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Abstract

Leaky-bucket transactions can be regarded as income transfers allowing for transaction costs. In its most rudimentary form, leaky-bucket transactions trace out the maximum "leakage" of transaction costs before income inequality is exacerbated, or—alternatively—before a welfare loss is experienced. This notion suggests that part of the income transfer should reach the transferee in order to keep the degree of income inequality or social welfare intact.

However, in general, this conjecture is theoretically wrong. Rather there exists a unique benchmark such that it holds only for transfers among income recipients below the benchmark. When both are above the benchmark, the transferee has to be given more than the amount taken from the transferor, and when they are on opposite sides of the benchmark, both should experience an income loss. These three cases cover progressive transfers only. Three more cases apply to regressive transfers, and six more cases apply to income gains. Each of these twelve cases is covered by the present paper.

Yet experimental research shows poor empirical evidence for this theory. Subjects' perceptions of maintaining the degree of income inequality rather follow a simple precept: If someone gains income, the other person involved should be positively compensated, and if someone loses income, the other person involved should be negatively compensated. This expresses sort of compensating justice rather than restoration of the former degree of income inequality according to the orthodox theory. Compensating justice is, however, at variance with the transfer principle.

JEL Classification: D31, D63, C91.

It seems that Okun (1975, pp. 91-95) was the first who investigated the Pigou-Dalton transfer principle in the face of transaction costs. The pure transfer principle states that an order preserving transfer from a richer to a poorer income recipient should decrease income inequality. Okun started out with an approval of the pure transfer principle, but remarked that in reality "the money must be carried from the rich to the poor in a leaky bucket. Some of it will simply disappear in transit, so the poor will not receive all the money that is taken from the rich." [Okun (1975, p. 91).]

Then, in some kind of thought experiment, Okun (1975, p. 91) put the following question to an outside ethical observer: "I shall not try to measure the leak now, because I want you to decide how much leakage you would accept and still support the Tax-and-Transfer Equalization Act." Later on, Okun (1975, p. 94) states his own view: "Since I feel obliged to play the far-fetched games that I make up, I will report that I would stop at the leakage of 60 percent in this particular example."

Thus, Okun wants the outside ethical observer to trace out the maximum leakage of a transfer from richer to poorer income recipients which would just be tolerable to him or her. Although Okun is not explicit about the measure to be applied to determine the maximum leakage, we will not go astray in assuming that he had the observer's judgment on the social welfare of the society in mind. More precisely, if an amount δ is taken from a richer income recipient and is transferred to a poorer income recipient, then, in the absence of transaction costs, the gain in social welfare due to the increase in income of the poorer income recipient exceeds the loss of social welfare due to the decrease in income of the richer income recipient. This results from the transfer principle. The leaky-bucket case means, however, that the poorer income recipient gets less, say an amount $\varphi < \delta$, instead of δ. Let $\gamma \doteqdot \min \varphi$. Then the maximum tolerable leakage, $(\delta - \gamma)$, results from the minimum amount of φ, *viz.* $\gamma > 0$, that the poorer income recipient should receive such that no loss in social welfare is experienced.

However, Okun does not stop here. He states that the tolerated leakage is an increasing function of the income level of the transferors: "If the proposed tax were to be imposed only on the handful of wealthiest American families with annual incomes above \$ 1 million, you might well support the equalization up to a much bigger leakage. *In fact, some people would wish to take money away from the super-rich even if not one cent reached the poor.*" (Okun (1975, p. 94); our emphasis.)

It is, in particular, this last statement which deserves closer attention. An ethical observer who subscribes to this judgment considers a decrease in the income of a super-rich income recipient by δ to *increase* social welfare even if the poor income recipients do not get anything, *i.e.*, $\varphi = 0$. Note, however, that leaky-bucket transactions imply that $\gamma \neq 0$. This is easy to be seen: when such outside ethical observer is asked whether (s)he would approve of a tax which pinches the super-rich income recipients by δ, but, due to constitutional reasons of tax universality, has to tax also the poor income recipients at φ, then this observer should endorse such a tax provided that $0 <\mid \varphi \mid<\mid \gamma \mid$.[1] In other words, taxing the rich income recipients at δ and the poor at γ just maintains the level of social welfare.

So far, we have interpreted the outside ethical observer's judgment in terms of social welfare, which seems to us what Okun had primarily in mind. But this is not the only basis of judgements on income distributions. *Income inequality measures* are perhaps the more primitive concept to judge income distributions. Moreover, income inequality measures and social welfare functions are ordinally equivalent, as was shown by Blackorby and Donaldson (1978) and Dagum (1990, 1993).

In terms of income inequality measures the maximum tolerable leakage is defined such that the income inequality as measured by a particular inequality measure is not exacerbated, *i.e.*, the income inequality measure remains constant. Preliminary reasoning would suggest that leakages are only acceptable if they leave part of the transfer for the transferee.[2]

However, Seidl (2001), Hoffmann (2001), and Lambert and Lanza (2006) showed that this conjecture is theoretically wrong. Rather there exists a *unique benchmark* as a function of the income inequality measure and the income distribution applied, such that the above conjecture holds only when both parties involved in a progressive transfer have incomes *below* the benchmark. When both parties have incomes *above* the benchmark, then the transferee has to receive even a higher amount than that taken from the transferor in order to maintain the degree of income inequality. When the parties lie on *opposite sides* of the benchmark, then also the "transferee" should suffer an income loss in order to maintain the degree of income inequality. More formally, this is stated in the Leaky-Bucket Theorem [Theorem 10].

Recall that Okun invited responses of subjects to determine the extent of the tolerable leakage. This calls for experiments.[3] As to the full experimental design, note that transfers are not confined to progressive transfers (*i.e.*, going from richer to poorer income recipients). Transfers can rather occur according to four categories: The income of an income recipient may either be *increased* or *decreased* by a certain amount δ, and the other income recipient, whose income should be adapted by an amount γ, may either be *richer* or *poorer* than the first one. For each of these four categories the respective incomes can lie above, below, or on opposite sides of the benchmark. This makes twelve cases to be analyzed.[4] The experimental design used in this paper investigates all four categories, *i.e.*, all twelve cases.

The paper is arranged as follows: Section I gives an appraisal on the theory of leaky-bucket transactions, Section II describes the experimental design and procedure, Section III presents the results, and Section IV concludes.

I Theory

In this section we describe the leaky-bucket theory as required by our experimental design.[5] The number of income recipients is assumed to be finite; all incomes are assumed to be different and strictly positive. The anonymity axiom holds. The inequality measures are assumed to be continuously differentiable.

Thus, we consider only income distributions $y = \{y_i \mid i = 1, \ldots, n;\ 1 < n < \infty\}$ such that, by the anonymity axiom, we can focus on an increasing arrangement of incomes, $0 < y_1 < y_2 < \ldots < y_n < \infty$. More generally, income inequality measures are denoted by $I : \mathbb{R}^n_{++} \to \mathbb{R}_+$.[6]

Definition 1: $I(\cdot)$ is scale invariant if $I(y) = I(\lambda y)$ for all $y \in \mathbb{R}^n_{++},\ \lambda > 0$. $I(\cdot)$ is translation invariant if $I(y) = I(y + \zeta e)$ for all $y \in \mathbb{R}^n_{++}$ and for all $\zeta \in \mathbb{R}$ such that $y_1 + \zeta > 0$, where e denotes the unit vector.

Lemma 2: If a differentiable income inequality measure $I(\cdot)$ is scale invariant, then

$$\sum_{i=1}^{n} \frac{\partial I}{\partial y_i} y_i = 0.$$

PROOF: Consider a scale variation of y, λy, $\lambda > 0$. Then $I(\lambda y) \equiv I(y)$ for all $\lambda > 0$ implies

$$\frac{\partial I(\lambda y)}{\partial \lambda} = \sum_{i=1}^{n} \frac{\partial I(\lambda y)}{\partial \lambda y_i} y_i \equiv 0 \text{ for all } \lambda > 0. \text{ Hence, } \lim_{\lambda \to 1} \frac{\partial I(\lambda y)}{\partial \lambda} = \sum_{i=1}^{n} \frac{\partial I(y)}{\partial y_i} y_i = 0.$$

Q.E.D.

LEMMA 3: If a differentiable income inequality measure $I(\cdot)$ is translation invariant, then

$$\sum_{i=1}^{n} \frac{\partial I}{\partial y_i} = 0.$$

PROOF: By translation invariance $I(y) = I(y + \zeta e)$ for all $\zeta \in \mathbb{R}$ such that $y_1 + \zeta > 0$. This implies

$$\frac{\partial I(y + \zeta e)}{\partial \zeta} = \sum_{i=1}^{n} \frac{\partial I(y + \zeta e)}{\partial y_i} \equiv 0 \text{ for all } \zeta \in \mathbb{R},\ y_1 + \zeta > 0.$$

Hence

$$\lim_{\zeta \to 0} \frac{\partial I(y + \zeta e)}{\partial \zeta} = \sum_{i=1}^{n} \frac{\partial I(y)}{\partial y_i} = 0.$$

Q.E.D.

DEFINITION 4: A differentiable income inequality measure $I(\cdot)$ is inequality averse if

$$\frac{\partial I}{\partial y_j} < \frac{\partial I}{\partial y_k} \text{ for } y_j < y_k.$$

REMARK 5: For *inequality sympathy* (which was often observed in experiments[7]) the inequality signs in Definition 4 are reversed.

DEFINITION 6: $I(\cdot)$ satisfies the Pigou-Dalton transfer principle if

$$I(y + \tau e_k - \tau e_j) < I(y) \text{ and } I(y - \tau e_k + \tau e_j) > I(y) \text{ for all } y_k < y_j,$$

where e_i denotes an $n-$dimensional vector with a 1 on the $i-$th position and zeros everywhere else, and τ is such that $0 < \tau < \min_{i,\ell \in \{1,\dots,n\}} \mid y_i - y_\ell \mid$.

THEOREM 7: For differentiable inequality averse income inequality measures which are scale or translation invariant there exists a benchmark y^*, $y_1 < y^* < y_n$ such that $\frac{\partial I}{\partial y_i} < 0$ for all $y_i < y^*$, and $\frac{\partial I}{\partial y_i} > 0$ for all $y_i > y^*$.

Proof: As all $y_i > 0$, Lemmata 2 and 3 demonstrate that not all $\dfrac{\partial I}{\partial y_i}$ can be positive. By inequality aversion $\dfrac{\partial I}{\partial y_i}$ increases monotonically as y_i increases. Hence, there exists a benchmark, y^*, such that $\dfrac{\partial I}{\partial y_i} < 0$ for all $y_i < y^*$ and $\frac{\partial I}{\partial y_i} > 0$ for all $y_i > y^*$.

Q.E.D.

Remark 8: It seems that the existence and properties of benchmarks have first been noticed and analyzed by Seidl (2001) and Hoffmann (2001). The most comprehensive study is due to Lambert and Lanza (2006). Lambert and Lanza (2006, Theorem 1, p. 255) prove the existence of a benchmark by way of Lorenz consistency of an income inequality measure. Recall that a Lorenz curve is scale invariant by definition. Hence, we hold that Theorem 7 is somewhat more general, as it covers also the case of translation invariance.

Remark 9: Hoffmann (2001, p. 238) considers the benchmark as sort of natural dividing line between the "relatively poor" and the "relatively rich" and, hence, treats it as the relative poverty line. Notice, however, that the benchmark depends both on the income distribution and on the income inequality measure applied. Hence, the poverty line, too, would depend on the income distribution and on the income inequality measure applied. While the former dependence is judicious, the latter dependence would make the set of the poor dependent on the income inequality measure applied, which is somewhat odd for a poverty line.

We are now ready to state the basic theoretical results of the leaky-bucket theory. Note that they are independent of the particular income inequality measure applied; it is just assumed to be differentiable, inequality averse, and scale invariant. These results will then be subject to experimental testing, which, however, requires assumptions as to the income inequality measures applied.

