Working Paper Series

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ECINEQ WP 2006 – 34

ECINEQ 2006-34
March 2006

www.ecineq.org

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Abstract

Within a model where the parents make the decisions relating to their children's education, we show that skill dynamics normally results in a sub-optimal situation involving income per capita. This derives from an under-education trap that is endogenously generated. When sub-optimality is caused by a lack of human capital at the steady state, a minimum wage or a redistribution policy makes it possible to increase output per capita and to reduce inequality because both increase the educated share of the population by raising certain households above the trap. These policies only need to be implemented over one period of time, i.e. one generation. Moreover, the sooner they are laid down, the more efficient these policies become. Finally, the income per head at the steady state is higher when individuals have naive expectations rather than when they have perfect predictions. Several simulations are performed that illustrate and corroborate these findings.

Keywords: Education, Inequality, Minimum wage, Redistribution.
JEL Classification: I 21, I 28, J 24.

[*] This research programme has been funded by the C.N.R.S. and the 'Plan Etat –Région (Nord-Pas-de-Calais'.

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1 Introduction

Since the seminal works by Becker (1974), Becker and Tomes (1976, 1979) and Tomes (1981), the analysis of intergenerational human capital mobility has experienced considerable developments. When the positive marginal impact of parents' education on the human capital of their offspring is monotonously decreasing, all dynasties tend to the same human capital in the long term (Loury, 1981). Imperfections on the credit market can only slow this convergence down. However, in cases of non convexity, and particularly when imperfections on the credit market are combined with threshold effects (minimum consumption, fixed cost of education), there is scope for under education traps (Barham and Boadway, 1995; Galor and Tsiddon, 1997): a number of dynasties are indefinitely maintained in a low skilled position and inequality is a lasting characteristic of the steady state. In such situations, government interventions could increase the number of families that opt for higher education. Apart from educational policy that is not analysed here, government intervention may then try to narrow the constraint on education by increasing the income of low paid workers. Two means of public interventions are then typically analysed, i.e. the setting of a minimum wage and a redistribution policy. However, both policies produce two opposite impacts on the education decision, in that they firstly boost education by releasing its financing constraint, but they also reduce the incentive to educate by lowering the relative return to skill.

From a traditional neoclassical perspective, both the minimum wage and redistribution have unwilling outcomes. The minimum wage normally results in unemployment of the less skilled, except in the unlikely case of monopsony on the labour marker (Stigler, 1946). More recent analyses have shown that positive effects of a minimum wage on employment may appear in cases of efficiency wage (Manning, 1995, Rebitzer and Taylor, 1995) and matching approaches (Lang & Kahn, 1998). In addition, in a dynamic perspective, the setting of a minimum wage may foster education and training, and thereby boost growth (Cahuc and Michel, 1995; Agell and Lomerud, 1997; Ravn and Sorensen, 1999; Ragacs, 2004).

The disadvantage of redistribution normally stems from its negative impact on labour supply due to taxation, which creates a trade-off between growth and post-tax inequality. In a political economy perspective, this mechanism has been utilised to show that pre-tax inequality hampers growth because it incites the median voter to demand more redistribution (Alesina & Rodrik, 1994). However, here again, certain approaches have shown that redistribution can boost growth by promoting education and training (Barham and Boadway, 1995; Orazem and Tesfatsion, 1997).

Finally, the human capital intergenerational mobility as well as the impact on it of public intervention depends to a large extent on who takes the decision on education. When this decision belongs to the parents (Durlauf, 1996; Glomm, 1997; Nordblom, 2001; Viaene and Zilcha, 2001), the approach points to the impact of parents' income on children's education. The decision may also be taken by children (Lucas, 1988, Galor and Zeira, 1993, Barham and Boadway, 1995[1]; Galor and Tsiddon, 1997) or it may be the result of a bargaining process involving both parents and children (Glomm and Ravikumar, 1992; Orazem and Tesfatsion, 1997).

In the model developed in this paper, the children's education is decided by the parents. This is based on the hypothesis that there is no access to credit for youths, particularly for those from low income families. This assumption aims at modelling the situation of a large number of European countries, but it is not consistent with the North American situation. We show that the 'natural' skills dynamics normally leads to a sub-optimal situation regarding per capita income. This is due to an under-education trap that is endogenously determined. When sub-optimality is caused by a lack of human capital, egalitarian public policies that raise certain households above the trap swell the educated share of the population, and thereby per capita income. Two policies are analysed and simulated, i.e., the setting of a minimum wage and a redistributive action. Both are transitory because they increase the number of skilled households for all the subsequent periods and thereby render their continued implementation unnecessary. Once executed, these policies increase both efficiency and equality, but redistribution is the most efficient because the minimum wage induces unemployment for similar tax burden and skill upgrading. In addition, we show that, without public intervention, naïve expectations lead to a higher per capita income at the steady state than perfect predictions.

The paper is organised as follows. Education behaviour is studied in Section 2. Section 3 describes production and equilibrium on the factor markets. The main characteristics of the steady state and some of the features of the transitional dynamics are examined in Section 4. Section 5 describes public policies and the mechanisms these generate. Simulation exercises are implemented and the main results of the model are discussed in Section 6.

2 Households and education

An individual lives through two periods of time. When he is young, he firstly receives basic education from his parents over sub-period $(1-\theta)$. Following this, i.e. over the remaining sub-period θ, he either works, or prolongs his education. As an adult, he works one period of time and he is paid according to his skill.

One household comprises 1 adult (parent) and 1 youngster (child), and the child's education is decided and financed by his parent because, by assumption, children have no access to credit facilities.

There are *M* dynasties formed of the successive generations belonging to the same family.

[1] In Barham and Boadway (1995), children take the decision, but this decision depends on the parents' capacity to fund education because the market for credit is imperfect.

'Parent (i,t)' denotes generation t of dynasty i taken as a parent, and 'child (i,t)' generation t of dynasty i taken as a child, where t refers to the period when the individual becomes adult (parent). A family thus comprises parent (i,t) and child (i,t+1).

Parents are provided with different levels of human capital. Human capital is initially distributed across parents (dynasties) over interval $\left[1,\hat{h}\right]$, with $\hat{h} \leq \overline{h}*$, $\overline{h}*$ being the highest human capital at the steady state. The human capital of parent (i,t) is denoted h_{it}, and his income I_{it}.

There are 2 types of occupation, skilled and unskilled.

2.1. Parents' behaviour and spending on education

As an adult, an individual works one period of time and he can choose between being employed as an unskilled or as a skilled worker, whatever his endowment with human capital. We denote w_{Ht} the wage *per unit of human capital*, and w_{Lt} the wage per unit of unskilled labour, at time t. If he is employed as a skilled worker, individual (i,t) receives income $I_{it} = w_{Ht}h_{it}$, and he receives income $I_{it} = w_{Lt}$ when employed as an unskilled worker.

Lemma 1: At time t, all workers whose human capital is lower (higher) than w_{Lt} / w_{Ht} are employed in unskilled (skilled) occupations, and $w_{Lt} \geq w_{Ht}$.

Proof: Individual (i,t) only decides to fill a skilled position if $w_{Ht}h_{it} \geq w_{Lt}$, i.e. $h_{it} \geq w_{Lt} / w_{Ht}$. As a consequence, skill level w_{Lt} / w_{Ht} separates households with skilled occupations from those with unskilled occupations. Any skilled worker provided with human capital h is such that $w_{Ht}h > w_{Lt}$. Since $h \geq 1$, then $w_{Lt} \geq w_{Ht}$.

Definition 1: The *unit skill premium* at time t is the ratio w_{Ht} / w_{Lt} of the wage of one unit of human capital working one unit of time on the wage of one unit of time in an unskilled occupation.

Note that $w_{Ht} / w_{Lt} < 1$ because $w_{Lt} \geq w_{Ht}$ (Lemma 1). In addition, the skill premium of a skilled worker provided with human capital h is then $w_{Ht}h / w_{Lt} > 1$ since, as he has a skilled occupation, $h > w_{Lt} / w_{Ht}$.

The human capital of child (i,t+1) is determined by the following education function:

$$h_{it+1} = \begin{cases} \beta h_{it}^{\eta} & \text{if } s_{it} = 0 \\ \delta h_{it}^{\gamma} s_{it}^{\sigma} & \text{if } 0 < s_{it} < \lambda^{1/\sigma} \\ \delta h_{it}^{\gamma} \lambda & \text{if } s_{it} \geq \lambda^{1/\sigma} \end{cases} \quad (1)$$

with : $0 < \eta < \gamma < 1$, $0 < \sigma < 1$, $1 \leq \beta < \delta$, $\delta\lambda > 1$.

Expressions h_{it}^{η} and h_{it}^{γ} denote the intra-family externalities (h_{it} is the human capital of child (i,t+1)'s parent), and s_{it} the resource that parent (i,t) allocates to his child's education.

βh_{it}^{η} is the human capital of child (i,t+1) at the end of the basic education time, which depends on his parent's human capital.

Inequality $\delta\lambda > 1$ is a necessary condition for human capital not to decrease over time when parents educate their children. Indeed, since $h_{it+1} = \delta h_{it}^{\gamma} s_{it}^{\sigma} < \delta h_{it}^{\gamma} \lambda$ and $h_{it}^{\gamma} < h_{it}$, inequality $\delta\lambda < 1$ would always induce lower human capital for educated children than for their parents.

Feature $s_{it} \geq \lambda^{1/\sigma} \Rightarrow h_{it+1} = \delta h_{it}^{\gamma} \lambda$ signifies that there is an upper limit $\lambda^{1/\sigma}$ in education spending above which higher expenditure will produce no impact on the children's human capital. This assumption is consistent with the empirical finding that, after controlling for parents' skills, an increase in parents' incomes provides no impact on the children's human capital level for the highest incomes (Shea, 2000). Since spending more than $\lambda^{1/\sigma}$ does not increase children's human capital, condition $s_{it} \leq \lambda^{1/\sigma}$ always applies.

