The dynamics of personal income distribution and inequality in the United States

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Abstract

We model the evolution of age-dependent personal income distribution and inequality as expressed by the Gini ratio. In our framework, inequality is an emergent property of a theoretical model we develop for the dynamics of individual incomes. The model relates the evolution of personal income to the individual's capability to earn money, the size of her work instrument, her work experience and aggregate output growth. Our model is calibrated to the single-year population cohorts as well as the personal incomes data in 10- and 5- year age bins available from March Current Population Survey (CPS). We predict the dynamics of personal incomes for every single person in the working-age population in the USA between 1930 and 2011. The model output is then aggregated to construct annual age-dependent and overall personal income distributions (PID) and to compute the Gini ratios. The latter are predicted very accurately - up to 3 decimal places. We show that Gini for people with income is approximately constant since 1930, which is confirmed empirically. Because of the increasing proportion of people with income between 1947 and 1999, the overall Gini reveals a tendency to decline slightly with time. The age-dependent Gini ratios have different trends. For example, the group between 55 and 64 years of age does not demonstrate any decline in the Gini ratio since 2000. In the youngest age group (from 15 to 24 years), however, the level of income inequality increases with time. We also find that in the latter cohort the average income decreases relatively to the age group with the highest mean income. Consequently, each year it is becoming progressively harder for young people to earn a proportional share of the overall income.

Key words: income dynamics, income distribution, inequality, age profiles, Gini, United States.
JEL classifications: D01, D31, E17, J1, O12.

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1 Introduction

Social fairness and welfare is often analyzed in the framework of income inequality. For that reason the study of income distribution has occupied a special place in economic research. The main questions of interest are how to describe and explain income differences among individuals. When thinking about these, two practical concerns arise. The first is empirical - the measurement of income and elicitation of empirical facts about its distribution. The second is theoretical - can the properties of observed income distribution be explained using causal relationships. Multiple attempts to tackle these issues have been made, however their full reconciliation proves to be quite difficult.

There is some consensus regarding the broader empirical properties of income distribution and inequality. The stylized facts include growing income variance within a cohort as it ages, disproportionately high top income shares, long tails and skewness to the right, lower median earning relatively to the mean, which in turn varies across groups of individuals with different features. Neal and Rosen (2000) offer a detailed account of these. Mincer (1974) emphasizes the striking difference between income age profiles of earners, especially the varying pick income age for different education groups. He also noted large scatter of income across genders, races, family statuses, educational levels and professions. Overall, the stylized empirical facts about personal incomes evidence large heterogeneity in income paths, which likely depend on individual circumstances.

As attempts to explain the differences among individuals and overall income distributions, a number of theories have been proposed. Among them, the models of human capital are arguably the most established (Becker, 1964; Mincer, 1958). In these models, variations in earnings are explained by distribution of schooling and post-school investments in human capital. The latter is however not observed directly and hence certain distributional properties have to be assumed in order to justify positive skewness of earnings. In addition, this makes empirical testing of the models problematic (Mincer, 1974). For this reason human capital theory is primarily praised for its analytical convenience. Various other models have been proposed to explain differences in earnings, including selection, learning, sorting and matching, which mostly analyze the wage setting mechanisms by looking at supply and demand for labor (Neal and Rosen, 2000). As an alternative explanation for the causes of distributional variations, Hartog (1981) proposes a multicapability theory of income, suggesting that the differences are driven by various personal qualities such as intellect, physical ability and social skills. He provides some empirical description of how these capability factors distributed among individuals within and between occupational fields and claims that earning are determined by how much certain qualities combinations are valued by various jobs.

Aside from theoretical models, a different line of research has emerged attempting to capture the shape of the overall distribution. Various parametric functions have been proposed to fit income distributions or parts thereof. For instance, a battery of empirical examinations, starting from Pareto (1897), find that high incomes follow a power law or Pareto distribution. However, there is no consensus about the distributional form of the low and middle incomes, which usually represent around 90% of the mass. Among notable attempts are Champernowne (1953, 1973) and Salem and Mount (1974), who showed that gamma functions provide a relatively good fit for the overall income distribution. The most recent contributions, including Dragulescu and Yakovenko (2001), Silva and Yakovenko (2005) and Yakovenko (2012), show that incomes in the United States and the United Kingdom below the Pareto region are distributed exponentially. Souma (2001) and Souma and Nirei (2005) find that a log-normal fits better for the Japanese data. These studies suggest a two-class distribution of incomes, where the majority of incomes is described either by an exponential or a log-normal and the very top incomes follow a power law, thereby proclaiming universality of the structure.

A special attention within the analysis of income distribution has more recently been drawn to inequality (Atkinson, 1997). In particular, methodologies for measuring top incomes shares have been gaining momentum and has caused considerable debate about the inequality trends. Piketty (2003), Piketty and Saez (2003), Atkinson (2005), Atkinson et al. (2011) and Burkhauser et al. (2012) among others provide measures of income shares for top percentiles of earners. These measures provide detailed account of the level and trends in the disproportionate amount of total income concentrated in the hands of a small proportion of population. However, no attempt is made to explain a wide range of other stylized facts, mentioned earlier. Most importantly, the dynamics of the age-dependent distributions is not considered. For complete understanding of the underlying income structure and its evolution, it is essential to know how the relative position of each individual in the distribution changes in time, within and between age cohorts, income and social groups. A complete dynamic structure of income distribution is then only attainable with a model that tracks individual movements in time relatively to all other individuals.

So far, a majority of proposed approaches for modeling and explaining income dynamics have been constrained in scope and coverage. Theoretical models either do not provide full justification for the multitude of the observed facts or are difficult to test empirically, while comprehensive estimations of income distribution and inequality measures are not based on any rigorous theory. To our knowledge, there have so far been no successful attempts to reconcile the two and produce a micro-founded model for personal income dynamics, which could be directly estimable using available data and produce robust predictions for all possible features of income distribution and inequality at both micro and macro level. In this paper we present just that kind of model.

We develop a theoretical model for the evolution of personal income, which was previously described in a different formulation in Kitov (2005a). Our model is based on two broad individual characteristics - ability to earn money and the work instrument, simply described as a job. The ability to earn money can be directly linked to the notion of human capital or capability, as described by Hartog, whereas work instrument is related to the demand side of the labor force market and should be understood as a relative value and earning potential of the job. The dynamics of personal income is governed by a first order differential equation that allows for a variety of shapes of income paths depending on individual characteristics. This flexibility permits us to reconcile cross-sectional and intertemporal stylized facts when the model is calibrated to income data from March Current Population Survey (CPS) and population data from the U.S. Census Bureau between 1947 and 2011. From the distribution of personal incomes implied by the model we predict Gini coefficients and compare them to those reported in CPS. This empirical test allows us to check how our micro model for individual income growth performs when aggregated to a macro level.

The rest of the paper is organized as follows. Section 2 introduces the model and discusses its key features and emergent properties. Next, section 3 describes the CPS tabulated data set with the emphasis on age-dependent differences in income and inequality dynamics. In section 4 we propose a methodology for calibrating out micro-founded model to the aggregated CPS data. Section 5 presents the model predictions for the age-dependent and overall Gini indexes, compares them to the measured characteristics of personal income distribution in various age groups. Finally, section 6 concludes.

2 Model

We start by presenting our model for the dynamics of personal income. The principal assumption we make in this version of the model is that each and every individual above fifteen years of age has a capability to work or earn money. To obtain money income, individuals may use one or several means or tools from the full set of available options that may includes paid job, government transfers, bank interest, capital gain, inter-family transfers, and others. The U.S. Census Bureau (2006) includes 42 money income components, and in what follows we employ their definition and measurements. It is important to stress that some principle sources of income are not included in the CB definition, which results in the observed discrepancy between aggregate (gross) personal income (GPI), as reported by the Bureau of Economic Aanalysis and the gross money income calculated by the CB.

In this section we summarize the formulation of a theoretical model, originally described in Kitov (2005a), and present it's closed form solution in a simplified setting. We start by assuming that the rate of money income, i.e. the overall income a person earns per unit time, $M(t)$, is proportional to her/his capability to earn money, σ. Since no person is isolated from the surrounding world - in this setting the economy, the work (money) s/he produces dissipates [1] through the interactions with the outside world, thereby decreasing the final income rate. The counteractions of external agents, which might be people or some externalities, determines the price of the goods and services a person creates. The price depends not on some absolute measure of the quality of the goods, but on the aggregate opinion of all the agents on relative merits of the producers, not goods, expressed in monetary units.

For example, the magic of famous brands provides a significant increase in incomes for their owners without proportional superiority in quality because people appreciate the creators not goods. As a whole, an equilibrium system of prices arises from the aggregate opinions on relative merits of each and every person, and not from the physical quantities and qualities of goods and services, as often assumed in economics. Personal incomes are thus ranked in a fixed hierarchy, which, when expressed in monetary units, is transformed in the dynamic system of prices. Since the hierarchy of incomes is fixed, the amounts and qualities of goods can only reorder individuals not change the final aggregate price of everything produced – GDP.

