Working Paper Series

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ECINEQ WP 2009 – 130

ECINEQ 2009-130
September 2009

www.ecineq.org

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Abstract

A widely accepted criterion for pro-poorness of an income growth pattern is that it should reduce a (chosen) measure of poverty by more than if all incomes were growing equiproportionately. Inequality reduction is not generally seen as either necessary or sufficient for pro-poorness. Because empirical income distributions fit well to the lognormal form, lognormality has sometimes been assumed in order to determine analytically the poverty effects of income growth. We show that in a lognormal world, growth is pro-poor in the above sense if and only if it is inequality-reducing. It follows that lognormality may not be a good paradigm by means of which to examine pro-poorness issues.

Keywords: poverty, growth, pro-poorness, lognormal distribution.
JEL Classification: I32, D63, D31

* I thank Francois Bourguignon, Florent Bresson, Jean-Yves Duclos, Essama Nssah, Stephen Jenkins, Magne Mogstad and Arjan Verschoor for their helpful comments on an earlier draft.

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Introduction

The pro-poor growth literature has lately departed from that on the growth-inequality relationship, and now focuses on the income elasticity of poverty according to various measures. See Foster and Székely (2000, pp. 60-62) for a discussion of this trend (and an opposing suggestion). Pro-poor income growth is now generally conceived, in Klasen's (2008) words, as "growth that is particularly beneficial to the poor". Klasen goes on to comment that "there is considerable debate whether (or to what extent) growth is still 'pro-poor' when it is accompanied by only one of the two conditions, that is, positive income growth of the poor or pro-poor distributional change" (page 420).

According to a line of study that began with Kakwani and Pernia (2000), pro-poorness requires that the incomes of the poor grow faster than those of the rich; see also Kakwani et al. (2004). Ravallion and Chen (2003) take a different approach, arguing that growth is pro-poor if it involves poverty reduction for some choice of a poverty index; see also Kraay (2006) on this. Because of its focus on relative gains, the first interpretation is referred to as a relative approach to assessing the pro-poorness of economic growth, while the second is considered an absolute approach. Osmani (2005) argues for a recalibrated absolute approach, whereby economic growth is considered pro-poor if it achieves an absolute reduction in poverty *greater than would occur in a benchmark case*. In a sense, then, this is also a relative approach. Distributional neutrality is becoming generally adopted as the benchmark. The Osmani approach, in conjunction with a distribution-neutral benchmark, in essence requires that income growth for the poor should exceed the average growth in percentage terms, thereby reducing poverty more than across-the-board benchmark growth would. This relative requirement is clearly much stronger than the absolute one of positive income growth of the poor, which only guarantees that poverty will reduce.[1] However there is a distinction between 'income growth for the poor which exceeds the average growth in percentage terms' and 'inequality-reducing growth', as inequality theorists well appreciate. Something would be lost were the two kinds of growth to come down to the same thing in poverty analysis: we would lose the ability to conduct nuanced investigation of the pro-poorness, growth and inequality nexus.

The lognormal form, which, some authors argue, fits passingly well to empirical income distribution data (Harrison 1981, Cowell 1999; but see also Bandourian et al. 2003 for a contrary view) and has attractive analytical properties, has been used as an aid to the understanding of pro-poorness by a number of authors (see Bouguignon 2003; Epaulard 2003; Klasen and Misselhorn 2006; López and Servén 2006; Kalwij and Verschoor 2006, 2007). Bourguignon recognizes that there is a "complex yet identity-related relationship between mean income growth and poverty change" (*op. cit.* p. 6), but says that it is too demanding and "cumbersome" to require microdata for each country, and he introduces the lognormality assumption as a functional approximation. Kalwij

[1] Foster and Szekely (2000), cited above, themselves propose an approach to measuring pro-poorness which comes down to assessing the extent to which growth in per capita income is accompanied by growth in social welfare and reduction of inequality, both measured using the methodology of Atkinson (1970). They conclude that, after "extensive empirical application involving household surveys from 20 countries over a quarter century ... growth is good for the poor. However, it seems that it is even better for other sectors of society".

and Verschoor (2006) do the same, but in their 2007 paper they suggest using lognormality only to identify the determinants of a poverty elasticity "without imposing the functional form itself", holding that on account of its good fit, the lognormal distribution "contains valuable information about ...determinants" (op. cit., p. 809).

