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**To Segregate or to Integrate:
Education Politics and Democracy**

David de la Croix
Matthias Doepke

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To Segregate or to Integrate: Education Politics and Democracy*

David de la Croix[†]

Department of Economics and CORE, Université Catholique de Louvain

Matthias Doepke

University of California, Los Angeles, CEPR, and NBER

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Abstract

The governments of nearly all countries are major providers of primary and secondary education to its citizens. In some countries, however, public schools coexist with private schools, while in others the government is the sole provider of education. In this study, we ask why different societies make different choices regarding the mix of private and public schooling. We develop a theory which integrates private education and fertility decisions with voting on public schooling expenditures. In a given political environment, high income inequality leads to more private education, as rich people opt out of the public system. Comparing across political systems, we find that concentration of political power can lead to multiple equilibria in the determination of public education spending.

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[†] **Correspondence address:** David de la Croix, Université catholique de Louvain, Place Montesquieu 3, B-1348 Louvain-la-Neuve, Belgium. Matthias Doepke, Department of Economics, UCLA, 405 Hilgard Ave, Los Angeles, CA 90095. E-mail addresses: delacroix@ires.ucl.ac.be, doepke@econ.ucla.edu

1 Introduction

Public schooling is one of the most widespread social policies around the world today. Starting with industrializing European nations in the nineteenth century, nearly all countries have introduced compulsory schooling laws and public funding of education. However, despite the near universal involvement of the government, private and public funding of education continue to coexist. The share of private education funding varies greatly across countries, from only 1.9 percent of total spending in Norway, to 44.5 percent in Chile (1998, see Section 6). Institutional arrangements concerning the funding of private schools also differ across countries. Private schools can be partly supported by the government as in France or New Zealand, or even entirely publicly funded as in Belgium, or rely exclusively on private funding as in the United States (Toma 1996).

Recently, the OECD Programme for International Student Assessment (PISA), which assesses the knowledge and skills of 15-year-old students in a cross-section of countries, has sparked an intense debate on the relative merits of different education systems. A central question in this debate is why education systems differ so much across countries in the first place. Are there particular country characteristics that explain the choice of an integrated education system instead of a regime with segregation between public and private schools? Why is it that even countries that appear similar in their overall characteristics sometimes end up with widely different education systems? What role does the distribution of political power in a country play for education politics?

The aim of this study is to provide a positive theory of education systems that can be used to address these questions. We develop an analytically tractable framework that integrates political determination of the quality of public schools with private education and fertility decisions. Parents have the choice of sending their children to tax-financed public schools or, alternatively, to opt out of the public system and provide private education to their children. A key feature of our political economy setup is that it allows for bias in the political system, in the sense that the weight of a certain group of voters in political decision making may be larger or smaller than its relative size in the population. In particular, we contrast outcomes under an even distribution of political power, such as in a representative democracy, with outcomes when a political bias gives a disproportionate share of power to the rich.

We find that when political power is evenly distributed, a unique political equilib-

rium exists. Perhaps not surprisingly, the extent of public versus private funding depends on the income distribution. Parents decide to send their children to a private school if they would like to endow their children with an education of a much higher quality than what is provided by the public system. In a society with little inequality, the preferred education level varies little in the population, so that all parents use public schooling. For increasing levels of inequality, an increasing share of richer people chooses private education for their children.¹

In our model, the ultimate motive behind public education provision is redistribution. At first sight, one might expect that the equilibrium tax rate should increase with inequality, because a widening income distribution implies a greater demand for redistribution. However, this demand effect is counteracted by an opposite force: as inequality increases, more and more parents opt out of the public system entirely, reducing the funding requirements for public schooling. The latter effect turns out to dominate, so that the tax rate declines with inequality. This result provides a contrast to a literature where a greater degree of inequality motivates more redistribution through higher taxes (see Alesina and Rodrik 1994, Persson and Tabellini 1994, and in particular Gradstein and Justman 1997 in an application to education). As Bénabou (2000), among others, has pointed out, the empirical evidence is generally in favor of our model's prediction, i.e., more unequal countries tend to redistribute less.²

Another novel feature of our analysis concerns the efficiency properties of public education. In our theory, fertility decisions are endogenous. Consistent with empirical evidence, the theory predicts that poorer parents who use public schools have more children than parents who choose costly private schools. By raising their fertility rate relative to what they would choose if they had to pay for their children's education, the public-school parents impose a fiscal externality on all taxpayers. This externality would not be present in a model with fixed fertility.

The results described so far, pertaining to democratic countries with an even distribution of political power, do not yet offer an explanation for why sometimes countries with similar general characteristics end up with very different education systems. This changes once we introduce bias into the political system. If political

¹This echoes the result of Besley and Coate (1991). Assuming that quality is a normal good, households who opt out of the public sector are those with higher incomes.

²Bénabou (2000) also develops a model that is consistent with this observation, albeit through a different mechanism. One of the driving factors for his result is an uneven distribution of political power. The opting-out decision in our model provides an alternative to Bénabou's mechanism.

power is concentrated among the rich, as in many dictatorships, multiple equilibria with different education systems can arise. In particular, a given population may choose either a high quality of public education with everyone participating in the public system, or a low quality of public education with the rich sending their children to private schools. The source of multiplicity is a complementarity between the rich voters' choice of whether to participate in public education, and their preferred quality of public schools. Interestingly, this complementarity works entirely through the political system. Previous authors have emphasized peer effects as a source of strategic complementarities in education, which we abstract from here.³

At first sight, the multiplicity-of-equilibria result may seem to suggest that countries with a biased political system can experience frequent and sudden shifts in the makeup of their education systems. However, a more appropriate real-world interpretation of this result is path dependence and persistence of education regimes in such countries.⁴ The multiplicity result relies on the assumption that voters commit to their fertility and schooling regime choices (i.e., public or private) before the electorate determines the quality of public schooling. By committing to their choices, the voters create groups with specific interests for or against public education that are fixed *ex post*. It is these fixed long-term choices that give rise to self-fulfilling expectations regarding the quality of public schooling. To assess the empirical content of the multiplicity result, one therefore needs to consider the degree to which the households in a given country are locked into particular fertility or schooling choices. Fertility clearly is an irreversible choice that has to be made years before children enter a school. The reversibility of the schooling choice, in contrast, may vary substantially across countries or regions. If, for example, educational segregation is linked to residential segregation, switching from a private to a public school would also require moving to a different neighborhood or city. This implies high switching costs that lock households into a particular choice for extended periods. In such a country, we would expect the schooling regime to persist over time. If, on the other hand, public and private schools were located right next to each other in the same neighborhoods, switching costs would be less of an issue.

The main predictions of our theory are consistent with a set of stylized facts on public and private schooling in a cross section of countries. Using micro data from the

³Variable political power also plays a role for the multiple steady result in Bénabou (2000), although the mechanism is entirely different.

⁴The model presented here does not have an explicit dynamic structure; dynamics are discussed in the working paper version of this paper, de la Croix and Doepke (2003b).

OECD PISA program, we confirm that in a large set of countries high-income households are more likely to use private education, while their fertility rates are lower. Comparing across countries, high inequality is associated with a larger share of private schooling. Countries with a high degree of educational segregation, in turn, tend to have larger fertility differentials across social classes. Concerning the role of political power, we turn to the relationship between democracy and education funding. If we interpret democracies as countries with an even distribution of political power, while non-democracies are biased to the rich, our model implies that there is more scope for variation in education systems in non-democracies than in democracies. Indeed, using a cross section of 158 countries, we find that the variance of public spending across countries is smaller for democracies than for non-democracies.

Our paper relates to a number of branches of the literature. A number of authors have addressed the choice of public versus private schooling within a majority voting framework (see Stiglitz 1974, Glomm and Ravikumar 1998, Epple and Romano 1996b, and Glomm and Patterson 2002). A recurring theme in this literature is the argument that if there are private alternatives to public schools, voters' preferences may not be single-peaked, so that a majority voting equilibrium may fail to exist. In contrast, we rely on probabilistic voting as the political mechanism, which yields a fully tractable theory of education regimes in which voting equilibria are guaranteed to exist. Moreover, our probabilistic voting setup is not restricted to democracies, since we can analyze what happens if the political system is biased to the rich.

Our model makes predictions for the link between inequality in a country and the resulting education system and quality of education. A similar objective is followed by Fernández and Rogerson (1995), who consider a model where education is discrete (zero or one), partially subsidized by the government, and voters decide on the extent of the subsidy. Fernandez and Rogerson emphasize that in unequal societies, the poor may forgo education entirely. Since all voters are taxed, in this case public education constitutes a transfer of resources from the poor and the rich to the middle class, echoing the findings of Epple and Romano (1996a). While these arguments are highly relevant for the case of post-secondary education, at the primary and secondary level participation rates are high even for poor children in most countries. Thus, at these levels the choices that we model here (private versus public education and the quality of public education) may be the more important margins.

A different branch of the literature takes the schooling regime (public or private) as given, and analyzes the economic implications of each regime. Glomm and Ravikumar (1992) contrast the effects of public and private schooling systems on growth and inequality. In a country with little inequality, a fiscal externality created by public schooling leads to lower growth under public schooling than under private schooling. In unequal societies, however, public schooling can dominate, since more resources are directed to poor individuals with a high return on education. Similar conclusions are derived by de la Croix and Doepke (2004) in a framework which emphasizes the interdependence of fertility and education decisions of parents. The model of Glomm and Ravikumar (1992) has been extended by Bénabou (1996) to allow for local interactions between agents, such as neighborhood effects and knowledge spillovers. Our work advances over these papers by endogenizing the choice of the schooling regime as a function of the income distribution and the political system.

In the next section, we introduce our model and analyze the political equilibrium. Section 3 describes how in a democratic country the choice of a schooling regime and the quality of schooling depend on the income distribution. Section 4 generalizes the voting process to allow for unequal political power. We show that multiple equilibria can arise in societies dominated by the rich. In Section 5 we analyze alternative timing assumptions. In Section 6 we confront the testable implications of the model with empirical evidence. Section 7 concludes. All proofs are contained in the mathematical appendix.

2 The Model Economy

Preferences and technology

The model economy is populated by a continuum of households of measure one. Households are differentiated by their human capital endowment x , where x is the wage that a household can obtain in the labor market. People care about consumption c , their number of children n , and their children's education h . The utility func-

tion is given by:⁵

$$\ln(c) + \gamma [\ln(n) + \eta \ln(h)]. \quad (1)$$

Notice that parents care both about child quantity n and quality h . The parameter $\gamma \in \mathbb{R}_+$ is the overall weight attached to children. The parameter $\eta \in (0,1)$ is the relative weight of quality.⁶ As we will see below, the tradeoff between quantity and quality is affected by the human capital endowment of the parent and by the schooling regime.

