Abstract

We analyse the impact of the structure of meritocratic education systems upon the emergence and self-reproduction of the elite. We build an intergenerational model in which tertiary education comprises elite and standard universities, with different expenditure per student and selection patterns. Even if education differences between families are initially low, this system generates an enlargement of these differences resulting in the emergence of an elite group that is to a large extent self-reproducing. The higher the relative funding of elite universities, the higher the elite self-reproduction, and the lower social mobility. The increases in the shares of the middle class in the population augment social mobility for the generation when they occur but lessen mobility for the subsequent generations. These findings provide explanations for several observed facts and a theoretical basis for the ‘Great Gatsby Curve’.

Keywords: Education, Elite, Selection, Social mobility, Social stratification.

1. Introduction

We analyse the impact of the structure of tertiary education, particularly the distinction between elite and standard universities and the differences in their selection procedures, upon social stratification and social mobility at the top, i.e., between the middle class and the ‘elite’.

Since World War II, education levels and education systems have known dramatic changes in advanced economies. These changes have critically modified both the social and the educational structure of these societies.

After the war, advanced economies have firstly experienced a major transformation of their education systems that has led to mass tertiary education. The increase in the minimum dropout age, the multiplication of universities and tertiary establishments, and the increase in the public funding allocated to education have participated in this change. In most advanced economies, the share in a generation of the individuals pursuing tertiary education has shifted from less that 10% in the decade following the war to 60% and more in the present period. This increase began in the late fifties / early sixties in the US and in the sixties in Western European countries, and it carried on during the sixties and seventies. A new increase occurred in the last two decades. In most countries, mass tertiary education has however come with the emergence of a dual system characterised by the concomitance of both standard and elite universities. Standard universities have been opened to a large public of students and their selection has been rather smooth. In contrast, ‘elite’ universities have recruited a narrow number of students with a very severe selection to entry. Albeit different in their strictness, both selection procedures are ‘meritocratic’ in the sense that they intend to select students according to their ability and excellence, and not to their social origin. In addition, the expenditure per student has always been critically higher in the elite universities compared to the standard ones, and expenditure per student has decreased with mass tertiary education in a majority of countries.

The changes in education have been accompanied by substantial modifications in the social structure. In most advanced countries, the shares of primary sector and of blue collars in the working population have decreased, and those of white collars, middle skilled and high skilled positions have increased. The weight of the middle class has critically risen in the employed population. This rise has been significant in the sixties and seventies. As a consequence, the households in the middle class have known a noteworthy improvement of their position. The sixties and seventies have seen both children from the lower classes to upgrade their skill and
join the middle class, and children from the middle class to join the upper class. These changes created a general optimism because all the component of the society could expect to go up the social ladder and benefit from an improvement of their conditions. This feeling was reinforced by the continuous decrease in inequality experienced by all advanced countries in the sixties and seventies. From the mid-fifties to the mid-seventies in the US and the mid-eighties in most Western European countries, the top income share (top 10%, 1% and 0.1%) knew a significant downward shift. The richest accepted this loss because the continuous increase in total income made their income to grow as well. From the eighties, this picture has however radically changed. Firstly, the downward trend in inequality has known a spectacular reversal and inequality has increased, sometimes critically, in all advanced countries. The most striking change concerns the top incomes. From 1980 to 2007, the share of the top 1% in total income shifted from 8.2% to 18.3% in the US, 6.6 to 15.4% in the UK, 8.1 to 14.6% in Canada, 7.6 to 9% in France and 4.5 to 6.9% in Sweden (World Top Income database, http://g-mond.parisschoolofeconomics.eu/topincomes/). In the US, the top 1% captured 58% of the increase in total income from 1976 to 2007, and this percentage attains 65% over the period 2002-2007 (Piketty and Saez, 2003; series updated to 2010 by the authors). As regards the US, Haskel et al. (2012) even show that from 1991 to 2010, (i) there were little differences in income growth across the different skill groups, (ii) the increase in real income is almost fully concentrated in the very top of the income distribution, and (iii) both the low skill and very high skill did better than the medium skill. This provides a picture in which the middle class does not benefits from the continuous increase in the education level, whereas a very narrow number of very high incomes receive most of the income growth. The worsening of the position of the middle class and the huge improvement of the elite’s position could be considered acceptable if, in the same time, social mobility had increased so that the probability to go up the social ladder and enter the club of these happy few had risen. If it is too early to have a definite diagnosis on this point, several works suggest that, quite the opposite, social mobility tends to decrease and the self-reproduction of the elite to increase (Machin, 2007, for the UK; Cahuc et al., 2011, and BenHalima et al., 2013, for France). Based on data from Corak (2012 and 2013) that reveal a positive relationship between income inequality (measured by the Gini coefficient) and social immobility (measured by the intergenerational earnings elasticity) in a cross sections of countries, the figure that links inequality to intergenerational mobility was called by Alan
Krueger (2012) ‘the Great Gatsby curve’. If true, this curve predicts that the rise in inequality observed in most advanced economies should come with lower social mobility.

In this paper, we develop an intergenerational model in which the division of tertiary education between standard and elite universities generate a social stratification in which the upper class (the ‘elite’) is to a large extent self-reproducing. After basic education, an individual characterised by its personal ability and family backgrounds can enter a standard university if s/he possesses a minimum skill level, and an elite university if s/he belongs to the best pupils. The barrier to enter the elite university is much higher, but both selection processes are meritocratic in the sense that they are based on ability and cultural achievement, and not on social origins. We show that such a system divides the society in several groups depending on their degree course. The mobility between the middle class (persons graduated from the standard university) and the elite (graduated from the elite university) depends on the difference in expenditure per student between the two types of tertiary establishments. The higher this difference, the lower social mobility. This comes from the fact that belonging to the narrow percentage of the best pupils accepted in the elite university depends on two factors, i.e., personal ability and intra-family transferred skills. Thus, the most able children from the middle class can overtake the least able children from the elite in spite of higher family-transferred cultural backgrounds of the latter. The increase in expenditure per student in the elite university to the detriment of standard ones increases the skill level of the elite families compared to the middle class, which in turn augments the weight of the cultural background transferred inside the family compared to the impact of personal ability. Another key finding is that an increase in the share of the middle class in the population increases social mobility in the generation when it occurs, but it lowers the upward mobility of the middle class later on. This is because the middle class enlargement firstly necessitates the passage of young from the lower to the middle class. But it afterwards increases the weight of the middle class children, reducing thereby their probability to join the elite. These findings offer a rather lightening reading of certain changes in social mobility between the middle class and the elite since World War 2. They also provide an explanation for the ‘Great Gatsby curve’.

