The decompositions of rank-dependent poverty measures using ordered weighted averaging operators

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Abstract

This paper concerns with rank-dependent poverty measures and shows that an ordered weighted averaging, hereafter OWA, operator is underlying in the definition of these indices. The dual decomposition of an OWA operator into the self-dual core and the anti-self-dual remainder allows us to propose a decomposition for all the rank-dependent poverty measures in terms of incidence, intensity and inequality. In fact, in poverty fields it is well known that every poverty index should be sensitive to the incidence of poverty, the intensity of poverty and to the inequality among the poor individuals. However, the inequality among the poor can be analyzed in terms of either incomes or gaps of the distribution of the poor. And depending on the side we focus on, contradictory results can be obtained. Nevertheless, the properties inherited by the proposed decompositions from the OWA operators obliges the inequality components to measure equally the inequality of income and inequality of gap overcoming one of the main drawbacks in poverty and inequality measurement. Finally, we provide an empirical illustration showing the appeal of our decompositions for some European Countries in 2005 and 2011.

Key words: Aggregation functions; OWA operators; Dual decomposition; Rank-dependent poverty measure; Intensity, Incidence and Inequality among the poor.

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1 Introduction

Recently, there has been an increasing interest by scholars of applying the Ordered Weighted Averaging, hereafter OWA, operators to several contexts, and in particular, into the economic field. Specifically, the decomposition of the OWA operators as the sum of two parts, proposed by García-Lapresta and Marques Pereira [16] and called as the self-dual core and the anti-self-dual remainder have been applied in a social framework (see García-Lapresta et al. [17] and Aristondo et al. [2], among others). In this paper we show that there exist a connection between a class of poverty measures and OWA operators.

In the evaluation of poverty, two different steps must be taken into account: the identification of the poor and the aggregation of poverty information in a numerical value. The identification problem has been solved by considering an income threshold, the so-called poverty line, which partitions a society into poor, whose incomes fail the poverty line, and non-poor, whose incomes are above the poverty line. On the other hand, the aggregation step should essentially be the choice of an appropriate poverty measure. Since the seminal work of Sen [25], a great number of poverty measures have been introduced in the literature, see Clark et al. [10], Chakravarty [8], Foster, Greer and Thorbecke [15], Shorrocks [26], Kakwani [21] among others.

This paper focus on the class of rank-dependent poverty measures. These poverty measures are those indices defined on poverty gaps for which the weight for each individual depends on its position in the distribution. We prove that the normalized version of all the rank-dependent poverty measures, such as the poverty gap ratio, the two popular indices introduced by Sen [25], the index and the consequent class of indices proposed by Thon ([27] and [28]), the Kakwani index [20], the Shorrocks index [26] and the S-Gini class introduced by Weymark ([29]), can be interpreted as OWA operators.

We adopt the methodology proposed by García-Lapresta and Marques Pereira [16] decomposing the OWA operators, underlying in the poverty measures, as the sum of the self-dual core and the anti-self-dual remainder of an OWA operator. In particular, we show that the self-dual core and the anti-self-dual remainder can be reinterpreted as a measure of intensity of poverty and the inequality among the poor, respectively. In fact, Sen [25] mention in his seminal work that every poverty index should combine three components: the incidence of poverty, the intensity of poverty and the inequality among the poor. In other word, any poverty measure should be a function of the number of poor people in the society, the incidence, the extent of the shortfall of the poor,

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1 The poverty gap of an individual below the poverty line is the difference between the poverty line and its income level. For the ones above the poverty line it is 0.
the intensity, and finally, it should take into account the inequality among the poor. Therefore, we will be able to decompose all the rank dependent poverty measures into their three underlying components.

Firstly, we normalize all the rank-dependent poverty measures in order to transform them into OWA operators, and then, we decompose the OWA operators into the self-dual core and the anti-self-dual remainder. The incidence component is obtained from the normalization factor and from the OWA decomposition the other two parts are obtained, the intensity and the inequality components.

The archetypical measures for incidence and intensity are the headcount ratio and the income gap ratio, respectively. However, the inequality among the poor component can also be measured using different inequality measures. In addition, inequality of the poor could refer to the inequality of the income of the poor or to the inequality of the gap of the poor. A crucial requirement in the measurement of inequality is the Pigou-Dalton principle. The axiom requires that a transfer of income from a poor individual to a richer one entails an increase in the inequality of the society. This axiom could be interpreted as the counterpart of Sen’s transfer axiom which demands that a regressive transfer of income, a transfer from a poorer to a richer individual when both are poor, has to increase the level of poverty. However, a regressive transfer of income could be also interpreted as a regressive transfer of shortfall. That is, a transfer of income from a richer to a poorer individuals entails a transfer of gap from the richer on shortfalls, to the richer on shortfalls. However, if we focus on shortfalls, the richer on shortfalls is now the poorer on incomes, and the poorer on shortfalls is the now richer on incomes. As a consequence, as we have mentioned, the inequality component of a poverty measure decomposition could be defined in terms of incomes or shortfalls. In this respect, in the literature, different poverty decompositions have been proposed, also for the same poverty index, in terms of incomes or gaps of the poor, see Osberg and Xu [24] and Aristondo et al. [3].

In addition, the choice between income and gap inequality is not innocuous and different choices between income and gap may lead to contradictory results. That is, the inequality of incomes and the inequality of gaps may have opposite results for the same distribution. However, from the properties of OWA operators inherited by their components, we can conclude that the anti-self-dual remainder, namely the inequality component in our decompositions, is a perfect complementary indicator, that is, an inequality measure that measures equally the inequality of income and the inequality of gap.

To work with a poverty index that is decomposable into the above mentioned

\[ \text{headcount ratio} \] is the percentage of poor people and the \[ \text{income gap ratio} \] is the mean of the relative gap of the poor.
components is very important, especially for policy makers. In fact, policy makers will be able to analyze the sources of changes in poverty focusing on their three components. In addition, these decompositions allow us to obtain consistent results for incomes and gaps when analyzing the inequality among the poor individuals.

