Wealth Inequality, Unequal Opportunities and Inefficient Credit Markets*

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June 2013

Abstract

This paper investigates the impact of heterogeneous wealth on credit allocation from an egalitarian opportunity and an efficiency point of view. Due to DARA, poor entrepreneurs, other things equal realize better projects. This notwithstanding, due to bidimensional hidden information they may be rationed out or obtain a loan only at the cost of cross subsidizing bad projects realized by rich entrepreneurs. In the first case inefficiency arises in the form of insufficient investment, in the second in the form of inefficient projects being realized. An egalitarian redistribution of endowments may lead to perfect screening, no inefficiencies in the allocation of credit and EOp.

Keywords: Wealth, Credit, Cross-subsidization, DARA, Equality of Opportunity

JEL classification: D31; D82; G21

*We are indebted to Alberto Bisin, Alberto Martin, Maitreesh Ghatak, Pierre-Olivier Gourinchas, Alireza Naghavi, Marco Pagano and Tommaso Oliviero for useful comments. We are also grateful to the participants at the Conference of the Association for Public Economic Theory (APET) in Taipei (June, 2012), the Workshop on Institutions, Individual Behaviour and Economic Outcomes in Argentiera, Sardinia (September, 2012), the audience of the CSEF seminar in Naples (September, 2012), the XXI International Conference on Money, Banking And Finance (Rome, December 2012) and the Royal Economic Society Conference (Royal Holloway, April 2013). Responsibility for mistakes remains entirely ours.

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1 Introduction

The credit market is supposed to transfer resources from savers without an entrepreneurial option, to illiquid would-be entrepreneurs, creating surplus in the process. However imperfect information may interfere with a fair and efficient allocation of credit. Collateral may be required for incentive or screening purposes putting the relatively poor entrepreneurs at a disadvantage. We investigate the properties of equilibria in credit markets with heterogeneous wealth entrepreneurs characterized by DARA, under the Equality of Opportunity benchmark and explore the relationship with correlated inefficiencies in credit allocations.

The ideal of a society in which no one suffers disadvantage on grounds of unequal opportunities is widely upheld as desirable. The requirement extends far beyond the vague injunction to eschew public sphere discrimination as it implies a central normative investigation for deciding on which grounds one might justify responsibility-sensitive policy interventions. The idea of competing on equal terms was formalized in the Equality of Opportunity (EOp) principle which requires the distinction between unchosen circumstances and individual choices (see among egalitarian philosophers, Rawls (1971) and Sen (1973)). The former are terms imposed on an individual in ways that she could not have influenced or controlled; these terms are just given. The latter, instead, constitute the personal responsibility of individuals. Main exogenous circumstances for instance include the wealth inherited and early environment provided by parents and, in general, all the features of the world in which one finds oneself prior to any opportunity for responsible choice (Roemer, 1998). In the last 20 years the opportunity egalitarian literature has extended on the measurement of inequality of opportunity as a tool to implement an efficiency-enhancing redistribution.\footnote{A variety of measures was adopted with the aim of separating ‘inequality of opportunity’ and ‘responsibility sensitive inequality’, and was applied mainly in the context of income inequality but also in taxation, education, health. For a recent survey, see Pignataro (2012).}

The EOp principle requires that jobs and options to borrow money for investment purposes, such as starting a business, should be open to all young applicants. However a strong evidence demonstrates unequivocally that entry in the credit market is heavily wealth-dependent, and that potential investment is constrained by personal and family wealth (Evans and Jovanovich, 1989). Most empirical contributions lie in the field of development economics,
where inefficiencies in credit market are likely to be tantamount (for comprehensive surveys, see Benabou, 1996, Banerjee, 2002; Banerjee and Duflo, 2010). Wealth dependence can be the product of many factors, among which endogenous risk preferences (Cressy, 2000) or myopic consumption, but most recent evidence refutes the hypotheses that these factors are the sole relevant (see for instance Kan and Tsai, 2006 and Berg, 2012). The likely explanation is that imperfect information may force the bank to choose on the basis of collateral provision. This in turn may be due to the need to control for opportunistic entrepreneurial behavior or to screen better applicants in presence of hidden information. We build an imperfect information model that fits well with most stylized facts about credit markets in developing economies as reported by Banerjee and Duflo (2010), in particular with the fact that rich people borrow more (wealth dependence) and pay lower interest rates. We will use a further stylized fact, e.g., that monopoly power does not appear to be a cause for high interest rates, as an assumption.\footnote{The last fact reported, high lending rates, is outside the scope of this paper.}

Our model entails bidimensional heterogeneity among individuals with hidden information and moral hazard. Potential entrepreneurs differ for both circumstances and personal responsibility. Circumstances are perfectly represented by the ex-ante endowed wealth, while the individuals’ responsibility variable is codified as effort aversion (an indicator of preferences) affecting the measure of the chosen actions (effort) the individual takes. Effort aversion affects the individual willingness to supply effort and therefore measures (inversely), other things equal, the propensity of individual to work hard. We want to investigate the properties of the equilibrium in a framework where both features are unobservable by competitive lenders. We investigate in particular whether moral hazard and adverse selection involve the violation of the equality of opportunity principle.