Definition 2: The '*saturation point*' is the income for which the optimal expenditure on higher education is exactly equal to the highest efficient expenditure on education $\lambda^{1/\sigma}$.

To paint an accurate picture of education, it is convenient to rewrite the education function in the following fashion: $h_{it+1} = \underbrace{\beta h_{it}^{\eta}}_{Basic\ Education} \times \underbrace{\frac{\delta}{\beta} s_{it}^{\sigma} h_{it}^{\gamma-\eta}}_{Higher\ Education}$, with $s_{it} \geq \left(\frac{\beta}{\delta}\right)^{1/\sigma} h_{it}^{\frac{\eta-\gamma}{\sigma}}$. At the end of basic education, child (i,t+1) attains human capital βh_{it}^{η} which depends on his parent's human capital and on the efficiency of basic education β. If further education is not funded, the child directly enters the labour market. If his parent funds higher education, they must firstly pay an entry cost $\hat{s}_{it} = (\beta/\delta)^{\frac{1}{\sigma}} h_{it}^{\frac{\eta-\gamma}{\sigma}}$ that is increasing with the child's human capital at the end of the basic education.

This feature captures the fact that children provided with higher human capital get access to better and dearer universities. Note that (i) when parents only pay the fixed cost of higher education, their children's human capital remains at the level reached at the end of basic education, and (ii) the higher the parent's skill and income, the higher the entry cost for his child's higher education. When a parent funds higher education, his child's human capital h_{it+1} thus depends, (i) on the level $\beta h_{it}{}^{\eta}$ obtained from his basic education, (ii) on the intra-family externality in the capture of higher education $h_{it}{}^{\gamma-\eta}$, (iii) on the total education expense $s_{it} \geq \hat{s}_{it}$, and (iv) on ratio $\delta / \beta > 1$ that measures the efficiency of higher education.

Lemma 2: A dynasty that continuously does not fund higher education tends to human capital $\beta^{\frac{1}{1-\eta}}$.

Proof: $\beta^{\frac{1}{1-\eta}}$ is the stable steady state of dynamics $h_{it+1} = \beta h_{it}{}^{\eta}$.

Parent (*i*,*t*)'s total income is used for his own consumption (c_{it}) and for his child's education expense s_{it} (consumption, materials, education fees etc.):

$$I_{it} = c_{it} + s_{it} \tag{2}$$

with:

$$I_{it} = \begin{cases} w_{Ht} h_{it} & \text{if} \quad h_{it} \geq w_{Ht} / w_{Lt} \\ w_{Lt} & \text{if} \quad h_{it} < w_{Ht} / w_{Lt} \end{cases} \tag{3}$$

Parent (*i*,*t*)'s utility function depends on his consumption and his child's future reward for education:

$$U_{it} = (1-a)\log c_{it} + a \log I^{e}_{it+1} \tag{4}$$

Coefficient $a < 1$ measures parents' altruism and superscript *e* denotes an expected value.

Parents' altruism is here measured by the reward for education, i.e., $I^{e}_{it+1} = w^{e}_{Ht+1} h_{it+1}$ if the child is employed as a skilled worker, and $I^{e}_{it+1} = w^{e}_{Lt+1}$ if he is employed as an unskilled worker. As it does not depend on the child's utility function, there is no dynastic altruism. This reflects the complexity of calculating the distribution of human capital for all the subsequent periods when households are heterogeneous. In addition, it must be noted that the parent's utility does not depend on his child's total income. Indeed, if this income is $w^{e}_{Ht+1} h_{it+1}$ for children who pursue higher education, it is $w_{Lt}\theta + w^{e}_{Lt+1}$ for those children who directly join the labour market after basic education. But these two values are not comparable since the former does not integrate the implicit income received by the child pursuing higher education when the parent freely provides him with food, accommodation etc. We then suppose that it is the value of the child's human capital, represented by his income once adult, that is accounted for by parents. This is similar to considering the offspring's human capital when his income is proportional to his human capital (Glomm, 1997).

Parent (*i*,*t*) maximises his utility (4) subject to the income constraint (2), the education function (1) and his child's expected income. Given the discontinuity, the decision process is sequential. The parent firstly calculates the solution of this programme for $I^e_{it+1} = w^e_{Ht+1}h_{it+1}$, and then verifies that this is his optimum. Indeed, the parent provides income for education only if it makes the child fill a skilled position, i.e., if $w^e_{Ht+1}h_{it+1} > w^e_{Lt+1}$. Moreover, even in this case, the programme solution may not be optimal. In fact, the parent must compare the utility deriving from his maximisation programme assuming $I^e_{it+1} = w^e_{Ht+1}h_{it+1}$, with that corresponding to zero expense for education, i.e. all income being spent on consumption, and $I^e_{it+1} = w^e_{Lt+1}$ if the non educated child becomes an unskilled worker and $I^e_{it+1} = w^e_{Ht+1}\beta h_{i,t}{}^{\eta}$ if he obtains a skilled occupation even without higher education.

In summary, the household decision process comprises 3 stages: (*i*) determining the solution of the maximisation programme with $I^e_{it+1} = w^e_{Ht+1}h_{it+1}$, (*ii*) verifying that this solution is consistent with a skilled position for the child, and (*iii*) verifying that the parent's utility provided by this solution is higher than that with no education expense ($I_{it} = c_{it}$). We show that condition (*ii*) is also met when condition (*iii*) is fulfilled.

Lemma 3: The condition for parent (*i*,*t*), to fund his child's education is:

(i) $h_{it}{}^{\gamma} > \dfrac{1}{\delta s_{it}{}^{\sigma}}\left(\dfrac{I_{it}}{I_{it}-s_{it}}\right)^{\frac{1-a}{a}}\dfrac{w^e_{Lt+1}}{w^e_{Ht+1}}$ if the child has an unskilled occupation when not funded

(ii) $h_{it}{}^{\gamma-\eta} > \dfrac{\beta}{\delta s_{it}{}^{\sigma}}\left(\dfrac{I_{it}}{I_{it}-s_{it}}\right)^{\frac{1-a}{a}}$ if the child has a skilled occupation when not funded

Proof: See Appendix 1.

Lemma 4: If he educates his child, parent (*i*,*t*) spends for education:

(i) $s_{it} = \dfrac{\sigma a}{1+\sigma a - a}I_{it}$ if he is below the saturation point $\left(I_{it} \le \dfrac{1+\sigma a - a}{\sigma a}\lambda^{1/\sigma}\right)$

(ii) $s_{it} = \lambda^{1/\sigma}$ if he is above the saturation point $\left(I_{it} > \dfrac{1+\sigma a - a}{\sigma a}\lambda^{1/\sigma}\right)$

Proof: See Appendix 2.

Note that any parent whose income I_{it} is lower (higher) than $\frac{1+\sigma a-a}{\sigma a}\lambda^{1/\sigma}$ is below (above) the saturation point.

2.2. Intergenerational Skill Mobility

Proposition 1: *Child* (i,t+1)*'s human capital is:*

$$h_{it+1}=\begin{cases}(1)\quad \beta h_{it}^{\eta} & \textit{if his parent does not fund higher education}\\ (2)\quad \delta\left(\frac{\sigma a}{1+\sigma a-a}w_{Lt}\right)^{\sigma}h_{it}^{\gamma} & \textit{if } \frac{\sigma a w_{Lt}}{1+\sigma a-a}<\lambda^{1/\sigma} \textit{ and his unskilled parent funds higher education}\\ (3)\quad \delta\left(\frac{\sigma a}{1+\sigma a-a}w_{Ht}\right)^{\sigma}h_{it}^{\gamma+\sigma} & \textit{if } \frac{\sigma a w_{Ht}h_{it}}{1+\sigma a-a}<\lambda^{1/\sigma} \textit{ and his skilled parent funds higher education}\\ (4)\quad \delta\lambda h_{it}^{\gamma} & \textit{if } \frac{\sigma a w_{Ht}h_{it}}{1+\sigma a-a}\geq\lambda^{1/\sigma} \textit{ and his skilled parent funds higher education}\end{cases}$$

Proof: By assumption for case (1), and by inserting $s_{it}=\frac{\sigma a}{1+\sigma a-a}I_{it}$ or $s_{it}=\lambda^{1/\sigma}$ into $h_{it+1}=\delta h_{it}^{\gamma}s_{it}^{\sigma}$ in the 3 other cases.

Definition 3: The functions that relate child (i,t+1)'s human capital to his parent's human capital for given values of wages and of the model parameters are called '*Intergenerational Skill Mobility*' (ISM).