To attain human capital, children have to be educated by teachers. The wage of teachers equals the average wage in the population, which is normalized to one.⁷ Parents can choose from two different modes of education. First, there is a public schooling system, which provides a uniform education s to every student. Education in the public system is financed through an income tax v ; apart from the tax, there are no direct costs to the parents. The schooling quality s and the tax rate v are determined through voting, to be described in more detail later. Parents also have the possibility of opting out of the public system. In this case, parents can freely choose the education quality e , but they have to pay the teacher out of their own income. Since education e is measured in units of time of the average teacher, the total cost of educating n children privately is given by ne . We assume that education spending is tax deductible. While tax deductibility of education expenditures varies across countries, deductibility simplifies the analysis because it implies that taxation does not distort the choice between quantity and quality of children.⁸ Apart from the education expenditure, raising one child also takes fraction $\phi \in (0,1)$ of an adult's time. The budget constraint for an adult with wage x is given by:

$$c = (1 - v) [x(1 - \phi n) - ne]. \quad (2)$$

Education is thus either private, e , or public, s . Effective education can be expressed

⁵The logarithmic utility function is chosen for simplicity; any utility function representing homothetic preferences over the bundle (c, n, h) would lead to the same results. Also, while we focus on a static framework here, the working paper version of this paper extends the analysis to a dynamic setting where today's children are tomorrow's adults.

⁶The parameter η cannot exceed 1 because the parents' optimization problem would not have a solution. More specifically, utility would approach infinity as parents choose arbitrarily high levels of education and arbitrarily low levels of fertility. A similar condition can be found in Moav (2005).

⁷The important assumption here is that the cost of education is fixed, i.e., all parents face the same education cost regardless of their own wage. The level of the teacher's wage is set to the average wage for convenience.

⁸The same result would obtain if parents educated their own children (or at least had the option to do so), because then the parent's teaching efforts would reduce their taxable income.

as the maximum of the two: $h = \max\{e, s\}$. Of course, parents who prefer public education will choose $e = 0$.

The consumption good is produced by competitive firms using labor as the only input. We assume that the aggregate production function is linear in effective labor units. The production setup does not play an important role in our analysis; the advantage of the linear production function is that the wage is fixed. Assuming a uniform distribution of human capital over the interval $[1 - \sigma, 1 + \sigma]$, output is given by:

$$Y = \int_0^\infty x g(x) dx - \int_0^\infty h(x) g(x) dx.$$

where the density function is given by $g(x) = 0$ for $x < 1 - \sigma$ and if $x > 1 + \sigma$, and $g(x) = 1/(2\sigma)$ for $1 - \sigma \leq x \leq 1 + \sigma$. In the first term, x is the marginal product of each type, and the second term are the effective labor units used for teaching children.⁹

Substituting the budget constraint (2) into the utility function (1) allows rewriting the utility of a given household as:

$$u[x, v, n, e, s] = \ln(1 - v) + \ln(x(1 - \phi n) - ne) + \gamma \ln n + \gamma \eta \ln \max\{e, s\}.$$

Timing of events and private choices

The level of public funding for education s is chosen by a vote among the adult population. The voters' preferences depend on their optimal fertility and education choices (n and e), which are made before voting takes place. In making these choices, agents have perfect foresight regarding the outcome of the voting process. This timing is motivated by the observation that public education spending can be adjusted frequently, while fertility cannot. Similarly, the choice between public versus private education entails substantial switching costs, especially when educational segregation is linked to residential segregation.¹⁰

At given expected policy variables v and s , the utility function u is concave in n . Within each group, some agents may choose public schooling, in which case their

⁹The uniform distribution of human capital is chosen for simplicity; other distributions would lead to similar results. In particular, in the probabilistic voting model described below (unlike the standard majority voting model) there is no special significance to the relative positions of median and mean income.

¹⁰In Section 5 we will explore the implications of alternative timing assumptions.

fertility rate is denoted n^s , while others opt for private education; fertility for those in private schools is denoted as n^e . All parents planning to send their children to the public school choose the same fertility level:

$$n^s = \arg \max_n u[x, v, n, 0, s] = \frac{\gamma}{\phi(1 + \gamma)}. \quad (3)$$

The households planning to provide private schooling chose:

$$\begin{aligned} n &= \arg \max_n u[x, v, n, e, s] = \frac{x\gamma}{(1 + \gamma)(e + \phi x)}, \\ e[x] &= \arg \max_e u[x, v, n, e, s] = \frac{\eta\phi x}{1 - \eta}. \end{aligned} \quad (4)$$

Private spending on education depends positively on the wage x . Highly skilled parents have a comparative advantage in terms of raising educated children. Indeed, since the cost of children is a time cost, children are cheap for the low skilled individuals. In contrast, the cost for education quality is a resource cost, which is more affordable for educated, high-income parents.

Notice that e is independent of the outcome of the voting process, implying that the timing of choosing e does not affect the results (in contrast, we will see in Section 5 that it does matter when the basic choice between public and private schooling is made). Replacing the optimal value for $e[x]$ in the fertility equation we find:

$$n^e = \frac{\gamma(1 - \eta)}{\phi(1 + \gamma)}. \quad (5)$$

From equations (3) and (5) we see that parents choosing private education have a lower fertility rate.¹¹

Lemma 1 (Constant parental spending on children)

For given s , v and x , parental spending on children (and therefore taxable income) does not depend on the choice of private versus public schooling, and is equal to $\frac{\gamma}{1+\gamma} x$.

Lemma 1 implies that the tax base does not depend on the fraction of people participating in public schooling. This property will be important for generating uniqueness of equilibrium. The lemma relies on three assumptions: homothetic prefer-

¹¹This result has a testable implication. In countries where public education is dominant, fertility differentials between high- and low-skilled mothers should be smaller than in countries with a large share of private education, which is what we find in Section 6 (the education of the mother is relevant here, because usually most of the time cost of rearing children is borne by the mother).

ences, tax deductible education spending, and endogenous fertility. With endogenous fertility, parents choosing private schools have fewer children, keeping their total budget allocation to children in line with those choosing public schools.¹² This is a typical feature of endogenous fertility models.

A first result is that parents with high human capital are more demanding in terms of expected public education quality, or, in other words, child quality is a normal good:

Lemma 2 (Opting out decision)

There exist an income threshold:

$$\tilde{x} = \frac{1 - \eta}{\delta \phi \eta} E[s] \quad \text{with: } \delta = (1 - \eta)^{\frac{1}{\eta}} \quad (6)$$

such that households strictly prefer private education if and only if $x > \tilde{x}$.

Here $E[s]$ is expected quality of public schooling. An implication of the above lemma is that if some people with income x choose the public regime, all people with income $x' < x$ will strictly prefer public schooling. Similarly, if at least some people with income x opt out of public education and choose private education, all households with income $x' > x$ choose it as well.

We denote the fraction of children participating in the public education system as:

$$\Psi = \begin{cases} 0 & \text{if } \tilde{x} < 1 - \sigma \\ \frac{\tilde{x} - (1 - \sigma)}{2\sigma} & \text{if } 1 - \sigma \leq \tilde{x} \leq 1 + \sigma \\ 1 & \text{if } \tilde{x} > 1 + \sigma \end{cases} \quad (7)$$

The political mechanism

The public education system operates under a balanced-budget rule:

$$\int_0^{\tilde{x}} n^s s g[x] dx = \int_0^{\tilde{x}} v(x(1 - \phi n^s)) g[x] dx + \int_{\tilde{x}}^{\infty} v(x(1 - \phi n^e) - e[x]n^e) g[x] dx, \quad (8)$$

¹²With fixed fertility, the resources allocated to children would be $x\phi n$ with public education and $x\phi n/(1 - \eta)$ with private education. However, even with fixed fertility a constant tax base could be achieved through an endogenous labor supply setup.

with total spending on public education on the left-hand side and total revenues on the right-hand side. After replacing fertility and education by their optimal value, this constraint reduces to:

$$v = \Psi \frac{\gamma}{\phi} s. \quad (9)$$

Since the level of schooling and taxes are linked through the budget constraint, the policy choice is one dimensional.

The level of public expenditures, and hence taxes, is chosen through probabilistic voting. Assume that there are two political parties, p and q . Each one proposes a policy s^p and s^q . The utility gain (or loss) of a voter with income x if party q wins the election instead of p is $u[x, v^q, n, e, s^q] - u[x, v^p, n, e, s^p]$. Instead of assuming that an adult votes for party q with probability one every time this difference is positive (as in the median voter model), probabilistic voting theory supposes that this vote is uncertain. More precisely, the probability that a person votes for party q is given by

$$F(u[x, v^q, n, e, s^q] - u[x, v^p, n, e, s^p]),$$

where F is an increasing and differentiable cumulative distribution function. This function captures the idea that voters care about an “ideology” variable in addition to the specific policy measure at hand, i.e., the quality of public schooling. The presence of a concern for ideology, which is independent of the policy measure, makes the political choice less predictable (see Persson and Tabellini (2000) for different formalizations of this approach). The probability that a given voter will vote for party q increases gradually as the party’s platform becomes more attractive. Under standard majority voting, in contrast, the probability of getting the vote jumps discretely from zero to one once party q offers a more attractive platform than party p .

Since the vote share of each party varies continuously with the proposed policy platform, probabilistic voting leads to smooth aggregation of all voters’ preferences, instead of depending solely on the preferences of the median voter. Party q maximizes its expected vote share, which is given by $\int_0^\infty g[x]F(\cdot)dx$. Party p acts symmetrically, and, in equilibrium, we have $s = s^q = s^p$. The maximization program of each party implements the maximum of the following weighted social welfare function:¹³

$$\int_0^\infty g[x] (F)'(0) u[x, v, n, e, s]dx.$$

¹³This result was first derived by Coughlin and Nitzan (1981). The same framework can also be derived within the setup of lobbying models, see Bernheim and Whinston (1986).

The weights $(F)'(0)$ capture the responsiveness of voters to the change in utility. In particular, a group that has little ideological bias cares relatively more about economic policy. Such groups are therefore targeted by politicians and enjoy high political power. In addition, political power may also depend on other features of the political system such as voting rights. We will capture the political power of each person by a single parameter $\theta[x]$. This includes the extreme cases of representative democracy with equal responsiveness, and dictatorship of the rich ($\theta[x] = 0$ for x below a certain threshold). Accordingly, the objective function maximized by the probabilistic voting mechanism is given by:

$$\Omega[s] \equiv \int_0^{\tilde{x}} u[x, v, n^s, 0, s] \theta[x] g[x] dx + \int_{\tilde{x}}^{\infty} u[x, v, n^e, e[x], 0] \theta[x] g[x] dx. \quad (10)$$

The maximization is subject to the government budget constraint (8).

