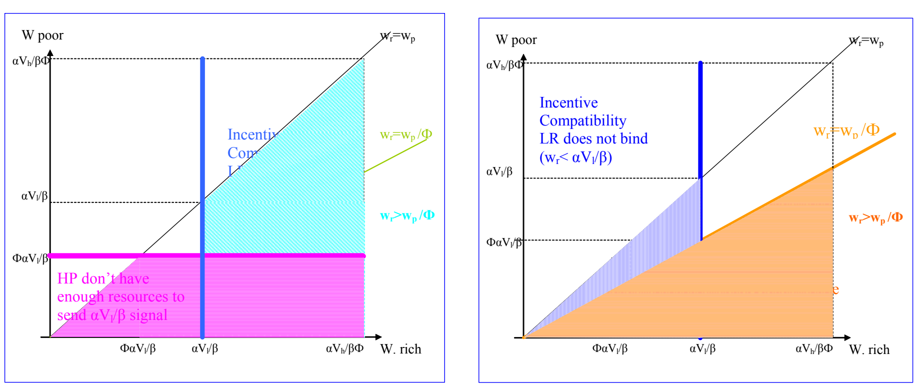

Figure 2 shows the graphical representation of (2) and (3) above: in both cases there is an area of wealths where a separating equilibrium does not exist.

3.3 Existence of Separating Equilibrium by needs and Partially Separating Equilibria

In the region of wealths where there does not exist a separating equilibrium, the principal can look for an alternative signaling structure allowing three types of applications, A_1, A_2 and A_3, with two types of agents pooling in one of the three alternatives as showed in Figure 3. Let $A_3 = 0$, no application submitted, as intuitive since lowest signal is likely to be identified with lowest need and capacity agents and will not be awarded the project, and the two remaining candidate application qualities, $A_1 > A_2 > 0$, can be interpreted as standard application (A_2) and application with Appendix (A_1). Given that only one instrument is available and the two unobservable characteristics affect the set of available signals for each agent, *Belief-Action Parity,* as in Esteban and Ray (2000, 2003, 2006), is required.

Definition 5 *Belief-Action Parity states that if the beliefs are such that expected profitability is identical after two announcements, permissions should be allocated to them with equal probability.*

Belief-Action parity implies that no more than two positive announcements can be served with positive probability. The proof is intuitive: if two announcements are served with positive probability, the one that has greater expected profitability should be fully served; If three positive announcements where served, two of them would be fully served, and no agent would send the higher one.

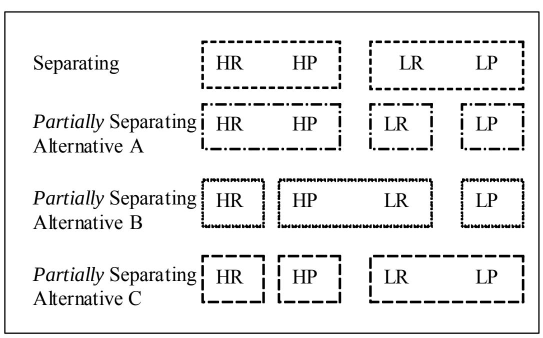

Figure 3

At **Alternative A**, all needy agents send the same application, and LR and LP separate themselves. This can only be a candidate equilibrium when

$$\beta < \alpha < \beta + (1 - \beta)(1 - \sigma) = 1 - \sigma + \sigma\beta$$

since otherwise LR would not get any project and so would have no incentive to separate from LP, and there are not enough projects to cover up to the LP.

Let the belief and award probabilities be such that

$$\mu(A_1) = 1 \rightarrow p(A_1) = 1 \tag{5}$$

$$\mu(A_2) = 0 \rightarrow p(A_2) = \frac{\alpha - \beta}{\sigma(1-\beta)} \tag{6}$$

$$\mu(A_3) = 0 \rightarrow p(A_3) = 0 \tag{7}$$

i.e. all high types get a project, and the remaining projects go to LR, since belief to be high type is zero for both A_2 and A_3. This equilibrium only exists, as the separating considered, when $w_r/\phi_l \leqslant w_p/\phi_h$, so does not provide an improvement for the possible wealth distributions where needy and non needy agents can be separated.

Proposition 6 (Partially Separating Equilibrium Alt. A) *At a partially separating equilibrium Alt. A, probability of getting the project for LR is greater than in the separating equilibrium, and their application cost is equal or greater than the application cost of the high types on the separating equilibrium, as long as* $[\alpha - \beta - \sigma(1-\beta)] > 0$.

For the needy agents, they always get the project $(\alpha > \beta)$ *but their application cost is greater or equal than in a totally separating equilibrium.*

And the set of wealths for which there does not exist a partially separating alt. A coincides with the set where there does not exist a separating.

The Partially Separating Equilibrium (alt. A) has the following structure. Let A_1, A_2 and A_3 be the application standards and let $p(A_1)$, $p(A_2)$ and $p(A_3)$ be as in (5), (6) and (7). For α such that $\beta < \alpha < 1 - \sigma + \sigma\beta$, $A_3 = 0$

i. For $w_p < p(A_2)V_l$, Incentive Compatibility for LP can not bind so $A_2 = w_p/\phi_l$.

a. For $w_r < (1 - p(A_2))V_l + w_p$, Incentive Compatibility for LR can not bind so $A_1 = w_r/\phi_l$ as long as $w_r/\phi_l \leqslant w_p/\phi_h$

b. For $w_r \geqslant (1 - p(A_2))V_l + w_p$, $A_1 = (1 - p(A_2))V_l + w_p$ whenever $w_p/\phi_h \geqslant (1 - p(A_2))V_l + w_p$

ii. For $V_l \geqslant w_p \geqslant p(A_2)V_l$, $A_2 = \frac{p(A_2)V_l}{\phi_l}$.

a. For $w_r < (1 - p(A_2))V_l + p(A_2)V_l$, Incentive Compatibility for LR can not bind so $A_1 = w_r/\phi_l$ as long as $w_r/\phi_l \leqslant w_p/\phi_h$

b. For $w_r \geqslant (1-p(A_2))V_l+p(A_2)V_l$, $A_1 = (1-p(A_2))V_l + \frac{p(A_2)V_l}{\phi_l}$ whenever $w_p/\phi_h \geqslant (1 - p(A_2))V_l + \frac{p(A_2)V_l}{\phi_l}$

iii. For $w_p \geqslant V_l$, $A_2 = \frac{p(A_2)V_l}{\phi_l}$.

a. For $w_r < (1 - p(A_2))V_l + \frac{p(A_2)V_l}{\phi_l}$, Incentive Compatibility for LR can not bind so $A_1 = w_r/\phi_l$ as long as $w_r/\phi_l \leqslant w_p/\phi_h$ and $A_1 \geqslant \frac{V_l}{\phi_l}$

b. For $w_r \geqslant (1 - p(A_2))V_l + \frac{p(A_2)V_l}{\phi_l}$, $A_1 = \frac{V_l((1-p(A_2))\phi_l + p(A_2))}{\phi_l}$ whenever $w_p/\phi_h \geqslant V_l$ and $A_1 \geqslant \frac{V_l}{\phi_l}$.