Theorem 10 (Leaky-Bucket Theorem): Let $I(\cdot)$ denote a differentiable, inequality averse, and scale or translation invariant income inequality measure with y^* as its benchmark. Consider two income recipients, j and k, and assume that j's income is changed by δ, where $\mid \delta \mid < \min_{i,\ell \in \{1,\ldots,n\}} \mid y_i - y_\ell \mid$. Determine the change γ of k's income such that $\Delta I = 0$. Then:

$$\gamma \doteq -\frac{\partial I / \partial y_j}{\partial I / \partial y_k} \delta,$$

which means:

Benchmark	$\delta > 0$		$\delta < 0$	
	$y_j < y_k$	$y_j > y_k$	$y_j < y_k$	$y_j > y_k$
$y^* > y_j, y_k$	$\gamma < -\delta$	$-\delta < \gamma < 0$	$\gamma > -\delta$	$-\delta > \gamma > 0$
$y^* < y_j, y_k$	$-\delta < \gamma < 0$	$\gamma < -\delta$	$-\delta > \gamma > 0$	$\gamma > -\delta$
y^* between y_j and y_k	$\gamma > 0$	$\gamma > 0$	$\gamma < 0$	$\gamma < 0$

PROOF: By differentiability we have

$$\frac{\partial I}{\partial y_j} \delta + \frac{\partial I}{\partial y_k} \gamma \doteq 0, \text{ which implies } \gamma = -\frac{\partial I / \partial y_j}{\partial I / \partial y_k} \delta.$$

The cells in the above table result from Definition 4 and Theorem 7.

Q.E.D.

Notice that the leaky-bucket theory is not at variance with the transfer principle. Note the difference: the transfer principle focuses on *changes* in the degree of income inequality when transaction costs are absent, *i.e.*, when the income of one income recipient is changed by δ, the income of a corresponding income recipient is changed by $-\delta$. Then, by Definition 4, income inequality is *decreased* for progressive transfers [*i.e.*, if either $\delta < 0$ and $y_j > y_k$ or if $\delta > 0$ and $y_j < y_k$] and *increased* for regressive transfers [*i.e.*, if either $\delta < 0$ and $y_j < y_k$ or if $\delta > 0$ and $y_j > y_k$]. By contrast with the transfer principle, the leaky-bucket theory focuses on transfers with leakages which *maintain* the degree of income inequality, i.e., normally we have $\delta \neq -\gamma$.

II The Experiment

The main goal of our experiment is testing the validity of the leaky-bucket theory as established in Theorem 10. As there is no way of extracting the benchmarks (whose existence was before long not even known to the experts) directly from our subjects, we had to take another route. Hence, we asked our subjects for the assessments of their γ's in the twelve cells of the table in Theorem 10. Subjects faced a change of j's income by

δ and had to determine the change in k's income, γ, that would re-establish the same degree of income inequality as in the situation before j's income change. The data gained in this way were then used to test the leaky-bucket theory.

A The Experimental Design

The experiment was conducted at the KiEEL laboratory (Kiel Experimental Economics Laboratory) at the University of Kiel, Germany. Subjects were volunteers recruited from the students of different departments of this university. Many of them were not acquainted with income distributions. Therefore, subjects received a short training in handling income distributions before coming to the laboratory. (The training program is provided in the Appendix.) In this training, the concepts of income distributions, scale invariance, and the transfer principle were explained. Subjects were alerted that income distributions may be evaluated along different criteria: First, they may be judged according to the subjects' perceptions of being *more equally or more unequally distributed*. Second, they may be judged according to warranting a greater GDP for the economy, irrespective of the distribution of the incomes. Third, they may be judged according to securing higher aggregate social welfare, where we stressed that social welfare had to be judged according to the values of the beholders. Fourth, they may be judged according to the subjects' preferences for income distributions if their income positions were determined under an ex-ante veil of ignorance. We asked subjects to focus on the first criterion.

Then the instructions of the experiment were explained to the subjects. (A transcript with the instructions is provided in the Appendix.) The experimental design involved the income distribution [€500, €750, €1000, €1250, €1500, €1750, €2000], which was proposed to represent the monthly incomes of seven income recipients (*e.g.*, students). Using Abbink and Sadrieh's (1995) Ratimage toolbox, these incomes were presented in the upper half of a computer screen as illustrated in Figure 1. The same diagram was displayed on the lower half of the screen. Upon pressing any key, the income of an income recipient was increased or diminished by €100 (standing proxy for our δ in Theorem 10). At the same time, the income of another income recipient was set equal to zero as an indicator that the subject was required to adapt this income.

The upper half of the screen continued to present the original diagram highlighting now the incomes involved in the corresponding question, reminding the subjects of the original income distribution.

The subject was asked to adapt the income set to zero in the lower half of the screen such that the *degree of income inequality of the original income distribution would be re-established in the new one*[8]. Every decision of the subject was shown in the diagram on the screen, and the subject was asked either to confirm the answer or to try out some other decisions. The subject was allowed to play round as long as he or she wanted to do so before making his or her decision definitive.[9] In total, 84 combinations [7×6, each for +€100 and −€100] were presented to each subject in a random order. Subjects were also told that the change in an income recipient's income by ±€100 was only caused by some chance event beyond the control of any party.[10]

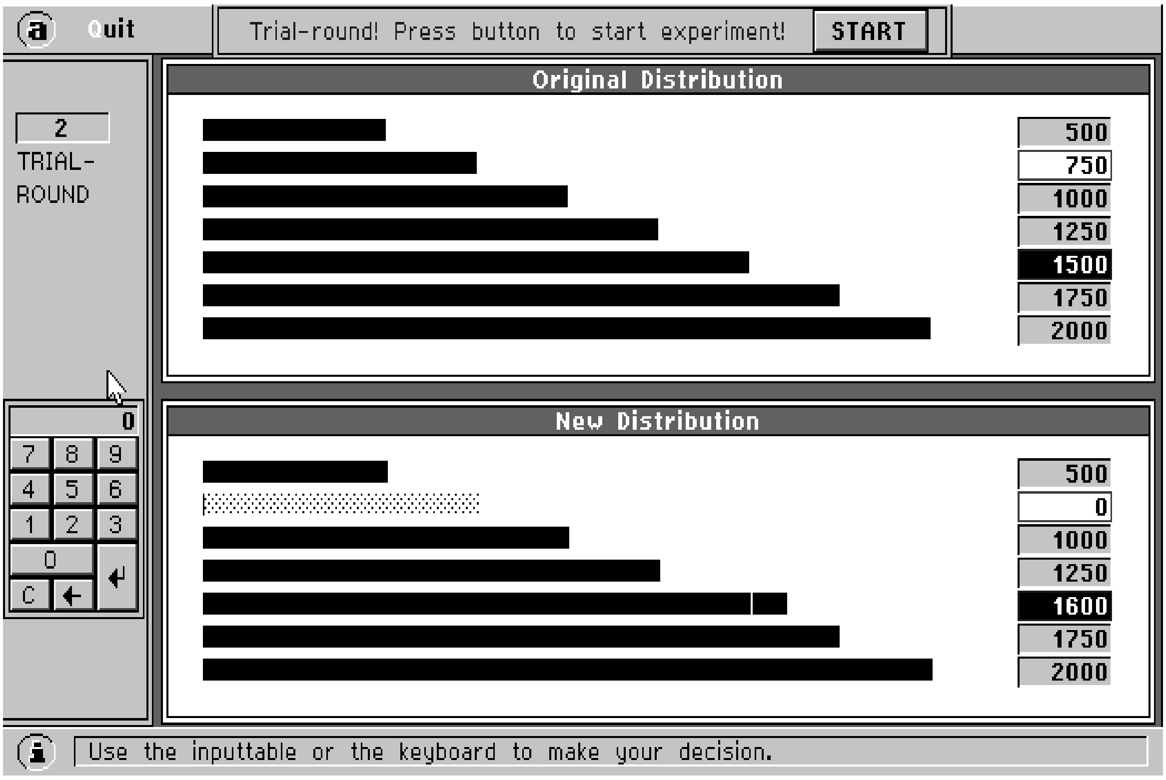


Figure 1: Screenshot

This experiment required much attention and effort on the part of the subjects. To induce subjects to participate in our experiment, we relied on their interest in the topic, since we did not know of an incentive-compatible payment régime that would not have biased subjects' responses.[11] We ended up with the data of 41 subjects.[12]

B The Numerical Benchmarks

Note from Theorem 7 that the benchmark is implicitly defined by setting the partial derivatives equal to zero. Now, the partial derivatives are functions of both the income distributions and the parameters of the respective income inequality measures applied. As the income distribution involved is given by our experimental design, the partial derivatives are functions of the parameters of the applied income inequality measures only. The graph of the income levels for which the partial derivatives assume the value zero for the respective parameter values of the income inequality measure is the graph of the benchmarks. For the computation of the graphs of the benchmarks we selected the most common income inequality measures, *viz.* the entropy income inequality measure, the extended Atkinson income inequality measure, and the extended Gini inequality measure.[13] The respective graphs of the benchmark functions are shown in Figure 2.

Figure 2 shows that the function of the benchmarks for the entropy inequality measure is S-shaped as a function of the parameter c, that the function of the benchmarks for the extended Atkinson inequality measure is ogival-shaped (counter-S-shaped) as a function of the parameter θ, and that the function of the income categories for the extended Gini inequality measure is decreasingly convex-shaped as a function of the parameter ν.

Inserting the seven incomes of our experimental design as benchmarks into the equations for the benchmarks (whose graphs are shown in Figure 2)[14] allows us, by monotonocity of the benchmark functions, to compute the associated parameter values for the seven incomes to become the benchmarks. They are summarized in Table 1.

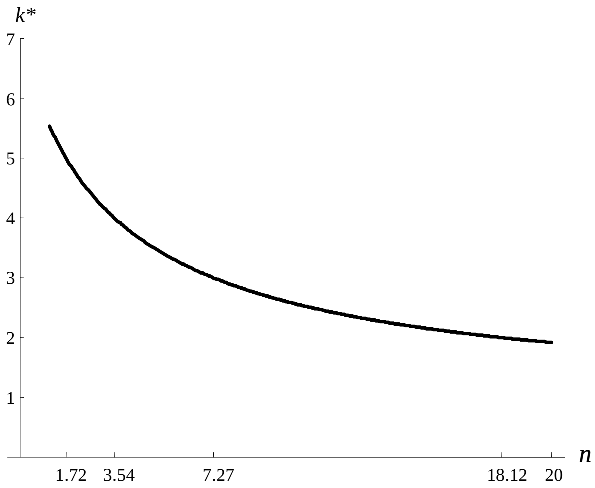
Measures	Income levels y_i						
	500	750	1000	1250	1500	1750	2000
Atkinson's θ	3.2×10^7	6.77	3.11	1	1.68	8.88	3.3×10^6
Gini's ν [b]	[a]	18.12	7.27	3.54	1.72	[a]	[a]
Entropy inc. inequ. c	-3.26×10^6	-5.77	-2.11	0	2.68	9.88	3.2×10^6

[a] Not defined.

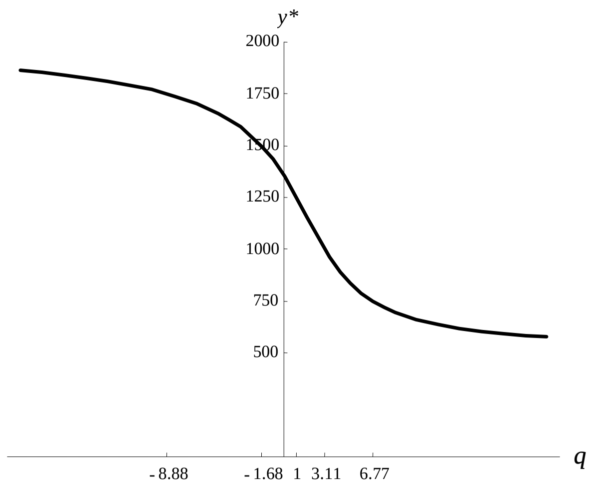
[b] Rank positions converted into income levels.

