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Abstract

This paper presents the advantages of taking into account the distribution of the *individual* wage gap when analyzing female wage discrimination. The limitations of previous approaches such as the classic Oaxaca-Blinder and the recent distributive proposals using quantile regressions or counterfactual functions are thoroughly discussed. The new methodology presented here relies on Jenkins' (1994) work and proposes the use of poverty and deprivation literature techniques that are directly applicable to the measurement of discrimination. In an empirical application, we quantify the relevance of the *glass ceiling* and *sticky floor* phenomena in the Spanish labor market.

Keywords: children distributive analysis, economics of gender, wage discrimination, glass ceiling, *sticky floor*.
JEL Classification: J16, J31, J71.

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The lower wages paid to female workers in comparison with males can easily be checked empirically in most labor markets. The fact that these wage differentials are not justified in terms of labor productivity is usually known as *gender wage discrimination*. In spite of the increasing interest on these matters since the classical works of Oaxaca (1973) and Blinder (1973), not much is yet known about how this phenomenon affects the different subgroups of female workers.

Very recent lines of research aim to include distributional aspects in the study of gender discrimination. Some of them propose the use of quantile regressions in the estimation of wage equations in order to increase the number of points in the earnings distribution at which the wage gap is evaluated (for example García, Hernández and López-Nicolás (2001) and Gardeazábal and Ugidos (2005) among others). Other proposals include a variety of techniques to estimate counterfactual earnings distribution functions in order to compare them with the original wage distribution and quantify the effects of wage differentials throughout the whole earnings range (see Fortin and Lemieux (1998) and Bonjour and Gerfin (2001)). Certainly both approaches allow us to obtain more information from the observed wage distributions than the classical methodology. Nevertheless, both avoid considering the individual dimension of discrimination, i.e. each individual's discrimination experience.

This was addressed by Jenkins (1994), who underlined the need to consider the following two issues in any analysis of wage discrimination: 1) how to identify which individuals suffer discrimination and in what quantity; and 2) how to sum up the wage gaps using an index that verifies a set of desirable normative properties. He proposed to analyze the distribution of individual wage gaps using the theoretical advances in poverty research. Indeed, poverty and discrimination have strong similarities. More precisely, both imply some income or wage gap: either individual income does not provide a *minimum level of resources* (Atkinson, 1998), or similarly, the female wage is below what she would receive *if she was male but otherwise had identical attributes*. From this perspective, the wage gap reveals itself as genuinely individual, implying that its distribution should play a crucial role when quantifying the aggregate level of discrimination.

Discrimination analysis should focus on the definition of indicators that summarize individual wage gaps by using weights that take into account the different discriminatory experiences of female workers. The classical methodology, widely used in empirical work, limits the analysis to the calculation of the *mean* wage gap. This implicitly means estimating all the individual wage gaps and aggregating them, attaching the same weight to each gap, independent of its relative relevance or value within the wage distribution. This does not seem a good idea from a distributive point of view, since the aggregation process includes value judgments in an obscure way. The aim of this paper is to propose a normative framework for the study of wage discrimination based on the literature on poverty and deprivation, which provides us with indicators that are explicit in incorporating the necessary judgments about how to aggregate wage gaps. These indices will permit the ranking of a list of women's earnings' distributions in terms of their discrimination level and the comparison of the discriminatory experiences of women with different personal characteristics (education, occupation, marital status, wage, age, etc.). In doing so, we rely on previous works by Jenkins (1994) and Shorrocks (1998).

Our contribution is fourfold. First, we examine the limitations of recent distributional approaches for the analysis of wage discrimination. Second, we discuss the normative properties that any discrimination measure should satisfy when aggregating individual wage gaps, and we suggest a minimal set of them on which to reach a wide agreement. Next, we propose different discrimination measures, taken from the poverty literature, that are consistent with the above properties. These measures allow us to quantify both absolute and relative (with respect to different reference wages) discrimination. Third, unlike Jenkins (1994), we propose the use of quantile regressions to identify and estimate individual wage gaps. It is shown that the estimation of wage equations by means of quantile regressions and the use of normative measures of discrimination *á la Jenkins* are not exclusive but complementary techniques.[1] Fourth, in order to provide empirical evidence on the theoretical contribution of the paper, we contrast the advantages of our approach using data from a Spanish survey on wages. This last exercise allows us to identify those subgroups of female workers who suffer the highest discrimination levels. We show not only the existence of *glass ceilings* and *sticky floors* in the Spanish labor market, but we also quantify the contribution of each of them to total discrimination.[2] We should underline that, even if we recurrently refer to gender wage discrimination, the contributions of this paper are readily applicable to any other source of discrimination such as race, sexual orientation, nationality, age, religion, citizenship, etc.

The paper is organized as follows. Section 1 presents the classic approach to the measurement of discrimination and gives a sound justification of the importance of considering distributive aspects in discrimination measurement. In Section 2 we discuss the limitations of a variety of distributional techniques recently used in the measurement of wage discrimination. Section 3 presents our theoretical proposal detailing its main contributions. Section 4 provides two alternative empirical procedures to estimate individual wage gaps. In Section 5, we provide empirical evidence on the advantages of our techniques on a sample of Spanish wages micro-data. Finally, Section 6 concludes by presenting our main findings.

1. THE RELEVANCE OF THE DISTRIBUTIVE APPROACH IN ANALYSING WAGE DISCRIMINATION

1.1 Wage discrimination: The identification problem

Usually, gender wage discrimination is identified as the difference in earnings between male and female workers who are otherwise identical in their attributes and thus in their expected productivity. In order to detect its presence and to measure its relevance, researchers have traditionally estimated wage equations conditional on a list of variables which, *a priori*, are potential determinants of the individual's salary.

Thus, two separated *mincerian* log wage equations for males and females are commonly estimated:

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[1] Note that the counterfactual functions approach does not identify the discriminated female workers and does not provide us with individualized wage gaps. In that case Jenkins's analysis becomes impossible.

[2] Usually, the literature has identified the existence of a *glass ceiling* when the gender pay gap is significantly larger at the top of the wage distribution. In contrast, Arulampalam, Booth and Bryan (2004), after Booth, Francesconi and Frank (2003), identified as a *sticky floor* when the gender wage gap is significantly larger at the bottom of the wage distribution.

$$\omega_{m_i} = \ln(y_{m_i}) = Z'_{m_i}\beta_m + u_{m_i}$$
$$\omega_{f_i} = \ln(y_{f_i}) = Z'_{f_i}\beta_f + u_{f_i} \qquad [1]$$

where m refers to males, f to females, y_i stands for the i^{th} worker hourly wage, ω_i is the natural logarithm of y_i, Z'_i is the vector of characteristics, β are the characteristics' rates of return, and u_i is the corresponding error term.[3]

1.2 Wage discrimination: The aggregation problem

Traditionally based on OLS estimations of these wage equations, discrimination has been evaluated in the mean distribution of the characteristics, and has thus quantified the wage discrimination suffered by the *mean* female worker when compared to the *mean* male worker. This is precisely the approach proposed by Oaxaca (1973) and Blinder (1973) in their seminal articles and which has been recurrently utilized in the literature ever since. In the original Oaxaca-Blinder decomposition, the mean observed wage gap is divided into two components: a first component, A, would quantify the labor market premium on the mean differences in characteristics between genders, while the second component, B, would show how differently the labor market rewards workers with a different gender evaluated at the mean female characteristics:

$$\overline{\ln(y_m)} - \overline{\ln(y_f)} = (\overline{Z'_m} - \overline{Z'_f})\hat{\beta}_m + \overline{Z'_f}(\hat{\beta}_m - \hat{\beta}_f) = \mathrm{A} + \mathrm{B}\ .$$

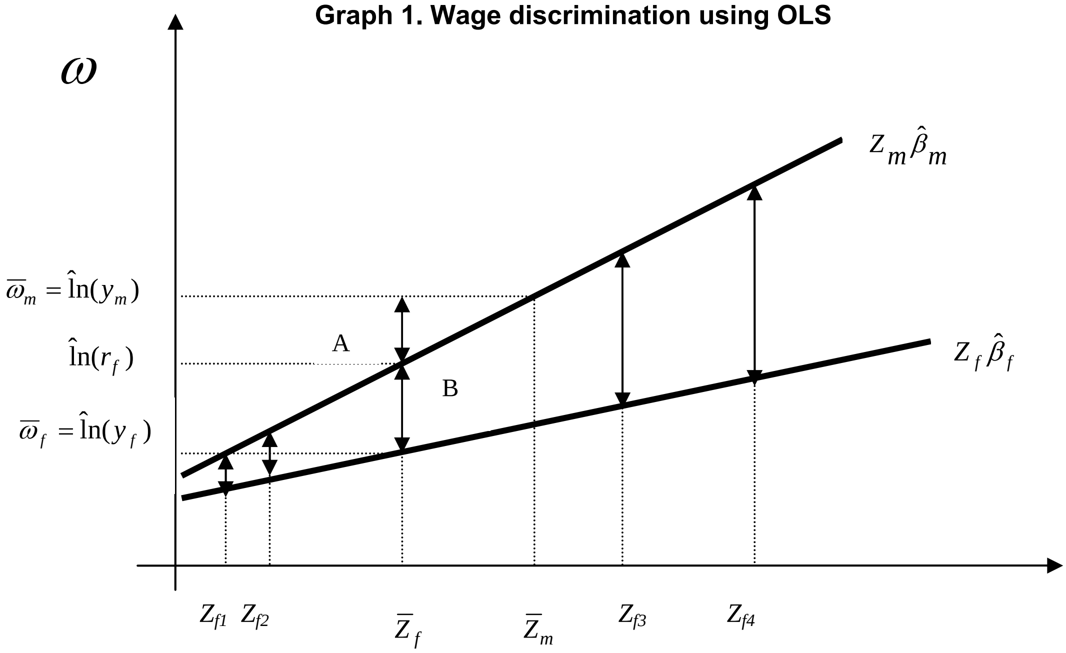
Graph 1 shows, in the one-dimensional case, that the second component denotes the wage penalty the mean female worker faces given that she has a different remuneration of attributes compared to males.[4] Even if seldom noted, it is easy to check that B is the mean of the individual differences between predicted male and female log wages

[3] In his classic survey Cain (1986) offers a detailed reference to the most important theories that attempt to explain discrimination and discusses models based on Mincer (1974). Recently, in Kunze (2000) we find a revision of the most relevant empirical literature in trying to achieve a consistent estimation of the parameters in wage equations.

