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Measuring inequality of well-being with a correlation-sensitive multidimensional Gini index*

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Abstract

We propose to measure inequality of well-being with a multidimensional generalization of the Gini coefficient. We derive two inequality indices from their underlying social evaluation functions. These functions are conceived as a double aggregation functions: one across the dimensions of well-being, and another across the individuals. They differ only with respect to the sequencing of aggregations. We argue that the sequencing that does not exclude the Gini index to be sensitive to the correlation between the dimensions is more attractive. We illustrate both Gini indices using Russian household data on three dimensions of well-being: expenditure, health and education.

Keywords: multidimensional inequality, single parameter Gini index, correlation increasing majorization, Russia.
JEL classification: D63, I31, O52.

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I. Introduction

Individual well-being is a multidimensional notion.[1] Indeed, individuals care about many different aspects of their lives, including their material standard of living, health and education. These non-monetary dimensions are neither freely tradable nor perfectly correlated with income. Yet, most analyses of inequality have confined themselves to the analysis of one sole dimension. If we want to take the multidimensionality of individual well-being seriously, we need to incorporate the various dimensions explicitly into the analysis of inequality. The question then becomes how best to do this.

A straightforward approach is to consider each relevant outcome separately, that is to study the evolution of inequality dimension by dimension (examples of this approach are by Atkinson et al. 2002, World Bank 2005, and Fahey et al. 2005 amongst others).[2] This method clearly goes beyond a sole focus on income and may provide additional insights. However, by construction it leads us to ignore the interrelationships and possible correlation between the dimensions of well-being. To neglect the interrelationships between these dimensions would be to abandon one of the primary motivations for a multidimensional approach to inequality (Atkinson and Bourguignon 1982).[3]

An example might help to clarify the problem we face. After the collapse of the Soviet Union, the Russian society underwent a fundamental transition from a centrally planned to a free market economy. Moreover, Russia was hit by a severe financial crisis in August 1998. We focus on two stylized facts caused by these dramatic events. First, income inequality followed an inverse U-shaped pattern, increased dramatically initially, peaking in 1998 and decreasing thereafter, though not yet reaching the pre-crisis level (Gorodnichenko et al. 2008). Over the same period, health inequality increased considerably (Blam and Kovalev 2006) as well as educational inequality (Smolentseva 2007). In other words, the evolution of inequality in the three dimensions diverged throughout the period. Second, with the transition to a market economy in Russia, access to social services increasingly hinges on households' ability to pay for them (Blam and Kovalev 2006, Lokshin and Ravallion 2008, Smolentseva 2007). Therefore, we witness an increase in the correlation between income, health and education. In this context, focusing on a sole

[1] See, for instance, Rawls (1971), Sen (1985) and Streeten (1994).

[2] In other words, the approach is to analyze inequalities in income and inequality in health separately. It should be noted that we refer here to inequality *within* each dimension – so that inequalities in health are understood in terms of differences between people in health status – and not to the covariation of health and income – sometimes referred to as the 'income gradient' – as in Wagstaff et al. (1991), Wagstaff (2002), Wilkinson (1996) or Lokshin and Ravallion (2008).

[3] In addition, when the dimensions are plenty or go in very different directions, it might prove thorny to attain a complete picture of overall inequality if following a dimension-by-dimension approach.

dimension or analyzing each dimension separately will lead to overlook important aspects of the multidimensional distribution of well-being in the country.

Our goal is to find a multidimensional inequality index that combines the information on inequality of the different dimensions while incorporating that on the correlation between them. In this essay, we present two such indices, both multidimensional generalizations of the Gini coefficient, and illustrate them using Russian data. While we argue that only one of these indices is best suited for the task, the contrasts between them reveal an interesting aspect of recent Russian economic history.

The one-dimensional Gini coefficient is probably the best known inequality index in economics. Apart from its interpretation related to the area above the Lorenz curve or as the sum of all pairwise distances between the individuals, the one-dimensional Gini coefficient can be obtained from a rank-dependent social evaluation function. Such function attaches welfare-weights to individuals that depend on their position in the total distribution. The present essay generalizes this latter approach to derive a multidimensional Gini coefficient from its underlying multidimensional rank-dependent social evaluation function.[4]

The multidimensional social evaluation function underlying the inequality indices presented in this essay combines two distinct aggregations in an explicit way: one aggregation across dimensions and one across individuals. The aggregation across dimensions of well-being leads to an index of well-being and depends on the preferences of the society (or ethical observer) over the relative importance of the different dimensions.[5] An essential feature of the aggregation across individuals is its rank-dependence. The sequencing of both aggregations leads to two alternative procedures (Dutta et al. 2003, Pattanaik et al. 2008, bring a similar point). In the first procedure, one first aggregates across all individuals in each dimension, resulting in a vector of dimension-specific summary statistics, which are then aggregated in the second step. The second procedure is its mirror-image and aggregates first across the dimensions to come to a well-being index for each individual, which are in a second step aggregated over all individuals. An example of the first procedure is the recent derivation of a multidimensional Gini index by Gajdos and

[4]For a recent survey on the normative approach to derive multidimensional inequality measures, the reader is referred to Weymark (2006). Measures of multidimensional inequality lead to a complete ordering, but require agreement about the set of underlying value judgements. An alternative approach, based on multidimensional stochastic dominance allows for some disagreement about the value judgements, but leads inevitably to an incomplete ordering, so that some societies can be ordered with respect to their well-being inequality, but others can not. This indecisiveness may be informative in its own right, but for policy purposes a complete ordering is more convenient. Therefore we focus in this paper on the derivation of a measure of well-being inequality. Trannoy (2006) provides a recent survey of the multidimensional dominance literature.

[5]The well-being function is thus common to all members of society and permits well-being judgements that are not purely subjective but interpersonally justifiable and comparable (Gaspart 1998). A potential concern with the 'objective' well-being approach is that it is overly paternalistic or perfectionist, since it represents the preferences of the external observer about what constitutes a good life for the individuals, and not the preferences of the individuals themselves (Fleurbaey 2005). On the impossibility of combining differences in individual preferences with multidimensional egalitarianism, see Fleurbaey and Trannoy (2003).

Weymark (2005). We will argue that the second procedure is more attractive, since it allows the resulting measure to comply with a multidimensional distributional concern about the correlation between the dimensions, which is not the case for the first procedure.[6] The importance of correlation between the dimensions in the analysis of multidimensional inequality has been suggested by Atkinson and Bourguignon (1982), Rietveld (1990), and Tsui (1999). Besides the normative approach, two other broad strategies have been followed to generalize the Gini coefficient into multiple dimensions, each extending an alternative one-dimensional definition of the Gini coefficient.[7] Koshevoy and Mosler (1996) introduced the *Lorenz zonoid* as an m-dimensional generalization of the standard Lorenz curve. From the volume of the Lorenz zonoid, a multidimensional Gini coefficient can be derived naturally (Koshevoy and Mosler 1997). An alternative strategy is followed by Arnold (1987), Koshevoy and Mosler (1997) and Anderson (2004), who extend the definition based on the sum of all distances between pairs of individuals. In particular, they propose a multidimensional *distance measure* to measure the pair-wise distances between the vectors of outcomes. Both approaches represent a mathematical or geometrical extension of the one-dimensional Gini coefficient. As such, they lack normative content in the sense that inequality cannot be readily interpreted in terms of welfare losses. In addition, the properties satisfied by these indices are not made explicit and are sometimes hard to unravel. We thus regard them as less attractive to measure well-being inequality. The rest of the essay is structured as follows. Section II surveys some attractive properties for the aggregation across individuals and dimensions. Depending on the sequencing of the aggregation two multidimensional Gini social evaluation functions are derived. Multidimensional distributional concerns are introduced in section III, paying special attention to a multidimensional generalization of the one-dimensional Pigou-Dalton transfer principle and the effect of changes in correlation between dimensions. Both multidimensional Gini social evaluation functions are compared with respect to their compliance to these distributional concerns. From the resulting social evaluation function a multidimensional single parameter Gini inequality index is derived in section IV. Section V illustrates the use of this index based on Russian household data. Section VI concludes the essay.

II. Two multidimensional S-Gini social evaluation functions

In this section we derive two alternative social evaluation functions to compare multidimensional distributions. Given the multidimensional setting, the social evaluation functions involve a double aggregation. One aggregation is over the dimensions of well-being and the other is across the

[6] In addition, Dutta et al. (2003) argue that this approach is more in line with the conceptual framework of welfare economics.

[7] Many alternative definitions and interpretations of the one-dimensional Gini coefficient exist in the literature. See Sen and Foster (1997), Anand (1983) and Yitzhaki (1998). Arnold (2005) gives a survey of the recent developments on multidimensional generalizations of the Gini coefficient. See also List (1999).

individuals. The two social evaluation functions presented will differ in the sequencing of both aggregations, but are equivalent in terms of the properties imposed to each aggregation. We present the two sequences of aggregation, the set of attractive properties, and the resulting social evaluation functions.

We assume that there are m relevant dimensions of well-being (e.g. income, health and educational outcomes) for a population of n individuals.[8] Each distribution matrix X in $\mathbb{R}^{n\times m}_{++}$ represents a particular distribution of the outcomes for the n individuals in the m dimensions. An element of the distribution matrix x^i_j denotes the outcome of individual i in dimension j. A row of matrix X refers to the outcomes of one individual and a column refers to the outcomes in one dimension. Distribution matrices can be compared by making use of a social evaluation function $W_{n\times m}$ that maps a positive $n \times m$ distribution matrix to the positive real line. As mentioned above, the social evaluation function carries out a double aggregation. The aggregation over the m dimensions (columns) of well-being will be performed by aggregation function W_m. It can be interpreted as an index of multidimensional well-being. The other aggregation, carried out by function W_n, is across the n individuals (rows) and can be interpreted as a standard one-dimensional social evaluation function. The sequencing of both aggregations will turn out to play a crucial role in the following.

