Bequest Motives, Estate Taxes, and Wealth Distributions in Becker-Tomes Models with Investment Risk

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Abstract

I introduce investment risk into Becker and Tomes (1979) model and reinvestigate the impacts of bequest motives and estate taxes on wealth inequality. Contrasting with Becker and Tomes (1979) and Davies (1986), I find that bequest motives increase wealth inequality and estate taxes reduce wealth inequality. Furthermore in my model the impacts of estate taxes on wealth inequality does not depend on the redistribution of tax revenues. Different from Davies (1986) I also find that economic growth decreases wealth inequality.

JEL classification: D31; H20

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1 Introduction

I set up a heterogeneous agents overlapping generations (OLG) model with idiosyncratic investment shocks and labor efficiency shocks to investigate the impacts of bequest motives and estate taxes on wealth distributions.

I introduce idiosyncratic investment risk into Becker and Tomes (1979) model. I find that in an economy with altruism bequest motives and idiosyncratic investment risk, bequest motives increase wealth inequality.\(^1\) In terms of policy implications, I find that estate taxes reduce wealth inequality. These findings contrast with those of Becker and Tomes (1979) and Davies (1986). They highlight the different impacts of investment shocks and labor efficiency shocks on wealth distributions. I also find that, as in Benhabib et al. (2011), idiosyncratic investment risk generates the fat tail of the wealth distribution in my model.\(^2\)

Furthermore, I investigate why the effects of bequest motives and estate taxes on wealth inequality depend on the idiosyncratic investment risk. Following Davies (1986) I write the stationary wealth distribution as a weighted sum of ancestors’ luck. With investment return luck, the weighted average of labor income luck is revised. Without investment return luck, the effect of labor income shock is temporary since it is additive. The effect of investment risk is persistent since it is multiplicative. Thus with investment return luck, we revise the lag structure effect of estate taxes in Davies (1986).

Becker and Tomes (1979) set up a model in which parents maximize their utility by choosing optimal investments in the human and nonhuman capital of children and other members. The theory recognizes that endowments and market rewards depend on luck.

Davies (1986) presents a Becker-Tomes model and uses a decomposition technique to study the impact of estate taxes on the wealth distribution in the long run. Davies (1986) separates the inheritance effect (the lag structure effect, in his terminology) and the redistribution effect (the "transfer effect", in his terminology). The wealth of the current generation can be expressed as a weighted average of the labor income luck in the past of the lineage. The inheritance of bequests decreases wealth inequality in the long run through averaging the labor income luck.\(^3\) For example, a father’s good luck can compensate the grandfather’s bad luck. The inheritance effect of an estate tax increases wealth inequality because it reduces the averaging force of inheritance. The redistribution effect is due to the lump-sum transfer. A higher transfer causes lower inequality of the wealth distribution.\(^4\) However, the calculations of Davies (1986) show

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\(^1\)This model is different from Benhabib et al. (2011). There they use a "joy-of-giving" bequest motive. Thus that model is a nonaltruistic model.

\(^2\)The mechanism to generate the fat tail of the wealth distribution depends on bequest motives. It is the accumulation of the investment risk across generations that produces the fat tail.

\(^3\)This averaging effect does not require that the labor income luck is independent and identically distributed (i.i.d.). The effect also holds for luck correlated across generations.

\(^4\)This intuition comes from the mathematical result that \(X + a\) Lorenz dominates \(X + b\) for any non-negative random variable \(X\) with a finite positive mean and \(a > b > 0\) (See Theorem 3.A.25 of
that it is hard to find "realistic" parameters such that the increase of the estate tax can cause a higher transfer. Both the inheritance effect and the transfer effect imply that a higher estate tax increases wealth inequality. Atkinson (1980) finds that estate taxes increase wealth inequality in a model with 'joy of giving' bequest motives.

Bossmann et al. (2007), using a two-period overlapping generations (OLG) general equilibrium model, find that estate taxes reduce wealth inequality in the long run. They find that bequest motives cause the wealth distribution to become more equal because of the increase in aggregate wealth which overcompensates the increase in the variance of wealth such that the coefficient of variation falls.