Table 1: Zero Positions of Partial Derivatives Evaluated at $y_i,\ i = 1, \ldots, 7$.

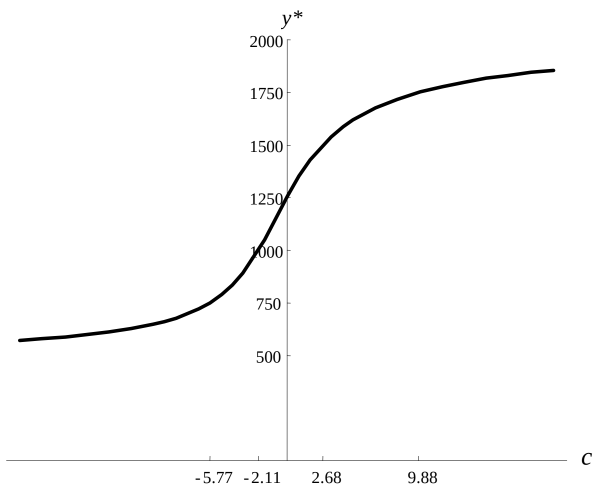
Table 1 shows us that only benchmarks between €750 and €1500 are associated with reasonable values of the parameters of the most common income inequality measures. All other benchmarks require excessive parameter values of inequality attitudes, which were never evidenced in empirical research. Therefore, we can restrict the benchmarks to lie in the open interval (€750, €1500).



(a) Extended Gini Inequality Measure



(b) Atkinson Inequality Measure



(c) Entropy Inequality Measure

Figure 2: Benchmarks for the Different Inequality Measures

III Results

Recall that we asked our subjects how the 84 income values of the y_k's should be adapted. These data allowed us to compute the γ's. They represent the data set of our analysis.

Summarizing all cases of Theorem 10, the theoretically correct sign of γ is determined by

$$\text{sign}\,\gamma \doteq \text{sign}\left(\left[|y_j - y_k| - \max\left\{|y_j - y^*|, |y_k - y^*|\right\}\right] \times \delta\right). \tag{1}$$

This formula means that, if y_j and y_k are on the *same* side of the benchmark, then $|y_j - y_k| < \max\{|y_j - y^*|, |y_k - y^*|\}$, and γ should be negative for $\delta > 0$ and positive for $\delta < 0$. If y_j and y_k are on *opposite* sides of the benchmark, then $|y_j - y_k| > \max\{|y_j - y^*|, |y_k - y^*|\}$ and γ should be positive for $\delta > 0$ and negative for $\delta < 0$.

Notice that we had no direct data on subjects' benchmarks. As our subjects did not know Theorem 10, they would not have understood queries for their benchmark. Thus, we were left to the message of Table 1, demonstrating that values of benchmarks outside the interval (€750, €1500) are extremely unlikely. This allowed us to assume for all benchmarks the condition €750 $< y^* <$ €1500. Given this assumption, we could partition our stimulus set into three subsets:

(a) *Same side:* The incomes involved lie certainly on the *same side* of the benchmark. This covers the stimuli $(y_i, y_k) \in$ {(500,750), (750,500), (1500,1750), (1750,1500), (1500,2000), (2000,1500), (1750,2000), (2000,1750)}, both for $\delta > 0$ and $\delta < 0$, *i.e.*, 16 stimuli.

(b) *Opposite sides:* The incomes involved lie certainly on *opposite sides* of the benchmark. This covers the stimuli $(y_i, y_k) \in$ {(500,1500), (1500,500), (500,1750), (1750,500), (500,2000), (2000,500), (750,1500), (1500,750), (750,1750), (1750,750), (750,2000), (2000,750)}, both for $\delta > 0$ and $\delta < 0$, *i.e.*, 24 stimuli.

(c) *Benchmark?:* The remaining 44 stimuli, for which there was *no a priori reasoning* on which side of the benchmark the incomes involved lie.

Descriptive statistics of our results are provided in Tables 2 and 3.

Stimulus subsets	$\delta > 0$				$\delta < 0$			
	$y_j < y_k$		$y_j > y_k$		$y_j < y_k$		$y_j > y_k$	
	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
(a) Same side	18.64	97.54	45.66	86.23	-56.37	101.15	-2.52	84.42
(b) Opposite sides	39.75	121.53	72.63	84.80	-94.42	118.85	12.76	90.36
(c) Benchmark?	25.34	109.97	54.16	85.81	-52.53	96.10	2.37	87.89
Aggregate	28.19	111.36	57.81	86.07	-65.19	105.52	4.40	88.09

Table 2: Means and Standard Deviations for the Values of γ.

According to the leaky-bucket theory, we should have observed for the subset (a) negative γ's for $\delta > 0$ and positive γ's for $\delta < 0$. However, Table 2 shows us that the opposite holds for the mean value of the γ's. For the subset (b) we should have observed positive γ's for $\delta > 0$ and negative γ's for $\delta < 0$. Table 2 demonstrates that this is evidenced by the data for the mean value of the γ's, except for the case $\delta < 0$, $y_j > y_k$. The subset (c) covers the case for which we cannot rely on a priori reasoning about the benchmarks. Provided that subjects' benchmarks are symmetrically distributed around €1250, the leaky-bucket theory would demand a mean value of the γ's near zero. However, Table 2 exhibits positive mean values of the γ's for $\delta > 0$, and negative mean values of the γ's for $\delta < 0$. Whereas the mean values of γ are significantly different from zero for $\delta > 0$, the mean value of γ is significantly different from zero for $\delta < 0$ and $y_j < y_k$ only. For $\delta < 0$ and $y_j > y_k$ a Wilcoxon test shows that the mean value of γ is not significantly different from zero. With the exception of this latter case, the overall results confirm that positive γ's dominate for $\delta > 0$, and negative γ's for $\delta < 0$.

Stimulus subsets		$\delta > 0$						$\delta < 0$					
		$y_j < y_k$			$y_j > y_k$			$y_j < y_k$			$y_j > y_k$		
		pos.	neg.	zero	pos.	neg.	zero	pos.	neg.	zero	pos.	neg.	zero
(a) Same side	n	95	59	10	115	39	10	44	112	8	59	95	10
	%	57.92	35.97	6.09	70.12	23.78	6.09	26.83	68.29	4.88	35.98	57.93	6.09
(b) Opposite sides	n	148	77	21	205	27	14	45	192	8	101	125	20
	%	60.16	31.31	8.53	83.33	10.98	5.69	18.37	78.37	3.26	41.06	50.81	8.13
(c) Benchmark?	n	274	142	35	342	90	18	105	328	18	183	232	36
	%	60.75	31.49	7.76	76.0	20.0	4.0	23.28	72.73	3.99	40.58	51.44	7.99
Aggregate	n	517	278	66	662	156	42	194	632	34	343	452	66
	%	60.05	32.29	7.67	76.98	18.14	4.88	22.56	73.49	3.95	39.84	52.50	7.67

Table 3: Absolute Numbers and Percentages of the γ's.

In Table 3, we present the absolute numbers and the percentages of positive, negative, and zero values of the γ's for the four basic cases for the subsets (a), (b), and (c). These results reinforce the behavioral pattern observed in Table 2. They show the preponderance of positive income compensations for income gains ($\delta > 0$) and negative income compensations for income losses ($\delta < 0$). Broadly speaking, subjects' perceptions to maintain the degree of income inequality seem to follow a simple precept: If someone gains income, the other person involved should be positively compensated, and if someone loses income, the other person involved in the transfer should be negatively compensated. This expresses a sort of *compensating justice* rather than restoration of former degree of income inequality, according to the leaky-bucket theory as explained in Theorem 10. Whenever the leaky-bucket theory coincides with the compensating-justice hypothesis, then it is empirically evidenced (*e.g.*, for incomes lying on opposite sides of the benchmark), otherwise it is empirically declined.

Moreover, Tables 2 and 3 reveal a particular pattern of compensating justice:

(i) When an income recipient receives an extra income of €100 ($\delta > 0$), our subjects hold that the other income recipient should also receive an extra income ($\gamma > 0$) in order to maintain the degree of income inequality. However, when the other income recipient is poorer, he or she should receive considerably more extra income than when he or she is richer.

(ii) When an income recipient experiences an income loss of €100 ($\delta < 0$), our data show that, when the other income recipient is richer, he or she should also experience an income loss ($\gamma < 0$). However, when the other income recipient is poorer, our subjects hold that his or her income should stay put in order to maintain the degree of income inequality.

When screening the data for the various stimuli, these tendencies become more pronounced the poorer or richer the involved income recipients are.

More formally, let $\mid \delta \mid < \min_{i,\ell \in \{1,\dots,n\}} \mid y_i - y_\ell \mid$, let γ_k and γ_i denote the γ's for income recipients k and i, respectively, and let $y_j \to y_j + \delta$. Then we have for $\delta > 0$ and $k < j < i$ that $\gamma_k > \gamma_i > 0$; moreover, the γ_k's increase and the γ_i's decrease as j increases. For $\delta < 0$ and $k < j < i$ we have: $\gamma_k > \gamma_i$, $\gamma_i < 0$, and both the γ_k's and the γ_i's increase as j

increases (for this case the γ_k's are not necessarily negative). When the primary changes in income move to the extremes of our experimental income distribution, this tendency becomes less clear-cut.

In order to capture these qualitative findings in quantitative terms, we used a logarithmic equation. We finally settled on this functional form because all parameters with the exception of one are significant at the 10% significance level.[15]

$$(2) \qquad \gamma^g_{st} = \alpha^g + \beta^g(\ln y_j - \ln y_k)_t + \varepsilon^g_{st}, \quad s = 1, \ldots, 41;\ t = 1, \ldots, 21$$

using the panel-data estimation method with random effects for the four categories $g = 1, \ldots, 4$ resulting from $\delta \gtrless 0$ and $y_j \lesseqgtr y_k$. The results are presented in Table 4 (bold-faced coefficients are significant at the 10 per cent level).

Cases		α	β
$\delta > 0$	$y_j < y_k$	12.1203 (0.347)	**-26.883** (0.001)
	$y_j > y_k$	**34.441** (0.000)	**39.097** (0.000)
$\delta < 0$	$y_j < y_k$	**-29.978** (0.012)	**59.304** (0.000)
	$y_j > y_k$	**-18.7177** (0.065)	**38.787** (0.000)

Table 4: Simple Compensating Justice [equation (2)]
(p-values in parentheses).

This table shows us that the distance of the logs of the incomes has a particularly pronounced impact on γ, when either the richer income recipient gains δ, or when the poorer income recipient loses δ. But the general tendency of compensating justice is also present for the other two cases. For instance, the coefficient estimates for Equation (2) show that, when an income recipient with an income of €1000 [€1250] gains €100, then an income recipient with an income of €500 should be compensated with €61.54 [€70.27], and an income recipient with an income of €2000 should be compensated with €30.75 [€24.76]. (Note that the additive constant of the last figures is nonsignificant.) When an income recipient with an income of €1000 [€1250] loses €100, then an income recipient with an income of €500 should gain €8.17 [€16.82], whereas an income recipient with an income of €2000 should lose €62.09 [€57.85].

Summarizing the results of our leaky–bucket experiment, we found that the leaky–bucket Theorem 10 lacks empirical support. Rather compensating justice holds (Tables 2 and 3). Only when the leaky-bucket theory concurs with the compensating-justice hypothesis, it enjoys empirical support. This holds only for transfers between income recipients whose incomes lie on opposite sides of the benchmarks (*i.e.*, the last line of the table in Theorem 10). For transfers between income recipients on the same side of the benchmark the leaky-bucket theory is empirically not supported. Compensating justice proves to be the better hypothesis to describe subjects' behavior in case of transfers with transaction costs.