Our characterizing assumptions are that individuals’ wealth is not publicly observable, while agents exhibit decreasing absolute risk aversion (DARA, hereafter). This first assumption requires some discussions\footnote{For a fuller analysis of this assumption see our previous paper Coco and Pignataro (2012) where it was first introduced.}. In most of the credit market literature (with the exceptions of Stiglitz and Weiss, 1992 and Coco and Pignataro, 2012) wealth is supposed to be observable while entrepreneurial ability is not. However in other fields (for example tax evasion),
the idea that financial income and wealth positions of individuals are common knowledge would be considered rather odd. The idea that a bank official can assess the extent of one individual’s wealth is even stranger. In most papers the assumption of common knowledge on wealth possibly proxies for the belief that there are no reasons to conceal one’s wealth. In this paper this is not the case because decreasing risk aversion may turn wealth into a bad signal. Relative to our former paper (Coco and Pignataro, 2012) however here we assume heterogeneity and asymmetric information on the responsibility variable. This makes the model more realistic and allows us to discuss the equilibria in terms of the EOp paradigm.

This bidimensional asymmetric information and the ensuing moral hazard complicate considerably the game. In a situation where individuals are risk averse, their willingness to bear risk is an important additional channel through which the distribution of wealth determines the contract form with the efficiency and the equity properties of equilibrium. In particular DARA gives personal endowment a new role in providing incentives that can mitigate or exacerbate information problems. More wealth (and less risk aversion) negatively impacts on effort provision (see Newman, 2007). Adverse selection on wealth types is therefore endogenously generated by different optimal levels of effort along the distribution of wealth. As a consequence poor individuals end up as hard-working agents, other things equal. Moreover for each wealth class, preferences on effort aversion affect effort provision as well and the two dimensions interact in a complex manner.

We consider a contract space in terms of collateral and interest rate. Risk aversion and effort aversion (through their consequent effort choice) determine the willingness to post collateral and therefore the existence and form of equilibrium. Risk aversion influences the willingness to post collateral both directly and through effort choice (moral hazard) in different directions. When the moral hazard channel dominates, no equilibrium exists where poor entrepreneurs can be served. Instead when the direct effect of risk aversion prevails, we discover a unique pure strategy subgame perfect equilibrium in the screening game. For some preferences, only some poor entrepreneurs are excluded from the market. Different risk classes (rich and poor) may be pooled at a single contract in equilibrium, where cross-subsidization occurs not only between wealth classes but also between different effort aversion

\footnote{See Aney et al. (2012) for a case of bidimensional heterogeneity with observable wealth.}
types. The credit market equilibrium is then characterized by inequality of opportunity and inefficiency in contrast with the traditional trade-off. The rich are charged a low rate of interest (relative to their risk) even if they are characterized by high level of effort aversion, while poor borrowers (with low effort aversion) are charged too high an interest rate. We then demonstrate that inequality of opportunity and the perverse redistribution prospect are always associated to inefficiency. When hard working poor individuals are excluded from credit lines, some potential surplus is not realized. On the other side when cross subsidization occurs some surplus-wasting projects are carried out.

Of course the result that poor entrepreneurs may be rationed out or served at worse term contracts has already been discussed in the literature. However exclusion or worse term on credit result from the inability to write incentive compatible contracts in the presence of ex-post moral hazard (see for example Banjeree and Newman, 1993) also in connection with the existence of a quasi-fixed lending cost (for a useful survey Banjeree and Duflo 2010). In this case borrowing is obviously constrained by the amount of assets owned. A discussion of whether ex-ante or ex-post moral hazard is more important in the credit market is widely beyond the scope of this paper, but certainly most of the literature of the credit market uses our setting. One could also contend that the assumption of ex-post moral hazard, that is the possibility of strategic bankruptcy and flight of the borrower is more likely and relevant in a developed and mobile context rather than in developing countries. Also the possibility of endogenous cross-subsidization has appeared in the theoretical literature (e.g. among others, Banerjee and Newman, 1993; Black and de Meza, 1997; de Meza and Webb, 1999, 2000; Ghatak et al., 2007; Martin, 2009; Parker, 2003), but not between different wealth classes. Therefore, in our opinion, the interplay between equity, implicit redistribution and efficiency has not been discussed satisfactorily so far.

Our modeling strategy follows the literature on ex-ante imperfect information in the credit market (de Meza and Webb, 1987) and in particular the theory of collateral use. Inefficient levels of investments may occur notwithstanding collateral (Bester, 1985; Besanko and Thakor, 1987; for survey see Coco, 2000) serving as a screening device. Most related to our work are the papers by Stiglitz and Weiss (1992) and Coco (1999). These papers demonstrate the impossibility of screening by collateral in the credit market with two classes of borrowers with different risk attitudes. Risk preferences and project quality interact through moral hazard in conflicting ways, so that
collateral is not a meaningful signal of project quality. Finally, our paper is related to Gruner (2003) who considers a setting where rich borrowers crowd out productive poor ones. He suggests that an ex-ante complete redistribution of endowments, by inducing the substitution of rich entrepreneurs with poor ones, may lead to a Pareto-improvement due to a rise in the risk-free interest rate.

The structure of the paper is as follows. Section 2 introduces the baseline model while section 3 discuss the characterization of the loan contracts. The inequality of opportunity equilibrium is instead investigated in section 4. Concluding remarks follow in section 5.