[4] In Graph 1 the mean female and male worker is $\overline{Z}_f$ and $\overline{Z}_m$, respectively, and different individual female workers in the population are associated with Z_{f1}, Z_{f2}, Z_{f3} and Z_{f4}.

estimated for each woman in the population.[5] The choice of the male wage structure as the non-discriminatory reference is equivalent to considering discrimination as the disadvantage of any group with respect to the most favored group. This would not be true in the case of choosing some other reference.

Graph 1. Wage discrimination using OLS



Whatever the non-discriminatory remuneration structure of reference, the use of the wage distribution mean is a large waste of information. In the first place, the mean does not allow for differences in the discriminatory experience at different points of the wage distribution. Furthermore, and most importantly, it implies assuming that to give the same weight to each different individual discrimination experience is a desirable way of aggregating wage gaps, independently of the actual degree of discrimination suffered by each individual. This all imposes, implicitly and in an obscure way, value judgments that are rather implausible from a normative point of view. There has been little, if any, discussion in the literature on the adequacy of these assumptions. Most probably this

[5] Thus, $B = \sum_i \left(\overline{Z}_{f_i}\hat{\beta}_m - \overline{Z}_{f_i}\hat{\beta}_f \right) / n$, being n the total number of female workers. Notice that if there are women enjoying negative wage gaps, $\overline{Z}_{f_i}\hat{\beta}_m < \overline{Z}_{f_i}\hat{\beta}_f$, positive and negative gaps would offset each other in B.

has been due to the attractive mathematical properties of the mean and also to the general lack of discussion of normative implications in discrimination measurement.[6]

2. THE LIMITATIONS OF RECENT DISTRIBUTIVE APPROACHES

Recently, a number of papers have introduced a variety of econometric techniques in order to incorporate distributive aspects in the comparative analysis of wage distribution. Since the Juhn, Murphy and Pierce (1991, 1993) seminal papers,[7] a large list of works have suggested that the market remuneration to individual endowments is not constant along the wage range. Buchinsky (1994) presented empirical evidence using quantile regressions in the study of the evolution of wages in the US. Di Nardo, Fortin and Lemieux (1996) quantified the effects generated by the change in the distribution of workers' characteristics on wage density using non-parametric regression techniques to estimate counterfactual wage distributions (which permited them to combine one period's population attributes with the returns structure of another). More recently, in their analysis of Portuguese wage inequality, Machado and Mata (2001) used quantile regressions to model the conditional wage distribution on workers' characteristics allowing for the measurement of different returns for each attibute at different points of the wage range.

Within the studies that aim to measure gender wage discrimination, Blau and Khan (1996, 1997) explained the international differences in female wage deficiency and their evolution in time using the methodology proposed by Juhn, Murphy and Pierce (1991).[8] Fortin and Lemieux (1998) decomposed at various wage percentiles changes in the US gender gap using *rank regressions*. Bonjour and Gerfin (2001) applied the methodology proposed by Donald, Green and Paarsch (2000) to decompose the wage gap in Switzerland. Most recently, other papers have used quantile regressions in order to decompose the gender wage gap at different points of the wage distribution. Examples of this are Reilly (1999) and Newell and Reilly (2001) in the analysis of ex-communist

[6] In this context, we agree with Jenkins (1994) in defending that the study of discrimination should aim to rely on measurements that allow us to identify the differences in results when we incorporate, explicitly, different judgments in the aggregation of individual information.
[7] These authors use OLS regressions in providing alternative disaggregation of the estimated and counterfactual wage differences for different time periods.
[8] This methodology allowed them to take into account the role played by the wage structure in the explanation of the gender wage gap.

countries in transition, Albrecht, Björklund and Vroman (2003) in their study of the *glass-ceiling* in Sweden, Albrecht, Van Vuuren and Vroman (2004) in their work for the Netherlands,[9] García, Hernández and López-Nicolás (2001), Gardeazábal and Ugidos (2005), and De la Rica, Dolado and Llorens (2005) in their works for Spain,[10] and Arulampalam, Booth and Bryan (2005) exploring the gender pay gap over the European Union[11]

We maintain that all these recent approaches to the analysis of discriminatory practices are a clear improvement on other previous ones but present, nevertheless, some important limitations in measuring discrimination from a distributive point of view. In some cases, problems arise from the conceptual confusion of the distributive aspects of measurement with the distributive effects of discrimination. Also, the fact that all the procedures proposed try to avoid incorporating value judgements in the aggregation of the different discriminatory experiences, under a normative shelter, make much more difficult any comparison on the matter. Given the interest on both issues: distributive aspects and value judgements in the aggregation of the gaps, we should pay special attention to the arguments that sustain them.

2.1 The comparison of conditional wage distributions: distributive aspects and conceptual errors in measuring discrimination

In order to provide an illustration of the problems that arise when using counterfactual distribution functions in the estimation of wage discrimination, suppose that we know that the female wage distribution without discrimination is r_f and we compare it with the observed female wage distribution y_f. When moving from y_f to r_f, it will not come as a

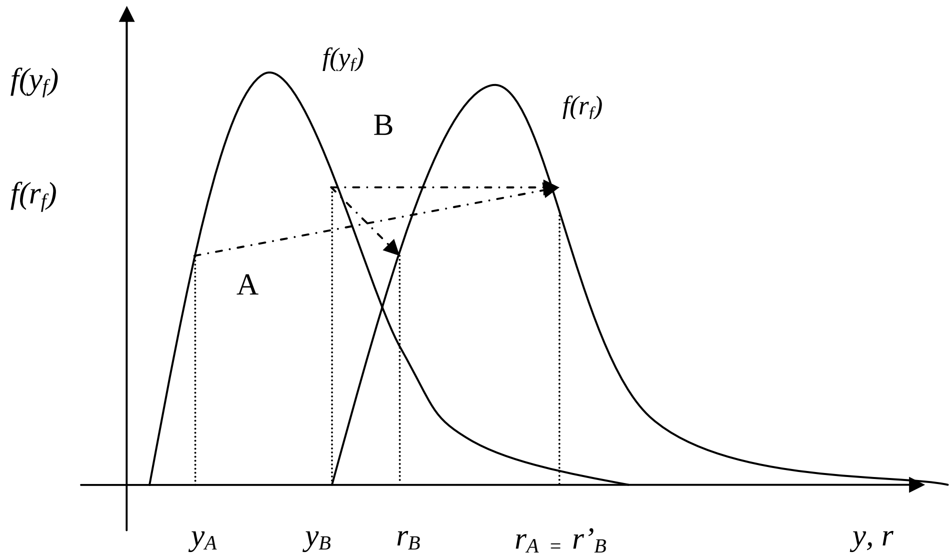
[9] In both cases, authors use techniques developed by Machado and Mata (2005) where quantile regressions are used in order to estimate counterfactual density functions.

[10] In García, Hernández and López-Nicolás (2001) female wage discrimination in the Spanish labour market increases along the wage range both in absolute and in relative terms (in relation to the total wage gap). These authors use instrumental variables in order to endogenise education and other econometric techniques that allow us to avoid a selection bias. In contrast, Gardeazábal and Ugidos (2005) obtain that relative female wage discrimination in Spain decreases as wages increase. Here authors estimate the discrimination at each quantile using the "corresponding" quantile characteristics and not mean population characteristics, as García, Hernández and López-Nicolás (2001) did. De la Rica, Dolado and Llorens (2005), instead, used (even if partially) Albrecht, Björklund y Vroman (2003) proposal and identify the highest wage discrimination levels for Spain in the last deciles of the female wage distribution for those women with a high level of education.

[11] Other recent works that have tackled distributive issues from significantly more simple methodologies are, *inter alia*, Gerry, Kim and Li (2004), Méndez and Hernández (2001) and Vartiainen (2002).

surprise that some female workers change their relative positions. This could imply that the earnings differentials between both distributions, evaluated at each quantile, would not show the true differences in discriminatory experiences of female workers. Let us show an example**:** Suppose that we depart from such a wage distribution as the density function $f(y_f)$ on the left hand side of Graph 2 and, once we eliminate direct wage discrimination, the new density function moves uniformly to the right, $f(r_f)$. In this particular case, the distributive analysis using quantile differences would conclude that all female workers experience the same absolute level of discrimination, whatever their wage.

Graph 2. Wage discrimination using counterfactual densities



Nevertheless, this may not be necessarily true. It may be the case, as depicted in the graph, that all type A women, who initially earned y_A, earn r_A when the discriminatory component is eliminated. Additionally, a similar number of those female workers who were earning y_B could be experiencing a lower wage change once we eliminate discrimination and thus appear in r_B. The rest of type B women would reach the same wage level as females in A, the level $r'_B = r_A$. Obviously, the level of discrimination suffered by group A is much larger than that suffered by group B, but neither the study of the differences in the mean nor the comparison of quantile counterfactual distributions would detect it. In other words, when comparing density functions we are not only quantifying discrimination but also the re-orderings in the wage distribution. In this way, this measurement of discrimination is *contaminated* in the presence of mobility between quantiles.

The comparison of the mean, the variance or quantiles of the wage distribution functions does not allow us to consider the individual discriminatory experience. This strategy makes it impossible to assure that a certain decile suffers more or less discrimination than another, given that the women who were initially placed in each of them may not be the same women once individual discrimination is taken into account. Nevertheless various techniques in the literature on gender wage discrimination are based implicitly on the assumption that these are the same women. Clearly, these papers should observe caution in the interpretation of some of their results.[12] The use of these techniques should remain within the interesting study of the distributive effects of discrimination. However, these effects should not be understood as levels of discrimination at different points of wage distribution.[13]

2.2 The need for normative measures of wage discrimination

It is important to be aware that neither the methodologies based on conditional wage distribution functions nor those using quantile regressions consider how to weight the different levels of discrimination estimated throughout the wage range. Implicitly, they avoid the construction of a single aggregated indicator. This decision may be argued as adequate in the aim of incorporating the least number of value judgments possible in the analysis.[14] To provide measurements of discrimination at different quantiles without any aggregation criterion implies solving the judgments issue in a trivial way: no aggregation is undertaken and therefore no value judgments are incorporated. We should be aware, however, that this strategy makes it rather difficult to compare discrimination levels between distributions (apart from the trivial case in which a given wage distribution presents more discrimination in all estimated quantiles).