The analysis is carried out from an intergenerational modelling of the impact of the educational system. In the economic literature, the intergenerational analysis of education and human capital formation and of their impact upon social stratification has known a large
development since the pioneering work of Becker and Tomes (1979)\textsuperscript{1}. Within intergenerational models in which human capital is accumulated through education, several factors can generate a lasting division of the society between high skilled and low skilled individuals: a fixed cost of education with a credit constraint (Galor and Zeira, 1993; Barham et al., 1995); a $S$-shaped education function (Galor and Tsiddon, 1997b); local externalities and neighbourhood effects (Benabou, 1993, 1994, 1996; Durlauf, 1994, 1996). The effects of the structure of educational systems upon social polarization, and particularly the influence of public expenditure for primary, secondary and higher education, were modelled and analysed by Driskill and Horowitz (2002), Su (2004), Bertocchi and Spagat (2004) and Chusseau and Hellier (2011). From a partition between basic and secondary education, the latter being divided into vocational and general studies, Bertocchi and Spagat (2004) propose a model that determines social stratification at the different stages of economic development. Chusseau and Hellier (2011) build an intergenerational model with three educational cycles (compulsory basic education, vocational studies and university, with a selection to enter the latter) that can generate very different social stratifications depending (i) on the public funding allocated to each cycle, (ii) on the initial distribution of human capital across dynasties, and (iii) on the severity of the selection procedure. However, even if they generate social stratification, these approaches are not centred on the analysis of the elite and mobility at the top.

In the empirical literature, intergenerational social mobility has been estimated by the impact of the income and education of one generation (the parents) upon the income and education of the next (the children). A high impact indicates a low intergenerational mobility. This abundant literature reveals the high and significant impact of family backgrounds upon educational achievement and income. However, intergenerational mobility critically differs across advanced countries, the lowest mobility being found in the US, followed by the UK and Continental Europe, Nordic countries displaying the highest mobility (Chusseau and Hellier, 2013, for a review).

The economic analysis of the elite is still limited (Brezis and Temin, 2008, for a review). This literature has focused on the impact of (i) the recruitment and the training of elites, and (ii) the influence of the elites’ networks upon economic growth, social mobility and inequality. Brezis and Temin (2008) point to the fact that globalization tends to a uniformity of the elites at the

\textsuperscript{1} Chusseau and Hellier (2013) for a review.
international level. This uniformity could foster social immobility and inequality between the elite and other social groups (Etzioni-Halevy, 1997). It also tends to promote uneven access to education by slowing down the investment in human capital of the majority if it reinforces the self-reproduction of the elite (Easterly, 2001). The distinction between high-quality universities (elite universities) and standard universities reinforces and perpetuates self-reproduction of the elite, even if the selection process is based on meritocratic entrance exams (Arrow et al., 2000; Brezis and Crouzet, 2006).

2. The model

We consider successive generations. Each individual has one child and a dynasty comprises the successive generations linked by a parent-child relationship. The number of dynasties is constant and normalised to 1. So, the number of individuals inside a social group represents the share of this group in the total population. The individual of the $t$-th generation of dynasty $i$ is called *individual* $(i,t)$, and her/his human capital (skill) at the end of education time is denoted $h_{it}$.

When born, the individual is endowed with a minimal skill $l$. When young, s/he receives education. The education activity produces cognitive as well as non-cognitive skills. Education comprises two phases, i.e., basic and higher education. All individuals firstly follow the compulsory basic education. At the end of basic education, individual $(i,t)$ has the skill level $h_{it}^B$ and s/he can either join directly the labour market or enter the university. This can be the standard university $S$ or the elite university $E$.

Both basic and higher education are freely provided by the government. This assumption makes it possible to focus on the sole family-related and personal abilities as determining the differences between individuals.

To enter the standard university ($S$), the individual must possess the minimum skill $\hat{h}$ at the end of basic education. To enter the elite university ($E$), s/he must belong to the top proportion $\alpha$ in terms of skill at the end of basic education.

An individual's education depends on three determinants:

1) The real public expenditure per pupil (or per student) in the education cycle s/he follows, i.e. in basic education and in the standard and the elite universities.
2) Her/his family-related ability that displays the impact of intra-family externalities. These externalities act through several channels: the intra-family direct transmission of human capital; the intra-family transmission of capacity to learn, i.e., capacity of analysis, capacity to organise the studying activity etc.; information about the education system, the education strategies etc. All these channels improve the catching of the education provided for by the government. The intra-family externalities are directly linked to the parent's skill.

3) The individual's personal innate ability $$a_{it}$$, which is randomly distributed across individuals within each generation and is independent from family backgrounds. We assume that personal ability belongs to the segment $$[a, \bar{a}] \subset \mathbb{R}$$, and that abilities are equiprobable in this segment.

In addition, the individual's performance in higher education depends on her/his human capital level at the conclusion of basic education.

The individual firstly maximises her/his lifetime labour income, and s/he subsequently allows this income for the different possible allocations according to her/his utility function. Labour income is increasing with the individual’s skill level. This level is higher when having a university degree than when having basic education only, and when having an elite university degree than a standard university degree (which is shown hereafter). Hence, individuals always prefer to enter the university than to remain at the basic education level, and they all prefer to enter an elite university than a standard one. The individual’s choice is depicted in Figure 1.

![Figure 1. The individuals’ choice](image)
2.1. Basic Education

Individual \((i,t)\)'s skill at the end of basic education \(h_{it}^B\) depends on public expenditure per pupil in basic education, on her/his parent’s human capital that measures the intra-family externalities and on her/his personal ability. The basic education function is:

\[
h_{it}^B = \delta_B a_{it} (h_{it-1})^\eta
\]

where \(0 < \eta < 1\), \(a_{it} \in [\underline{a}, \bar{a}]\) is individual \((i,t)\)'s personal ability, \(h_{it-1}\) her/his parent’s skill and \(\delta_B > 0\) depicts the positive impact of public expenditure per pupil in basic education \(g_B\) upon the educational achievement: \(\delta_B = \delta_B(g_B)\), with \(\partial \delta_B / \partial g_B > 0\).

We assume \(\delta_B > l^{1-\eta} / a\), which ensures that the skill attainment at the end of basic education is higher than when the individual was born.

Without loss of generality in what follows, we assume that \(\delta_B = 1\) at the initial time and as long as the basic education policy is not modified. We consequently write:

\[
h_{it}^B = a_{it} (h_{it-1})^\eta
\]

2.2. Higher Education

The higher education functions (in standard and elite universities) depend on the individual's human capital at the end of basic education and on public expenditure per student in each type of university. We assume identical functions for both \(S\) and \(E\). Thus, the difference in education efficiency between the two types of university only depends on public funding per student that is always assumed higher in \(E\) than in \(S\). As a consequence, individuals always prefer to be in the elite than in the standard university. From these assumptions, higher education determines the individual skill \(h_{it}^H\) through the following function:

\[
h_{it}^H = \delta_H h_{it}^B = \delta_H a_{it} (h_{it-1})^\eta
\]

where \(h_{it}^H\) is individual \((i,t)\)'s skill level at the end of higher education and \(\delta_H > 1\) depicts the impact of public expenditure per student upon higher education. As for basic education, \(\delta_H\) increases with public expenditure per student in higher education.
Consequently, the two higher education functions corresponding to the standard university and the elite university are:

$$h^S = \delta_S h^B = \delta_S a (h_{it-1})^\eta$$  \hspace{1cm} (3)$$

$$h^E = \delta_E h^B = \delta_E a (h_{it-1})^\eta$$  \hspace{1cm} (4)$$

with $\delta_S < \delta_E$.

As personal abilities are distributed in the segment $[a, \bar{a}]$, then there is an infinite number of functions $h^j = \delta_j a (h_{it-1})^\eta$, $j = B, S, E$.