2 The model

Consider a one period competitive credit market populated by entrepreneurs owning projects with risky income streams. Each project requires both (fixed) investment capital $K$ and effort supplied by the entrepreneur. Specifically the uncertain revenue from an investment can take one of the two values, $Y$ in the event of successful state with a certain level of probability $p(e)$ and zero in case of failure with probability $(1 - p(e))$ where $e \in [0, e]$ denotes the amount of effort. Returns to effort are positive and diminishing as usual, i.e., $p'(e) > 0$ and $p''(e) < 0$. In more general terms, a higher level of effort $e$ results in a project whose returns first-order stochastically dominates ($FOSD$) the project returns with lower levels of effort (De Meza and Webb, 1987). Utility for the would-be borrowers is a concave increasing function that exhibits decreasing absolute risk aversion ($DARA$, hereafter), i.e., $d(-U''(w)/U'(w))/dw < 0$ and $U(w = 0) = -\infty$. Each agent has a different amount of illiquid wealth $w_i, i \in [R, P]$ for rich and poor respectively, which are both insufficient to achieve full collateralization, $w_i < (1 + r)K$. This implies the need to borrow the whole amount of capital, $K$, in order to undertake the investment projects. Moreover let us denote $X = (1 + r)K$ as the total repayment where $r$ is the interest rate required by the bank for an amount of collateral $c$. Individuals differ also because of a scalar indexed effort aversion $\mu_j, j \in [L, H]$ which can be respectively low or high. We assume that a project realized by entrepreneurs with a low effort aversion ($L$) may have a positive net value, while high effort aversion ($H$) always induces
such a low level of effort that its net return is negative\(^6\). As a consequence projects of types—\(H\) should not be undertaken from a social perspective, as they produce less than the resources employed. Moreover an equilibrium with separation of types—\(H\) can be ruled out from the outset. While simplifying considerably the picture this assumption is quite realistic in delivering a world in which some potential entrepreneurs are basically looters (in the words of Akerlof et al., 2003) and could only realize their projects when obtaining pooling contracts with positive net present value projects/entrepreneurs.

Besides being characterized by \(L\) (low) or \(H\) (high) effort aversion, entrepreneurs differ also for their endowment and they may be either of type \(R\) (rich) or \(P\) (poor). The two feature of the borrowers are distributed independently in the population and therefore if \(\lambda_{ij} \in i j \{R, P\} \times \{L; H\}\), is the proportion of borrowers of class \(ij\) where \(\lambda_{ij}/\lambda_i = \lambda_{-ij}/\lambda_{-i}\). In other terms the proportion of low and high aversion to effort type is the same in the rich and poor group of the population.\(^7\) The borrower’s wealth \(w_i\), her own effort aversion \(\mu_j\) and her consequent effort choice \(e(w_i, \mu_j)\) are known to the individual but not observable by a competitive lender.

The expected utility of a borrower \(ij\) equals the expected net revenue from the project minus the cost of effort:

\[
U_{ij}(X, c) = p(e_{ij})U(Y - X + w_i) + (1 - p(e_{ij}))U(w_i - c) - \mu_j e_{ij} \quad (1)
\]

Intuitively, if a lender can observe a borrower’s level of effort and can enforce an effort contingent contract, then there is no moral hazard and a first-best outcome will emerge. If such a contract is not possible then moral hazard persists and the lender must infer \(e^*(w_i, \mu_j)\), the participating borrowers’ optimal level of effort as a function of wealth \(w\) and effort aversion \(\mu\). Bertrand competition in the credit market implies that the payment

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\(^6\)This assumption is introduced to simplify the treatment, but it is not restrictive. As we are going to see in the next sections the contracts designed could be rewritten as contracts in which entrepreneurs at \(H\)–levels has a positive expected return with a potential separation among wealth classes. We adopt the former characterization because it makes contracts easier to analyze delivering a more tractable framework in the inequality of opportunity perspective.

\(^7\)This assumption rules out the possibility that being rich or poor affects inherently your ability/willingness to work hard. We think this is the fairest non-biased assumption at last when one considers this model to be descriptive of a market for first time borrowers, whose wealth is inherited. The assumption is sustained form evidence in Evans and Jovanovich (1989) who reject the hypothesis that wealth dependence in the decision to become entrepreneur of richer individuals is driven by an intrinsically higher ability.
specified by the contract must be such that a competitive lender just expects to break even and so eq. (2) is equal to zero. Now consider the case of ex-ante asymmetric information. Lenders know the wealth distribution of borrowers, but are not able to distinguish the particular borrower’s wealth when a loan application is made. We assume zero risk-free interest rate and an infinitely elastic supply of funds in the deposit market. In such a scenario it is known that the standard optimal form of finance would be equity, but assuming unverifiable ex-post returns makes debt the only feasible form of finance (see de Meza and Webb, 2000). For a single borrower, a bank’s expected profit from accepting an application for a contract from a type—ij is given by:

$$
\pi_{ij} = p(e_{ij})X + (1 - p(e_{ij}))c - K
$$

In the successful state entrepreneurs pay back the amount borrowed X with probability $p(e_{ij})$ otherwise the banks keep the amount of money put up as collateral c. Entrepreneurs and banks sign a contract of the general form $\{X, c\}$. We seek subgame perfect Nash equilibria of the following two-stage screening game à la Rothschild and Stiglitz (1976). In the first stage, banks compete for the pool of customers whose type is unknown to them. They may potentially offer applicant borrowers a menu of loan contracts $\{X_{ij}, c_{ij}\} \in ij \{R, P\} \times \{L; H\}$. Then entrepreneurs weigh the pros and cons of entering the market and, if so, choose their preferred offer (one) among those offered. Therefore formally a Nash equilibrium here is a set of contracts such that (1) each bank earn nonnegative profits on each contract and (2) there exists no other set of contract that would earn positive profits in aggregate if offered in addition to the original set, with each individual contract in the set earning nonnegative profits. We restrict our attention to pure strategy equilibria.