Therefore, we provide an empirical illustration of how our decompositions could be a good instrument for policy makers since allow them to better understand the sources that cause poverty. Using individual level data from the European Union Survey on Income and Living Conditions (EU-SILC), we compute some rank-dependent indices and their decompositions for 25 countries in two different periods, 2005 and 2011. These five countries have been chosen because their increment or reduction on poverty is derived from different causes. That is, for example, an increase on poverty can be due to an increment on the number of poor, an increase on the intensity of poverty, a growth on the inequality among the poor or some combination of the three. For this reason, we believe that our proposal allows us to identify the source that causes a variation in the poverty index and in consequence to address more efficiency policy aiming at reducing poverty.

Our paper in naturally related to two different strands of the literature. Firstly, it concerns with the literature on aggregation function and on OWA operators and in particular on their decomposition into the self-dual core and the anti-self-dual remainder. Beliakov et al. [6] prove that a family of composed aggregation functions generalizes the Bonferroni mean. García-Lapresta et al. [17] prove that the normalized version of a class of poverty measures based on the one-parameter family of exponential means can be decomposed using the dual decomposition of an aggregation function into a self-dual core and anti-self-dual remainder. This decomposition let them to decompose this poverty family into the three poverty components that can be interpreted as incidence, intensity and the inequality among the poor, respectively. Aristondo et al. [2] prove that the decomposition of the Sen poverty index in terms of the head-count ratio, the mean of the normalized poverty gap and the Gini index, can also be achieved using the dual decomposition of OWA operators. On the other hand, Aristondo et al. [1] analyze the welfare and illfare functions associated with three classical inequality measures, namely the Gini index, the Bonferroni index and the De Vergottini index, proving that these functions can be interpreted as OWA operators and decomposing them in the corresponding two factors.

Secondly, it is associated with the literature on poverty measurement and, in particular, with the decomposition of poverty measures in terms of what Jenkins and Lambert [19] called the three 'I's of poverty, incidence, intensity and inequality among the poor. And also with the correlated research on the consistent decomposition of poverty indices in terms of incomes or gaps.
In this respect, in the literature, different poverty decompositions have been proposed in terms of incomes or gaps of the poor, see Osberg and Xu [24] and Aristondo et al. [3]. This difficulty also arises in other economic fields in which bounded variables are involved. For instance, Erreygers [13] characterizes two indicators which measure achievements and shortfall inequality equally. In turn, Lambert and Zheng [22] introduce a weakener version ensuring that achievement and shortfall inequality ranking should not be reversed. Lasso de la Vega and Aristondo [23] and Aristondo and Lasso de la Vega [4] propose a unified framework in which income and gap distributions can be jointly analyzed.

The remainder of the paper is organized as follows. Section 2 presents basic notation and definition for poverty measurement and the definition of rank-dependent poverty measures completes this section. A description of the OWA operator and its properties are provided in Section 3. This section also includes the definition of the self-dual core and the anti-self-dual remainder of OWA operators and its decomposition. Our proposal for the decomposition of rank-dependent poverty measures based on incidence, intensity and inequality concludes the section. Section 4 provides an empirical illustration, and Section 5 concludes with a summary of our results and concluding remarks.

2 Poverty measures

In what follows, we introduce basic notations and definitions used in poverty measurement.

2.1 Notations and definitions

Points in $[0, \infty]^n$ are denoted $x = (x_1, \ldots, x_n)$, with $1 = (1, \ldots, 1)$, $0 = (0,\ldots,0)$. Accordingly, for every $x \in [0,\infty]$, we have $x \cdot 1 = (x,\ldots,x)$. Given $x, y \in [0,\infty]^n$, by $x \geq y$ we mean $x_i \geq y_i$ for every $i \in \{1,\ldots,n\}$, and by $x > y$ we mean $x \geq y$ and $x \neq y$. With this notation, $x$ represents the income distribution vector of a population of $n \geq 2$ individuals such that $x_i$ stands for the income of $i$-th individual. The set of income distributions is $D = \bigcup_{n \in \mathbb{N}} [0,\infty]^n$. For a given $x \in D$, let us denote by $x_{(1)} \leq \cdots \leq x_{(n)}$ and $x_{[1]} \geq \cdots \geq x_{[n]}$ the non decreasing and non-increasing rearrangement of the coordinates of $x$, respectively. In particular, $x_{(1)} = \min\{x_1,\ldots,x_n\} = x_{[n]}$ and $x_{(n)} = \max\{x_1,\ldots,x_n\} = x_{[1]}$. A permutation $\sigma$ on $\{1,\ldots,n\}$ is denoted as $x_\sigma = (x_{\sigma(1)},\ldots,x_{\sigma(n)})$ and the arithmetic mean as $\mu(x) = (x_1 + \cdots + x_n)/n$.

As defined by Sen [25], the analysis of poverty involves two steps: the identifi-
cation of the poor and the aggregation of their individual poverty levels into a composite poverty measure. The identification requires the choice of a poverty line \( z \in (0, \infty) \) that establishes a cut-off point for poor and non poor. An individual \( i \) is identified as poor if \( x_i < z \) and as non poor if \( x_i \geq z \). We denote by \( q = q(x, z) \) the number of the poor people in the society. For a distribution \( x \), we define the poor distribution and its mean as, \( x_q = (x_{(1)}, \ldots, x_{(q)}) \) and \( \mu(x_q) = \frac{x_{(1)} + \cdots + x_{(q)}}{q} \), respectively.

The second step, the aggregation, assigns a numerical value to each distribution that determines the overall level of poverty. That is, a poverty measure is a non-constant function \( P : D \times [0, \infty) \rightarrow \mathbb{R} \) whose value \( P(x, z) \) denotes the degree of intensity of the poverty associated with an income distribution \( x \) and the poverty line \( z \).

### 2.2 Poverty measures

The most common measure of poverty is the so-called headcount ratio, defined as the ratio between the number of poor people in society \( q \) and the total number of individuals \( n \).

\[
H = H(x, z) = \frac{q}{n} \tag{1}
\]

It ranges from zero (nobody is poor) to one (everybody is poor). This poverty measure satisfies the focus, replication invariance and symmetry axioms. However, the most important limitation is that it does not capture neither how far the poor are from a given poverty line (intensity of the poverty), nor the inequality among the poor since it violates both the monotonicity and transfer axioms.