We denote $H^j_a (h_{t-1}) = \delta_j a (h_{t-1})^\eta$ the function that binds the skill level at the end of the education cycle $j$ to the parent’s skill $h_{t-1}$ for a personal ability $a$. The shapes of curves $H^j_a (h_{t-1})$, $j = B, S, E$, are depicted in Figure 2. Note that all the curves $H^j_a (h_{t-1})$ move upwards when public expenditures per student $\delta_j$ increase. Function $H^j_a (h_{t-1})$ is denoted $H_j (h_{t-1})$ for $a = a$ and $\overline{H}_j (h_{t-1})$ for $a = \bar{a}$. All the education functions $H^j_j (h_{t-1})$ are thus located between curves $H_j (h_{t-1})$ and $\overline{H}_j (h_{t-1})$.

Figure 2. The $j$-education path and the $j$-steady segment, $j = B, S, E$
**Definition 1.** We call \( j \)-education path, \( j = B, S, E \), the set of curves \( H^j_d(h_{t-1}) \) that are situated between the curves \( H_j(h_{t-1}) \) and \( H_j(h_{t-1}) \) for given education expenditures \( \delta_j \).

The \( j \)-education path gathers all the possible skills an individual can reach at the end of the education cycle \( j \). Obviously, the position of the individual inside this path depend on her/his personal ability and parent’s skill. Figure 2 depicts the set of curves \( H^j_d(h_{t-1}) \) and thus the \( j \)-education path, \( j = B, S, E \).

### 2.3. Skill and social segmentation

Provided that individuals always prefer university to basic education and the elite university to the standard university, and because of the selection patterns to enter each university, we can state the following

**Lemma 1.** Individual \((i, t)\)'s human capital \( h_{it} \) is equal to:

1) \( h^B_{it} \) if \( \hat{h}^B_{it} < \hat{h} \);

2) \( h^S_{it} \) if \( \hat{h}^B_{it} \geq \hat{h} \) and \( h^B_{it} \) is too low for belonging to the \( \alpha \) highest human capital at the end of basic education;

3) \( h^E_{it} \) if \( \hat{h}^B_{it} \geq \hat{h} \) and \( h^B_{it} \) belongs to the \( \alpha \) highest human capital at the end of basic education.

**Definition 2.** Within each generation, we call:

1) **Lower class** the set of individuals who have only followed basic education,

2) **Middle class** the set of individuals who have a standard university degree, and

3) **Elite** those with an elite university degree.

### 3. Education and social mobility

The *Education system* \( (\hat{h}, \alpha, \delta_B, \delta_S, \delta_E) \) is the set that combines (i) the selection procedures \( \hat{h} \) and \( \alpha \), and (ii) the triplet of public expenditures per student in the three education cycles
\[ \delta_j = \delta_j(g_j), j = B, S, E. \] Identically, the Higher education policy \( \left( \hat{h}, \alpha, \delta_B, \delta_E \right) \) is the set that is defined by (i) the selection procedures, and (ii) the expenditure per student in each university.

### 3.1. Steady segments

Consider the dynamics \( h_t = \delta_j(a(h_{t-1})^{\eta}), h_{t-1} > 0 \). The unique steady state of this dynamics is \( h_a^j = \left(a \delta_j\right)^{1/(1-\eta)}. \) Consequently, the education-\( j \) dynamics \( h_t = H_a^j(h_{t-1}) \) corresponding to the different \( a \in [\underline{a}, \bar{a}] \) determine a set of steady states that form the segment \( \left[ \left(a \delta_j\right)^{1/(1-\eta)}, \left(\bar{a} \delta_j\right)^{1/(1-\eta)} \right] \). The construction of this segment is depicted in Figure 2. There are thus three sets of steady states of this type, each of which relates to one of the three education cycles \( B, S \) and \( E \). We denote \( h_j = \left(a \delta_j\right)^{1/(1-\eta)} \) and \( \bar{h}_j = \left(\bar{a} \delta_j\right)^{1/(1-\eta)} \).

**Definition 3.** The segment \( \left[h_j, \bar{h}_j\right] = \left[\left(a \delta_j\right)^{1/(1-\eta)}, \left(\bar{a} \delta_j\right)^{1/(1-\eta)} \right] \) is called \( j \)-steady segment, \( j = B, S, E. \)

![Figure 3. Education paths and steady segments](image)
The education dynamics engender the three steady segments $[h_B, \bar{h}_B] = [a^{1/(1-\eta)}]$, $[h_S, \bar{h}_S] = [(\delta_S a)^{1/(1-\eta)}, (\delta_S \bar{a})^{1/(1-\eta)}]$ and $[h_E, \bar{h}_E] = [(\delta_E a)^{1/(1-\eta)}, (\delta_E \bar{a})^{1/(1-\eta)}]$. The three education paths and the three steady segments ($B$, $S$ and $E$) are depicted in Figure 3.

**Lemma 2.** Consider an individual who stands in the $j$-steady segment $[h_j, \bar{h}_j], j = B, S, E$. Then, her/his child is also in the $j$-steady segment if s/he follows the $j$ study.

**Proof.** Assume a parent with a skill inside the segment $[h_j, \bar{h}_j]$. Then, her/his child’s highest possible skill is $\bar{h}_j$ (corresponding to the highest possible parent’s skill and the highest possible child ability), and her/his child’s lowest possible skill is $h_j$ (with the lowest parent’s skill and the lowest possible child’s ability). Hence, the child’s skill also belongs to segment $[h_j, \bar{h}_j]$.

### 3.2. Entry to the university

The condition for individual $(i, t)$ to enter the university is that her/his attainment at the end of basic education is higher than $\hat{h}$. Given the basic education function $h_B^B = a_B (h_{it-1})^\eta$, this is thus only possible if $a_B (h_{it-1})^\eta \geq \hat{h}$.

When the minimum level necessary to enter the university $\hat{h}$ is lower than the lower limit of the $B$-steady segment, i.e. $\hat{h} < a^{1/(1-\eta)}$, then all dynasties can enter the university after a limited number of generations. When this minimum level is higher than the upper limit of the $B$-steady segment, i.e. $\hat{h} > a^{1/(1-\eta)}$, then (i) all the dynasties with an initial human capital lower than $\hat{h}$ remain perpetually under-educated (they never enter the university) and (ii) all the dynasties with an initial human capital higher than $\hat{h}$ enter the university from the first generation, and all their offspring also pursue higher education. The dynamics is less clear if the minimum level necessary to enter the university is inside the $B$-steady segment, i.e. if $a^{1/(1-\eta)} < \hat{h} < a^{1/(1-\eta)}$. Once a dynasty is located above $\hat{h}$, then this dynasty perpetually follows higher education. In addition,
all dynasties should sooner or later pass above $\hat{h}$ when time tends toward infinite\(^2\). This can however take a large number of generations, particularly when $\hat{h}$ is close to $\bar{a}^{1/(1-\eta)}$.