3 Characterization of loan contracts

3.1 Agents’ preference map

In the standard explanation of separation among classes (Bester, 1985), the whole weight of screening is borne by the amount of collateral required on

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8The general structure of our model uses the definition of pure-strategy equilibria proposed by Bester (1985).
contracts to sort good and bad risk. In the present multidimensional context, instead, the banks’ statistical inference problem is more complicated. A borrower signing a contract which requires posting higher level of collateral may belong to rich or poor classes (unless collateral exceeds the poor’s wealth) but at the same time she can be relatively highly-averse to supply effort, influencing adversely the performance of the contract.

We start by looking at the effect of moral hazard. Using eq. (1), the first order condition for the borrower’s optimal choice of effort $e^*(w_i, \mu_j)$ is given by:

$$p'(e_{ij})U(Y - X + w_i) - p'(e_{ij})U(w_i - c) = \mu_j$$  (3)

Eq. (3) shows that the borrower supplies effort until the expected value of marginal effort equals its marginal cost. The maximization conditions are satisfied since the probability of success $p(e_{ij})$ is concave. Rearranging eq. (3), the optimal choice of effort $e^*_{ij}(w_i, \mu_j)$ is described by:

$$p'(e^*_{ij}) = \frac{\mu_j}{U(Y - X + w_i) - U(w_i - c)}$$  (4)

From straightforward comparative statics it follows that $\frac{de^*_{ij}}{dY} > 0; \frac{de^*_{ij}}{dc} > 0; \frac{de^*_{ij}}{dX} < 0$ as is customary in moral hazard models. On one side a higher amount of collateral reflects higher penalty in case of failure providing incentives in effort. On the other side a higher repayment negatively impacts the borrower’s return in case of success, but not in the case of failure. This reduces incentives to supply effort.

As shown in Newman (2007) it may conceivably be argued that more wealth and less risk aversion worsen the moral hazard issue. In particular in our model the adverse selection on wealth types generated as a function of different choices of effort is combined with the role attributed to effort aversion. A multidimensional moral hazard effect changes as a combined function of $DARA$ and effort aversion.

We define for any class of wealth, the marginal borrower as the individual who is indifferent between exiting and remaining active in the credit market. As a direct consequence the marginal set is defined as the set of individuals indifferent between two options along the wealth distribution. Thus, one can show that there exists a negative relation between effort and wealth, i.e., the
marginal effort is lower, the higher is the wealth of individuals:

\[ \frac{de^*_{ij}}{dw} < 0 \]  

(5)

Proof. See the Appendix

while at the same time, a negative correspondence between effort and aversion is established,

\[ \frac{de^*_{ij}}{d\mu} < 0 \]  

(6)

Proof. See the Appendix

which implies that individuals with a higher effort aversion also display a higher probability of default due to moral hazard. Since the marginal individuals capture the lowest share of project expected returns, their choice of effort is farthest from the socially efficient value.

Because of decreasing risk aversion, moral hazard impacts more heavily on wealthier borrowers and of course the effect is heavier for people with larger effort aversion. We can now state the following result:

Lemma 1: Given a certain effort aversion \( \mu_j \), marginal poor entrepreneurs are the first to exit from the market:

\[ \frac{d\mu}{dw} |_{\nu_{ij}(\cdot)=0} > 0 \]  

(7)

Condition (7) is crucial for at least two reasons. First it constitutes a signal about the possibility of inequality of opportunity in the market. The hard-working agents are excluded from the market due to their initial conditions and independently from their level of effort aversion. Second, concerning the role of private information on wealth, Coco and Pignataro (2012) demonstrate that decreasing risk aversion may turn wealth (and availability to post collateral) into a bad signal. In this extended version with multidimensional hidden information, another effect occurs because of private information on effort aversion. In order to catch the idea, let us suppose (as proposed in the model) four classes of entrepreneurs \( ij \in \{R, P\} \times \{L; H\} \). In this setting rich borrowers (both types) could benefit from not signaling their wealth because of an implicit cross subsidy they would earn in a pooling with hard working poor entrepreneurs.
3.2 Single-crossing preferences

To explore the types of equilibria that may arise in this context, it is useful to draw a diagrammatic representation of the equilibrium. Using (1) and from the Envelope Theorem, we know that the slope of an indifference curve of a borrower in the \((X, c)\)-space is:

\[
s_{ij}(X, c) = \frac{dX}{dc} < 0
\]  

(8)

**Proof. See the Appendix**

representing the marginal rate of substitution between income in the two states at a certain contract \((X, c)\). The first element in order to establish the possibility of separating equilibria is the slope of the indifference curves in the \((X, c)\) space with respect to the wealth dimension of borrowers. In this respect we try to distinguish the direct effect of risk preferences from the impact of moral hazard. We rewrite the slope of the indifference curve in (8) as:

\[
s_{ij}(X, c) = \frac{dX}{dc} = M_{ij}(w)R_i(w)
\]

where \(M_{ij}(w) = -\frac{(1-p(c_{ij}))}{p(c_{ij})}\) while \(R_i(w) = \frac{U'(W^{FS}_i)}{U'(W^{FS}_i)}\) where \(W^{FS}_i = Y - X + w_i\) and \(W^{F}_i = w_i - c\). The curvature of the indifference curve with respect to changes in wealth is then:

\[
\frac{\partial}{\partial w} (s_{ij}(X, c)) = M_{ij}(w)R'_i(w) + M'_{ij}(w)R_i(w) \geq 0
\]  

(9)