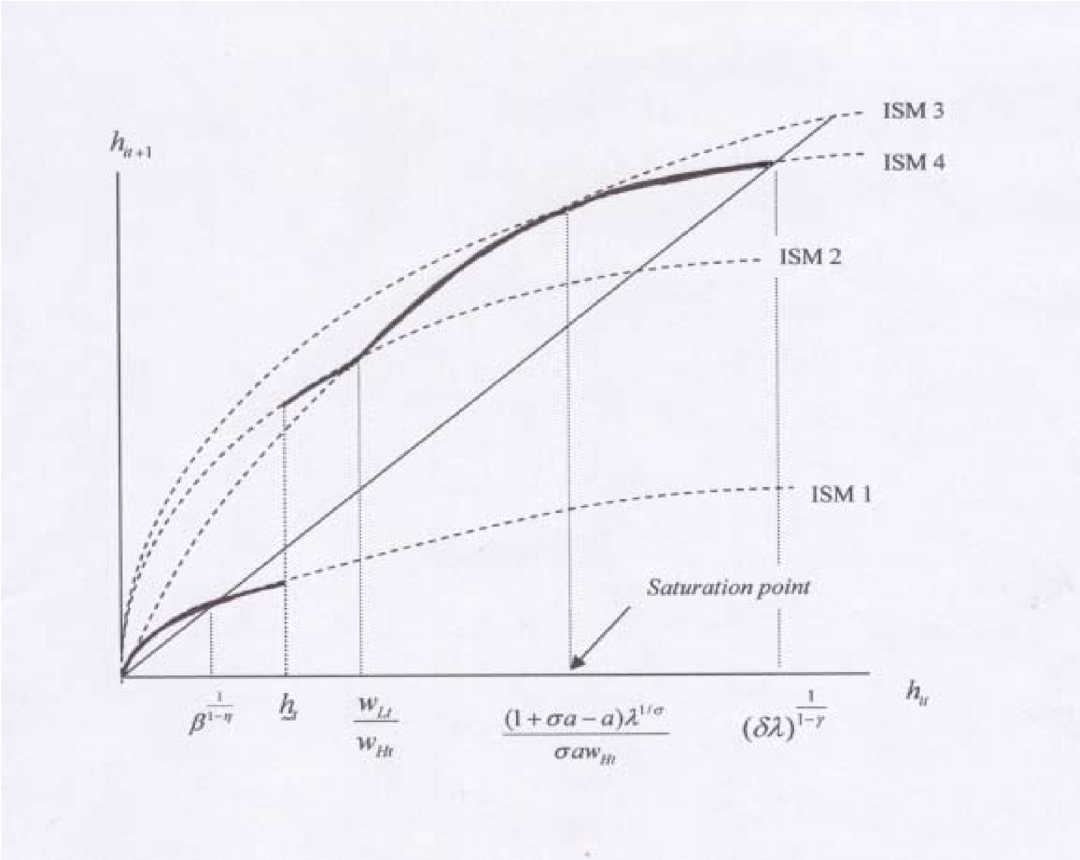
Proposition 1 shows that there are 4 possible ISMs, and thus 4 related curves (Table 1).

Table 1: The four regimes (ISMs)

ISM	**ISM 1**	**ISM 2**	**ISM 3**	**ISM 4**
Interval of human capital	$[1,\underset{\sim}{h}_t[$	$[\underset{\sim}{h}_t, w_{Lt}/w_{Ht}[$	$\left[\frac{w_{Lt}}{w_{Ht}},\left(1+\frac{1-a}{\sigma a}\right)\frac{\lambda^{1/\sigma}}{w_{Ht}}\right[$	$\left[\left(1+\frac{1-a}{\sigma a}\right)\frac{\lambda^{1/\sigma}}{w_{Ht}},(\delta\lambda)^{\frac{1}{1-\gamma}}\right]$
Equation	$h_{it+1}=\beta h_{it}^{\eta}$	$h_{it+1}=\delta\left(\frac{\sigma a}{1+\sigma a-a}w_{Lt}\right)^{\sigma}h_{it}^{\gamma}$	$h_{it+1}=\delta\left(\frac{\sigma a}{1+\sigma a-a}w_{Ht}\right)^{\sigma}h_{it}^{\gamma+\sigma}$	$h_{it+1}=\delta\lambda h_{it}^{\gamma}$

Figure 1 pictures the 4 curves for $\gamma+\sigma<1$. The bold line depicts the operative part of each curve.

Figure 1: Intergenerational Skill Mobility



Let us suppose (this feature is proved further on) that there is a threshold $\underset{\sim}{h}_t$ such that all parents with human capital higher than $\underset{\sim}{h}_t$ fund their children's education, and all parents with human capital lower than $\underset{\sim}{h}_t$ do not. The latter dynasties are then situated on curve ISM 1. All parents (i,t) such that $h_{it} > \underset{\sim}{h}_t$ fund their children's education, and all these children will have more human capital than their parents if $h_{it} < (\delta\lambda)^{\frac{1}{1-\gamma}}$. For families in which parents' human capital belongs to interval $\left[\underset{\sim}{h}_t, w_{Lt}/w_{Ht}\right[$, i.e., having unskilled positions, the intergenerational increase in skill follows function ISM 2. When parents' human capital belongs to interval $\left[\frac{w_{Lt}}{w_{Ht}}, \left(1+\frac{1-a}{\sigma a}\right)\frac{\lambda^{1/\sigma}}{w_{Ht}}\right[$, i.e., parents having a skilled occupation and below the saturation point[2], the intergenerational increase in human capital follows relation ISM 3, with a quicker increase than in the previous case. Finally, when parents are above the saturation point, the intergenerational increase in skill follows relation $h_{it+1} = \delta\lambda h_{it}^{\gamma}$, with steady state $h^* = (\delta\lambda)^{\frac{1}{1-\gamma}}$.

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[2] $h_t = \left(1+\frac{1-a}{\sigma a}\right)\frac{\lambda^{1/\sigma}}{w_{Ht}}$, i.e., the intersection of curves ISM3 and ISM4, is human capital at the saturation point.

2.3. Conditions

Lemma 3 provides the conditions for parent (*i*,*t*) to fund his child's higher education according to the position the child will occupy if not funded. As a matter of fact, parent (*i*,*t*)'s decision to fund his child's higher education depends on three elements:

(i) his occupation (skilled/unskilled),

(ii) the position of his income regarding the saturation point (below/above), and

(iii) the occupation his child will obtain if not funded (skilled/unskilled).

Table 2 depicts the conditions for parent (*i*,*t*) to fund his child's education according to his situation in terms of the three elements above. These conditions are established by inserting into Lemma 3, (i) $s_{it} = \frac{\sigma a}{1+\sigma a - a} I_{it}$ if the parent is below the saturation point $\left(\frac{\sigma a}{1+\sigma a - a} I_{it} \leq \lambda^{1/\sigma} \right)$, (ii) $s_{it} = \lambda^{1/\sigma}$ if the parent is above the saturation point $\left(\frac{\sigma a}{1+\sigma a - a} I_{it} > \lambda^{1/\sigma} \right)$, and (iii) the value of I_{it} according to the parent's occupation.

Table 2: The conditions for parent (*i*,*t*) to fund his child's education

	Parent (*i*,*t*)'s characteristics	Conditions
(1)	Parent (*i*,*t*) has an unskilled occupation He is beneath the saturation point If not educated, his child has an unskilled position	$h_{it}^{\gamma} > c \frac{w_{Lt+1}^{e}}{w_{Ht+1}^{e}} w_{Lt}^{-\sigma}$
(2)	Parent (*i*,*t*) has a skilled occupation He is beneath the saturation point If not educated, his child has an unskilled position	$h_{it}^{\gamma+\sigma} > c \frac{w_{Lt+1}^{e}}{w_{Ht+1}^{e}} w_{Ht}^{-\sigma}$
(3)	Parent (*i*,*t*) has a skilled occupation He is beneath the saturation point If not educated, his child has a skilled position	$h_{it}^{\gamma+\sigma-\eta} > \beta c w_{Ht}^{-\sigma}$
(4)	Parent (*i*,*t*) has a skilled occupation He is above the saturation point If not educated, his child has an unskilled position	$h_{it}^{\gamma} > \frac{1}{\delta\lambda} \left(\frac{w_{Ht} h_{it}}{w_{Ht} h_{it} - \lambda^{1/\sigma}} \right)^{\frac{1-a}{a}} \frac{w_{Lt+1}^{e}}{w_{Ht+1}^{e}}$
(5)	Parent (*i*,*t*) has a skilled occupation He is above the saturation point If not educated, his child has a skilled position	$h_{it}^{\gamma-\eta} > \frac{\beta}{\delta\lambda} \left(\frac{w_{Ht} h_{it}}{w_{Ht} h_{it} - \lambda^{1/\sigma}} \right)^{\frac{1-a}{a}}$

with $c \equiv \frac{1}{\delta} \left(\frac{1+\sigma a - a}{1-a} \right)^{\frac{1-a}{a}+\sigma} \left(\frac{1-a}{\sigma a} \right)^{\sigma}$

The case in which the parent has an unskilled occupation and his non educated child a skilled one is not possible for normal values of the parameters. In addition, we suppose that the model parameters are such (i) that all unskilled parents are below the saturation point, and (ii) that skilled parents can attain the saturation point, i.e., ISM 4 always cuts the 45°line before ISM 3.

The five conditions in Table 2 are ranked according to parents' skill level. However, cases (3) and (4) exclude each other. In fact, since parents related to (3) have lower human capital than those related to (4), the children without higher education of the former cannot have more human capital than the children without higher education of the latter.

All parents depend on one of these five conditions. Inside each of the 5 sets of parents, if the related condition is fulfilled for parent (*i*,*t*), then it is also fulfilled for any parent whose human capital is higher than h_{it}. This is because an upward shift in the hierarchy of parents' human capital always increases the left hand side of the related inequality whereas it exerts no impact on (cases 1, 2 and 3), or it decreases (cases 4 and 5), its right hand side.

Proposition 2: *At any time t, if non educated children have unskilled occupations, there is one human capital threshold* $\underset{\sim}{h}_t$ *above which all parents fund their children's education, and under which all parents do not.*

Proof: If we denote $]n]$ the fact that condition (*n*), *n* = 1…5, is met by at least one of the parents who depend on it, and $[n]$ the fact that (*n*) is fulfilled by all parents who depend on it, we may show (see Appendix 3): $]1] \Rightarrow [2]$, $]2] \Rightarrow [3]$, $]2] \Rightarrow [4]$, and $]4] \Rightarrow [5]$. We also know that within each set of parents, if the related condition is fulfilled for one of them, then it is fulfilled for all parents with higher human capital. Consequently, if parent (*i*,*t*) funds his child's education, all parents with human capital higher than h_{it} do the same: there is one threshold at each period above which all parents fund their children's education, and below which they do not.