Proof is provided at Appendix B. The structure of the proof, common to all the three-signals equilibriums considered, builds as follows:

1. Either incentive compatibility for LP binds or resources constraint for LP binds, with $A_2 = p(A_2)V_l$ (ii) or $A_2 = w_p/\phi_l$ (i) respectively. And in (iii) need to account for incentive compatibility so that LP does not want to mimic the high types.

2. Either incentive compatibility for LR binds (case (b)) or resources constraint binds for LR (case (a)), for any possible situation on LP incentive compatibility constraints.

3. And need to check that HL has enough resources to submit A_1 that satisfies all the incentive compatibility constraints described above.

At **Alternative B**, HR agents separate themselves, HP and LR pool and LP do not apply, i.e. HP and LR that are not separable when $w_r/\phi_l \geqslant w_p/\phi_h$ are pooling, but HR can separate themselves. Alternative B is only feasible when $\sigma\beta < \alpha < \beta + \sigma(1-\beta)$. If $\alpha > \beta + \sigma(1-\beta)$, A_1 and A_2 should award both projects with probability one, and no agent would be willing to send A_1. For $\alpha < \sigma\beta$, A_2 and A_3 would give the same zero probability of being awarded the project and no agent would present the more expensive higher quality application A_2. Let the beliefs and probabilities of being awarded the project be such that

$$\mu(A_1) = 1 \rightarrow p(A_1) = 1 \tag{8}$$

$$\mu(A_2) = \frac{\beta(1-\sigma)}{\beta + \sigma - 2\beta\sigma} \rightarrow p(A_2) = \frac{\alpha - \beta\sigma}{\beta + \sigma - 2\beta\sigma} \tag{9}$$

$$\mu(A_3) = 0 \rightarrow p(A_3) = 0 \tag{10}$$

i.e. a project is awarded to all Applications with Appendix, that are send by HR, and the remaining projects are awarded randomly among the regular applicants (HP and LR).

Proposition 7 (Partially Separating Equilibrium Alt. B) *In a partially separating equilibrium Alt.B, HP send a signal smaller or equal than in the separating equilibrium and is awarded the project with smaller probability when $\alpha < 1/2$, and $\beta < 1/2$. For $\alpha < \beta$, probability of being awarded the project for a HP is greater in the separating equilibrium whenever $\alpha > \beta^2/(2\beta - 1)$. For $\alpha > \beta$, probability of being awarded the project for a HP is smaller than in the separating equilibrium whenever $\sigma > 1/2\beta$.*

Let A_1, A_2 and A_3 be the application standards and let $p(A_1)$, $p(A_2)$ and $p(A_3)$ be as in (8), (9) and (10). Assume $\sigma\beta < \alpha < \beta + \sigma(1-\beta)$ and $w_p \geqslant V_l$

i. For $V_l > w_p \geqslant p(A_2)V_l$ and $\min(\frac{w_r}{\phi_l}, \frac{w_p}{\phi_h}) \geqslant \frac{V_h(1-p(A_2))}{\phi_h} + p(A_2)V_l$, and for $w_p \geqslant V_l$ and $\min(\frac{w_r}{\phi_l}, \frac{w_p}{\phi_h}) \geqslant \frac{V_h(1-p(A_2))}{\phi_h} + p(A_2)V_l \geqslant V_l$, partially separating equilibrium is given by

$$A_1 = \frac{V_h(1-p(A_2))}{\phi_h} + V_l p(A_2),\ A_2 = V_l p(A_2),\ A_3 = 0$$

ii. For $w_p < p(A_2)V_l$, and $\min(\frac{w_r}{\phi_l}, \frac{w_p}{\phi_h}) \geqslant \frac{V_h(1- p(A_2))}{\phi} + \frac{w_p}{\phi_l}$,partially separating equilibrium is given by

$$A_1 = \frac{V_h(1 - p(A_2))}{\phi_h} + \frac{w_p}{\phi_l},\ A_2 = w_p/\phi_l,\ A_3 = 0$$

iii. For $C_l > w_p \geqslant p(A_2)V_l$ and $\min(\frac{w_r}{\phi_l}, \frac{w_p}{\phi_l}) < \frac{V_h(1- p(A_2))}{\phi_h} + p(A_2)V_l$, and for $w_p \geqslant V_l$ and $\min(\frac{w_r}{\phi_l}, \frac{w_p}{\phi_l}) < \frac{V_h(1- p(A_2))}{\phi_h} + p(A_2)V_l$,partially separating equilibrium is given by

$$A_1 = \max(\frac{w_r}{\phi_l}, \frac{w_p}{\phi_h}),\ A_2 = V_l p(A_2),\ A_3 = 0$$

iv. For $w_p < p(A_2)V_l$, and $\min(\frac{w_r}{\phi_l}, \frac{w_p}{\phi_h}) < \frac{V_h(1- p(A_2))}{\phi_h} + w_p$,partially separating equilibrium is given by

$$A_1 = \max(\frac{w_r}{\phi_l}, \frac{w_p}{\phi_h}),\ A_2 = \frac{w_p}{\phi_l},\ A_3 = 0$$

Proof is provided in Appendix B. Comparing A_2 to the equilibrium application on the separating equilibrium we obtain the values of the parameters for which a partially separating in beneficial for the HP agents.