2.1 An equation for income dynamics

The model is primarily motivated by the observed paths of individual income growth and their variations. The evolution of income is described by a phase of concave growth in the initial stage of work experience, a satiation during prime working age and an eventual decline following the pick income. Given individual differences, these three stages may occur with different rates. Similar growth trajectories have been studied before and so we draw our model from many analogous processes observed in the natural sciences. An example from physics is a bulk heating of a body accompanied by cooling through its surface. For a uniform distribution of heating sources, the energy released in the body is proportional to its volume, or cube of characteristic linear size, and the energy lost through its surface is proportional to the square of the linear size. In relative terms, the energy balance or the ratio of cooling and heating is

[1] The conventional economic term for the process would be depreciation, but we feel that the physical term is more appropriate in this case

inversely proportional to the linear size. As a result, a larger body undergoes faster heating because it loses relatively less energy and also reaches a higher equilibrium temperature.

Applying this intuition to income, we realize that the rate of dissipation of income must be proportional to the attained income level per unit time. The rate of change in $M(t)$ is thus inversely proportional to the size of the means, or an instrument, used to earn money - we call this $\Lambda\,(t)$. Also, recall that $M(t)$ is proportional to$\sigma\,(t)$, the ability to earn money[2]. So, following the analogy from physics, we can write the following ordinary differential equation for the dynamics of income depending on the work experience, t:

$$\frac{dM(t)}{dt} = \sigma(t) - \frac{\alpha}{\Lambda(t)}M(t) \tag{1}$$

where $M(t)$ is the rate of money income denominated in dollars per year $[\$/y]$[3], t is the work experience expressed in years $[y]$, $\sigma(t)$ is the capability to earn money, and is a feature of an individual $[\$/y^2]$; $\Lambda(t)$ is the size of the earning means - a feature of the income source of an individual $[\$/y]$, and α is the dissipation coefficient $[\$/y^2]$. We also assume that $\sigma\,(t)$ and $\Lambda\,(t)$ are mutually independent - that is a person's ability is unrelated to her work instrument. Now, since we chose t to denote work experience, rather than a person's age or a fixed year, it is natural to assume that $M(0) = 0$, which is the initial condition for (4). At the initial point, $t = 0$, when the person reaches the working age (15 years old in the USA) her income is zero and then changes according to (4) as $t > 0$. Note that both $\sigma\,(t)$ and $\Lambda\,(t)$ can vary with t. This means that (4) has to be solved numerically, which is the approach we apply to calibrate the model to data in section 4. Before proceeding to the calibration stage, we first make simplifying assumptions, under which the model has a closed-form solution, which is discussed next.

2.2 Earning capability and instrument

For simplicity of notation, which will be explained in the subsequent subsection, we introduce a modified capability to earn money:

$$\Sigma\,(t) = \frac{\sigma\,(t)}{\alpha}$$

From this point onwards we will omit the word "modified" and refer to $\Sigma\,(t)$ simply as earning capability or ability.

For the purpose of completeness of the model we introduce a second time flow, τ, which represents calendar years. The time flow for work experience, t, and calendar years, τ, relate to each other in a natural fashion. For a simple illustration, consider a person that turns 15 in a year τ_0, i.e. her work experience is $t_0 = 0$. By year τ this person will have $t = \tau - \tau_0$ years of work experience. Consequently, τ is a global parameter that applies to everyone, whereas t is an individual characteristic and changes from person to person. We allow Σ and Λ to also depend on τ, thereby introducing differences in income capability and instrument among age cohorts. In other words, the model captures cross sectional and intertemporal variation in both parameters. Here, we also make a final simplifying assumption by letting $\Sigma\,(t, \tau_0)$ and

[2]Our understanding of ability could be interpreted in the spirit of human capital models, but we intentionally avoid making direct comparison here and leave it for future investigations.

[3]We present dimensions of each characteristic to check whether the equation is balanced.

$\Lambda(t, \tau_0)$ evolve as a square root of the growth rate in aggregate output per capita. Capability and instrument for fixed τ_0 thus evolve according to:

$$\Sigma(\tau_0, t) = \Sigma(\tau_0, t_0) \sqrt{\frac{Y(\tau)}{Y(\tau_0)}} = \Sigma(\tau_0, t_0) \sqrt{\frac{Y(\tau_0 + t)}{Y(\tau_0)}} = \Sigma(\tau_0, t_0)\, dY(\tau_0, t) \tag{2}$$

and

$$\Lambda(\tau_0, t) = \Lambda(\tau_0, t_0) \sqrt{\frac{Y(\tau)}{Y(\tau_0)}} = \Lambda(\tau_0, t_0) \sqrt{\frac{Y(\tau_0 + t)}{Y(\tau_0)}} = \Lambda(\tau_0, t_0)\, dY(\tau_0, t) \tag{3}$$

where $\Sigma(\tau_0, t_0)$ and $\Lambda(\tau_0, t_0)$ are the initial values of capability and instrument for a person with zero work experience in year τ_0; $Y(\tau_0)$ and $Y(\tau)$ are the aggregate output per capita values in the years τ_0 and τ, respectively and $dY(\tau_0, t) = Y(\tau)/Y(\tau_0) = Y(\tau_0 + t)/Y(\tau_0)$ is the cumulative GDP growth. Note that the initial values $\Sigma(\tau_0, t_0)$ and $\Lambda(\tau_0, t_0)$ essentially depend on the year when a person turns 15 only, τ_0, since the initial work experience is fixed at $t = 0$ for all individuals irrespective of when they start working. Consequently, we can restrict our attention to the initial values of capability and instrument as functions of the underlying initial year: $\Sigma(\tau_0)$ and $\Lambda(\tau_0)$, respectively. When we multiply equations (2) and (3) we notice that the product $\Sigma(\tau_0, t_0)\Lambda(\tau_0, t_0)$ evolves with time in line with growth of real GDP per capita. We call $\Sigma\Lambda$ the capacity to earn money, which means that $\Sigma(\tau_0, t_0)\Lambda(\tau_0, t_0)$ is the initial capacity.

Equation (1) can now be re-written to account for the dependence on the initial year, τ_0:

$$\frac{dM(\tau_0, t)}{dt} = \alpha\left(\Sigma(\tau_0, t) - \frac{1}{\Lambda(\tau_0, t)} M(\tau_0, t)\right) \tag{4}$$

Note that when we fix τ_0 and restrict our attention to an arbitrary person with growing work experience t, we return to our original equation (1). Moreover, the path of income dynamics depends on τ_0 only through the influence of the latter on the initial earning capability and instrument. In other words τ_0 only determines the starting position of the income rate and not the trajectory of the income path, which is completely described by equation (1).

It is natural to assume that the capability to earn money, $\Sigma(\tau_0, t)$, and the size of earning means, $\Lambda(\tau_0, t)$ are bounded above and below. Then they must have positive minimum values among all persons, $k = 1, \ldots, N$, with the same work experience t in a given year τ: $\min_k \Sigma_k(\tau_0, t) = \Sigma_{min}(\tau_0, t)$ and $\min_k \Lambda_k(\tau_0, t) = \Lambda_{min}(\tau_0, t)$, respectively, where $\Sigma_i(\tau_0, t)$ and $\Lambda_i(\tau_0, t)$ are the parameters corresponding to each individual in an economy. We can now introduce the relative and dimensionless values of the defining variables in the following way:

$$S(\tau_0, t) = \frac{\Sigma(\tau_0, t)}{\Sigma_{min}(\tau_0, t)} \tag{5}$$

and

$$L(\tau_0, t) = \frac{\Lambda(\tau_0, t)}{\Lambda_{min}(\tau_0, t)} \tag{6}$$

where $S(\tau_0, t)$ and $L(\tau_0, t)$ are the dimensionless capability and instrument, respectively, and are measured relatively to a person with the minimum values for these parameters.

Kitov (2005b) describes how a discrete uniform distribution for S and L results from a calibration process, here we only present the final outcome. Namely, we allow the relative initial values of $S(\tau_0, t_0)$ and $L(\tau_0, t_0)$, for any τ_0 and t_0, to take discrete values from a sequence of integer numbers ranging from 2 to 30. Overall, there are 29 different values of $S(\tau_0, t_0)$ and $L(\tau_0, t_0)$: $S_1(\tau_0, t_0) = 2, \ldots, S_{29}(\tau_0, t_0) = 30$, and similarly for $L(\tau_0, t_0)$.

The largest possible relative value of $S_{max} = S_{29} = L_{max} = L_{29} = 30$ is only 15 times larger than the smallest - $S_{min} = S_1$ and $L_{min} = L_1$. In the model, the minimum values Σ_{min} and Λ_{min} are chosen to be two times smaller than the smallest possible values of L_1 and S_1 respectively. Because the absolute values of variables Σ_i, Λ_j, Σ_{min} and Λ_{min} evolve with time according to the same law described in (2) and (3), the relative and dimensionless variables $S_i(\tau_0, t)$ and $L_j(\tau_0, t)$, $i, j = 1, \ldots, 29$, do not change with time thereby retaining the discrete distribution of the relative values. This means that the distribution of the relative capability to earn money and the size of the earning means is fixed over calendar years and age cohorts. The rigid hierarchy of relative incomes is one of the main implications of the model and is supported empirically in Kitov (2005a,b) for the period between 1994 and 2002.