We start by assuming that all individuals have the same political power, i.e. $\theta[x] = 1$, implying that the weight of a given group in the objective function is simply given by its size. The role of this assumption will be further investigated in Section 4. It can be checked that $\Omega[s]$ is strictly concave. Replacing \tilde{x} by $2\sigma\Psi + 1 - \sigma$ into the objective, taking the first-order condition for a maximum, and solving for s yields:

$$s = \frac{\eta\phi}{1 + \gamma\eta\Psi} \equiv s[\Psi]. \quad (11)$$

From this expression we can see that s is decreasing in the participation rate Ψ : when more children participate in public schools, spending per child is reduced. Looking at the corresponding tax rate,

$$v = \frac{\eta\gamma\Psi}{1 + \gamma\eta\Psi'}, \quad (12)$$

we observe that a rise in participation is followed by a less than proportional rise in taxation. Since, by Lemma 1, the taxable income is unaffected by increased participation, this translates into lower spending per child. To see the intuition for this result, consider the consequences of increasing Ψ for a given s . In the welfare function maximized by the political system, the increase in Ψ leads to a proportional increase in the marginal benefit of increasing schooling s , since more children benefit from public education. The marginal cost of taxation, in contrast, increases more than proportionally, since the higher required taxes reduce consumption and increase marginal utilities. To equate marginal costs and benefits, an increase in Ψ is therefore met by a reduction in s .

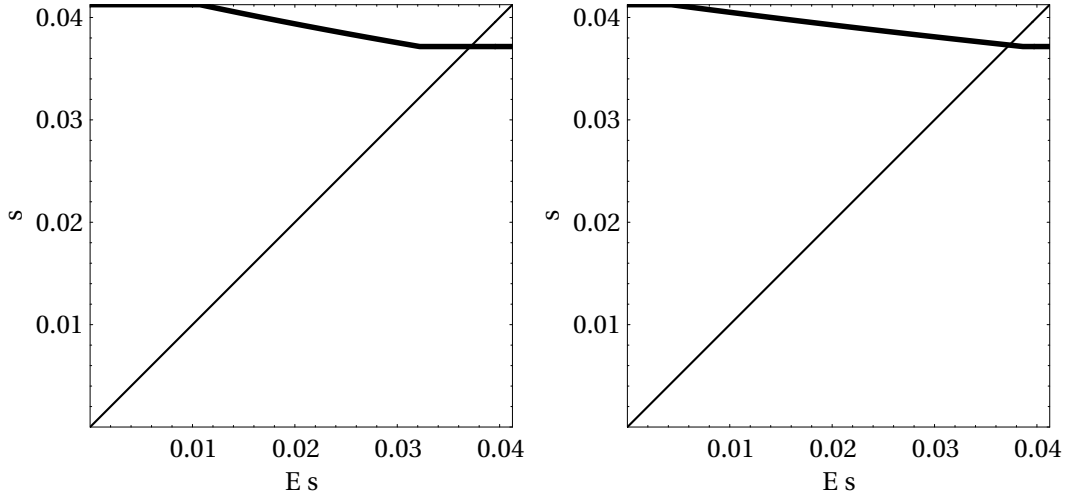


Figure 1: The fixed point with $\sigma = 0.5$ (left) and $\sigma = 0.8$ (right)

The equilibrium

So far, we have taken the participation rate Ψ as given, and solved for the corresponding voting outcome concerning the quality of public schools. In equilibrium, the choice whether or not to participate in public schooling has to be optimal. In the definition of equilibrium we will use an earlier result: the incentive to use private schooling is increasing in income (Lemma 2). As a consequence, any equilibrium is characterized by an income threshold \tilde{x} such that people choose public education below \tilde{x} and private education above \tilde{x} . This leads to the following definition of an equilibrium:

Definition 1 (Equilibrium)

An equilibrium consists of an income threshold \tilde{x} satisfying (6), a fertility rule $n = n^s$ for $x \leq \tilde{x}$ and $n = n^e$ for $x > \tilde{x}$, a private education decision $e = 0$ for $x \leq \tilde{x}$ and $e = e[x]$ for $x > \tilde{x}$, and aggregate variables (Ψ, s, v) given by equations (7), (11) and (12), such that the perfect foresight condition holds:

$$E[s] = s. \quad (13)$$

Proposition 1 (Existence and Uniqueness of Equilibrium)

An equilibrium exists and is unique.

To see the intuition for the result, notice that participation in public schooling is a continuously increasing function of expected school quality through equations (7)

and (6). Actual school quality, in turn, is a continuous and decreasing function of participation. Combining these results, we can construct a continuous and decreasing mapping from expected to actual school quality. This mapping has a unique fixed point, which characterizes the equilibrium.

The uniqueness result relies on endogenous fertility. If one assumes, to the contrary, that fertility is exogenous and constant, Lemma 1 no longer holds, and the tax basis increases with participation Ψ . If the tax-basis effect is sufficiently pronounced, the actual schooling level will no longer decrease in participation, and the equilibrium mapping may fail to have a unique fixed point.

Figure 1 shows two numerical examples of the fixed point mapping. The chosen parameters are: $\gamma = 0.4$, $\eta = 0.55$, $\phi = 0.075$. The implied fertility levels are $n^e = 1$, $n^s = 2.22$. In the left panel, $\sigma = 0.5$, and we have $s = 0.034$ and $\Psi = 1$. In the right panel, $\sigma = 0.8$, and we have $s = 0.037$ and $\Psi = 0.96$.

3 Comparing the Education Regimes

Depending on the coverage of the public education system, we have three cases to consider.

Regime	Ψ
Public	1
Segregation	$\in (0, 1)$
Private	0

In the public regime, all children go to public school. Under segregation, the most skilled parents send their children to private school, while others use public schools. In the private regime, everybody attends private schools. We first derive the conditions under which each education regime arises. The following proposition summarizes the results.

Proposition 2 (Occurrence of education regimes)

The private regime is not an equilibrium outcome.

Whether public schooling can arise in equilibrium depends on the preference parameters γ and η . Let $\hat{\gamma} = (1 - \delta - \eta)/(\delta\eta)$.

If $\gamma > \hat{\gamma}$, public education is not an equilibrium outcome and $\Psi < 1/2$ for any σ .

If $\gamma < \hat{\gamma}$, the public regime prevails if and only if

$$\sigma \leq \hat{\sigma} = \frac{1 - \eta}{(1 + \gamma\eta)\delta} - 1.$$

Otherwise, we have segregation with $\Psi > 1/2$.

Let us first explain why the private regime cannot be an equilibrium outcome. When participation is very low ($\Psi \rightarrow 0$), high quality public education can be provided at very low tax levels. The quality of public schools is then sufficiently high ($s \rightarrow \eta\phi$) for the poorest parents to prefer public over private education.

To see whether a public regime can arise, we have to look at the preferences of the richest person. If this person has a high income relative to the average (high σ), the preferred education quality is sufficiently larger than publicly provided schooling for private education to be optimal. The effect of inequality on segregation is established in the next proposition. The public regime arises only if the income distribution is sufficiently compressed, so that the preferred education level varies little in the population. From now on we restrict attention to the region of the parameter space where the public regime can occur for a sufficiently compressed income distribution, and where at any time at least half the population is in public schools.

Assumption 1 *The model parameters satisfy:*

$$\gamma < \hat{\gamma} \equiv \frac{1 - \delta - \eta}{\delta\eta}.$$

As in nearly all countries participation in public schools far exceeds 50 percent, this is the empirically relevant case.¹⁴

Proposition 3 (Inequality and segregation) *Under Assumption 1, an increase in inequality leads to a lower share of public schooling, a higher quality of public schooling, and lower taxes:*

$$\frac{\partial \Psi}{\partial \sigma} \leq 0, \quad \frac{\partial s}{\partial \sigma} \geq 0, \quad \frac{\partial v}{\partial \sigma} \leq 0.$$

The inequalities are strict if a positive fraction of parents already uses private schools.

¹⁴Put differently, using the calibrated parameter values $\eta = 0.6$ and $\phi = 0.075$ from de la Croix and Doepke (2003a), Assumption 1 can be read as a condition on fertility n^s (see Equation (3)). The condition imposes $n^s < 7.79$ per person, which requires fertility per woman to be smaller than 15.6 children.

Higher income inequality leads to lower participation in public schools and more segregation (Ψ closer to $1/2$) if the majority of the population is in public schools. Intuitively, in this case an increase in inequality raises the income of the marginal person (who was just indifferent between private and public schooling before the increase in inequality). As a consequence, the preferred level of education increases, and this person now strictly prefers private schooling. The lower participation in public schooling after an increase in inequality also implies that the tax rate goes down. Thus, despite the increased demand for redistribution, everybody is taxed less as more parents opt out of the public schooling system.

The preferences of households at the income threshold \tilde{x} are linked to the relative quality of public versus private schooling. At the threshold, households are indifferent between both types of schools. This implies that the quality they receive from public schools is lower than the one from private schools, since the gap between the two has to compensate for higher costs of private education. This result is consistent with the literature devoted to the estimation of the relative quality of private education, correcting for the effect of higher social class of the pupils in the private sector. The majority of the results suggest that controlling for sample selectivity reduces the achievement advantage of private school students over public school students, but does not eliminate it.¹⁵

Notice that if Assumption 1 is satisfied, the unique equilibrium in a economy without inequality ($\sigma = 0$) is public schooling. The result may seem surprising at first sight, because public schooling implies a fiscal externality. In an equitable society, the social optimum would be pure private schooling. The reason why public schooling arises nevertheless is linked to our timing assumptions. If parents commit to a schooling choice before the schooling quality is set, a hold-up problem arises. From an ex ante perspective, it would be socially optimal for everybody to commit to using private schools. Ex post, however, if some parents decide to go for public schools anyway, the political system will provide a quality of public schooling that makes this decision optimal after the fact. Thus, the expectation that a certain public service will be provided creates a constituency that ensures that the service will be provided in reality.

¹⁵See Kingdon (1996) for India, Bedi and Garg (2000) for Indonesia, Alderman, Orazem, and Paterno (2001) for Pakistan, Vandenberghe and Robin (2003) for Brazil and Neal (1997) for Catholic U.S. schools. Some other studies find no difference between private and public schools performances (see Goldhaber (1996)).

4 Political Power and Multiple Equilibria

In this section, we relax the earlier assumption that each member of the population carries equal weight in the voting process. We will see that if political power is concentrated among high-income individuals, multiple equilibria can arise.