**Proof. See the Appendix**

Here \(M_{ij}(w)R'_i(w)\) captures the risk preference effect while \(M'_{ij}(w)R_i(w)\) captures the impact of moral hazard. Not surprisingly (9) has an ambiguous sign. On one side, the effect of (decreasing) risk aversion makes the indifference curve flatter as wealth increases. On the other side the negative impact of moral hazard makes it steeper. Indeed, for a given project choice, due to decreasing absolute risk aversion, rich individuals require a smaller reduction in the repayment rate to compensate for an increase in collateral (e.g. they are more willing to post collateral). Whenever the impact of moral hazard prevails as in eq. (10), rich individuals put such a low level of effort, and their probability of success diminishes by so much that their trade-off
between collateral and interest rate becomes worse than poor people’s one, notwithstanding their lower risk aversion:

\[
\frac{\partial e_{ij}}{\partial w} > p(e_{ij})(1 - p(e_{ij}))(A(W^S_i) - A(W^F_i))
\]  

(10)

**Proof.** See the Appendix

Note that this ambiguity in general means that the single crossing property of indifference curves which is a necessary condition to ensure the possibility of separation does not hold as a general rule.

Now let us investigate the impact of effort aversion on the marginal rate of substitution between repayment \(X\) and collateral \(c\). Independently by the amount of endowed wealth, entrepreneurs characterized by effort aversion—\(L\) display a relative preference for posting more collateral compared to the ones characterized by effort aversion—\(H\) at any point in the space \((X, c)\) due to their higher success probability. Thus the impact of effort aversion is always the same and this implies that independently by which one of the two effects prevails the slope of indifference curves with high effort aversion should be steeper.

The heterogeneity between these two unobservable elements identifies four classes of entrepreneurs. When \(\frac{\partial}{\partial w}(s_{ij}(X, c)) > 0\), for instance, the direct impact of decreasing risk aversion exceeds the effect of moral hazard\(^9\). Thus intuitively the marginal cost of the repayment is globally lower for rich agents or lower effort aversion, holding the other characteristic constant.

Let us now consider the slope of the isoprofit curve for a bank lending to the borrower of class \(ij\) only:

\[
\frac{dX}{dc}\bigg|_{\pi_{ij}} = -\frac{(1 - p(e_{ij}))(dp(e_{ij})/dc)(X - c)}{p(e_{ij}) + (dp(e_{ij})/dX)(X - c)}
\]  

(11)

where \(\pi_{ij}\) is the bank’s expected profit from the borrower of class \(i\). Since \(dp(e_{ij})/dX\) is negative, (11) could in principle be positive. Note that this becomes more likely for high values of \(X\) and lower values of \(c\) (see Coco, 1999). We may immediately note that, by construction, under this information structure, individuals with a larger wealth (higher risk from the point

\(^9\text{When } \frac{\partial}{\partial w}(s_{ij}(X, c)) < 0, \text{ the procedure is analogous at least for type—RH and type—PL.}\)
of view of banks) may prefer contracts that are actuarially fair for poor individuals due to decreasing risk aversion.

4 Inequality of opportunity equilibrium

In the absence of asymmetric information, the equilibrium is trivial. Poor and rich borrowers with low effort aversion take the contract while individuals with higher effort aversion are excluded from the market due to negative expected return. Here instead the search for equilibria is quite complex. Interaction of preferences and feasible contracts in the dimensional space makes it difficult. Our starting point has to be the fact that given the assumptions we have both positive and adverse selection of contract terms (particularly collateral). Hence an increase in collateral for example can lead to exit of high-effort averse types (poor or rich depending on the initial contract) but also more simply of poor entrepreneurs with low effort aversion. What we know for sure is that, given a certain level of effort-aversion, poor entrepreneurs exit first because of decreasing risk aversion. And conversely that, for a given wealth-type, individuals with higher effort aversion exit first. An important point concerns the relative slope of the indifference curves of different types. As we observed above the relative slope of poor vs rich types, keeping constant their effort aversion, cannot be ascertained a priori as it depends on the relative strength of the risk aversion versus the incentive effect. Instead we know that within a wealth class less effort-averse types are necessarily more willing to post collateral at any contract (e.g., flatter indifference curves), thus suggesting that separation within wealth classes would be in principle feasible. But separation of wealth types is not feasible given the violation of the single crossing property. The procedure we will use in order to discover potential equilibria is to analyze separately the cases where risk aversion prevails on the incentive effect and viceversa. We will start the analysis of possible equilibria looking at portions of the contract space where all types participate.

As a starting point, let us look at Figure 1 where we can note the participation constraint of poor entrepreneurs with high effort-aversion type—PH denoted $PC_{PH}$ and that of the poor entrepreneurs with low effort aversion $PC_{PL}$ in the contract space.

In area $A$ below the $PC_{PH}$, every type participates at any contract. How-
ever because all types-$H$ participate, it is quite unlikely that a feasible zero-profit pooling contract could be offered. However suppose there exists a potential global pooling zero-profit contract at $C_1$ (or any point in area $A$). This contract is not an equilibrium as we know that type-$PL$ can always be attracted by an appropriate higher-collateral contract due to flatter indifference curves and it will definitely deliver higher profits to the bank$^{10}$.

Hence a pure pooling equilibrium offered to all types is not feasible. However we know that a separating equilibrium with a contract for the type-$H$ is never feasible by definition (the surplus would be too low due to negative expected return). Hence no contract is feasible in area $A$.

Now suppose that there is a contract $C_2$ in area $B$, above the $PC_{PH}$, where a bank breaks even with a contract with the 3 remaining types ($PL, RH, RL$). Again to explore the possibility that this pooling contract is an equilibrium we should analyze the relative slopes of indifference curves of different types. We know that for a competitive lender, types-$L$ represent the ones delivering more profits at any given contract and that rich borrowers with low effort aversion types ($I_{RL}$) have definitely flatter indifference curves than rich borrowers with high effort aversion types ($I_{RH}$). So it is always possible to offer a contract separating $RL$-types and delivering positive profits. A three-types pooling can be excluded as well by competition.