Pestieau and Possen (1979) use a progressive property or estate tax to generate a stationary wealth distribution.\(^5\) The stationary distribution is lognormal. They show that the greater the degree of progressivity of the estate tax, the lower the wealth inequality. Both our model and theirs have multiplicative investment shocks. However, we study a different tax scheme from theirs. They study how the degree of progressivity of the estate tax influences wealth inequality while we investigate how the tax rate of a linear estate tax affects wealth inequality.\(^6\)

I summarize the related literatures in Table 1.

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The rest of the paper is organized as follows. Section 2 contains the basic set-up of our model with investment risk. Section 3 presents a simple Becker-Tomes with only

\(^5\)The tax scheme in Pestieau and Possen (1979) has the form

\[ S_A = pS_B^c \]

where \( p \geq 1, 0 \leq c \leq 1 \), \( S_A \) represents the after tax estate, and \( S_B \) the estate before taxes. The lower the value of \( c \), the greater the degree of progressivity of the tax. The constant \( p \) is the instrument through which the government returns the tax revenues to the economy.

\(^6\)The mechanisms to generate stationary distributions in the two models are also different. Pestieau and Possen (1979) use a progressive property or estate tax, while we use a Kesten process.
labor efficiency risk. I investigate the impacts of bequest motive on wealth distribution in an economy with investment risk in section 4. I study the impacts of estate taxes on wealth distributions in section 5. Section 6 concludes the paper. Section 7 contains proofs. Section 8 discusses the effect of economic growth on wealth inequality.

2 A Becker-Tomes model with investment risk

Agents live for one period. At the end of the period, the agent dies and gives birth to one child. The population keeps constant. Its size is normalized to be 1.

At the beginning of his life the agent receives an inheritance $I_t$ left by his father. The estate tax rate is $b$. His after-tax inheritance is $(1 - b)I_t$. The agent’s labor earnings $H_t$ follows a stochastic process.

Assumption 1: $\{H_t\}$ is stationary and ergodic.

From Assumption 1, we know that $H_t$’s are not necessarily independent and identically distributed. Our model includes the interesting and realistic case of labor efficiency processes correlated across generations.8

Assumption 2: $H_t > 0$, $E(H_t) = 1$, and $Var(H_t) < \infty$.

The agent also receives a lump-sum transfer $G_t$ from the government. The agent’s after-tax wealth is

$$L_t = H_t + (1 - b)I_t + G_t.$$

The agent’s consumption is $C_t$. He leaves bequests $B_t$ to his child. His budget constraint is

$$C_t + B_t = L_t.$$

The agent has a gross interest rate $\tilde{R}_{t+1}$. $\tilde{R}_{t+1}$ is independent and identically distributed (i.i.d.) along generations. Thus $I_{t+1} = \tilde{R}_{t+1}B_t$. Both $H_t$ and $\tilde{R}_{t+1}$ are i.i.d. across agents in the same generation.

The agent has altruism bequest motives. He cares about the total wealth of his child. Another strand of literature, for example Atkinson (1980), Bossmann et al. (2007, and Benhabib et al. (2011), uses "joy-of-giving" bequest motives.9 The agent’s problem is

$$\max_{C_t, L_{t+1}} \frac{C_t^{1-\gamma}}{1-\gamma} + \chi \frac{L_{t+1}^{1-\gamma}}{1-\gamma}$$

7We use $\{x_t\}$ to represent a sequence.

8Davies (1986) uses a mean-reverting process

$$H_t = (1 - v)\bar{H} + vH_{t-1} + \varepsilon_t$$

with $0 < v < 1$.

9We will discuss how the the different modellings of bequest motives influence the impacts of estate taxes on wealth inequality in section 5.
\[ s.t. \quad C_t + \frac{L_{t+1}}{\bar{R}_{t+1}(1 - b)} = Z_t \]
\[ Z_t = L_t + \frac{H_{t+1} + G_{t+1}}{\bar{R}_{t+1}(1 - b)} \]

where \( \chi \) is the bequest motive intensity and \( \gamma > 0 \) is the coefficient of relative risk aversion.