At **Alternative C**, HR separate from HP, and LR and LP pool. This can only be an equilibrium when $\sigma\beta < \alpha < \beta$, since otherwise there can not be separation between HR and HP. But we can not find consistent beliefs according to Belief Action Parity: both the signals send by HR and the send by HP would have the same prior probability of being send by a needy agent, and so should be awarded the project with the same probability. The only alternative would be to allow high types to choose between high signal and getting the project for sure, or a smaller signal and getting the project with smaller probability. That would leave same condition as the Incentive Compatibility constraints of Alt.B, but at the moment of separating HP from LR, the same problems as in a separating equilibrium with $\alpha < \beta$ would appear. Alternative C would only imply greater application costs and bias allocation of projects to the rich among the needy agents, with no improvement on the wealth distributions where separation of needs can be implemented.

4 Social Welfare Maximization

Let us define the Social Welfare Function as the sum of the agents' utility from the projects minus application costs. The principal's objective is to maximize social welfare subject to his budget constraint and to the existence of a separating equilibrium in needs, i.e. maximize the matching of needs and projects allocation subject to application and in-kind award costs, project costs and budget constraint, and to the constraints on the existence of a separating equilibria.

4.1 Principal's budget constraint:

We can distinguish four parts on the principal's cost:

1. Unit cost of providing the project, that we denote by C.
2. Cost of evaluating the applications, $E(a,\phi)$, that depends both on the size of the applications received a and on the complexity of the application ϕ[6]. We assume $E_a(a,\phi) > 0$ and $E_{\phi\phi}(a,\phi) > 0$, $E_{aa}(a,\phi) > 0$ and $E_\phi(a,\phi) < 0$, higher quality and more complex applications have higher evaluation costs, and $E_{a\phi}(a,\phi) < 0$, marginal cost of evaluating a bigger application is decreasing with the application complexity ϕ.
3. The cost of designing the application procedure so that marginal cost of the application is lower for needy agents, $G(\phi_l - \phi_h)$, that is increasing and convex function on the difference $(\phi_l - \phi_h)$. This cost can also be interpreted as the cost of performing more random evaluations.
4. Cost of in-kind aid provision to reduce the value of the project for the non-needy agents, $F(V_h - V_l)$.This cost is for each of the projects awarded since all of them receives same share in-kind. $F()$ is an increasing and concave function of the in-kind share provided.

Without loss of generality we set $\phi_l = 1$ and assume V_h is independent of the in-kind share of the project. We assume V_l has a lower bound $V_{min} > 0$ to ensure there is some interest from the non-needy agents to apply even when the project is totally provided in-kind. The principal chooses $(\alpha^*, \phi_h^*, V_l^*)$, i.e. size of the application, complexity and share of in-kind award of the project, that maximize the Social Welfare Function given his budget constraint.

Let N denote the type of the agent $(N = h, l)$ and let $f(w, N)$ be the joint distribution function of capacity and needs. The Social Welfare function has the form:

$$W = \int_N \int_w \left[p(a^*(w,N))V(N) + (w - \phi(N)a^*(w,N))\right] f(w,N)dwdN \tag{11}$$

and the principal's budget constraint is given by

$$B = \int_N \int_w E(a^*(w,N), \phi(N)) f(w,N)dwdN + \alpha\left[C + F(V_h - V_l)\right] + G(\phi_l - \phi_h) \tag{12}$$

The principal's budget constraint determines the number of projects that is feasible to award to maximize Welfare given the technology available to evaluate applications and to set ϕ_h and V_l. For each set of parameters, we define budgets $\hat{B}$ and $\check{B}$ as the smallest budgets that maximize welfare providing $\hat{\alpha} = (w_r\beta)/V_H$ and $\check{\alpha} = (w_r\beta)/V_{\min}$ projects respectively (i.e. incentive compatibility bins). And we define $\tilde{B}$ as the minimum budget that provides $\alpha^* = \beta$ projects.

[6] Following the example of the student's term paper, the cost of grading increases with the number of pages of the paper and with the dificulty.

4.2 Existence of a separating equilibrium in needs constraint:

Whenever LR resources constraint binds, $w_r < \frac{\alpha V_l}{\beta}$, to ensure that LR are not able to mimic HP's signal, we need the additional constraint $\phi_h \leqslant w_p/w_r$. When is LR incentive compatibility constraint that binds, $w_r > \frac{\alpha V_l}{\beta}$ we need to make sure HP are able to send the signal so $\phi_h \leqslant w_p\beta/\alpha V_l$ is required.

4.3 Optimal application rules and in-kind share of projects:

Let us now look at the optimal (ϕ_h, V_l, α) triplet for each range of budgets. Let $\bar{w}$ be average capacity in the applicant pool. When a separating equilibrium exists, $(W - \bar{w})$ is independent of the existence of variability in capacity for $w_r = w$.

Claim 8 *Threshold Budget $\hat{B}$ ($\check{B}$), defined as the smallest budget to provide $\hat{\alpha} = (w_r\beta)/V_H$ ($\check{\alpha} = (w_r\beta)/V_{\min}$) projects, is increasing in the unit cost of the project and in the share of needy agents. It is independent of share of poor agents, non-increasing in lowest capacity level and increasing in highest capacity level.*

Proof. Budget $\hat{B}$ ($\check{B}$) is defined as the utility maximizing cost to provide $\hat{\alpha} = (w_r\beta)/V_H$ ($\check{\alpha} = (w_r\beta)/V_{\min}$). At $\hat{\alpha}$ ($\check{\alpha}$) incentive compatibility for LR binds, and $a^* = w_r = \frac{\hat{\alpha}V_h}{\beta}$ ($a^* = w_r = \frac{\check{\alpha}V_{\min}}{\beta}$). For a separating equilibrium to exist, we need $\phi_h^* \leqslant w_p/w_r$, Budget is given by

$$\begin{aligned}
\hat{B} &= \beta E(w_r, \phi_h^*) + \frac{w_r\beta}{V_H}C + G(1-\phi_h^*) \\
\check{B} &= \beta E(w_r, \phi_h^*) + \frac{w_r\beta}{V_{\min}}C + G(1-\phi_h^*) + F(V_h - V_{\min})
\end{aligned}$$

that is increasing in β and C, increasing in w_r and non-increasing in w_p, given ϕ_h^* that is implicitly defined by

$$-\frac{V_h}{C}\left[\beta E_{\phi_h}(w_r, \phi_h^*) - G'(1-\phi_h^*)\right] = \gamma_1 + \beta w_r$$