The use of the lognormal form in poverty and growth analysis nevertheless has its critics. Bresson (2009, p. 268) opines that the lognormal "..can be seen as a peculiar choice since these authors choose to set aside all the 20th century debates on the statistical distributions of income," and goes on to show that the systematic use of the lognormal in cross-county analysis "may lead to an overestimation of the growth and inequality elasticities" (p. 290), pointing out also in Bresson (2008) that resort to the lognormal "collapses the heterogeneity of both the shape of observed income distributions and the spectrum of its potential evolutions".

Setting aside these strong reservations, in this paper we push the analysis of the lognormal form a little further, and in a slightly new direction. If distributional change is constrained by the fact that lognormality must be preserved, what in fact *are* the implications? Can we learn anything for the *general* study of pro-poorness from the relationship between poverty, growth and inequality changes which goes on *within the lognormal model*?

Analysis

Let B be the lognormal income distribution $LN(\theta_B, \sigma_B)$ (B denoting 'before growth'), so that an income $x_B \in LN(\theta_B, \sigma_B)$ takes the form $x_B = \exp\{\theta_B + n\sigma_B\}$ where $n \sim N(0,1)$. The mean is $\mu_B = \exp\{\theta_B + \frac{1}{2}\sigma_B^2\}$ (corresponding, incidentally, to the income value at $n = \frac{1}{2}\sigma_B$: see on for the significance of this). After distributional change which preserves the lognormal form, let the new distribution be A. Assuming that people have not changed their relative positions in the income distribution during the growth experience – an assumption which is convenient and also prevalent in the pro-poorness measurement literature[2] - the income value $x_A = \exp\{\theta_A + n\sigma_A\}$ now occupies the position (rank) in distribution A which x_B had in distribution B. Let

(1) $$\frac{\mu_A}{\mu_B} = 1 + g$$

define the proportional growth rate g. If growth is positive, $g > 0$; if there has been recession rather than positive growth, $g < 0$. Assuming $g \neq 0$, the function $q(x)$ defined by

(2) $$q(x_B) = \frac{\left(\frac{x_A}{x_B} - 1\right)}{g} \quad \forall x_B$$

describes the growth pattern: $q(x)$ is an (arc) elasticity measuring the percentage change in income x per 1% change in the mean during the growth process, assuming no rank changes. It follows from (1) and (2) that

(3) $$\ell n\left\{\frac{1 + gq(x_B)}{1+g}\right\} = (\sigma_A - \sigma_B)\left(n - \tfrac{1}{2}(\sigma_A + \sigma_B)\right)$$

(see the Appendix). This formula defines the growth experience of an income x_B in terms of the underlying realization $n \sim N(0,1)$ which defines it and the lognormal spread parameters σ_B and σ_A which determine income inequality.

A number of insights stem from (3). First, if $\sigma_A = \sigma_B$ then $q(x) \equiv 1 \;\; \forall x$ i.e. growth is equiproportionate: there has been a scale change in all incomes, and that is all. Second, if $\sigma_A \neq \sigma_B$ then there is a unique income value, call it x_0, which experiences the same proportional growth rate as the mean: $q(x) = 1 \Leftrightarrow x = x_0$ where, from (3), x_0 corresponds to $n = \frac{1}{2}(\sigma_A + \sigma_B)$. Since

[2] See Jenkins and Van Kerm (2006) and Grimm (2007) in respect of the new issues which must be confronted in measuring pro-poorness of growth when there is mobility among the poor, i.e. when some who are initially poor, as well as some who are not, cross the poverty line.

$\frac{1}{2}(\sigma_A + \sigma_B) > \frac{1}{2}\sigma_B$, and $\frac{1}{2}\sigma_B$ is the $N(0,1)$ realization corresponding to the mean μ_B in the initial distribution, *this critical income value is above the mean*.