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[12] Some recent works that suffer from this problem within the literature of gender wage discrimination are Albrecht, Björklund and Vroman (2003) and Bonjour and Gerfin (2001).

[13] The decomposition of wage discrimination using quantile regressions does not suffer from this problem. This technique quantifies the level of discrimination experienced by females situated at different wage quantiles, but it does not evaluate the wage difference between them and those that occupy the same position in the non-discriminatory distribution.

[14] As it was maintained by Gardeazábal and Ugidos (2005) in their introductory section.

We argue here that in the distribution literature there are valuable options that incorporate judgments in a very reasonable way. The Lorenz dominance criterion aggregates income levels in order to compare different income distributions in terms of inequality under a minimum number of value judgments on which there has been an agreement.[15] This adds robustness, but incompleteness, to the orderings. In those cases in which the Lorenz criterion cannot order functions, complete inequality indices (Gini, Theil or Atkinson index) are unavoidable. The latter incorporate a larger number of value judgments than the Lorenz criterion but allow us to undertake slightly more delicate orderings. Often, the results offered by complete indices do not coincide, but differences between them are not at all random but consistent with their particular normative properties. A deep analysis of these permits us the best comprehension of the analyzed phenomenon.

Jenkins' (1994) approach advances in this direction and proposes discrimination measures that allow for the aggregation of wage gaps.[16] Our proposal extends his approach incorporating some improvements. We propose a normative framework in which to insert a discrimination measurement following the literature on deprivation.

3. NORMATIVE DISCRIMINATION MEASURES

So far we have shown that, firstly, when analyzing discrimination we should focus on the "experience of each individual." Given the bi-dimensional nature of this information, summarized by (y_{f_i}, r_{f_i}), any measure that tries to quantify it should be written as a function of $(r_{f_i} - y_{f_i})$, rather than as a function of r_{f_i} and y_{f_i} taken separately. Secondly, we need to aggregate these individual experiences. This implies

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[15] Basically resumed in two axioms: *symmetry* (or *anonymity*) and the *Pigou-Dalton Principle of Transfers*.

[16] A number of papers have used the indices just as proposed by Jenkins (1994). We know of the empirical works of Makepeace, Paci, Joshi and Dolton (1998), Denny, Harmon and Roche (2000), Gustafsson and Li (2000), Hansen and Wahlberg (2001) and Ullibarri (2003). In Favaro y Magrini (2003), differently from the rest, we find some criticisms to Jenkins' approach and authors propose the estimation of bivariant density functions as an alternative. In our opinion this is not an alternative to Jenkins' techniques but a useful descriptive tool previous to the deeper distributive analysis of discrimination we present here. It is true, however, that this could be complemented with some index which aggregates wage changes experienced by females (for example an index based on transition matrices). In doing this we incorporate *ad hoc* value judgements associated with the index's aggregation properties in an obscure way.

taking value judgments into account, and these are, necessarily, of a subjective nature. Is this a problem? Not if we accept that discrimination is a *bad thing* in the same way that poverty or the duration of unemployment are. Hence the question is: what properties should a measure of discrimination satisfy? The literature on economic poverty has widely accepted a list of normative properties as satisfactory requirements for any poverty measure. We believe that these same properties are also adequate in the case of the study of wage discrimination. Let us discuss our proposal in detail.

3.1 Normative properties of discrimination indices

Consider two vectors of individual wage gaps, x_f and x'_f, where $x_f =$ $(r_{f_1} - y_{f_1}, ..., r_{f_n} - y_{f_n})$, and $x'_f = (r'_{f_1} - y'_{f_1}, ..., r'_{f_s} - y'_{f_s})$, being y_{f_i} and r_{f_i} female wages with and without discrimination, and being *n* and *s* respectively the total number of female workers in each distribution. $d(x_f)$ represents the level of discrimination, which corresponds to distribution $\boldsymbol{x_f}$ for a given measure *d*. The minimal set of normative properties or axioms that *d*(**.**) should satisfy are the following:

1) *Continuity Axiom*. $d(\boldsymbol{x_f})$ must be a continuous function for any vector of wage gaps in its domain, $\boldsymbol{x_f}$.

2) *Focus Axiom*. If we can obtain $\boldsymbol{x'_f}$ from $\boldsymbol{x_f}$ by rises in wages of non-discriminated women, then $d(\boldsymbol{x'_f}) = d(\boldsymbol{x_f})$.

3) *Symmetry* (or *Anonymity) Axiom*. If $\boldsymbol{x'_f}$ can be obtained from $\boldsymbol{x_f}$ by a finite sequence of permutations of individual wage gaps, then $d(\boldsymbol{x'_f}) = d(\boldsymbol{x_f})$.

4) *Replication Invariance Axiom*. If we can obtain $\boldsymbol{x'_f}$ from $\boldsymbol{x_f}$ by replications of the population, then $d(\boldsymbol{x'_f}) = d(\boldsymbol{x_f})$.

5) *(Weak) Monotonicity Axiom*. If $\boldsymbol{x'_f}$ can be obtained from $\boldsymbol{x_f}$ by increasing the discrimination level of a woman, then $d(\boldsymbol{x'_f}) > d(\boldsymbol{x_f})$.

6) *(Weak) Transfer Axiom*. If we can obtain $\boldsymbol{x'_f}$ from $\boldsymbol{x_f}$ by a sequence of "regressive transfers" between two discriminated female workers, so that the one with the highest discrimination suffers an increase in her wage gap equal to the decrease experienced by the other, then $d(\boldsymbol{x'_f}) > d(\boldsymbol{x_f})$.

The *Continuity Axiom* is a reasonable property for any index in order to guarantee that small changes in wage gaps do not lead to large changes in discrimination levels. The *Symmetry Axiom* guarantees that the index does not favor any particular woman, and the *Replication Invariance Axiom* is a technical property that allows for comparisons between distributions of different size. The two other final axioms lead to two basic properties. The *Monotonicity Axiom* refers to discrimination intensity, so that a worsening in the position of a discriminated woman yields a higher level of aggregate discrimination. The *Transfer Axiom* implies that a higher inequality level between discriminated women, in terms of their discrimination sharing, leads to an increase in the discrimination index.

Finally, the *Focus Axiom* requires the index to be dependent on the distribution of discriminated women while disregarding the wage level of the rest of the female workers. This does not mean that measures verifying this axiom are necessarily independent of the existence of women with wage advantages with respect to male workers,[17] but it does require that these salary advantages are not taken into account when measuring aggregate discrimination. We consider this axiom essential in order to properly aggregate individual discrimination. Suppose hypothetically that we find a labor market in which 40 percent of females suffer from a \$100 wage discrimination while another 40 percent earn a \$100 more than their equivalent males. An index that compensates these differences would measure the same discrimination in this labor market than in one in which all females earn exactly the same as identical males.[18] We consider these two situations as clearly different because we believe that discrimination is a form of individual (rather than group) deprivation, just like poverty or unemployment are. In all those cases, it is straightforward that an individual's

[17] In fact, the share of these women over total female workers will be taken into account in all indices that verify *continuity*, *monotonicity* and *replication invariance axioms* (see Zheng (1997) for the poverty case). Other things equal, the larger the share of discriminated women the larger discrimination will be.
[18] This is the case in the measurement of discrimination using the Oaxaca-Blinder decomposition.

deprivation situation cannot be counterbalanced by the lack of deprivation of others.[19] The advantage of our approach is that it provides tools in order to aggregate individual discrimination using an index with reasonable normative properties. Further, it provides a framework to fully characterize discriminated individuals in a society given that it could be the case that some types of discrimination only appear in certain occupations or sectors and not in others. This definitely helps us in deepening the knowledge about discrimination in all possible settings.[20]

Accepting the axioms above, we will be able both to construct discrimination profiles by accumulating individual wage gaps and to develop some dominance criteria to rank wage distributions according to their discrimination level. Next we will be able to make a correspondence between these rankings and those obtained by using complete discrimination indices that also satisfy these properties. This is the case in the inequality and poverty fields, where there are valuable theorems that establish a relationship between the income distribution ranking obtained by Lorenz or TIP's dominance criteria and those obtained by complete inequality and poverty indices compatible with those criteria. Thus, by using a minimal set of judgments, summarized in the above properties, we will be able to identify particular empirical cases where the discrimination distribution ranking is independent of the index chosen, since all indices yield the same result. This makes our analysis of discrimination significantly more robust.

This line of research was opened by Jenkins (1994) when he used the Inverse Generalized Lorenz Curve (IGLC) in the discrimination field, and defined discrimination indices consistent with its dominance criterion.[21] Later, in the deprivation field, Shorrocks (1998) generalized these relationships in the continuous

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[19] This is similar to considering that the existence of famous Gypsy musicians or African-American athletes should not offset the inferior economic position of most individuals from their ethnic or racial group.

[20] Note that our approach allows also for the analysis and characterization of non-discriminated women. Alternatively, one could address the analysis of male discrimination using female wage structure as a reference although, presumably, it would be rather small given that only a minority of men, if any, would appear to be discriminated.

[21] This curve represents the per capita cumulative sum of wage gaps, on absolute values, for each cumulative proportion of women, once they have been ranked from higher to lower absolute wage gap. Note that Jenkins (1994), when defining the IGLC on absolute values of $\boldsymbol{x_f}$, does not impose the *focus axiom*. However, as it has been shown, it seems reasonable to redefine the variable, the dominance criterion and the indices he proposes taking that axiom into account.

case and summarized previous results obtained by different authors in the deprivation field.[22] In what follows, we extend this analysis and propose the use of Discrimination Curves and discrimination indices that will be defined so as to satisfy the above axioms.