Comparing both expressions we find that $\hat{B} < \check{B}$. ■

Proposition 9 *For $\tilde{B} > B > \hat{B}$, $w = w_r = a^*$, $V_l^* = V_h$. There exists a budget level $B_w \geqslant \hat{B}$ such that ϕ_h^* is independent of projects awarded and percentage of rich agents for all greater budgets. Share of budget spend in designing the procedure decreases with budget and number of projects awarded.*

Proof. For $B < \tilde{B}$, $\alpha < \beta$, and for $B > \hat{B}$, $\hat{\alpha} > (w_r\beta)/V_H$ and Incentive compatibility LR does not bind, $w_r < \frac{\alpha V_h}{\beta}$ and $w = w_r = a^*$. This is the cheapest signal, and only needy agents are served. The principal's maximization problem is

$$\max_{\phi_h, V_l, \alpha} W - \bar{w} = \alpha V_h - \phi_h \beta w_r$$

$$s.t.\ B = \beta E(w_r, \phi_h) + \alpha \left[C + F(V_h - V_l)\right] + G(1 - \phi_h) \quad (\lambda) \tag{13}$$

$$\phi_h \leqslant w_p / w_r \ (\gamma_1) \tag{14}$$

$$w_r < \frac{\alpha V_h}{\beta} \ (\gamma_2) \tag{15}$$

We see that V_l does not appear in $(W - \bar{w})$ and in the application cost, so $V_l^* = V_h$. The First Order Conditions are:

$$-\beta w_r - \lambda \left[\beta E_{\phi_h}(w_r, \phi_h) - G'(1 - \phi_h)\right] - \gamma_1 = 0$$

$$V_h - \lambda C + \gamma_2 \frac{V_h}{\beta} = 0$$

$$\rightarrow \text{either } \gamma_2 = 0 \text{ and } \lambda = \frac{V_h}{C} > 0$$

$$\rightarrow \text{ or } \gamma_2 > 0,\ \alpha^* = \frac{\beta w_r}{V_h} \text{ and } \phi_h^* \text{ determined by (13)}$$

For the first case, we get that (ϕ_h^*, α^*) are given by (16) and (17) whenever $\phi_h^* \leqslant w_p / w_r$

$$-\frac{V_h}{C}\left[\beta E_{\phi_h}(w_r, \phi_h^*) - G'(1 - \phi_h^*)\right] = \gamma_1 + \beta w_r \tag{16}$$

$$\alpha^* = \frac{B - \beta E(w_r, \phi_h^*) - G(1 - \phi_h^*)}{C} \tag{17}$$

and $\phi_h^* = w_p / w_r$ when (14) binds. From (16) we find that ϕ_h^* is independent of the number of projects awarded and independent on the percentage of rich agents in the population. Marginal application cost for high types, ϕ_h^*, is non-decreasing with poor agent's wealth and decreasing with the cost of the project C. From (16) and (17) together, we find that number of projects awarded increases with ϕ_h^* and decreases with the project cost C. Since ϕ_h^* is independent of α, the share of resources spend on project design and evaluation is independent of the number of projects awarded: the more resources the more projects awarded when $[\beta E(w_r, \phi_h^*) + G(1 - \phi_h^*)]$ has been covered.

For $\gamma_2 > 0$ to hold, it should be the case that it is optimal to spend the additional funds in decreasing ϕ_h instead of awarding more projects. That could only be the case if

$$0 < V_h - \beta w_r < \lambda \left[C + \beta E_{\phi_h}(w_r, \phi_h) - G'(1 - \phi_h)\right]$$

and since $\left[\beta E_{\phi_h}(w_r, \phi_h^*) - G'(1 - \phi_h^*)\right]$ is decreasing in ϕ_h, there exists a budget $B_w \geqslant \hat{B}$ such that ϕ_h^* is given by (16) and (15) does not bind. ∎

Proposition 10 *For $B < \hat{B}$, $a^* = \frac{\alpha V_l}{\beta}$. and either $V_l^* = V_{\min}$ or $E_a(\frac{\alpha V_h}{\beta}, \phi_h) < F'(0)$ and $V_l^* = V_h$. Number of projects awarded is bounded above by $\hat{\alpha} = (w_r \beta)/V_h$ or $\check{\alpha} = (w_r \beta)/V_{\min}$, α^* decreases with ϕ_h^* and V_l^* and increases with B. This implies that expenditure in application design increases with the budget and projects awarded.*

Proof. For $B < \hat{B}$, $\hat{\alpha} < (w_r \beta)/V_H$ what implies $w_r \geqslant \frac{\alpha V_h}{\beta}$, separating application is $a^* = \frac{\alpha V_l}{\beta}$ and Social Welfare is given by

$$\max_{\phi_h, V_l, \alpha} W - \bar{w} = \alpha V_h - \phi_h \alpha V_l$$

$$B = \beta E(\frac{\alpha V_l}{\beta}, \phi_h) + \alpha \left[C + F(V_h - V_l)\right] + G(1 - \phi_h) \ \ (\lambda) \tag{18}$$

$$\phi_h \leqslant \frac{w_p \beta}{\alpha V_l} \ (\gamma_1) \tag{19}$$

$$w_r \geqslant \frac{\alpha V_l}{\beta} \ (\gamma_2) \tag{20}$$

$$V_l \leqslant V_{\min} \ (\rho) \tag{21}$$

Whenever incentive compatibility for LR (20) binds, we need to add the constraint that HP have enough resources to send the signal (19). Moreover, we establish a lower bound for V_l to ensure low type agents would consider to apply. We need to consider four situations depending on constraints (19) , (20) and (21) binding. First Order Conditions are given by:

$$-\alpha V_l - \lambda \left[\beta E_{\phi_h}(\frac{\alpha V_l}{\beta}, \phi_h) - G'(1 - \phi_h)\right] - \gamma_1 = 0 \tag{22}$$

$$-\alpha \phi_h - \lambda \alpha \left[E_a(\frac{\alpha V_l}{\beta}, \phi_h) - F'(V_h - V_l)\right] - \gamma_1 \frac{w_p \beta}{\alpha V_l^2} - \rho - \gamma_2 \frac{\alpha}{\beta} = 0 \tag{23}$$

$$[V_h - \phi_h V_l] - \lambda \left[E_a(\frac{\alpha V_l}{\beta}, \phi_h) V_l + C + F(V_h - V_l)\right] - \gamma_1 \frac{w_p \beta}{\alpha^2 V_l} - \gamma_2 \frac{V_l}{\beta} = 0 \tag{24}$$

We need to consider two cases for (23): whenever $E_a(\frac{\alpha V \min}{\beta}, \phi_h) > F'(V_h - V_{\min})$, this condition is not satisfied, the principal wants to decrease V_l as much as possible, so either (21) binds or $E_a(\frac{\alpha V_{\min}}{\beta}, \phi_h) < F'(V_h - V_{\min})$. We know that $D(V_l) = E_a(\frac{\alpha V_l}{\beta}, \phi_h) - F'(V_h - V_l)$ is increasing in V_l, so as long as $E_a(\frac{\alpha V_{\min}}{\beta}, \phi_h) > F'(0)$, some resources are spend on decreasing V_l.