The income ranges in which the proportional growth rates that are experienced are higher/lower than the overall growth rate g can also be identified from (3). These ranges differ depending on whether there is positive growth or recession:

(4a) If $g > 0$ & $\sigma_A < \sigma_B$ then $q(x) \gtrless 1 \Leftrightarrow x \lessgtr x_0$

(4b) If $g > 0$ & $\sigma_A > \sigma_B$ then $q(x) \gtrless 1 \Leftrightarrow x \gtrless x_0$

(4c) If $g < 0$ & $\sigma_A < \sigma_B$ then $q(x) \gtrless 1 \Leftrightarrow x \gtrless x_0$

(4d) If $g < 0$ & $\sigma_A > \sigma_B$ then $q(x) \gtrless 1 \Leftrightarrow x \lessgtr x_0$

In particular, in times of positive growth in a lognormal world, all below-average incomes are necessarily growing more quickly than the mean growth rate if inequality is falling, and they are necessarily growing more slowly than the mean growth rate if inequality is rising. In times of recession, $q(x) > 1$ signifies that income x *falls* more quickly than the mean rate of decline, in the general *decrease* in income values that is taking place. Hence if recession is accompanied by rising inequality in a lognormal world, all below-average incomes are necessarily falling more quickly than the mean rate of decline, and if recession is accompanied by falling inequality, all below-average incomes are necessarily falling more slowly than the mean rate of decline.

These mechanisms place severe limitations on what can happen at the individual level during growth or recession, if distributional change is such that the lognormal form is preserved! Despite the lognormal distribution’s passable fit to real-world data, one would hardly want to argue that individual growth/recession experiences in the real world should necessarily follow the patterns described in (4a)-(4d).

What about the pro-poorness of income growth/recession in a lognormal world? The view that distributional change should be judged pro-poor if it reduces poverty by more than if all incomes were growing at a (benchmark) uniform rate g is quite widely accepted. To be classified as pro-poor in this sense, a growth pattern $q(x)$ should lead to more poverty reduction than the benchmark growth pattern, call it $q_0(x) \equiv 1 \ \forall x$. In times of positive growth, this property is assured if $q(x) > 1$ for all incomes up to the poverty line, since then all poor incomes grow by more in absolute terms than they would in the benchmark scenario. In times of recession, pro-poorness is assured if $q(x) < 1$ for all incomes up to the poverty line – for then all poor incomes fall by less, and as a result poverty increases by less, than in the benchmark situation.[3]

A non-destitute society is one whose poverty line is below the mean income (Cowell, 1988, p.159). If a non-destitute society has a lognormal income distribution, then its poverty line *a fortiori* lies below the critical income level x_0 which features in (4a)-(4d). If, in such a society, the lognormal form is preserved during growth or recession, then (4a) to (4d) apply:

Theorem 1

In a lognormal world, and in times of positive growth or recession, distributional change in any non-destitute society is unambiguously pro-poor if inequality falls and unambiguously anti-poor if inequality rises.[4]

[3] The only assumption here is that overall poverty falls when, all else equal, a poor person's income rises (i.e. his/her income shortfall from the poverty line decreases). This is true of all members of the additive and separable class of poverty indices used, for example, by Kakwani and Pernia (2000). By the same token, positive growth is unambiguously anti-poor if $q(x) < 1$ for all incomes up to the poverty line, and recession is unambiguously anti-poor if $q(x) > 1$ for all incomes up to the poverty line. But it is not necessary that $[q(x) - 1]$ should have the same sign for all incomes up to the poverty line in order that, *for a particular poverty index in this class*, growth be counted as pro-poor (or, indeed, anti-poor). In Essama-Nssah and Lambert (2009), pro-poorness measurement *à la* Osmani, with the distribution-neutral benchmark, is systematized in terms of the growth pattern function $q(x)$.

[4] The inequality change referred to here is in the strong sense of a Lorenz improvement or worsening, since the spread parameter σ determines the Lorenz curve for a lognormal income distribution independently of the location parameter θ, and lognormal Lorenz curves do not intersect. The pro-poorness criterion is that of Osmani (2005) with distribution-neutral benchmark.

This perhaps surprising result brings us full circle. In a lognormal world, there is no difference between pro-poor growth and inequality-reducing growth! To the extent that the pro-poor literature has lately departed from that on the growth-inequality relationship, this departure clearly must refer to a non-lognormal world.[5]

What are the implications of our analytics for the other approaches to pro-poorness measurement? Foster and Szekely (2000) approach pro-poorness directly in terms of inequality reduction (recall footnote 1). In a lognormal world, this is identical to the widely accepted approach we have described. Ravallion and Chen's (2003) 'absolute' approach to the pro-poorness of growth fares slightly differently.