3.2 Dominance relations between Discrimination Curves

Let us define $g(x_f)$ to be the vector of individual wage discrimination, which corresponds to the vector of individual wage gaps, $x_f = (r_{f_1} - y_{f_1}, r_{f_2} - y_{f_2}, ..., r_{f_n} - y_{f_n})$:

$$g_i(x_f) = \max\left\{ (r_{f_i} - y_{f_i}), 0 \right\}$$

The Discrimination Curve represents for each $0 \le p \le 1$ the sum of the first 100*p percent of $g_i(x_f)$ values divided by the total number of female workers, n, once these have been ranked from a higher to a lower wage discrimination level. Hence, $g(x_f) = (g_1, g_2, ..., g_n)$ satisfies that $g_1 \ge g_2 \ge ... \ge g_n$, and for each value of $p = k/n$ the curve can be written as:

$$D_p(g(x_f)) = \sum_{i=1}^{k} \frac{g_i(x_f)}{n}$$

where k is any integer number such that $k \le n$.[23] The Discrimination Curve is the IGLC defined for $g(x_f)$ rather than for absolute values of wage gaps, $|x_f|$, as in Jenkins (1994). The latter implies, counter-intuitively, considering positive and negative wage gaps as equivalent.

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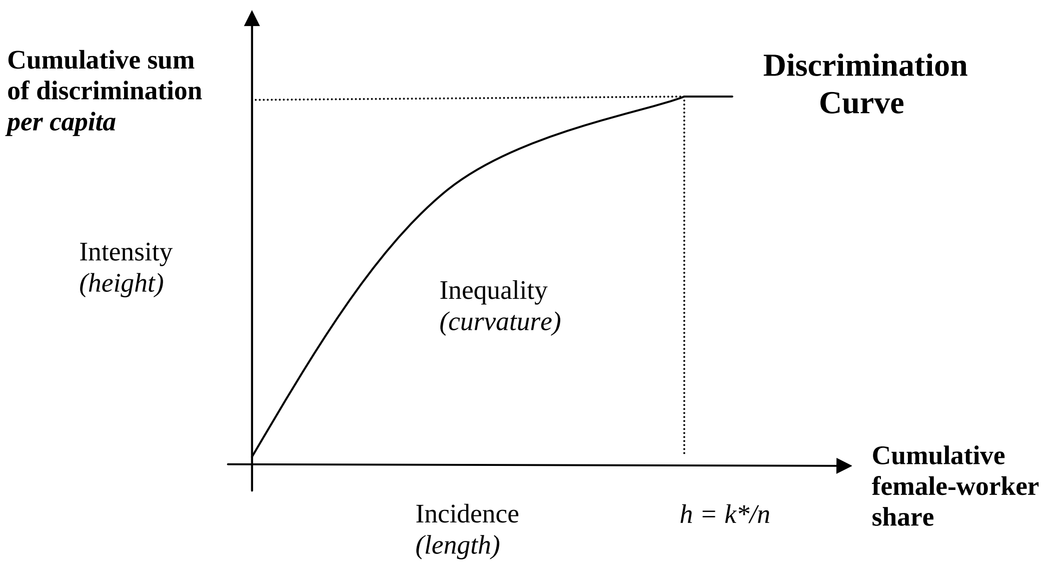
[22] These results are derived from works by Spencer and Fisher (1992), and their “absolute rotated Lorenz curve”, and from Jenkins and Lambert (1997, 1998) and their TIP (“Three ‘I’s of Poverty”) curves in the analysis of poverty. Jenkins (1994) refers to the “inverse generalised Lorenz curve” in the analysis of discrimination, while Shorrocks (1993) applied that approach to the unemployment duration profiles. Also, Blanke and Shorrocks (1994) use this approach to study the length of time spent in poverty.

[23] In the continuous case we would consider a measure of individual wage discrimination, given by variable ***u***, distributed in the female population as the distribution function F. Afterwards, and following Shorrocks (1998), we would define the discrimination curve D(F;·) as:

$$D(F; p) = \int_{F^{-1}(1-p)}^{\infty} u dF(u) = \int_{1-p}^{1} F^{-1}(q) dq, \quad p \in [0,1]$$

D(g) accumulates individual discrimination levels, from higher to lower discrimination, divided by *n*. As shown in Graph 3,[24] *D(g)* is a positive, increasing and concave function; where $D_0(g)=0$, $D_1(g)=\bar{g}$, and takes a constant value when we consider the last discriminated woman, k^*. The shape of the above curve provides us with useful information. First, it shows the *incidence* of discrimination so that to identify the proportion of discriminated women, we only need to know the percentile where the curve becomes a horizontal line, $h=k^*/n$. Second, it informs us about its *intensity*, since the height of the curve is the accumulated discrimination averaged by the number of female workers. Third, it also shows the *inequality* aspect of the discrimination distribution by the degree of concavity of the curve before point *h*.[25]

Graph 3. Discrimination Curve



Definition of dominance in discrimination. Given two wage discrimination distributions, g^1 y g^2, we would say that:

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[24] This is an adaptation of Figure 1 in Jenkins and Lambert (1997), where the properties of the TIP curves are shown to measure aggregate poverty.

[25] Note that if all discriminated women suffered the same absolute discrimination level, the first part of the curve would be a straight line where the slope would be equal to the common individual discrimination.

g^1 dominates g^2 in a discriminatory sense if

$$g^1 \neq g^2 \quad \text{and} \quad D_p(g^1) \leq D_p(g^2) \quad \text{for any} \quad p \in [0,1]$$

It is straightforward then to show that this dominance criterion is closely linked to the six properties mentioned above: the *continuity axiom* (small changes in g yield small changes in the curve); the *focus axiom* (the curve becomes horizontal when the first non-discriminated woman is included in the calculation of the cumulative share, so that the wage advantage of these women is not taken into account); the *symmetry axiom* (since the only aspect of female workers considered is their discrimination level, which makes impossible to identify them); the *replication axiom* (the curve does not change when the initial population has been replicated); the *monotonicity axiom* (the curve turns upwards when the discrimination level of any woman increases); and the *transfer axiom* (the curve increases its degree of concavity when the discrimination is more unevenly distributed while the average discrimination level is kept unchanged).

Thus, we can establish a relationship between dominance in the discriminatory sense and the set of aggregate indices, $d^*(x_f)$, that satisfy in $g(x_f)$ the *continuity*, *focus*, *monotonicity*, *symmetry*, *transfer* and *replication invariance axioms*.

Theorem:[26]

For any pair of wage discrimination distributions, g^1 and g^2 , it follows that,

$$g^1 \text{ dominates } g^2 \text{ in a discriminatory sense}$$

$$\Leftrightarrow$$

$$d(x_f^1) < d(x_f^2) \quad \text{for any} \quad d(\cdot) \in d^*$$

Hence, a higher discrimination curve leads, unambiguously, to a higher discrimination level for an extensive set of discrimination indices. Notice that this theoretical result also makes it possible to quantify the differences in discrimination between two wage

[26] This result was first shown in Shorrocks (1993), where it was used to study the duration of unemployment, and in Jenkins and Lambert (1993) in the poverty field. This work established the basis for later results on TIP curves (Jenkins and Lambert (1997, 1998)). The continuous case is shown in Shorrocks (1998). Jenkins (1994) first used this approach in the wage discrimination field, where he

distributions without using complete indices. This stems from Theorems 4 and 5 in Jenkins and Lambert (1998). Let us consider, for example, that wage distribution B has a higher discrimination level than A. In this case, it can be helpful to increase observed wages, y_f, in distribution B, by multiplying them by a number higher than 1, while keeping r_f constant. This entails decreasing individual wage gaps proportionally, so that later we can check if the initial relationship of dominance still holds. If it holds, we could repeat the exercise to determine the larger interval (written in terms of wages in distribution B) where distribution A has lower discrimination levels. In this way we can study how robust and intense is our initial result even without using complete discrimination indices.

3.3 Complete indices consistent with dominance discrimination

Since the dominance criterion is not always able to give us conclusive results in empirical applications (the estimated discrimination curves can cross) it is interesting to explore some of the indices belonging to d^*. We are interested in those that satisfy both the normative axioms above and any other property that may be of special interest for empirical analysis, such as *decomposability*.

Additive Decomposability. Consider a partition within $\boldsymbol{x_f}$, where $n_1 + n_2 + \ldots + n_J = n$ are the sizes of J subpopulations, $\boldsymbol{x_f^{(1)}}$, $\boldsymbol{x_f^{(2)}}$, ..., $\boldsymbol{x_f^{(J)}}$. A discrimination index d is said to be additively decomposable if:

$$d(x_f) = \sum_{j=1}^{J} \left(\frac{n_j}{n} \right) d(x_f^{(j)}) .$$

This property suggests that it may be desirable to decompose overall discrimination as the weighted sum of subpopulation discrimination levels. However, this is not a widely accepted criterion in the poverty field if, for example, we consider that the poverty level in a group cannot be independent of that in other groups. Despite this serious criticism, the above property is clearly very helpful in most empirical applications, since it allows us to measure the contribution of each population group, j, to the total level of detected

defined wage discrimination as the difference, in absolute terms, between the wages estimated with and without discrimination.

discrimination, $(n_j/n)d(x_f^{(j)})$. This means that we can study discrimination for different female characteristics. Thus not only can we classify women by earnings (as in the quantile estimations mentioned above) but also by any other variables, such as education level, age, or geographical location.

Jenkins (1994) proposed the use of different *families* of aggregate discrimination indices. If they were conveniently defined over $\boldsymbol{x_f}$, instead of $|r_f - y_f|$ as he initially proposed, the main difference of Jenkins' approach with respect to our proposal would be the *transfer axiom*. Jenkins shows a preference for the use of indices that do not satisfy this axiom.[27] In fact, the family of decomposable indices that he uses in his empirical analysis, J_α, is a concave function that depends on the relative individual discrimination level (with respect to the average wage) :

$$J_\alpha = \sum_{i \in f} \omega_i \,(1 - d_i^{-\alpha}) = 1 - \sum_{i \in f} \omega_i d_i^{-\alpha}$$

where $d_i = 1 + |r_{f_i} - y_{f_i}| / \bar{r}_f$ is the normalised wage gap, $\omega_i = \left(y_{f_i} / n\bar{y}_f\right)$ is the earnings rate of individual *i*, and $\alpha > 0$, where α is a parameter which represents the discrimination aversion degree of the index: the higher the parameter value, the higher the weight of larger wage gaps. Note that the concavity of this function guarantees that these indices take values between 0 and 1, which is a good property.[28] However, this property also means that given a constant aggregate wage gap, the more discrimination is focused on fewer women, the lower the discrimination level will be. It follows that evenness in the distribution of discrimination will increase the value of the index. This is inconsistent with what is generally assumed in other forms of relative deprivation like poverty, where increasing the level of deprivation of a more deprived person should have a larger impact on the aggregate level of deprivation than when the increase affects a less deprived person. In our view, that should be the case for discrimination too and thus we propose the use of indices that satisfy the *transfer axiom*.