IV Conclusion

During the last 110 years, admirable advances have been made in the fields of theoretical and empirical research of income inequality measurement, and the related field of concentration measurement.[16] Comprehensive information on this research can be gained from a great number of excellent surveys and textbooks.[17] Yet it is only a bit more than a decade since disillusion with the popular acceptance of central axioms of income inequality measurement began to undermine faith in the validity of inequality measurement. A number of questionnaire and experimental studies showed poor acceptance of central distributional axioms such as scale invariance, the income equalizing effects of income translations, the population principle, Pareto-dominance, Lorenz-dominance, and the transfer principle.[18] Even for the simplest experimental designs in terms of numbers, the acceptance rates of these axioms hardly exceed some 40%. This seems to be caused by response-mode effects: if the axioms are presented in verbal form, agreement rises up to some 60%.[19] Subjects seem to have difficulties in transforming verbal convictions into numbers.

The theoretical analysis of leaky-bucket transactions, which can be seen as a generalization of the transfer principle with transaction costs, is of recent origin (Seidl (2001); Hoffmann (2001); Lambert and Lanza (2006)). It has opened up new avenues of analysis. Rather than tracing out the maximum leakage of transaction costs such that a transfer still "pays at the margin",[20] the theory has shown a plethora of possible results. In Theorem 10 we showed that leaky-bucket transactions encompass twelve cases each of which entails different results. Only one of them covers the traditional case of transfers which allows the transferee to receive a positive fraction of the transfer taken from a richer transferor.

Our experimental research showed that leaky-bucket theory is poorly evidenced by the data. Subjects rather follow some notion of compensating justice: If an income recipient loses income, the other income recipient involved should be negatively compensated, and if an income recipient gains income, the other income recipient involved should be positively compensated. Whenever the leaky-bucket theory coincides with the compensating-justice hypothesis, then it is confirmed (*e.g.*, for incomes lying on opposite sides of the benchmark), otherwise it is declined.

Our central finding is that compensating justice asks for a higher compensation for the poorer income recipient (as compared to richer income recipients) in the case of income gains, and a lower loss (or even a small gain) for the poorer income recipient (as compared to richer income recipients) in the case of income losses. When screening the data for the various stimuli, these tendencies become more pronounced the poorer or richer the involved income recipients are.

The compensating-justice hypothesis, whose empirical validity was demonstrated in this paper, is at variance with the transfer principle. Moreover, any income inequality measure which satisfies *leaky-bucket consistency* (that is, the transaction costs associated with a progressive transfer which maintains the degree of income inequality must not exceed the amount of the transfer itself) violates the transfer principle, too. Although compensating justice captures subjects' inequality perceptions for transfers with transaction costs much better than leaky-bucket theory does, it cannot be expressed by an income inequality measure except by constant income inequality measures. For the respective impossibility theorems and a possibility theorem see Lasso de la Vega and Seidl (2007).

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Notes

[1]This case was termed the leaky-bucket paradox by Seidl (2001).

[2]These beliefs were entertained, *e.g.*, by Atkinson (1970, p. 5), Okun (1975, pp. 91–95), Jenkins (1991, pp. 28–29), and Amiel et al. (1999, pp. 87–89).—Notice the difference from the pure transfer principle: whereas the pure transfer principle focuses on *changes* in the degree of income inequality, the transfer principle with leakages focuses on the *maintenance* of the degree of income inequality.

[3]For related work see Amiel et al. (1999) and Beckman et al. (2003a; 2003b).

[4]Even the seminal theoretical paper of Lambert and Lanza (2006), which treats leaky buckets in terms of inequality measures exclusively, focuses on progressive transfers only, which covers just three out of twelve possible cases. In contrast to that, Hoffmann (2001, pp. 239-240) focuses on income increases of a particular income recipient and on regressive transfers.

[5]Therefore, we allow only for strictly positive incomes, which are different for different income recipients. For the general theory see Lambert and Lanza (2006).

[6]The reader might wonder why we do not confine the range of income inequality measures to the unit interval. Yet there are some recognized income inequality measures which do not satisfy this condition, most notably the entropy income inequality measure.

[7]This is evidenced by the widespread empirical rejection of the transfer principle; see Amiel and Cowell (1992; 1994a; 1998; 1999b, p. 118; 2000) and Harrison and Seidl (1994a,b).

[8]In the software they could enter any integer number between zero and 2,200.

[9]Already before embarking on the phase of real data collection, subjects were invited to trial plays in order to become fully acquainted with the program. Only when they pressed a “start”-button did the actual experiment begin. Moreover, an experimenter was present all the time who could be addressed in case of questions or problems.

[10]Other experimenters, *e.g.*, Amiel et al. (1999, pp. 94-95), stated explicitly in their instructions that some amount was *taken* from a person and ask for the minimum amount that should be *given* to some

other person to make the transfer worthwhile. This addressed an act of redistribution and appealed to subjects' desire for transfers or feelings of social envy rather than to their perception of the degree of income inequality. We were only interested in the latter aspect.

[11]It is noteworthy that subjects were not rolled in for the experiment, but individually showed up for the experiment in the laboratory at any time during one calendar week. They knew beforehand that they were not going to receive any financial or non-financial reward for their participation in the experiment. On average they took one hour to complete all questions. In fact, they were allowed to leave without completion, but only one subject did so.

[12]Two typing errors were eliminated. Therefore the case numbers in the aggregate line of Table 3 sum to 861 for $\delta > 0$, $y_j < y_k$ and for $\delta < 0$, $y_j > y_k$, whereas they sum to 860 for $\delta > 0$, $y_j > y_k$ and for $\delta < 0$, $y_j < y_k$.

[13]Note that these common measures serve only as auxiliary means for the purpose of estimating realistic intervals for the benchmarks. This enables us to properly analyze our experimental data. The alternative of asking subjects directly for their benchmarks is not viable, because the existence and the working of the benchmarks was until recently not even known to the experts. Hence we could not assume that subjects were knowledgeable of the benchmarks. As the mathematics to show the graphics in Figure 2 and the values in Table 1 is extensive and serves only the purpose of a proper calibration of the experimental benchmarks, we have omitted it from this presentation. It is presented in the Appendix.

[14]See the preceeding footnote.

[15]We tried also some other equations, *e.g.*, depending on the differences of the plain incomes, or using dummies for the income position of the income recipient whose income is changed by δ. However, no substantial differences in the results were observed. Therefore, we settled on equation (2), which allows for an immediate interpretation and intuition. Other estimates are provided in the Appendix.

[16]Cf., *e.g.*, Pareto (1895), Lorenz (1905), Gini (1912; 1914), Dalton (1920), Bonferroni (1930), Herfindahl (1950), Champernowne (1952; 1974), Amato (1968), David (1968), Kolm (1969), Atkinson (1970), Piesch (1975), Fishburn and Willig (1984), Ok (1995).

[17]Cf., *e.g.*, Cowell (1977; 2000), Nygård and Sandström (1981), Kanbur (1984), Foster (1985), Lambert (1989), Chakravarty (1990), Jenkins (1991), Champernowne and Cowell (1998), Silber (1999).

[18]Cf. Amiel and Cowell (1992; 1994a,b; 1998; 1999a,b; 2000); Ballano and Ruiz-Castillo (1993); Harrison and Seidl (1994a,b); Bernasconi (2002); Traub et al. (2007); Camacho-Cuena and Seidl (2007).

[19]Our experiment was carried out in terms of numbers. It did not seem viable to us to ask respective questions in verbal form because of the greater intricacy of leaky-bucket transactions.

[20]See, *e.g.*, Atkinson (1970, p. 5), Okun (1975, pp. 91–95), Jenkins (1991, pp. 28–29), and Amiel et al. (1999, pp. 87–89).

LEAKY BUCKETS VERSUS COMPENSATING JUSTICE: AN EXPERIMENTAL INVESTIGATION

Income Inequality Measures to Calibrate the Benchmarks

by Eva Camacho-Cuena, Tibor Neugebauer, and Christian Seidl

To interpret our data in the main paper, we use three common income inequality measures, *viz.* entropy[1], extended Atkinson[2], and extended Gini[3].

Note that these common measures serve only as auxiliary means for the purpose of estimating realistic intervals for the benchmarks. This enables us to properly analyze our experimental data. The alternative of asking subjects directly for their benchmarks is not viable, because the existence and the working of the benchmarks was until recently not even known to the experts. Hence we could not assume that subjects were knowledgeable of the benchmarks.

Income inequality measures may either process incomes only in increasing (or decreasing) order, or may process them in any order. The former ones are called *positional* income inequality measures, the latter ones are called *nonpositional* income inequality measures.[4] We shall focus on two families of nonpositional inequality measures, *viz.* the entropy class and the extended Atkinson class of income inequality measures, and on the extended Gini class for positional income inequality measures.

Definition 11: The entropy class of income inequality measures is defined as:[5]

$$\begin{aligned} I_{Ec}(y) &= \tfrac{1}{c(c-1)}\left[\tfrac{1}{n}\sum_{i=1}^{n}\left(\tfrac{y_i}{\mu}\right)^c - 1\right] && \text{for } c \neq 0,1; \\ I_{E1}(y) &= \tfrac{1}{n}\sum_{i=1}^{n}\tfrac{y_i}{\mu}\ln\left(\tfrac{y_i}{\mu}\right) && \text{for } c = 1; \\ I_{E0}(y) &= \tfrac{1}{n}\sum_{i=1}^{n}\ln\left(\tfrac{\mu}{y_i}\right) && \text{for } c = 0; \end{aligned}$$

where μ denotes the mean income.

Definition 12: The extended Atkinson class of income inequality measures is defined as:

$$\begin{aligned} I_{A\theta}(y) &= 1 - \left[\tfrac{1}{n}\sum_{i=1}^{n}\left(\tfrac{y_i}{\mu}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} && \text{for } \theta > 0,\ \theta \neq 1; \\ I_{A1}(y) &= 1 - \left[\prod_{i=1}^{n}\left(\tfrac{y_i}{\mu}\right)\right]^{\frac{1}{n}} && \text{for } \theta = 1; \\ I_{A0}(y) &= 1 - \prod_{i=1}^{n}\left(\tfrac{y_i}{\mu}\right)^{-\frac{y_i}{n\mu}} && \text{for } \theta = 0; \\ I_{A\theta}(y) &= 1 - \left[\tfrac{1}{n}\sum_{i=1}^{n}\left(\tfrac{y_i}{\mu}\right)^{1-\theta}\right]^{-\frac{1}{1-\theta}} && \text{for } \theta < 0. \end{aligned}$$

Remark 13: The plain Atkinson income inequality measure is defined as $I_{A\theta}(y) = 1 - \frac{\mu_\theta}{\mu}$ for $\theta > 0$, where μ_θ denotes the equally distributed equivalent income. In the Atkinson definition μ_θ is just the generalized mean of the standardized incomes with parameter $\theta > 0$. For negative θ the plain Atkinson income inequality measure is not defined for inequality aversion. One could associate it with inequality sympathy, which would, however, yield negative values for the inequality measure. An elegant mathematical escape from this difficulty is the extended Atkinson family as proposed by Lasso de la Vega and Urrutia (2006). They define the Atkinson income inequality measure for $\theta \leqslant 0$ as $I_{A\theta}(y) = 1 - \frac{\mu}{\mu_\theta}$.

The so defined extended Atkinson income inequality measure satisfies inequality aversion for all real values of θ and, thus, also the transfer principle. However, it does not satisfy transfer sensitivity for the branch $\theta \leqslant 0$; it shares this property with the entropy income inequality measure for the branch $c \geqslant 1$. Moreover, both income inequality measures are not invertible, that is they associate two parameter values with the same inequality indicator. Note that, contrary to the entropy inequality measure, the extended Atkinson income inequality measure has the advantage that its range is the unit interval. However, $I_{A\theta}(y),\ \theta \leqslant 0$, raises problems of economic intuition.[6]

Definition 14: The extended Gini class of income inequality measures is defined as:

$$I_{G\nu}(y) = \frac{1}{n}\sum_{i=1}^{n}\left\{1 + n\left[\left(\frac{n-i}{n}\right)^{\nu} - \left(\frac{n-i+1}{n}\right)^{\nu}\right]\right\}\frac{y_i}{\mu},\ \nu > 1.$$

For $\nu = 2$, the extended Gini measure becomes the orthodox Gini coefficient.