The probability for a person to get an earning means of relative size L_j is constant over all 29 discrete values of the size and the same is valid for S_i. That means that in a given year τ, all people with the same work experience t, of age 15 or over are distributed uniformly among the 29 groups for the relative ability and instrument to earn money, respectively. Thus, the relative capacity for a person to earn money is distributed over the working age population as the product of independently distributed S_i and L_j:

$$S_i(\tau_0, t)\, L_j(\tau_0, t) \in \left\{ \frac{2 \times 2}{900}, \ldots, \frac{2 \times 30}{900}, \frac{3 \times 2}{900}, \ldots, \frac{3 \times 30}{900}, \ldots, \frac{30 \times 30}{900} \right\}$$

Overall, there are $29 \times 29 = 841$ values of the normalized capacities available between 4/900 and 900/900. Some of these cases seem to be degenerate (for example, $2 \times 30 = 3 \times 20 = 4 \times 15 = \cdots = 30 \times 2$), but since Σ and Λ have different influences on income growth in (4), each of the 841 combinations of $S_i L_j$ define a unique time history of income rate dynamics. The model implies that even though no individual future income trajectory is predefined, it can only be chosen from the set of 841 predefined individual paths for each single year of birth, or equivalently initial work year τ_0.

2.3 Solution with fixed parameters

Since the model contains time varying parameters, we use numerical methods to solve it and calibrate to data, as described in the subsequent section. However, in order to better understand the behavior of the system, we first consider a simplified case when $\Sigma(\tau_0, t)$ and $\Lambda(\tau_0, t)$ are constant over t. It is a plausible assumption since these two variables evolve very slowly with time, relatively to the observed income growth, and so can be neglected. Note that in the following exposition we fix τ_0 and so income evolution trajectories are a function of work experience t only. Now, given constant Σ and Λ, as well as the initial condition $M(0) = 0$, the general solution of equation (1) is as follows

$$M(t) = \Sigma\Lambda \left(1 - \exp\left(-\frac{\alpha}{\Lambda} t\right)\right) \tag{7}$$

The solution in (7) indicates that personal income rate depends on work experience, capability to earn money, the size of the means used to earn money and economic growth through an exponential function.

We can re-arrange equation (7) in order to construct dimensionless and relative measures of income. We first substitute in the product of the relative values S_i and L_j and the time dependent minimum values Σ_{min} and Λ_{min} for Σ and Λ. Note that for notational brevity we omit the dependence of parameters on time and experience. We also normalize the equation to the maximum values Σ_{max} and Λ_{max} in a given calendar year, τ, for a given work experience, t. The normalized equation for the rate of income, $M_{ij}(t)$, of a person with capability i, S_i, and the size of earning means j, L_j, where $i, j \in \{2, \ldots, 30\}$ is as follows:

$$\frac{M_{ij}(t)}{S_{max}L_{max}} = \Sigma_{min}\Lambda_{min}\left(\frac{S_i}{S_{max}}\right)\left(\frac{L_j}{L_{max}}\right)\left(1 - \exp\left\{-\left(\frac{\alpha}{\Lambda_{min}L_{max}}\right)\left(\frac{1}{L_j/L_{max}}\right)t\right\}\right) \quad (8)$$

or more compactly:

$$\tilde{M}_{ij}(t) = \Sigma_{min}\Lambda_{min}\tilde{S}_i\tilde{L}_j\left(1 - \exp\left\{-\left(\frac{1}{\Lambda_{min}}\right)\left(\frac{\tilde{\alpha}}{\tilde{L}_j}\right)t\right\}\right) \quad (9)$$

where

$$\begin{aligned} \tilde{M}_{ij}(t) &= \frac{M_{ij}(t)}{S_{max}L_{max}} \\ \tilde{S}_i &= \frac{S_i}{S_{max}} \\ \tilde{L}_i &= \frac{L_i}{L_{max}} \\ \tilde{\alpha} &= \frac{\alpha}{L_{max}} \end{aligned}$$

and $S_{max} = L_{max} = 30$. Note that Σ and Λ are treated as constant during a given calendar year, but evolve according to (2) and (3) as a function of work experience. The term $\Sigma_{min}(\tau_0, t)\Lambda_{min}(\tau_0, t) = dY(\tau_0, t)$ then corresponds to the total (cumulative) growth of real GDP per capita from the start point of a personal work experience, τ_0 ($t_0 = 0$), and vary for different years of birth. This term might be considered as a coefficient defined for every single year of work experience because this is a predefined exogenous variable. Consequently, it is possible to measure personal income in units of minimum earning capacity, $\Sigma_{min}(\tau_0, t)\Lambda_{min}(\tau_0, t)$, for each particular starting year τ_0. Equation (9) becomes dimensionless and the coefficient changes from $\Sigma_{min}(\tau_0, t_0)\Lambda_{min}(\tau_0, t_0) = 1$ in line with real GDP per capita. In the next subsection we provide simulations of the individual income trajectories under the assumption of constant parameters and compare them to the calibrated version, where the output growth is taken into account and the parameters are allow to vary. The details of the calibration procedure will follow in section 4.

2.4 Decay of income

The exponential growth trajectory of income described by equation (1) clearly does not present a full picture of income evolution with age. As numerous empirical observations show, average income among the population reaches its peak at some age and then starts declining. This is seen in individual income paths, for instance presented in Mincer (1974), as well as from the evolution of mean incomes in the 10-year age groups as defined by CPS, see figure 3. In our model this effect naturally result from setting the money earning capability $\Sigma(t)$ to zero[4] at some critical work experience, $t = T_c$. The solution of (4) for $t > T_c$ then becomes:

$$\tilde{M}_{ij}(t) = \tilde{M}_{ij}(T_c) \exp\left\{-\left(\frac{1}{\Lambda_{min}}\right)\left(\frac{\tilde{\gamma}}{\tilde{L}_j}\right)(t - T_c)\right\} \tag{10}$$

and by substituting in the solution from (9) we can write the following decaying income trajectories for $t > T_c$:

$$\tilde{M}_{ij}(t) = \Sigma_{min}\Lambda_{min}\tilde{S}_i\tilde{L}_j\left(1 - \exp\left\{-\left(\frac{1}{\Lambda_{min}}\right)\left(\frac{\tilde{\alpha}}{\tilde{L}_j}\right)T_c\right\}\right)\exp\left\{-\left(\frac{1}{\Lambda_{min}}\right)\left(\frac{\tilde{\gamma}}{\tilde{L}_j}\right)(t - T_c)\right\} \tag{11}$$

The first term in (11) is the level of income rate attained at time T_c. The second term represents an exponential decay of the income rate for work experience above T_c. The exponent index $\tilde{\gamma}$ represents the rate of income decay that varies in time and is different from $\tilde{\alpha}$. It was shown in Kitov (2005a) that the exponential decay of income rate above T_c results in the same relative, when normalized to the maximum income for this calendar year, income rate level at the same age. This means that the decay exponential can be obtained according to the following relationship:

$$\tilde{\gamma} = -\frac{\ln C}{A - T_c} \tag{12}$$

where C is the constant relative level of income rate at age A. Thus, when the current age reaches A, the maximum possible income rate $\tilde{M}_{ij}$ (for $i = 29$ and $j = 29$) drops to C. Income rates for other values of i and j are defined by (9). For the period between 1994 and 2002, empirical estimates of the parameters in (12) are $C = 0.72$ and $A = 64$ years (see Kitov, 2005a for details).

Consequently, people can only use their earning instrument, which keeps growing with time, but their capability remains at zero - income starting from some predefined (but growing with τ) point in time, T_c is a function of τ_0 and t, and experiences exponential decay. As an alternative, we could claim there exists a strong external process, which forces the exponential fall on top of the original capability. Initial exponential growth and eventual decay, however, do not complete the model. We still need to introduce special treatment for the very top incomes, that have been shown to follow Pareto distribution in multiple empirical studies - this is discussed in the following subsection.

2.5 Pareto distribution for top incomes

Because the exponential term in (7) includes the size of earning means growing as the square root of the real GDP per capita, longer and longer time is necessary for a person with the maximum relative values S_{29} and L_{29} to reach the maximum income rate (see figure 6. In order to account for top incomes, which evolve according to a power law, we need to assume that there exists some critical level of income rate that separates the two income classes:

[4]We realize that the implication of the model about people reaching a certain age completely lose their ability to earn money is very strong and will potentially cause substantial criticism due its political and ethical implications. We would like to stress that this is merely a feature of the model that allows us to match the empirical observations about income trajectories.

exponential and Pareto. We will refer to this level as Pareto threshold income, $M_p(t)$. Below this threshold, in the sub-critical income, personal income distribution (PID) is accurately predicted by our model for the evolution of individual income. Above the Pareto threshold, in the super-critical income zone, PID is governed by a power law. Any person reaching the Pareto threshold can obtain any income in the distribution with a rapidly decreasing probability governed by a power law.