Let us describe two procedures. In the first procedure, we first aggregate across the different individuals by making use of W_n in each dimension. This step obtains for each dimension a summary statistic and generates a single m-dimensional row vector. Then this row vector is aggregated using W_m. Kolm (1977) calls this first procedure a *specific* one. In the second procedure, the order of aggregation is reversed: the first step attaches to each individual a level of well-being and generates an n-dimensional column vector; in the second step this column vector is aggregated using W_n. Following Kolm (1977) this second procedure will be referred to as an *individualistic* one. The following diagram summarizes the social evaluation functions obtained by both procedures:

$$W^1_{n\times m} \quad : \quad \mathbb{R}^{n\times m}_{++} \to \mathbb{R}_{++} : X \longmapsto W^1_{n\times m}(X) = W_m\left[W_n(x_1), \ldots, W_n(x_m)\right],$$
$$W^2_{n\times m} \quad : \quad \mathbb{R}^{n\times m}_{++} \to \mathbb{R}_{++} : X \longmapsto W^2_{n\times m}(X) = W_n\left[W_m(x^1), \ldots, W_m(x^n)\right].$$

Both procedures split the complex multidimensional aggregation into two (easier) one-dimensional aggregations, which have been studied extensively in the literature before, see Ebert (1988) for an overview. We present and discuss a list of interesting properties for a generic one-dimensional

[8] Throughout the analysis, the number of dimensions m is assumed to be fixed, whereas we allow for variable population size n. We do not discuss which dimensions of well-being should be included, rather we assume that these are either obtained by a democratic process or given by philosophical reasoning like the primary goods defined by (Rawls 1971), the list of 'functionings' proposed by (Nussbaum 2000), or the basic needs approach advocated by (Streeten 1994).

aggregation function $W_k : \mathbb{R}^k_{++} \to \mathbb{R}_{++}$ that maps a positive vector $x = (x_1, \ldots, x_k)$ on the positive real line. The aggregation functions W_m and W_n are examples of such aggregation functions. The properties crystallize different value judgements on how the aggregation should be done. We do not claim that the set of properties laid down here is the only one possible, *au-contraire*, but we suggest that it represents an attractive set for the problem of measuring societal well-being. We restrict ourselves to continuous aggregation functions, so that the result is not overly sensitive to small changes in one of its entries, for instance caused by measurement errors.

Property 1. ***Monotonicity (MON)*** *For all x, y in $\mathbb{R}^k_{++}$: if $y > x$, then $W_k(y) > W_k(x)$.*[9]

Property 2. ***Symmetry (SYM)*** *For all x in $\mathbb{R}^k_{++}$, and all $k \times k$ permutation matrices P : $W_k(Px) = W_k(x)$.*

Property 3. ***Normalization (NORM)*** *For all $\lambda > 0 : W_k(\lambda 1_k) = \lambda$.*[10]

Monotonicity captures the intuition that all entries of x are desirable. If a vector is obtained by increasing at least one entry of another vector, it should be preferred to the initial one. Monotonicity is an attractive property for aggregation functions across dimensions as well as across individuals. Symmetry states that any information other than the quantities stated in the entries of x are unimportant in the aggregation. Symmetry is an attractive property in the aggregation across individuals since it assures an impartial treatment of all individuals. In the aggregation across dimensions, however, one might want to treat the dimensions differently to give priority to certain dimensions. Therefore we will not impose symmetry in the aggregation across dimensions. Normalization makes sure that whenever all entries of x are equal to λ, the result should be λ as well.

Let K be the set of all k entries and let L be a subset of K. The next property we introduce is separability of the subset L.

Property 4. ***Separability (SEP)*** *For all x, x', y, y' in $\mathbb{R}^k_{++}$: if there is an $L \subset K$ such that for all l in L $x_l = y_l$ and $x'_l = y'_l$ whereas for all k in $K \backslash L$, $x_k = x'_k$ and $y_k = y'_k$, then $W_k(x) > W_k(y) \Leftrightarrow W_k(x') > W_k(y')$.*

Separability is a practical property, since it imposes that in the comparison of two vectors, the magnitude of the 'unconcerned' entries in L should not matter. An example helps to clarify. Let W_m aggregate across three dimensions of well-being: income, health and education, and suppose

[9] Let the vector inequality $y > x$ denote that $y_l \geq x_l$ holds for all its entries $l = 1, \ldots, k$ and $y_l > x_l$ for at least one entry l.

[10] Let 1_k denote a k-dimensional vector with all entries equal to 1.

two individuals who have the same outcome in the income dimension and different outcome levels in health and education. Separability asserts that the exact level of income is not important to order the individuals with respect to their well-being. Separability implies, for instance, that the marginal rate of substitution between health and education is independent of the income level of the individuals. It is a commonly made, but arguably strong property to aggregate across dimensions of well-being (see Deaton and Muellbauer 1980 for a discussion). Moreover, separability excludes all considerations about the position or rank of the entries in the total vector. Yet, in recent work on self-reported well-being it has been documented that individuals do not only care about the levels of their outcomes, but also about the relative position *vis-à-vis* other individuals in the distribution (Ferrer-i-Carbonell 2005, Luttmer 2005).

To allow considerations about the positions to play a role in the aggregation across individuals, we use a weakening of the separability property, which states that the comparison of two vectors is not affected by the magnitude of common entries in both vectors *as long as the initial ranking is maintained*. This property allows us to take both the level and position in the distribution into account. Let us define therefore the set of ordered vectors $\widetilde{\mathbb{R}}^k_{++}$. The first entry x_1 of an ordered vector has the highest level, the second entry the second highest; and so forth.

Property 5. ***Rank-dependent Separability (RSEP)*** *For all x, x', y, y' in $\widetilde{\mathbb{R}}^k_{++}$: if there is an $L \subset K$ such that for all l in L $x_l = y_l$ and $x'_l = y'_l$ whereas for all k in $K \backslash L$, $x_k = x'_k$ and $y_k = y'_k$, then $W_k(x) > W_k(y) \Leftrightarrow W_k(x') > W_k(y')$.*

Next, we impose three invariance properties. These properties specify which transformations or standardization procedures of the data will leave the ordering of two vectors x and y unaffected. In any multidimensional analysis the transformation and standardization of the data is an essential step to make the potentially very different dimensions comparable.[11]

Property 6. ***Weak ratio-scale invariance (WSI)*** *For all x, y in $\mathbb{R}^k_{++}$ and all positive λ : $W_k(x) > W_k(y)$ if and only if $W_k(\lambda x) > W_k(\lambda y)$.*

Property 7. ***Strong ratio-scale invariance (SSI)*** *For all x, y in $\mathbb{R}^k_{++}$ and all positive diagonal matrices $\Lambda : W_k(x) > W_k(y)$ if and only if $W_k(\Lambda x) > W_k(\Lambda y)$.*

Property 8. ***Weak translation invariance (WTI)*** *For all x, y in $\mathbb{R}^k_{++}$ and all $\kappa : W_k(x) > W_k(y)$ if and only if $W_k(x + \kappa 1_k) > W_k(y + \kappa 1_k)$.*

[11] On the issue of making meaningful comparisons of multidimensional outcome vectors, see Ebert and Welsch (2004). Alternatively, we could assume the data to be standardized from the beginning to leave the invariance to the standardization outside the characterization of the measure. We prefer to incorporate the standardization into the characterization given its potential effects on the final result, see e.g. Decancq, Decoster and Schokkaert (2009).

Property 6, weak ratio-scale invariance, states that a rescaling of all entries of the two vectors x and y with the same positive number does not affect their ordering. Doubling all outcomes, for instance, should not lead to a reordering of both vectors. This property is standard in the aggregation across individuals and assures, for instance, that W_n is unaffected by a general inflation. In the aggregation across dimensions, it is an appealing property when the units of measurement of the entries are the same, for example, when the entries are different sources of income or incomes at different points in time. We will impose this property to the aggregation functions both across dimensions W_m and across individuals W_n.

Property 7, the strong ratio-scale invariance, is a much stronger property and requires that a rescaling of all entries of x and y should not lead to a reordering. This rescaling factor is allowed to differ across the entries of vectors. Strong ratio-scale invariance is especially useful in the aggregation across dimensions when the variables are expressed in very different units of measurement, such as income in dollars and education in years. The property allows the entries of both vectors to be standardized by an entry-specific rescaling such as a division by their respective mean (that is, e.g. individual income divided by the mean income, and individual education by the mean education level of the distribution).[12] However, this property will turn out to be fairly restrictive in terms of the functions satisfying it.

Property 8, weak translation invariance, imposes that the ordering of two vectors by W_k is not affected if a common amount is added to all entries. We will impose weak translation invariance together with weak ratio-scale invariance to the aggregation across individuals to come to a parsimonious aggregation function which is invariant to common linear transformations of the entries.

The final two properties will allow us to compare k-dimensional vectors with a variable size k. Since we assume the number of relevant dimensions m to be fixed throughout the analysis, these properties will only be imposed on the aggregation function across individuals W_n. Together they permit the comparisons of distributions of different population sizes. We say that z is a replication of x if z is obtained by cloning x l-times, so that $z = (x, x, \ldots, x)$(l-times).

Property 9. ***Replication invariance (REP)*** *For all x in $\mathbb{R}^k_{++}$ and all z in $\mathbb{R}^{lk}_{++}$ which is a replication of $x : W_{lk}(z) = W_k(x)$.*

Property 10. ***Restricted aggregation (RA)*** *For all x in $\widetilde{\mathbb{R}}^k_{++}$:*
$W_k(x) = W_k\left(W_l\left(x_1, \ldots, x_l\right), \ldots, W_l\left(x_1, \ldots, x_l\right), x_{l+1}, \ldots, x_k\right)$.

[12] This property does not restrict the standardization to using the same central tendency measure for each dimension. One could, for instance, choose to standardize cardinal variables (such as income) by their respective mean and ordinal variables (such as self-reported health status) by their median, as suggested by Allison and Foster (2004).

If we impose replication invariance to W_n, the aggregation takes place on a per-capita basis. Replication invariance has been introduced in the literature on one-dimensional inequality measurement by Dalton (1920). Restricted aggregation asserts that the aggregation of the total vector is equivalent to an aggregation in which the outcomes of the better-off subgroup containing l entries are first aggregated into one aggregate. This property has been studied by Donaldson and Weymark (1980).

The result below summarizes the properties imposed to the aggregation function across dimensions W_m and derives the sole class of functions satisfying them all.

Proposition 1. *A continuous aggregation function* $W_m : \mathbb{R}^m_{++} \to \mathbb{R}_{++}$ *satisfies*
(a) *MON, NORM, SEP and WSI, if and only if for each* x *in* $\mathbb{R}^m_{++}$ *we have*

$$W_m(x) = \left(\sum_{j=1}^{m} w_j (x_j)^{\beta} \right)^{(1/\beta)}, \tag{1}$$

where $w_j > 0$ *for all* j *and* $\sum_{j=1}^{m} w_j = 1$,

(b) *MON, NORM and SSI, if and only if for each* x *in* $\mathbb{R}^m_{++}$ *we have*

$$W_m(x) = \prod_{j=1}^{m} {x_j}^{w_j}, \tag{2}$$

where $w_j > 0$ *for all* j *and* $\sum_{j=1}^{m} w_j = 1$.