With the intention of analyzing the individual shortfall, normalized gaps are introduced. For incomes that are below the poverty line, the normalized gap is the relative distance between the income value and the poverty line and for

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3 Donaldson and Weymark [12] define two different ways to identify the poor: the weak and the strong definition. In particular, we use the weak form.

4 The focus axiom demands independence of the index from the non-poor people. The replication invariance axiom claims that replications of the distribution do not change the index value and the symmetry axiom entails that the name and the position do not matter.

5 The Monotonicity axiom requires an increase in poverty with a reduction in the income level of a poor individual. The Transfer axiom states that a poverty measure decreases with a (progressive) transfer of income from a poor to another poorer individual.
incomes above, it is zero. Formally:

\[ g_i = \max \left\{ \frac{z - x_i}{z}, 0 \right\} \]

We denote the censored normalized income gap vector as \( g = (g_1, \ldots, g_n) \) and the normalized gap vector of the poor as \( g_q = (g_1, \ldots, g_q) \). If we compute the mean of the normalized poverty gaps, we obtain an intensity measure of poverty, defined as:

\[ M = M(x, z) = \mu(g_q) = \frac{1}{q} \sum_{i=1}^{q} g_i \]

where \( g_1 \geq \cdots \geq g_q \) and \( x(1) \leq \cdots \leq x(q) < z \). However, this index does not reflect the inequality among the poor. In addition, even if the poverty gap measure satisfies the *monotonicity axiom*, it violates the *transfer axiom*.

### 2.2.1 Rank-dependent poverty measures

In what follows, we review the distribution sensitive or rank-dependent poverty measures. In economic literature, rank-dependent poverty measures are those poverty measures whose individual’s weight depends only on its place in the distribution with respect to the others.

A poverty measure \( P(x, z) \) is rank-dependent if for each income distribution \( x \) in \( D \) and any fixed poverty line \( z \):

\[ P(x, z) = \sum_{i=1}^{q} w_i(q, n) \frac{z - x(i)}{z} = \sum_{i=1}^{q} w_i(q, n) g_i \]

where \( w_1(q, n) \geq w_2(q, n) \geq \cdots \geq w_q(q, n) \). In addition, if the weights decrease strictly then the *transfer axiom* is satisfied.

The most simple index is the so-called *poverty gap ratio*, hereafter *PGR* which represents the mean of the poverty gap ratios among all individuals in the population.

\[ PRG(x, z) = \sum_{i=1}^{q} \frac{1}{n} g_i \]

It is the average of the censored normalized income gap vector. This index satisfies the *focus, replication invariance, symmetry* and *monotonicity axioms*. However, it violates the *transfer axiom*.

Among the indices that depends on the individual ranking, Sen [25] proposes a new poverty measure that places a greater weight on the income of a poor
individual. We refer to it as the Sen poverty index, $S : [0, \infty)^n \times (0, \infty) \to [0, 1]$. Formally, it is defined as follows:

$$S(x, z) = \frac{1}{n(q + 1)} \sum_{i=1}^{q} 2(\frac{1}{n(q + 1)} - i) g[i]$$  \hspace{1cm} (5)

$S(x, z)$ is a weighted sum of the normalized gaps, where the poorest of the group has a weight of $q$ and the richest has a weight equal to 1. This measure satisfies the *focus, symmetry and scale invariance axioms*. However, it violates the *continuity, replication invariance and transfer axioms*.\(^6\)

Sen proposes a second index, a *replication invariance* version, $S'$, approximating the $S$ index by its asymptotic value when the population size is large. In the literature the $S'$ index is the official version of the Sen index.\(^7\)

$$S'(x, z) = \frac{1}{nq} \sum_{i=1}^{q} 2(\frac{1}{n(q + 1)} - i) g[i]$$  \hspace{1cm} (6)

However, the violation of the *transfer axiom* and the lack of *continuity* do not disappear. The fact that the Sen index does not satisfy the *transfer axiom* motivated Thon [27] to propose a variant of Sen’s poverty measure, replacing the rank of the poor individual among the poor, with the rank of each poor individual among the total population distribution.

$$T(x, z) = \frac{1}{n(n + 1)} \sum_{i=1}^{q} 2(\frac{1}{n(n + 1)} - i) g[i]$$  \hspace{1cm} (7)

This simple change allows the Thon index, $T$, to satisfy the *continuity* and *transfer axioms*. However, this measure is a violator of the *replication invariant axiom*.

Afterwards, Shorrocks [26] proposes a new poverty measure, $SST$, that overcomes the lack of the *replication invariance* property suffered by the Thon index.

$$SST(x, z) = \frac{1}{n^2} \sum_{i=1}^{q} 2(\frac{1}{n} - i) g[i]$$  \hspace{1cm} (8)

The Shorrocks index is similar to the Thon poverty measure since for greater values of $n$ and $q$ the Thon measure becomes the Shorrocks measure. Further...
thermore, this measure also overcomes the lack of the replication invariance property suffered by the Thon poverty measure. Retaining that a poverty measure should also be sensitive to the absolute rank of the individuals involved in any income transfer, Kakwani [20] proposes a weakness of the Sen index by raising the weighting function to the power of \( k \).

\[
K_k(x, z) = \sum_{i=1}^{q} \frac{q(q+1-i)^k}{n \sum_{i=1}^{q} i^k} g_{[i]} \quad k \geq 0
\]  

In particular, for \( k = 0 \), \( K_k(x, z) \) reduces to the first Sen index.

The Thon class [28], \( T_\tau \) index, is based on the idea that transfers should have the same impact irrespective of their location. For \( \tau = 2 \) we have the SST index. Formally:

\[
T_\tau(x, z) = \sum_{i=1}^{q} \frac{\tau n + 1 - 2i}{(\tau - 1)n^2 g_{[i]}} \quad \tau \geq 2
\]

Finally, the S-Gini class of poverty measures, \( S_\sigma \), is obtained by combining the S-Gini social welfare measure proposed by Donaldson and Weymark [11] and Chakravarty’s [8] welfare-based poverty measure.

\[
G_\sigma(x, z) = \sum_{i=1}^{q} \left[ \left( \frac{n + 1 - i}{n} \right)^\sigma - \left( \frac{n - i}{n} \right)^\sigma \right] g_{[i]} \quad \sigma \geq 1
\]

In particular, for \( \sigma = 1 \) and \( \sigma = 2 \) we have the PGR and SST indices, respectively.