[27] Even though he offers theoretical results for both cases depending on the sign and value of a parameter.
[28] These numbers represent, respectively, the lowest and highest discrimination level.

Taking into account all the above, we consider that it is not necessary to define new discrimination indices, as Jenkins suggests, but only to make good use of those with the best normative properties within the poverty literature.[29] Therefore, if we adapt the family indices proposed by Foster, Greer and Thorbecke (1984) to measure [absolute] discrimination we can write a discrimination index such that:

$$d_{\alpha}(x_f) = \left(\frac{1}{n}\right) \sum_{i=1}^{k*} (x_{f_i})^{\alpha} \quad , \quad \alpha \geq 0$$

where $k*$ denotes again the number of discriminated female workers and α is the discrimination aversion parameter. For the special case $\alpha = 0$ the index is a head-count measure of the incidence of discrimination among women, for $\alpha = 1$ it accounts for the average level of discrimination per woman. Further, it is well known that for values of α strictly higher than 1 these indices satisfy our normative requirements, $d_{\alpha} \in d^*$, and are additively decomposable.[30]

3.4 Absolute versus relative discrimination

An additional issue in the measurement of discrimination is whether to use a relative rather than an absolute approach. In order to do this we need to define new indices, dr_{α} , which would be a function of the wage gap vector normalized with respect to some average wage, for example the mean female wage without discrimination, $\bar{r}_f$:[31]

$$dr_{\alpha}(x_f / \bar{r}_f) = \left(\frac{1}{n}\right) \sum_{i=1}^{k*} (x_{f\,i} / \bar{r}_{f})^{\alpha}$$

Another interesting possibility consists in normalizing each female wage gap individually by dividing it by her earnings without discrimination, $v_{f_i} = x_{f_i} / r_{f_i}$. This

[29] Zheng (1997 and 2000) offers a survey of the main poverty indices and also of the theorems which link those indices with poverty orderings based on deprivation profiles.
[30] It would also be interesting to measure discrimination adapting our approach to the use of different poverty indices that satisfy other normative properties such as those proposed by Sen (1976) or Hagenaars (1987). The latter would allow us to measure discrimination as the social welfare loss it causes.
[31] Another possibility would be to use the mean observed wage, $\bar{y}_f$.

implies that the critical point is no longer the average wage but, instead, the highest discrimination level that each woman could suffer: [32]

$$dr_\alpha(v_{f_i}) = \left(\frac{1}{n}\right) \sum_{i=1}^{k*} (v_{f_i})^\alpha$$

In order to guarantee that these indices satisfy the same properties as $d_\alpha(x_f)$, we need to redefine the discrimination curves on the normalized wage discrimination vector, D($\Gamma(x_f / \bar{r}_f)$) or D($\Gamma(v_f)$), where

$$\Gamma_i\left(\frac{x_f}{\bar{r}_f}\right) = \max\left\{ \left(\frac{r_{f_i} - y_{f_i}}{\bar{r}_f}\right),\ 0 \right\} \quad \text{and} \quad \Gamma_i(v_f) = \max\left\{ \left(\frac{r_{f_i} - y_{f_i}}{r_{f_i}}\right),\ 0 \right\}$$

reformulating the dominance criterion and the theorem in a consistent way. This point was missed by Jenkins (1994), and implies an inconsistency in his Results 1 and 2 when relating them to J_α and R_v indices. Hence, the Normalised Discrimination Curve, $D(\Gamma)$, which maintains the same graphic characteristics than $D(g)$, can be written as:

$$D_p(\Gamma) = \sum_{i=1}^{k} \frac{\Gamma_i}{n}$$

once the vector Γ have been ranked from higher to lower relative wage discrimination: $\Gamma_1 \geq \Gamma_2 \geq ... \geq \Gamma_n$.

Definition of dominance in normalised discrimination. Given two normalised discrimination vectors, Γ^1 and Γ^2, we say that:

Γ^1 dominates Γ^2 in a discriminatory sense if

[32] The role played by r_{f_i} in this kind of normalization is similar to that of the poverty line in the deprivation literature. Hence, by dividing the individual wage gap by r_{f_i}, we do something similar to what is done in the poverty literature when constructing relative poverty gaps by using individual poverty lines for each household (depending on its size, composition, location,...).

$$\Gamma^1 \neq \Gamma^2 \text{ and } D_p(\Gamma^1) \leq D_p(\Gamma^2) \text{ for any } p \in [0,1]$$

The dominance theorem for the relative case could be stated as follows:

Theorem (relative case):

For any pair of normalised discrimination distributions, Γ^1 and Γ^2, it follows that,

$$\Gamma^1 \text{ dominates } \Gamma^2 \text{ in a discriminatory sense}$$

$$\Leftrightarrow$$

$$dr(x_f / \bar{r}_f)^1 < dr(x_f / \bar{r}_f)^2 \quad \text{for any } dr(\cdot) \in dr^*$$

$$[\, dr(v_f^1) < dr(v_f^2) \quad \text{for any } dr(\cdot) \in dr^* \,]$$

being $dr^*(\cdot)$ the discrimination indices set which satisfies the aforementioned axioms in $\Gamma(x_f / \bar{r}_f)$ [or $\Gamma(v_f)$].

When comparing distributions with the same average wage, estimated without discrimination, the orderings derived from relative indices do not differ from those of the absolute case. The differences will appear when the means differ. In this case, the relative approach implies comparisons of individual discrimination levels given as a proportion of their respective mean, which implies neglecting the differences between both distributions' mean.[33]

4. ESTIMATING INDIVIDUAL WAGE GAPS

In order to implement the above analysis we need to estimate y_{f_i} and r_{f_i}. We only know about Jenkins' (1994) proposal based on OLS estimations of *mincerian* wage equations for men and women. Following the previous expressions [1] we can predict both the estimated wage of a female worker, $\hat{y}_{f_i}$, and her potential wage if her attributes were remunerated as if she was male, $\hat{r}_{f_i}$:

[33] These differences are, however, crucial in the absolute case.

$$\hat{y}_{f_i} = \exp(Z'_{f_i}\hat{\beta}_f + \hat{\sigma}_f^2/2)$$
$$\hat{r}_{f_i} = \exp(Z'_{f_i}\hat{\beta}_m + \hat{\sigma}_f^2/2)$$

where $\hat{\sigma}_f^2$ is the estimated variance of u_f.[34] The conditional wage gap $(\hat{r}_{fi} - \hat{y}_{fi})$ reflects the estimated wage discrimination experienced by a female worker *i*, being $(\hat{r}_f - \hat{y}_f)$ the distribution of the estimated discrimination in the female workers group.[35]

Alternatively we propose to estimate individual wage gaps by quantile regressions. When log wage equations are estimated by quantile regressions, exp($Z'_{fi}\hat{\beta}_f^q$) represents the conditional quantile *q* of female wage distribution y_f:

$$\hat{y}_{f_i}^q = \exp(Z'_{fi}\hat{\beta}_f^q)$$
$$\hat{r}_{f_i}^q = \exp(Z'_{fi}\hat{\beta}_m^q)$$

In the latter case, $\hat{y}_{f_i}^q$ could be calculated by attaching to each working woman *i* those coefficients estimated in the female quantile regression, q_i^*, which minimize her individual residual; and $\hat{r}_{f_i}^q$ would be computed for each woman *i* using the *male* wage structure at her quantile q_i^*. In this way, what we would be actually doing for each woman is selecting her predicted wage, $\hat{y}_{f_i}^{q*}$, as the closest to her actual wage, y_{f_i}, and comparing it with a male wage, $\hat{r}_{f_i}^{q*}$, estimated for a hypothetical man with her

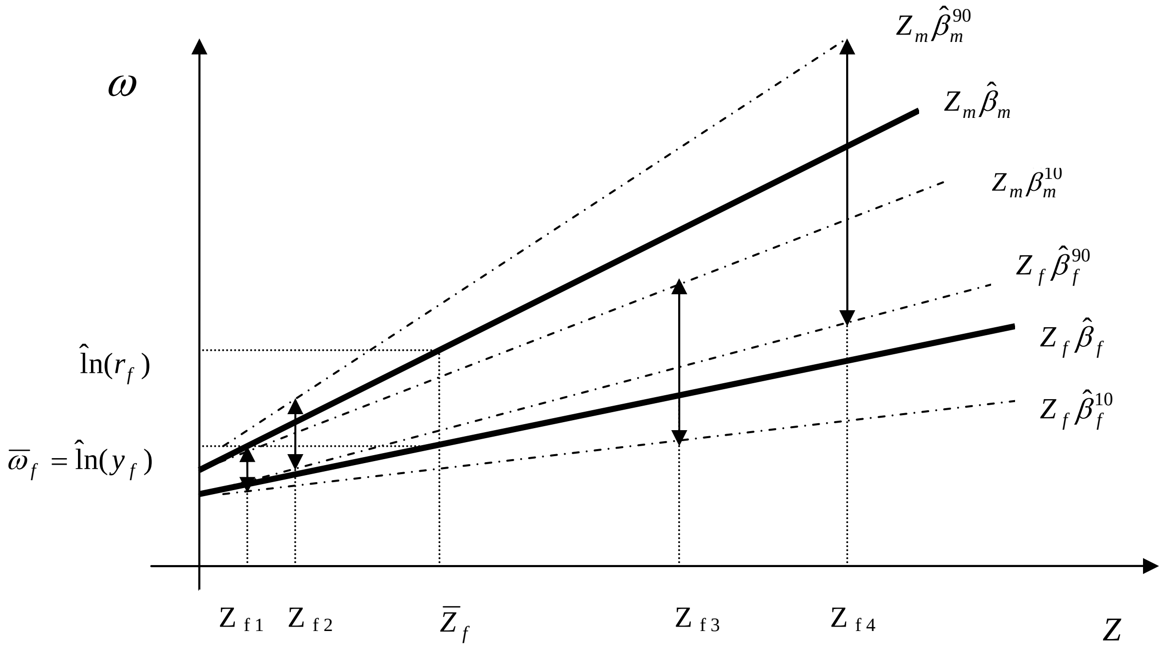
---

[34] Exp($Z'_{fi}\hat{\beta}_f + \hat{\sigma}_f^2/2$) is the expected value of the log-normal variable, y_f, conditional to Z_{fi} in the OLS regression. Note that in Jenkins (1994), there is a mistake because ($\hat{\sigma}_f^2/2$) was dropped out in the above expression. In the second equation we could substitute ($\hat{\sigma}_f^2/2$) by ($\hat{\sigma}_m^2/2$), and use the male variance of residuals. We have checked that in doing so, our empirical results would not change.