Proof. See for (a) Blackorby and Donaldson (1982, theorem 2) and for (b) Tsui and Weymark (1997, theorem 4) □

Result (a) defines the aggregation across dimensions to be a Constant Elasticity of Substitution (CES) function, where parameter β reflects the degree of substitutability between the dimensions of well-being. In particular, β is related to the elasticity of substitution between the dimensions σ and equals $1 - 1/\sigma$. When $\beta = 1$, the dimensions of well-being are seen as perfect substitutes. As β tends to $-\infty$, the dimensions tend to perfect complementarity; at the extreme, individuals are judged by their worst outcomes.[13] Result (b) (itself a limiting case of the previous when $\beta = 0$) is in some respects disappointing and reveals how restrictive the requirement of strong ratio-scale invariance can be. In the presence of the other properties, strong ratio-scale invariance is only satisfied by a so-called Cobb-Douglas well-being function which has unit elasticity of substitution, that is, $\sigma = 1$. Other elasticities between the dimensions cannot be obtained without relaxing this property. One has to make a painful trade-off here: it is impossible to obtain results that are robust to alternative rescaling procedures of the different dimensions if one desires more flexibility

[13]This extreme case is excluded by the monotonicity property and should be considered as a limiting case.

in the functional form. In other words, the functional flexibility of W_m can only be obtained when one can rely on a theoretically sensible and ethically justifiable standardization procedure of the original dimensions.

The weighting scheme $w = (w_1, \ldots, w_m)$ consists of the weights w_j, which are all positive and sum to 1 and reflects the relative importance of the different dimensions. In interplay with parameter β and the standardization chosen, the weights determine the marginal rates of substitution or trade-offs between the dimensions (Decancq and Lugo 2008).

The well-being functions characterized by proposition 1 are popular measures of well-being in the literature. The Human Development Index advocated by the UNDP, for instance, is a special case of expression (1) with $\beta = 1$, and weights w_j equal to 1/3.[14] Other examples may be found in the literature on multidimensional inequality measurement: Maasoumi (1986), for instance, derives a CES well-being function based on different considerations rooted in information theory.

An analogous result can be obtained for the properties imposed on W_n, the aggregation across individuals.

Proposition 2. *A continuous aggregation function $W_n : \mathbb{R}^n_{++} \to \mathbb{R}_{++}$ satisfies MON, SYM, NORM, RSEP, WSI, WTI, REP and RA if and only if for each x in $\mathbb{R}^n_{++}$ we have*

$$W_n(x) = \sum_{i=1}^{n} \left[\left(\frac{r_i}{n}\right)^{\delta} - \left(\frac{r_i - 1}{n}\right)^{\delta} \right] x_i, \tag{3}$$

where $\delta > 0$ and r_i is a shorthand for $r_i(x)$, that is the rank of individual i on the basis of the levels of vector x.

Proof. See Ebert (1988, proposition 10). □

The class of aggregation functions obtained is a convenient one, since it consists of weighted averages whose welfare weights and the associated value judgements are captured by a single parameter δ, the bottom-sensitivity of the aggregation function. If δ equals 1, the aggregation function becomes an unweighted average in the utilitarian tradition. The higher δ, the more weight is given to the bottom of the distribution, with as limiting case $\delta = +\infty$, which leads to a Rawlsian aggregation function where only the worse-off individual is counted for the social evaluation.[15] For values of δ between 0 and 1 the best-off individuals are given more weight. The standard Gini social evaluation function is obtained by setting $\delta = 2$.

[14]Original variables (GDP *per capita*, life expectancy, school enrolment and literacy rate) are standardized by a linear rescaling so that their minimum value is 0 and the maximum is 1. Additionally, GDP *per capita* is first transformed by a logarithmic transformation, to allow the marginal rates of substitutions between life expectancy and GDP *per capita*, and educational outcomes and GDP *per capita* to vary over the range of GDP *per capita* values.

[15] Again, this extreme case is excluded by the monotonicity property and should be considered as a limiting case.

By substituting the obtained one-dimensional aggregation functions (1) and (3) into the two initial two-step procedures we obtain the following two multidimensional social evaluation functions:

$$W^1_{n\times m}(X) \;=\; \left[\sum_{j=1}^{m} w_j \left(\sum_{i=1}^{n} \left[\left(\frac{r^i_j}{n}\right)^{\delta} - \left(\frac{r^i_j - 1}{n}\right)^{\delta}\right] x^i_j\right)^{\beta}\right]^{(1/\beta)}, \tag{4}$$

where $\delta > 0$, all weights $w_j > 0, \sum_{j=1}^{m} w_j = 1$, and r^i_j is a shorthand for $r^i_j\,(x_j)$, that is the rank of individual i in dimension j. Similarly,

$$W^2_{n\times m}(X) \;=\; \sum_{i=1}^{n} \left[\left(\frac{r^i}{n}\right)^{\delta} - \left(\frac{r^i - 1}{n}\right)^{\delta}\right] \left(\sum_{j=1}^{m} w_j (x^i_j)^{\beta}\right)^{(1/\beta)}, \tag{5}$$

where $\delta > 0$, all weights $w_j > 0, \sum_{j=1}^{m} w_j = 1$, and r^i is a shorthand for $r^i\,(s)$, the rank of individual i on the basis of s the vector of well-being levels, with $s^i = \left(\sum_{j=1}^{m} w_j (x^i_j)^{\beta}\right)^{(1/\beta)}$. The first alternative, $W^1_{n\times m}$ is a special case of the multidimensional generalized Gini social evaluation functions proposed by Gajdos and Weymark (2005). The second social evaluation function, $W^2_{n\times m}$ has, to the best of our knowledge, not yet been introduced in the literature. In the following section we will compare both aggregation procedures with respect to their sensitivity to two specific multidimensional distributional concerns and investigate the empirical differences based on a Russian data set.

At this point two concerns might be brought to attention. The first is that both social evaluation functions impose the same degree of substitution between all pairs of dimensions. However, one could reasonably argue that this should not be the case. A possible solution is to adopt a nested approach which first aggregates subsets of dimensions using expression (1) – each with a different parameter β – and then aggregates these subsets using again the same expression. The second concern is that the social evaluation functions derived make use of the same level of bottom sensitivity for all dimensions. Yet, as Anand (2002, p.485) argues, given that health has both intrinsic and instrumental value "we should be more averse to, or less tolerant of, inequalities in health than inequalities in income". A solution, albeit an imperfect one, is to transform the original dimensions before the aggregation is done (for instance by a logarithmic transformation of the health variable).

III. Multidimensional Distributional Concerns

The reader will note that so far we have not introduced any property that captures distributional concerns. In the standard one-dimensional analyses, distributional sensitivity is obtained by imposing some form of the *Pigou-Dalton* transfer principle. The principle states that a transfer of income from a poorer to a richer individual leads to a social inferior situation. Some proposals have been made to generalize the one-dimensional Pigou-Dalton principle to the multidimensional setting. In this section we focus on two popular generalizations and investigate their effect within the multidimensional framework of the previous section. The two distributional concerns affect both aggregation functions W_n and W_m at the same time and are therefore defined as properties of the multidimensional social evaluation function $W_{n\times m}$. In view of the empirical analysis in the following section, we are especially interested in the parameter-restrictions imposed by the distributional concerns on the parameters β and δ in expression (4) and (5).

The first distributional concern asserts that if a uniform mean-preserving averaging is carried out in all dimensions, the resulting distribution matrix is socially preferred to the original one. In the literature, the concern is referred to as *uniform majorization* (Kolm 1977, Marshall and Olkin 1979, Tsui 1995, Weymark 2006) and its formalization is rooted in multidimensional majorization theory. A uniform mean-preserving averaging can be obtained by applying the same bistochastic transform to all dimensions.[16]

Property 11. ***Uniform Majorization (UM)*** *For all distribution matrices X and Y in $\mathbb{R}^{n\times m}_{++}$: if $Y = BX$ for some $n \times n$ bistochastic matrix $B, Y \neq X$ and Y is not a permutation of X, then $W_{n\times m}(Y) > W_{n\times m}(X)$.*

An example can clarify uniform majorization. Consider the following matrices that summarize the outcomes of three individuals (rows) for two dimensions of well-being (columns),

$$B = \begin{bmatrix} 0.75 & 0.25 & 0 \\ 0.25 & 0.75 & 0 \\ 0 & 0 & 1 \end{bmatrix}; X = \begin{bmatrix} 50 & 80 \\ 90 & 20 \\ 10 & 50 \end{bmatrix} \text{ and } Y = \begin{bmatrix} 60 & 65 \\ 80 & 35 \\ 10 & 50 \end{bmatrix},$$

where indeed $Y = BX$ so that Y is obtained from X by a bistochastic transform. Uniform majorization imposes that distribution matrix Y should be socially preferred to X.

Atkinson and Bourguignon (1982) identify another distributional concern. They argue that a multidimensional social evaluation should be sensitive to the correlation between the dimensions of well-being. Epstein and Tanny (1980) introduced the concept of a correlation increasing transfer in the literature of risk and uncertainty. We use here the definition advanced by Tsui (1999) where a correlation increasing transfer is defined as a rearrangement of the outcomes of two individuals such that one individual gets the highest outcomes in all dimensions and the other the lowest.

Definition 1. ***Correlation Increasing Transfer (CIT)*** *For all distribution matrices X and Z, Z is obtained from X through a CIT if $X \neq Z$, X is not a permutation of Z, and there are two*

[16] A bistochastic transform of a distribution matrix X involves a premultiplication of the distribution matrix by a bistochastic matrix, which is a square nonnegative matrix with all row and column sums equal to 1.

individuals k and l such that (i) $z_j^k = max\{x_j^k, x_j^l\}$ for all dimensions j, (ii) $z_j^l = min\{x_j^k, x_j^l\}$ for all dimensions j and (iii) $z^i = x^i$ for all $i \notin \{k, l\}$.