We call under-education trap the situation in which the successive generation of a dynasty indefinitely remain at the basic education level, i.e., they never integrate higher education. From the discussion above we can infer the following

**Lemma 3.** Consider a dynasty $i$ with an initial human capital $h_{i0} < \hat{h}$. Then:

1) This dynasty is in an under-education trap if $\hat{h} > \bar{a}^{1/(1-\eta)}$;

2) This dynasty enters the university after a limited number of generations if $\hat{h} < a^{1/(1-\eta)}$;

3) This dynasty enters the university but this can require a very large number of generation if $a^{1/(1-\eta)} < \hat{h} < \bar{a}^{1/(1-\eta)}$.

### 3.3. Social mobility

We now analyse the intergenerational dynamics of education by taking into account the selection patterns to enter both types of university. To do this, let us start from a situation is which the middle class parents are in the $S$-steady segment $[h_S, \bar{h}_S]$ and the parents in the elite group in the $E$-steady segment $[h_E, \bar{h}_E]$, which will occur sooner or later.

For social mobility to exist, certain children from the middle class (whose parents have a standard university degree) must enter the elite university and certain children from the elite must enter the standard university, falling thereby in the middle class. These moves depend on the skill level of children at the end of basic education, which determines their rank and hence the type of university they can join.

At the end of basic education, all the children from the middle class have a skill level situated in segment $[a(h_S)^\eta, \bar{a}(\bar{h}_S)^\eta] = [(a\delta_S)^{1/(1-\eta)}, (\bar{a}\delta_S)^{1/(1-\eta)}]$. As a matter of fact, $a(h_S)^\eta$ is the lowest possible skill of middle class children at the end of basic education (corresponding to the lowest ability $a$ and the lowest parent skill $h_S$), and $\bar{a}(\bar{h}_S)^\eta$ is the highest possible skill of

\(^2\)Because the probability never to go through segment $[\hat{h}, \bar{a}^{1/(1-\eta)}]$ tends towards 0 when $t \to \infty$.\)
middle class children at the end of basic education (corresponding to the highest ability $\bar{a}$ and the highest parent skill $\bar{h}_S$). Similarly and for the same reasons, all the children from the elite have a skill level situated in segment $\left[a(h_E)^\eta, \bar{a}(\bar{h}_E)^\eta\right] = \left[(a\delta_E)^{\eta\eta}, (\bar{a}\delta_S)^{\eta\eta}\right]$ at the end of basic education. The condition for children from the middle class to enter the elite university is that, at the end of basic education, the most educated child from the middle have a higher level than the least educated child from the elite, i.e., $\bar{a}(\bar{h}_S)^\eta > a(h_E)^\eta$. Otherwise, all the children from the elite have a higher level that those from the middle class at the end of basic education. Thus, the former all enter the elite university and let thereby no opportunity for the middle class children to enter the elite. This establishes the following

Lemma 4. Assume that the middle class parents are in the $S$-steady segment and the elite parents in the $E$-steady segment. Then the condition for social mobility is:

$$\delta_E < \left(\bar{a}/a\right)^{1/\eta}\delta_S$$

(5)

Proof. There is social mobility if children from the middle class can enter the elite, i.e.,

$$a(h_E)^\eta < \bar{a}(\bar{h}_S)^\eta \iff a^{1/(1-\eta)}\delta_E^{\eta/(1-\eta)} < \bar{a}^{1/(1-\eta)}\delta_S^{\eta/(1-\eta)} \iff \delta_E < \left(\bar{a}/a\right)^{1/\eta}\delta_S.$$

It is clear that, when condition (5) is not fulfilled, children from the middle class will remain in the middle class and children from the elite will remain in the elite. Both groups are then fully insulated from each other. In contrast, when condition (5) is met, there is room for social mobility at the top.

Definition 4. Assume that condition (5) is fulfilled. The segment $\left[a(h_E)^\eta, \bar{a}(\bar{h}_S)^\eta\right] = \left[(a\delta_E)^{\eta\eta}, (\bar{a}\delta_S)^{\eta\eta}\right]$ is called mobility segment.

The mobility segment is the set in which the attainments of the middle class children and of the elite children overlap at the end of basic education. It is thus the set in which social mobility

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3 Since the number of children selected to enter the elite university equal the number of children from the elite.
can occur. The case in which the mobility segment does exist and thus in which children from the middle class enter the elite university is depicted on Figure 4.

Finally note that, when the $S$-steady segment $\left[ (\delta_S a)^{(1-\eta)}, (\delta_S a)^{(1-\eta)} \right]$ and the $E$-steady segment $\left[ (\delta_E a)^{(1-\eta)}, (\delta_E a)^{(1-\eta)} \right]$ overlap, the mobility segment does exist\(^4\).

3.4. Steady state

A steady state of an education dynamics is characterised by two features:

1) The members of the three social groups are all situated in their steady segment at each generation, i.e., the $B$-steady segment for the lower class, the $S$-steady segment for the middle class and the $E$-steady segment for the elite.

2) The number of dynasties inside each social class remains constant over time. This does not mean that the dynasties indefinitely remain in the same segments. When the mobility segment exists, dynasties inside the middle class can move to the elite \textit{et vice versa}. In contrast, when the lower class exists, the dynasties within this class indefinitely remain inside\(^5\).

\(^4\) $(\delta_S a)^{(1-\eta)} > (\delta_E a)^{(1-\eta)} \Rightarrow \delta_E < (\bar{a} / a) \delta_S \Rightarrow \delta_E < (\bar{a} / a)^{1/\eta} \delta_S$ since $\bar{a} > a > 0$ and $0 < \eta < 1$.

\(^5\) Dynasties can leave the lower class only if $\bar{h}_B > \bar{h}$, which makes all the dynasties leave the lower class when $t \to \infty$. 
Formally, we can thus give the following

**Definition 5.** We call *steady state of the education system* \( \left( \hat{h}, \alpha, \delta_B, \delta_S, \delta_E \right) \) a situation in which:

1) either \( \bar{h}_B = \bar{a}^{1/(1-\eta)} < \hat{h} \) and all the dynasties within the under-education trap have a human capital inside the \( B \)-steady segment, or \( \hat{h} < \bar{a}^{1/(1-\eta)} \) and all dynasties enter the university,

2) the size of each social group remains unchanged from one generation to the next, and

3) at each generation, all the dynasties inside the middle class have a human capital inside the \( S \)-steady segment and all the dynasties within the elite have a human capital inside the \( E \)-steady segment.

From the above definition, we can establish

**Lemma 5.** Consider an education system \( \left( \hat{h}, \alpha, \delta_B, \delta_S, \delta_E \right) \) characterised by the education functions (2)-(4). Then, whatever the initial distribution of skills across dynasties, this education system tends towards a steady state.

Proof. Appendix A.

It must be noted that:

1) If \( \bar{a}^{1/(1-\eta)} > \hat{h} \), then (i) there is one unique steady state which is fully defined by the limits of the \( S \)- and \( E \)-steady segments (depending on \( \delta_S, \delta_E, \bar{a} \) and \( \bar{a} \)) and the share \( \alpha \), (ii) the lower class vanishes and (iii) the middle class accounts for the share \( 1-\alpha \) of the population.

2) When \( \bar{a}^{1/(1-\eta)} < \hat{h} \), then there is a large number of possible steady states depending on the initial distribution of skills across the dynasties. The dynasties that are initially below the skill level \( \left( \hat{h} / \bar{a} \right)^{1/\eta} \) will perpetually remain in the under-education trap. Thus, the proportion of the middle class inside the population at the steady state \( \beta \) is lower than \( (1-\alpha) \).