[35] We are assuming that individual women's residual wages are unaffected by discrimination, thus the characteristics included in *Z* explain the full phenomenon. Therefore, two women with identical observed characteristics will present the same level of estimated discrimination. Obviously, as in Oaxaca-Blinder decomposition, any misspecification of the model drives to measurement errors in our discrimination level.

characteristics and situated in the same relative ranking within the conditional male wage distribution (as shown in Graph 4).[36]

Graph 4. Wage discrimination using Quantile Regressions



5. AN EMPIRICAL ANALYSIS: THE CASE OF SPAIN

In this section, we show the advantages of our approach identifying those female workers who suffer the highest discrimination levels in the Spanish labor market. Thus, we will compare aggregate discrimination levels estimated by OLS and Quantile Regressions (QR) for males and females using a sample of private sector employees.[37] The purpose here is also to explore to what extent OLS and QR results differ, not only because they estimate at different points of the wage distribution, but also because they yield different aggregate levels of discrimination.

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[36] This is an *ad hoc* choice that might be forcing the interpretation of this type of estimates, but we consider that it appears most reasonable to measure individual discrimination comparing women and men with the same characteristics and situated at the same position in their corresponding conditional wage distributions.

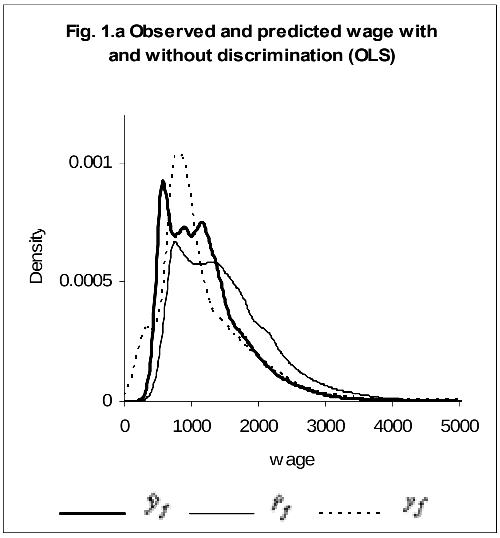
[37] Data come from the *Encuesta de Estructura Salarial* (Survey of Wage Structure) undertaken by the *Instituto Nacional de Estadística* (INE) in 1995. This survey covers employees in firms with ten or more workers and does not include any wage information for employees in Agriculture, Public Administration, Health Services or Education. Those individuals who did not work the entire month or who worked part-time were removed from the sample. The final number of observations for analysis are 27,085 women and 100,208 men.

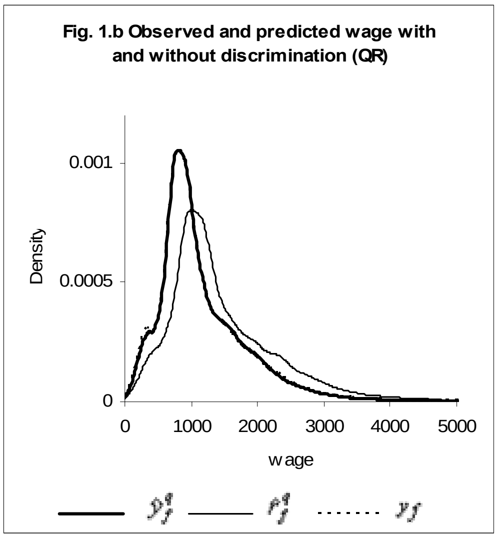
The variable to be explained is the logarithm of hourly wage, and explanatory variables are those usually included in the related literature and available in the database: tenure, (potential) experience, level of education, region of residence, type of contract, occupation (one digit National Classification of Occupations 1994), firm size, type of collective agreement, firm property (public or private) and the market at which most of the firm production is destined (international, national or local).[38] Coefficients are reported in Table A1 in the Appendix. Once we check that wage regressions results are roughly consistent with those in other previous empirical analyses, we construct wage distributions for working women, estimated with and without discrimination. These estimates are denoted respectively by $\hat{y}_f$ and $\hat{r}_f$ in the OLS case, and $\hat{y}_f^q$ and $\hat{r}_f^q$ in the quantile case.[39]

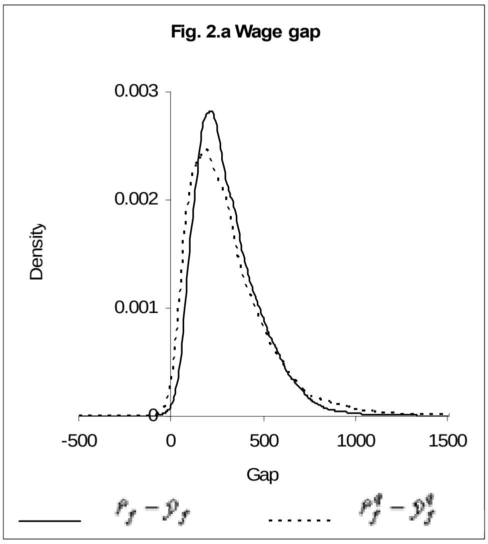
Descriptive statistics for wages and conditional wage gaps estimated with both models are reported in Table A2 in the Appendix. A first interesting result is that the average absolute wage gap (315.1 in OLS and 319.5 in QR) is 26 percent with respect to the observed average wage (1,188), both in OLS and QR estimations. This turns out to be relatively high compared to estimations on female wage discrimination for other developed countries in the literature.[40]

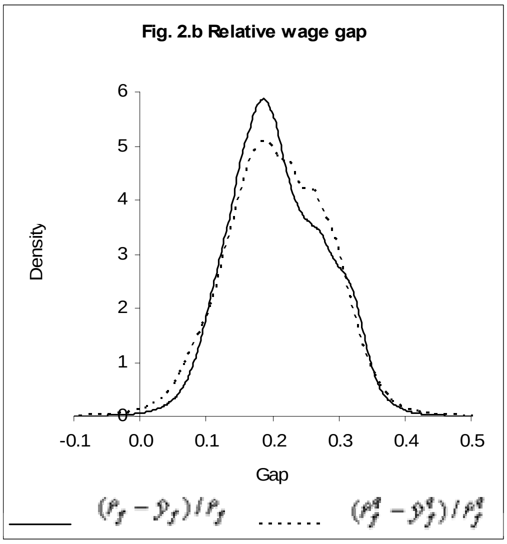
The non-parametric *kernel* wage and wage gaps densities are depicted in Figures 1a to 2b. Observed wages result in a more accurate fit using QR, especially evident in the lower tail, and thus showing a greater dispersion in QR than in OLS. Furthermore, as it is shown in Table A2 in the Appendix, QR also presents a greater dispersion of wage gap density estimations in the absolute case, even if not so much in the relative case when each individual wage gap is normalized by $\hat{r}_{f_i}$ and $\hat{r}_{f_i}^q$.

[38] It was not possible, however, to control for other relevant workers' personal characteristics such as marital status or the presence of children in the household. Furthermore, this database only contains working women and does not allow controlling for *selection bias.*
[39] We compute quantile regressions in ten different points of the distribution (exactly at the middle quantile within each decile: i.e. 5^{th}, 15^{th}, 25^{th},…., 95^{th}).
[40] However, we should be cautious in making comparisons when studies follow different methodological approaches. Recent international comparisons of gender pay gaps are European Commission (2002), Blau and Kahn (2003), and Hernández and Méndez (2005).









In order to compare discrimination levels captured by both procedures from a normative and distribute point of view, absolute and normalized discrimination curves are depicted in Figures 3a and 3b. The figures show that OLS gender gap distribution dominates QR in discrimination. Thus, our second result is that, in the Spanish case, QR discrimination is always larger than OLS for all discrimination indices fulfilling the axioms proposed (in both absolute and relative cases).[41] We can check this just by looking at the bottom three discrimination measures reported in Table 1. This table includes also a couple of additional interesting indices despite the fact that they do not verify all the axioms

[41] Notice that our notion of *relative discrimination* is based on the ratio of the estimated discrimination to the individual wage without discrimination. This should be taken into account when comparing our results with previous evidence.

proposed. Most precisely, the first row reports the *headcount* ratio, or proportion of discriminated women which, using both methods, is larger than 99%, so virtually all females in the sample earn less than a similar male.[42] Further, the second index shows that the estimated amount of money one should transfer in order to remove discrimination using QR is also larger than using OLS.