The mechanisms driving the power law distribution and defining the threshold are not well understood not only in economics but also in physics for similar transitions. The absence of the explicit description of the driving mechanisms does not prohibit using well-established empirical properties of the Pareto distribution in the U.S. – constancy of the exponential index through time and the evolution of the threshold in sync with the cumulative value of the real GDP per capita (Piketty and Saez, 2003; Yakovenko, 2003; Kitov, 2005b, 2006). Therefore we include the Pareto distribution with empirically determined parameters in our model for the description of the PID above the Pareto threshold. The power law distribution of incomes implies that we do not need to follow each and every individual income as we did in the sub-critical income zone. All we need to know the number of people in the Pareto zone, i.e. the number of people with incomes above the Pareto threshold, as defined by relationships (8) and (11).

The initial dimensionless Pareto threshold is found to be $M_p(t_0) = 0.43$ (Kitov, 2005a) and evolves proportionally to growth in real output per capita:

$$M_p(t) = M_p(t_0)\frac{Y(t)}{Y(t_0)} \tag{13}$$

When personal income reaches the Pareto threshold, it undergoes a transformation and obtains a new quality to reach any income with a probability described by the power law distribution. This approach is similar to that applied in the modern natural sciences involving self-organized criticality. Due to the exponential (with a small negative index) character of income rate growth the number of people with incomes distributed according to the Pareto law is very sensitive to the threshold value. However, people with high enough S_i and L_j can eventually reach the threshold and obtain an opportunity to get rich, i.e. to occupy a position at the high-income end of the Pareto distribution[5].

2.6 Model finalized

Now, the model is finalized. Personal incomes in the sub-critical zone are proportional to the earning capacity S_iL_j - individual income grows in time according to equation (9) until the person reaches critical age T_c, above which an exponential decay according to (11) is observed. When income reaches the Pareto threshold, at any point in time, incomes can take any value with a probability declining with the income level according to a power law. Clearly, if a given income trajectory has not reached the Pareto threshold before T_c, the probability to

[5]Here one can introduce a concept distinguishing the below-threshold (sub-critical) and the above-threshold (super-critical) behavior of earners. Using the analogy from statistical physical, (Yakovenko, 2003) associates the sub-critical interval for personal incomes with the Boltzmann-Gibbs law and the extra income in the Pareto zone with the Bose condensate. In the framework of geomechanics, as adapted to the modeling of personal income distribution (Kitov, 2005a), one can distinguish between two regimes of tectonic energy release (Rodionov et al., 1982) – slow sub-critical dissipation on inhomogeneities of various sizes and fast energy release in earthquakes. The latter process is more efficient in terms of tectonic energy dissipation and the frequency distribution of earthquake sizes also obeys the Pareto power law.

enter the super-critical zone falls to zero, because it starts to decay exponentially. Personal income above the Pareto threshold at critical work experience starts to decrease and can break the Pareto threshold from above at some point, in which case a backward transition to the sub-critical level is observed.

Every year, each single year cohort above 15 years of age are divided into 841 groups according to their capacity to earn money - that is every year the total number of 15 year-olds is divided into 841 groups, 16 year-olds are divided into 841 groups and so on. Any new generation has the same distribution of S_i and L_j as the previous one, but different initial values of Σ_{min} and Λ_{min}, which evolve proportionally to the real GDP per capita annual growth rate. Consequently, the actual PID depends on the population distribution in the same age cohorts. The population age structure is taken as an exogenous parameter. The critical work experience, T_c also grows proportionally to the square root of real GDP per capita. Based on the independent measurements of the population age distribution and GDP one can easily model the evolution of the PID below and above the Pareto threshold.

Since the model defines the evolution of all individual incomes, with the knowledge of all exogenous parameters we can calibrate the model to data and test whether it can reproduce empirical observations about the overall PID. In this paper we chose to calibrate the model to the U.S. Census Bureau March CPS income data and compare our model predictions to the Gini coefficient for personal incomes for any given age, as estimated from the internal CPS data. Note that estimates of Gini indexes from the public CPS data will be different to the ones reported by CPS. Next, we present the data employed for calibrating model parameters and later discuss the calibration procedure.

3 Data

3.1 CPS age-dependent incomes

We have retrieved all data on personal money income from the Census Bureau's web site in various digital formats as well as the scanned copies of paper reports for the years between 1947 and 2011. The latter readings were converted into a digital format. The data set includes sixty five annual personal income distributions, the estimates of mean income in various age groups, and the estimates of Gini ratio made by the CB. The PIDs are given in income bins, the number and widths of which have been varying since 1947. Figure 1 displays the evolution of bin counting. Since 2000, the number of bins for the working age population as a whole is 45 and 42 for the ten-year-wide age groups. The bin with the highest incomes is open-ended and includes all persons with incomes above the maximum income threshold. We should also point out the well-known problem of topcoding of high incomes, which may significantly affect the overall estimates (Larrimore et al., 2008; Burkhauser et al., 2011). This paper, however, presents a preliminary attempt to test our model and so we avoid discussion about correcting topcoded incomes and their cell means. For annual output per capita growth rates we use data from the Bureau of Economic Analysis are and before 1929 we use the Maddison historical data base.

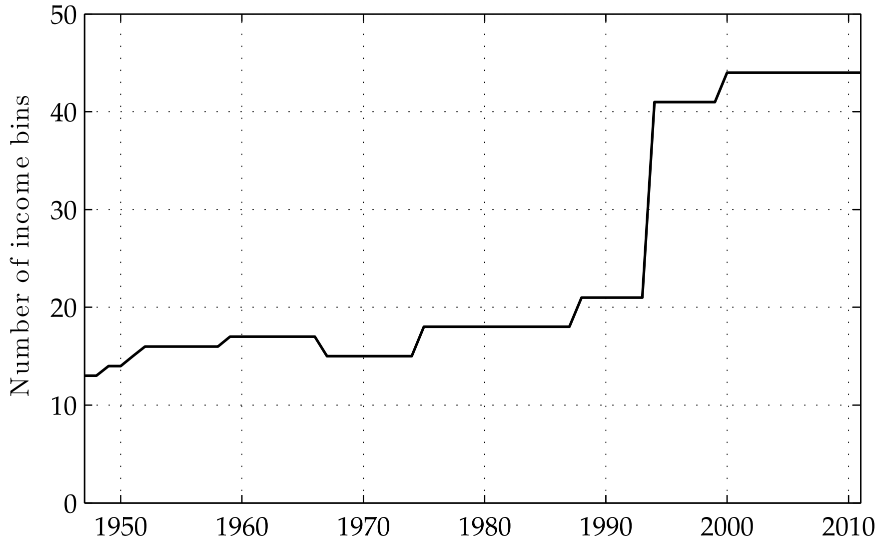

Figure 1: The number of money income bins. Since 2000, the number of bins for the whole population was extended to 45 with the highest (open-end) income bin starting from $250,000, while the age dependent high-income bin starts at $100,000.

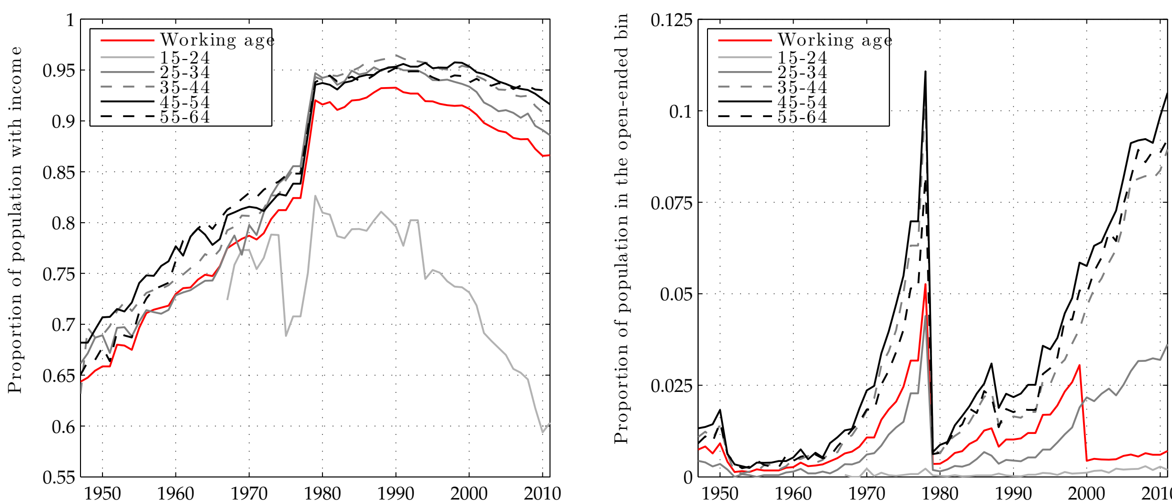

Figure 2: *Left panel:* The portion of population with income according to the CB definition in various age groups in the USA since 1947. Total – above 15 years of age. In the group between 15 and 24 years of age, the portion has been falling since 1979. Notice the break in the distributions between 1977 and 1979 induced by large revisions implemented in 1980 – "Questionnaire expanded to show 27 possible values from 51 possible sources of income." *Right panel:* The portion of population in the open-end high income bin.