Based on the notion of a correlation increasing transfer, the second distributional concern can be formalized. It says that a distribution matrix Z that is obtained from X by any correlation increasing transfer, is socially inferior. This concern is called *correlation increasing majorization.* In other words, if two distribution matrices have identical marginal distributions, the one with lower correlation between the dimensions is preferred. Correlation increasing majorization captures the idea of compensating inequalities among different dimensions, hence implicitly assuming that dimensions are substitutes.[17]

Property 12. ***Correlation Increasing Majorization (CIM)*** *For all distribution matrices X and Z in $\mathbb{R}_{++}^{n\times m}$: if Z is obtained from X by a correlation increasing transfer then $W_{n\times m}(X) > W_{n\times m}(Z)$.*

Consider the following example,

$$X = \begin{bmatrix} 50 & 80 \\ 90 & 20 \\ 10 & 50 \end{bmatrix} \text{ and } Z = \begin{bmatrix} 50 & 20 \\ 90 & 80 \\ 10 & 50 \end{bmatrix}.$$

Distribution matrix Z is obtained from X by a correlation increasing transfer between the first two individuals. The first individual in Z gets the lowest outcomes in all dimensions, whereas the second individual in Z gets the highest outcomes of the first two individuals of X. Correlation increasing majorization imposes that X is preferred to Z.

We investigate the impact of introducing both distributional concerns to the two multidimensional S-Gini social evaluation functions derived in the previous section summarized in expression (4) and (5). We start by the social evaluation function resulting from the first procedure $W_{n\times m}^1$.

Proposition 3. *A continuous double aggregation function $W_{n\times m}^1 : \mathbb{R}_{++}^{n\times m} \to \mathbb{R}_{++}$, where W_n satisfies MON, SYM, NORM, RSEP, WSI, WTI, REP and RA; and W_m satisfies MON, NORM, SEP and WSI,*
i) satisfies UM if and only if $W_{n\times m}^1$ satisfies equation (4) with $\delta > 1$,
ii) cannot satisfy CIM.

Proof. See appendix A. □

[17] Bourguignon and Chakravarty (2003) suggest that depending on the nature of the dimensions, the opposite could be considered. Here, dimensions are considered 'substitutes' or 'complements' according to the Auspitz-Lieben-Edgeworth-Pareto (ALEP) definition, in terms the second cross-partial derivative of the utility function (Atkinson 2003).

The intuition for the first part of this result is the following: the bistochastic transform leads to more equally distributed dimensions, so that any aggregation across individuals with a preference for equality (that is, $\delta > 1$) leads to larger summary statistic than the one corresponding to the initial distribution. Monotonicity of the aggregation across dimensions assures that the distribution matrix after the mean preserving averaging is preferred. The impossibility result involving correlation increasing majorization is a special case of a result that has been formally proven by Gajdos and Weymark (2005, proposition 10). Intuitively, a dimension-specific summary statistic looses all information about the individual outcomes and hence also about the correlation between the dimensions, so that a social evaluation function derived according to the first procedure is always insensitive to any correlation increasing transfer.

Imposing both distributional concerns on the social evaluation function $W^2_{n\times m}$ obtained by the second procedure, leads to the following restrictions on the parameter-space.

Proposition 4. *A continuous double aggregation function* $W^2_{n\times m} : \mathbb{R}^{n\times m}_{++} \to \mathbb{R}_{++}$*, where* W_m *satisfies MON, NORM, SEP and WSI; and* W_n *satisfies MON, SYM, NORM, RSEP, WSI, WTI, REP and RA,*
i) satisfies UM if and only if $W^2_{n\times m}$ *satisfies equation* (5) *with* $\beta < 1$ *and* $\delta > 1$,
ii) satisfies CIM if and only if $W^2_{n\times m}$ *satisfies equation* (5) *with* $\delta > \delta'$*, where* δ' *is a threshold depending on the initial matrix, the correlation increasing transfer,* w *and* β*.*

Proof. See appendix A. □

For both distributional concerns to be satisfied, the degree of substitutability β should be smaller than 1 and the bottom-sensitivity of the aggregation across individuals W_n should be "large enough", that is larger than a lower-bound δ' which depends on the initial matrix X, the weighting scheme w and the degree of substitutability β. The result is summarized in figure 1. The quadrant where $\beta < 1$ and $\delta > 1$ represents the pairs of parameters (β, δ) for which uniform majorization is satisfied. Correlation increasing majorization is satisfied in the light-shaded area, above lower-bound δ' represented by the full line. In general, the exact location of the lower bound depends on the initial distribution matrix X, the distribution matrix after correlation increasing transfer Z, the weighting scheme w and the degree of substitutability β. However, in the quadrant where $\beta > 1$ and $\delta > 1$ holds, correlation increasing majorization is always satisfied.

The difference between both aggregation procedures concerning their compliance with both distributional concerns is essential. Concerning uniform majorization, the first procedure imposes no restrictions on the degree of substitutability in the aggregation across dimensions, whereas

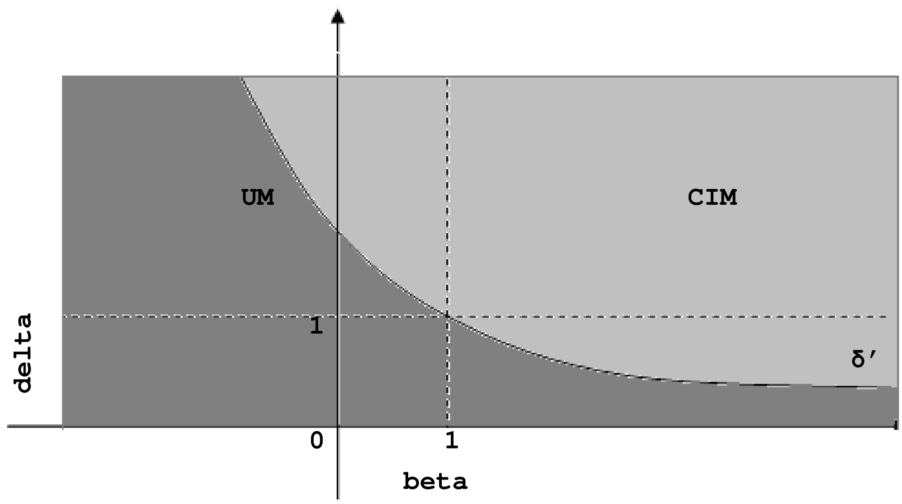

FIGURE 1. Compliance of a multidimensional S-Gini Social welfare function $W^2_{n\times m}$ with both distributional concerns for different β and δ parameters.

the second procedure does. More importantly, aggregating according to the first procedure excludes *a-priori* any compliance with correlation increasing majorization, whereas an aggregation according to the second procedure allows correlation increasing majorization to be satisfied for a specific subset of the parameter-space above lower-bound δ'. Unfortunately, this subset depends on the data at hand, which is quite inconvenient from an applied perspective.[18]

Therefore, we consider a weakening of correlation increasing majorization, the so called *unfair rearrangement* principle. According to this principle, the sequence of correlation increasing transfers that makes one individual top-ranked in all dimensions, another individual second ranked in all dimensions and so forth, leads to social inferior situation. It has been introduced in the literature on multidimensional inequality by Dardanoni (1996). Instead of requiring that any correlation increasing transfer leads to an inferior distribution matrix, the unfair rearrangement principle restricts attention to one specific sequence of correlation increasing transfers.

Property 13. ***Unfair Rearrangement Principle (URP)*** *For all distribution matrices X and Z^* in $\mathbb{R}^{n\times m}_{++}$: if Z^* is obtained from X by the sequence of correlation increasing transfers that makes one individual in Z^* top-ranked in all dimensions, another individual second ranked in all dimensions and so forth, then $W_{n\times m}(X) > W_{n\times m}(Z^*)$.*

[18]A conservative procedure to get an idea about the lower bound δ' is to simulate all possible matrices that can be obtained by a CIT from a given distribution matrix X, and selecting the maximal lower bound that assures CIM to hold. For realistic data sets this is a computationally intense exercise and moreover this procedure easily leads to the extreme values for the lower bound δ' so that the intersection with the area where UM holds is virtually empty (computations for the empirical example in section V are available upon request).

Again, an example can clarify this principle,

$$X = \begin{bmatrix} 50 & 80 \\ 90 & 20 \\ 10 & 50 \end{bmatrix} \text{ and } Z^* = \begin{bmatrix} 10 & 20 \\ 90 & 80 \\ 50 & 50 \end{bmatrix}.$$

In distribution matrix Z^* the first individual is bottom-ranked in all dimensions, the second individual is top-ranked in all dimensions and the third one is middle-ranked, so that the dimensions are perfectly correlated. Practically, by restricting our scope to the specific sequence of correlation increasing transfers leading to an unfair rearrangement, the minimal bottom sensitivity δ^* can be obtained such that for all $\delta > \delta^*$, an unfair rearrangement of a given distribution matrix X for a given weighting scheme w and a degree of substitutability β leads to a decrease in societal well-being. Note that for any given X, w and β, the lower bound on the bottom sensitivity for compliance with the unfair rearrangement principle δ^* is smaller or equal to the lower bound on the bottom sensitivity for compliance with the correlation increasing principle δ'. In the empirical section, we obtain estimates for δ^* for a series of parameters β conditional on the data set at hand X and the weighting scheme w.

IV. Two Multidimensional S-Gini Inequality Indices

In the one-dimensional normative approach, a relative inequality index is derived from its underlying social evaluation function as the fraction of total welfare wasted due to inequality (Atkinson 1970, Kolm 1969, Sen 1973). In a seminal article, Kolm (1977) generalizes the one-dimensional definition to the multidimensional setting by defining multidimensional inequality to be the fraction of the aggregate amount of each dimension of a given distribution matrix that could be destroyed if every dimension of the matrix is equalized while keeping the resulting matrix socially indifferent to the original matrix (see also Weymark 2006). Formally, a multidimensional relative inequality index $I(X)$ is defined as the scalar that solves

$$W_{n\times m}\left(\left(1 - I(X)\right) X_{\mu}\right) \quad = \quad W_{n\times m}\left(X\right) \ , \tag{6}$$

where X_μ is the equalized distribution matrix defined such that all the elements in the $j-$th column of the matrix are the dimension-wise mean $\mu(x_j)$. By substituting $W^1_{n\times m}$ obtained in (4) in expression (6), the following multidimensional S-Gini inequality index I^1 can be obtained,

$$I^1(X) = 1 - \frac{\left[\sum_{j=1}^{m} w_j \left(\sum_{i=1}^{n}\left[\left(\frac{r_j^i}{n}\right)^{\delta} - \left(\frac{r_j^i - 1}{n}\right)^{\delta}\right] x_j^i\right)^{\beta}\right]^{(1/\beta)}}{\left(\sum_{j=1}^{m} w_j \mu(x_j)^{\beta}\right)^{1/\beta}}, \tag{7}$$

which belongs to the class of multidimensional generalized Gini indices proposed by Gajdos and Weymark (2005, proposition 10). Based on the social evaluation function $W^2_{n\times m}$ summarized in expression (5) and the definition of a relative inequality index in expression (6) , I^2, another multidimensional S-Gini inequality index can be derived as follows,

$$I^2(X) = 1 - \frac{\sum_{i=1}^{n}\left[\left(\frac{r^i}{n}\right)^{\delta} - \left(\frac{r^i-1}{n}\right)^{\delta}\right]\left(\sum_{j=1}^{m} w_j (x_j^i)^{\beta}\right)^{1/\beta}}{\left(\sum_{j=1}^{m} w_j \mu(x_j)^{\beta}\right)^{1/\beta}}. \tag{8}$$

Both multidimensional inequality indices are generalizations of the *S-Gini inequality index* and lead to the one-dimensional S-Gini index for m equal to 1 (Donaldson and Weymark 1980, Kakwani 1980, Yitzhaki 1983).[19] By inspection of expressions (7) and (8) it is clear that their difference arises from two elements. First, the sequence of the summations across individuals and dimensions differs and second, the weights attached to each individual are different. The inequality index I^1 uses individual weights depending on r_j^i, that is their rank in the distribution of each dimension j. In contrast, in the inequality index I^2, the individuals' weight depends on r^i, their rank in the overall distribution of well-being. Both elements may lead to very different empirical results, when I^1 and I^2 are computed for the same distribution matrix X. *A priori* it is even hard to predict which of both indices will show more inequality.