The optimal policy functions are
\[
C_t = \frac{1}{1 + \left[ \frac{\bar{R}_{t+1}(1 - b)}{\bar{R}_{t+1}(1 - b)} \right]^{\frac{1}{\gamma}} \chi^{\frac{1}{\gamma}}} Z_t
\]
and
\[
L_{t+1} = \frac{\bar{R}_{t+1}(1 - b)}{1 + \left[ \frac{\bar{R}_{t+1}(1 - b)}{\bar{R}_{t+1}(1 - b)} \right]^{\frac{2}{\gamma}} \chi^{-\frac{1}{\gamma}}} Z_t
\]

Thus the individual wealth accumulation process contains a multiplicative shock and an additive shock
\[ L_{t+1} = d_{t+1} L_t + \varepsilon_{t+1} \tag{1} \]

where
\[
d_{t+1} = \frac{\bar{R}_{t+1}(1 - b)}{1 + \left[ \frac{\bar{R}_{t+1}(1 - b)}{\bar{R}_{t+1}(1 - b)} \right]^{\frac{2}{\gamma}} \chi^{-\frac{1}{\gamma}}} \tag{2}
\]
\[
\varepsilon_{t+1} = \frac{1}{1 + \left[ \frac{\bar{R}_{t+1}(1 - b)}{\bar{R}_{t+1}(1 - b)} \right]^{\frac{2}{\gamma}} \chi^{-\frac{1}{\gamma}}} (H_{t+1} + G_{t+1}) \tag{3}
\]

Investment risk introduces a multiplicative shock to the wealth accumulation process. This is the most important difference between our model and Becker and Tomes (1979).

The CRRA utility functions bring us the linear policy functions. The linear relationship has two advantages: (1) It reduces the difficulty of aggregation. And (2) it gives us the linear process (1), which plays the crucial role in our analysis of wealth distributions.

The government taxes inheritance and redistributes the revenue to all agents. The government budget is
\[ G_t = b \int I_t dj \]
where \( \int dj \) represents the aggregation for all agents.
2.1 The stationary wealth distribution

In order to guarantee that the process (1) has a stationary wealth distribution with observed fat tail, we need more assumptions.

Assumption 3: \( E(\log d_{t+1}) < 0 \).

The stationary wealth distribution is

\[
L_\infty = \sum_{t=1}^{\infty} \left( \prod_{i=1}^{t-1} d_i \right) \varepsilon_t
\]

(4)

with the assumption that \( \prod_{i=1}^{0} d_i = 1 \).

Assumption 4: \( 1 < E(d_{t+1})^2 < \infty \). The conditional law of \( \log d_t \), given \( d_t \neq 0 \), is nonarithmetic. And \( P(\varepsilon_t = (1 - d_t)c) < 1 \), for \( \forall c \in \mathbb{R} \).

As in Benhabib et al. (2011) we have

Theorem 1 Under assumptions 3 and 4, the individual wealth has a unique stationary distribution with an asymptotic Pareto tail of exponent \( 1 < \mu < 2 \) such that

\[
E (d_{t+1})^\mu = 1
\]

(5)

i.e.

\[
\lim_{x \to \infty} \frac{P(L_\infty > x)}{x^{-\mu}} = C
\]

where \( C > 0 \) is a constant.

Theorem 1 shows that investment risk causes the fat tail of wealth distribution. And labor earnings shocks do not influence the tail of wealth distribution. Those agents who keep drawing good luck of investment shocks become the rich people in our model. As in Benhabib et al. (2011), this shows the different impacts of labor earnings shocks and investment shocks on wealth distributions.

Observing that U.S. data shows that \( \mu = 1.49 \) we will restrict \( \mu < 2 \). However \( \mu < 2 \) implies that the stationary wealth distribution does not have a finite variance. Thus we could not use the coefficient of variation as our inequality measure, even though Becker and Tomes (1979) and Bossmann et al. (2007) use it. We will use the Pareto exponent as our inequality index when we investigate the comparative statics in sections 4, 8, and 5.

Theorem 2 The higher \( d_t \), the lower the Pareto exponent \( \mu \) of the wealth distribution.

Comparison of theorem 2 and the results of Becker and Tomes (1979) and Davies (1986) shows that the impacts of investment shocks on wealth distribution are different from those of labor efficiency shocks. A higher \( d_t \) implies that wealth accumulates faster for a good draw of investment shock. That brings a fatter tail of the wealth distribution.
3 The Becker-Tomes model without investment risk

In a Becker-Tomes model, there is only labor efficiency risk. The young agent consumes $c_{1,t}$. He divides the income into two parts: consumption, $c_{2,t+1}$, and bequest left for his child, $b_{t+1}$. The agent has an altruistic bequest motive. The agent’s problem is

$$\max_{c_t, L_{t+1}} \frac{C_t^{1-\gamma}}{1-\gamma} + \chi \frac{L_{t+1}^{1-\gamma}}{1-\gamma}$$

subject to

$$c_t + \frac{L_{t+1}}{R(1-b)} = Z_t$$

where $Z_t = L_t + \frac{H_{t+1} + G_{t+1}}{R(1-b)}$. Note that now the gross interest rate $R$ is a constant.