It is important to note that the parameters \( k \), Kakwani index, \( \tau \), Thon class, and \( \sigma \), S-Gini class, may be chosen according to society’s preference for the sensitivity of the measure to an income transfer at different income positions. In fact, the measures are more sensitive to the bottom of the distribution for higher values of the parameters. That is, for high values of the parameters we are going to be measuring extreme poverty.

3 Alternative decompositions using OWA operators

In the last years, there has been an increasing interest in decomposing indices in terms of the three components of poverty. However, the inequality among the poor component could refer both to the inequality of the income
of the poor and to the inequality of the gap of the poor. And the choice between the inequality index of the poor incomes, and the inequality index of the normalized income gaps of the poor is not innocuous, since it may lead to contradictory conclusions. To overcome this drawback, we propose alternative decompositions on incidence, intensity and inequality for the rank-dependent poverty indices based on ordered weighted averaging (OWA) operators. From the basic properties of OWA operators the inequality term of these decompositions will be a consistent inequality measure, measuring equally the inequality of incomes and gaps.

3.1 Definitions

We summarize basic notations on aggregation functions, OWA operators and the decomposition of OWA operators into the self-dual core and the anti-self-dual remainder.

In this section we restrict the domain to \([0, 1]^n\). We begin by defining standard properties of real functions defined on \([0, 1]^n\). For more details see Fodor and Roubens [14], [7], [5], [18] and Garcia-Lapresta and Marques Pereira [16].

**Definition 1** Let \(A : [0, 1]^n \rightarrow \mathbb{R}\) be a function.

1. \(A\) is symmetric if \(A(x_\sigma) = A(x)\), for any permutation \(\sigma\) on \(\{1, \ldots, n\}\) and all \(x \in [0, 1]^n\).
2. \(A\) is monotonic if \(x \geq y \Rightarrow A(x) \geq A(y)\), for all \(x, y \in [0, 1]^n\). Moreover, \(A\) is strictly monotonic if \(x > y \Rightarrow A(x) > A(y)\), for all \(x, y \in [0, 1]^n\).
3. \(A\) is invariant for translations if \(A(x + t \cdot 1) = A(x)\), for all \(t \in \mathbb{R}\) and \(x \in [0, 1]^n\) such that \(x + t \cdot 1 \in [0, 1]^n\). On the other hand, \(A\) is stable for translations if \(A(x + t \cdot 1) = A(x) + t\), for all \(t \in \mathbb{R}\) and \(x \in [0, 1]^n\) such that \(x + t \cdot 1 \in [0, 1]^n\).
4. \(A\) is invariant for dilations if \(A(\lambda \cdot x) = A(x)\), for all \(\lambda > 0\) and \(x \in [0, 1]^n\) such that \(\lambda \cdot x \in [0, 1]^n\). On the other hand, \(A\) is stable for dilations if \(A(\lambda \cdot x) = \lambda A(x)\), for all \(\lambda > 0\) and \(x \in [0, 1]^n\) such that \(\lambda \cdot x \in [0, 1]^n\).
5. \(A\) is idempotent if \(A(x \cdot 1) = x\), for all \(x \in [0, 1]\).
6. \(A\) is compensative if \(x_{(1)} \leq A(x) \leq x_{(n)}\), for all \(x \in [0, 1]^n\).
7. \(A\) is self-dual if \(A(1 - x) = 1 - A(x)\), for all \(x \in [0, 1]^n\).
8. \(A\) is anti-self-dual if \(A(1 - x) = A(x)\), for all \(x \in [0, 1]^n\).
9. \(A\) is S-convex if \(A(\lambda y) \leq A(x)\), for all \(x, y \in [0, 1]^n\) where \(y\) is obtained from \(x\) by a progressive transfer. Moreover a function \(A\) is strictly S-convex if strict inequality holds.

**Definition 2** Let \(\{A^{(k)}\}_{k \in \mathbb{N}}\) be a sequence of functions, with \(A^{(k)} : [0, 1]^k \rightarrow \mathbb{R}\) and \(A^{(0)}(x) = x\) for every \(x \in [0, 1]\). We say that \(\{A^{(k)}\}_{k \in \mathbb{N}}\) is invariant
for replications if it holds that
\[ A^{(mn)}(\underbrace{x, \ldots, x}_m) = A^{(n)}(x), \]
for all \( x \in [0, 1]^n \) and any number of replications \( m \geq 2 \) of \( x \).

**Definition 3** A function \( A : [0, 1]^n \rightarrow [0, 1] \) is called an \( n \)-ary aggregation function if it is monotonic and \( A(0) = 0, A(1) = 1 \). An aggregation function is said to be strict if it is strictly monotonic.\(^8\)

It is well-known that every aggregation function is compensative. However, aggregation functions in general don’t satisfy neither self-duality nor anti-self-duality properties. Nevertheless, self-duality, anti-self-duality and stability for translations are important properties of aggregation functions and will play an important role in this paper.

### 3.2 Dual decomposition and OWA operators

We know that any aggregation function \( A \), can be decomposed as the sum of a self-dual function and an anti-self-dual function as follows:

**Definition 4** Let \( A : [0, 1]^n \rightarrow [0, 1] \) be an aggregation function. The functions \( \tilde{A}, \hat{A} : [0, 1]^n \rightarrow [0, 1] \) define as
\[
\hat{A}(x) = \frac{A(x) - A(1 - x) + 1}{2} \tag{12}
\]
\[
\tilde{A}(x) = \frac{A(x) + A(1 - x) - 1}{2} \tag{13}
\]

are called the core and the remainder of aggregation function \( A \), respectively.