Figure 2 illustrates the coverage of two extremes of the PIDs - people without income and people with the highest incomes included in the open-ended bin. The share of people with income according to the CB definition has been increasing since 1947, reached its peak in 1987 and has been decreasing since 1999. The share of population in the highest income bin varies in time and differs between the age groups. The Census Bureau has to adjust the bin structure and to increase the lower boundary of the upper income bin when the portion of people reaches 10%. In 2011, this limit was practically reached in three age groups and so we expect a new revision to the bin structure any time soon. Otherwise, the income distribution of rich people, which is described by a Pareto law, is not resolved.

Figure 3 illustrates an important characteristic of the observed PID evolution - the age at which the peak average income is reached has been increasing in time. In 1948, people with work experience between 20 and 29 years (i.e. 35 to 44 years of age) had the highest mean income. In 2011, the peak moved to the age group with 40 to 49 years of work experience. The change in the age of peak income is a defining parameter of our model, as will be described in the following section. Figure (3) underlines the growing difficulty for younger people to get jobs. In 1948, people were reaching 0.8 of the peak income after 10 years of work, whereas in 2011 this number increased to 16 years, given the approximations within the 10-year age groups and a cubic spline interpolation between groups. Unfortunately, with a growing real GDP per capita, younger people will suffer an increasing difficulty since the peak age rises as its square root. The right panel in figure 3 shows income in age groups normalized to the income of the group with the higher mean income in each year. The declining trends for the two youngest groups clearly indicate that their average incomes have been falling relatively to the group with the highest incomes. Given that the average across all ages has been growing in the last 20 years, this emphasis the declining proportion of incomes attributed to the youngest cohorts in the population, as they earn less compared to the more experienced.

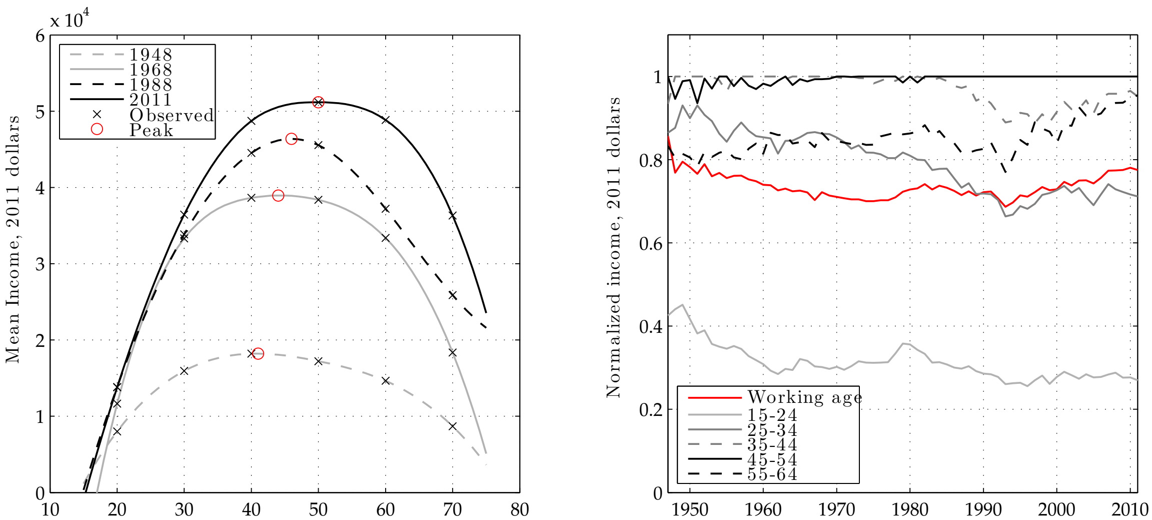

Figure 3: The evolution of mean personal income. *Left panel*: years 1948, 1960, 1974, 1987, and 2011- mean income normalized to peak value in these years. *Right panel*: Mean income in various age groups normalized to peak value in a given year. The age of peak mean income changes with time.

3.2 Age-dependent Gini ratios

One of the most popular aggregate measure of income inequality is the Gini ratio, G. This measure is characterized by a number of advantages such as relative simplicity, anonymity, scale independence, and population independence. On the other hand, the Gini ratio belongs to the group of operational measures - its evolution in time is not theoretically linked to macroeconomic variables and the differences in Gini ratio observed between various countries are not well understood. These problems make the Gini ratio more useful in political and social discussions, but not in economics as a quantitative science.

Figures 4 depict the evolution of Gini ratio in time as reported by the Census Bureau for the years after 1994 and in figure 5 we present our estimates from the PIDs since 1947. The evolution of the Ginis is related to economic growth and changes in the age structure. Kitov (2008b) also demonstrated that the empirical estimates of Gini ratios converge to the theoretical ones when all individuals in the working age population have income. Such convergence might be clearly observed in the age-dependent PIDs, since the portion of population without income decreases with age.

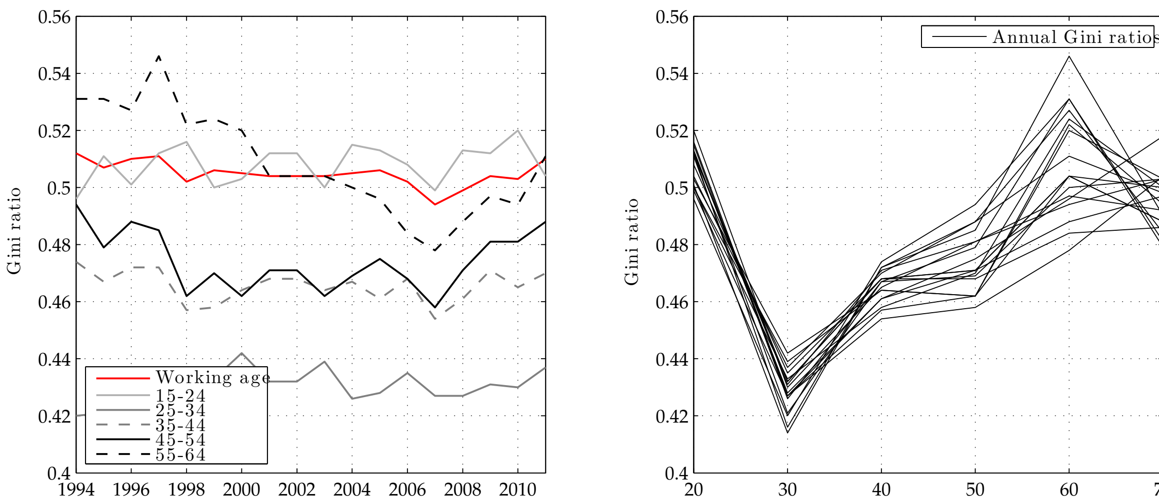

Figure 4: Left panel: The evolution of Gini ratio in various age groups since 1994. Central panel: The age dependence of Gini index in ten-year-wide groups for the years between 1994 and 2011. Right panel: The age dependence of Gini index in five-year-wide groups for selected years between 1998 and 2011. The age of the peak Gini ratio increases with time.

Information on each and every individual income is not available form the CB database. In this situation, Gini ratio can be estimated from data. For example, if (X_i,Y_i) are the values obtained from the CPS, with the X_i indexed in increasing order $(X_{i-1} < X_i)$, where X_i is the cumulative proportion of the population variable, and Y_i is the cumulative proportion of the income variable, then the Lorenz curve can be approximated on each interval as a straight line between consecutive points and the resulting approximation for G will be as follows:

$$G = 1 - \sum_{i=1}^{n} (X_i - X_{i-1})(Y_{i-1} + Y_i), \text{ for } i = 1, \ldots, n \tag{14}$$

In addition, one can approximate the Lorenz curve between consecutive points (X_i, Y_i) using exponential splines or, where appropriate, the power law for the interpolation of the underlying PIDs, as proposed by Dragulescu and Yakovenko (2001). The choice of an appropriate function for the PID interpolation reveals an important problem of the CPS - the usage of the same income bins during relatively long periods of time. The growth rate of nominal GDP in the U.S. has been high - more people obtained larger incomes above the predefined upper income limit in CPS questionnaire and found themselves in the group "$MAX and over". Consequently, the coverage of population below and above the Pareto threshold, which has been also proportionally growing, has been changing significantly. This variation in the coverage might potentially result in an increasing or decreasing overall resolution and corresponding bias in the estimations of Gini ratio. In addition, under-coverage of the highest incomes is a potential source of on-going discussion about increasing income inequality in the U.S. The absence of good quantitative estimates may result in a wrong interpretation of income inequality.