It must be noted that (i) at any time, threshold $\underset{\sim}{h}_t$ is determined by one of the five considered conditions, and (ii) $\underset{\sim}{h}_t$ changes over time since it depends on the distribution of skills and on the wages of both the current period and the subsequent period. For example, if there is at least one unskilled parent who funds his child's education, then this threshold is given by condition (1) in Table 2: $\underset{\sim}{h}_t = \left(c \frac{w^e_{Lt+1}}{w^e_{Ht+1}} w_{Lt}^{-\sigma} \right)^{\frac{1}{\gamma}}$.

3 Production and equilibrium

3.1. Production

There are two factors of production, unskilled labour *L* and skilled labour *H*.

L consists of simple occupations for which no skill is required. In *L*, one individual's unit of working time represents 1 unit of labour.

H consists of skilled occupations of different complexity and perfectly substitutable. Individuals' occupation complexity is proportional to their human capital, and one individual's contribution to *H* is assumed to be equal to his human capital.

The economy produces one good, the price of which is 1, with the Cobb-Douglas technology:

$$Y_{jt} = H_{jt}^{\alpha} L_{jt}^{1-\alpha} \tag{5}$$

Where Y_{jt}, H_{jt} and L_{jt} respectively denote firm *j*' s production and use of skilled and unskilled labour at time *t*.

The amount of skilled and unskilled labour utilised in production are:

$$H_t = H_{1t} + H_{2t} \tag{6}$$

$$L_t = M_{Lt} + \theta M'_{Lt} \tag{7}$$

The total of skilled labour H_t (Relation 6) includes the human capital of parents who occupy skilled positions $H_{1t} = \sum_{h_{it} > w_{Lt}/w_{Ht}} h_{it}$ and the human capital of children who do not pursue higher education and nevertheless possess enough human capital to be employed as skilled workers $H_{2t} = \sum_{\underset{\sim}{h}_t \geq h_{it} \geq (w_{Lt}/\beta w_{Ht})^{1/\eta}} \theta\beta h_{it}^{\eta}$.

The total of unskilled labour L_t (Relation 7) includes the parents who occupy unskilled positions $M_{Lt} = \dim\{h_{it},\ h_{it} \leq w_{Lt}/w_{Ht}\}$ and the time children who do not continue into higher education and do not have human capital enough to be employed in skilled occupations spend as unskilled workers: $\theta \times M'_{Lt}$, $M'_{Lt} = \dim\left\{h_{it} \text{ such that: } h_{it} \leq \underset{\sim}{h}_t \text{ and } \beta h_{it}^{\eta} \leq w_{Lt}/w_{Ht}\right\}$ being the number of households who do not provide higher education to their children, and whose non educated children are employed in an unskilled occupation at time *t*.

The firm's profit maximisation programme determines the factor demands, and thus wages and the unit skill premium at equilibrium:

$$w_{Lt} = (1-\alpha)\left(\frac{H_t}{L_t}\right)^{\alpha} \tag{8}$$

$$w_{Ht} = \alpha\left(\frac{L_t}{H_t}\right)^{1-\alpha} \tag{9}$$

$$\frac{w_{Ht}}{w_{Lt}} = \frac{\alpha}{1-\alpha}\frac{L_t}{H_t} \tag{10}$$

3.2. Transitional equilibria

At any time, production, the distribution of parents between skilled and unskilled occupations, as well as the amount of each factor may be calculated from Equations (6), (7) and (10). This equilibrium directly depends on the distribution of households across the set of available skills because this distribution determines both w_{Ht} / w_{Lt} and the amount of unskilled labour supplied by the youth $\theta \times M'_{Lt}$.

The determination of w_{Ht} / w_{Lt}, H_t and L_t is rather complex. However, it will appear in the simulation exercises that all the children who do not go into higher education have an unskilled position, and that, except in the first period, all the children with unskilled parents do not pursue higher education. The different transitional equilibria are then determined by the 3 equations:

$$H_t = \sum_{h_{it} > \frac{w_{Ht}}{w_{Lt}}} h_{it} \tag{11}$$

$$L_t = (1+\theta)\dim\{h_{it},\ h_{it} \le w_{Ht} / w_{Lt}\} \tag{12}$$

$$\frac{w_{Ht}}{w_{Lt}} = \frac{1-\alpha}{\alpha} \times \frac{\sum_{h_{it} \ge w_{Ht} / w_{Lt}} h_{it}}{(1+\theta)\dim\{h_{it},\ h_{it} \le w_{Ht} / w_{Lt}\}} \tag{13}$$

4 Steady state and human capital dynamics

4.1. The steady state

Let $m_L \equiv M_L / M$ denote the proportion of unskilled households in the population at the steady state, and thus $m_H = 1 - m_L$ that of skilled households.

Proposition 3: *There is a continuum of steady states corresponding to all the proportions of skilled household* m_H *belonging to a certain interval* $\left[\underline{m}_H *, \overline{m}_H *\right]$ *and characterised by the following features:*

(1) All skilled workers have the same human capital $\overline{h}* = (\delta\lambda)^{\frac{1}{1-\gamma}}$.

(2) All unskilled workers have the same human capital $\underline{h}* = \beta^{\frac{1}{1-\eta}}$.

(3) Skilled and unskilled labour are respectively $\overline{H}* = m_H (\delta\lambda)^{\frac{1}{1-\gamma}} M$ *and* $\overline{L}* = (1+\theta) m_L M$.

(4) Production per household $y* \equiv Y*/M$ *is* $y* = (1+\theta)^{1-\alpha} (\delta\lambda)^{\frac{\alpha}{1-\gamma}} \left(m_H\right)^{\alpha} \left(m_L\right)^{1-\alpha}$.

(5) The unit skill premium is $\dfrac{w_H *}{w_L *} = \dfrac{\alpha}{1-\alpha}(1+\theta)(\delta\lambda)^{-\frac{1}{1-\gamma}} \dfrac{m_L}{m_H}$, *and the skill premium* $\dfrac{w_H * \overline{h} *}{w_L *} = \dfrac{\alpha}{1-\alpha}(1+\theta)\dfrac{m_L}{m_H}$.

Proof: Feature (1) results from the steady state of function ISM 4. Feature 2 derives from the steady state of ISM 1. Feature (3) is calculated from (14), (15) and (16) and from the fact that all skilled workers have the same human capital $(\delta\lambda)^{\frac{1}{1-\gamma}}$. Feature (4) is obtained by inserting $\overline{H}*$ and $\overline{L}*$ as defined in Feature (3) into $Y = H^{\alpha} L^{1-\alpha}$, and Feature (5) by inserting $\overline{L}*$ and $\overline{H}*$ in Relation (13). Interval $\left[\underline{m}_H *, \overline{m}_H *\right]$ is built as follows: $\underline{m}_H *$ is the lowest value of $m_H *$ consistent with unskilled workers not funding their children's higher education, and $\overline{m}_H *$ the highest $m_H *$ consistent with skilled workers funding their offspring's higher education. The analytical determination of $\underline{m}_H *$ and $\overline{m}_H *$ are described in Appendix 4.

Proposition 4: *The share of skilled households in the population that maximises the product per capita at the steady state is* α.

Proof: $\frac{\partial y^*}{\partial m_L} = (1+\lambda)^{1-\alpha}(\delta\lambda)^{\frac{\alpha}{1-\gamma}}\left(\frac{1-m_L}{m_L}\right)^{\alpha}\left(1-\frac{\alpha}{1-m_L}\right) = 0 \Rightarrow m_H = 1-m_L = \alpha$

Two features resulting from Propositions 3 and 4 may be pointed out:

(i) For $m_H = \alpha$, i.e. for the share of skilled households in the population that maximises income per capita at the steady state, the skill premium at the steady state is $\frac{w_H \bar{h}^*}{w_L} = 1+\theta > 1$, which means that the least restrictive condition for skilled parents to fund their children's education is fulfilled.

(ii) Since $\frac{w_H{}^* \bar{h}^*}{w_L{}^*} = \frac{\alpha}{1-\alpha}(1+\theta)\frac{m_L}{m_H}$, the higher m_H, the lower inequality between skilled and unskilled workers.

Three situations may be distinguished from the position of α with regard to interval $\left[\underline{m}_H{}^*, \overline{m}_H{}^*\right]$:

(i) If α belongs to $\left[\underline{m}_H{}^*, \overline{m}_H{}^*\right]$, all shares m_H situated into $\left[\underline{m}_H{}^*, \alpha\right[$ are sub-optimal and correspond to higher inequality than for $m_H = \alpha$, and all shares m_H into $\left]\alpha, \overline{m}_H{}^*\right]$ are sub-optimal and correspond to lower inequality than for $m_H = \alpha$.

(ii) If $\alpha < \underline{m}_H$, $\underline{m}_H$ is the attainable share of skilled labour that is the best in terms of per capita income, and it is the least egalitarian in interval $\left[\underline{m}_H{}^*, \overline{m}_H{}^*\right]$.

(iii) If $\alpha > \overline{m}_H$, $\overline{m}_H$ is the attainable share of skilled labour that is the best in terms of per capita product, and it is the most egalitarian in interval $\left[\underline{m}_H{}^*, \overline{m}_H{}^*\right]$.