As a particularly simple form of variable political power, we consider outcomes with a minimum-income restriction for voting. There is now a threshold \bar{x} such that only individuals with income $x \geq \bar{x}$ are allowed to vote.¹⁶ All individuals above the threshold continue to carry equal weight in the voting process. This formulation captures property restrictions on voting, which were common in the early phases of many democracies. Similar cases of political exclusion can also arise from literacy requirements, age restrictions on voting (given that young people tend to be relatively poor), citizenship restrictions (assuming that recent immigrants are poorer on average than the native population), and political mechanisms other than voting (such as lobbying and bribery) that favor the rich in either democracies or non-democracies.

The objective function of the political system is now given by:

$$\Omega[s] \equiv \int_{\bar{x}}^{\max\{\bar{x}, \bar{x}\}} u[x, v, n^s, 0, s] g[x] dx + \int_{\max\{\bar{x}, \bar{x}\}}^{\infty} u[x, v, n^e, e[x], 0] g[x] dx. \quad (14)$$

Replacing \bar{x} by $2\sigma\Psi + 1 - \sigma$ into the objective, taking the first-order condition for a maximum, and solving for s yields:

$$\begin{aligned} \text{If } \Psi \geq \frac{\bar{x} - (1 - \sigma)}{2\sigma}, \quad s &= \frac{\eta\phi((1 - \sigma) - \bar{x} + 2\sigma\Psi)}{\Psi(\gamma\eta((1 - \sigma) - \bar{x} + 2\sigma\Psi) + (1 + \sigma) - \bar{x})}. \quad (15) \\ \text{If } \Psi < \frac{\bar{x} - (1 - \sigma)}{2\sigma}, \quad s &= 0. \end{aligned}$$

Hence, with a biased political system corner solutions with a schooling quality of zero may arise. The equilibrium schooling quality s satisfies:

$$s = \max \left\{ \frac{\eta\phi(1 - \sigma - \bar{x} + 2\sigma\Psi)}{\Psi(1 + \sigma - \bar{x} + \gamma\eta(1 - \sigma - \bar{x} + 2\sigma\Psi))}, 0 \right\}. \quad (16)$$

¹⁶Similar results would obtain if individuals below the threshold had positive, but sufficiently low weight in the political mechanism.

The corresponding tax rate is still given by (12). A few properties of (16) are of interest here. First, we have $s = 0$ for $\Psi \leq \bar{x} - (1 - \sigma)/2\sigma$. Second, we have:

$$s = \max \left\{ \frac{\eta\phi}{\frac{(1+\sigma-\bar{x})\Psi}{(1-\sigma-\bar{x}+2\sigma\Psi)} + \gamma\eta\Psi}, 0 \right\} \leq \frac{\eta\phi}{1 + \gamma\eta\Psi},$$

where the right-hand side is the schooling level that arises with equal political power as given by (11). Finally, for $\Psi = 1$ we have

$$s = \frac{\eta\phi}{1 + \gamma\eta},$$

just as in the case with equal political power. In the new formulation with variable voting power, the existence of equilibrium can still be proven. However, the equilibrium is no longer necessarily unique. It is also no longer true that the private regime never exists (as we showed in Proposition 2 for the democratic case of an even distribution of political power). In fact, in the biased political system pure private schooling is always an equilibrium outcome.

As soon as \bar{x} is smaller than \bar{x} , i.e., all voters expect to send their children to private schools, the chosen school quality is zero. Intuitively, these voters care only about taxes, but not about the quality of public schools that they do not use. As a consequence, private schooling becomes attractive to all parents.

To show that multiple equilibria can arise, we concentrate on the parameter space where public schooling is the unique equilibrium when $\bar{x} = 0$. We establish that in this case there are at least three equilibria.

Proposition 4 (Multiplicity of equilibria for $\bar{x} > 1 - \sigma$)

If \bar{x} , γ , and σ satisfy the conditions

$$\bar{x} > 1 - \sigma, \quad \gamma < \hat{\gamma}, \quad \text{and} \quad \sigma \leq \hat{\sigma} = \frac{1 - \eta}{(1 + \gamma\eta)\delta} - 1,$$

there are at least three equilibria. One is a private regime, one a public regime, and one features segregation.

In Figure 2 we take the same parameter values as in Figure 1 and we set $\sigma = 0.5$ and $\bar{x} = 0.7$, which implies that the bottom 20% have no political power. There are

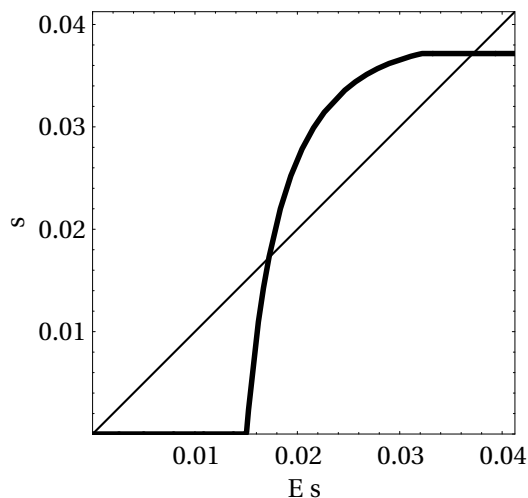


Figure 2: The fixed point with multiple equilibria ($\sigma = 0.5, \bar{x} = 0.7$)

now three fixed points: $s = 0$, $s = 0.017$, and $s = 0.037$, with participation $\Psi = 0$, $\Psi = 0.3$, and $\Psi = 1$.

The possibility of multiple equilibria exists because we assume that people have to decide on fertility and public versus private schooling before the vote on the quality of public education takes place. If all decisions were taken simultaneously, the voting process would lead to the same outcome as the weighted social planning problem, which is generically unique. Pre-commitment generates multiplicity in this setting, but not in the version with equal political weights, because there is now a strategic complementarity between the education choices of skilled people through the quality of public schools.¹⁷ When everybody with political power uses private schools, an given individual does not want to switch to the public system, since the quality of the public schooling is low. If, however, all voters were to switch together to the public system, they would vote for a much higher quality of public schools; in that case it would be rational to stay in the public system. Here, the political bias towards the rich offsets the increased cost of taxation resulting from higher participation in public schooling. Provided that there is a strong concentration of political power, the model can account for the fact that some countries with similar general characteristics choose very different educational systems.¹⁸

¹⁷When actions are strategic complements, the utility of those taking the action depends positively on how many people take the action. Classic examples are Matsuyama (1991) for increasing returns, Katz and Shapiro (1985) for network externalities, and Diamond and Dybvig (1983) for bank runs.

¹⁸A number of authors have derived similar multiplicity results in other applications of voting models. In Saint-Paul and Verdier (1997) there is majority voting on a capital income tax. If political power is unequally distributed, and is biased in favor of households having better access to

The next proposition shows that despite the possibility of multiple equilibria, the coverage of public schooling is never higher in societies dominated by the rich than in democracies:

Proposition 5 (Coverage of public education as a function of \bar{x})

Let Ψ_0 be the equilibrium coverage of public education for $\hat{x} = x_0$, and v_0 the corresponding tax rate. If Ψ_1 and v_1 are an equilibrium coverage and a tax rate for a $\hat{x} = x_1 > x_0$, then we have

$$\begin{aligned}\Psi_1 &\leq \Psi_0, \\ v_1 &\leq v_0.\end{aligned}$$

In summary, if the rich wield more power than the poor, multiple equilibria may arise. In any such equilibrium, the coverage of and spending on public education cannot be higher than in the outcome with equal political weights. Entirely private education systems are always possible.

While we have established the results of this section for the extreme case where households with an income below \bar{x} do not wield any political power, the results generalize to an environment where these households have some positive weight in the political system, but lower than the weight of households with income above \bar{x} . In particular, It is easy to establish that, for any \bar{x} , a pure private regime exists for the weight of low-income people sufficiently small. As before, if in this case the unique equilibrium in the model with an even distribution of political power is pure public schooling, at least three equilibria exist when the poor have less political power.

5 Outcomes with Government Commitment

So far we have assumed that the level of government spending on education is determined after private households have decided whether to send their children to

world capital markets, expectations-driven multiple equilibria can arise. In a dynamic majority voting framework, Hassler et al. (2003) assume that young agents base their education decisions on expectations over future redistribution. Self-fulfilling expectations can lead to either high or low redistribution equilibria. Finally, there are other political economy models that do not have indeterminacy of equilibrium but display multiple steady states (see for example Bénabou 2000 or Doepke and Zilibotti 2005). Initial conditions, as opposed to self-fulfilling expectations, determine which steady state the economy approaches.

private or public schools. In this section we analyze an alternative timing assumption, namely, the voters elect a government which pre-commits to a given overall spending level on education, while households can make their schooling choice conditional on this spending. Even though under each timing assumption people have perfect foresight, we will see that timing makes an important difference. For the following analysis, we return to the assumption of an even distribution of political power, i.e. $\bar{x} = 0$.

In the new timing, the government sets the tax rate at the beginning of the period. Since the tax base is independent of the schooling choice, this is equivalent to determining total spending on public education. After the tax is set, parents choose fertility and public versus private education for their children. Public schooling per child will then be simply given by the ratio of pre-committed total spending to the number of children in public schools. Since the government has perfect foresight, the problem can be solved backwards by first determining individual decisions as a function of policies, and then choosing policies taking this dependency into account.

Since fertility choices conditional on schooling are not affected by taxes, fertility rates will be as before determined by equations (3) and (5). Private education spending is also unaffected by the new timing of decisions and is given by equation (4). The participation decision is determined by the threshold defined in Lemma 2, which is now defined in terms of actual schooling quality s :

$$\tilde{x}[s] = \frac{1 - \eta}{\delta\phi\eta} s \quad (17)$$

We also redefine the endogenous fraction of children participating in the public education system as a function of actual quality:

$$\Psi[s] = \begin{cases} 0 & \text{if } \tilde{x}[s] < 1 - \sigma \\ \frac{\tilde{x}[s] - (1 - \sigma)}{2\sigma} & \text{if } 1 - \sigma \leq \tilde{x}[s] \leq 1 + \sigma \\ 1 & \text{if } \tilde{x}[s] > 1 + \sigma \end{cases} \quad (18)$$

From Equation (9), the link between taxes and expenditures is given by:

$$v = \Psi[s] \frac{\gamma}{\phi} s \quad (19)$$

The objective function modeling the voting process is the same as before, but Ψ and

\tilde{x} are now endogenous:

$$\Omega[s] \equiv \int_0^{\tilde{x}[s]} u[x, v, n^s, 0, s] g[x] dx + \int_{\tilde{x}[s]}^{\infty} u[x, v, n^e, e[x], 0] g[x] dx. \quad (20)$$

The structure of the problem is similar to a standard Ramsey (1927) problem, where the government chooses optimal taxes taking into account the reaction of private agents. Once again three regimes are possible: public schooling ($\Psi[s] = 1$), segregation ($1 > \Psi[s] > 0$), and private schooling ($\Psi[s] = 0$). In the segregation case, the first-order condition for optimization is as follows:

$$\begin{aligned} \int_0^{\tilde{x}[s]} \left(\frac{\partial u}{\partial v} \frac{\partial v}{\partial s} + \frac{\partial u}{\partial s} \right) g[x] dx + \int_{\tilde{x}[s]}^{\infty} \left(\frac{\partial u}{\partial v} \frac{\partial v}{\partial s} + \frac{\partial u}{\partial s} \right) g[x] dx \\ + \int_0^{\tilde{x}[s]} \frac{\partial u}{\partial v} \frac{\partial v}{\partial \Psi} \frac{\partial \Psi}{\partial s} g[x] dx + \int_{\tilde{x}[s]}^{\infty} \frac{\partial u}{\partial v} \frac{\partial v}{\partial \Psi} \frac{\partial \Psi}{\partial s} g[x] dx = 0. \end{aligned}$$

The first line of this optimality condition is the same as the one we get in the problem without commitment. The second line is a new term which arises from the endogenous dependency of Ψ on s . This term is always negative. The new negative term implies that under government commitment, the optimal s is lower than under no-commitment, as long as the solution for Ψ is interior. Intuitively, the government now takes into account that a marginal increase in s increases the number of families who use public schools. On the margin this lowers the value of the objective function, since the marginal family is just indifferent between private and public schooling, but imposes a fiscal burden on the rest of the population once it switches from private to public school.