For any possible combination of constraints (19) , (20) and (21) binding we find that $\alpha \leqslant (w_r \beta)/V_h$. When neither (19) nor (21) bind, putting together the First Order Conditions and taking into account that $\gamma = 0$, we obtain

$$F'(V_h - V_l)\left[V_h - \phi_h V_l\right] - \phi_h \left[C + F(V_h - V_l)\right] = E_a(\frac{\alpha V_l}{\beta}, \phi_h) \tag{25}$$

$$\left[F'(V_h - V_l) - E_a(\frac{\alpha V_l}{\beta}, \phi_h)\right] \alpha V_l = \phi_h \left[G'(1 - \phi_h) - \beta E_{\phi_h}(\frac{\alpha V_l}{\beta}, \phi_h)\right] \tag{26}$$

that with the budget constraint determine $(\alpha^*, \phi_h^*, V_l^*)$. We find that α^* is decreasing in ϕ_h^* and V_l^* and increasing in B. This implies that ϕ_h^* and V_l^* decrease with budget available. ■

Claim 11 *For $B > \tilde{B}$, more projects than needy agents are awarded, and (ϕ_h^*, V_l^*) equal the optimal choices when Incentive Compatibility does not bind for LR agents and $B_w < B < \tilde{B}$,.*

Proof. For $B > \tilde{B}$, $\alpha > \beta$, $w < \frac{(1-\alpha)V_h}{(1-\beta)}$ or $w_r < \frac{(1-\alpha)V_h}{(1-\beta)}$ and $w = w_r = a^*$, Social welfare is given by

$$\max_{\phi_h, V_l, \alpha} W - \bar{w} = \beta V_h + (\alpha - \beta)V_l - \phi_h \beta w_r$$
$$\text{s.t. } B = \beta E(w_r, \phi_h) + \alpha\left[V + F(V_h - V_l)\right] + G(1 - \phi_h)$$
$$\phi_h \leqslant w_p / w_r$$
$$w_r < \frac{(1-\alpha)V_l}{(1-\beta)}$$

We see that both $[W - \bar{w}]$ and resources available increase with V_l, so $V_l^* = V_h$. and ϕ_h^* has same form as when Incentive Compatibility does not bind for LR agents and $\alpha < \beta$ since first order conditions coincide.

To increase budget over the one that covers all needy agents only generates waste of resources since application cost is independent of number of projects awarded for this level of wealth. ■

Summarizing, there exist three budget levels $\hat{B} < \check{B} < \tilde{B}$ and a number of applications awarded $\hat{\alpha} = w_r\beta/V_h < \check{\alpha} = w_r\beta/V_{\min} < \tilde{\alpha} = \beta$ such that:

1. For $B \leqslant \hat{B}$, as budget increases applications awarded increase and ϕ_h is non-increasing. Incentive compatibility for LR binds, either $E_a(\frac{\alpha V_h}{\beta}, \phi_h) < F'(0)$ and $V_l^* = V_h$ or $V_l^* = V_{\min}$ and $a^* = \frac{\alpha^* V_l^*}{\beta} < w_r$

2. For $\hat{B} < B < \check{B}$, either $E_a(\frac{\alpha V_h}{\beta}, \phi_h) < F'(0)$, $V_l^* = V_h$ and Incentive Compatibility for LR does not bind, and application is constant at w_r, or $V_l^* = V_{\min}$ and $a^* = \frac{\alpha^* V_l^*}{\beta} < w_r$

3. For $\tilde{B} > B > \hat{B}$ incentive compatibility for LR does not bind, application is constant at w_r and ϕ_h^* is given by

$$\beta w_r = -\frac{V_h}{C}\left[\beta E_{\phi_h}(w_r, \phi_h^*) - G'(1 - \phi_h^*)\right]$$

whenever $B_w \geqslant \hat{B}$ or by $\phi_h^* = w_p/w_r$ otherwise. For $B_w \geqslant \hat{B}$, share of resources spend on the application design, $\left[\beta E(w_r, \phi_h^*) + G(1 - \phi_h^*)\right]/B$, is decreasing with B, and decreasing in the population's share of needy agent's α.

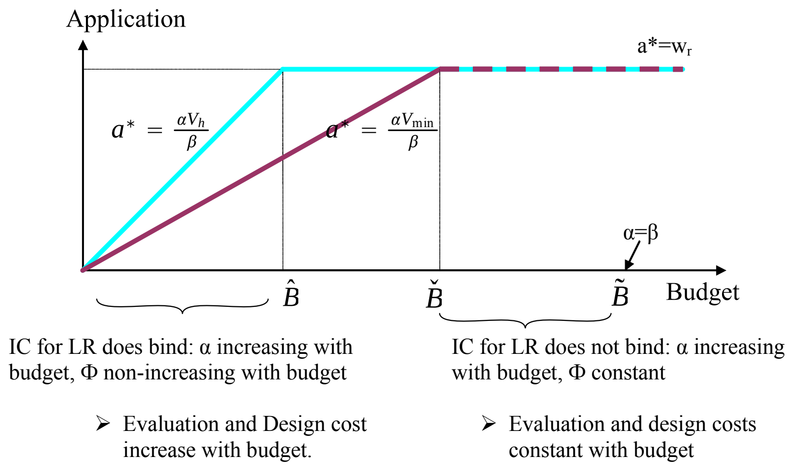

Figure 4

Figure 4 presents the optimal instruments for the principal for different sizes of his budget: there exists a threshold in the donor's budget such that separating signal is increasing in number of projects for budgets below and constant afterwards. For smaller budgets, all instruments are used in the optimal mechanism when costs are low enough. Separating signal and complexity of the application process increase with number of projects awarded and value of the project for non-needy agents. For bigger budgets, separating application is independent of number of projects awarded. Marginal cost of applying for needy agents is independent of the number of projects awarded. Share of resources spend in the mechanism decreases with budget.