4.2. Transitional dynamics

The transitional dynamics completely depends on the initial distribution of human capital. As a consequence, there is no one single model of transitional dynamics, but an infinity of possible paths depending on this initial distribution and on the model parameters. In addition, the transitional dynamics also depends on parents' expectations on wages for the next generation. We explore 2 types of expectations, i.e., (i) rational expectations with perfect information, and (ii) naïve expectations. The first type of expectations results in a perfect prediction of forthcoming wages. In the second type, we suppose that the unit skill premium for the next generation is assumed to be equal to that of the current generation by all parents.

Perfect predictions

If parents perfectly predict wages in the next period (generation), we have: $w^e_{Ht+1} = w_{Ht+1}$, $w^e_{Lt+1} = w_{Lt+1}$, and $\frac{w^e_{Ht+1}}{w^e_{Lt+1}} = \frac{w_{Ht+1}}{w_{Lt+1}}$.

Proposition 5: *With perfect predictions, the transitional dynamics is characterised either by a decrease in the unit skill premium, or by a decrease in the number of households having skilled occupations, or even both.*

Proof: Firstly suppose that the number of households employed in skilled positions remains constant over time. Since their human capital increases, H_t also increases, whereas L_t remains constant. The unit skill premium w_H / w_L thereby decreases. If this decrease leads certain households not to educate their children because it makes the condition for parents to fund their offspring's education more restrictive, then the number of households with skilled occupations decreases.

Naïve expectations

In the case of 'naïve expectations', every parent anticipates that the following generation will know the same unit skill premium as the present one. This does not mean that they consider wages as being unchanged, but that they suppose that the relative return from education remains broadly constant between *t* and (*t*+1). In this case, we may write: $\frac{w^e_{Ht+1}}{w^e_{Lt+1}} = \frac{w_{Ht}}{w_{Lt}}$

Proposition 6: *When ratio H_t / L_t increases, the condition for skilled parents to fund their offspring's higher education is less restrictive with naïve expectations than with perfect expectations if children without higher education have unskilled occupations.*

Proof: Because, (i) in conditions (2) and (4) in Table 2, the right hand sides of the related inequalities increase with w^e_{Lt+1} / w^e_{Ht+1}, and (ii) when H_t / L_t increases with time, then $\frac{w_{Lt+1}}{w_{Ht+1}} > \frac{w_{Lt}}{w_{Ht}}$.

Proposition 6 shows that naïve expectations may shape several features:

(i) More parents should normally fund their children's higher education.

(ii) The amount of labour available for production then decreases because some children who would have worked in case of perfect prediction now pursue higher studies.

(iii) Certain individuals may choose unskilled positions when they become adults albeit having received higher education.

(iv) At the steady state, the number of households with skilled occupations may be higher (but not lower), and inequality lower (but not higher), than in the case of perfect expectations.

Once the model's parameters are given, the emergence out of the previous features depends on the initial distribution of human capital across households. We show in Section 6 that they can be produced with reasonable values in the parameters and in the human capital distribution.

Under education Trap

Definition 4: Dynasty *i* falls into an *under-education trap* if, once parent (*i,t*) has decided not to fund his child's higher education, all the following generations of dynasty *i* do not fund their child's higher education.

For unskilled dynasty *i* to stand in an under-education trap, it is necessary that threshold $\tilde{h}_t$ does not decrease fast enough to reach dynasty *i*'s decreasing human capital. Finally note that a dynasty that falls into an under-education trap achieves human capital $\underline{h} = \beta^{\frac{1}{1-\eta}}$ at the steady state.

5 Public policies

By assuming no public intervention, we have established (i) that the model dynamics may result in a *continuum* of steady states characterised by the share of skilled households m_H over interval $[\underline{m}_H *, \overline{m}_H *]$, (ii) that the corresponding inequality is $\frac{w_H \overline{h} *}{w_L} = \frac{\alpha}{1-\alpha}(1+\lambda)\frac{m_L}{m_H}$, and (iii) that the value of m_H that maximises the per capita income is α. When m_H is lower than α, a rise in the share of skilled household could both increase per capita income and reduce inequality. In a large number of situations, there is thus room for public intervention. The problem is to know (i) if such policies do really exist, and, if so, (ii) which of them is the most efficient.

We now analyse the possible impacts of 2 types of public intervention, i.e. the setting of a minimum wage and a redistribution policy.

The implementation of social policies must be analysed from their 3 impacts on: (i) the parents' income and occupation (skilled/unskilled), (ii) the conditions for parents to fund higher education, and (iii) the children's skill level. As a matter of fact, the impact of the social policy on children's human capital directly depends on its impacts on both parents' situation and the condition to fund higher education.

5.1. Minimum wage

The setting of a minimum wage $\underline{w}_{Lt}$ that is higher than the value $\hat{w}_{Lt}$ of w_{Lt} at the full employment equilibrium results, (i) in an increase in the skill intensity in production (Relation 11), (ii) in a decrease in the real wage per unit of skill (Relation 12), (iii) in a decrease in the unit skill premium (Relation 13), and (iv) in moving those workers whose earnings belong to interval $\left[\hat{w}_{Lt}, \underline{w}_{Lt}\right]$ from skilled to unskilled occupations (or to unemployment). Moreover, for those parents who decide to fund higher education with or without minimum wage, their children's human capital is higher when they have an unskilled occupation, and lower when they have a skilled occupation (see Proposition 1).

The minimum wage modifies parents' decisions to fund education through several channels.

It firstly increases the income of all unskilled parents, and it may thereby incite some of them (those with the highest human capital) to fund their children's education.

The minimum wage secondly causes unemployment because (i) it lowers the demand for unskilled labour, and (ii) it increases the supply of unskilled labour as fewer households now work as skilled workers. Both these effects are however tempered by the fact that a section of the youths that would have worked without a minimum wage now go into higher education. In addition, the emergence of unemployed workers implies the setting of unemployment compensations.

Thirdly, the minimum wage displaces threshold $\underset{\sim}{h}_t$ that depends on both present and future wages of workers employed in skilled and unskilled positions. This third impact is clearly the most complex to analyse because the minimum wage generates opposite mechanisms.

Let us firstly suppose that individuals have perfect predictions. For parents who have unskilled occupations, or become unskilled because of its setting, the minimum wage firstly releases condition $h_{it}^{\gamma} > c\dfrac{w_{Lt+1}}{w_{Ht+1}w_{Lt}^{\sigma}}$ (see Table 2) because it increases w_{Lt}. Nevertheless, it also lowers the skill premium of the next period (w_{Ht+1}/w_{Lt+1}), thereby making the condition more restrictive in the case of perfect expectations. It firstly does so if the minimum wage is maintained during the subsequent period. We shall however suppose that the minimum wage is not maintained for the next generation, which is a normal assumption because, as mentioned thereafter, it is a more efficient tool when it is applied rapidly and its impact render its continuation unnecessary. Even in this case, the minimum wage normally results in a decrease in the skill premium of the period after because it augments the number of skilled workers. This may curtail the positive effect of the minimum wage on the funding of their child's education by unskilled parents. In the case of skilled parents, the condition to fund their child is $h_{it}^{\gamma+\sigma} > c\dfrac{w_{Lt+1}}{w_{Ht+1}w_{Ht}^{\sigma}}$. Given that it decreases the present wage per unit of skill, the setting of a minimum wage makes the condition more restrictive. However, if the minimum wage persuades certain unskilled parents to fund their child's education, all the skilled parents will do the same (Proposition 2), and the tightening of the constraint caused by the trap produces no effect.

Considering parents who decide to fund higher education, their child's human capital is thus higher for households having unskilled occupations, and lower for households having skilled occupations. Moreover, since w_{Ht} decreases, certain skilled households who would have spent $s_{it} = \lambda^{1/\sigma}$ for education without minimum wage now curb this spending. The number of parents spending $\lambda^{1/\sigma}$ for their child's education decreases.

When expectations are naive, threshold $\underset{\sim}{h}_t$ goes up with the setting of a minimum wage. Indeed, in the five possible cases described in Table 2, $\underset{\sim}{h}_t$ increases with w_{Lt} and decreases with w_{Ht}. As a consequence, a minimum wage tends to discourage education when expectations are naïve.

5.2. Redistribution

Redistribution consists in transferring a proportion of total income from the well paid to the low paid workers. To be equitable, a redistribution framework must comply with certain features: (i) the net transfer must decrease with the pre-redistribution income, and (ii) the redistribution system must not modify the hierarchy of incomes. This means that if households *i* and *j* are such that *i*'s income is higher than *j*'s before redistribution, this hierarchy must not be inverted after redistribution. In this paper, redistribution consists in levying an income tax on all workers at rate τ and paying a lump sum transfer $f = \tau\bar{I}_t$ to all workers, $\bar{I}_t$ being the average income at time *t*. This redistribution pattern ensures a balanced budget and it meets both features required for equity.

Redistribution typically modifies skill dynamics. Since it increases the income of parents situated under the average wage, their expense for education normally increases and some of them may now decide to fund their child's education. Conversely, the incomes of parents with earnings higher than the average wage decrease. They consequently spend less on their child's education.

It can be noted that a redistribution policy may produce several undesirable outcomes.

Firstly, if the skill corresponding to the average wage is lower than threshold $\underset{\sim}{h}_t$, there is a scope for situations where redistribution shifts certain parents from education funding to no education funding.

This is however a very unlikely occurrence given that the human capital corresponding to the average wage is normally higher than $\underset{\sim}{h}_t$.

Secondly, a redistribution policy normally slows down the transitional dynamics towards the steady state because all households above the average wage suffer a decrease in their after tax income, thereby lowering their education expense. This undesirable result may be limited when the redistribution policy is implemented for one generation only. In fact, it is more efficient to concentrate redistribution on one generation because, the later the government intervenes, the lower the human capital of dynasties that do not fund higher education, and the more difficult and more costly redistribution becomes.