However, from proposition 4 it follows that $I^2(X) < I^2(Z^*)$ whenever $\delta > \delta^*$ (i.e. when the unfair rearrangement principle holds) where Z^* is the distribution matrix introduced in the previous section obtained from X by an unfair rearrangement. Since I^1 is not sensitive to changes in correlation, it always holds that $I^1(X) = I^1(Z^*)$. When all dimensions are perfectly correlated, the ranks r_j^i equal r^i for all j dimensions. Moreover, if β equals 1 both aggregations across individuals and dimensions are weighted averages so that the order can be switched without affecting the results. Hence, $I^1(Z^*) = I^2(Z^*)$ whenever $\beta = 1$. In sum, in the specific case for $\delta > \delta^*$ and $\beta = 1$, I^2 shows always less inequality than I^1. In the general case, however, the comparison between both indices depends on the inequality *within* each dimension and the correlation structure *between* the dimensions as will be illustrated in the next section based on a Russian data set.

V. Empirical Illustration: Russian Well-Being Inequality Between 1995 and 2003

As an empirical illustration, we consider the evolution of inequality in Russia between 1995 and 2003. During this period, the Russian Federation underwent a fundamental transition and was hit by a severe financial crisis. Many studies have documented that inequality in expenditures increased over the period preceding the crisis (Gorodnichenko et al. 2008). Others have studied

[19] The recently proposed multidimensional Gini index by Banerjee (2008) belongs to the class of indices given by expression (8) . In particular, it is obtained when setting β equal to 1; δ equal to 2 and the weighting scheme w by the first normalized principal component of distribution matrix X.

the effects of the transition on other aspects of well-being such as health or education (Moser et al. 2005, Blam and Kovalev 2006, Smolentseva 2007). We will analyze the evolution of well-being inequality by including, in addition to expenditure data, other dimensions such as health and education.

Data come from the Russian Longitudinal Monitoring Survey (RLMS), a series of nearly annual, nationally representative surveys designed to monitor the effects of Russian reforms on health and economic welfare of households and individuals.[20] We use three indicators for the respective dimensions of well-being: equivalent real household expenditures, a constructed health indicator and years of schooling. Equivalent real household expenditures are widely used in the literature as an indicator of material standard of living.[21] We use the square root of household size as the equivalence scale. For the health dimension, the RLMS is particularly rich in objective health indicators. We aggregate eight of these indicators to obtain a composite index of health status. The weights attached to each indicator are derived from an ordinal logit regression of self-assessed health status on the objective health indicators.[22] By this procedure, the constructed measure is as close as possible to the self-reported health status of the individual, while making sure that individuals with the same objective characteristics obtain the same health measure. Third, years of schooling is constructed combining the information on the highest grade reported and the completion of higher education.

We restrict the analysis to adults with complete information in all three indicators, leaving us with a sample of approximately 6,000 individuals in each wave (see table 3 in appendix B for summary statistics). Table 4 in appendix B includes two members of the class of one-dimensional S-Gini inequality indices, the standard Gini coefficient ($\delta = 2$) and a more bottom sensitive Gini index ($\delta = 5$). Given that the sample represents a fraction of the total Russian population, we use a bootstrapping procedure to compute the standard errors of the inequality indices and their respective intervals at 95% confidence level.[23]

[20] The RLMS data are obtainable from: http://www.cpc.unc.edu/projects/rlms/. The survey (phase II) has been carried out annually since 1994 (with the exception of 1997 and 1999) and is still ongoing.

[21] We use the original household expenditure variable, where nominal values are corrected for yearly inflation and use June 1992 as the reference year. No other adjustment is made. For a detailed discussion on possible adjustments, see Gorodnichenko et al. (2008).

[22] More precisely, for every individual, the composite index of health is the predicted value of a pooled ordered logit health regression with the following explanatory variables: indicators of diabetes, heart attack, anaemia or other health problems; indicators of a recent medical check-up, hospitalization or operation in the last three months; life-style indicators such as smoking, regular exercises or jogging and age and gender dummies. All variables are highly significant and have the expected sign. The results can be found in table 2 in appendix B. The predicted values from this regression are linearly transformed such that the most unhealthy individual over the considered period obtains slightly more than 0 and the healthiest almost 1. A similar procedure has been used by van Doorslaer and Jones (2003) and Nilsson (forth.). See also Lokshin and Ravallion (2008) for a similar procedure applied to the RLMS data using an OLS regression.

[23] From the original sample repeatedly (1000 times) a new sample is drawn with replacement, of the same size as the original sample. For each of the 1000 new samples, we keep track of all the computed S-Gini inequality indices and report the interval that contains 95% of the results.

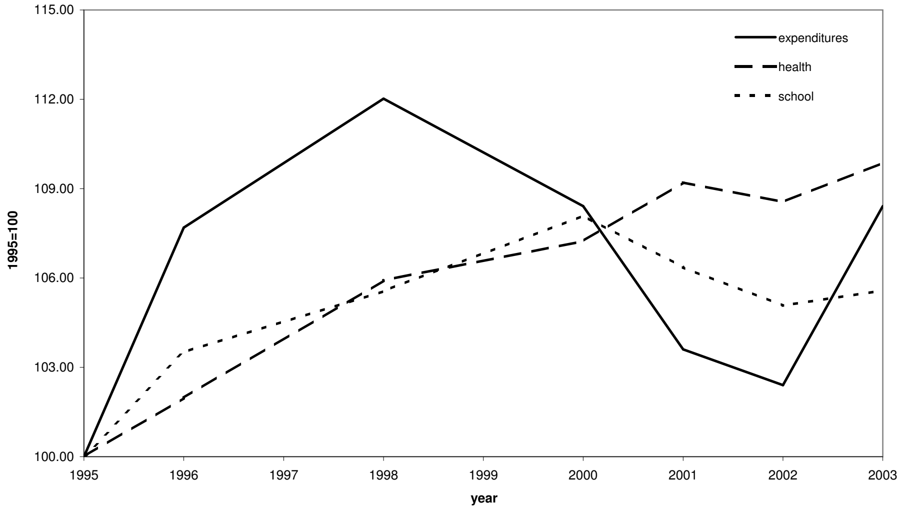

Figure 2. The evolution of Russian inequality measured by a dimension-by-dimension S-Gini inequality index (for $\delta = 2$).

Source: Authors' calculations based on the RLMS 1995-2003.

Figures 2 and 3 show the evolution of the S-Gini inequality index of the three dimensions separately for a bottom sensitivity parameter of 2 and 5. The inequality indices are normalized such that 1995 equals 100 to compare changes in inequality more easily.[24] Income inequality increases approximately by 10% between 1995 and 1998. After the financial crisis income inequality decreases continuously, though still not reaching 1995 values, and rises again in 2003. These findings are in line with the literature (Gorodnichenko et al. 2008). By contrast, health inequality increases throughout the period and educational inequality remains relatively stable. In short, all three dimensions experience rather distinct patterns, highlighting the need for a multidimensional approach to the analysis of the distribution of well-being.

In order to compute both multidimensional Gini indices presented in the preceding section, we are inevitably forced to make four crucial choices about the parameters. These are first, on the appropriate standardization for each dimension; second, on the degree of substitutability β; third, on the weighting scheme $w = (w_1, \ldots, w_m)$ and fourth, on the bottom sensitivity parameter δ. In the present illustration we standardize every dimension by dividing the outcomes by its respective mean in 2000. We set β equal to 0, so that the aggregation across dimensions follows expression (2). The reason for this choice is to make results robust to alternative standardization procedures involving a dimension-wise rescaling. As already explained, this is an attractive property when the indicators are of very different nature as it is the case here. Third, we use equal dimension

[24]This normalization is particularly helpful given that the variables used for each dimension differ in their bounds and measurement characteristics. More precisely, income is virtually unbounded and continuous while health is continuous and bounded by construction (between 0 and 1) and education is discrete and bounded (between 1 and 16).

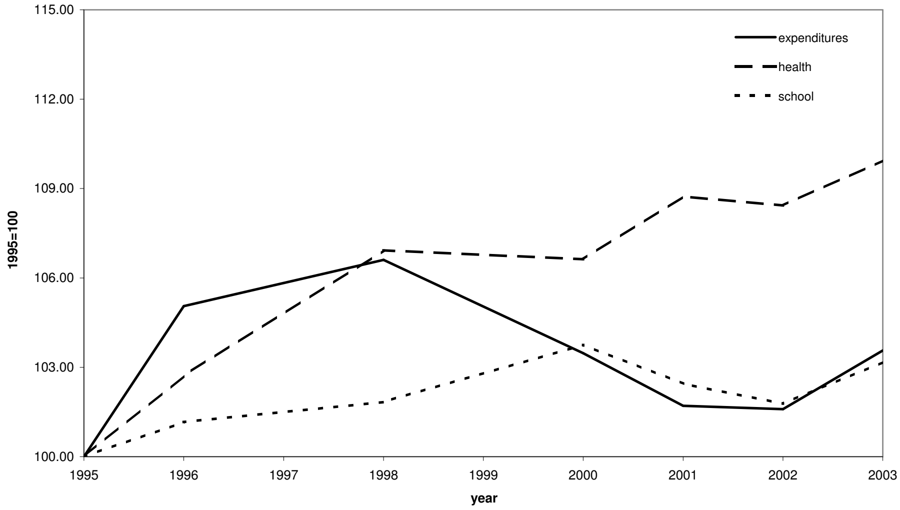

FIGURE 3. The evolution of Russian inequality measured by a dimension-by-dimension S-Gini inequality index (for $\delta = 5$).