Any further analysis needs some additional assumptions about the relative slope of indifference curves of type-$RH$ ($I_{RH}$) and type-$PL$ ($I_{PL}$). If the incentive effect prevails ($\frac{\partial}{\partial w} (s_{ij}(X,c)) < 0$), then the slope of $I_{PL}$ will be flatter than that of $I_{RL}$ and therefore necessarily also of $I_{RH}$. In this case a contract that steals from the candidate three-types pooling the type-$RL$ also steals the type-$PL$, making it all the more profitable. So the only possible equilibrium contract set in this area would be a contract separating type-$RH$ from the rest. However separating the type-$RH$ is impossible because their projects deliver negative return. In this case no set of contracts in area $B$ of figure 3 can represent an equilibrium, meaning that type-$PL$ will be excluded from the market. In this case a separating contract for the type-$RL$ excluding all other types will be devised by the bank$^{11}$. As a consequence poor entrepreneurs are systematically excluded from the market due to their inability to distinguish themselves from type-$RH$. Inequality

$^{10}$Other types can be attracted as well depending on the relative slope but what really matters is that the additional contract is not preferred by low wealth/high effort aversion types.

$^{11}$This is feasible considering that types-$RH$ exit first.
of opportunity is apparent.

If on the other side the risk aversion effect prevails, then indifference curves \((I_{PL})\) could in principle be steeper than \((I_{RH})\). In this case a pooling contract can be offered to two types—RH and −PL (see the contract \(C_3\) in Figure 2 on the zero profit line \((O_{PL/RH})\) for the type−RH and type−PL while freeing the type−RL for a higher collateral ‘fair’\(^{12}\) contract (on the separating contract line \(O_{RL}\)), like \(C_4\). This contract lies necessarily on the \(PC_{PH}\) line. Any contract above this participation constraint would not be a stable equilibrium because an additional contract northwest of this one could always steal the more profitable \((PL)\)-types. Contract \(C_3\) instead cannot be broken as moving northwest would necessarily bring in also the worse \((PH)\) types and the additional contract would not be profitable.

Now let’s examine the features of this equilibrium. At the pooling contract \((C_3)\) types−PL are systematically cross subsidizing types−RH. This means that the terms of the contract will be penalizing for poor hard working types. In this case rich types get access to credit both if they are effort averse and if they aren’t. Therefore inequality of opportunity follows immediately. Note that by assumption rich types−H carry out their project due to cross subsidization but they are actually burning some surplus.

As a final step we discuss the equilibria we found, particularly the last one, on the basis of available evidence on credit markets in developing countries. Banerjee and Duflo (2010) list some stylized established facts:

\(a\) Richer people borrow more (wealth dependence);
\(b\) Richer people pay lower interest rates;
\(c\) These divergences in interest rates are not driven by differences in default rates;
\(d\) Lending rates vary widely in the same credit market;

Of course both our equilibria are consistent with fact \(a\), while the second one (the only one where poor entrepreneur participate) is obviously consistent with fact \(b\). Poor entrepreneurs pay the higher interest rate, while rich ones pay the high or the low one depending on their quality (e.g., effort aversion). The second equilibrium is also clearly consistent with fact \(c\). Poor entrepreneurs pay a higher interest rate, notwithstanding their low default rates, mainly because they subsidize bad quality types. The same equilibrium is also compatible with fact \(d\)\(^{13}\).

\(^{12}\)Meaning with no cross-subsidy.

\(^{13}\)Note that in developed countries the risk premia on loans are usually low and not very
Finally note that inequality of opportunity in equilibrium results in poor entrepreneurs being, on average, necessarily of better 'quality' (effort aversion in our specification) than richer ones. To our knowledge the only test for this hypothesis has been performed by Evans and Jovanovich (1989), who find that 'the correlation of entrepreneurial ability and assets is negative and statistically significant'.

5 Concluding remarks

The aim of this paper has been to investigate whether equality of opportunity actually holds in a credit market with heterogeneity on unobservable circumstances and responsibility, respectively wealth and effort aversion, and to discuss the relation between violation of EOp and efficient credit allocation. Only if all equally-qualified applicants get credit whatever their endowment situation, the ideal of equality of opportunity holds. In our setting equality of opportunity requires that individuals with the same level of the responsibility (same effort aversion) variable be offered the same opportunities.

We show that the impact of moral hazard depends on both wealth and effort aversion of individual borrowers. Effort decreases in both variables. In particular because of decreasing risk aversion, moral hazard results in rich borrowers supplying less effort to their projects compared to the poor ones. The combination of these features delivers a complex environment where the search for equilibria is particularly difficult. We demonstrate that any equilibrium entails some forms of inequality of opportunity. In some cases no poor entrepreneur gets credit due to her inability to separate herself from worse (rich) entrepreneurs. Some surplus is lost as a consequence. Under other restrictions instead a partial separating equilibrium with cross-subsidization between classes exists, where not only equality of opportunity is violated, but poor entrepreneurs with a higher level of responsibility (lower effort aversion) cross-subsidize the rich ones with lower responsibility. Access to the credit market is thus paid by hard-working poor entrepreneurs with a perverse redistribution ‘tax’. The additional consequence of the subsidy is that negative surplus projects are carried out. In this case efficiency and equity violations occur jointly. This last equilibrium is consistent with most consensus micro-evidence on credit markets particularly in developing countries.