In the simulation carried out in the next section, the redistribution policy will only be implemented over the first period and the induced increase in skilled labour will automatically reduce inequality during the following periods.

Finally, as for the minimum wage, a redistribution policy moves certain households from a no funding to a funding education decision. It thus lowers the skill premium of the following generation, thereby pushing threshold $\underset{\sim}{h}_t$ up. It is thus likely that over a certain value of τ the redistribution policy has no further impact on families' education decision.

6 Simulations and discussion

We now implement a number of simulations that aim to compare the steady state and the transitional dynamics according (i) to the type of expectations (perfect or naïve), and (ii) to the behaviour of the social planner (no intervention, minimum wage, redistribution).

We firstly describe the values of the parameters and the initial distribution of human capital selected for the simulation exercises. Secondly, we examine the characteristics of the different simulated scenarios. The main results of the simulations are finally presented and discussed.

6.1. Parameters and initial conditions

Table 3 depicts the values of the model parameters used in the simulations and Table 3 the initial distribution of human capital.

Table 3: values of the parameters

Parameters	β	σ	δ	a	θ	λ	γ	η	α
values	2.4	0.3	4.825	0.3	0.3	0.7	0.55	0.25	0.5

Coefficient α is equal to 0.5. This indicates that half of total income goes to skilled workers, which is consistent with the proportion calculated in the long term for the US and the UK (Lindert, 2000). Coefficient γ, i.e. the elasticity of children's human capital with respect to that of their parents, takes value 0.55, which is consistent with several empirical estimations (Solon, 1999). η = 0.25 indicates that 45% of this intra-family externality is captured at the end of basic education. θ = 0.3 signifies that basic education accounts for 70% of children' available time. The elasticity of higher education with respect to parents' expenditure on education, that is only effective for incomes below the saturation point, is σ = 0.3, which is consistent with Shea's calculations (2000). The altruism indicator *a* is chosen equal to 0.3. Coefficients β = 2.4 and δ = 4.825 indicate that productivity is slightly higher in basic education than in higher education ($\delta / \beta = 2.0125$). Finally, λ is selected so as to have a saturation point that has a non negligible influence on the skill dynamics.

It may be noted that, for the selected parameters, (i) skilled workers and unskilled workers respectively possess human capital 14.95 and 3.2 at the steady state, and (ii) the optimal proportion of skilled workers at the steady state is 50%. In addition, $\underline{m}_H{}^* = 0.365$ and $\overline{m}_H{}^* = 0.888$, which shows that coefficient $\alpha = 0.5$ belongs to interval $\left[\underline{m}_H{}^*, \overline{m}_H{}^*\right]$.

Table 4: Initial conditions

Human capital at initial time	Weight (%)	Distribution
Low	70	Uniform distribution over interval $\left]1.4\right[$
Medium	20	Uniform distribution over interval $\left[4,8\right[$
High	10	Uniform distribution over interval $\left[8,15\right[$

There are 100 dynasties (or: sets of dynasties, each of them accounting for 1% of the total). We distinguish 3 groups of human capital at initial time (Table 4). 70% of the parents are initially endowed with low human capital and they are uniformly distributed over the interval of human capital $\left]1,4\right[$. 20% of the parents are initially endowed with medium human capital and uniformly distributed over the interval of human capital $\left[4,8\right[$. Finally, 10% of the parents are initially endowed with high human capital and uniformly distributed over interval $\left[8,15\right[$. This distribution is fairly representative of the situation of Europe throughout the 60s (OECD, 1986).

6.2. The Scenarios

Six scenarios are successively simulated: no government intervention with perfect predictions (denoted NIPP) and with naïve expectations (NINE); the setting of a minimum wage with two possible distributions of the derived unemployment across households (scenarios MW1 and MW2); two redistribution policies (RP1 and RP2) [3].

Minimum wage

Individuals have perfect predictions and the minimum wage is only set over one period of time, i.e. for the first generation.

Two scenarios with a minimum wage are distinguished according to the distribution of the unemployment induced. In the first scenario (MW1), unemployment is uniformly distributed across unskilled workers, whereas it only concerns unskilled workers with the lowest human capital in the second (MW2). This latter case may be justified by assuming that the most educated unskilled workers are more efficient in the job search activity. All things being equal, the first scenario produces more unemployment and less educated children than the second. In fact, when unemployment is uniformly distributed across unskilled workers, some of them who would otherwise have funded their child's education due to the minimum wage will not do so because they are unemployed. Their children then join the unskilled labour force, thereby increasing unemployment.

The minimum wage is set at the level which allows the highest educational gain, i.e. the greatest number of children pursuing higher education. Indeed, as pointed out in Section 5.1., the impact of a minimum wage on parents' income, and thereby on their capacity to fund education, is twofold since it increases their pre-tax gains but also increases the tax burden they endure. Given that Scenario MW2 results in lower unemployment than MW1, it is possible to implement a higher minimum wage and a higher tax rate in MW2 than in MW1. As a consequence, the implemented minimum wage respectively causes an increase of 10% and 15% in the wage of unskilled workers in MW1 and MW2, compared with equilibrium without intervention. The respective related tax burdens (in proportion of total income) are 9% and 13.3%.

Redistribution

The redistribution policy is only implemented over the first period and it is organised as presented in section 5.2. We analyse two definitions of the tax rate τ . In a first scenario (RP1), τ is selected so as to reach the same proportion of skilled workers at the steady state as in MW2. This makes it possible to compare the outcomes of redistribution with that of the more efficient minimum wage policy, i.e. MW2. In a second scenario (RP2), we determine the lowest marginal tax rate τ that results in a steady state as close as possible to the optimal situation, i.e. $m_H = \alpha$.

[3] Other simulations were implemented that may be obtained from the authors.

6.3. Results and discussion

Table 5 depicts the main outcomes of the 5 scenarios.

Without public intervention and with perfect predictions, the share of households having a skilled occupation at the steady state is 35%, which is well beneath the optimal proportion (50%). The different public policies, as well as no intervention with naïve expectations, cause (i) a significant rise in the proportion of skilled dynasties at the steady state, (ii) an increase the related per capita income, and (iii) a reduction in inequality. The two shapes of the redistribution policy lead to identical tax rates, and thus identical outcomes.

Table 5: The Scenarios' outcomes

<table>
<tr><th colspan="2"></th><th>NIPP</th><th>NINE</th><th>MW1</th><th>MW2</th><th>RP1 & 2</th><th>Optimum</th></tr>
<tr><td rowspan="2">Product per household</td><td>Period 1</td><td>1.47</td><td>1.43</td><td>1.15</td><td>1.06</td><td>1.45</td><td rowspan="2">2.2025</td></tr>
<tr><td>Steady state</td><td>2.1</td><td>2.16</td><td>2.14</td><td>2.16</td><td>2.16</td></tr>
<tr><td rowspan="2">Unit skill premium</td><td>Period 1</td><td>0.296</td><td>0.283</td><td>0.249</td><td>0.237</td><td>0.291</td><td rowspan="2">0.087</td></tr>
<tr><td>Steady state</td><td>0.163</td><td>0.132</td><td>0.143</td><td>0.132</td><td>0.132</td></tr>
<tr><td colspan="2">Skill premium at the steady state</td><td>2.41</td><td>1.95</td><td>2.12</td><td>1.95</td><td>1.95</td><td>1.3</td></tr>
<tr><td rowspan="2">m_H *</td><td>Period 1</td><td>40</td><td>40</td><td>30</td><td>28</td><td>40</td><td rowspan="2">50</td></tr>
<tr><td>Steady state</td><td>35</td><td>40</td><td>38</td><td>40</td><td>40</td></tr>
<tr><td colspan="2">Unemployment rate[4], period 1, %</td><td>0</td><td>0</td><td>25.7</td><td>34.3</td><td>0</td><td>-</td></tr>
<tr><td colspan="2">Unemployment compensation**</td><td>0</td><td>0</td><td>33</td><td>33</td><td>0</td><td>-</td></tr>
<tr><td colspan="2">Tax rate, period 1, % of total income</td><td>0</td><td>0</td><td>9.0</td><td>13.3</td><td>13.2</td><td>-</td></tr>
<tr><td colspan="2">Minimum wage*</td><td>no</td><td>no</td><td>1.001</td><td>1.055</td><td>no</td><td>-</td></tr>
</table>

* Instead of 0,918 for the lowest wage without public intervention. ** As a % of the minimum wage

In all scenarios except NINE, the distribution of dynasties between those who fund their children's higher education and those who do not, takes place at the first period. As a consequence, the distribution of dynasties between skilled and unskilled occupations is fully realised at the second period. *There is an under education trap* because all the dynasties that do not fund higher education during the first period will remain unskilled later on. When a minimum wage or a redistribution policy is implemented over the first period, certain parents fund their children's education whereas they would otherwise not do so. The related dynasties will then remain in skilled occupations over time, and thereby increase the share of skilled labour in the population at the steady state. The resulting increase in human capital is logically higher when unemployment only concerns those unskilled workers with the lowest human capital (MW2) because, when all unskilled workers are equally affected by unemployment (MW1), certain households do not finance their children's education because they are unemployed.

The cost of each policy is rather different. The setting of a minimum wage causes unemployment and places a rather high tax burden on the first generation of skilled workers. Scenarios MW2 and RPs

[4] The unemployment rate is the ratio of the unemployed population to the working population.

demonstrate the same results in terms of human capital and output per head from the second period up to the steady state, but the former at the expense of unemployment, a lower per capita income and a similar tax burden than the latter in the first period. This shows that redistribution must be considered as a better policy than the setting of a minimum wage to boost income and reduce inequality. Moreover, when compared with the case of non intervention and perfect prediction, both policies trigger a decrease in the per capita income over a number of generations. Indeed, as more children pursue higher education, unskilled labour available for production decreases and several periods are necessary for this to be offset by the increase in human capital. Output per head outstrips its level in scenario NIPP after 7 generations for scenario MW1, and 8 generations for MW2 and RPs.