Source: Authors' calculations based on the RLMS 1995-2003.

weights ($w_j = 1/3$) for simplicity.[25] Finally, we will compare two values for the bottom sensitivity parameter δ, the first corresponding to the standard Gini coefficient ($\delta = 2$) and the other giving higher weight to individuals at the bottom of the distribution ($\delta = 5$).

The selection of the parameters is an essential and difficult step in the computation of any multidimensional index of inequality. Inevitably, the parameters imply value judgements about the nature of well-being and the contribution of each dimension to it. It is thus important to make these choices in an explicit and clear way so that they can be open to public scrutiny. To clarify what our parameter-choices mean in terms of the well-being function, we present the implied marginal rates of substitution in table 1. The average Russian in 2000, who spends 4,457 rubles, attends 6 years of school and has a health indicator of 0.635 (on a scale of 0 to 1) is willing to pay about 702 rubles for an increase of 0.1 on the health scale (which means, for instance, roughly 600 rubles for not being hospitalized) or 718 rubles for an extra year of education.[26]

We first check whether the selected parameters lead to compliance of the indices with the distributional concerns introduced in section III. Following the conditions obtained in proposition 3, index I^1 satisfies uniform majorization for the parameters chosen and it is by construction not sensitive to the correlation between the dimensions. Index I^2 satisfies uniform majorization for the parameters selected. We check the unfair rearrangement principle for a wide range of

[25]On the issue of setting weights in multidimensional measures of well-being and deprivation, see Decancq and Lugo (2008).

[26]A more detailed overview of the distribution of marginal rates of substitution between income and health for individuals with a different incomes and health status can be found in table 5 in appendix B. These implied marginal rates of substitution should ideally be compared and to alternative studies based on questionnaires or market behavior. Unfortunately, we are not aware of these data in a Russian context.

TABLE 1. Implied marginal rates of substitution (MRS) between the dimensions of well-being.

MRS	Expenditures	Schooling	Health
Expenditures	1		
Schooling	-718	1	
Health	-702	-0.98	1

Source: Russian Longitudinal Monitoring Survey, 2000.

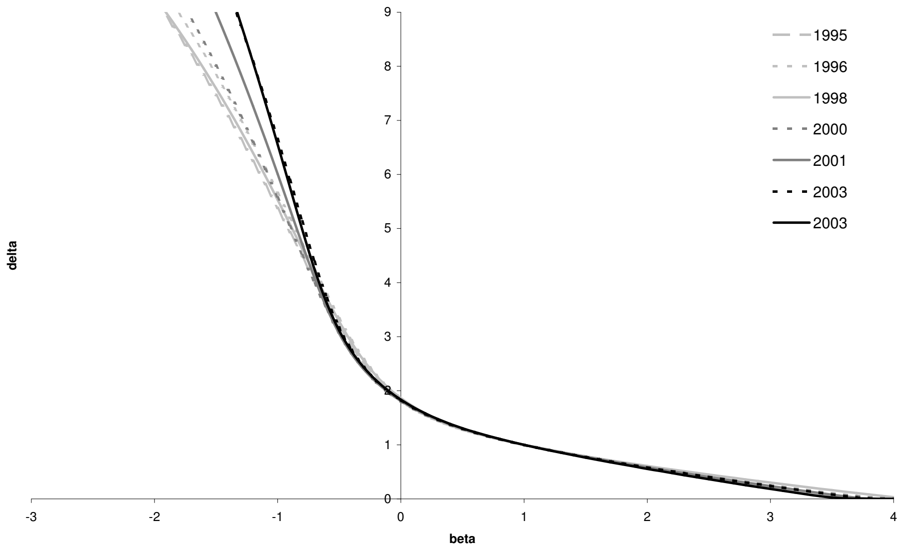

FIGURE 4. Compliance of the Russian data set with the unfair rearrangment principle for different years and different β and δ parameters.

Source: Authors' calculations based on the RLMS 1995-2003.

parameters and summarize the results in figure 4. This figure depicts a part of the normative space given by the degree of substitutability β and bottom sensitivity δ. In every point of the normative space, multidimensional inequality of the distribution matrix at hand X is compared to the corresponding Z^*, the distribution matrix obtained by an unfair rearrangement. The curves connect the points where $I^2(X)$ equals $I^2(Z^*)$ for a given year. Above the curve, the unfair rearrangement principle is satisfied and below the curve it is not. Hence, for a degree of substitutability β of 0 and a bottom sensitivity parameter δ equal to 2 or 5, it is clear that the unfair rearrangement principle is satisfied for all years.

Table 6 in appendix B summarizes the evolution in multidimensional well-being inequality using I^1 and I^2, with bootstrapped confidence intervals. The losses due to well-being inequality are about 30% for δ equal to 2 and 50% for δ equal to 5. Well-being inequality increases during

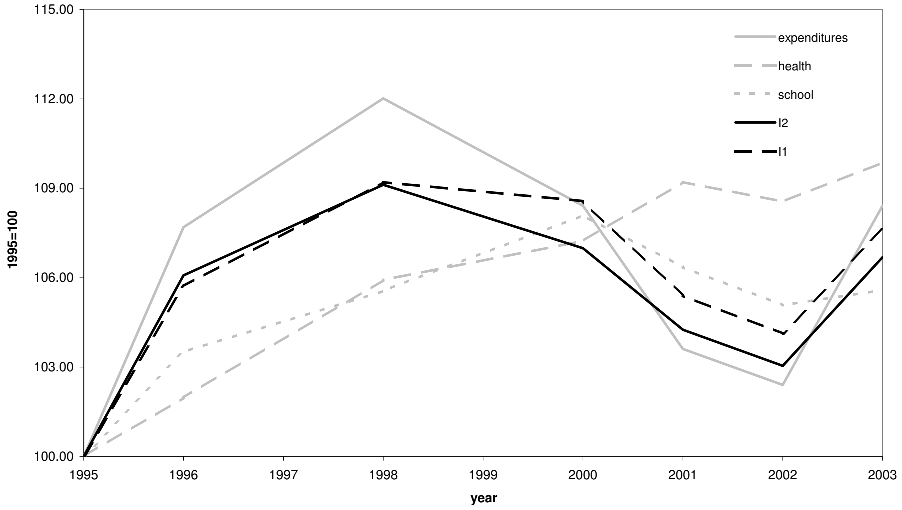

FIGURE 5. The evolution of Russian inequality measured by two multidimensional S-Gini inequality indices (for $\delta = 2$).

Source: Authors' calculations based on the RLMS 1995-2003.

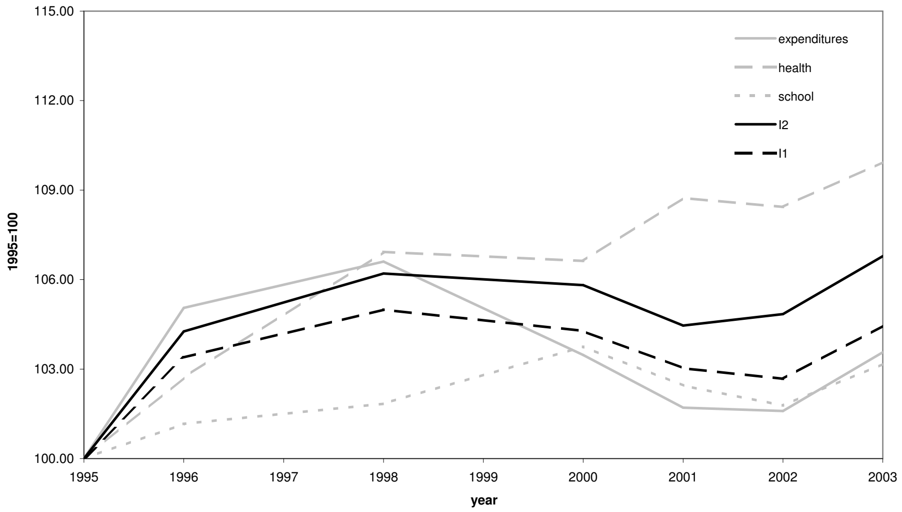

FIGURE 6. The evolution of Russian inequality measured by two multidimensional S-Gini inequality indices (for $\delta = 5$).

Source: Authors' calculations based on the RLMS 1995-2003.

the first four years and shows afterwards a U-shaped pattern. This pattern is consistent for both indices and both δ parameters. In figures 5 and 6, the evolution of multidimensional well-being inequality according to I^1 is depicted by the full black line and the evolution of I^2 by the dashed line. Again, all figures are normalized such that 1995 equals 100. For reference, the grey lines depict the dimension-wise trends in inequality. Note that the multidimensional inequality follows a pattern similar to the one income inequality for δ equal to 2 (figure 5), but that for the more bottom sensitive index (figure 6) the relative evolution resembles much more the one of health inequality. In other words, in this empirical example the multidimensional analysis tells a different story compared to an analysis focussing on just one dimension.

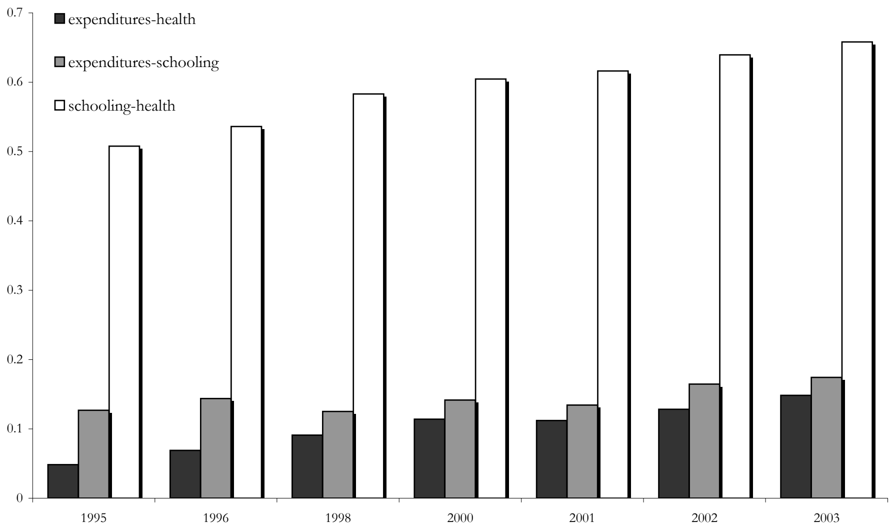

FIGURE 7. The evolution of the correlation between the Russian dimensions of well-being measured by the Spearman rank correlation coefficient.