**4. Self-reproduction of the elite**

To combat under-education, two policies (or a combination of both) are possible, i.e., (i) an increase in the expenditure per pupil in basic education that moves the \( B \)-steady segment upwards and (ii) a decrease in the selection threshold \( \hat{h} \).
We now focus on the more complex issue of the mobility between the middle class and the elite, and thus on the elite self-reproduction.

4.1. Mobility rates

A straightforward measurement of the upward mobility of the children from the middle class consists in computing the proportion of those who join the elite group. Identically, the downward social mobility of the children from the elite can be measured by the proportion of those who join the middle class, and the elite self-reproduction by the proportion of children from the elite who remain in this group.

**Definition 5.** We call:

1) *Elite replacement rate* the proportion $\rho$ of children from the elite who fall in the middle class and *elite self-reproduction rate* the proportion $\pi = 1 - \rho$ of children from the elite who remain in the elite.

3) *Middle class upward mobility rate* the proportion $\mu$ of children from the middle class who enter the elite.

We place ourselves at the steady state and we assume that the mobility segment does exist (condition (5) is fulfilled). Hence, at the end of basic education:

1) all the children are inside segment $\left[\left(\frac{1}{1 - \eta}a\delta_S^\eta\right), \left(\frac{1}{1 - \eta}a\delta_E^\eta\right)\right]$;

2) all the middle class children are inside segment $\left[\left(\frac{1}{1 - \eta}a\delta_S^\eta\right), \left(\frac{1}{1 - \eta}\bar{a}\delta_S^\eta\right)\right]$;

3) all the elite children are inside the segment $\left[\left(\frac{1}{1 - \eta}a\delta_E^\eta\right), \left(\frac{1}{1 - \eta}\bar{a}\delta_E^\eta\right)\right]$;

4) mobility occurs inside the mobility segment $\left[\left(\frac{1}{1 - \eta}a\delta_E^\eta\right), \left(\frac{1}{1 - \eta}\bar{a}\delta_S^\eta\right)\right]$.

Let $f_S(h)$ and $f_E(h)$ be respectively the density function of the middle class children’s skills at the end of basic education and the density function of the elite children’s skills at the end of basic education. Then:

1) $f_S(h)$ is defined on segment $\left[\left(\frac{1}{1 - \eta}a\delta_S^\eta\right), \left(\frac{1}{1 - \eta}\bar{a}\delta_S^\eta\right)\right]$, and $\int f_S(h)dh = 1$.
2) $f_E(h)$ is defined on segment $\left[(a\delta_E)\frac{1}{1-\eta}, (\bar{a}\delta_E)\frac{1}{1-\eta}\right]$, and 
\[
\int f_E(h)dh = 1.
\]

Note that functions $f_S(h)$ and $f_E(h)$ typically change from one generation to the next.

We successively study the impact on social mobility (i.e., on the mobility rates) of:
1) a change in public expenditure per student in both universities, and
2) a change in the selection procedures, i.e., in the weight of each social group.

The mobility rates vary at each generation and they depend on the distribution of abilities across children and on the distribution of children in the set of skill attainments at the end of basic education. We know that abilities are equiprobable in the segment $[a, \bar{a}]$.

To study the impact changes in the education system, we assume that there is a continuum of middle class children on their segment $\left[(a\delta_S)\frac{1}{1-\eta}, (\bar{a}\delta_S)\frac{1}{1-\eta}\right]$ at the end of basic education, and a continuum of elite children on their segment $\left[(a\delta_E)\frac{1}{1-\eta}, (\bar{a}\delta_E)\frac{1}{1-\eta}\right]$ at the end of basic education, and that $(a\delta_E)\frac{1}{1-\eta} < (\bar{a}\delta_S)\frac{1}{1-\eta}$, i.e., the mobility segment does exists. In addition, we assume that the density functions $f_S(\bullet)$ and $f_E(\bullet)$ that define the respective distributions of the middle class and the elite children in their post-basic education segments are given. This permits to compare situations with different education systems. We however make no particular assumptions on the shape of these functions.

**Lemma 6.** There is at each generation a unique $h_\alpha$ inside the mobility segment such that all the children with a skill level above (below) $h_\alpha$ at the end of basic education enter the elite university (the standard university).

**Proof.** Appendix B.

The existence and uniqueness of $h_\alpha$ mean that all the middle class children with a skill level higher than $h_\alpha$ at the end of basic education enter the elite university and become thereby members of the elite. Symmetrically, all the elite children with a skill level lower than $h_\alpha$ at the
end of basic education enter the standard university and ‘fall’ thereby in the middle class. This generates social mobility. This also allows calculating the mobility rates. Denoting respectively $f_S(h)$ and $f_E(h)$ the density functions attached to the skill attainments of children from the middle class and for the elite at the end of basic education, we have:

$$
\mu = \int_{h_a}^{(1/(1-\eta))} f_S(h)dh ; \quad \rho = \int_{h_a}^{(1/(1-\eta))} f_E(h)dh ; \quad \pi = \int_{h_a}^{(1/(1-\eta))} f_E(h)dh
$$

Finally note that $h_a$ is the value such that $\mu\beta = \rho\alpha$.

### 4.2. Public expenditures in higher education

**Proposition 1.** At the steady state, an increase (decrease) in the expenditure per student in the elite university $\delta_E$ reduces (augments) the middle class mobility rate $\mu$ and the elite replacement rate $\rho$, and it thereby augments (reduces) the elite self-reproduction rate.

**Proof.** Appendix C.

Proposition 1 shows that a change in the higher education policy that fosters the funding of the elite university does not only increase the educational level of the elite compared to the middle class. It also lowers in social mobility. Fewer children from the middle class can now join the elite, i.e., the self-reproduction of the elite is reinforced. This derives from the fact that, with the rise in the funding allocated to the elite university, the middle class will turn out to possess less human capital relative to the elite. As a consequence, fewer highly able children from the middle class will overtake the human capital level of the low able children from the upper class because of the growing difference in intra-family externalities between the two social groups. Here, the intra-family externality is the key channel by which social mobility is lessened and self-reproduction of the elite reinforced.

**Proposition 2.** At the steady state, an increase (decrease) in the expenditure per student in the standard university $\delta_S$ augments (reduces) the middle class mobility rate $\mu$ and the elite replacement rate $\rho$ and it lessens (increases) the elite self-reproduction rate. This increases (decreases) social mobility.

**Proof.** Appendix C.
Logically, the impact of an increase in the expenditure per student in the standard university \( \delta_S \) is the exact opposite of the impact of an increase in \( \delta_E \). The reason is similar. Increasing \( \delta_S \) raises the skill level of the middle class, providing their children with higher intra-family skill transfers and thus a better position in the competition for the entry in the elite university.

It can finally be reminded that, if the increase in \( \delta_E \) and/or the decrease in \( \delta_S \) are sufficiently high so that \( \delta_E > (\bar{a} / \underline{a})^{1/n} \delta_S \), this results in full social immobility at the top, i.e., a total self-reproduction of the elite (Lemma 5).