Fig. 3.a Absolute Discrimination Curves

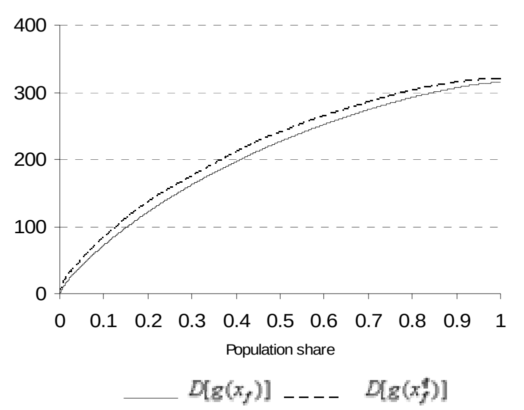


Fig. 3.b Normalized Discrimination Curves

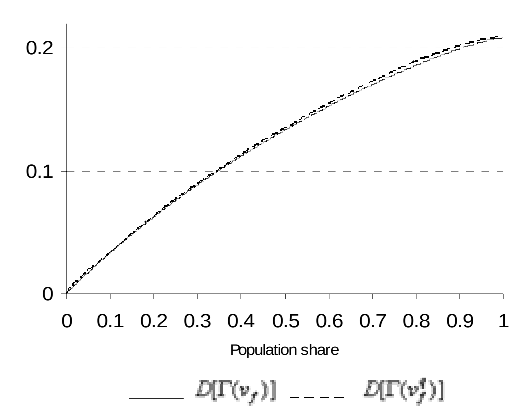


Table 1. Indices of Discrimination

	Absolute OLS	QR		Normalized OLS	QR
$d_{\alpha=0}$	0.9988	0.9962	$dr_{\alpha=0}$	0.9988	0.9962
$d_{\alpha=1}$	315.11	319.61	$dr_{\alpha=1}$	0.208	0.209
$d_{\alpha=2}$	136,000	166,000	$dr_{\alpha=2}$	0.049	0.050
$d_{\alpha=3}$	8.E+07	1.E+08	$dr_{\alpha=3}$	0.012	0.013
$d_{\alpha=4}$	7.E+10	2.E+11	$dr_{\alpha=4}$	0.003	0.004

In order to deepen the distributive analysis, we divide female workers in deciles defined by their observed wages and calculate absolute and relative discrimination curves separately for each group, using the above individual wage gaps estimated over the total population of male and female workers. From Figures 4a and 4b, it is clear that absolute discrimination increases as wages grow in both OLS and QR estimations. If we drew the curves for relative discrimination by deciles, however, we would see that they show an ambiguous pattern due to the appearance of some crosses. In any case, we can assert, differently from the absolute case, that relative discrimination is larger at the bottom

[42] Remember that this survey does not include any Public Administration employees whose wages would presumably decrease discrimination in the Spanish case.

than at the top of the female wage distribution. This is a third finding of our analysis and would be consistent with the existence of a *sticky floor* phenomenon in the Spanish labor market**.**

Fig. 4a. Absolute Discrimination Curves by deciles OLS

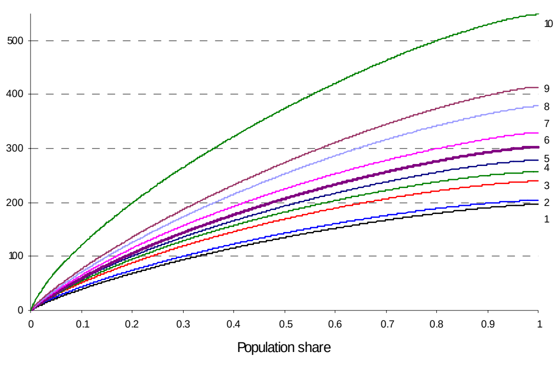


Fig. 4b. Absolute Discrimination Curves by deciles QR

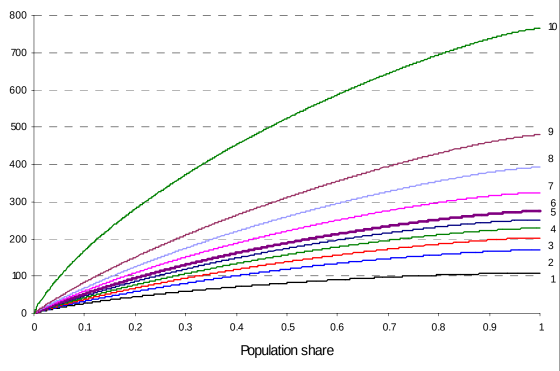


Fig. 5 Discrimination by deciles: $dr_{\alpha=2}$ ratio (average = 1)

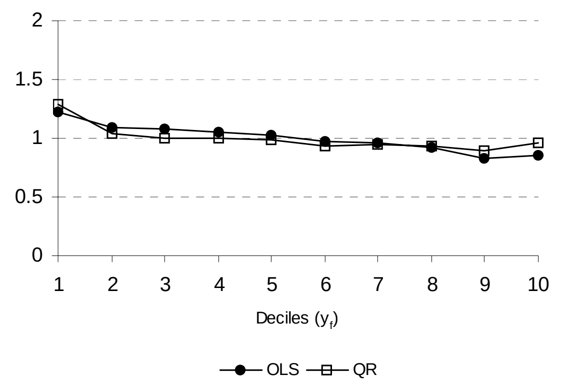
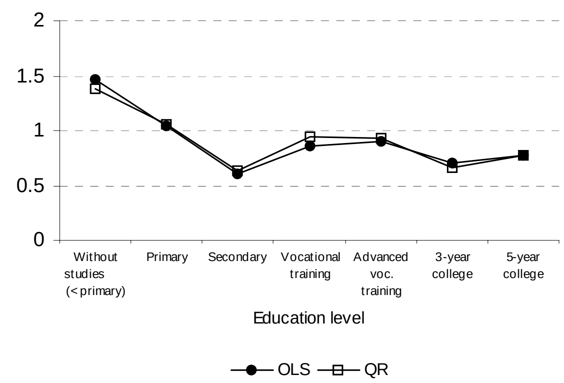


Fig. 6 Discrimination by education: $dr_{\alpha=2}$ ratio (average = 1)



Aiming for a more explicit result on the ranking of relative discrimination by deciles, we propose the use of additively decomposable indices of relative discrimination. This strategy clearly offers less robust results but provides us with some evidence for intermediate deciles whose discrimination curves cross. Figures 5 displays for index $dr_{\alpha=2}$, the ratio of within-group discrimination (for each decile) to average discrimination. A ratio value above (below) one indicates a level of discrimination larger (smaller) than average. Both OLS and QR estimations show very similar patterns. We observe that females in the first decile experience the largest relative discrimination. As we move along the wage distribution, discrimination decreases slightly (with the exception of the last decile). Similarly, when separating females by their education levels, relative discrimination is much higher than average for those without studies and lower for those with higher studies (see Figure 6). However, in this second case the evolution of discrimination along the educational career decreases but without a clear pattern.

Trying to move onwards in this analysis, we follow the strategy in De la Rica, Dolado and Llorens (2005) and break the sample into those females holding a university degree and the rest. Figures 7a and 7b present relative discrimination results by deciles for these two groups, here again we use the overall population average discrimination as a reference.[43] The results for females without university studies (the largest group) resemble the slightly decreasing pattern of total female workers (see Figure 5), now including the last decile too. In contrast, among females with a university degree, relative discrimination has a considerably different pattern: it surprisingly increases with the wage level.[44] This increase is even sharper for the last decile when using QR. Thus, among the more skilled women, it is the group of top-wage female earners that face the largest relative discrimination level. This interesting result indicates the existence of a *glass ceiling* for some female employees.[45]

Fig. 7a Discrimination by deciles non-university degree: $dr_{\alpha=2}$ ratio
(overall population average = 1)

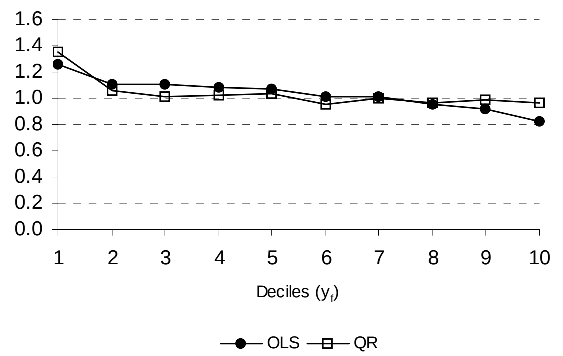
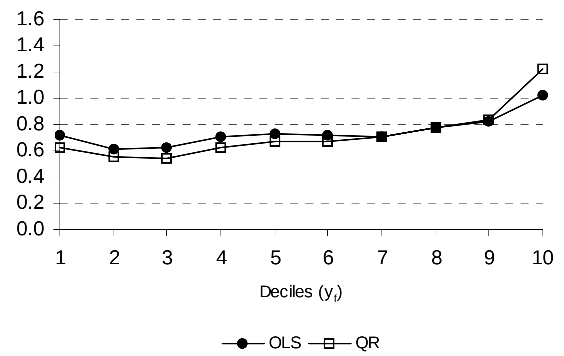


Fig. 7b Discrimination by deciles university degree: *$dr_{\alpha=2}$* ratio
(overall population average = 1)



[43] Note that deciles are constructed for each sub-population. Table A3 in the Appendix shows the demographic weight of each group in the overall population deciles.
[44] Note, however, that its highest value is lower than that experienced by low-wage women without a university degree.
[45] It is relevant to emphasize that this result is not only associated with the index dr_2, but it can be obtained with other discrimination indices due to dominance by deciles in their respective normalized curves.

Therefore, while there seems to be a *sticky floor* for low educated women in the Spanish labor market, for the highest educational group there also seems to be a *glass ceiling*. De la Rica, Dolado and Llorens (2005) obtained a similar result by decomposing the gender wage gap in different percentiles using quantile regressions. The advantage of our approach compared to theirs is that our approach allows us to quantify and compare both phenomena. We do this by calculating the contribution of each group's relative discrimination to the whole discrimination level and present our results in Table 2. In the first column, we include the demographic weight of each subgroup of female workers, (n_j/n)*100, in the second and fifth columns we present their relative discrimination results, $dr_{\alpha=2}(x_f^{(j)})$, and finally we detail their contribution to the whole discrimination level, $(n_j/n)*dr_{\alpha=2}(x_f^{(j)})$ (both in absolute and percentage values for OLS and quantile regressions).