Definition 15: A general income inequality measure defined on Y, $I : \mathbb{R}^n_{++} \rightarrow \mathbb{R}_+$, is defined as

$$I(y) = f\left[\frac{1}{n}\sum_{i=1}^{n} w(i)u\left(\frac{y_i}{\mu}\right)\right],$$

where the $w(i)$'s denote some weights, and $u(\cdot)$ denotes the utility of income normalized for mean income.

Theorem 16: We have

(i) for nonpositional income inequality measures

$$\frac{\partial I}{\partial y_k} \gtreqless 0 \Leftrightarrow y_k \gtreqless y^* = \mu \times (u')^{-1}\left[\frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{\mu}u'\left(\frac{y_i}{\mu}\right)\right].$$

(ii) for positional income inequality measures:

$$\frac{\partial I}{\partial y_k} \gtreqless 0 \Leftrightarrow w(k) \gtreqless I(\cdot),$$

Proof: Lambert and Lanza (2006, pp. 255-256, 257, and 261).

Theorem 17: Let α stand for c and $1-\theta$, *i.e.*, $\alpha = c = 1-\theta$. Then the benchmarks for α coincide for the entropy and the extended Atkinson income inequality measures.

PROOF: From the definitions of the inequality measures it is immediate that

$$I_A(y) = \begin{cases} 1 - [1 - \alpha(1-\alpha)I_E(y)]^{\frac{1}{\alpha}}, & \alpha < 1,\ \alpha \neq 0; \\ 1 - e^{-I_E(y)}, & \alpha = 0; \\ I_A(y) = 1 - \frac{1}{I_E(y)}, & \alpha = 1; \\ I_A(y) = 1 - [1 - \alpha(1-\alpha)I_E(y)]^{-\frac{1}{\alpha}}, & \alpha > 1. \end{cases}$$

Differentiating both sides of these equations partially with respect to y and setting the partial derivatives equal to zero for $y = y^*$ shows

$$\left.\frac{\partial I_A(y)}{\partial y}\right|_{y=y^*} = 0 \quad \Leftrightarrow \quad \left.\frac{\partial I_E(y)}{\partial y}\right|_{y=y^*} = 0.$$

Q.E.D.

THEOREM 18: The benchmarks y^* are

(i) for the entropy and the extended Atkinson income inequality measures computed from $\left.\frac{\partial I}{\partial y}\right|_{y=y^*} = 0$:

$$y^* = \begin{cases} \mu & \text{for } \alpha = 0; \\ \mu \prod_{i=1}^{n} \left(\frac{y_i}{\mu}\right)^{\frac{y_i}{n\mu}} & \text{for } \alpha = 1; \\ \mu \left[\frac{1}{n}\sum_{i=1}^{n} \left(\frac{y_i}{\mu}\right)^{\alpha}\right]^{\frac{1}{\alpha-1}} & \text{for } \alpha \neq 0, 1; \end{cases}$$

(ii) for the extended Gini income inequality measure:

$y_k \leqslant y^* \leqslant y_{k+1}$ such that $w(k) \geqslant I_G(y) \geqslant w(k+1)$,
where $w(k) = 1 + n\left[\left(\frac{n-i}{n}\right)^{\nu} - \left(\frac{n-i+1}{n}\right)^{\nu}\right]$.

PROOF: Calculate $\frac{\partial I}{\partial y}\,|_{y=y^*} = 0$ for the entropy and the extended Atkinson income inequality measure as stated in Definitions 11 and 12 using the notation of Theorem 17. The proof for the extended Gini income inequality measure follows from Theorem 16 and Seidl (2001).

Q.E.D.

REMARK 19: Recall that $\alpha = c = 1 - \theta$. Thus, when plotting the benchmarks as functions of the conventional distributional parameters, the graphs are mirror images of each other. For instance, for the extended Atkinson income inequality measure, our experimental income distribution associates the benchmark of €1500 with the parameter value of $\theta = -1.68$. For the entropy income inequality measure, the benchmark of €1500 is associated with the parameter value $c = 2.68$. For the benchmark of €1000 we have $\theta = 3.11$ and $c = -2.11$. Although re-defining the distributional parameters[7] would show identical plots of the benchmarks, we decided to stick to the conventional parameters of the respective income inequality measures.

REMARK 20: The relationship between the entropy and the extended Atkinson income inequality measures is, however, nonlinear as the proof of Theorem 17 shows us. This implies that the values of these income inequality measures are different even if we define $c = 1 - \theta$. They convey different messages; otherwise we could dispense with one of them.

Theorems 16 and 18 show that the benchmarks are functions of *all* incomes and of the parameters of the income inequality measures. Instead of finitely many incomes one can alternatively work with continuous income distributions. Then the benchmarks for continuous income distributions are functions of the parameters of the density functions of the continuous income distributions and of the parameters of the income inequality measures [see Hoffmann (2001, pp. 245-248)]. As our paper focuses on an experiment with only seven incomes, we do not deal with continuous income distributions.

It is well known that income inequality measures have associated social welfare functions, $W : \mathbb{R}^n_{++} \to \mathbb{R}$. In general, a social welfare function is a decreasing function $F(\cdot)$ of an income inequality measure: $W(y) = F[I(y)]$. Hence, the social welfare function inherits the property of inequality aversion with reversed inequality signs.

As $F(\cdot)$ is arbitrary otherwise, any $I(y)$ defines a whole family of social welfare functions. Once the transformation $F(\cdot)$ has been agreed upon, the benchmark for the social welfare function is implicitly defined by

$$\left.\frac{\partial W}{\partial y}\right|_{y=y^*} = 0 \quad \Leftrightarrow \quad \left.\frac{\partial F}{\partial I}\frac{\partial I}{\partial y}\right|_{y=y^*} = 0.$$

A convenient way of formulating a social welfare function was proposed by Blackorby and Donaldson (1978, pp. 69-70). They suggest to first multiply an income inequality measure by (-1) to change its welfare implication from a decreasing to an increasing scale, then add 1 to normalize for the value of 1 for income equality, and finally multiply by the mean income μ to enter an efficiency component.[8] Following this approach, we have $W(y) = [1 - I(y)]\mu$, and

$$\left.\frac{\partial W}{\partial y}\right|_{y=y^*} = 0 \quad \Leftrightarrow \quad \left.\frac{\partial I}{\partial y}\right|_{y=y^*} = \frac{1 - I(y)}{n\mu}.$$

For income inequality measures whose values are confined to the unit interval, $\frac{1-I(y)}{n\mu} > 0$, and the benchmark of the associated social welfare function is, by Theorem 7, higher than the benchmark of the associated income inequality measure.

Because the class of admissible social welfare functions has the power infinity, and because we asked our subjects only for their income inequality perceptions, we did not consider social welfare functions in the main paper any further.

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Notes

[1]The entropy income inequality measures were developed by Bourguignon (1979), Cowell (1980), and Shorrocks (1980; 1984) to identify income inequality measures which are decomposable for different homogenous subgroups of income recipients. Decomposition occurs with respect to a within-group component (a weighted sum of the inequality indices of the subgroups), and a between-group component (the inequality measure evaluated by assigning every member of a subgroup the mean income of the respective subgroup). Entropy income inequality measures are, too, driven by an inequality-aversion parameter. They are generalizations of two non-parameterized income inequality measures originally proposed by Theil.

[2]This income inequality measure is based on the concept of the *equally distributed equivalent income*, which was foreshadowed by Champernowne (1952), and first formulated by Kolm (1969). Atkinson (1970) re-established this concept and made it the centerpiece of his famous income inequality measure. It was extended by Lasso de la Vega and Urrutia (2006).

[3]The Gini coefficient was introduced by Gini (1912; 1914). It can be expressed in several different ways. For a concise survey see, e.g., Anand (1982, appendix). The Gini coefficient may, akin to the Atkinson income inequality measure, be extended to include an inequality-aversion parameter. Pioneering work was done by Donaldson and Weymark (1980), Weymark (1981), Yitzhaki (1983), and Chakravarty (1988).

[4]When nonpositional income inequality measures satisfy symmetry, consequently the ordering of an income distribution becomes immaterial.

[5]I_{E1} is Theil's T inequality measure, I_{E0} is Theil's L inequality measure. For $c = 2$, $I_{E2} = \frac{1}{2}V^2$, where V is the coefficient of variation of the incomes.

[6]As $\mu_\theta > \mu$ for $\theta \leqslant 0$, inequality sympathy obtains and, therefore, $I_{A\theta}(y) = \frac{\mu_\theta - \mu}{\mu_\theta}$ can be interpreted as the relative monetary gain due to the prevailing unequal income distribution rather than as an indicator of the relative welfare loss in monetary terms due to income inequality. A related interpretation may concern the entropy inequality measure for the branch $c \geqslant 1$.

[7]This was, for instance, done by Lasso de la Vega and Urriarte (2006).

[8]For further work on the relationship between income inequality measures and social welfare functions see Dagum (1990; 1993).

LEAKY BUCKETS VERSUS COMPENSATING JUSTICE: AN EXPERIMENTAL INVESTIGATION

Alternative Estimates of Equation 2

by Eva Camacho-Cuena, Tibor Neugebauer, and Christian Seidl

In our paper we used a logarithmic equation to capture our qualitative findings in quantitative terms. We finally settled on this functional form because all parameters with the exception of one are significant at the 10% significance level. Alternatively, we used a functional form of the plain differences of the income levels of the income recipients involved. This is the equation:

$$(3) \qquad \gamma_{st}^{g} \;=\; \alpha^{g} + \beta^{g}(y_j - y_k)_t + \varepsilon_{st}^{g}, \quad s = 1, \ldots, 41;\; t = 1, \ldots, 21$$

using the panel-data estimation method with random effects for the four categories $g = 1, \ldots, 4$ resulting from $\delta \gtrless 0$ and $y_j \lesseqgtr y_k$. The results are presented in Table 5 (bold-faced coefficients are significant at the 10 per cent level).

Cases		α	β
$\delta > 0$	$y_j < y_k$	**23.16033** (0.076)	-0.0074954 (0.330)
	$y_j > y_k$	**38.59084** (0.000)	**0.0287423** (0.000)
$\delta < 0$	$y_j < y_k$	**-20.40951** (0.091)	**0.0674103** (0.000)
	$y_j > y_k$	-9.889199 (0.336)	**0.0214383** (0.000)

Table 5: Simple Compensating Justice [equation (3)] (p-values in parentheses).

This table shows us that the income distance has a much higher impact on γ, when either the richer income recipient gains δ, or when the poorer income recipient loses δ. For instance, the coefficient estimates for Equation (3) show that, when an income recipient gains €100, a poorer income recipient is compensated with €45.78 for the minimum distance of €250, and with €81.70 for the maximum distance of €1500. When an income recipient loses €100, a richer income recipient's income should be reduced by €37.26 for the minimum distance of €250, and by €121.52 for the maximum distance of €1500.

For the remaining two cases, the tendency of compensating justice is much weaker. When an income recipient is given €100, a richer income recipient should receive a small positive compensation. Notice that the coefficient of the distance is small and nonsignificant. When an income recipient loses €100, a poorer income recipient should only lose income if his or her income is no more distant than €250 (note again that the negative constant is nonsignificant).