**4.3. Changes in the selection patterns**

Changes in the selection procedures (\( \hat{h} \) and \( \alpha \)) result in changing the weight in the population of the middle class (\( \beta \)) and of the elite (\( \alpha \)). Consequently, the analysis of such changes will be made by assuming changes in \( \beta \) and \( \alpha \).

**Proposition 3.** An increase (decrease) in the share \( \beta \) of the middle class in the population without change in the share \( \alpha \) of the elite causes at the steady state:

1) a decrease (increase) in the elite self-reproduction rate, and

2) a decrease (increase) in the middle class upward mobility rate.

Proof. Appendix D.

Firstly note that the case in which the density of the middle class is higher in the top extremity of its steady segment than on average is very unlikely. This is a result of all the simulations implemented with reasonable values of the model parameters. In addition, this feature is logic when the mobility segment is not too large (which is typically the case) and gathers thereby a limited share of the middle class children. This is because, at each generation, the most skilled children from the middle class enter the elite and are replaced by less skilled children from the elite. This lowers the number of middle class individuals in the highest skill attainments.

Proposition 3 shows that the enlargement of the middle class tends to jeopardise its upward mobility. This is because, as the middle class gets bigger, the number of its children candidate to the elite university augments whereas the number of selected students remains unchanged. In
addition this enlargement also diminishes the elite self-reproduction. This is because more children from the middle class will accede to the elite, lessening thereby the number of elite children who remain in their group. So, the increase in the weight of the middle class (without increase in the weight of the elite) jeopardises both the positions of the middle class and of the elite. Even if there are more middle class children who rise to the elite, their proportion however decline because of the increase in the middle class population. In addition, this reduces the number of elite children who stay in the elite.

It must however be underlined that these results concern the steady state. In fact, the generation when the middle class enlargement occurs is typically better off compared to their parents. This is because: 1) the enlargement of the middle class operates by the bottom, i.e., children from the lower class accede to the middle class, being thus in a better social position than their parents; 2) the top side of the middle class and the bottom side of the upper class are not impacted yet during this first generation, and the mobility between the two groups is thus unchanged compared to what it would have been without enlargement. On average, the increase in $\beta$ is experienced as an improvement compared to the previous generation in the middle class (because of the improvement of its bottom side) and as no change by the elite. It is only in the following generations that the middle class suffers a decrease in its upward mobility and the elite a decrease in its self-reproduction.

**Remark:** At the steady state, we know that two cases are possible. The existence of a lower class is conditioned by inequality $\hat{h} \geq \overline{h}_B$. If $\hat{h} < \overline{h}_B$, then the lower class vanishes sooner or later and $\beta \rightarrow 1 - \alpha$. This means that passing from one steady state with $\beta_1 < 1 - \alpha$ to another with $\beta_2$ such that $\beta_1 < \beta_2 < 1 - \alpha$ requires a two-shot change in higher education policy. Threshold $\hat{h}$ must firstly be lowered under $\overline{h}_B$ (but remaining above $h_B$) to make certain children from the lower class enter the standard university, and subsequently be elevated above $\overline{h}_B$ to prevent the whole of the lower class children to enter the university and join the middle class.

**Proposition 4.** An increase (decrease) in the share $\alpha$ of students recruited by the elite university with $\beta$ constant increases (decreases) both the middle class upward mobility rate and the elite self-reproduction rate

**Proof.** Appendix D.
4.4. From equality to social polarization

We now start from one initial state (generation 0) in which all individuals possess the same human capital $h_0$, $h_0 < \hat{h} < \bar{h}_B$, and we study the dynamics of skill distribution and social stratification that derive from the education system $(\hat{h}, \alpha, \delta_B, \delta_s, \delta_E)$. This makes it possible to depict how, from a purely egalitarian starting point, the society tends to differentiate because of intra-family transmission of skill and personal ability. If personal ability is the main driver of educational achievement at the beginning of the process, the position of the family becomes subsequently essential, particularly once a number of dynasties have reached the minimum human capital $\hat{h}$ at the end of basic education. This comes from the selection procedure in the elite university that naturally tends to give an advantage to the children from the elite over other children. We assume that a certain number of generations is necessary for the most educated dynasties (these whose successive generations have all benefited from a high personal ability) to reach human capital $\hat{h}$ required to enter the university at the end of basic education.

At generation 0, all individuals possess the same skill. From the first generation, the distribution of skill begins to diverge because of differences in personal ability. From the second generation, this divergence tends to increase because of the combination of differences in personal ability and intra-family externality. At the $t$-th generation, $t > 1$, each dynasty can be characterised by the sequence of abilities of its successive generations $1, \ldots, t$. Each dynasty $i$ has an ‘ability history’ at generation $t$, which is the vector $A_i = (a_{i1}, a_{i2}, \ldots, a_{it}) \in [\underline{a}, \overline{a}]^t$ of the abilities of the successive generations of this dynasty. From a certain generation $\theta$, there is at least one dynasty whose human capital at the end of basic education is higher than $\hat{h}$ and certain dynasties enter the university. As long as the proportion of dynasties that enter the university is lower than $\alpha$, all universities are elite universities. Assume that, from generation $\theta'$, the proportion of children who enter the university becomes higher than $\alpha$. Then, tertiary education becomes divided between the elite university and the standard university, which divides the individuals with tertiary education degrees between the middle class and the elite. The higher the difference in expenditure per student between both types of university, the most self-reproducing the elite, and the fewer the number of children from the middle class who go up the social ladder.
The above description shows that, even if we start from a fully egalitarian situation with an educational system which is universalist in the sense that the rules and selection procedures apply equally to all children whatever their origin, this system typically leads to social polarization and to the emergence of an elite that is to a large extent self-reproducing. This derives from the combination of an elite university that recruits the best students of each generation and of intra-family externalities that makes the children from the elite better equipped to integrate the elite university because they benefit from the skill externalities provided for by their parents inside the family.

5. Discussion and conclusion

The model developed in this article exhibits the following major results:

1) An educational system characterised by the division of higher education between elite universities selecting a narrow share of the students and standard universities whose recruitment is based on a minimum skill level at the end of compulsory education leads to social stratification between the middle class and the elite, the latter being to a large extent self-reproducing.

2) The higher the difference in expenditure per student between the elite and standard universities, the lower the upward social mobility of the middle class, and the more self-reproducing the elite group. From a certain difference in expenditures, social mobility vanishes and the elite become fully insulated from the rest of the society.

3) An increase in the share of the middle class in the population (without change in the share of the elite) lowers social mobility, except for the generation when this increase occurs.

4) An enlargement of the elite (i.e., an increase in the proportion of elite households in the population) also jeopardises social mobility at the steady state.

The core of the analysis lies in the respective intensity of personal ability ($a_t$) and intra-family skill transfers (measured by the parents’ skill $h_{t-1}$). When the differences in skill between the middle class and the elite is not too large because of limited differences in allocations between elite and standard universities, then the difference in intra-family transfers between the two groups remains limited and personal ability prevails, which results in social mobility. In contrast, when the allocations to elite universities are high compared to standard universities, the
differences in intra-family transfers is high and social backgrounds prevail, which boost the self-reproduction of elites.