With naïve expectations, the improvement at the steady state is reached at the expense of lower per capita incomes and a waste of investment in human capital during the first periods of the transitional dynamics[5]. A number of parents fund higher education for children who will turn out to be unskilled once they become adults. This waste of education expenditure comes with lower per capita income since the children pursuing higher education do not participate in production.

Finally note that the policies are more efficient when they are implemented rapidly, i.e., over the first period. As threshold $\underset{\sim}{h}_t$ (the human capital over which parents decide to fund education) is initially higher than $\beta^{\frac{1}{1-\eta}}$ (i.e. the steady state without funding higher education) , then the dynasties that would fund higher education only because of government intervention (minimum wage or redistribution) display a drop in their human capital from generation to generation as long as these policies are not implemented, thus making government intervention more difficult and costly. In any case, after a certain time, the human capital of these dynasties becomes too small for these policies to be efficient.

The above discussion shows that:

(i) Both redistribution and a minimum wage increase the share of skilled workers in the working population, and thereby output per head and equality at the steady state. However, redistribution is more efficient because it makes it possible to reach the same goal without unemployment and with a lower tax burden and a higher production during the first period.

(ii) These policies only need to be implemented over one period (generation).

(iii) The sooner they are carried out, the more efficient and the less costly they are.

(iv) Equality and efficiency are now compatible goals in terms of government intervention.

(v) Naïve expectations results in higher per capita income at the steady state. This nevertheless comes at the cost of inefficient over-education for several generations.

[5] In scenario NINE, per capita output outstrips its level in scenario NIPP after 9 generations.

All these outcomes stem from the under education trap which is the key element in our model. As pointed out by Piketty (2000), such traps emerge in cases of non-convexity. These may result from the combination of imperfect market for credit and a threshold, e.g. an exogenous minimal expense for education (Galor and Zeira, 1993). In our model, this threshold is endogenously determined and it changes over time. If there is a minimal expense for higher education, this has a limited impact on the formation of the trap because this expense increases with parents' income[6]. The main cause of the trap is the fact that, if their children cannot get a skilled occupation even when being educated, parents have no incentive to educate them. This behaviour is linked to the production function in which, unlike a large number of models dealing with this problem, skilled and unskilled labour are not perfect substitutes and workers are not always paid in proportion to their human capital. In fact, when employed in an unskilled occupation, an individual receives the same wage whatever his human capital, whereas his wage is proportional to his human capital in skilled occupations. This logically means that parents do not fund education when they foresee that their child will occupy an unskilled position. The basic rationale is close to that of Barham and Boadway (1995), except that (i) the production function is specified, (ii) there is a skill dynamics and a steady state, (iii) the outcome of public intervention in terms of output per head can thereby be analysed.

7 Conclusion

We have shown that, within a model where education is decided by the parents, the 'natural' dynamics of skill typically leads to a sub-optimal steady state in terms of income per capita. This results from an under education trap that divides up the population between skilled and unskilled workers at the steady state according to the initial distribution of human capital. When sub-optimality comes from a lack of skilled workers, egalitarian policies such as the setting of a minimum wage or redistribution make it possible both to increase output per head and reduce inequality because it moves certain households out of the trap. These policies may be limited to one period of time (generation) because they create the very conditions that make them unnecessary. Moreover, naïve expectations tend to increase the number of parents that fund their children's education, and thus per capita income and equality at the steady state. Simulations have been provided about that illustrate and corroborate these findings. Obviously, other policies are possible, particularly education policies which are not analysed here. Considering educational policies would nevertheless require an extension to the approach so as to explicitly model the allocation of resources to educational activities.

[6] Indeed, parents with higher incomes are more educated and their children have more human capital at the end of basic education. Their cost of entry in higher education is thus more expensive (see sub-section 2.1)

Appendix

1. Proof of Lemma 3: *The condition for parent* (i,t)*, to fund his child's education is*:

(i) $h_t^{i\ \gamma} > \frac{1}{\delta s_{it}^{\ \sigma}}\left(\frac{I_{it}}{I_{it}-s_{it}}\right)^{\frac{1-a}{a}} \frac{w_{Lt+1}}{w_{Ht+1}}$ *if the child has an unskilled position when not funded*

(ii) $h_t^{i\ \gamma-\eta} > \frac{\beta}{\delta s_{it}^{\ \sigma}}\left(\frac{I_{it}}{I_{it}-s_{it}}\right)^{\frac{1-a}{a}}$ *if the child has a skilled position when not funded*

Parents only fund education if it brings their child to a skilled position, i.e. $h_{it+1} > \frac{w_{Lt+1}}{w_{Ht+1}}$ with $h_{it+1} = \delta s_{it}^{\ \sigma} h_{it}^{\ \gamma}$ (Proposition 1), and thus $h_{it}^{\ \gamma} > \frac{w_{Lt+1}}{\delta w_{Ht+1} s_{it}^{\ \sigma}}$.

In addition, parents finance their child's education only if it provides them with higher utility than a situation with no education funding, i.e. $U_{it}(s_{it} > 0) > U_{it}(s_{it} = 0)$. Inserting (1) and (2) into (4) yields: $U_{it}(s_{it} > 0) = (1-a)\log(I_{it} - s_{it}) + a\log w_{Ht+1}\delta h_{it}^{\ \gamma} s_{it}^{\ \sigma}$. Moreover, $U_{it}(s_{it} = 0) = (1-a)\log I_{it} + a\log w_{Lt+1}$ if the non educated child has an unskilled position because then $c_{it} = I_{it}$ and $I_{it+1} = w_{Lt+1}$, and $U_{it}(s_{it} = 0) = (1-a)\log I_{it} + a\log w_{Ht+1}\beta h_{it}^{\ \eta}$ if the non educated child has a skilled position ($c_{it} = I_{it}$ and $I_{it+1} = w_{Ht+1}\beta h_{it}^{\ \eta}$). Condition $U_{it}(s_{it} > 0) > U_{it}(s_{it} = 0)$ may thus be written: $h_{it}^{\ \gamma} > \left(\frac{I_{it}}{I_{it}-s_{it}}\right)^{\frac{1-a}{a}} \frac{w_{Lt+1}}{w_{Ht+1}\delta s_{it}^{\ \sigma}}$ in the first case, and $h_{it}^{\ \gamma-\eta} > \frac{\beta}{\delta s_{it}^{\ \sigma}}\left(\frac{I_{it}}{I_{it}-s_{it}}\right)^{\frac{1-a}{a}}$ in the second case. In addition, (i) since $\left(\frac{I_{it}}{I_{it}-s_{it}}\right)^{\frac{1-a}{a}} > 1$, inequality $h_{it}^{\ \gamma} > \left(\frac{I_{it}}{I_{it}-s_{it}}\right)^{\frac{1-a}{a}} \frac{w_{Lt+1}}{w_{Ht+1}\delta s_{it}^{\ \sigma}}$ entails $h_{it}^{\ \gamma} > \frac{w_{Lt+1}}{\delta w_{Ht+1} s_{it}}$, if the child has an unskilled position when not funded, and (ii) if the child has a skilled position when not funded, he also has a skilled position when funded. As a consequence, conditions (i) and (ii) of Lemma 3 include condition $h_{it}^{\ \gamma} > \frac{w_{Lt+1}}{\delta w_{Ht+1} s_{it}^{\ \sigma}}$.

2. Proof of Lemma 4: *If he educates his child, parent (i,t) spends for education*:

(i) $s_{it} = \frac{\sigma a}{1+\sigma a - a} I_{it}$ if $\frac{\sigma a}{1+\sigma a - a} I_{it} \le \lambda^{1/\sigma}$

(ii) $s_{it} = \lambda^{1/\sigma}$ if $\dfrac{\sigma a}{1+\sigma a - a} I_{it} > \lambda^{1/\sigma}$

By inserting (2), $I_{it+1} = w_{Ht+1} h_{it+1}$, and (1) into (4), the optimisation programme becomes: $\max\limits_{s_{it}} u_{it} = (1-a)\log(I_{it} - s_{it}) + a\log(w_{Ht+1}\delta h_{it}{}^{\gamma} s_{it}{}^{\sigma})$. The optimal expense for education determined by this programme is: $s_{it} = \dfrac{\sigma a}{1+\sigma a - a} I_{it}$. Finally, since the education expense cannot be higher than $\lambda^{1/\sigma}$, $s_{it} = \lambda^{1/\sigma}$ if $\dfrac{\sigma a}{1+\sigma a - a} I_{it} > \lambda^{1/\sigma}$.