Our qualitative results suggest that γ does not only depend on the *income distance*, but also on the *income position* of the income recipient whose income is changed by δ. This led us to estimate a modified version of Equation (3) which includes dummies representing the income levels of the income recipient whose income is changed by δ:

$$(4) \quad \gamma^g_{st} \;=\; \alpha^g + \beta^g_0(y_j - y_k)_t + \beta^g_1\Delta^g_{1t} + \beta^g_2\Delta^g_{2t} + \beta^g_3\Delta^g_{3t} + \beta^g_5\Delta^g_{5t} + \beta^g_6\Delta^g_{6t} + \beta^g_7\Delta^g_{7t} + \varepsilon^g_{st}\,,$$

$$s = 1, \ldots, 41;\; t = 1, \ldots, 21,$$

where $\Delta_i = 0$ for $i \neq j$ and $\Delta_i = 1$ for $i = j$. Equation (4) is calibrated for the median income $y_4 =$ €1250, which implies that $\Delta_4 = 0$. Using the panel-data estimation method, we estimated again four equations for $g = 1, \ldots, 4$ corresponding to the four categories resulting from $\delta \lesseqgtr 0$ and $y_j \lesseqgtr y_k$. In Table 6 we present the results (bold-faced coefficients are significant at the 10 per cent level).

We define the behavioral pattern as observed in Table 6 as *graded* compensating justice. It operates in this way:

- When a poorer income recipient j experiences an extra income of €100 (first line of Table 6), a richer income recipient k is first awarded a fixed amount of €19.68. This amount is diminished by 2.09% of the income difference between the two income recipients. The increase in income of the richer income recipient k is higher, the poorer the income recipient j is who gets €100. This obviously allows higher compensations to the richer income recipient. Thus, the values of the dummies decrease if better endowed income recipients receive €100. Notice that, although not all dummies are significant, their pattern conforms with the expectations of the compensating-justice hypothesis.

- When a richer income recipient j experiences an extra income of €100 (second line of Table 6), a poorer income recipient k should even be higher compensated than the richer income recipient in the former case. He or she is awarded a fixed amount of €37.89 plus 4.48% of the income difference between the two parties. If j's income is below €1,250, the poorer income recipient k gets an additional premium. For income recipients j with incomes above €1,250, the poorer subject k seems to perceive some overshooting due to the 4.48% of the income difference. This is corrected that by a negative amount which further decreases as the income position of the richer income recipient increases.

- When a poorer income recipient j experiences an income decrement of €100 (third line of Table 6), then, according to the hypothesis of compensating justice, the income of a richer income recipient k should also experience a loss. Table 6 shows that this income loss should be made up by a fixed amount of €18.22 plus 8.19% of the income difference. Overshooting due to the 8.19% of the income difference is counteracted by another amount for income recipients j with incomes below €1,250. For income recipients j with income exceeding €1,250, the income loss is reinforced.

- When a richer income recipient j experiences an income loss of €100 (fourth line of Table 6), a poorer income recipient k should experience an increase of income amounting to 4.25% of the income difference plus an amount which decreases as the income position of the richer income recipient increases. Calculations show that, with the exception of the lowest incomes among the poorer income recipients, this yields an income loss for the poorer income recipient, too.

Cases		α	β_0	β_1 $y_j = 500$	β_2 $y_j = 750$	β_3 $y_j = 1000$	β_5 $y_j = 1500$	β_6 $y_j = 1750$	β_7 $y_j = 2000$
$\delta > 0$	$y_j < y_k$	19.67573	**0.0208962**	**49.374013**	**31.18177**	11.87218	-7.486005	-14.44789	–
		(0.174)	(0.016)	(0.000)	(0.001)	(0.228)	(0.524)	(0.339)	–
	$y_j > y_k$	**37.89269**	**0.0448488**	–	**23.40732**	8.545121	-7.500397	**-20.3**	**-19.98902**
		(0.001)	(0.000)	–	(0.050)	(0.361)	(0.340)	(0.008)	(0.009)
$\delta < 0$	$y_j < y_k$	-18.22181	**0.0819304**	10.89055	**20.05978**	11.47097	-.07494253	**-52.58828**	–
		(0.173)	(0.000)	(0.210)	(0.021)	(0.196)	(0.944)	(0.000)	–
	$y_j > y_k$	-0.3070848	**0.0425491**	–	11.96249	-2.600058	**-24.44466**	**-35.76574**	**-37.82178**
		(0.979)	(0.000)	–	(0.305)	(0.777)	(0.001)	(0.000)	(0.000)

Table 6: Graded Compensating Justice [equation (4)] (p-values in parentheses).

The equivalent of Equation (4) with the logs of the incomes is Equation (5):

$$(5)\ \gamma^g_{st} \ = \ \alpha^g + \beta^g_0(\ln y_j - \ln y_k)_t + \beta^g_1\Delta^g_{1t} + \beta^g_2\Delta^g_{2t} + \beta^g_3\Delta^g_{3t} + \beta^g_5\Delta^g_{5t} + \beta^g_6\Delta^g_{6t} + \beta^g_7\Delta^g_{7t} + \varepsilon^g_{st}\,,$$

$$s = 1, \ldots, 41;\ t = 1, \ldots, 21,$$

where $\Delta_i = 0$ for $i \neq j$ and $\Delta_i = 1$ for $i = j$. Like Equation (4) it is also calibrated for the median income $y_4 =$ €1250, which implies that $\Delta_4 = 0$. Using the panel-data estimation method, we estimated again four equations for $g = 1, \ldots, 4$ corresponding to the four categories resulting from $\delta \lesseqgtr 0$ and $y_j \lesseqgtr y_k$. In Table (7) we present the results (bold-faced coefficients are significant at the 10 per cent level).

Table (7) shows us that using the logs of the incomes provides a slight improvement: we now observe 17 coefficients which are significant at the 10% level instead of 15. However, the gain is minor: whereas the coefficients in Table (6) perform better for the case $\delta > 0$ and $y_j > y_k$, the coefficients in Table (7) perform better for the case $\delta < 0$ and $y_j < y_k$.

Summarizing the results of our leaky–bucket experiment, we found that the leaky–bucket Theorem 10 lacks empirical support. Rather compensating justice holds (Tables 2 and 3). Only when the leaky-bucket theory concurs with the compensating-justice hypothesis, it enjoys empirical support. This holds only for transfers between income recipients whose incomes lie on opposite sides of the benchmarks (*i.e.*, the last line of the table in Theorem 10. For transfers between income recipients on the same side of the benchmark the leaky-bucket theory is empirically not supported. Compensating justice proves to be the better hypothesis to describe subjects' behavior in case of transfers with transaction costs.

Cases		α	β_0	β_1 $y_j = 500$	β_2 $y_j = 750$	β_3 $y_j = 1000$	β_5 $y_j = 1500$	β_6 $y_j = 1750$	β_7 $y_j = 2000$
$\delta > 0$	$y_j < y_k$	17.5969	**25.3923**	**57.4799**	**34.4478**	12.8339	-7.6336	-14.2024	–
		(0.221)	(0.035)	(0.000)	(0.001)	(0.197)	(0.517)	(0.349)	–
	$y_j > y_k$	**36.8519**	**42.6570**	–	18.3642	5.4844	-3.8104	**-12.499**	**-7.7987**
		(0.001)	(0.001)	–	(0.120)	(0.556)	(0.625)	(0.094)	(0.009)
$\delta < 0$	$y_j < y_k$	**-22.292**	**110.004**	**49.3771**	**36.3893**	**16.7120**	-2.4636	**-53.6739**	–
		(0.084)	(0.000)	(0.000)	(0.000)	(0.063)	(0.817)	(0.000)	–
	$y_j > y_k$	-3.4241	**44.3415**	–	7.7379	-5.2727	**-21.1125**	**-28.6746**	**-26.6766**
		(0.763)	(0.000)	–	(0.501)	(0.561)	(0.005)	(0.000)	(0.000)

Table 7: Graded Compensating Justice [equation (5)] (p-values in parentheses).

LEAKY BUCKETS VERSUS COMPENSATING JUSTICE: AN EXPERIMENTAL INVESTIGATION

Training of Subjects

by Eva Camacho-Cuena, Tibor Neugebauer, and Christian Seidl

We recruited subjects with different backgrounds. In order to provide a common basic understanding of income distributions, they underwent a training which should alert them to the problems of income inequality without influencing their opinions. They were just alerted that sometimes different views are possible. The motivation part illustrates just our considerations for the presentation of the respective problems. They were not disclosed to our subjects.

PROBLEM 1: Consider two income distributions:

$$A = [10, 20, 30, 50, 70, 80, 90]$$

$$B = [50, 50, 50, 50, 50, 50, 50]$$

QUESTION: Which of the two income distributions is more equally distributed?

MOTIVATION: Obviously B is more equally distributed than A, although some subjects might prefer to live in a society with income distribution A. Problem 1 was intended to convey the notion of “more equally distributed” versus “preferred” income distributions.

PROBLEM 2: Consider income distribution:

$$C = [100, 200, 300, 500, 700, 800, 900]$$

and compare it with B.

QUESTIONS:

(i) Which of the two income distributions is more equally distributed? [Obviously B.]

(ii) Which of the two income distributions generates more income for the economy? [Obviously C.]

(iii) Which of the two income distributions generates more welfare for the economy? [The answer depends on the subject’s social welfare function. For welfarist social welfare functions, C generates more welfare. If equality preferences enter the social welfare function, B might also emerge as generating higher welfare.]

(iv) Would you rather live in a society with income distributions B or C if your own income position will be later on determined by chance? [The answer depends on the subject’s distributional preferences and on his or her risk attitude.]

MOTIVATION: This problem should alert subjects that a different focus of evaluating income distributions may ask for different responses who apparently conflict, but are nevertheless mutually consistent.

PROBLEM 3: Consider three income distributions:

$$A = [10, 20, 30, 50, 70, 80, 90]$$

$$D = [10, 20, 30, 50, 70, 70, 90]$$

$$E = [10, 10, 30, 50, 70, 80, 90]$$

QUESTIONS:

(i) Is D more equally or more unequally distributed than A?

(ii) Is E more equally or more unequally distributed than A?

MOTIVATION: In D, as compared with A, the second richest person loses 10 monetary units. In E, as compared with A, the second poorest person loses 10 monetary units. There is not right or wrong answer; it is up to the view of the beholder whether D or E is more equally or more unequally distributed than A. This example was chosen to alert subjects to the situation that richer or poorer income recipients might lose income, which may be considered differently. As D has a move most probably above the benchmark and E below the benchmark, a subject bound to the transfer principle should consider D to be more unequally distributed and E to be more equally distributed than A.

PROBLEM 4: Consider three income distributions:

$$A = [10, 20, 30, 50, 70, 80, 90]$$

$$F = [10, 30, 30, 50, 70, 70, 90]$$

$$G = [20, 20, 30, 50, 70, 80, 80]$$

QUESTIONS:

(i) Is F more equally or more unequally distributed than A?

(ii) Is G more equally or more unequally distributed than A?

(iii) Is D more equally or more unequally distributed than F?

MOTIVATION: Problem 4 is an exercise in the transfer principle. F comes about from A if 10 monetary units are transferred from the second richest to the second poorest income recipient. G comes about from A if 10 monetary units are transferred from the richest to the poorest income recipient. G can come about from F in two ways: (i) the second poorest transfers 10 monetary units to the poorest income recipient, and the richest transfers 10 monetary units to the second richest income recipient; thus, we have two progressive transfers. (ii) The richest transfers 10 monetary units to the poorest income recipient, and the second poorest transfers 10 monetary units to the second richest income recipient; thus, we have a progressive and a regressive transfer. Hence, the transition from F to G can be seen in different ways. If procedure invariance holds, then both views should yield the same result. If procedure invariance is violated, both views can yield different results. Problem 4 was chosen to alert subjects to possibly different views of the same result.

PROBLEM 5: Consider three income distributions:

$$A = [10, 20, 30, 50, 70, 80, 90]$$
$$H = [5, 20, 30, 50, 70, 70, 90]$$
$$J = [15, 20, 30, 50, 70, 80, 80]$$

QUESTIONS:

(i) Is H more equally or more unequally distributed than A?