In the private and public regimes, participation $\Psi[s]$ is locally independent of s , as long as the marginal family strictly prefers its current schooling choice. The additional term is therefore zero, and hence the optimal schooling choice of s does not depend on government commitment. The following proposition summarizes the result.

Proposition 6 (Equilibrium with commitment)

An equilibrium with commitment exists. Public school quality is lower than or equal to the level reached without commitment. The inequality is strict if participation Ψ satisfies: $0 < \Psi < 1$.

Existence is guaranteed because the objective function is continuous on a compact

set. The equilibrium is not guaranteed to be unique, however, because the objective function is not globally concave. In particular, it has kinks at the values of s corresponding to $\tilde{x}[s] = 1 - \sigma$ and $\tilde{x}[s] = 1 + \sigma$. However, multiplicity occurs only for knife-edge cases.

If we extend this model to concentrated political power as in Section 4, we no longer get generic multiplicity of equilibria. Under the original timing, multiple equilibria arose as self-fulfilling prophecies. With government commitment, the government moves first and chooses the generically unique equilibrium that maximizes the objective function.

To summarize, we see that the relative timing of the decisions taken by individual households and the government has an important bearing on the positive implications of our theory. Which timing, then, should be considered to be the most realistic? Clearly, there is no general answer to this question, as political decision horizons can vary substantially from country to country. It is useful, however, to consider as a benchmark the common case of a government that adjusts the education budget (which determines the quality of public schooling) at an annual frequency. As far as fertility decisions are concerned, the realistic assumption then is that households move before the government. Children generally enter school at age six, so that at the very minimum, six years pass from the fertility decision until schooling actually begins. It is hard to imagine that the government commits to a schooling quality more than six years ahead of time, without any possibility of later adjustments.

In contrast, matters are less clear cut when it comes to the choice of an education system (i.e., whether to send one's child to public or private school). We have already analyzed the case where parents make this choice before the government decides on school quality. What would happen if households chose fertility before the vote on schooling quality, but could adjust their choice of public versus private schooling after the vote? As we will see, this framework in which households make at least one decision before the government leads to similar implications as our original timing where the government moves last.

In this intermediate case, when households choose fertility, they do so under perfect foresight regarding the future quality of schools. There will be an income threshold \bar{x} below which people have large families (corresponding to the expectation of public schooling). The objective of the voting process takes three different forms depending on how the threshold for private education $\tilde{x}[s]$ compares to the threshold

for small families \bar{x} . For $\bar{x} < \tilde{x}[s]$, it is given by:

$$\Omega[s] = \int_0^{\bar{x}} u[x, v, n^s, 0, s]g[x]dx + \int_{\bar{x}}^{\tilde{x}[s]} u[x, v, n^e, 0, s]g[x]dx + \int_{\tilde{x}[s]}^{\infty} u[x, v, n^e, e[x], 0]g[x]dx,$$

for $\bar{x} = \tilde{x}[s]$, we have

$$\Omega[s] = \int_0^{\tilde{x}[s]} u[x, v, n^s, 0, s]g[x]dx + \int_{\tilde{x}[s]}^{\infty} u[x, v, n^e, e[x], 0]g[x]dx, \quad (21)$$

and for $\bar{x} > \tilde{x}[s]$, we have:

$$\Omega[s] = \int_0^{\tilde{x}[s]} u[x, v, n^s, 0, s]g[x]dx + \int_{\tilde{x}[s]}^{\bar{x}} u[x, v, n^s, e[x], 0]g[x]dx + \int_{\bar{x}}^{\infty} u[x, v, n^e, e[x], 0]g[x]dx.$$

If $\bar{x} \neq \tilde{x}[s]$, as in the previous case the first-order condition for optimality has an additional term related to the marginal impact of s on $\tilde{x}[s]$. In equilibrium, however, agents have perfect foresight, and $\bar{x} = \tilde{x}[s]$ will hold. In this case, consider the s that maximizes the objective function holding $\tilde{x}[s]$ constant at \bar{x} , as in our original timing. In Equation (21), in a neighborhood around this s , it is the case that the marginal effect of a change in s on \tilde{x} is zero. The reason is that agents below \bar{x} have chosen large families in expectation of using public schools, whereas families above $\bar{x} = \tilde{x}[s]$ have chosen small families in expectation of private schooling. Families close to the threshold therefore strictly prefer their expected schooling choice to the alternative. Thus for $\bar{x} = \tilde{x}[s]$ which occurs in equilibrium, the first-order condition is as in our original timing. If the solution is interior, this implies that the outcome has to be the same.¹⁹

To summarize, whether the model generates multiplicity of equilibria crucially depends on the timing of private decisions relative to the determination of government policy. If parents make decisions that lock them into specific choices for their children for a long time, whereas the government can adjust the quality of public education more frequently, the possibility of multiple equilibria is present. In

¹⁹Depending on parameters, however, under the intermediate timing there can also be additional corner solutions. The original and intermediate timing lead to the same equilibria if the lock-in effect through fertility is sufficiently strong. The strength of the lock-in effect, in turn, depends on the fertility differential between parents with children in public and private schools.

the real world, the strength of these lock-in effects would depend on a number of features that are not explicitly modeled in our theory. For example, in some countries educational segregation is linked to residential segregation, i.e., there are some districts where mostly rich people live who use private schools, whereas poorer districts are served by public schools. In such an environment, a switch in the type of schooling would also entail a switch of residence, and maybe even a switch of jobs if the distances are sufficiently large. Clearly, in such an environment the lock-in into a particular schooling type would be much stronger than in a country where private and public schools exist right next to each other, with few hurdles to switching schools.

6 Stylized Facts versus Model Predictions

In this section we document some basic facts regarding the relationship between schooling regimes, inequality, and fertility in a number of countries. Our theory makes predictions on how schooling choices and fertility vary across households within a given country, and on how aggregate measures of the quality and extent of private and public schooling depend on country characteristics. To examine the household-level predictions, we use micro data from the OECD Programme for International Student Assessment (PISA). For cross-country evidence, we use data from the OECD and the World Bank Development Indicators on public and private education spending.

6.1 Micro Data from the PISA Program

The international database of the OECD Programme for International Student Assessment (PISA) comprises data collected in the year 2000 on 15 year-old students in the principal industrialized countries. It includes a student questionnaire collecting information about the student's family and a school questionnaire that covers information on the extent of public funding and the public or private administration of schools.

We focus on four variables in the PISA database. The International Socioeconomic Index (ISEI) captures the attributes of occupations that converts parents' education into income. It was obtained by mapping parents' occupational codes onto an index

of occupational status, developed by Ganzeboom, De Graaf, and Treiman (1992). This index provides a rough measure of household income and human capital. Using the index, students were assigned numbers ranging from 16 to 90.²⁰ The second student-specific variable in our data set is the number of siblings, which is obtained by summing the answers to the question on how many brothers and sisters students have older than themselves, younger than themselves or of the same age.

At the school level, our data set contains information on the funding sources of schools and whether a school is publicly or privately run. For school funding, our variable is the percentage of total funding that stems from government sources, as opposed to fees paid by parents, benefactors, or other sources of income. The school type is determined by the question “Is your school a public or a private school.” Here public schools are those managed by a public education authority, a government agency, or a governing board that is appointed either by the government directly or elected by public franchise.

For each country, we have computed average school characteristics for different social groups. For each group we report the average share of education spending covered by public sources, the share of students in private schools, and the average fertility rate. Tables 1–3 provide the results.

Let us now compare the predictions of our model with this data set. A first prediction is that parents with a high social status should exhibit a stronger preference for private education. In particular, in countries where the quality of public education is relatively low, we should observe more children from high social classes in private schools. In fact, among the 27 countries, we observe a negative relation between public subsidization and social class in 18 cases. In eight cases, the subsidy rate is the same across social status, while in one case (Hungary), we find an increasing relationship. As far as the private/public distinction is concerned, there is a clear positive relationship between social status and private school attendance in 19 out of the 26 countries for which these data are available. The highest segregation levels are observed in Australia, Brazil, Mexico, and Spain. In all these countries, the difference between the subsidization rate of schools attended by rich children and the one of the schools attended by poorer children exceeds 15 percent.

²⁰Typical occupations between 16 and 35 points on the ISEI scale include small-scale farmer, metalworker, mechanic, taxi or truck driver, and waiter/waitress. Between 35 and 53 index points, the most common occupations are bookkeeping, sales, small business management, and nursing. As the required skills increase, so does the status of the occupation. Between 54 and 70 points, typical occupations are marketing management, teaching, civil engineering, and accounting. In the top quartile from 71 to 90 points, occupations include medicine, university teaching, and law.