As in Davies (1986), the optimal policy functions are

$$C_t = \frac{1}{1 + [R(1-b)]^{\frac{\gamma-1}{\gamma}} \chi^{\frac{1}{\gamma}}} Z_t$$

and

$$L_{t+1} = \frac{R(1-b)}{1 + [R(1-b)]^{\frac{\gamma-1}{\gamma}} \chi^{-\frac{1}{\gamma}}} Z_t$$

Thus we have the wealth accumulation process

$$L_{t+1} = \delta L_t + \theta (H_{t+1} + G_{t+1}) \quad (6)$$

where

$$\delta = \frac{R(1-b)}{1 + [R(1-b)]^{\frac{\gamma-1}{\gamma}} \chi^{-\frac{1}{\gamma}}}$$

and

$$\theta = \frac{1}{1 + [R(1-b)]^{\frac{\gamma-1}{\gamma}} \chi^{-\frac{1}{\gamma}}}.$$

The government budget is $G_t = b \int I_t dj$ where $\int dj$ represents the aggregation for all agents.

3.1 The stationary wealth distribution

We assume that $0 < \delta < 1$. From equation (6) we know that the unique stationary distribution of wealth is

$$L_\infty = \theta \sum_{t=1}^{\infty} \delta^{t-1} (H_t + G_t). \quad (7)$$
By writing the stationary wealth distribution in the form of (7), Davies (1986) separates the lag structure effect and the "transfer effect" of redistribution on wealth inequality. The channel through which redistribution influences $\delta$ in (7) is called the lag structure effect. The channel through which redistribution influences $H_t + G_t$ in (7) is called the "transfer effect".

Let $W(\cdot)$ be inequality measures defined over relative wealth that obey the Pigou-Dalton "principle of transfers." Davies (1986) shows

**Proposition 3** (Davies, 1986) $W(L_\infty) < W(L'_\infty)$ where

1. $L_\infty = \theta \sum_{t=1}^{\infty} \delta^{t-1} (H_t + G_t)$;
2. $L'_\infty = \theta' \sum_{t=1}^{\infty} (\delta')^{t-1} (H_t + G_t)$;
3. $0 < \delta' < \delta < 1$.

In order to see the intuition, we rewrite the expression of $L_\infty$

$$L_\infty = \frac{\theta}{1 - \delta} \sum_{t=1}^{\infty} (1 - \delta)\delta^{t-1} (H_t + G_t).$$

Let $X = \sum_{t=1}^{\infty} (1 - \delta)\delta^{t-1} (H_t + G_t)$. Note that $X$ is a weighted average of $\{H_t + G_t\}$. This average helps to cancel the uncertainty of $\{H_t + G_t\}$. Proposition 3 shows that the higher $\delta$ the stronger the average effect.

Now $d_t$ is a stochastic variable while $\delta$ is deterministic in equation (6). The higher $d_t$ the higher the mean of wealth in the stationary distribution. This is similar to the effect of increase of $c_4$ on the mean of wealth distribution in Bossmann et al. (2007). When it comes to the effect of increasing $d_t$ on the dispersion of the wealth distribution, we find the difference between processes (1) and (6).

### 4 Bequest motives and wealth distributions

In order to investigate the impacts of bequest motives on wealth inequality, we set $b = 0$. Thus $G_t = 0$.

From the expression (2) we know that $d_{t+1}$ increases with bequest motive $\chi$. Applying theorem 2 we have

**Proposition 4** In an economy with idiosyncratic investment risk, the higher the bequest motive $\chi$ the fatter the tail of the wealth distribution.

Propostion 4 shows that the higher the bequest motive intensity, the higher the wealth inequality. Benhabib et al. (2011), using a "joy-of-giving" bequest motive, shows that in an economy with idiosyncratic investment risk, the higher the bequest motive the fatter the tail of the wealth distribution.

Note that $\delta$ increases with bequest motive $\chi$. Applying proposition 3, we have
Proposition 5 In an economy without idiosyncratic investment risk, the higher the bequest motive $\chi$ the more equal the wealth distribution.