When we have capacity unobservability, capacity dispersion affects optimal marginal cost of application and affects budget threshold. But optimal mechanism is independent of the share of high capacity agents in the population. Higher constant capacity and higher capacity for the richest agents implies higher separating signal and less projects awarded for any given donor's budget. Higher difference between agent's capacities imposes stricter constraints on ϕ_h, and so number of projects awarded for a given budget is non-increasing in $(w_r - w_p)$

5 Conclusions

We present a signaling model with two unobservables (need and income) and only one signal, where the principal can choose, at a given cost, the marginal cost of the signal for needy and non-needy agents and the value of the project for non-needy agents.

We find that there is a budget threshold so that signal is increasing for budgets smaller than the treshold and constant afterwards. This implies that resources spend on the design of the mechanism increase with budget until a treshold and are constant afterwards, as is the application cost for needy agents.

We find that it is capacity inequality, the difference in capacity between rich and poor, and not the share of poor agents that has a burden on the project design. The greater inequality, the greater is the minimum application complexity (in terms of marginal cost of application for needy agents) that we need in order to obtain a separating equilibrium.

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A Appendix: Signaling Equilibrium with constant capacity

Proof: Pooling Equilibrium Constant Capacity. Let a^* be the pooling quality of an application, and let the application rules be

$$\begin{aligned} \mu(a) &= 0 \text{ for } a < a^* \rightarrow & p(a) = 0 \\ \mu(a) &= \beta \text{ for } a^* \leqslant a \leqslant w/\phi_l \rightarrow & p(a) = \alpha \\ \mu(a) &= 1 \text{ for } a > w/\phi_l \rightarrow & p(a) = 1 \end{aligned}$$

We need to check that neither high nor low type agents want to deviate from the pooling equilibrium. Incentive compatibility constraints for the High type are given by

$$aV_h + (w - \phi_h a^*) \geqslant w \implies a^* \leqslant \frac{\alpha V_h}{\phi_h} \tag{27}$$

$$\alpha V_h + (w - \phi_h a^*) \geqslant V_h + (w - \phi_h(a^* + \varepsilon_H)) \Longrightarrow \varepsilon_H \geqslant \frac{V_h(1-\alpha)}{\phi_h} \tag{28}$$

where (27) shows maximum needy agents are willing to pay to be in the pool, and (28) shows maximum they are willing to pay over the pooling signal to be identified as a high type. For a^* to be send and (28) to be satisfied, we need that

$$\phi_h \left[\frac{w}{\phi_l} - a^* \right] > V_h(1-\alpha)$$

the extra-payment to signal $a = w/\phi_l$ is greater than what he is willing to pay to separate himself.

For the low type, incentive compatibility constraint is

$$\alpha V_l + (w - \phi_l a^*) \geqslant w \implies a^* \leqslant \frac{\alpha V_l}{\phi_l} \tag{29}$$

Putting together all the incentive compatibility constraints we find that the candidate for a pooling equilibrium should satisfy

$$0 \leqslant a^* \leqslant \min \left(\frac{w}{\phi_l} - \frac{V_h(1-\alpha)}{\phi_h}, \frac{\alpha V_l}{\phi_l} \right)$$

and $a^* = 0$ is a candidate with no resources lost in the application process when $w > \frac{V_h(1-\alpha)\phi_l}{\phi_h}$. For smaller wealth, needy agents always want to separate and there is no pooling equilibrium. ■

Proof: Separating Equilibirum Constant Capacity. Let (a_H^*, a_L^*) be the candidate separating applications for high and low type respectively, and let the application procedure be

$$\begin{aligned}\mu(a) &= 0 \text{ for } a < a_H^* \\ &\rightarrow \quad \tilde{p}(a) = 0 \text{ if } \alpha < \beta \text{ and } p(a) = \frac{\alpha - \beta}{1 - \beta} \text{ if } \alpha > \beta \qquad (30) \\ \mu(a) &= 1 \text{ for } a \geqslant a_H^* \\ &\rightarrow \quad \tilde{p}(a) = \frac{\alpha}{\beta} \text{ if } \alpha < \beta \text{ and } p(a) = 1 \text{ if } \alpha > \beta \qquad (31)\end{aligned}$$

First we need to show that low types are not willing to apply. To prepare any application of quality smaller than a_H^* does not increase their probability of being awarded the project, so choice is between not applying and getting the projects with a probability according to (30) or try to mimic the high type. Incentive compatibility constraint for the low type is given by

$$\tilde{p}(\tilde{a}_H^*)V_l + (w - \phi_l \tilde{a}_H^*) \leqslant w \implies \tilde{a}_H^* \geqslant \frac{\tilde{p}(\tilde{a}_H^*)V_l}{\phi_l} \text{ if } \alpha < \beta \qquad (32)$$

$$p(a_H^*)V_l + (w - \phi_l a_H^*) \leqslant p(0)V_l + w \implies a_H^* \geqslant \frac{[p(a_H^*) - p(0)]\, V_l}{\phi_l} \text{ if } \alpha > \beta \qquad (33)$$

For the high type, he is willing to send a signal a_H^* when

$$\tilde{p}(\tilde{a}_H^*)V_h + (w - \phi_h \tilde{a}_H^*) \geqslant w \implies a_H^* \leqslant \frac{\tilde{p}(\tilde{a}_H^*)V_h}{\phi_h} \text{ if } \alpha < \beta$$

$$p(a_H^*)V_h + (w - \phi_h a_H^*) \geqslant p(0)V_h + w \implies a_H^* \leqslant \frac{[p(a_H^*) - p(0)]\, V_h}{\phi_h} \text{ if } \alpha > \beta$$

For $\alpha < \beta$, $\tilde{a}^* = \alpha V_l / \beta \phi_l$ is the maximum signal low types would send, (32) binds, and is only feasible when

$$\frac{w}{\phi_h} \geqslant \frac{w}{\phi_l} \geqslant \frac{\alpha V_l}{\beta \phi_l}$$