\( \hat{A} \) is self-dual and it is called the self-dual core of aggregation function \( A \). Note that \( \hat{A} \) is also an aggregation function since \( \hat{A}(0) = 0, \hat{A}(1) = 1 \) and \( \hat{A} \) satisfies monotonicity. On the other hand, \( \tilde{A} \) is called the anti-self-dual remainder of the aggregation function \( A \) since it satisfies the anti-self-duality property. However, it is not an aggregation function since \( \tilde{A}(0) = \tilde{A}(1) = 0 \). Moreover, \(-0.5 \leq \tilde{A}(x) \leq 0.5 \) for every \( x \in [0, 1]^n \). For more information see García-Lapresta and Marques Pereira [16].

**Remark 1** Following García-Lapresta and Marques Pereira [16] any aggregation function can be decomposed as the sum of the self-dual core, \( \hat{A} \), and the anti-self-dual remainder, \( \tilde{A} \), as follows \( A(x) = \hat{A}(x) + \tilde{A}(x) \).

As mentioned, the self-dual core \( \hat{A} \) is also an aggregation function and inherits\(^8\) For simplicity, the n-parity is omitted whenever it is clear from the context.
from the aggregation function \( A \) the properties of continuity, idempotency, compensativeness, symmetry, strict monotonicity, stability for translations, and invariance for replications, whenever \( A \) has these properties.

On the other hand, the anti-self-dual remainder \( \tilde{A} \) is not an aggregation function. In this case, the anti-self-dual remainder \( \tilde{A} \) inherits from the aggregation function \( A \) the properties of continuity, symmetry, invariance for replications, and also (strict) \( S \)-convexity, whenever \( A \) has these properties.

Following with the definitions, Yager [30] introduces OWA operators that are special aggregation functions similar to weighted means.

**Definition 5** Given a weighting vector \( \boldsymbol{w} = (w_1, \ldots, w_n) \in [0,1]^n \) satisfying \( \sum_{i=1}^{n} w_i = 1 \), the OWA operator associated with \( \boldsymbol{w} \) is the aggregation function \( A_{\boldsymbol{w}} : [0,1]^n \rightarrow [0,1] \) defined as follows,

\[
A_{\boldsymbol{w}}(\boldsymbol{x}) = \sum_{i=1}^{n} w_i \, x[i]
\]

In addition, following Chakravarty [9], if the weights are ordered in a (strictly) non-increasing way, \( (w_1 > \cdots > w_n) \) \( w_1 \geq \cdots \geq w_n \), then the OWA operator \( A_{\boldsymbol{w}} \) will be (strictly) \( S \)-convex.

**Remark 2** García-Lapresta and Marques Pereira [16] prove that the self-dual core and the anti-self-dual remainder of an OWA operator can be defined in the same way as follows:

\[
\hat{A}_{\boldsymbol{w}}(\boldsymbol{x}) = \sum_{i=1}^{n} \frac{w_i + w_{n-i+1}}{2} \, x[i]
\]

\[
\tilde{A}_{\boldsymbol{w}}(\boldsymbol{x}) = \sum_{i=1}^{n} \frac{w_i - w_{n-i+1}}{2} \, x[i]
\]

In particular, The self-dual core \( \hat{A}_{\boldsymbol{w}} \) is an OWA operator since \( \sum_{i=1}^{n} (w_i + w_{n-i+1})/2 = 1 \). However, the anti-self-dual remainder \( \tilde{A}_{\boldsymbol{w}} \) is not an OWA operator since \( \hat{A}_{\boldsymbol{w}} \) is not an aggregation function and \( \sum_{i=1}^{n} (w_i - w_{n-i+1})/2 = 0 \).

### 3.3 How to decompose

The rank-dependent poverty indices discussed in the previous section do not automatically fulfill the requirements described in Definition 5. In fact, their sum of the weights differs from 1. To overcome this drawback, let us introduce a normalization factor \( C_P \) for each poverty measure \( P \) and we denote by \( A_P(\mathbf{g}_\circ) \) the corresponding normalized poverty index.
Table 1 shows for each poverty index, the corresponding normalization factor and the OWA operator.

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<thead>
<tr>
<th>P</th>
<th>$C_P$</th>
<th>$A_P(g_q)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PGR</td>
<td>$H$</td>
<td>$\sum_{i=1}^{q} \frac{1}{q} g[i]$</td>
</tr>
<tr>
<td>$S$</td>
<td>$H$</td>
<td>$\sum_{i=1}^{q} \frac{2(g+1-i)}{q(q+1)} g[i]$</td>
</tr>
<tr>
<td>$S'$</td>
<td>$H$</td>
<td>$\sum_{i=1}^{q} \frac{2(q+0.5-i)}{q} g[i]$</td>
</tr>
<tr>
<td>$T$</td>
<td>$H\frac{n(2-H)+1}{n+1}$</td>
<td>$\sum_{i=1}^{q} \frac{2(n+1-i)}{q(2n-q+q)} g[i]$</td>
</tr>
<tr>
<td>$SST$</td>
<td>$H(2-H)$</td>
<td>$\sum_{i=1}^{n} \frac{2(n+0.5-i)}{q(2n-q)} g[i]$</td>
</tr>
<tr>
<td>$K_h$</td>
<td>$H$</td>
<td>$\sum_{i=1}^{q} \frac{(q+1-i)^h}{q} g[i]$</td>
</tr>
<tr>
<td>$T_r$</td>
<td>$H(\tau-H)$</td>
<td>$\sum_{i=1}^{q} \frac{\tau n+1-2i}{q(\tau n-q)} g[i]$</td>
</tr>
<tr>
<td>$G_{\sigma}$</td>
<td>$1-(1-H)^{\sigma}$</td>
<td>$\sum_{i=1}^{q} \frac{\lambda(tn-i)^{\sigma}(tn-i)^{\alpha}}{\mu^2(tn-i)\mu^2(tn-i)} g[i]$</td>
</tr>
</tbody>
</table>

It is easy to prove that $A_P(g_q)$ is an OWA operator applied to the normalized gap vector. In consequence, each rank-dependent poverty measure $P$ can be written as the product of the normalization factor and the normalized poverty index, as follows:

$$P = C_P A_P(g_q)$$

Proposition 1 shows all the properties satisfied by the OWA operator $A_P(g_q)$.