This model provides explanations to the observed facts underlined in the introduction. The post-World War 2 period experienced a rapid democratization of tertiary education and an increase in the share of executives and top executives (the elite) in the working population. These changes have been at the beginning twice beneficial to the middle class. Firstly, the middle class newcomers had parents from the lower class, and their access to university and skilled positions was indubitably a promotion to them. Second, the enlargement of the elite allowed the more able and more educated children from the middle class to join the elite. Consequently, both sides of the middle class experienced a social promotion, resulting in a general optimism of its members. However, these pro-middle class changes were transitional. Once individuals from the lower class have joined the middle class, their children will not continue going up the social ladder. Contrarily, the longer term (and steady state) negative impact of the enlargement of the middle class now lessens the middle class upward mobility (Proposition 3), except if the enlargement of the elite counteracts this move (Proposition 4). As the increase in the weight of the middle class has been higher than that of the elite, we can logically suspect that the negative impact of the first is prevailing. In addition, the growing gap between the elite and standard universities in terms of the expenditure per student restrains the middle class upward mobility and boosts the self-reproduction of the elite group.

Finally, the model predicts that growing differences in expenditure per student between the elite and the standard universities result in both (i) higher differences in skill levels between the middle class and the elite, and (ii) a lower social mobility. Since the first outcome means higher inequality, the model provides an explanation for the ‘great Gatsby curve’ put forward by Corak (2012 and 2013) and Krueger (2012). Contrary to the usual explanation in which inequality explains low mobility, particularly through education costs with imperfections on the credit market, our model shows that both higher inequality and lower mobility can derive from the very structure of the education system when the efficiency gap between elite and standard universities increases. It thus predicts that the countries with the more elitist educational systems will show both higher inequality and lower mobility. This is in line with the great Gatsby curve in which the hierarchy in social immobility replicates better the hierarchy in higher education ‘elitism’ than the hierarchy in inequality. Finally, our explanation is better tailored to explain mobility at
the top whereas the explanation through imperfections on the credit market fits better with the explanation for inequality at the bottom (under education).

In summary, the model provides a rather lightening reading of the observed developments in terms of social mobility between the middle class and the elite. Obviously, other factors had and still have an impact on the link between education and social mobility. In particular, our model assumes that higher education is free of charges for the students. This is to focus on the sole impact of the selection procedure. However, the increase in higher education fees has been substantial in several countries. This typically encourages the self-reproduction of the elite, particularly when the credit market is imperfect. In addition the ongoing globalization dynamics tend to displace the tax burden from the elite to the middle class because of internationally mobile tax bases, which again jeopardises the position of the middle class. Finally, intra-university skill externalities and networking reinforce the elite self-reproduction.

References

Appendix A. An education system always tends towards a steady state

A steady state is characterised by 3 features: 1) either \( \bar{a}^{1/(1-\eta)} < \hat{h} \) and all the dynasties within the under-education trap have a human capital inside the \( B \)-steady segment, or \( \hat{h} < \bar{a}^{1/(1-\eta)} \) and all dynasties enter the university; 2) the number of individuals in each social group remains constant from one generation to the next; 3) at each generation, all the dynasties inside the middle class have a human capital inside the \( S \)-steady segment and all the dynasties inside the elite have a human capital inside the \( E \)-steady segment.

We know that, either \( \hat{h} > \bar{a}^{1/(1-\eta)} \) and all the dynasties initially below the skill level \( \bar{h} \) remain perpetually inside the education trap, or \( \bar{h} < \bar{a}^{1/(1-\eta)} \) and all the dynasties sooner or later enter the university. Hence, the first feature of the steady state is always fulfilled in the long term.

As the first feature is fulfilled, the number of dynasties inside the lower class is unchanged. In addition, the number of individuals inside the elite is given and equal to \( \alpha \) at each generation. Consequently, the number of individuals inside the middle class is also constant at each generation. Feature 2 is thus verified.

The third feature is obviously fulfilled when the mobility segment is nil because (i) all the dynasties enter sooner or later in a steady segment, and (ii) they all perpetually remain in this
segment once they are inside (because of the construction of steady segments and since the mobility segment is nil).

We must now prove that the second feature is also fulfilled when the mobility segment exists (i.e., there is social mobility between the middle class and the elite). To show this, we start from a situation in which all the middle class members are in the S-segment and all the elite members in the E-segment. We then show that (i) if a dynasty inside the middle class moves to the elite, then this dynasty moves from the S-segment into the E-segment, and (ii) if a dynasty inside the elite moves to the middle class, then this dynasty moves from the E-segment into the S-segment.

Consider child \( i \) from the middle class who enters the elite university with the skill attainment \( h_i^E \) at the end of basic education. The lowest possible skill at the end of basic education of a child from the elite is \( \left( \frac{1}{1-\eta} S \right) \). Hence, \( h_i^E > \left( \frac{1}{1-\eta} E \right) \) and child \( i \)’s skill at the end of higher (elite) education is \( \delta_E h_i^E > \delta_E \left( \frac{1}{1-\eta} E \right) = \left( \frac{1}{1-\eta} E \right) \). The highest possible skill at the end of basic education of a child from the elite is \( \left( \frac{1}{1-\eta} E \right) \). This value is above the upper limit of the mobility segment, which induces \( h_i^E < \left( \frac{1}{1-\eta} E \right) \) and thus \( \delta_E h_i^E < \delta_E \left( \frac{1}{1-\eta} E \right) = \left( \frac{1}{1-\eta} E \right) \). Hence \( \delta_E h_i^E < \left( \frac{1}{1-\eta} E \right) \). Hence \( \delta_E h_i^E < \left( \frac{1}{1-\eta} E \right) \), which establishes that individual \( i \) is inside the E-steady segment.

Consider now child \( i \) from the elite who enters the standard university with the skill attainment \( h_i^S \) at the end of basic education. The highest possible skill at the end of basic education of a child from the middle class is \( \left( \frac{1}{1-\eta} S \right) \). Hence, \( h_i^S < \left( \frac{1}{1-\eta} S \right) \) and child \( i \)’s skill at the end of higher (elite) education is \( \delta_S h_i^S < \delta_S \left( \frac{1}{1-\eta} S \right) = \left( \frac{1}{1-\eta} S \right) \). The lowest possible skill at the end of basic education of a child from the elite is \( \left( \frac{1}{1-\eta} S \right) \). This value is below the lower limit of the mobility segment, which induces \( h_i^S > \left( \frac{1}{1-\eta} S \right) \) and thus \( \delta_S h_i^S > \delta_S \left( \frac{1}{1-\eta} S \right) = \left( \frac{1}{1-\eta} S \right) \). Hence \( \delta_S h_i^S > \left( \frac{1}{1-\eta} S \right) \), which establishes that individual \( i \) is inside the S-steady segment.