variable across loans (see Black and De Meza, 1997).
Finally note that wealth heterogeneity is the very factor impeding the perfect screening of types. In absence of wealth heterogeneity, effort aversion is correctly (e.g., inversely) correlated with the willingness to post collateral in order to obtain screening. Therefore when full ex-ante redistribution of wealth is possible, leading to uniform wealth levels in the population of entrepreneurs, willingness to post collateral correctly signals low effort aversion and the project’s quality. Perfect screening is in principle possible and only good projects, those carried out by low effort aversion entrepreneurs, are realized in equilibrium. Hence a perfectly egalitarian ex-ante redistribution of resources improves efficiency because it ensures that good projects, and only good projects, are carried out, thus avoiding also waste from realization of negative surplus projects. Contemporaneously, and by definition, in this equilibrium entrepreneurs get credit on the basis solely of the responsibility variable. This intervention would therefore improve final allocations both on distributive grounds, at least under the equality of opportunity benchmark, and on efficiency grounds.
6 Figures

Figure 1: No equilibrium in areas A and B

Figure 2: Partial separating equilibrium with cross-subsidization.
7 Appendix (not for publication)

A) Proof of eq. (5):
Starting by eq. (3) and assuming that \( W_i^S = Y - X + w_i \) and \( W_i^F = w_i - c \), we can simply rewrite that:

\[
\frac{\partial U}{\partial e_{ij}} = p'(e_{ij}) \left( U(W_i^S) - U(W_i^F) \right) - \mu_j
\]

By the Implicit Function theorem and due to decreasing absolute risk aversion, we simply observe that:

\[
\left[ p''(e_{ij}) \left( U(W_i^S) - U(W_i^F) \right) \right] de + \left[ p'(e_{ij}) \left( U'(W_i^S) - U'(W_i^F) \right) \right] dw = 0
\]

which implies:

\[
\frac{de_{ij}}{dw} = -\frac{p'(e_{ij}) \left( U'(W_i^S) - U'(W_i^F) \right)}{p''(e_{ij}) \left( U(W_i^S) - U(W_i^F) \right)} < 0
\]

B) Proof of eq. (6):
With a similar procedure shown in eq. (5), we may write:

\[
\left[ p''(e_{ij}) \left( U(W_i^S) - U(W_i^F) \right) \right] de - d\mu = 0
\]

\[
\left[ p''(e_{ij}) \left( U(W_i^S) - U(W_i^F) \right) \right] de = d\mu
\]

indicating,

\[
\frac{de_{ij}^*}{d\mu} = \frac{1}{p''(e_{ij}) \left( U(W_i^S) - U(W_i^F) \right)} < 0
\]

C) Proof of eq. (7):
In the marginal set, individuals have utility equal to zero. By the envelope theorem and differentiating equation with respect to \( \mu \) and \( w \), we observe that:

\[
[-e_{ij}] d\mu + \left[ p(e_{ij})U'(W_i^S) + (1 - p(e_{ij})U'(W_i^F) \right] dw =
\]

\[
[-e_{ij}] d\mu = - \left[ p(e_{ij})U'(W_i^S) + (1 - p(e_{ij})U'(W_i^F) \right] dw
\]

which implies that:

\[
\frac{d\mu}{dw} \bigg|_{u_{ij}(\cdot)=0} = -\frac{p(e_{ij})U'(W_i^S) + (1 - p(e_{ij})U'(W_i^F)}}{-e_{ij}} > 0
\]
D) Proof of eq. (8):
Starting by eq. (1):

\[ U_{ij} = p(e_{ij})U(W_i^S) + (1 - p(e_{ij}))U(W_i^F) - \mu_j e_i \]

By envelope theorem and differentiating with respect to \( X \) and \( c \), it follows that:

\[
\left[ -p(e_{ij})U'(W_i^S) \right] dX - \left[ (1 - p(e_{ij}))U'(W_i^F) \right] dc = 0
\]

\[
\left[ -p(e_{ij})U'(W_i^S) \right] dX = \left[ (1 - p(e_{ij}))U'(W_i^F) \right] dc
\]

which implies that:

\[ s_{ij}(X, c) = \frac{dX}{dc} = -\frac{(1 - p(e_{ij}))U'(W_i^F)}{p(e_{ij})U'(W_i^S)} < 0 \]

D) Proof of eq. (9):
We can again rewrite the slope of the indifference curve as:

\[ s_{ij}(X, c) = \frac{dX}{dc} = M_{ij}(w)R_i(w) \]

where \( M(w) = \frac{(1-p(e_{ij}))}{p(e_{ij})} \) while \( R_i(w) = \frac{U'(W_i^F)}{U'(W_i^S)} \). The curvature of the indifference curve with respect to change in wealth is then:

\[
\frac{\partial}{\partial w} \left( \frac{dX}{dc} \right) = M_{ij}(w)R'_i(w) + M'_ij(w)R_i(w)
\]

where \( M(w)R'(w) \) captures the effect of risk preference while \( M'(w)R(w) \) explains the moral hazard effect. First, let us solve \( M(w)R'(w) \):
\[ M_{ij}(w)R_i(w) = -\frac{(1 - p(e_{ij}))}{p(e_{ij})} \left[ \frac{U''(W_i^F)U'(W_i^S) - U''(W_i^S)U'(W_i^F)}{(U'(W_i^S))^2} \right] = \]
\[ = -\frac{(1 - p(e_{ij}))}{p(e_{ij})} \left[ \frac{U''(W_i^F)}{U'(W_i^S)} - \frac{U''(W_i^S)U'(W_i^F)}{(U'(W_i^S))^2} \right] = \]
\[ = -\frac{(1 - p(e_{ij}))}{p(e_{ij})} \frac{1}{U'(W_i^S)} \left[ U''(W_i^F) - \frac{U''(W_i^S)U'(W_i^F)}{U'(W_i^S)} \right] = \]
\[ = -\frac{(1 - p(e_{ij}))}{p(e_{ij})} U'(W_i^S) \left[ U''(W_i^F) - U''(W_i^S) \right] \]