5 Estate taxes and wealth distributions

In section 4 we find that the impacts of bequest motive on wealth distribution depend on the properties of idiosyncratic shocks. A higher bequest motive increases wealth inequality in an economy with investment shocks, while Davies (1986) show that a higher bequest motive decreases wealth inequality in an economy with labor efficiency shocks. Becker and Tomes (1979) show that taxing bequests increases wealth inequality in an economy with only labor earnings shocks. In this section, we investigate the impacts of estate taxes on wealth inequality in an economy with investment shocks.

Proposition 6 $d_{t+1}$ decreases with the estate tax rate $b$. Thus in an economy with idiosyncratic investment risk, the higher the estate tax rate $b$ the thinner the tail of the wealth distribution.

Benhabib et al. (2011) shows that a higher estate tax causes a thinner tail of the wealth distribution in a model with 'joy of giving' bequest motives. We show here a higher estate tax implies a thinner tail of wealth distribution in an altruistic model. The discussions below also help us to understand the results of bequest motives and estate taxes in Benhabin et al. (2011).

Through simulations, Davies (1986) finds that in an economy without idiosyncratic investment risk, the higher the estate tax rate $b$ the less equal the wealth distribution. Proposition 6 shows a result of estate taxes different from Becker and Tomes (1979), Atkinson (1980), and Davies (1986). In these previous literatures, estate taxes increase wealth inequality in the long run. Investment risk plays a crucial role for these opposite results. Comparing expressions (4) and (7) we find that our model has a different lag structure effect. With investment return luck, the weighted average of labor income luck is revised, since some realizations of $d_t$ are greater than 1.

Davies (1986) also shows that this lag structure effect is valid in both altruistic and nonaltruistic model.10 This finding and that in Benhabib et al. (2011) also shows that the lag structure effect in economies with idiosyncratic investment risk is valid in both altruistic and nonaltruistic model, even though the effects in our model and in Davies (1986) are opposite.

5.1 Redistribution

Bossmann et al. (2007) find that taxing bequests decreases wealth inequality in an economy with only labor earnings shocks. Our results show that higher estate taxes

\[ \text{See the comments in the last paragraph of Davies (1986).} \]
are likely to dampen wealth inequality in an economy with investment shocks. But the mechanisms in these two papers are different.

In Bossmann et al. (2007) bequest motives reduce inequality. Thus the lag structure effect of estate taxes is to increase wealth inequality. This is in line with Becker and Tomes (1979) and Davies (1986). The reason why Bossmann et al. (2007) find the different results of estate taxes from those previous literatures is that the "transfer effect" of redistribution is to reduce wealth inequality while in those literatures the "transfer effect" of redistribution is to increase wealth inequality.\textsuperscript{11} And in Bossmann et al. (2007) the "transfer effect" dominates the lag structure effect.\textsuperscript{12}

Our results about the impacts of estate taxes on wealth inequality are different from Becker and Tomes (1979) and Davies (1986). In stead of revising the "transfer effect", our model revises the lag structure effect. And the finding that estate taxes reduce wealth inequality does not depend on whether tax revenues increase or decrease with an increase of the estate tax rate. As found in Bossmann et al. (2007), the direction of the "transfer effect" of estate taxes depends on how to model bequest motives. The difference between Bossmann et al. (2007) and the previous literatures of Becker and Tomes (1979) and Davies (1986) is that Bossmann et al. (2007) uses 'joy of giving' bequest motives while two other papers use altruism. The finding of our model, using different mechanism, does not depend on the modeling of bequest motives.

6 Conclusion

I introduce idiosyncratic investment risk into Becker and Tomes (1979) model. The results of the impact of bequest motives and economic growth on wealth inequality are reversed. Bequest motives increase wealth inequality and economic growth decreases it. The policy implications on wealth inequality in my model are divergent from those of Becker and Tomes (1979) and Davies (1986): Estate taxes reduce wealth inequality.

\textsuperscript{11} In Becker and Tomes (1979) and Davies (1986) the "transfer effect" of redistribution is to increase wealth inequality because tax revenues decrease with an increase of the estate tax rate.