**Proposition 1** The OWA operator $A_P(g_q)$ satisfies continuity, idempotency (hence, compensativeness), symmetry, monotonicity, stability for translations and $S$-convexity, for each rank-dependent poverty index $P$.

**Proof:** Following OWA literature we know that in general, OWA operators are continuous, idempotent (hence, compensative), symmetric and stable for translations. The positivity of weights implies monotonicity. Finally, as for PGR the weights are ordered in a non-increasing way and they are strictly decreasing for the others, then $S$-convexity is satisfied for PGR and strict $S$-convexity for the others. ■

Using the definition of the self-dual core and the anti-self-dual reminder of an OWA operator, we propose an alternative decomposition for all the rank-dependent poverty measures in terms of the three components of poverty. The following proposition formalizes our intuition.

**Proposition 2** All rank-dependent poverty measures can be rewritten as follows:

$$P(x, z) = C_P A_P(g_q) = C_P (\hat{A}_P(g_q) + \tilde{A}_P(g_q))$$  

where, for each index, $\hat{A}_P(g_q)$ and $\tilde{A}_P(g_q)$ are the self-dual core and the anti-self-dual remainder of $A_P(g_q)$ defined in (14) and (15) and reported in Table 2.
Proof: The proof is straightforward.

Table 2: Decompositions

<table>
<thead>
<tr>
<th>Index</th>
<th>( A_P(g_q) )</th>
<th>( \tilde{A}_P(g_q) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PGR</td>
<td>( M(x, z) )</td>
<td>0</td>
</tr>
<tr>
<td>S</td>
<td>( M(x, z) )</td>
<td>( \frac{n}{n+1} A_A(g_q) )</td>
</tr>
<tr>
<td>S'</td>
<td>( M(x, z) )</td>
<td>( A_A(g_q) )</td>
</tr>
<tr>
<td>T</td>
<td>( M(x, z) )</td>
<td>( \frac{nH(x, z)}{2 - nH(x, z)} A_A(g_q) )</td>
</tr>
<tr>
<td>SST</td>
<td>( M(x, z) )</td>
<td>( \frac{H(x, z)}{2 - nH(x, z)} A_A(g_q) )</td>
</tr>
<tr>
<td>( K_k )</td>
<td>( \sum_{i=1}^{q} \left( \frac{q+1-i^k}{2} \right) g[i] )</td>
<td>( \sum_{i=1}^{q} \left( \frac{q+1-i}{2} \right) g[i] )</td>
</tr>
<tr>
<td>( T_r )</td>
<td>( M(x, z) )</td>
<td>( \frac{H(x, z)}{2 - nH(x, z)} A_A(g_q) )</td>
</tr>
<tr>
<td>( G_o )</td>
<td>( \sum_{i=1}^{q} \left( \frac{n+1-i^o-(n-i)^o}{2(n^o-(n-i)^o)} \right) g[i] )</td>
<td>( \sum_{i=1}^{q} \left( \frac{n+1-i^o-(n-i)^o}{2(n^o-(n-i)^o)} \right) g[i] )</td>
</tr>
</tbody>
</table>

Remark 3 By Proposition 1 and Remark 1, \( A_P \) is idempotent, symmetric, strict monotonic and stable for translations. As mentioned before, the self-dual core \( \hat{A}_P \) inherits the properties of idempotency, symmetry, strict monotonicity and stability for translations from the OWA operator \( A_P \). The strictly monotonicity implies that \( \hat{A}_P \) is increasing in the gap of a poor person. The stability for translations means that equal absolute changes in all poor gaps lead to the same absolute change in \( \hat{A}_P \). However, \( \hat{A}_P \) is not S-convex, and then it goes against the Pigou-Dalton transfer principle. Consequently, these properties can be regarded as basic properties of a poverty intensity index.

In turn, the remainder \( \tilde{A}_P \) is symmetric, fulfils \( \tilde{A}_P(g_1, \ldots, g_q) = 0 \) if and only if \( g_1 = \cdots = g_q \). From Proposition 1, \( A_P \) is also strictly S-convex, and consequently the anti-self-dual reminder \( \tilde{A}_P \) satisfies the Pigou-Dalton transfer principle. Therefore, \( \tilde{A}_P \) can be interpreted as a measure of inequality among the poor individuals.

The previous proposition shows that all the rank-dependent poverty indices can be decomposed into the three components of poverty. Firstly, the incidence of poverty is captured by the headcount ratio, \( H(x, z) \). The intensity of poverty, the poverty depths of poor individuals in the society, is summarized by \( \hat{A}_P \). And the inequality among the poor, which is measured in terms of the gaps, is captured by the dispersion measure \( \tilde{A}_P \), which provides the sensitivity to the inequality among the poor.

The following proposition shows that the inequality component \( \hat{A}_P \) is consistent with respect to incomes and gaps, since it measures equally the both concepts.

\footnote{Pigou-Dalton is a crucial axiom in inequality measurement. It requires that a transfer from a rich person to a poorer one decreases inequality.}
Proposition 3 For inequality measure $\tilde{A}_P$, the following equivalence holds:

$$\tilde{A}_P(g_q) = \tilde{A}_P(x_q/z)$$  \hspace{1cm} (17)

Proof: Since $\tilde{A}_P$ is an anti-self-dual function it satisfies

$$\tilde{A}_P(x) = \tilde{A}_P(1 - x),$$

then

$$\tilde{A}_P(g_q) = \tilde{A}_P(1 - g_q) = \tilde{A}_P(x_q/z)$$

that concludes the proof. ■

As mentioned above, the main result of Proposition 3 is that the inequality components of all the rank-dependent poverty indices measure income and gap inequality equally.

The last proposition shows the invariance properties that are satisfied by the inequality measure $\tilde{A}_P$ defined before.

Proposition 4 The inequality measure $\tilde{A}_P$ is invariant for translations and stable for dilations.

The proof is straightforward.

That is, the $\tilde{A}_P$ component remains invariant if the gaps of all the poor individuals are increased by the same amount. Hence, $\tilde{A}_P$ can be considered an absolute inequality measure.