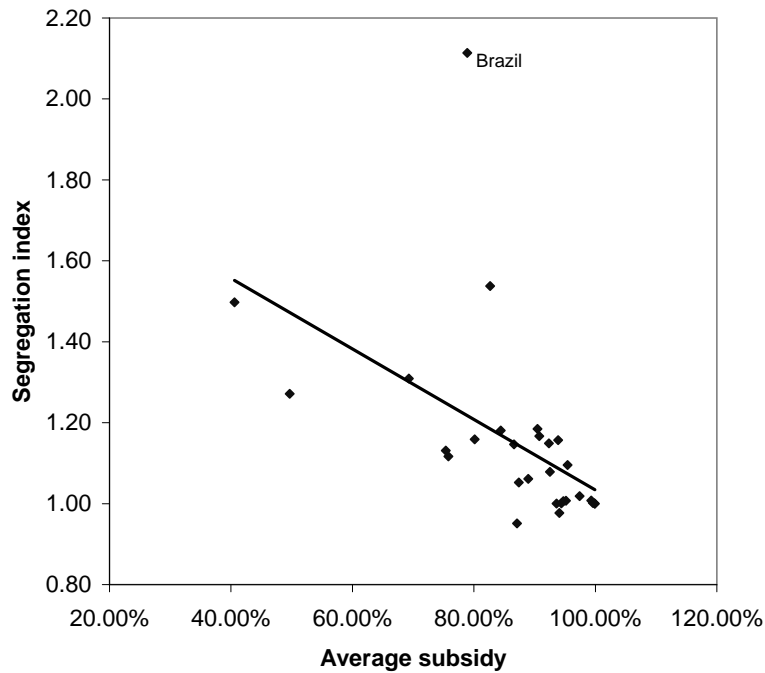


Figure 3: Segregation is associated with low subsidization

The group of countries for which the difference between the subsidization rate of schools attended by children with high social status and the one of the schools attended by poorer children is very low (typically below 5 percent) includes the Czech Republic, Denmark, Finland, Germany, Iceland, The Netherlands, Norway, and Russia. In all these countries, the level of subsidization is particularly high. This corresponds to the public regime of the theoretical model: if the quality of public education is sufficiently high, even the children from rich families will attend publicly funded schools.

Overall, the negative relationship between the average level of subsidization and the extent of segregation is quite strong. In Figure 3 we compare the overall level of subsidization with a segregation index computed as the ratio between the subsidization rate of the lowest social class over the one of the upper class. The correlation coefficient between these variables is -0.52 ($t\text{-stat} = -3.10$).

As far as fertility is concerned, in all the countries we find that the fertility of the lowest social group exceeds the fertility of the highest social group. In a majority of countries, the relationship between fertility and social status is monotonically decreasing; in some countries, it is U-shaped. The differential fertility is large in all the high-segregation countries except Spain, while in the countries with a strong public education sector, fertility differentials are lower.

Table 1: Data per Social Status (I)

Country	Social Status	Number of Observations	Subsidy Rate	% in Private Schools	Fertility
Australia	16-35	939	76.95%	NA	3.30
	36-53	1777	72.33%	NA	3.05
	54-70	1607	65.04%	NA	2.94
	71-90	542	58.77%	NA	2.83
Austria	16-35	874	95.52%	4.46%	2.81
	36-53	2213	91.60%	9.26%	2.58
	54-70	992	86.19%	16.23%	2.40
	71-90	276	80.62%	26.09%	2.63
Belgium	16-35	1558	89.21%	68.87%	3.01
	36-53	2388	88.08%	75.63%	2.59
	54-70	1573	85.46%	81.25%	2.66
	71-90	516	84.76%	83.33%	2.75
Brazil	16-35	1699	87.93%	2.35%	3.67
	36-53	831	79.52%	10.59%	3.36
	54-70	926	66.77%	23.00%	3.07
	71-90	125	41.60%	49.60%	2.86
Canada	16-35	6502	90.36%	0.00%	3.07
	36-53	9536	89.94%	0.00%	2.84
	54-70	9006	87.83%	0.00%	2.81
	71-90	2302	85.12%	0.00%	2.86
Czech Republic	16-35	1064	93.07%	7.33%	2.81
	36-53	2741	94.06%	6.20%	2.48
	54-70	1141	94.71%	6.22%	2.32
	71-90	309	95.25%	6.47%	2.19
Denmark	16-35	883	95.11%	20.50%	3.05
	36-53	1445	94.75%	23.18%	2.81
	54-70	1130	94.42%	24.78%	2.88
	71-90	271	94.50%	25.46%	2.87
Finland	16-35	1010	99.84%	1.58%	3.26
	36-53	1911	99.85%	3.30%	2.94
	54-70	1453	99.83%	3.51%	2.86
	71-90	381	99.78%	4.72%	2.88
France	16-35	1191	78.29%	22.50%	3.14
	36-53	1443	75.57%	21.83%	2.62
	54-70	929	73.36%	20.67%	2.61
	71-90	268	69.29%	26.12%	2.89

Table 2: Data per Social Status (II)

Country	Social Status	Number of Observations	Subsidy Rate	% in Private Schools	Fertility
Germany	16-35	887	97.89%	2.03%	2.77
	36-53	1970	97.84%	3.40%	2.46
	54-70	1105	96.56%	6.52%	2.38
	71-90	357	96.11%	7.84%	2.46
Greece	16-35	1278	85.63%	1.10%	2.68
	36-53	1447	87.02%	2.07%	2.44
	54-70	1074	82.60%	4.38%	2.31
	71-90	280	72.52%	15.71%	2.24
Hungary	16-35	1022	85.76%	3.82%	2.74
	36-53	2070	86.68%	4.01%	2.42
	54-70	1281	88.20%	3.51%	2.30
	71-90	261	90.18%	6.13%	2.38
Iceland	16-35	572	99.58%	0.00%	3.80
	36-53	1300	99.52%	0.62%	3.46
	54-70	926	99.10%	1.30%	3.37
	71-90	364	98.82%	1.92%	3.46
Ireland	16-35	990	94.68%	50.71%	3.94
	36-53	1463	91.43%	63.98%	3.52
	54-70	1040	87.83%	72.21%	3.33
	71-90	183	81.23%	78.69%	3.25
Italy	16-35	1391	79.47%	2.16%	2.45
	36-53	1995	74.50%	5.81%	2.30
	54-70	896	74.53%	5.80%	2.22
	71-90	281	71.17%	6.05%	2.34
Korea	16-35	1554	53.63%	47.23%	2.46
	36-53	1840	48.12%	50.00%	2.25
	54-70	803	46.47%	49.69%	2.18
	71-90	96	42.19%	45.83%	2.20
Mexico	16-35	1607	44.74%	3.17%	4.33
	36-53	1134	40.15%	17.20%	3.74
	54-70	619	34.52%	33.60%	3.39
	71-90	222	29.87%	49.55%	2.95
Netherlands	16-35	480	94.93%	72.92%	3.09
	36-53	739	94.59%	77.00%	2.80
	54-70	807	93.86%	76.70%	2.79
	71-90	153	94.85%	71.90%	2.86

Table 3: Data per Social Status (III)

Country	Social Status	Number of Observations	Subsidy Rate	% in Private Schools	Fertility
New Zealand	16-35	643	84.82%	0.78%	3.56
	36-53	1275	81.16%	3.61%	3.20
	54-70	1041	78.20%	5.86%	3.07
	71-90	339	73.19%	12.39%	2.86
Norway	16-35	418	99.57%	0.72%	3.40
	36-53	1737	99.71%	0.63%	2.98
	54-70	1148	99.53%	1.13%	2.99
	71-90	538	99.39%	1.12%	2.95
Poland	16-35	1052	94.78%	0.38%	3.07
	36-53	1449	92.08%	2.00%	2.75
	54-70	564	91.24%	4.08%	2.49
	71-90	156	82.51%	13.46%	2.45
Portugal	16-35	1843	89.10%	7.27%	2.67
	36-53	1501	86.67%	6.46%	2.33
	54-70	747	83.58%	8.17%	2.23
	71-90	278	77.73%	7.55%	2.28
Russia	16-35	1594	94.19%	0.00%	2.91
	36-53	2742	93.38%	0.00%	2.71
	54-70	1135	92.94%	0.00%	2.66
	71-90	828	94.15%	0.00%	2.44
Spain	16-35	2078	90.92%	24.01%	2.60
	36-53	2020	82.93%	42.48%	2.36
	54-70	902	73.21%	55.21%	2.40
	71-90	387	59.13%	68.22%	2.50
Switzerland	16-35	1290	98.28%	2.56%	2.93
	36-53	2398	96.20%	5.50%	2.58
	54-70	1351	93.73%	8.44%	2.54
	71-90	582	89.72%	13.40%	2.68
United Kingdom	16-35	1858	98.24%	0.65%	3.44
	36-53	3166	96.50%	2.46%	2.99
	54-70	2276	89.99%	8.92%	2.82
	71-90	856	84.93%	14.02%	2.82
United States	16-35	584	94.01%	4.11%	3.80
	36-53	840	92.42%	5.12%	3.54
	54-70	899	92.84%	5.01%	3.10
	71-90	202	87.18%	7.92%	3.02

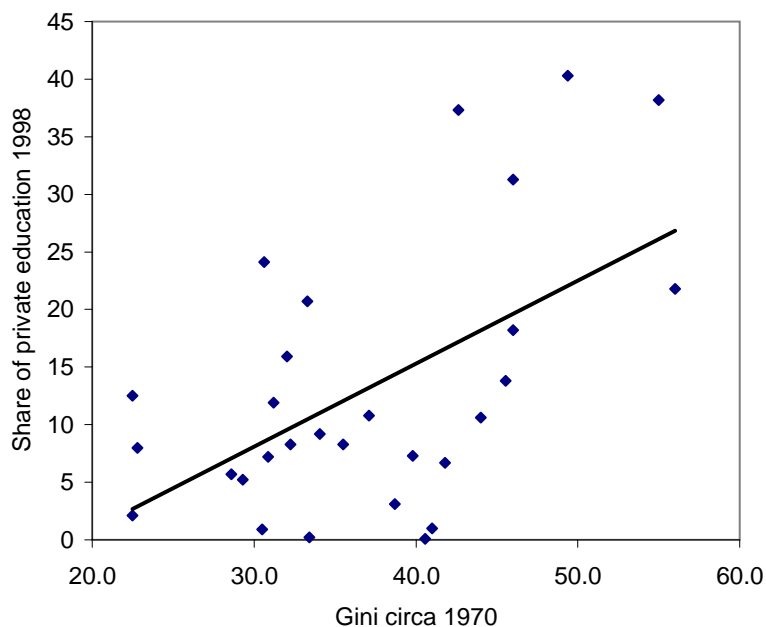


Figure 4: Inequality and education systems across countries

6.2 OECD Data on Private Funding

The literature contains few empirical studies on the determinants of the mix between public and private education across countries. One exception is James (1993). She computes from various sources the percentage of total enrollments that were in private schools around the year 1980 for 50 countries. By way of regression analysis, she concludes that the main determinants of the private/public mix are cultural factors such as religious competition and linguistic heterogeneity. However, the small number of observations compared to the number of explanatory variables casts some doubts on the robustness of the results.