\textsuperscript{12} See Wan and Zhu (2013).
References


7 Appendix A: Proofs

7.1 Proof of proposition 1

Proof: Let
\[ h(x) = \log E(d_t)^x. \]

By Assumption 4, \( E(d_t)^2 < \infty \). Then \( h(x) \) is a continuous function of \( x \in [0, 2] \).\(^\text{13}\) From Assumption 3 we know that \( E d_t < 1 \). Thus \( h(1) < 0 \). By Assumption 4, \( E(d_t)^2 > 1 \). Thus \( h(2) > 0 \). We know that \( h(x) \) is convex from page 158 of Loeve (1977). Thus there exists a unique \( \mu \in (1, 2) \) such that \( h(\mu) = 0 \), i.e. \( E(d_t)^\mu = 1 \). Note that \( E(d_t)^2 < \infty \) implies that \( E(d_t)^\mu \log^+(d_t) < \infty \) where \( \log^+(d_t) = \max\{0, \log(d_t)\} \). By assumption 4, the conditional law of \( \log d_t \), given \( d_t \neq 0 \), is nonarithmetic. Thus \( d_t \) satisfies the conditions of Lemma 2.2 in Goldie (1991).

Also \( E(n_t)^2 < \infty \) implies \( E(\varepsilon_t)^2 < \infty \). Thus \( E(\varepsilon_t)^\mu < \infty \) since \( 1 < \mu < 2 \). And by assumption 4, \( P(\varepsilon_t = (1 - d_t)c) < 1 \), for \( \forall c \in \mathbb{R} \). By Theorem 4.1 of Goldie (1991) we have proposition 1. \( \blacksquare \)

7.2 Proof of theorem 2

Proof: Suppose that \( d_t \geq \tilde{d}_t \) almost surely. Let \( \mu \in (1, 2) \) solves \( E(d_t)^\mu = 1 \) and \( \tilde{\mu} \in (1, 2) \) solves \( E(\tilde{d}_t)^{\tilde{\mu}} = 1 \). Thus
\[ \log E(d_t)^\mu = 0 \]
and
\[ \log E(\tilde{d}_t)^{\tilde{\mu}} = 0. \]

Note that
\[ E(d_t)^{\tilde{\mu}} \geq E(\tilde{d}_t)^{\tilde{\mu}}. \]

Thus
\[ \log E(d_t)^{\tilde{\mu}} \geq \log E(\tilde{d}_t)^{\tilde{\mu}} = 0. \]

Also by lemma ??, \( \log Ed_t < 0 \). From the proof of proposition 1 we know that \( \log E(d_t)^x \) is a continuous and convex function of \( x \). Thus \( \mu \leq \tilde{\mu} \). \( \blacksquare \)

\(^{13}\)For a sequence \( x_n \to x \), we have
\[ (d_t)^{x_n} \leq f(d_t) \]
where
\[ f(d_t) = \begin{cases} 1 & \text{if } d_t \leq 1 \\ (d_t)^2 & \text{if } d_t > 1. \end{cases} \]

Note that \( E(d_t)^2 < \infty \) implies that \( Ef(d_t) < \infty \). Thus using Lebesgue convergence theorem, we have
\[ \lim_{x_n \to x} h(x_n) = h(x). \]
7.3 Proof of proposition 6

Proof: Note that
\[
d_{t+1} = \frac{\tilde{R}_{t+1}(1 - b)}{1 + \left[\tilde{R}_{t+1}(1 - b)\right]^{\frac{1}{\gamma}} \chi^{-\frac{1}{\gamma}}} = \frac{\tilde{R}_{t+1}}{(1 - b)^{-1} + \left(\tilde{R}_{t+1}\right)^{\frac{\gamma - 1}{\gamma}} (1 - b)^{-\frac{1}{\gamma}} \chi^{-\frac{1}{\gamma}}}.
\]

Thus \(d_{t+1}\) decreases with \(b\). An application of theorem 2 gives us proposition 6. ■

8 Appendix B: Economic growth and wealth distributions

In order to investigate the impacts of economic growth on wealth distribution, we assume that
\[
E(H_{t+1}) = g.
\]

Applying theorem 2 to investigate the effect of bequest motives on wealth inequality, we have

Proposition 7 In an economy with idiosyncratic investment risk, the higher the economic growth rate \(g\) the thinner the tail of the wealth distribution.

The higher the bequest motive intensity, the lower the wealth inequality.

By using proposition 3, Davies (1986) shows

Proposition 8 In an economy without idiosyncratic investment risk, the higher the economic growth rate \(g\) the more equal the wealth distribution.