4 Empirical findings

In this section, we illustrate the methodology developed in the paper with data from European Union countries for two different years: 2005 and 2011. For this purpose, we use the European Union Survey on Income and Living Conditions (EU-SILC). The individuals are the unit of analysis and the household disposable income is the variable of interest. Therefore, the equivalence scale, defined as the square root of the number of individuals in each household, is here used to convert household income to individual one. In order to show an exhaustive study of poverty and its decomposition in the three components shown in this paper, we focus on five countries, namely Belgium, France, Italy, Sweden and United Kingdom.

In all these cases, for each country, in order to account for the differences in the purchasing power of different national currencies, monetary values are
converted to Purchasing Power Standards (PPS), including for those countries that share a common currency, for example the Euro. In addition, a time series exercise is done for the period 2005 – 2011, and in order to take into account for inflation, the monetary values are also deflated by the Harmonized Index of Consumer Prices (HICP), base 2005. We want to note that, these changes only affect the mean income of the poor and the poverty line, and not the relative poverty and inequality measures, as they are relative measures.

The last considerations we should do are the poverty line and poverty index elections. Accordingly, individuals are considered poor if they live in a household whose income is below a threshold.

However, before doing this elections, we will do a dominance study following Jenkins and Lambert (1998b) dominance criteria based on TIP curves. Let \( z \) be a poverty line, \( q \) the number of poor people and \( g \) the poverty gap vector. The TIP curve of the normalized poverty gap, denoted by \( TIP(g; p) \), where \( 0 \leq p \leq 1 \), plots against \( p \) the sum of the first \( 100p \) percent of \( g \)-values divided by the total number of receiving units. Thus \( TIP(g; 0) = 0 \) and for integer values \( l, l \leq n, TIP(g; p = l/n) = \frac{\sum_{i=1}^{l-1} g_i}{n} \). The curve is horizontal at all \( p \) corresponding to incomes at or above the poverty line. They provide unanimous poverty orderings for different poverty lines according to all poverty indices defined in terms of relative gaps satisfying some basic properties. That is, given two distributions \( x \) and \( y \), and choosing two corresponding poverty lines, \( z_x \) and \( z_y \), then, \( TIP(g_x; p) \geq TIP(g_y; p) \) for all \( p \in [0, 1] \) for the poverty lines \((z_x, z_y)\) if and only if \( P(x; \xi_x) \geq P(y; \xi_y) \) for all \((\xi_x, \xi_y) = (\lambda z_x, \lambda z_y)\) with \( \lambda \in [0, 1] \) and for all poverty measure \( P \) defined on normalized poverty gap vectors satisfying the axioms of focus, symmetry, replication invariance, monotonicity and transfer.

Consequently, we will start doing a dominance study between 2005 and 2011 computing the TIP curves for the selected five countries in each year. Usually, the poverty line is selected as a value between the 50 and the 60 percent of the median national equivalised income in each year. To carry out this dominance study we will take as the poverty line the 70 percent of the median national equivalised income in each year. In this case, if a dominance between the curves is concluded we will have poverty dominance for a range of poverty measures that fulfill the above mentioned axioms and all the poverty lines that are a lower proportion of the selected poverty lines.

We have drawn the TIP curves for the five countries in the two periods of study. The TIP curves for the selected five countries for the two periods of study are shown in Figure 1. Belgium, France, Sweden and United Kingdom are shown in a unique graph for every \( p \in [0, 1] \). However, the TIP curves for Italy in 2005 and 2011 seems to be very close and we have computed two graphs for different values of \( p \) in order to show it better.
Graphs show that none of the TIP curves intersect between 2005 and 2011. For the countries of Belgium, Italy and Sweden the TIP curves for 2011 are
above the \textit{TIP} curves of 2005, that is, the \textit{TIP} curves in 2011 for these three countries dominate the \textit{TIP} curves of 2005. Consequently, for every poverty lines that are a lower proportion of the 70 percent of the median income for each country and for every poverty measure satisfying \textit{focus, symmetry, replication invariance, monotonicity} and \textit{transfer} axioms the poverty values are higher in 2011 than in 2005.

An opposite result is obtained for France and United Kingdom. Poverty decreases between 2005 and 2011 for a range of poverty lines as mentioned before that are a lower proportion of the 70 percent of the median income for each country, and a set of poverty indices.

Now we have concluded that there exist dominance on poverty for each country between 2005 and 2011, we will try to find the sources of changes on poverty. For this purpose, we have computed some of the decompositions shown in this paper. Particularly, the poverty lines considered in this decompositions are fixed at the 60 percent of the median national equivalised income in each year.\textsuperscript{10} Additionally, standard errors for both the poverty measures and the corresponding poverty components, incidence, intensity and inequality, have been computed via a bootstrap procedure. More in details, to compute the bootstrap standard errors for all the estimators, we randomly re-sample with replacement the income variable, obtaining a new sample of size the same size denoted as $x^*$. Then, the new sample is used to compute each new index. Of course, having a new distribution implies the computation of a new poverty line. The procedure is thus repeated 1000 times. Finally, for each index, the bootstrap standard deviation has been computed as the sample standard deviation of the estimated values from the new samples.

The results for the poverty decompositions in the two periods are reported in Table 3 and Table 4. In particular, the Table 3 collect the decompositions for the five countries and the poverty measures of $S'$ and $SST$ between 2005 and 2011. Whereas, Table 4 is devoted to the decompositions of the poverty families of Kakwani and S-Gini. It is important to point out that at high values of the parameters of the Kakwani and the S-Gini family of indices, the poverty analysis corresponds to the analysis of extreme poverty. Therefore, Table 3 show the results for the overall poverty whereas Table 4 is devoted to the case of extreme poverty.