Source: Authors' calculations based on the RLMS 1995-2003.

We see that for δ equal to 2, I^2 shows a less pronounced relative change than I^1, whereas the opposite holds for $\delta = 5$. Both indices clearly diverge for the more bottom-sensitive indices ($\delta = 5$). And, although the difference is not statistically significant, both indices disagree about the change in inequality between 2001 and 2002 for δ equal to 5. To understand these observations, we need to turn to the essential difference between both indices, that is, the sensitivity to correlation between the dimensions of well-being. Figure 7 shows the three pair wise correlation coefficients between the dimensions of well-being, all of which show clear increase over the considered period (see also Decancq 2009).

From the analysis in the preceding section, it follows that I^1 is always insensitive to the correlation between the dimensions, so that the trend of I^1 captures only the evolution of the inequality *within* the dimensions. Index I^2 is additionally sensitive to the correlation *between* the dimensions. The more weight is given to the bottom of the distribution (the higher δ), the more correlation increasing transfers lead to a rise in the measurement of inequality. Thus, the more the observed increase in correlation (see also table 7 in appendix B) is translated in a higher inequality measurement by I^2. This additional aspect of multidimensional well-being inequality following from the increase in correlation leads to a sharper increase of inequality measured by I^2 when considerable weight is given to the bottom of the distribution and even offsets the small decrease in inequality in the separate dimensions captured by I^1 between 2001 and 2002.

This illustration highlights the difference between both multidimensional Gini inequality indices. Although they are both derived from a two-step social evaluation function respecting similar properties in each step, the sequencing of both steps makes them very different with respect to their sensitivity to correlation between the dimensions. As shown in this empirical example, this is not only a theoretical concern but also affects the empirical results. Finally, the comparison of both indices reveals the increase of correlation between the dimensions as a characteristic aspect of the Russian fast transition.

VI. Conclusion

In this paper we have proposed two indices of inequality that take the multidimensionality of well-being explicitly into account. The resulting indices are multivariate generalizations of the popular Gini coefficient, in which both the levels and the position of individuals in the overall distribution matter for the social evaluation. We followed a normative approach to derive the inequality indices from their underlying rank-dependent multidimensional social evaluation functions. These functions are, in turn, conceived as explicit two-step aggregation functions, one across attributes and another across individuals. In both steps, we imposed a set of properties to come to a single class of functions. We think that the set of properties suggested in this essay entails an acceptable compromise between functional flexibility and the parsimony needed to come to an applicable and practically manageable index of well-being inequality.

The sequencing of both aggregations turns out to be essential in terms of the underlying principles. Aggregating first across individuals and then across dimensions leads to an index which is *a-priori* insensitive to the correlation between the dimensions. In our view this is a serious drawback. The second procedure, in which we first aggregate across dimensions and then across individuals, leads to a index that can be sensitive to correlation between the dimensions for specific choices of parameter-values. We show that researchers who want to obtain a correlation-sensitive rank-dependent inequality index, have to be willing to give a (potentially) large weight to the bottom of the distribution.

To apply the multidimensional indices in a satisfactory way to real-world data, hard choices have to be made about the appropriate parameter values (on standardization, weighting, substitutability between the dimensions and bottom-sensitivity). Theoretical guidelines on how these parameter choices can and should be made are required. Furthermore, there is a pressing need to collect more and richer individual data including non-monetary dimensions of well-being together with monetary ones, so that multidimensional inequality can be analyzed in a way that is sensitive to the correlation between its dimensions.

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Appendix A. Proofs

Proposition 3. *A continuous double aggregation function $W^1_{n\times m} : R^{n\times m}_{++} \to R_{++}$, where W_n satisfies MON, SYM, NORM, RSEP, WSI, WTI, REP and RA; and W_m satisfies MON, NORM, SEP and WSI,*
i) satisfies UM if and only if $W^1_{n\times m}$ satisfies equation (4) *with $\delta > 1$,*
ii) cannot satisfy CIM.

Proof. For part i) concerning *uniform majorization* (UM), let $Y = BX$, so that $W^1_{n\times m}$ satisfies UM if and only if $W^1_{n\times m}(Y) > W^1_{n\times m}(X)$. By construction, in every dimension j it holds that $y_j = Bx_j$, so that for all strictly Schur concave aggregation functions across individuals it holds that $W_n(y_j) > W_n(x_j)$ (Marshall and Olkin 1979, Chapter 3). Strict Schur concavity of W_n is obtained by restricting δ to be larger than 1 (Ebert 1988, proposition 6). If in all dimensions $j, W_n(y_j) > W_n(x_j)$ holds, by monotonicity of W_m it is the case that $W_m(W_n(y_1), \ldots, W_n(y_m)) > W_m(W_n(x_1), \ldots, W_n(x_m))$.

Part ii) concerning *correlation increasing majorization* (CIM), follows straight from Gajdos and Weymark (2005, theorem 10). □

Proposition 4. *A continuous double aggregation function $W^2_{n\times m} : R^{n\times m}_{++} \to R_{++}$, where W_m satisfies MON, NORM, SEP and WSI; and W_n satisfies MON, SYM, NORM, RSEP, WSI, WTI, REP and RA,*
i) satisfies UM if and only if $W^2_{n\times m}$ satisfies equation (5) *with $\beta < 1$ and $\delta > 1$,*
ii) satisfies CIM if and only if $W^2_{n\times m}$ satisfies equation (5) *with $\delta > \delta'$, where δ' is a threshold depending on the initial matrix, the correlation increasing transfer, w and β.*

Proof. A double aggregation function $W^2_{n\times m}$ satisfies the required properties on W_m and W_n if and only if it can be written as:

$$W^2_{n\times m}(X) = \sum_{i=1}^{n} a^i W_m(x^i), \tag{9}$$

with $a^i = \left[\left(\frac{r^i}{n}\right)^\delta - \left(\frac{r^i-1}{n}\right)^\delta\right]$ and $W_m(x^i) = \left[\sum_{j=1}^m w_j (x^i_j)^\beta\right]^{(1/\beta)}$ for all i. Note that r^i is the rank of individual i based on the vector of well-being levels. In other words, $r^i = 1$, if individual i is the best off; $r^i = 2$ if the individual is the second best off; and so forth.

For the first part of the proof, Kolm (1977, theorem 6) shows that UM holds if and only if $W^2_{n\times m}$ satisfies the following three conditions: (a) W_m is strictly concave, (b) W_n is increasing and (c) W_n is strictly Schur concave.
Note that (a) is fulfilled if and only if $\beta < 1$. For all $\beta < 1$, W_m is strictly quasi-concave. Any strictly quasi-concave function taking only positive values that satisfies weak ratio-scale invariance is strictly concave (Pemberton and Rau 2007). Condition (b) is fulfilled by the monotonicity of W_n and condition (c) is fulfilled if and only if $\delta > 1$. (Ebert 1988, proposition 6)

For the second part, let Z be obtained from X by a CIT between two individuals k and l so that $W^2_{n\times m}$ satisfies *correlation increasing majorization* (CIM) if and only if

$$W^2_{n\times m}(X) > W^2_{n\times m}(Z)\,. \tag{10}$$

Let the two individuals k and l have initial well-being $W_m(x^k)$ and $W_m(x^l)$ with ranks r^k and r^l, respectively. Assume, without loss of generality, that $W_m(x^l) \leq W_m(x^k)$. After the CIT, individuals k and l obtain well-being measurements $W_m(z^k)$ and $W_m(z^l)$ and all other individuals remain unaffected, so that $W_m(x^i) = W_m(z^i)$ for $i \neq k, l$. The CIT may lead to some re-ranking so that individuals k and l obtain welfare weights $a^{k'}$ and $a^{l'}$, which are not necessarily equal to a^k and a^l.

From the monotonicity of W_m it follows that:

$$W_m(z^l) < W_m(x^l) \leq W_m(x^k) < W_m(z^k). \tag{11}$$

We have:

$$W^2_{n\times m}(Z) = a^1 W_m(x^1) + \ldots + a^{k'} W_m(z^k) + \ldots + a^{l'} W_m(z^l) + \ldots + a^n W_m(x^n).$$

We define for notational convenience also $\widetilde{W}^2_{n\times m}(Z)$, that is the social welfare of distribution matrix Z after the CIT, but using the initial welfare weights, hence:

$$\widetilde{W}^2_{n\times m}(Z) = a^1 W_m(x^1) + \ldots + a^k W_m(z^k) + \ldots + a^l W_m(z^l) + \ldots + a^n W_m(x^n).$$

In case there is no reranking, $W^2_{n\times m}(Z) = \widetilde{W}^2_{n\times m}(Z)$. In general, however, there may be some reranking caused by the CIT.

Let us define ε as the gain in well-being of individual k due to the CIT, that is $\varepsilon = W_m(z^k) - W_m(x^k)$ and likewise the loss in well-being of individual l is defined as $\eta = W_m(x^l) - W_m(z^l)$.

Substituting this in equation (9) and (10), we obtain, after some rearranging, the following condition for CIM to hold:

$$a^l \eta - a^k \varepsilon > W^2_{n\times m}(Z) - \widetilde{W}^2_{n\times m}(Z). \tag{12}$$

Of special interest for us are the conditions on the parameters δ and β, which are implicit in this equation.

Let us first consider three possible cases for the bottom sensitivity δ. For $\delta = 1$ the right-hand-side of (12) is always zero. For $\delta > 1$, the weights a^i are increasing in r^i, so that $a^k < a^l$ which makes the right-hand-side of (12) non-positive. This follows from the fact that in $W^2_{n\times m}(Z)$ the highest weight is given to the individual with the smallest well-being and so forth, whereas in $\widetilde{W}^2_{n\times m}(Z)$ some weights may be attached to other well-being measures due to reranking. For $\delta < 1$, the weights a^i are decreasing in r^i, so that $a^k > a^l$ and the right-hand-side of (12) is always non-negative by a similar reasoning.

Second, let us consider the degree of substitutability β. Again there are three cases of interest. For $\beta = 1$, $W_m(z^k) - W_m(x^k) = W_m(x^l) - W_m(z^l)$ or $\varepsilon = \eta$. For $\beta > 1$, W_m has negative cross-derivatives and hence $W_m(x^k) - W_m(z^k) > W_m(z^l) - W_m(x^l)$ or $\varepsilon < \eta$ (Marshall and Olkin 1979). For $\beta < 1$, W_m has positive cross-derivatives and hence $\varepsilon > \eta$.