For $w < \alpha V_l / \beta$, incentive compatibility does not bind and separating equilibrium exists for $\tilde{a}^* = w/\phi_l$, since would not be feasible for low types to mimic the high types. Symmetrically, when $\alpha > \beta$, we have that if $w \geqslant \frac{(1-\alpha)V_l}{(1-\beta)}$, $a^* = \frac{(1-\alpha)V_l}{(1-\beta)\phi_l}$ and if $w < \frac{(1-\alpha)V_l}{(1-\beta)}$, $a^* = \frac{w}{\phi_l}$. ■

B Appendix: Signaling Equilibrium with two capacity levels

Proof: Separating Equilibrium Two Capacity Levels. Let (a_H^*, a_L^*) be the candidate separating quality of an application for high and low type respectively, and let the application procedure be

$$\begin{aligned}\mu(a) &= 0 \text{ for } a < a_H^* \\ &\rightarrow \quad \tilde{p}(a) = 0 \text{ if } \alpha < \beta \text{ and } p(a) = \frac{\alpha - \beta}{1 - \beta} \text{ if } \alpha > \beta \qquad (34) \\ \mu(a) &= 1 \text{ for } a \geqslant a_H^* \\ &\rightarrow \quad \tilde{p}(a) = \frac{\alpha}{\beta} \text{ if } \alpha < \beta \text{ and } p(a) = 1 \text{ if } \alpha > \beta \qquad (35)\end{aligned}$$

First we need to show that low types are not willing to apply. To prepare any application of quality smaller than a_H^* does not increase their probability of being awarded the project, so choice is between not applying and getting the projects with a probability according to (34) or try to mimic the high type. Incentive compatibility constraints for non-needy high capacity are

$$\begin{aligned}(LR) \quad w_r &\geqslant \tilde{p}(a_H^*)V_l + (w_r - \phi_l a_H^*) \Longrightarrow a_H^* \geqslant \frac{\tilde{p}(a_H^*)V_l}{\phi_l} \text{ if } \alpha < \beta \qquad (36) \\ p(0)V_l + w_r &\geqslant p(a_H^*)V_l + (w_r - \phi_l a_H^*) \Longrightarrow a_H^* \geqslant \frac{[p(a_H^*) - p(0)]\, V_l}{\phi_l} \text{ if } \alpha > \beta\end{aligned}$$

and for the low capacity not needy (low types) are

$$\begin{aligned}(LP) \quad w_p &\geqslant \tilde{p}(a_H^*)V_l + (w_p - \phi_l a_H^*) \Longrightarrow a_H^* \geqslant \frac{\tilde{p}(a_H^*)V_l}{\phi_l} \text{ if } \alpha < \beta \\ p(0)V_l + w_p &\geqslant p(a_H^*)V_l + (w_p - \phi_l a_H^*) \Longrightarrow a_H^* \geqslant \frac{[p(a_H^*) - p(0)]\, V_l}{\phi_l} \text{ if } \alpha > \beta\end{aligned}$$

Incentive Compatibility constraints for each of the needy agents (high types) are

$$\begin{aligned}(HR) \quad & \tilde{p}(a_H^*)V_h + (w_r - \phi_h a_H^*) \geqslant w_r \Longrightarrow a_H^* \leqslant \frac{\tilde{p}(a_H^*)V_h}{\phi_h} \text{ if } \alpha < \beta \\ & p(a_H^*)V_h + (w_r - \phi_h a_H^*) \geqslant p(0)V_h + w_r \\ & \Longrightarrow a_H^* \leqslant \frac{[p(a_H^*) - p(0)]\, V_h}{\phi_h} \text{ if } \alpha > \beta \\ (HP) \quad & \tilde{p}(a_H^*)V_h + (w_p - \phi_h a_H^*) \geqslant w_p \Longrightarrow a_H^* \leqslant \frac{\tilde{p}(a_H^*)V_h}{\phi_h} \text{ if } \alpha < \beta \\ & p(a_H^*)V_h + (w_p - \phi_h a_H^*) \geqslant p(0)V_h + w_p \\ & \Longrightarrow a_H^* \leqslant \frac{[p(a_H^*) - p(0)]\, V_h}{\phi_h} \text{ if } \alpha > \beta\end{aligned}$$

For $\alpha < \beta$ we need also to check that HR do not want to deviate over HP to signal himself. Given (34) and (35) deviation does not increase probability of getting the project, and this is consistent with the assumption that the principal wants to award projects to needy agents giving preference to the poorest, so to signal wealth on top of need is not on the agent's best interest.

For $\alpha < \beta$, if $\frac{\alpha V_l}{\beta \phi_l} \leqslant \min(\frac{w_r}{\phi_l}, \frac{w_p}{\phi_h})$, incentive compatibility constraint (36) binds and $\tilde{a}^* = \frac{\alpha V_l}{\beta \phi_l}$ is the separating equilibrium. Whenever $\frac{w_p}{\phi_h} < \frac{w_r}{\phi_l} < \frac{\alpha V_l}{\beta \phi_l}$, incentive compatibility binds but HP can not send the signal $\tilde{a}^* = \frac{\alpha V_l}{\beta \phi_l}$ so no separating equilibrium exists since LR would be able to imitate all HP signals. If $\frac{w_p}{\phi_h} > \frac{\alpha V_l}{\beta \phi_l} > \frac{w_r}{\phi_l}$, (36) can not bind, so $\tilde{a}^* = \frac{w_r}{\phi_l} < \frac{w_p}{\phi_h}$. Whenever $\frac{w_p}{\phi_h} < \frac{w_r}{\phi_l} < \frac{\alpha V_l}{\beta \phi_l}$, incentive compatibility does not bind but HP can not send signal $\tilde{a}^* = \frac{w_r}{\phi_l}$, so no separating equilibrium exists. For $\alpha > \beta$, the argument is symmetric for $a^* = \frac{(1-\alpha)V_l}{(1-\beta)\phi_l}$. ■