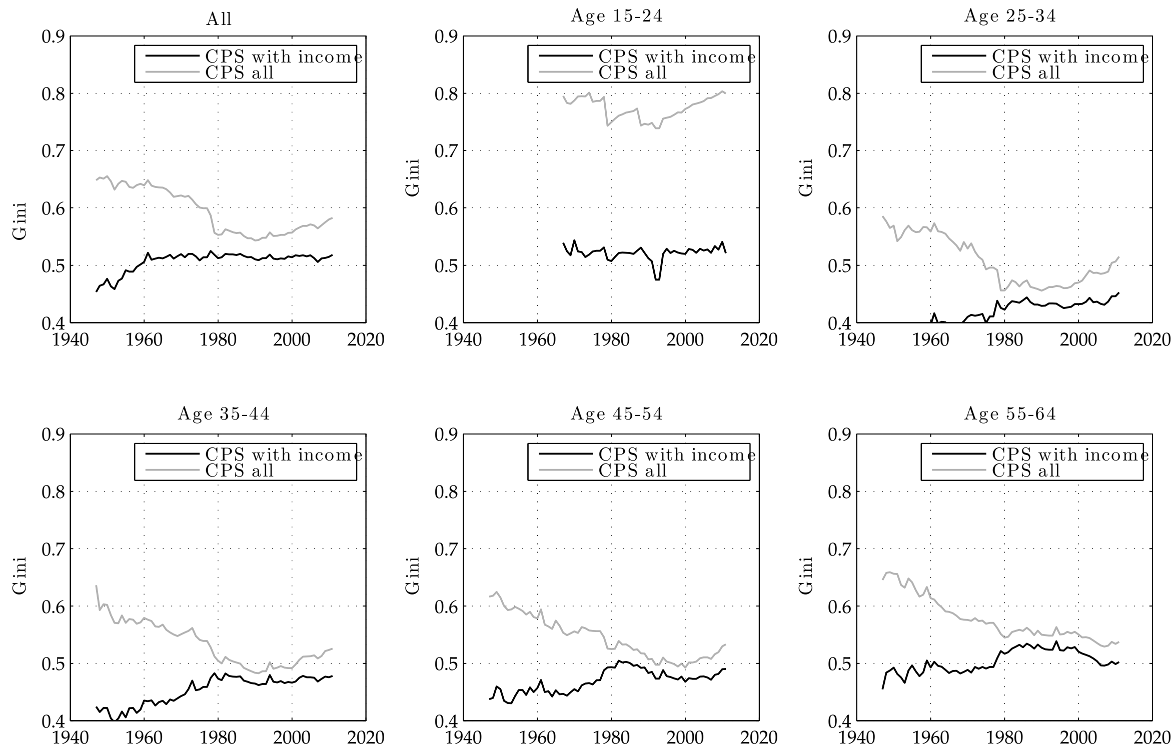

Figure 5: The evolution of Gini ratio in various age groups since 1947 as estimated from the distributions of personal income reported by the Census Bureau. The Gini ratio estimates explicitly reported by the since 1994 CB are also shown.

In the income range below the Pareto threshold, one can use quasi-exponential distribution and estimate mean income in the relevant bins. In the high-income zone, a power law approximation is a natural choice for the PIDs. Theoretically, the cumulative distribution function, CDF, for the Pareto distribution is defined by the following relationship:

$$F\left(x\right) = 1 - \left(\frac{x_m}{x}\right)^k \tag{15}$$

for all $x > x_m$, where k is the Pareto index. Then, the probability density function (pdf) is defined as

$$f\left(x\right) = \frac{k}{x}\left(\frac{x_m}{x}\right)^{k-1} \tag{16}$$

Functional dependence of the probability density function on income allows for the exact calculation of total population in any income bin, total and average income in this bin, and the input of the bin to the corresponding Gini ratio because the pdf defines the Lorenz curve. Thus, if populations are counted in some predefined income bins, then the relevant Lorenz curve can be constructed for a given value of the Pareto index, k. We use the relationship for $f(x)$ in the following calculations of empirical Gini ratios in the Pareto zone in all age groups. By definition, the Pareto threshold evolves proportionally to nominal GDP per capita, and does not depend on age.

The index k, however, depends on age (Kitov, 2008a). The evolution of the Pareto law index (slope) with age is as follows: $k = -1.91$for the age group between 25 and 34 years; $k = -1.48$ between 35 and 44; $k = -1.38$ between 45 and 54; $k = -1.14$ in the age group between 55 and 64. It is clear that index k declines with age. Obviously, a smaller index k corresponds to an elevated PID density at higher incomes and larger Gini ratio. The observed decrease in k with age deserves a special examination and should be inherently linked to some age-dependent dynamic processes above the Pareto threshold.

The declining k is a specific feature of the age-dependent PIDs, which is incorporated in our model. Kitov (2008b) estimates $k = -1.35$ for the population of 15 years of age and over, i.e. within the range of its change with age. One can expect, however, that the age-dependent and overall k might also undergo some changes over time. The latter index may vary just because of the changing age pyramid, i.e. changing input of various ages to the net k. For the empirical estimates of the Gini ratios carried out below, the observed variation in this index plays insignificant role because we use actual income distributions. For theoretical estimates, Gini ratios might be overestimated for the youngest age group and underestimated in the oldest age group when one uses $k = -1.35$.

The U.S. Census Bureau presents two versions of the PID – for total working age population and for population with some reported income. We have calculated empirical G-ratios in several (fixed) age groups between 1967 and 2011. Figure 5 displays the evolution of Gini ratio in all groups since 1947, except in the youngest one, for which the data is only available from 1967. The latter group is characterized by severe variations in measurement methodology and definition of income. This makes it difficult to distinguish actual and artificial features in the evolution of G in the youngest age group. The curves associated with all people aged in given ranges are marked "all", and those including only people with reported incomes – "with income". The major revision to income definition between 1977 and 1979, which dramatically increased the portion of people with income, induced sharp decrease in the curves named "all", and opposite changes in the curves "with income". For obvious reasons, the Gini ratios for people with income are systematically lower than those for the entire population.

One important feature of the empirical Gini curves was also mentioned in Kitov (2008b). Before 1977, the portion of population without income was big enough to introduce a significant bias in the estimates of Gini ratio. It was overestimated for the entire population and was underestimated for the population with income. The same effect is observed for the age-dependent Gini. Before 1977, one can observe large changes over time. After 1977, all curves are approximately horizontal, with only a slight decline. Hence, one can expect large deviation between these empirical curves and theoretical ones before 1977.

The age-dependent PID in the youngest group is characterized by a larger difference from the overall PIDs. Obviously, all individuals start with zero income and the initial part of income trajectory in time, as personal income observations show, is close to an exponential growth. In the mid-age groups, PIDs are similar to the overall PID. In the oldest age group, PID is also different and is closer to that in the youngest group. Accordingly, Gini ratio undergoes a substantial evolution from the youngest to the oldest age groups. One of the main goals of this paper is then to predict the evolution of age-dependent Gini ratios and explain the apparent difference between the age groups.

4 Results

In this Section, we will present the estimates of the optimal set of calibrated model parameters describing the observed PIDs, estimates of the mean income and compare our model predictions of the Gini ratios with the observed values estimated from the CPS data.

4.1 Simulating income paths

To being out description of the calibration procedure we should note that it is not possible to quantitatively estimate the value of the dissipation factor $\tilde{\alpha}$ using some independent measurements. Instead, a standard calibration procedure is applied. By definition, the maximum relative value of L_j is equal to 1 at the start point of the studied period, t_0. The value of $\Lambda_{min}(t_0)$ is also assumed to be 1. Thus, one can vary $\tilde{\alpha}$ in order to match predicted and observed PIDs, and the best-fit value of $\tilde{\alpha}$ is used for further predictions. The range of $\tilde{\alpha}/\Lambda_{min}$ from 0.09 to 0.04 approximately corresponds to that obtained in the modeling of the U.S. PIDs during the period between 1960 and 2002 (Kitov, 2005a). The actual initial value of $\tilde{\alpha}$ is found to be 0.086 for $t_0 = 1960$. The value of Λ_{min} changes during this period from 1.0 to 1.49 according to the square root of the real GDP per capita growth. The cumulative growth of the real GDP per capita from 1960 to 2002 is 2.22 times.

Figure 6 illustrates the difference between the closed form solution (9) with fixed parameters and the numerical solution with $\Sigma_{min}(\tau_0, t)\,\Lambda_{min}(\tau_0, t)$ varying in time proportionally to the GDP per capita. We use various capacity values to describe the income trajectory of a 75-year-old person in 1930 and 2011, i.e. someone with 60 years of work experience.