(ii) Is J more equally or more unequally distributed than A?

(iii) Is J more equally or more unequally distributed than H?

MOTIVATION: Again there is no right or wrong answer to these questions. H comes about from A by an income loss of 5 monetary units of the poorest income recipient and by an income loss of 10 monetary units of the second richest income recipient. This depicts income changes which point in the same direction and can serve as an example for graded compensating justice. J comes about from A by a gain of 5 monetary units of the poorest income recipient and a loss of 10 monetary units of the richest income recipient. This is a case which is at variance with graded compensating justice. J can come about from H in two ways: (i) the poorest income recipient receives 10 monetary units and the richest income recipient transfers 10 monetary units to the second richest income recipient. (ii) The richest income recipient transfers 10 monetary units to the poorest income recipient and the second richest income recipient receives 10 monetary units. If procedure invariance holds, then both views should yield the same result. If procedure invariance is violated, both views can yield different results.

LEAKY BUCKETS VERSUS COMPENSATING JUSTICE: AN EXPERIMENTAL INVESTIGATION – INSTRUCTIONS FOR THE EXPERIMENT
by Eva Camacho-Cuena, Tibor Neugebauer, and Christian Seidl

You are going to participate in an experiment in which your personal opinion is object of investigation. To avoid any external influence we would like to ask you to refrain from contacting other participants. Please take your decisions alone according to your discretion!

Do not hesitate to ask any question concerning these instructions. It is crucially important that you understand all details before you take any decision.

In the experiment your task is to compare two income distributions with each other. The first income distribution ("the original distribution") is given by the monthly income of 7 students as follows:

One student faces an income of	500 Euro,
a second student's income is	750 Euro,
a third student's income is	1000 Euro,
a fourth student's income is	1250 Euro,
a fifth student's income is	1500 Euro,
a sixth student's income is	1750 Euro,
a seventh student's income is	2000 Euro.

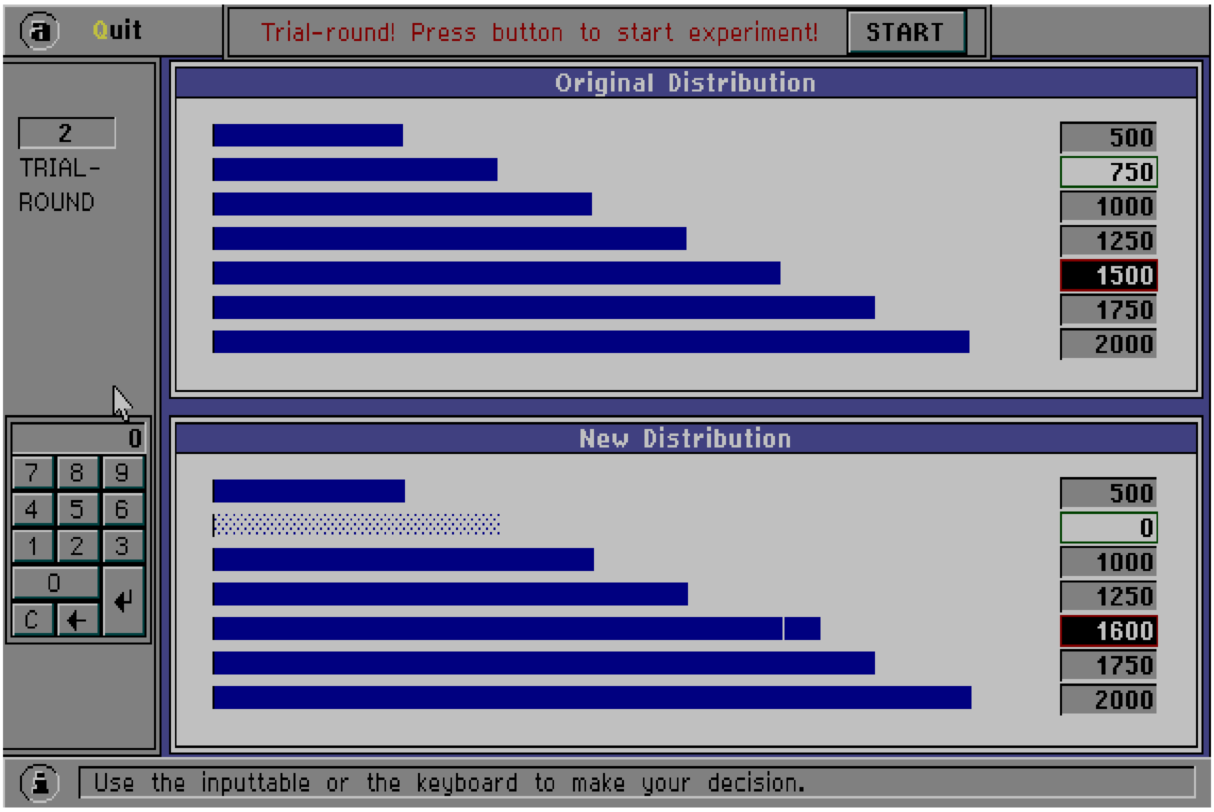
As you see, the students' incomes are very different. In other words, the income distribution exhibits a high degree of inequality.

The second income distribution ("new income distribution") is partly determined by you and by the computer. The computer picks the incomes of two students, the first of which is changed by 100 Euro. The student receives either 100 Euro more or less than in the original income distribution. You have to decide about the income to be assigned to the second student. On your screen this income is framed green on white background (green means "to be assigned"), the income of the first student is framed red on black background (red has the meaning "new income distribution").

Your task is one of determining the amount to be assigned to the second student such that the same degree of inequality of the original distribution is re-established in the new distribution. You surely understand that this task is a problem of (individual) opinion to which no objective solution can exist.

All your decisions you make alone at the computer. Before you start with the experiment you can run a trial of indeterminate length at the computer. As soon as you like to finish the trial and begin with the experiment you press the start button at the upper bound of your screen. During the trial run you may check whether you have understood the instructions entirely. Please ask the experimenter in case you have any doubts.

To insert your decision into the computer you have to choose an amount to be assigned to the second student by pressing the corresponding keys in the input table at the lower left bound of your screen and confirming the choice with the enter key (↵). Your entry will be displayed at the right side in the lower half of your screen within a window labelled *New Income Distribution*. It will be visualized through the length of blue bars in a histogram. As soon as you confirm any entry with the enter key the bar representing the income of the second student takes the corresponding length.



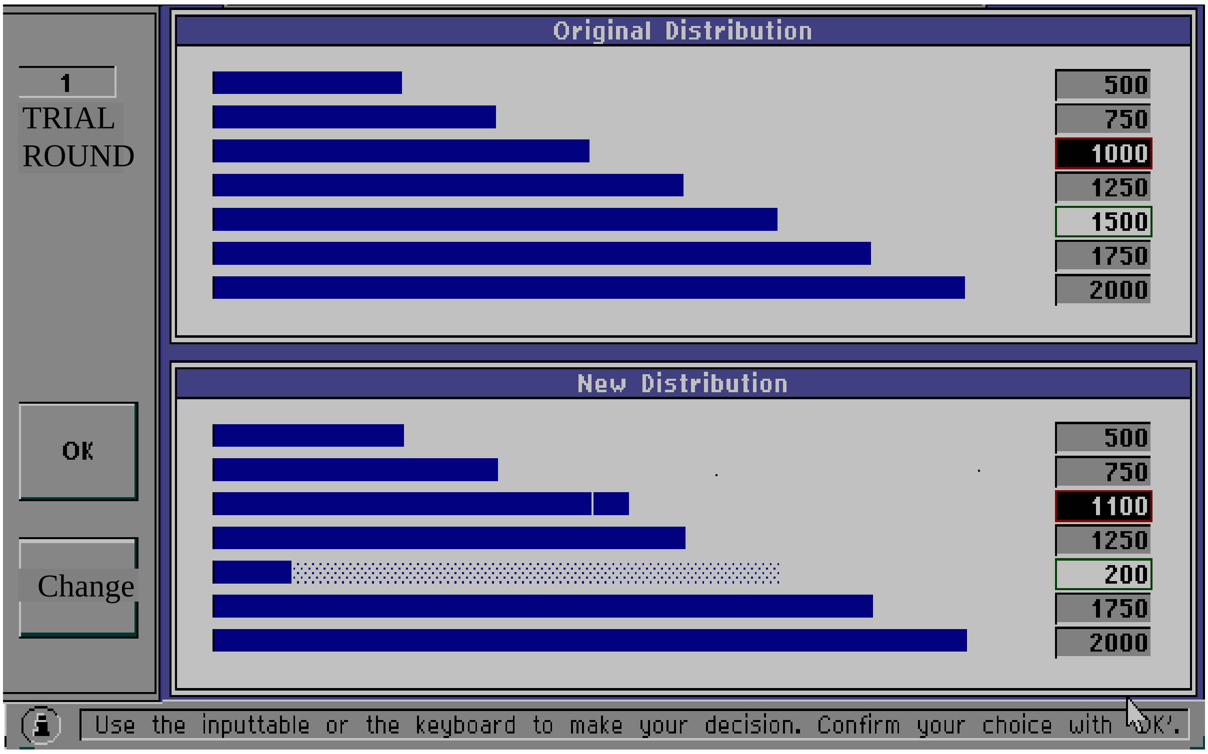
A blue bar represents the income of every student. If the income of one student differs between the new and the original income distribution the change will be visualized in the histogram. A white-blue bar represents a decrease in the new income distribution in comparison to the original one. If the income in the new income distribution is greater than in the original one, a white crossbar will indicate the original income in the histogram of the new income distribution.

To confirm your choice you have to press the button labelled “OK”. It will pop up on your display below the income table as soon as you have inserted a new income for the second student. Please press the button only if you are sure about your decision! -You will not be able to revoke your decision thereafter.- As long as you have not pressed the OK-button you can make a new entry anytime.

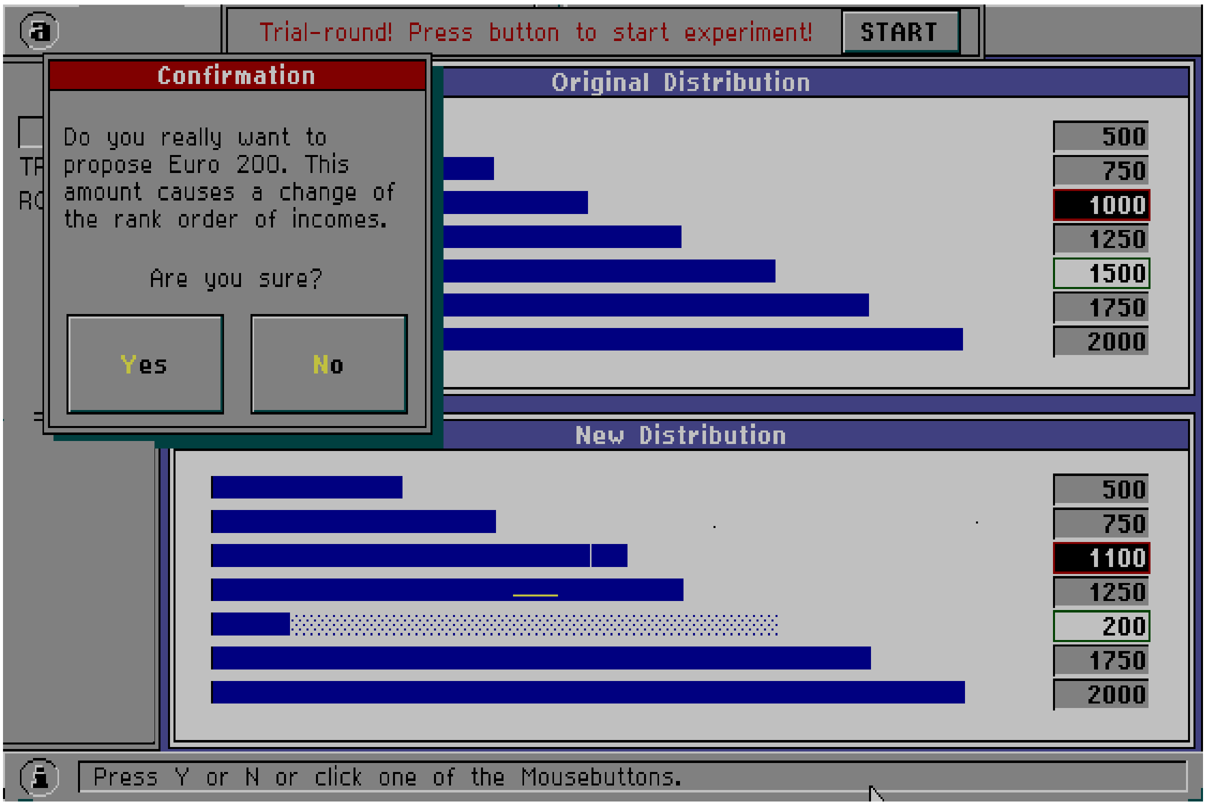
When you choose the income of the second student it is possible that the order of incomes within the students shifts. In other words, the second student can be assigned more (less) income than the students with greater (smaller) income in the original distribution. Shall you confirm such a choice with “OK” you will be prompted whether you are sure about it.