Starting with Belgium, Italy and Sweden, \textit{TIP} curves have concluded that poverty values should increase for a range of poverty measures. In fact, ev-

\textsuperscript{10} If the poverty line is established depending on the distribution then the focus axiom is violated. However, we follow the usual procedure to determine the poverty line as a percentage of the median income.
ery poverty measures grow up for these three countries between 2005 and 2011. However, the increase on poverty for these three countries have different sources. Starting with Belgium, in addition to the poverty measures, all the poverty components as the incidence, the intensity and the inequality also increase in this period. Following with Italy, the income gap ratio and the inequality among the poor also grow in this period. However, the headcount ratio, the percentage of poor people, decrease for this country between 2005 and 2011. Therefore, if we want to know the reason of the increase on the poverty level, we should focus on its three components: incidence, intensity and inequality among the poor. Concluding that, the growth on poverty is mainly due to increments in the intensity and inequality components. In fact, those components seem to compensate the reduction in the incidence of poor people. And if we look at Sweden, in addition to the poverty measures, the incidence also increase between 2005 and 2011. Nevertheless, the intensity and the inequality components fall in this period. In spite of having a decrease in the last two components, intensity and inequality, the incidence growth compensate these decrements concluding with an increase in poverty measures.

Focusing now in the other two countries, France and United Kingdom, the poverty measures fall down in the period of study, as exhibited by the TIP curves. However, the poverty decrease has different sources for the two countries. The incidence and the intensity components also decrease for the two countries. Nevertheless, the inequality components also decrease for United Kingdom but increase for France.

Following with the poverty analysis, as stressed before, Table 4 summarize the results of the analysis of extreme poverty for two of the five countries. The results for Italy, Belgium and United Kingdom related with extreme poverty are similar to those obtained for global poverty. Besides, we will focus on the other two countries. Focusing on France, the poverty measures fall down between 2005 and 2011 following TIP curves conclusion. Intensity components also decrease in this period but nothing can be concluded with respect to the inequality components. The two components related with Kakwani poverty indices decrease and the other two obtained from the S-Gini poverty measures increase. Finally, if we focus on Sweden, as stressed by the TIP curves, extreme poverty increases for Sweden between 2005 and 2011. On the other hand, inequality of the poor decreases in this period. However, nothing can be concluded for the intensity components, those related with Kakwani indices fall down in this period and the ones for the S-Gini indices grow up.
5 Concluding remarks

The recent literature on poverty measurement stresses the importance for a index to account for intensity, incidence and inequality. In this paper, we propose alternative decompositions for the rank-dependent poverty indices in terms of the above mentioned components using OWA literature. We have shown that an OWA operator is underlying in the definition of the rank-dependent poverty indices and the decomposition of the OWA operators in the self-dual core and the anti-self-dual remainder components have been applied. This decomposition have provided us a decomposition of all the rank-dependent poverty measures in terms of incidence, intensity and inequality among the poor. The properties inherited by each component from the OWA operators allow us to obtain consistent inequality components. That is, inequality measures that measures equally the inequality of the income of the poor, and the inequality of the gap of the poor. Finally, the illustration using EU-SILC data for some European Countries in 2005 and 2011 shows the gripping ability of our decompositions.

Since the decompositions we propose allow for consistent inequality components for the inequality among the poor, we believe that our proposal could be a good instrument for policy makers to better understand the sources that cause poverty.
<table>
<thead>
<tr>
<th></th>
<th>Poverty Indices</th>
<th>Inequality</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>z</td>
<td>H</td>
</tr>
<tr>
<td>Belgium</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2005</td>
<td>20%</td>
<td>0.2157*</td>
</tr>
<tr>
<td></td>
<td>[16.90%, 16.92%]</td>
<td>[0.2158, 0.2161]</td>
</tr>
<tr>
<td></td>
<td>[18.31%, 18.34%]</td>
<td>[0.2243, 0.2244]</td>
</tr>
<tr>
<td>2011</td>
<td></td>
<td>0.2243</td>
</tr>
<tr>
<td></td>
<td>[18.31%, 18.34%]</td>
<td>[0.2239, 0.2242]</td>
</tr>
<tr>
<td>France</td>
<td></td>
<td></td>
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<tr>
<td>2005</td>
<td>16.18%</td>
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<td>[16.19%, 16.20%]</td>
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<tr>
<td></td>
<td>[14.21%, 14.22%]</td>
<td>[0.2403, 0.2405]</td>
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<tr>
<td>2011</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>[14.21%, 14.22%]</td>
<td>[0.2403, 0.2405]</td>
</tr>
<tr>
<td>Italy</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2005</td>
<td>19.38%*</td>
<td>0.2944*</td>
</tr>
<tr>
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<td>[19.38%, 19.38%]</td>
<td>[0.2944, 0.2944]</td>
</tr>
<tr>
<td></td>
<td>[18.46%, 18.47%]</td>
<td>[0.3167, 0.3168]</td>
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<tr>
<td>2011</td>
<td></td>
<td>0.3167**</td>
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<tr>
<td></td>
<td>[18.46%, 18.47%]</td>
<td>[0.3167, 0.3168]</td>
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<tr>
<td>Sweden</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2005</td>
<td>14.51%*</td>
<td>0.2559*</td>
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<td>[14.50%, 14.53%]</td>
<td>[0.2560, 0.2567]</td>
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<tr>
<td></td>
<td>[18.23%, 18.26%]</td>
<td>[0.2527, 0.2531]</td>
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<tr>
<td>2011</td>
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<tr>
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<td>[18.23%, 18.26%]</td>
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<tr>
<td>United Kingdom</td>
<td></td>
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<tr>
<td>2005</td>
<td>20.57%</td>
<td>0.2902*</td>
</tr>
<tr>
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<td>2011</td>
<td></td>
<td>0.2749**</td>
</tr>
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<td>[18.43%, 18.45%]</td>
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</tr>
</tbody>
</table>

* Significant at the 0.05 probability level. ** Significant at the 0.01 probability level. *** Significant at the 0.005 probability level.

Note: Own elaboration from EU-SILC data and HICP from Eurostat web page.
Table 4: EU-SILC 2005-2011. Monetary Variable in PPS.

<table>
<thead>
<tr>
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<th>France</th>
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<tr>
<td></td>
<td>2005</td>
<td>2011</td>
<td>2005</td>
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<tr>
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<td>0.0704</td>
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<td>0.1891*</td>
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<td>0.2564*</td>
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<td>[0.2565, 0.2573]</td>
<td>[0.2540, 0.2545]</td>
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</table>

*, ** and *** Significant at the 0.05, 0.01 and 0.005 probability level, respectively.

Note: Own elaboration from EU-SILC data and HICP from Eurostat web page.

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