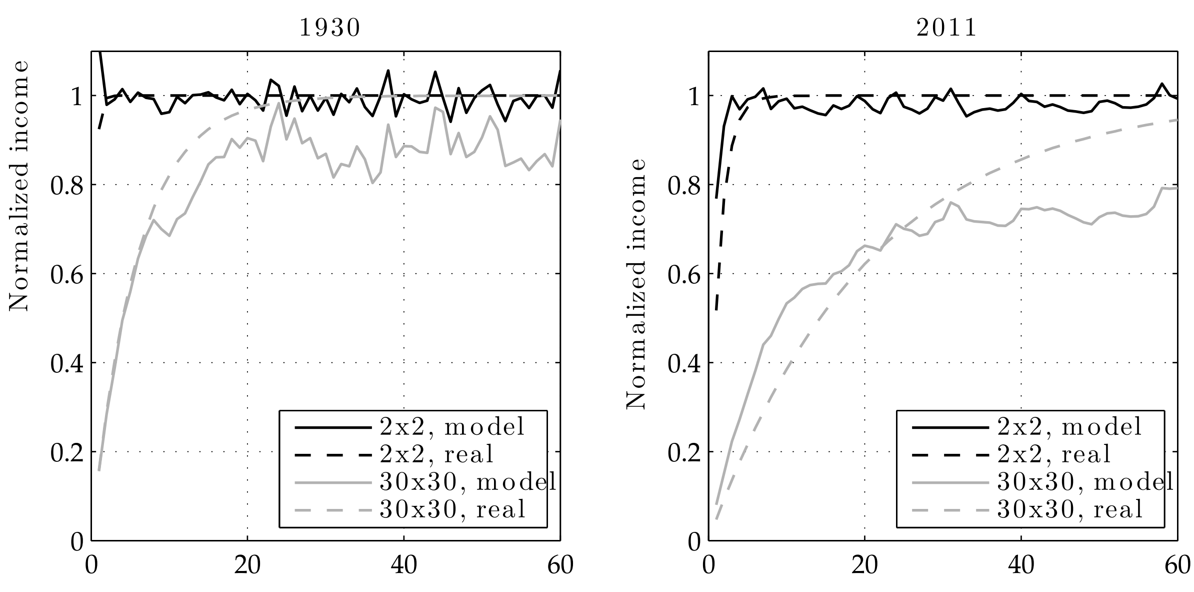

Figure 6: The evolution of personal income for different capacities for a 75-year-old person (work experience 60 years) in 1930 and 2011.

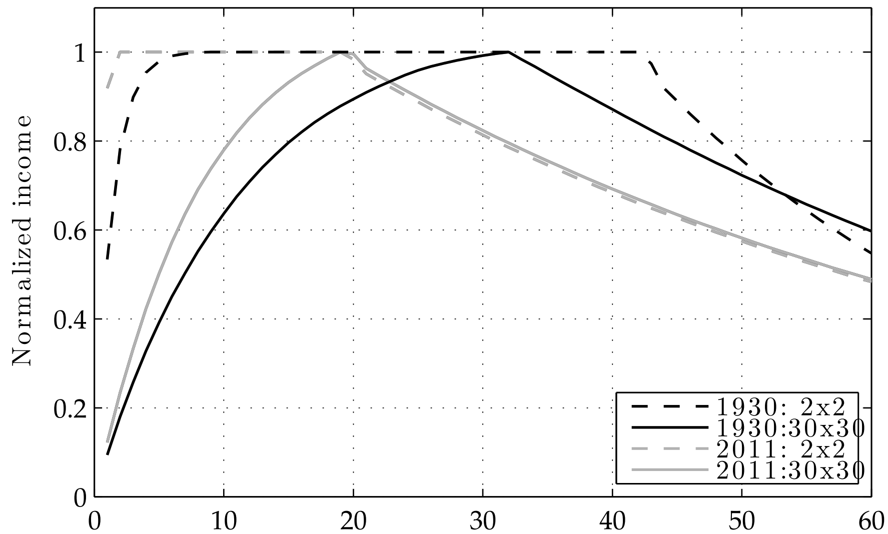

Figure 7: The change in the personal income distribution between 1930 and 2011 associated with growing T_c and larger earning tool, L. The exponential fall after T_c is taken into account.

Equation (9) represents the rate of income for a person with the defining parameters $S_i(\tau_0, t)$ and $L_j(\tau_0, t)$ at time t relative to the maximum possible personal income rate. The maximum possible income rate is earned by a person with $\tilde{S}_{29} = \tilde{L}_{29} = 1$ at time t. The coefficient Λ_{min}^{-1} in the exponential term in (9) evolves inversely proportional to the square root of real GDP per capita. This is the defining term of personal income evolution, which accounts for the differences between the start years of work experience. The numerical value of the ratio $\tilde{\alpha}/\Lambda_{min}$ is obtained by calibration to the initial year of the modeling. This calibration assumes that $\Lambda_{min}(\tau_0, t_0) = \Sigma_{min}(\tau_0, t_0) = 0$ at the start point of the modeling and only the dimensionless factor $\widetilde{\alpha}$ has to be empirically determined. In this case, the absolute value of $\tilde{\alpha}$ depends on the start year.

Figure 7 illustrates the exponential fall after T_c in 1930 and 2011 for two different work capacities and changing GDP per capita[6]. All curves are normalized to the picks of the respective trajectories for direct comparison on relative paths. In absolute scale, the income evolution path for an individual with a 30×30 earning capacity lies much higher than that of a 2×2 individual. Note also that for the individuals with equal capacities the pick income work experience grows with time (compare the solid lines).

4.2 Pareto incomes

There is a principal feature of the real PID, which is not described by the model so far, but has an inherent relation to the studied problem. The real income distribution spans the range from \$0 to several hundred million dollars, and the theoretical distribution extends only from \$0 to about \$100,000, i.e. the income interval used (Kitov, 2005a) to match the observed and

[6]A physical analog of such decay is cooling of a body, for example, the Earth. When all sources of internal heating (gravitational, rotational, and radioactive decay) disappear, the Earth only will be loosing the internal heat through the surface before reaching an equilibrium temperature with the outer space. This process of cooling is also described by an exponential decay because the heat flux from the Earth is proportional to the difference of the temperatures between the Earth's surface and the outer space.

predicted distributions. The power law distribution starting from the Pareto threshold income (from \$40,000 to \$60,000 during the last fifteen years) describes incomes of about ten per cent of the population. The theoretical threshold of 0.43 was introduced above, partly, in order to match this relative number of people distributed by the Pareto law. The model provides an excellent agreement between the real and theoretical distributions below the Pareto threshold. Above the threshold, the theoretical and real distributions diverge.

Above the Pareto threshold, the model distribution drops with an increasing rate to zero at about \$100,000. This limit corresponds to the absence of the theoretical capacity to earn money, S_iL_j, above 1. The dimensionless units can be converted into actual 2000 dollars by multiplying by a factor of \$120,000, i.e. one dimensionless unit costs \$120,000. The observed distribution decays above the Pareto threshold inversely proportional to income in the power of ≈ 3.5. Hence, actual and theoretical absolute income intervals are different above the Pareto threshold and retain the same portion of the total population ($\approx 10\%$). Thus, the total amount of money earned by people in the Pareto distribution income zone, i.e. the sum of all personal incomes, differs in the real and theoretical cases.

Total amount of money earned in the super-critical zone or extra income is of 1.33 times larger than the amount that would be earned if incomes were distributed according to the theoretical curve, in which every income is proportional to the capacity. This multiplication factor is very sensitive to the definition of the Pareto threshold. In order to match the theoretical and the observed total amount of money earned in the super-critical zone one has to multiply every theoretical personal income in the zone by a factor of 1.33. This is the last step in equalizing the theoretical and the observed number of people and incomes in both zones: sub- and super-critical. It seems also reasonable to assume that the observed difference in distributions in the zones is reflected by some basic difference in the capability to earn money.

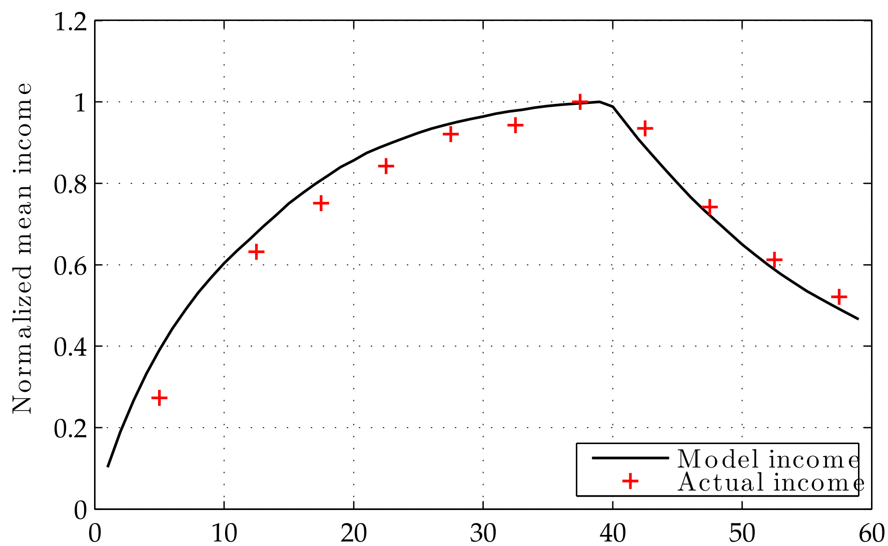

Figure 8: Normalized mean income in 1998: actual vs. modeled

4.3 Predicting Gini coefficients

The accuracy of theoretical estimates of Gini ratio is related to the quality of PIDs' prediction. Figure 9 indicates that the Gini ratios for the age groups over 34 years vary in a narrow range. This observation presumes that the underlying PIDs are very similar. Kitov (2007) demonstrated that the PIDs for the entire working age population (with income) for the years between 1967 and 2005 collapse practically to one curve when normalized to populations and nominal GDI (instead of GDP). Real GDP drives two key parameters in our model: critical work experience, T_c, and the size of earning tools, $\sigma(t)$. Despite the fact that GPI/GDP ratio has been varying over time since the start of the CPS we use real GDP per capita.