Proof: Partially Separating Alt. A. We want to show that (A_1, A_2, A_3) with $(p(A_1), p(A_2), p(A_3))$ is an equilibrium of the application game. The smallest application does not get the project, so the application is the minimum one, $A_3 = 0$. For the pooling types, HR and HP, Incentive compatibility constraints are given by

$$\begin{aligned} p(A_2)V_h + (w_r - \phi_h A_2) &\leqslant V_h + (w_r - \phi_h A_1) \\ p(A_2)V_h + (w_p - \phi_h A_2) &\leqslant V_h + (w_p - \phi_h A_1) \\ \Longrightarrow \phi_h(A_1 - A_2) &\leqslant V_h(1 - \ p(A_2)) \end{aligned} \tag{37}$$

and for the LR it is given by

$$\begin{aligned} p(A_2)V_l + (w_r - \phi_l A_2) &\geqslant V_l + (w_r - \phi_l A_1) \\ \Longrightarrow \phi_l(A_1 - A_2) &\geqslant V_l(1 - \ p(A_2)) \end{aligned} \tag{38}$$

Putting together (37) and (38) we get that

$$A_1 \geqslant \frac{(1 - p(A_2))V_l}{\phi_l} + A_2$$

Incentive compatibility for LP have the form

$$\begin{aligned} p(A_2)V_l + (w_p - \phi_l A_2) &\leqslant w_p \\ \Longrightarrow A_2 &\geqslant \frac{p(A_2)V_l}{\phi_l} \end{aligned} \tag{39}$$

$$\begin{aligned} V_l + (w_r - \phi_l A_1) &\leqslant w_r \\ \Longrightarrow A_1 &\geqslant \frac{V_l}{\phi_l} \end{aligned} \tag{40}$$

To derive the equilibrium, we need to consider when incentive constraints can bind given the resources. For LP, (39) binds when $w_p \geqslant p(A_2)V_l$. Otherwise, $A_2 = w_p/\phi_l$.Given A_2, we obtain A_1 from (38) and (40) as long as resources for LR allow for (38) to bind. Otherwise, $A_1 = w_r/\phi_l$ as long as $w_r/\phi_l \leqslant w_p/\phi_h$ and (40) are satisfied. ■

Proof: Partially Separating Equilibrium Alt. B. We want to show that (A_1, A_2, A_3) with $(p(A_1), p(A_2), p(A_3))$ is an equilibrium of the application game. The smallest application does not get the project, so the application is the minimum one, $A_3 = 0$.

For the HR applicants, incentive compatibility and resources constraints are:

$$V_h + (w_r - \phi_h A_1) \geqslant w_r \Longrightarrow A_1 \leqslant \frac{V_h}{\phi_h}$$
$$V_h + (w_r - \phi_h A_1) \geqslant\ p(A_2)V_h + (w_r - \phi_h A_2)$$
$$\Longrightarrow V_h(1 -\ p(A_2)) \geqslant \phi_h(A_1 - A_2) \tag{41}$$
$$A_1 \leqslant \min(\frac{V_h}{\phi_h}, \frac{w_r}{\phi_h}) \tag{42}$$

for the HP, we have

$$p(A_2)V_h + (w_p - \phi_h A_2) \geqslant w_p \Longrightarrow A_2 \leqslant \frac{p(A_2)V_h}{\phi_h}$$
$$p(A_2)V_h + (w_p - \phi_h A_2) \geqslant V_h + (w_p - \phi_h A_1)$$
$$\Longrightarrow \phi_h(A_1 - A_2) \geqslant V_h(1 -\ p(A_2)) \tag{43}$$
$$A_2 \leqslant \min(\frac{P(A_2)V_h}{\phi_h}, \frac{w_p}{\phi_h}) \tag{44}$$

Equations (42) and (44) show the resources constraints: agents are constrained by income and ability on the set of applications they can present. For the LR agents, the Incentive compatibility and resources constraints have the form:

$$p(A_2)V_l + (w_r - \phi_l A_2) \geqslant w_r \Longrightarrow A_2 \leqslant \frac{p(A_2)V_l}{\phi_l}$$
$$p(A_2)V_l + (w_r - \phi_l A_2) \geqslant V_l + (w_r - \phi_l A_1) \tag{45}$$
$$\Longrightarrow \phi_l(A_1 - A_2) \geqslant V_l(1 -\ p(A_2)) \tag{46}$$
$$A_2 \leqslant \min(\frac{P(A_2)V_l}{\phi_l}, \frac{w_r}{\phi_l}) \tag{47}$$

and for the LP agents,

$$w_p \geqslant C_l + (w_p - \phi_l A_1) \Longrightarrow A_1 \geqslant \frac{C_l}{\phi_l} \tag{48}$$
$$w_p \geqslant p(A_2)C_l + (w_p - \phi_l A_2) \Longrightarrow A_2 \geqslant \frac{p(A_2)C_l}{\phi_l} \tag{49}$$

We need to obtain the values of A_1 and A_2 that together with $p(A_1)$ and $p(A_2)$ form a signaling separating equilibrium where rich agents that value the project send A_1, and rich agents that do not value the project and poor agents that value it pool at A_2. Putting together (47) and (49) we obtain that both constraints can only be satisfied if both bind,

$$V_h(1 -\ p(A_2)) = \phi_h(A_1 - A_2)$$
$$A_1 = \frac{V_h(1 -\ p(A_2))}{\phi_h} + A_2 \tag{50}$$

constraint (44) is satisfied by A_1 and A_2 from (50). From (42) and (44) we find that for $w_p \geqslant V_l$ and $\min(\frac{w_r}{\phi_r}, \frac{w_p}{\phi_l}) \geqslant \frac{V_h(1-p(A_2))}{\phi_h} + p(A_2)V_l \geqslant V_l$, and for $V_l > w_p \geqslant p(A_2)V_l$ and $\min(\frac{w_r}{\phi_r}, \frac{w_p}{\phi_l}) \geqslant \frac{V_h(1-p(A_2))}{\phi} + p(A_2)V_l$, $A_2 = p(A_2)V_l$ and A_1 given by (50) are an equilibrium. For $w_p < p(A_2)V_l$, LP will not mimic any A_1 and A_2 greater than w_p/ϕ_l, so $A_2 = w_p/\phi_l$, and A_1 given by (50) are an equilibrium as long as $\max(\frac{w_r}{\phi_r}, \frac{w_p}{\phi_l}) \geqslant \frac{V_h(1-p(A_2))}{\phi_h} + w_p$. Otherwise, LR and HP can not mimic HR, incentive compatibility constraints (42) and (44) can not bind given the resources, and $A_1 = \max(\frac{w_r}{\phi_l}, \frac{w_p}{\phi_h})$ is enough for HR to separate themselves. ■