The OECD provides internationally comparable data on the relative proportions of public and private investment in education, distinguishing two levels of education, primary/secondary and tertiary, and covering the period 1985–1998. Apart from 1998, only OECD countries are covered. The number of countries varies from 6 in 1985 to 31 in 1998. In most countries, private sector expenditure is comprised mainly of household expenditures on tuition and other fees. The exception is Germany, where nearly all private expenditure is accounted for by contributions from the business sector to the system of apprenticeship at the upper secondary level. For primary and lower secondary education, there is little private funding in Germany.

In 1998, the data set contains information on a number of non-OECD countries (Nor-

	N.obs	Share of Private Funding	Differential Fertility
High (> 15)	6	29.50	1.94
Low (≤ 15)	9	7.35	1.51
Mean difference test			-0.42 (t-stat=-1.48)

Table 4: Differential fertility and percentage share of private funding

way, Israel, Uruguay, Czech Republic, Switzerland, Turkey, Argentina, Indonesia, Chile, Peru, Philippines, and Thailand). With observations on 31 countries, we can investigate whether inequality is a good predictor of private funding. Computing the correlation between Gini inequality coefficient estimated in 1970 by Deininger and Squire (1996) and private funding in 1998, we find that the correlation is positive and relatively strong (0.44 (t-stat=2.64) or 0.45 (t-stat=2.67) if Germany is excluded from the sample), even stronger when we consider only primary and secondary levels (0.55 (t-stat=3.55) or 0.59 (t-stat=3.87) if Germany is excluded). Figure 4 presents the cross plot of the private share in primary and secondary education with the Gini coefficient.²¹

An implication of our theory is that countries that use private resources in education more intensively should be characterized by higher fertility differentials. As pointed out by de la Croix and Doepke (2003a), such endogenous fertility differentials amplify the negative effect of inequality on growth. Defining differential fertility as the difference in fertility between women from the highest education group and women from the lowest group (those data from various surveys are summarized in Kremer and Chen 2002), the correlation coefficient between the level of private spending on education and the fertility difference is 0.49 (t-stat=2.03), which is significantly different from zero at the 5 percent level with 15 observations. To give an idea of the magnitude of the difference between fertility levels, Table 4 provides average fertility differentials for countries with high and low use of private resources in primary and secondary education. Here, given the low number of observations, the means difference test is not significant.

²¹A related relationship exists across U.S. states. The National Center for Education Statistics provides direct general education expenditures per capita of U.S. state and local governments for the years 1997 and 1998. While we do not have estimates of private expenditures, we can investigate whether income inequality is a good predictor of public spending on education. Computing the correlation between the Gini inequality coefficient estimated in 1979 (U.S. Census Bureau) and the logarithm of public spending on education per capita in 1997-1998, we find that the correlation is negative and strong, -0.46 (t-stat=-3.85).

6.3 Public Education Spending and Democracy

Although private spending on education is available for only few countries, more data is available for the counterpart of private spending, namely public spending on education (measured as a fraction of GDP) from the World Bank Development Indicators. Here we have a sample of 158 countries over the period 1970-2002 (with some missing observations). We divide the sample into three groups, based on their “level” of democracy. The democracy index is computed using the political right index from the Freedom in the World Country Ratings. This index is available for the same sample of countries and years; it lies on a one-to-seven scale, with one representing the highest degree of freedom.

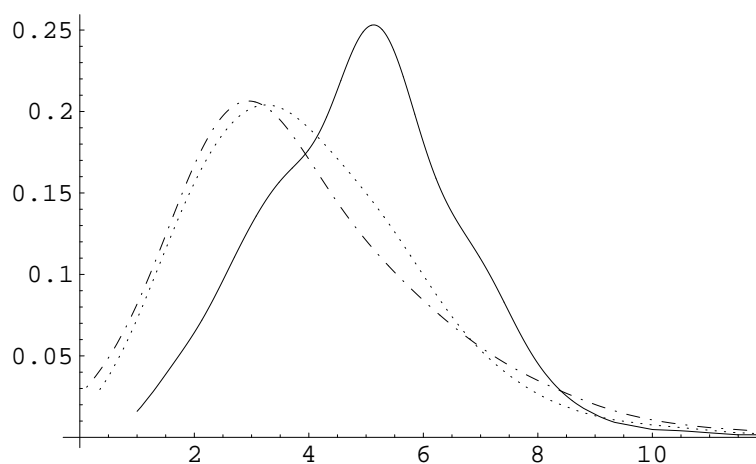
Table 5 displays the mean and variance of public spending on education for the three groups of observations. Variance is decreasing with democracy, the mean is increasing with democracy. This result is confirmed by inspecting the nonparametric density estimation of public spending as a share of GDP provided in Figure 5. The density for partially free and non-free countries displays a lower mean and a higher variance. Two-tailed tests of whether the differences are significant are provided in Table 6. The mean in democratic countries is significantly different from the one in the two other groups (medium and low), and the variance in the non-free countries is significantly higher than in the partially free-countries, which is itself higher than in the democracy group. The implications of the model when political power is allowed to be unequally distributed (see Propositions 4 and 5) are consistent with this set of facts. While we do not wish to claim that our mechanism is the only explanation for the observed high variance in the quality of public schooling in non-democracies, it is encouraging that the predictions of the model are in line with data in this dimension as well.

7 Conclusions

The education of its citizens is one of main areas of government intervention in every country in the world. At the same time, the government is generally not the only provider of education; education systems often display a juxtaposition of public and privately funded institutions. The degree of private involvement in the provision of education varies a great deal over different countries, going from fully public systems as in some European countries to segregated systems as in parts of

Democracy Index	Observations	Mean	Variance
High (= 1)	1020	4.96	3.08
Medium ($1 < x \leq 4$)	836	4.11	7.07
Low ($4 < x \leq 7$)	644	4.07	8.33

Table 5: Public spending on education



Legend: Free (solid line), partially free (dots), non-free (dashes).
 Note: Density estimation using the NonParametrix.m Package
 by Bernard Gress, 2004

Figure 5: Density of public education spending (% GDP)

	Mean Difference Test		Variance Ratio Test	
	Medium	Low	Medium	Low
High	0.85 [0.00]	0.89 [0.00]	0.44 [0.00]	0.37 [0.00]
Medium		0.03 [0.82]		0.85 [0.03]

Table 6: Mean and variance tests [P-value]

the U.S. In this paper, we try to understand how countries choose the mix of public and private education.

We first conclude that high inequality maps into a segregated education system. In a segregated system, the quality of public schools is sufficiently low for rich households to prefer paying for private schools to enhance the education of their children. When inequality is low, on the other hand, the rich decide to send their children to public schools, so that they avoid paying for education twice (first through taxes, second through private schools). The prediction of a strong relationship between inequality and the extent of public schooling is in line with empirical evidence. In both cross-country data and cross-state data in the U.S., we find that public spending on education is negatively related to income inequality.

Turning to the role of political power, we find that the quality and extent of public schooling generally increases with the political weight of the poor. In addition, if the poor are given at least equal weight in the political system, there is a unique equilibrium outcome. In societies that are politically dominated by the rich, on the other hand, multiple equilibria may arise. The reason is that when the rich are in charge, there is a complementarity between the number of rich people participating in public schools and their quality. For given initial conditions, such a country may either have a high-quality public schooling system in which many or all of the rich participate, or a low-quality system with all the rich using private schools. Despite the multiplicity, however, we find that spending on public education is never higher in a society dominated by the rich than in an otherwise identical economy where the poor have equal power. The model therefore provides an explanation for the observation that non-democratic countries spend on average a smaller fraction of GDP on public education than democracies, whereas the variance of spending across countries is higher.

While we have concentrated on cross-section evidence, another important question for future research is why public education was first introduced in the nineteenth century during the second phase of the Industrial Revolution. Galor and Moav (2006) argue that in this period capitalists started to have an interest in public education, because of complementarities between physical and human capital. Therefore, technological change strengthening this complementarity may have contributed to the introduction of public schooling. Galor, Moav, and Vollrath (2005) extend this analysis by distinguishing different sources of wealth. If land is less complementary to human capital than physical capital, a conflict of interest arises between land-

owners and capitalists. The outcome of this conflict depends on the distribution of wealth and land-ownership.

In our model, public schooling always arises if political power is equally shared. The theory therefore points to the expansion of voting rights in the nineteenth century as a key explanation for the introduction of public schooling. This still leaves open the question why voting rights were expanded in the first place. The theory of Galor and Moav (2006) offers one potential explanation. Acemoglu and Robinson (2000), however, point in a different direction: the rich shared power in order to avoid the threat of a revolution. In either case, given that the poor did gain political influence, in our model the introduction of public schooling is a necessary consequence. Once public education is in place, the size of the public system depends on the evolution of the income distribution. To this end, the model points to the declining income inequality observed around the turn of the century as a potential explanation for the large expansion of public education that followed its initial introduction.

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A Technical Appendix

Proof of Lemma 1: From budget constraint (2) total spending on children is given by $x\phi n + ne$. Substituting either $n = n^s$ and $e = 0$ or $n = n^e$ and $e = \eta\phi x / (1 - \eta)$ yields that

$$x\phi n + ne = \frac{\gamma}{1 + \gamma}x.$$

Taxable income therefore is

$$x(1 - \phi n) - ne = \frac{1}{1 + \gamma}x.$$

■

Proof of Lemma 2: We compute the level x such that a household with income x is indifferent between public and private by solving $u[x, v, n^s, 0, [E](s)] = u[x, v, n^e, e, 0]$:

$$\tilde{x} = \frac{1 - \eta}{\delta\phi\eta} [E](s) \quad \text{with: } \delta = (1 - \eta)^{\frac{1}{\eta}}. (6)$$

\tilde{x} is bigger than zero and depends positively on $[E](s)$. If x is greater (resp. smaller) than \tilde{x} , $u[x, v, n^s, 0, [E](s)]$ is smaller (resp. greater) than $u[x, v, n^e, e, 0]$, and the household prefers private (resp. public) education, which proves the Lemma. ■

Proof of Proposition 1: The result follows from an application of the Brouwer fixed point theorem. Given (11), the equilibrium expected schooling quality $E[s]$ and actual quality s lie in the interval:

$$E[s], s \in \left[\frac{\eta\phi}{1 + \gamma\eta}, \eta\phi \right] \quad (22)$$

We are now going to define a mapping Δ from $E[s]$ into s which maps this interval into itself. Since an equilibrium requires $E[s] = s$, a unique equilibrium exists if the mapping has unique fixed point.

Given expected schooling quality $E[s]$, according to Lemma 2 and equation (7) the fraction of families participating in public education is given by:

$$\Psi(E[s]) = \max \left\{ \min \left\{ \frac{1 - \eta}{2\sigma\delta\phi\eta} E[s] - \frac{1 - \sigma}{2\sigma}, 1 \right\}, 0 \right\}.$$

This function is (weakly) increasing in $E[s]$; the higher the expected quality of public education, the more parents are going to prefer using the public sector.