Let us define \( A(W) \) as the coefficient of decreasing absolute risk aversion, we can then rewrite \( M_{ij}(w)R_i(w) \) as:

\[ M_{ij}(w)R_i(w) = -\frac{(1 - p(e_{ij}))}{p(e_{ij})} U'(W_i^S) (A(W_i^S) - A(W_i^F)) = \]
\[ = \frac{dX}{dc} (A(W_i^S) - A(W_i^F)) > 0 \]

Since \( W_1 > W_2 \) and considering decreasing absolute risk aversion i.e. risk aversion decreases with wealth, \( A(W^F) > A(W^S) \) and that by construction that \( \frac{dX}{dc} \) is negative, we can surely say that the effect of risk preferences \( M_{ij}(w)R_i(w) \) is positive.

Then we can solve \( M_{ij}(w)R_i(w) \):

\[ M_{ij}'(w)R_i(w) = \left[ -\frac{p'(e_{ij}) \frac{\partial e_{ij}}{\partial w} p(e_{ij}) - (1 - p(e_{ij})) p'(e_{ij}) \frac{\partial e_{ij}}{\partial w}}{(p(e_{ij}))^2} \right] \frac{U'(W_i^F)}{U'(W_i^S)} = \]
\[ = \left[ \frac{p'(e_{ij}) \frac{\partial e_{ij}}{\partial w}}{(p(e_{ij}))} + (1 - p(e_{ij})) p'(e_{ij}) \frac{\partial e_{ij}}{\partial w} \right] \frac{U'(W_i^F)}{U'(W_i^S)} = \]
\[ = \frac{p'(e_{ij}) \frac{\partial e_{ij}}{\partial w}}{p(e_{ij})} \left[ 1 + \frac{(1 - p(e_{ij}))}{p(e_{ij})} \right] \frac{U''(W_i^F)}{U'(W_i^S)} = \]
\[ = \frac{p'(e_{ij}) \frac{\partial e_{ij}}{\partial w}}{p(e_{ij})} \frac{U'(W_i^S) - dX}{dc} = \]
\[ = \frac{p'(e_{ij}) \frac{\partial e_{ij}}{\partial w}}{p(e_{ij})} \frac{dX}{dc} - \frac{U'(W_i^F)}{U'(W_i^S)} < 0 \]
Therefore,

\[
\frac{\partial}{\partial w} \left( \frac{dX}{dc} \right) = \frac{dX}{dc} \left( A(W_i^S) - A(W_i^F) \right) - \frac{p'(e_{ij})}{p(e_{ij})} \frac{\partial e_{ij}}{\partial w} \left( \frac{dX}{dc} - \frac{U'(W_i^F)}{U(W_i^F)} \right) \leq 0
\]

As shown, the sign of eq. (7) is uncertain due to the combination of the positive effect of risk aversion \( \frac{dX}{dc} \left( A(W_i^S) - A(W_i^F) \right) \) and the negative moral hazard impact \(- \frac{p'(e_{ij})}{p(e_{ij})} \frac{\partial e_{ij}}{\partial w} \left( \frac{dX}{dc} - \frac{U'(W_i^F)}{U(W_i^F)} \right)\).

**D) Proof of eq. (10):**

After some algebraic manipulations,

\[
\frac{\partial}{\partial w} \left( \frac{dX}{dc} \right) = s_{ij} \left( A(W_i^S) - A(W_i^F) \right) - \frac{\partial e_{ij}}{\partial w} \frac{p'(e_{ij})}{p(e_{ij})} \left( s_{ij} - \frac{U'(W_i^F)}{U(W_i^F)} \right) =
\]

\[
= s_{ij} \left( 1 - p(e_{ij}) \right) \left( A(W_i^S) - A(W_i^F) \right) - \frac{\partial e_{ij}}{\partial w} \frac{p'(e_{ij})}{p(e_{ij})} \left( \frac{1 - p(e_{ij})}{p(e_{ij})} \right) s_{ij} + s_{ij} =
\]

\[
= s_{ij} \left( 1 - p(e_{ij}) \right) \left( A(W_i^S) - A(W_i^F) \right) - \frac{\partial e_{ij}}{\partial w} \frac{p'(e_{ij})}{p(e_{ij})} s_{ij} =
\]

\[
= s_{ij} \left( 1 - p(e_{ij}) \right) \left( A(W_i^S) - A(W_i^F) \right) - \frac{1}{p(e_{ij})} \left( U(W_i^S) - U(W_i^F) \right) =
\]

\[
= \frac{s_{ij}}{p(e_{ij}) \left( U(W_i^S) - U(W_i^F) \right)} \left( p(e_{ij}) (1 - p(e_{ij})) \left( A(W_i^S) - A(W_i^F) \right) - \frac{\partial e_{ij}}{\partial w} \right)
\]

The impact of moral hazard prevails if and only if:

\[
\frac{\partial e_{ij}}{\partial w} > p(e_{ij}) (1 - p(e_{ij})) \left( A(W_i^S) - A(W_i^F) \right)
\]
References