After having confirmed with OK you proceed to the next round by entering any key. Please take notice of the information given to you at the bottom line of the screen. There you are given instantaneous instructions about to do next.

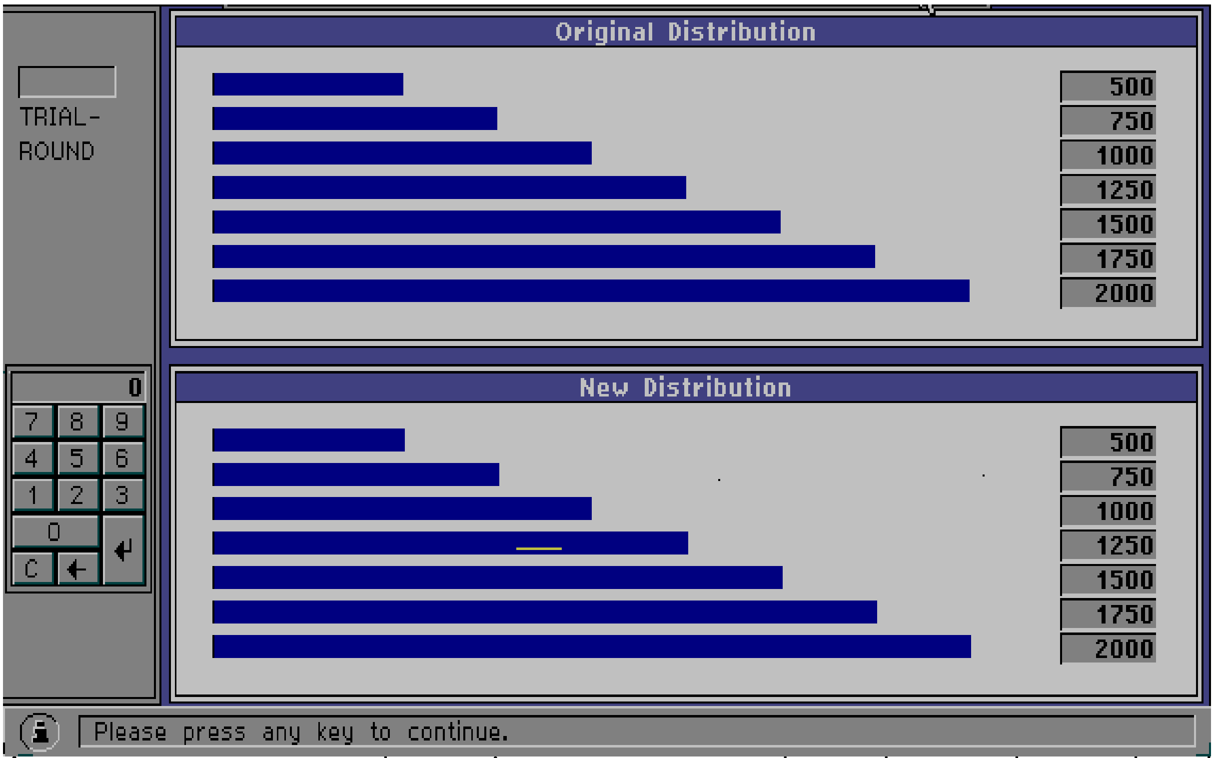
In total, the experiment consists of 84 rounds. That is, you must decide 84 times about what to assign to the second student to establish the same degree of inequality in both income distributions. As soon as you have reached the end please inform the experimenter.



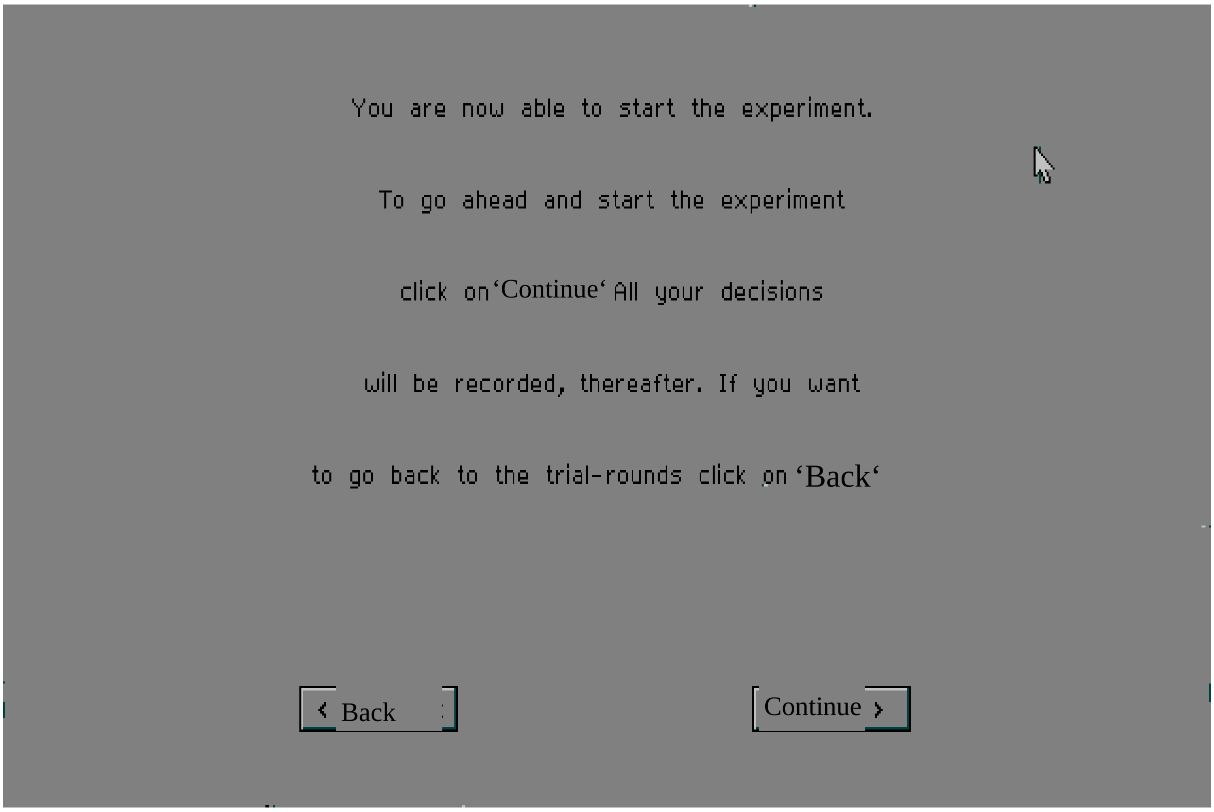
On the display you confirm your choice by pressing „OK“, or you decide to make another decision by pressing “Change”.



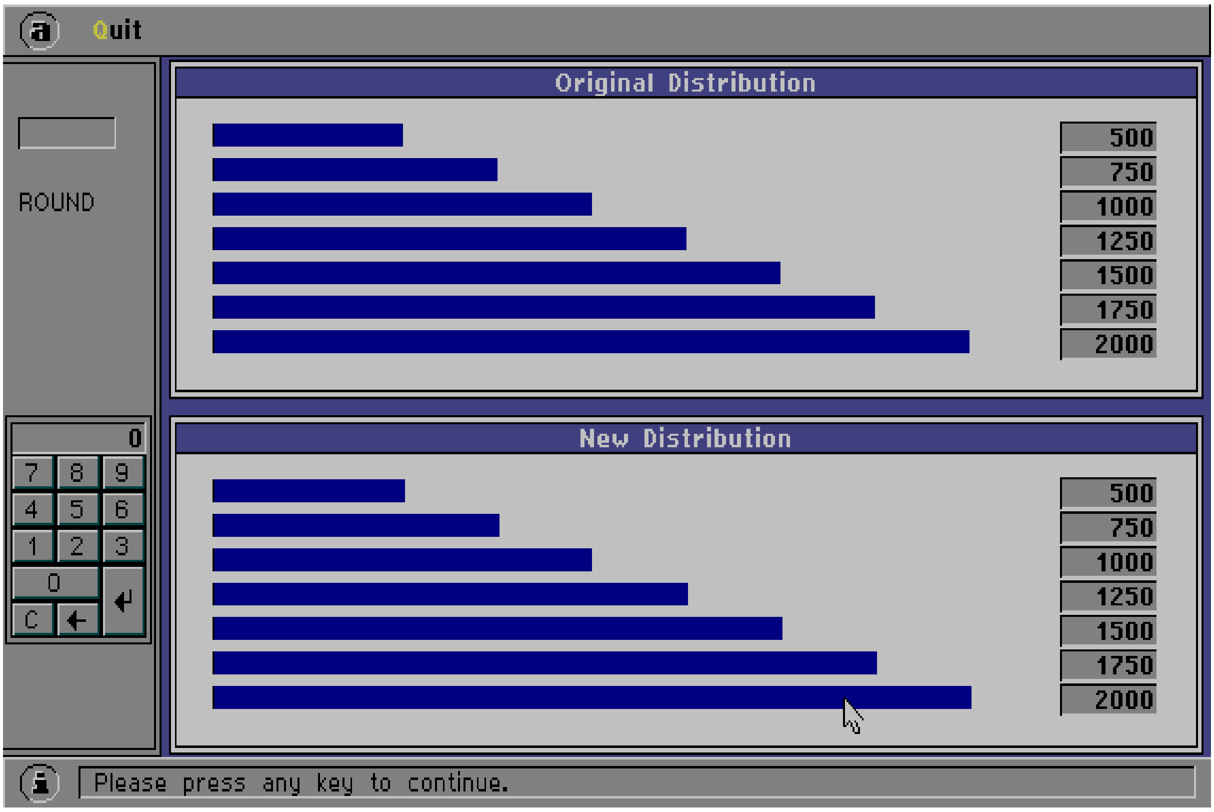
In this example, a confirmation induces a change in the ranks of the income distribution. The student who originally has the third highest income of 1500 Euro would receive 200 Euro in the new income distribution. In this case his income would be lower than the fourth highest income 1250. It is possible that you make a choice that affects the ranks of the income by mistake. Therefore, if your choice affects the ranks of the distribution you will be prompted on the screen. By a press on the button, you are able to change your choice or confirm it if you want to affect a change in the ranks of the income distribution.



As soon as you want to start the experiment you must press the start button at the upper bound of the computer screen (indicated in the figure by the mouse pointer).



As soon as you have pressed the start button, you will be asked for confirmation. You confirm by pressing "Continue >" or you abort by pressing "< Back" to return to the trial rounds.



As soon as you have confirmed your decision by „Continue >“, the first round of the experiment is going to start. Thereafter, all your decisions will be recorded.

Enjoy!