Figure 8 displays the theoretical and observed mean income curve for 1998. There is a good agreement between them, which is obtained with the standard model parameters. Note that here we use mean incomes in the more refined 5-year age groups, which provides a better resolution. However, model parameters and model predictions were calibrated to the 10-year age groups, since they are available for a much longer time span. Note that mean incomes first growth exponentially with age and then starts its decline after about 40 years of work experience. Out model can very accurately predict this development in other years as well, but not shown here for brevity.

Having observed that our model predictions can very well capture the evolution of true mean incomes, we produce the estimates of the Gini ratios from our model simulations. Figure 9 presents the evolution of the observed and predicted Gini ratio in four age groups (due to very high measurement discrepancies the youngest age group is omitted deliberately) and for the whole population over 15 years of age. For the sake of simplicity, we predicted Gini using the same index $k = -1.35$ for all age groups. Recall that this may not produce the most accurate results, since we have found that the Pareto exponential may vary both in time and across age cohorts.

Overall, we observe a good match between the observed and predicted curves, especially when the whole working age population is considered. This primarily concerns the evolution of the Gini coefficients for individuals with income. The curves for the entire population, that is those that include persons with and without income, lie above the predicted and observed "with income" lines. However, we observe that for the age groups with high ratios of individuals with income , 45-55 and 54-65, the three curves are much closer. Clearly, the two observed curves should converge as the proportion of people with income approaches unity, which should also result in our model predictions matching the observations much closer.

Even though there is some discrepancy between the predicted and observed age-dependent Ginis, the overall match is very close. This is emphasized by a practical convergence of our model curve and CPS estimates for the working age population Gini after 1965. The average margin of error is smaller than 0.01 and we can therefore conclude that our model does capture the overall structure of the U.S. personal income distribution quite well. This is a striking result, given that the only exogenous parameters of the models are output growth, population distribution and total incomes in the 10-year age groups. That is even with very limited data resolution on the aggregate level we were able to calibrate our micro-founded model, predict the income path trajectories for each person in the U.S. population in each of the analyzed year and by aggregating individual incomes produce the measures that have very good agreement with the observed trends in income inequality.

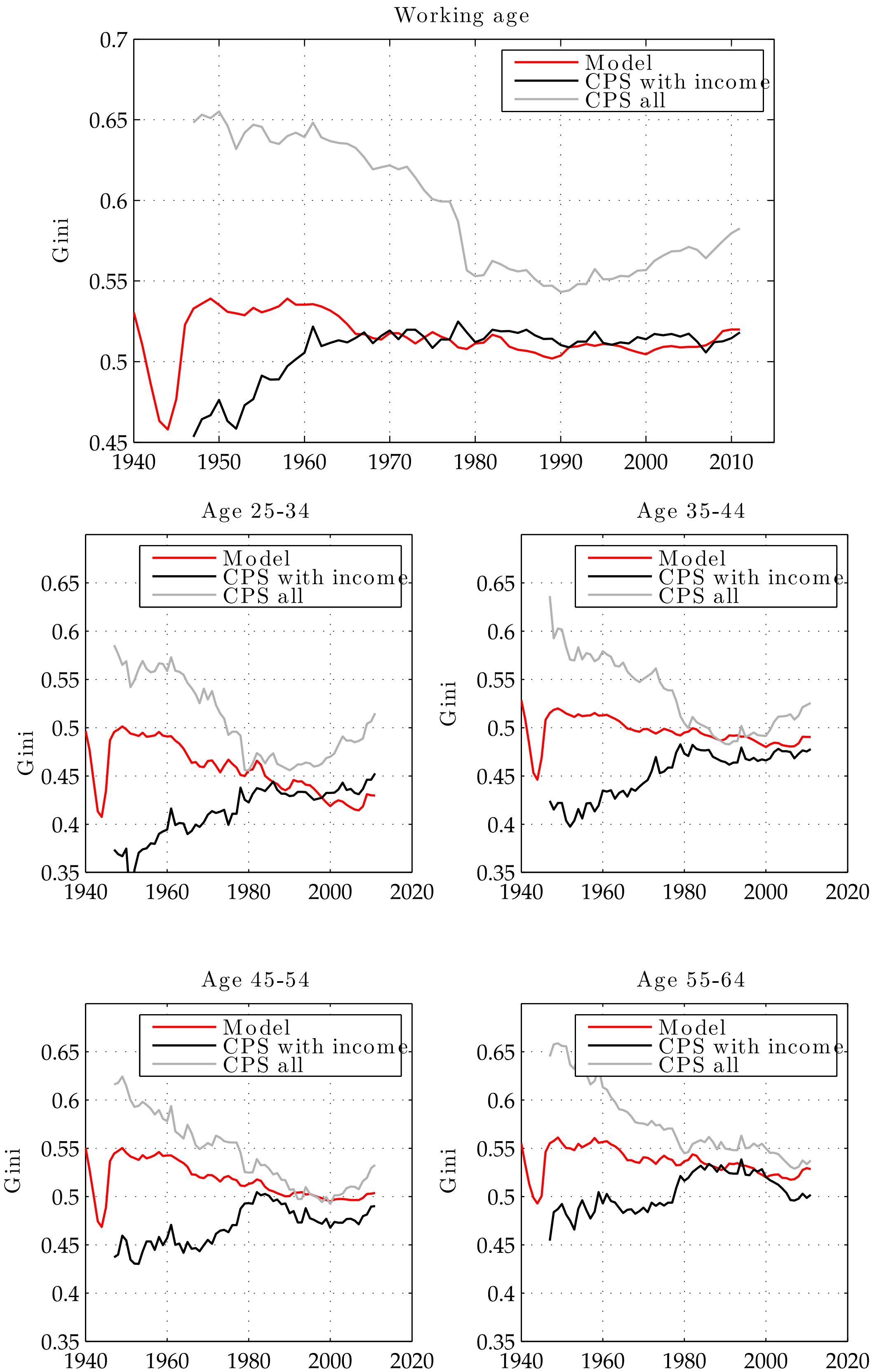

Figure 9: The evolution of the measured (blue lines) and predicted (red line) Gini ratios in various age groups.

5 Conclusion

This study was primarily carried out for further validation of the microeconomic model that describes the evolution of personal incomes in the United States. Our previous findings (Kitov, 2005b, 2006, 2007, 2008a) revealed some problems with the income definition, which did not allow a comprehensive description of the overall PID. The most important problem was that a large portion of population did not report any income. Another problem is a poor resolution of PIDs before 1977. Furthermore, in this paper we did not correct for topcoded incomes and did not make any additional adjustments to the censoring of incomes, as applied to by the Census Bureau to high incomes. We propose to use income imputation methodologies, similar to the the ones developed in Piketty and Saez (2003) and Burkhauser et al. (2011), in order to improve the resolution of top incomes. Moreover, given that CPS does not provide good data for the top end of the income distribution, it should be possible to obtain additional information from the tax returns data provided by the IRS. Reconciliation of CPS and IRS measurements should provide to be a relatively straight forward task when the income definitions are matched (Burkhauser et al., 2012). We leave this problem for further investigations.

In our model, every person above 15 years of age is assigned a non-zero income. That is we assume that any person that can legally work has some source of income. This discrepancy results in a significant deviation between observed and predicted Gini ratios for the age groups with a high proportion of individuals without incomes, especially in the younger cohorts. The age-dependent PIDs allow overcoming this discrepancy because the proportion of the population without income is very low (~2%) for ages over 45 years. Therefore, we find that the model predicts the Gini ratio in these age groups much more precisely. This paper confirms that the evolution of the Gini ratio for the years with a good PID resolution was accurately predicted (up to three decimal places). The predicted Gini for the overall population with incomes practically matches the one estimated directly from CPS since 1965.

As expected, the gap between the Gini ratio associated with the entire working population in a given age interval and that associated with people reporting income converge with the decreasing portion of people without income. The true Gini ratio had to be somewhere between these two estimates. For instance, in the group between 45 and 54 years of age, only 3% of the population is reported to have no income, which results in the observed gap between the respective Gini indices of less than 0.02. In all age groups, the model predicts slightly decreasing Gini ratios between 1967 and 2011. Nevertheless, the overall Gini is approximately constant, and any minor deviations are related to the economic growth and changes in the age structure of the American population.

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