3. Proof of Proposition 2

We start from the conditions for parents to fund higher education according to their characteristics:

	Parent (*i*,*t*)'s characteristics	Conditions
(1)	Parent (*i*,*t*) has an unskilled occupation He is beneath the saturation point If not educated, his child has an unskilled position	$h_{it}{}^{\gamma} > \dfrac{1}{\delta}\left(\dfrac{1+\sigma a - a}{1-a}\right)^{\frac{1-a}{a}+\sigma}\left(\dfrac{1-a}{\sigma a}\right)^{\sigma}\dfrac{w^{e}_{Lt+1}}{w^{e}_{Ht+1}} w_{Lt}{}^{-\sigma}$
(2)	Parent (*i*,*t*) has a skilled occupation He is beneath the saturation point If not educated, his child has an unskilled position	$h_{it}{}^{\gamma+\sigma} > \dfrac{1}{\delta}\left(\dfrac{1+\sigma a - a}{1-a}\right)^{\frac{1-a}{a}+\sigma}\left(\dfrac{1-a}{\sigma a}\right)^{\sigma}\dfrac{w^{e}_{Lt+1}}{w^{e}_{Ht+1}} w_{Ht}{}^{-\sigma}$
(3)	Parent (*i*,*t*) has a skilled occupation He is beneath the saturation point If not educated, his child has a skilled position	$h_{it}{}^{\gamma+\sigma-\eta} > \dfrac{\beta}{\delta}\left(\dfrac{1+\sigma a - a}{1-a}\right)^{\frac{1-a}{a}+\sigma}\left(\dfrac{1-a}{\sigma a}\right)^{\sigma} w_{Ht}{}^{-\sigma}$
(4)	Parent (*i*,*t*) has a skilled occupation He is above the saturation point If not educated, his child has an unskilled position	$h_{it}{}^{\gamma} > \dfrac{1}{\delta\lambda}\left(\dfrac{w_{Ht}h_{it}}{w_{Ht}h_{it} - \lambda^{1/\sigma}}\right)^{\frac{1-a}{a}}\dfrac{w^{e}_{Lt+1}}{w^{e}_{Ht+1}}$
(5)	Parent (*i*,*t*) has a skilled occupation He is above the saturation point If not educated, his child has a skilled position	$h_{it}{}^{\gamma-\eta} > \dfrac{\beta}{\delta\lambda}\left(\dfrac{w_{Ht}h_{it}}{w_{Ht}h_{it} - \lambda^{1/\sigma}}\right)^{\frac{1-a}{a}}$

1. *If condition (1) is fulfilled by at least one of its related parents, then condition (2) is fulfilled for all parents who depend on it.*

Assume that Condition (1) is fulfilled by at least one unskilled parent. It is then fulfilled by all skilled parents because they possess more human capital. As a consequence, all parents depending on Condition (2) fulfil condition (1): $h_{it}{}^{\gamma} > c\dfrac{w^{e}_{Lt+1}}{w^{e}_{Ht+1}} w_{Lt}{}^{-\sigma}$, whatever parent (*i*,*t*) who depends on Condition (2), with $c \equiv \dfrac{1}{\delta}\left(\dfrac{1+\sigma a - a}{1-a}\right)^{\frac{1-a}{a}+\sigma}\left(\dfrac{1-a}{\sigma a}\right)^{\sigma}$. As $w_{Ht}h_{it} > w_{Lt}$ (Lemma 1), then $\left(w_{Ht}h_{it}\right)^{-\sigma} < w_{Lt}{}^{-\sigma}$, and $h_{it}{}^{\gamma} > c\dfrac{w^{e}_{Lt+1}}{w^{e}_{Ht+1}}w_{Lt}{}^{-\sigma}$ induces $h_{it}{}^{\gamma+\sigma} > c\dfrac{w^{e}_{Lt+1}}{w^{e}_{Ht+1}}w_{Ht}{}^{-\sigma}$. Condition (2) is thus fulfilled by all parents who depend on it.

2. *If condition (2) is fulfilled by at least one of its related parents, then condition (3) is fulfilled for all parents who depend on it.*

Moving from condition (2) to condition (3) consists in moving from parents whose no educated children have unskilled positions to parents whose non educated children have skilled position, by remaining on the same ISM curve. Consequently, the most skilled parent related to (2) (parent (*k*,*t*)), who is also the least skilled parent related to (5), is such that: $w^{e}_{Ht+1}\beta h_{kt}{}^{\eta} = w^{e}_{Lt+1}$. By inserting $w^{e}_{Ht+1}\beta h_{kt}{}^{\eta} = w^{e}_{Lt+1}$ into condition (2), we find condition (3): $h_{kt}{}^{\gamma+\sigma-\eta} > \beta c w_{Ht}{}^{-\sigma}$, which is then verified for all parents who depend on it (since parent (*k*,*t*) is the parent with the lowest human capital inside this set of parents).

3. *If condition (2) is fulfilled by at least one of its related parents, then condition (4) is fulfilled for all parents who depend on it*

If (2) is fulfilled for one parent belonging to the related set of parents, it is also fulfilled for the parent (*k*,*t*) of that set with the highest human capital, whose expense for education is $\dfrac{\sigma a w_{Ht}h_{kt}}{1+\sigma a - a} = \lambda^{1/\sigma}$. By introducing $\lambda = \left(\dfrac{\sigma a}{1+\sigma a - a}w_{Ht}h_{kt}\right)^{\sigma}$ in condition (2), we obtain condition (4), which is thus fulfilled by parent (*k*,*t*). Since all parents belonging to the set related to condition (4) have higher human capital than parent (*k*,*t*), they all fulfil condition (4). Condition (4) is thus satisfied by all its related parents.

4. *If condition (4) is fulfilled by at least one of its related parents, then condition (5) is fulfilled for all parents who depend on it.*

Passing from condition (4) to condition (5) consists in moving from parents whose no educated children have unskilled positions to parents whose non educated children have skilled positions. Consequently, the most skilled parent related to (4) (parent (*k*,*t*)), who is also the least skilled parent related to (5), is such that: $w^{e}_{Ht+1}\beta h_{kt}{}^{\eta} = w^{e}_{Lt+1}$. If condition (4) is fulfilled for at least one of the parents who depend on it, it is then fulfilled by parent (*k*,*t*). By inserting $w^{e}_{Ht+1}\beta h_{kt}{}^{\eta} = w^{e}_{Lt+1}$ into condition (4), we find condition (5): $h_{kt}^{\gamma-\eta} > \frac{\beta}{\delta\lambda}\left(\frac{w_{Ht}h_{kt}}{w_{Ht}h_{kt}-\lambda^{1/\sigma}}\right)^{\frac{1-a}{a}}$, which is then verified for all parents who depend on it.

4. Determination of interval $\left[\underline{m}_H*, \overline{m}_H*\right]$

At the steady state, the population is divided in 2 different sets, i.e. that of skilled workers with human capital $\overline{h}* = (\delta\lambda)^{\frac{1}{1-\gamma}}$, and that of unskilled workers with human capital $\underline{h}* = \beta^{\frac{1}{1-\eta}}$. We show that:

$$\underline{m}_H* = \left[1+\frac{1}{1+\theta}\left(\frac{1}{\beta^{\frac{1}{1-\eta}}\delta}\frac{(1-\alpha)^{1-\sigma}}{\alpha}\left(1+\frac{\sigma a}{1-a}\right)^{\frac{1-a}{a}}\left(1+\frac{1-a}{\sigma a}\right)^{\sigma}\right)^{\frac{1}{1-\sigma\alpha}}\overline{h}*\right]^{-1};\ \overline{m}_H* = \left[1+\frac{1}{1+\theta}\left(\frac{\lambda^{1/\sigma}/\alpha\overline{h}*^{\alpha}}{1-\left(\beta/\overline{h}*^{1-\eta}\right)^{\frac{a}{1-a}}}\right)^{\frac{1}{1-\alpha}}\right]^{-1}$$

Because of Condition (5) in Proposition 2, the human capital of skilled parents at the steady state must meet inequality: $h_{it}^{\gamma-\eta} > \frac{\beta}{\delta\lambda}\left(\frac{w_{Ht}h_{it}}{w_{Ht}h_{it}-\lambda^{1/\sigma}}\right)^{\frac{1-a}{a}}$. Inserting $h_{it} = \overline{h}* = (\delta\lambda)^{\frac{1}{1-\gamma}}$ and $w_{Ht} = \alpha\left(\frac{\overline{L}*}{\overline{H}*}\right)^{1-\alpha} = \alpha\left(\frac{(1+\theta)(1-m_H)}{m_H(\delta\lambda)^{\frac{1}{1-\gamma}}}\right)^{1-\alpha}$ into this inequality yields:

$$m_H < \left[1+\frac{1}{1+\theta}\left(\frac{\lambda^{1/\sigma}/\alpha h*^{\alpha}}{1-\left(\beta/h*^{1-\eta}\right)^{\frac{a}{1-a}}}\right)^{\frac{1}{1-\alpha}}\right]^{-1} \equiv \overline{m}_H*$$

Because of Condition (1) in Proposition 2, the condition for unskilled parents with human capital $\underline{h}* = \beta^{\frac{1}{1-\eta}}$ not to fund their children's higher education at the steady state is: $\left(\underline{h}*\right)^{\gamma} < \frac{1}{\delta}\left(\frac{1+\sigma a-a}{1-a}\right)^{\frac{1-a}{a}+\sigma}\left(\frac{1-a}{\sigma a}\right)^{\sigma}\frac{w_{Lt+1}^{e}}{w_{Ht+1}^{e}}w_{Lt}^{-\sigma}$. By inserting $\frac{w_L}{w_H} = \frac{1-\alpha}{\alpha}(1+\theta)^{-1}(\delta\lambda)^{\frac{1}{1-\gamma}}\frac{m_H}{m_L}$ and $w_L* = (1-\alpha)\left(\frac{\overline{H}*}{\overline{L}*}\right)^{\alpha} = (1-\alpha)\left(\frac{m_H\overline{h}*}{(1+\theta)m_L}\right)^{\alpha}$ into this condition, we obtain:

$$m_H > \left[1 + \frac{1}{1+\theta}\left(\frac{1}{\delta\beta^{\frac{\gamma}{1-\eta}}}\left(\frac{1+\sigma a - a}{1-a}\right)^{\frac{1-a}{a}+\sigma}\left(\frac{1-a}{\sigma a}\right)^{\sigma}\frac{(1-\alpha)^{1-\sigma}}{\alpha}\right)^{\frac{1}{1-\sigma\alpha}}\bar{h}*\right]^{-1} \equiv \underline{m}_H *$$

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