This leaves us four possible combinations of parameters to be analyzed:

(1) $\delta \geq 1$ and $\beta \geq 1$ (with at least one inequality being strict).

If $\delta \geq 1$ and $\beta \geq 1$, it holds that $a^l\eta - a^k\varepsilon > 0$. Moreover, $W^2_{n\times m}(Z) - \widetilde{W}^2_{n\times m}(Z) \leq 0$ so that CIM is *always* satisfied.

(2) $\delta \leq 1$ and $\beta \leq 1$.

Since $\delta \leq 1$ and $\beta \leq 1$, it holds that $a^l\eta - a^k\varepsilon < 0$. Furthermore, since $W^2_{n\times m}(Z) - \widetilde{W}^2_{n\times m}(Z) \geq 0$, CIM can *never* be fulfilled.

(3) $\delta > 1$ and $\beta < 1$.

Since $\delta > 1$, it follows that $a^k < a^l$ and $W^2_{n\times m}(Z) - \widetilde{W}^2_{n\times m}(Z) \leq 0$. From $\beta < 1$, it follows that $\varepsilon > \eta$, so that inequality (12) is fulfilled for $a^l - a^k$ large enough, or $\delta > \delta'$, where δ' is a threshold depending on X, Z, w and β.

(4) $\delta < 1$ and $\beta > 1$.

From $\delta < 1$, it follows that $a^k > a^l$ and $W^2_{n\times m}(Z) - \widetilde{W}^2_{n\times m}(Z) \geq 0$. From $\beta > 1$ it follows that $\varepsilon < \eta$, so that equation (12) is fulfilled for $a^l - a^k$ large enough, or $\delta > \delta'$, where δ' is a threshold depending on X, Z, w and β.

In sum, CIM is satisfied for $\delta > \delta'$, where δ' is a threshold depending on the initial matrix, the correlation increasing transfer, w and β. $\square$

Appendix B. Tables

Table 2. Health status. Pooled ordered logit regression.

	Self-assessed health	
diabetes	-0.645***	(0.0456)
heart attack	-0.973***	(0.0542)
anemia	-0.596***	(0.0481)
health problem	-1.637***	(0.0212)
hospitalized	-0.771***	(0.0417)
check up	-0.216***	(0.0237)
operation	-0.230***	(0.0484)
smokes	-0.187***	(0.0221)
jogged	0.234***	(0.0466)
exercise	0.404***	(0.0296)
age 20	-0.276***	(0.0320)
age 30	-0.885***	(0.0328)
age 40	-1.481***	(0.0327)
age 50	-1.986***	(0.0368)
age 60	-2.567***	(0.0378)
age 70	-3.317***	(0.0430)
age 80	-4.036***	(0.0659)
age 90	-4.533***	(0.163)
male	0.520***	(0.0205)
N	58,166	
R^2 (*pseudo*)	0.2270	

Standard errors in parentheses

$* : p < 0.05, ** : p < 0.01, *** : p < 0.001$

Source: RLMS 1995-2003.

Table 3. Summary Statistics. Russia from 1995 to 2003.

Variables	**mean**	**std. dev.**	**minimum**	**maximum**
1995 (N = 5,011)				
Expenditures (in Rubles)	5,289	5,923	10	160,250
Schooling (in years)	5.01	3.72	1.00	16.00
Health status (between 0 and 1)	0.64	0.17	0.03	1.00
1996 (N = 5,305)				
Expenditures (in Rubles)	4,972	6,277	61	177,450
Schooling (in years)	5.35	4.02	1.00	16.00
Health status (between 0 and 1)	0.64	0.18	0.07	1.00
1998 (N = 5,717)				
Expenditures (in Rubles)	3,837	6,992	35	203,583
Schooling (in years)	5.73	4.29	1.00	16.00
Health status (between 0 and 1)	0.64	0.18	0.02	1.00
2000 (N = 6,221)				
Expenditures (in Rubles)	4,457	6,277	30	124,256
Schooling (in years)	6.20	4.60	1.00	16.00
Health status (between 0 and 1)	0.64	0.18	0.02	1.00
2001 (N = 7,047)				
Expenditures (in Rubles)	5,024	6,601	61	251,334
Schooling (in years)	6.53	4.72	1.00	16.00
Health status (between 0 and 1)	0.63	0.18	0.04	1.00
2002 (N = 7,648)				
Expenditures (in Rubles)	5,244	6,228	10	181,401
Schooling (in years)	6.79	4.81	1.00	16.00
Health status (between 0 and 1)	0.64	0.18	0.00	1.00
2003 (N = 7,700)				
Expenditures (in Rubles)	5,920	9,452	61	235,387
Schooling (in years)	7.05	4.94	1.00	16.00
Health status (between 0 and 1)	0.64	0.19	0.03	1.00

Source: RLMS 1995-2003

TABLE 4. Dimension-by-dimension inequality. Russia from 1995 to 2003.

	Expenditures				Health				Schooling			
	$\delta = 2$		$\delta = 5$		$\delta = 2$		$\delta = 5$		$\delta = 2$		$\delta = 5$	
Year	Index	Conf.Int.	Index	Conf.Int.	Index	Conf.Int.	Index	Conf.Int.	Index	Conf.Int.	Index	Conf.Int.
1995	0.416	[0.403; 0.431]	0.651	[0.639; 0.662]	0.152	[0.149; 0.155]	0.332	[0.325; 0.399]	0.351	[0.342; 0.360]	0.636	[0.626; 0.647]
1996	0.448	[0.434; 0.463]	0.684	[0.673; 0.695]	0.155	[0.152; 0.159]	0.341	[0.334; 0.349]	0.363	[0.355; 0.371]	0.644	[0.633; 0.654]
1998	0.466	[0.446; 0.491]	0.694	[0.680; 0.710]	0.161	[0.158; 0.164]	0.355	[0.348; 0.361]	0.371	[0.363; 0.377]	0.649	[0.638; 0.657]
2000	0.451	[0.436; 0.467]	0.673	[0.662; 0.685]	0.163	[0.159; 0.166]	0.354	[0.348; 0.361]	0.379	[0.374; 0.385]	0.661	[0.651; 0.670]
2001	0.431	[0.418; 0.446]	0.662	[0.652; 0.673]	0.166	[0.163; 0.169]	0.361	[0.355; 0.367]	0.373	[0.368; 0.379]	0.652	[0.643; 0.661]
2002	0.426	[0.414; 0.437]	0.661	[0.652; 0.671]	0.165	[0.162; 0.168]	0.360	[0.354; 0.366]	0.368	[0.364; 0.374]	0.648	[0.639; 0.657]
2003	0.451	[0.434; 0.467]	0.674	[0.663; 0.685]	0.167	[0.163; 0.169]	0.365	[0.359; 0.370]	0.371	[0.366; 0.375]	0.657	[0.648; 0.665]

Source: RLMS 1995-2003.

Table 5. Implied willingness to pay (in ruble) for not having to go to the hospital in 2000 for individuals with a different income (columns) and health status (rows).

MRS	1175	1655	2125	2619	3096	3719	4454	5612	8102
0.03	3320	4676	6004	7399	8747	10507	12584	15855	22890
0.1	996	1403	1801	2220	2624	3152	3775	4757	6867
0.2	498	701	901	1110	1312	1576	1888	2378	3434
0.3	332	468	600	740	875	1051	1258	1586	2289
0.4	249	351	450	555	656	788	944	1189	1717
0.5	199	281	360	444	525	630	755	951	1373
0.6	166	234	300	370	437	525	629	793	1145
0.7	142	200	257	317	375	450	539	680	981
0.8	124	175	225	277	328	394	472	595	858
0.9	111	156	200	247	292	350	419	529	763
1	100	140	180	222	262	315	378	476	687

Source: RLMS 1995-2003.

TABLE 6. Multidimensional inequality measured by two S-Gini indices for $\delta = 2$ and $\delta = 5$. Russia from 1995 to 2003.

	I^1				I^2			
	$\delta = 2$		$\delta = 5$		$\delta = 2$		$\delta = 5$	
Year	Index	Conf.Interval	Index	Conf.Interval	Index	Conf.Interval	Index	Conf.Interval
1995	0.315	[0.309; 0.322]	0.561	[0.555; 0.568]	0.329	[0.322; 0.336]	0.516	[0.508; 0.523]
1996	0.333	[0.327; 0.340]	0.580	[0.573; 0.586]	0.349	[0.342; 0.356]	0.538	[0.531; 0.545]
1998	0.344	[0.336; 0.355]	0.589	[0.581; 0.598]	0.359	[0.350; 0.369]	0.548	[0.540; 0.557]
2000	0.342	[0.335; 0.348]	0.585	[0.578; 0.591]	0.352	[0.346; 0.359]	0.546	[0.539; 0.553]
2001	0.332	[0.327; 0.339]	0.578	[0.572; 0.584]	0.343	[0.336; 0.350]	0.539	[0.532; 0.545]
2002	0.328	[0.324; 0.334]	0.576	[0.570; 0.582]	0.339	[0.334; 0.345]	0.541	[0.535; 0.548]
2003	0.339	[0.333; 0.346]	0.586	[0.580; 0.592]	0.351	[0.344; 0.358]	0.551	[0.544; 0.558]

Source: RLMS 1995-2003.

TABLE 7. Spearman rank correlation coefficient. Russia from 1995 to 2003.

1995 (N=5,011)	Expenditures	Schooling	Health
Expenditures	1		
Schooling	0.1268	1	
Health	0.0483	0.5079	1

1996 (N=5,305)	Expenditures	Schooling	Health
Expenditures	1		
Schooling	0.1438	1	
Health	0.0691	0.5361	1

1998 (N=5,717)	Expenditures	Schooling	Health
Expenditures	1		
Schooling	0.1252	1	
Health	0.0909	0.5832	1

2000 (N=6,221)	Expenditures	Schooling	Health
Expenditures	1		
Schooling	0.1417	1	
Health	0.1139	0.6046	1

2001 (N=7,047)	Expenditures	Schooling	Health
Expenditures	1		
Schooling	0.1344	1	
Health	0.1121	0.6163	1

2002 (N=7,648)	Expenditures	Schooling	Health
Expenditures	1		
Schooling	0.1647	1	
Health	0.1282	0.6394	1

2003 (N=7,700)	Expenditures	Schooling	Health
Expenditures	1		
Schooling	0.1743	1	
Health	0.1484	0.6582	1

Source: RLMS 1995-2003.

All correlation coefficients are different from zero at a 95% confidence level.