**Appendix B. Proof of the existence and uniqueness of \( h_\alpha \)**

Let \( h_\alpha \) be the skill achievement at the end of basic education such that all the children above \( h_\alpha \) enter the elite university. Thus, all elite children below \( h_\alpha \) enter the standard university and all middle class children above \( h_\alpha \) enter the elite university. Since the number of students in the elite university is constant, the number of elite children who enter the standard university is equal to the number of the middle class children who enter the elite university.
Let $m$ and $e$ be respectively the number of children from the middle class and from the elite in the mobility segment, and consider the variable $h$ defined on the mobility segment: 

$$h \in \left[ \left( a\delta_{E}^{\eta} \right)^{1/(1-\eta)}, \left( \bar{a}\delta_{S}^{\eta} \right)^{1/(1-\eta)} \right].$$

We define the following functions:

1) $E(h) =$ number of children from the elite in the segment $\left[ \left( a\delta_{E}^{\eta} \right)^{1/(1-\eta)}, h \right]$. On the mobility segment, $E(h)$ is continuous and monotonically increasing and from 0 to $e$.

2) $M(h) =$ number of children from the middle class in the segment $\left[ h, \left( \bar{a}\delta_{S}^{\eta} \right)^{1/(1-\eta)} \right]$. On the mobility segment, $M(h)$ is continuous and monotonically decreasing from $m$ to 0.

Hence, there is a unique $h_\alpha \in \left[ \left( a\delta_{E}^{\eta} \right)^{1/(1-\eta)}, \left( \bar{a}\delta_{S}^{\eta} \right)^{1/(1-\eta)} \right]$ such that $E(h_\alpha) = M(h_\alpha)$. The diagrammatic determination of $h_\alpha$ is depicted in Figure A1. This is the unique skill attainment at the end of basic education such that the number of children from the elite who do not enter the elite university $E(h_\alpha)$ equal the number of children from the middle class who enter the elite university $M(h_\alpha)$.

**Appendix C. Impacts of changes in public expenditures**

C1. *Increase in $\delta_{E}$*

An increase in the expenditure per student in the elite university $\delta_{E}$ moves the skill of all individuals in the elite upwards, which in turn increases the post basic education skill of their children. Let $h_{\alpha 1}$ be the thresholds to accede to the elite university before the increase in $\delta_{E}$. Then $h_{\alpha 1}$ is also the post-basic education skill attainment of the least skilled children from the middle class who enters the elite university before the increase in $\delta_{E}$. The increase in $\delta_{E}$ makes some children from the elite who were below $h_{\alpha 1}$ to move above $h_{\alpha 1}$ at the end of basic education. This induces:
1) An upward move of threshold $h_{\alpha}$ from $h_{a1}$ to $h_{a2} > h_{a1}$. This is because there are now more children above $h_{a1}$ whereas the number of children selected by the elite university remains constant ($\alpha$).

2) A decrease in the number of middle class children who enter the elite university (since the distribution of the middle class children is unchanged and $h_{\alpha}$ rises), and hence an increase in the number of children from the elite who enter the elite university.

Finally, we have:

1) an increase in the elite self-reproduction rate (since the number of elite children $\alpha$ is constant and the number of elite children entering the elite university rises);

2) a decrease in the middle class upward mobility rate (since the number of middle class children $\beta$ is constant and the number of those entering the elite university decreases).

C2. Increase in $\delta_S$

An increase in the expenditure per student in the standard university $\delta_S$ moves the skill of all individuals in the middle class upwards, which in turn increases the post basic education skill of their children. Let $h_{a1}$ be the thresholds to accede to the elite university before the increase in $\delta_S$. Then $h_{a1}$ is also the post-basic education skill attainment of the least skilled children from the elite who enters the elite university before the increase in $\delta_S$. The increase in $\delta_S$ makes children from the middle class who were below $h_{a1}$ to move above $h_{a1}$ at the end of basic education. This induces:

1) An upward move of threshold $h_{\alpha}$ from $h_{a1}$ to $h_{a2} > h_{a1}$.

2) An increase in the number of children from the middle class who enter the elite university, and hence a decrease in the number of elite children who enter the elite university (since the number of students in the elite university is constant).

Finally, we have:

1) a decrease in the elite self-reproduction rate (since the number of elite children $\alpha$ is constant);

2) an increase in the middle class upward mobility rate (since the number of elite children $\beta$ is constant).

Appendix D. Impacts of changes in the selection procedures

D1. Increase in $\beta$

We assume an increase in the weight of the middle class in the population $\beta$ without change in the weight of the elite $\alpha$. We also suppose no change in the density functions $f_S(h)$ and $f_E(h)$.

Let $h_{a1}$ be the thresholds to accede to the elite university before the increase in $\beta$. The increase in $\beta$ without change in $f_S(h)$ makes the number of middle class children to increase at each point of segment $\left[\left(\bar{a}\delta_S^\eta\right)^{1/(1-\eta)},\left(\bar{a}\delta_S^\eta\right)^{1/(1-\eta)}\right]$. Thus, the number of children from the middle class above $h_{a1}$ at the end of basic education increases. This induces:

1) An upward move of threshold $h_{\alpha}$ from $h_{a1}$ to $h_{a2} > h_{a1}$.
2) A decrease in the number of children from the elite entering the elite university, and hence a decrease in the elite self-reproduction rate.

3) A decrease in the middle class upward mobility rate \( \mu = \int_{h_\alpha} f_S(h)dh \) for a given density function \( f_S(h) \) since \( h_\alpha \) increases.

D1. Increase in \( \alpha \)
Assume an increase in the weight of the elite in the population \( \alpha \) without changes (i) in the weight of the middle class \( \beta \) and (ii) in the density functions \( f_S(h) \) and \( f_E(h) \). The number of middle class children and elite children entering the elite university are respectively

\[
\mu \times \beta = \beta \int_{h_\alpha} f_S(h)dh \quad \text{and} \quad \pi \times \alpha = \alpha \int_{h_\alpha} f_E(h)dh.
\]

Let \( h_{a1} \) be the thresholds to accede to the elite university before the increase in \( \alpha \). Without change in \( f_E(h) \), the shift in \( \alpha \) from \( \alpha_1 \) up to \( \alpha_2 > \alpha_1 \) makes the number of elite children in the segment \([h_{a1}, (\bar{\alpha} \delta_E)^{1/(1-q)}]\) to increase from \( \alpha_1 \int_{h_{a1}} f_E(h)dh \) up to \( \alpha_2 \int_{h_{a1}} f_E(h)dh \).

The corresponding increase is \( (\alpha_2 - \alpha_1) \int_{h_{a1}} f_E(h)dh \). Concurrently, the increase in the number of children recruited by the elite university is \( \alpha_2 - \alpha_1 \). For the number of middle class children who enter the elite university not to rise, we must have

\[
(\alpha_2 - \alpha_1) \int_{h_{a1}} f_E(h)dh \geq \alpha_2 - \alpha_1, \quad \text{i.e.,} \quad \int_{h_{a1}} f_E(h)dh \geq 1.
\]

This is impossible when the mobility segment does exist. As the number of middle class children is assumed constant, the middle class upward mobility rate increases.

As the middle class upward mobility increases, threshold \( h_\alpha \) must decrease and move from \( h_{a1} \) to \( h_{a2} < h_{a1} \). Assuming no change in the density function \( f_E(h) \), the elite self-reproduction rate moves from \( \int_{h_{a1}} f_E(h)dh \) up to \( \int_{h_{a2}} f_E(h)dh > \int_{h_{a1}} f_E(h)dh \) since \( h_{a2} < h_{a1} \). Hence, the elite self-reproduction rate increases.