Table 2. Discrimination by education groups

Groups	Population	OLS			Quantile Regressions		
		Within-group discrimination	Contribution to overall discrimination		Within-group discrimination	Contribution to overall discrimination	
	% of all women	$dr^{j}_{\alpha=2}$	Absolute	%	$dr^{j}_{\alpha=2}$	Absolute	%
Non-university degree	**88.6**	**0.050**	**0.044**	**91.5**	**0.051**	**0.045**	**91.8**
by decile							
1	8.9	0.061	0.0054	11.2	0.067	0.0059	11.9
2	8.9	0.053	0.0047	9.7	0.053	0.0047	9.4
3	8.9	0.054	0.0047	9.8	0.050	0.0045	9.0
4	8.9	0.052	0.0046	9.6	0.051	0.0045	9.0
5	8.9	0.052	0.0046	9.5	0.051	0.0045	9.2
6	8.9	0.049	0.0043	8.9	0.047	0.0042	8.5
7	8.9	0.049	0.0044	9.0	0.050	0.0044	8.9
8	8.9	0.046	0.0041	8.5	0.048	0.0042	8.5
9	8.9	0.044	0.0039	8.1	0.049	0.0043	8.8
10	8.9	0.040	0.0036	7.3	0.048	0.0043	8.6
University degree	**11.4**	**0.036**	**0.004**	**8.5**	**0.036**	**0.004**	**8.2**
by decile							
1	1.1	0.035	0.0004	0.8	0.031	0.0004	0.7
2	1.1	0.030	0.0003	0.7	0.028	0.0003	0.6
3	1.1	0.030	0.0003	0.7	0.027	0.0003	0.6
4	1.1	0.035	0.0004	0.8	0.031	0.0004	0.7
5	1.1	0.035	0.0004	0.8	0.033	0.0004	0.8
6	1.1	0.035	0.0004	0.8	0.033	0.0004	0.8
7	1.1	0.034	0.0004	0.8	0.035	0.0004	0.8
8	1.1	0.038	0.0004	0.9	0.039	0.0004	0.9
9	1.1	0.040	0.0005	0.9	0.041	0.0005	1.0
10	1.1	0.050	0.0006	1.2	0.061	0.0007	1.4
All women	**100**	**0.049**	**0.049**	**100**	**0.050**	**0.050**	**100**

In both cases, it can be seen that the highest educated women with the highest salaries bear much more relative discrimination than the highest educated women with the lowest salaries: 0.061 and 0.031 in quantile regressions, respectively. However, their contribution to overall discrimination only represents three decimal points over their demographic weight: 1.4 in comparison with 1.1 percent. In contrast, for female workers with the lowest wages and educational attainments, these percentages are 11.9 and 8.9, respectively. This means that although the *glass ceiling* phenomenon has a qualitative relevance, it is of a relatively small importance if we compare it with the *sticky floor* phenomenon.

6. CONCLUSIONS

In this paper we have detailed the advantages of analyzing wage discrimination from a distributive point of view, considering each individual discriminatory experience. We have exposed the limitations of using the classic approaches to the measurement of discrimination based on the analysis of the mean discriminatory experience and also of those that use some recent distributive methodologies based on quantile regressions and counterfactual wage distributions. Our theoretical contributions are two: First, we underline the imprecise measurement of discrimination using counterfactual functions. This is related to re-orderings as we move from the original wage distribution to a hypothetical non-discriminatory one. Second, and most importantly, we propose a new normative framework for the study of wage discrimination based on the poverty and deprivation literature. For the latter we provide a variety of improvements to Jenkins' (1994) approach to the identification and aggregation of individual discriminatory experiences by adding to its consistency and normative power.

The empirical application using Spanish data allows us to analyze the differences and similarities between OLS and quantile regressions in the measurement of aggregate discrimination from a distributive point of view. For this, we need to develop a new empirical procedure in order to estimate individual discriminatory wage gaps by quantile regressions. We should emphasize three basic results from this exercise. First, for the case of Spain, quantile regressions reveal a significantly higher level of aggregate discrimination compared to that detected using classical estimation techniques. Therefore, the choice between OLS and quantile regressions is all but innocuous from an aggregate point of view.

Second, in spite of the previous result, OLS and QR methods raise roughly similar discrimination patterns throughout the wage range for the Spanish case. These results show that simple estimation techniques, such as OLS regressions on the mean, can supply good outcomes for the measurement of discrimination from a distributive point of view if they are complemented with normative measures.

Finally, it seems clear that absolute discrimination increases with observed wages. However, conclusions are not so straightforward in the relative case. On the one hand, women with very low wages register significantly higher relative discrimination levels than the rest. On the other hand, those females who hold a University degree are a particular case. Those who are earning the highest salaries bear relative discrimination levels around the total wage distribution average, but much larger than all other female workers holding a university degree. This is all pointing to the existence of both *sticky floors* and *glass ceilings* in the Spanish labor market. The former has the highest quantitative relevance while the latter has a more qualitative significance.

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APPENDIX

Table A1. OLS and Quantile regressions estimates for hourly wage in logarithms

	Females						Males					
	OLS	QR at percentiles					OLS	QR at percentiles				
		5	25	45	75	95		5	25	45	75	95
Tenure	0.040	0.054	0.036	0.029	0.024	0.017	0.028	0.041	0.025	0.021	0.015	0.011
Tenure^2	-0.001	-0.001	-0.001	-0.001	0.000	0.000	-0.001	-0.001	0.000	0.000	0.000	0.000
Experience	0.024	0.014	0.017	0.020	0.024	0.028	0.032	0.025	0.027	0.029	0.033	0.037
Experience^2	-0.0003	-0.0002	-0.0002	-0.0003	-0.0003	-0.0003	-0.0004	-0.0004	-0.0004	-0.0004	-0.0004	-0.0005
Education [reference: Without studies or less than primary]												
Primary	0.065	0.014 *	0.047	0.044	0.065	0.089	0.046	0.041	0.024	0.041	0.054	0.072
Secondary	0.275	0.185	0.225	0.236	0.282	0.353	0.234	0.182	0.181	0.220	0.254	0.324
Vocational training	0.143	0.078	0.109	0.121	0.137	0.145	0.135	0.125	0.108	0.133	0.150	0.159
Advanced voc. training	0.234	0.171	0.196	0.197	0.225	0.322	0.243	0.206	0.209	0.238	0.261	0.280
3-year college	0.380	0.241	0.310	0.357	0.414	0.443	0.379	0.302	0.332	0.361	0.382	0.430
5-year college	0.570	0.343	0.474	0.523	0.625	0.703	0.582	0.439	0.503	0.561	0.610	0.679
Type of contract [reference: Fixed term contract]												
Indefinite contract	0.257	0.710	0.408	0.206	0.122	0.154	0.286	0.793	0.405	0.200	0.160	0.169
Occupation [reference: Non-qualified workers (9)]												
Managers	0.664	0.456	0.624	0.658	0.738	0.883	0.742	0.509	0.651	0.732	0.849	0.991
Professionals	0.540	0.503	0.553	0.523	0.516	0.616	0.495	0.432	0.487	0.488	0.512	0.614
Technicians	0.430	0.380	0.406	0.404	0.431	0.520	0.364	0.271	0.316	0.349	0.414	0.522
Clerks	0.219	0.250	0.228	0.206	0.210	0.257	0.191	0.184	0.168	0.183	0.208	0.267
Qualified (services)	0.149	0.184	0.172	0.144	0.112	0.111	0.063	0.095	0.070	0.058	0.049	0.122
Qualified (industry)	0.045	0.046 *	0.019 *	0.018 *	0.045	0.079	0.138	0.160	0.134	0.124	0.125	0.167
Operators	0.017 *	0.005 *	-0.003 *	-0.011 *	0.025	0.060	0.128	0.131	0.123	0.123	0.130	0.151
Size of the firm [reference: 10-19 workers]												
20-49 workers	0.010 *	0.012 *	0.008 *	0.019	0.022	0.041	0.063	0.059	0.046	0.056	0.085	0.092
50-99 workers	0.044	0.030 *	0.019 *	0.061	0.084	0.106	0.136	0.111	0.131	0.137	0.156	0.158
100-199 workers	0.116	0.074	0.100	0.128	0.135	0.176	0.179	0.152	0.191	0.189	0.195	0.196
> 200 workers	0.165	0.139	0.160	0.197	0.216	0.256	0.276	0.281	0.302	0.286	0.289	0.262
Type of labor agreement [reference: Firm labor agreement]												
National labor agreement	-0.072	-0.050	-0.104	-0.109	-0.105	-0.037	-0.066	-0.074	-0.087	-0.088	-0.074	-0.049
Sector or provincial agreement	-0.096	-0.063	-0.103	-0.122	-0.127	-0.071	-0.067	-0.055	-0.088	-0.094	-0.086	-0.061
Type of Sector [reference: Private sector]												
Public sector	0.140	0.243	0.032 *	0.076	0.210	0.144	0.027	0.167	0.061	0.049	-0.019 *	-0.061
Market [reference: Foreign market]												
Local-regional market	-0.057	-0.116	-0.066	-0.049	-0.046	-0.034	-0.016	-0.015 *	-0.019	-0.006 *	-0.007 *	-0.007 *
National market	-0.012 *	-0.030 *	-0.014 *	0.002 *	0.003 *	0.011 *	0.018	-0.023	0.007 *	0.017	0.030	0.060
Constant	5.938	5.101	5.783	6.110	6.330	6.421	6.009	5.073	5.804	6.147	6.379	6.580
R^2 or Pseudo-R^2	0.59	0.45	0.35	0.37	0.43	0.43	0.62	0.46	0.38	0.40	0.42	0.44
Observations	27,085						100,208					

* = coefficient is not significant at 10%. Coefficients for Regions omitted. OLS variances computed using White estimator.
Quantile regressions were performed also at percentiles 15, 35, 55, 65 and 85, not displayed for simplicity.

Table A2. Summary statistics: average and inequality

	Average	Theil (0)	Theil (1)	Theil (2)	Gini
Wages					
Observed					
y_f	1,188	0.182	0.175	0.210	0.320
Predicted by OLS					
$\hat{y}_f$	1,204	0.116	0.116	0.128	0.269
$\hat{r}_f$	1,519	0.111	0.110	0.122	0.262
Predicted by QR					
$\hat{y}_f^q$	1,177	0.166	0.160	0.185	0.308
$\hat{r}_f^q$	1,496	0.167	0.163	0.193	0.310
Conditional Wage Gaps					
Predicted by OLS					
absolute: $\hat{r}_f - \hat{y}_f$	315.1	0.176	0.163	0.185	0.315
relative: $(\hat{r}_f - \hat{y}_f)/\hat{r}_f$	0.208	0.070	0.061	0.059	0.196
Predicted by QR					
absolute: $\hat{r}_f^q - \hat{y}_f^q$	319.5	0.276	0.248	0.312	0.383
relative: $(\hat{r}_f^q - \hat{y}_f^q)/\hat{r}_f^q$	0.209	0.087	0.071	0.069	0.209

Note: Average values in pesetas.

Table A3. Women with and without a university degree
Percentage by decile of observed wage (*y*)

Decile of *y*	**Without a university degree**	**With a university degree**
1	94.4	5.6
2	96.1	3.9
3	96.4	3.6
4	95.8	4.2
5	93.8	6.2
6	93.4	6.6
7	89.3	10.7
8	85.9	14.1
9	80.0	20.0
10	60.7	39.3
Overall population	88.6	11.4