We can now use (11) to map the expected education quality $E[s]$ into the actual education quality s that would result from the political system if fraction $\Psi(E[s])$ of families participated in the public system. This education quality $s = \Delta(E[s])$ is

given by:

$$\Delta(E[s]) = \frac{\eta\phi}{1 + \gamma\eta\Psi(E[s])} = \frac{\eta\phi}{1 + \gamma\eta \max \left\{ \min \left\{ \frac{1-\eta}{2\sigma\delta\phi\eta} E[s] - \frac{1-\sigma}{2\sigma}, 1 \right\}, 0 \right\}}. \quad (23)$$

An equilibrium is characterized by a fixed point of $\Delta(E[s])$, i.e., a schooling level s that satisfies $s = \Delta(s)$, so that the schooling quality s that is expected by the parents is identical to the one actually implemented in the political process. Given (23), Δ is a continuous, weakly decreasing function mapping the closed interval given in (22) into itself. The mapping therefore crosses the 45-degree line exactly once, and a unique equilibrium exists. ■

Proof of Proposition 2: We first show that the private regime is not an equilibrium outcome. In a *reductio ad absurdum* argument, we would start by assuming the existence of such a case. We start from the optimal value of s given in Equation (11). From this equation, we observe that as Ψ tends to zero, s tends to $\eta\phi$. With this quality of public schooling, the equilibrium income threshold (from Equation (6) is: $\tilde{x}[\eta\phi] = (1 - \eta)/\delta$. For private education to be an equilibrium, this threshold should be lower than or equal to lowest income $1 - \sigma$. Given the definition of $\delta = (1 - \eta)^{1/\eta}$, we have that $\delta < 1 - \eta$ which implies $x[\eta\phi] > 1 > 1 - \sigma$. Hence, with a quality of public schools going to $\eta\phi$, it is always optimal for the poorest person to chose public education. Therefore, the private regime cannot arise in equilibrium.

If the public regime is an equilibrium, it has $\Psi = 1$ and $s = \eta\phi/(1 + \gamma\eta)$ (from Equation (11)). For this to be an equilibrium, we need the richest person to send his children to public school, which requires $\tilde{x}[\eta\phi/(1 + \gamma\eta)] \geq 1 + \sigma$. This condition can be written:

$$\frac{1 - \eta}{\delta(1 + \gamma\eta)} \geq 1 + \sigma.$$

If $\gamma \geq \hat{\gamma} = (1 - \delta - \eta)/(\delta\eta)$, the right hand side of the inequality is below 1, and the inequality can never be satisfied. This condition links the taste for children γ to the weight of education η independently of σ . If $\gamma < \hat{\gamma}$, the above inequality can be rewritten as a condition on σ , which in the condition of the Proposition.

It remains to show that $\gamma < \hat{\gamma} \Leftrightarrow \Psi > 1/2$. $\Psi = 1/2$ implies that the equilibrium is segregated. We solve for the equilibrium value of Ψ , which is given by:

$$\Psi = \frac{-\gamma\delta\eta(1 - \sigma) - 2\delta\sigma + \sqrt{8\gamma\delta\eta\sigma(1 - \delta(1 - \sigma) - \eta) + (\gamma\delta\eta(1 - \sigma) + 2\delta\sigma)^2}}{4\gamma\delta\eta\sigma}$$

There is only one value of γ for which this big expression is equal to 1/2, and it is $\hat{\gamma}$. Since Ψ is a continuous function of γ , and that we already know that it can be equal to 1 for certain values of σ if $\gamma < \hat{\gamma}$, we conclude that $\Psi > 1/2$ for $\gamma < \hat{\gamma}$ and $\Psi < 1/2$ for $\gamma > \hat{\gamma}$. ■

Proof of Proposition 3: In the interior regime (segregation), we have from (6) and

(7):

$$\Psi = \frac{\frac{1-\eta}{\delta\phi\eta}s - (1-\sigma)}{2\sigma}$$

Taking the derivative with respect to s , we obtain

$$\frac{\partial\Psi}{\partial\sigma} = \frac{2\sigma - 2\left[\frac{1-\eta}{\delta\phi\eta}s - (1-\sigma)\right]}{2\sigma} = \frac{1}{\sigma} \left(\frac{1}{2} - \Psi\right).$$

Hence,

$$\text{If } \Psi \in (0,1), \quad \frac{\partial\Psi}{\partial\sigma} > 0 \Leftrightarrow \Psi < \frac{1}{2}. \quad (24)$$

In the public regime,

$$\Psi = 1, \quad \Rightarrow \quad \partial\Psi/\partial\sigma = 0. \quad (25)$$

Proposition 2 together with (12), (24) and (25) imply Proposition 3. ■

Proof of Proposition 4: As in the proof of Proposition 1, we will proceed by analyzing the mapping from expected schooling quality $E[s]$ into actual quality s . Given (16), a schooling quality of zero is now possible, so that $E[s]$ and s lie in the following interval:

$$E[s], s \in [0, \eta\phi] \quad (26)$$

We are now going to define a mapping $\bar{\Delta}$ from $E[s]$ into s which maps this interval into itself. Since an equilibrium requires $E[s] = s$, any equilibrium corresponds to a fixed point of the mapping.

Given expected schooling quality $E[s]$, according to Lemma 2 and equation (7) the fraction of families participating in public education is given by:

$$\Psi(E[s]) = \max \left\{ \min \left\{ \frac{1-\eta}{2\sigma\delta\phi\eta} E[s] - \frac{1-\sigma}{2\sigma}, 1 \right\}, 0 \right\}. \quad (27)$$

This function is (weakly) increasing in $E[s]$; the higher the expected quality of public education, the more parents are going to prefer using the public sector.

We can now use (16) to map the expected education quality $E[s]$ into the actual education quality s that would result from the political system if fraction $\Psi(E[s])$ of families participated in the public system. This education quality $s = \bar{\Delta}(E[s])$ is given by:

$$\bar{\Delta}(E[s]) = \max \left\{ \frac{\eta\phi(1-\sigma-\bar{x}+2\sigma\Psi(E[s]))}{\Psi(E[s])(\gamma\eta(1-\sigma-\bar{x}+2\sigma\Psi(E[s]))+1+\sigma-\bar{x})}, 0 \right\}. \quad (28)$$

An equilibrium is characterized by a fixed point of $\bar{\Delta}(E[s])$, i.e., a schooling level s that satisfies $s = \bar{\Delta}(s)$, so that the schooling quality s that is expected by the parents is identical to the one actually implemented in the political process. Given $\Psi(E[s])$ and the properties of (16) stated in the text, $\bar{\Delta}$ is a continuous function mapping the

closed interval given in (26) into itself. Due to the Brouwer fixed point theorem, at least one equilibrium exists. Also notice that $\Psi = 0$ is always an equilibrium, since $\bar{\Delta}(E[0]) = 0$.

Now assume that conditions $\gamma < \hat{\gamma}$ and $\sigma < \hat{\sigma}$ are satisfied. We want to establish that in this case pure public schooling $\Psi = 1$ is an equilibrium. In (28), the schooling quality corresponding to $\Psi = 1$ is $s = \eta\phi/(1 + \gamma\eta)$. The first term inside the minimization in (27) satisfies for this s :

$$\begin{aligned} \frac{1 - \eta}{2\sigma\delta\phi\eta} s - \frac{1 - \sigma}{2\sigma} &= \frac{1 - \eta}{2\sigma\delta(1 + \gamma\eta)} - \frac{1 - \sigma}{2\sigma} \\ &\geq \frac{1}{2 \left(\frac{1 - \eta}{\delta(1 + \gamma\eta)} - 1 \right)} \left(\frac{1 - \eta}{\delta(1 + \gamma\eta)} - \left(2 - \frac{1 - \eta}{(1 + \gamma\eta)\delta} \right) \right) \\ &= \frac{1}{\frac{1 - \eta}{\delta(1 + \gamma\eta)} - 1} \left(\frac{1 - \eta}{\delta(1 + \gamma\eta)} - 1 \right) = 1, \end{aligned}$$

where $\gamma < \hat{\gamma}$ guarantees that the right-hand side of the first equation is positive, and the inequality follows from $\sigma \leq \hat{\sigma}$. The public education education quality $s = \eta\phi/(1 + \gamma\eta)$ is therefore mapped into $\Psi(s) = 1$, which, in turn, implies $\Psi(s) = s$. Therefore, if pure public schooling is an equilibrium under an equal division of political power, it is still an equilibrium with a voting threshold $\bar{x} < 1 - \sigma$. Since pure private schooling is always an equilibrium, we have at least two equilibria in this case. Next, notice that the slope of $\bar{\Delta}(s)$ is zero at $s = 0$, since for sufficiently small s we still have $\Psi = 0$. Also, at $s = \eta\phi/(1 + \gamma\eta)$ the slope of $\bar{\Delta}(s)$ is zero as well, since given $\sigma < \hat{\sigma}$ we have $\Psi = 1$ in a neighborhood of $s = \eta\phi/(1 + \gamma\eta)$. Given that $\bar{\Delta}(s)$ is continuous, it has to cross the 45-degree line at least one more time in between those values, implying the existence of a third equilibrium, which features segregation. ■

Proof of Proposition 5: We have previously established that:

$$\max \left\{ \frac{\eta\phi}{\frac{(1 + \sigma - \bar{x})\Psi}{(1 - \sigma - \bar{x} + 2\sigma\Psi)} + \gamma\eta\Psi}, 0 \right\} \leq \frac{\eta\phi}{1 + \gamma\eta\Psi},$$

where the left-hand side is the schooling level that arises for a given Ψ as a given by (16), and the right-hand side is the level that arises with equal political power as given by (11). Since the reverse mapping from a given schooling quality into Ψ does not depend on the distribution of political power, this inequality implies that the mappings Δ and $\bar{\Delta}$ defined in the proofs of Propositions 1 and 4 satisfy $\bar{\Delta}(s) \leq \Delta(s)$. Equilibria in the two environments corresponds to levels of s that satisfy $s = \bar{\Delta}(s)$ and $s = \Delta(s)$, respectively. It was established in the proof of Proposition 1 that $\Delta(s)$ is weakly decreasing and crosses the 45-degree line only once from above. Since $\bar{\Delta}(s) \leq \Delta(s)$, it then follows that $\bar{\Delta}$ cannot cross the 45-degree line above the unique crossing point for $\Delta(s)$, so that we have $\bar{s} \leq s$. Finally, the relationship

between s and v does not depend on the distribution of political power, and taxes are monotonically increasing in s , so that we have $\bar{v} \leq v$. ■