For Ravallion and Chen, growth is pro-poor if it involves poverty reduction for some choice of poverty index. They show that the mean growth rate of the poor measures the rate of decrease in the Watts index (suitably normalized, see their equation (4)). They term this measure "the rate of pro-poor growth"; if it is positive, the growth pattern counts as pro-poor for Ravallion and Chen. It is clear that in general an inequality enhancement can be accompanied by a reduction in an additive and separable poverty index such as the Watts. Ravallion and Chen's 'growth incidence curve' (henceforth GIC) shows "how the growth rate for a given quantile varies across quantiles ranked by income" (ibid., page 94). In our lognormal world, if *n(p) is* the $N(0,1)$ realization at percentile *p*, so that $x_B(p) = \exp\{\theta_B + n(p)\sigma_B\}$ is the income level at percentile *p* in the initial distribution, then $GIC(p) = gq(x_B(p))$. Thus, from (3) we have

$$(5) \qquad \ell n\left\{\frac{1+GIC(p)}{1+g}\right\} = (\sigma_A - \sigma_B)\Big(n(p) - \tfrac{1}{2}(\sigma_A + \sigma_B)\Big)$$

For percentiles p below the headcount in our non-destitute society, i.e. for income levels in distribution B which are below the mean, we have $n(p) < \frac{1}{2}\sigma_B$ as

[5] Later, we shall consider how inequality, growth and pro-poorness are related for some other parametric forms one might fit to income distribution data.

already explained. Hence at these percentiles, $GIC(p) \gtrless g$ according as $\sigma_A \lessgtr \sigma_B$.

What else, beyond the fact that $GIC(p) < g$, does (5) tell us if inequality increases? Suppose that the change in inequality is very small. Using the approximation that $\ell n(1+x) \approx x$ when x is small, we can draw out from (5) that

$$GIC(p) \approx g + (1+g)(\sigma_A - \sigma_B)\left(n(p) - \tfrac{1}{2}(\sigma_A + \sigma_B)\right) \tag{6}$$

For a sufficiently small inequality increase, the mean growth rate of the poor is non-negative (just integrate up to the headcount in (6), and then normalize by the headcount, to see this). Hence inequality-increasing growth can indeed count as pro-poor for Ravallion and Chen.

However, for the mean growth rate of the poor to exceed the aggregate growth rate *g* requires that $\sigma_A < \sigma_B$ – and then growth for each poor person exceeds benchmark growth, and theorem 1 applies. Interestingly, Ravallion and Chen note that the mean growth rate of the poor "... can be interpreted as the ordinary growth rate in the mean scaled up or down according to whether the distributional changes were pro-poor" (page 94). In a lognormal world, then, Ravallion and Chen's *pro-poor distributional changes* coincide exactly with *inequality-reducing changes.*

Concluding Discussion

The assumption of lognormality is "probably the most standard approximation of empirical distributions in the applied literature" (Bourguignon, 2003, page 11). For Klasen and Misselhorn (2006, page 3), "...the assumption of lognormality achieves the goal of providing a simple, yet powerful tool to assess and project poverty reduction depending on country circumstances." But is lognormality a good paradigm by means of which to examine pro-poorness issues? Given that inequality reduction and pro-poorness get conflated within a lognormal world, and that economists have worked hard in recent years to draw appropriate distinctions between these two concepts, our result will surely provide steam for the critics of lognormality.

If the functional form of the income distribution has only one spread parameter, it might seem intuitive that a rise (fall) in that spread parameter must mean that the incomes of the poor rise more slowly (faster) than those of the rest with positive growth. We have proved that this intuition holds exactly for the lognormal distribution. Clearly, then, the lognormal framework is inappropriate to model pro-poor growth as other than inequality-reducing growth. Since we can readily find cases where pro-poorness is associated with increasing or falling inequality in the real world[6], might a better-fitting model of the income distribution than the lognormal, with one or more additional spread parameters, lend itself to the nuanced analysis of the pro-poorness, growth and inequality nexus that we spoke of?

The displaced lognormal distribution has been found to correct for the negative skewness typically found in the distribution of log income, and was used by, for example, Gottschalk and Danziger (1985) to model income divided by the poverty line in their study of US growth and poverty. The Singh-Maddala (or Burr) distribution was found by McDonald (1984) to provide a better fit to US family nominal income for 1970-1980 than any other 2- or 3-parameter distribution he tried, and also better than some 4-parameter distributions (*ibid*., p. 659). Both of these distributions are reasonably tractable analytically, and in each case the direct link between pro-poorness and inequality reduction is broken (see the Appendix). Deeper analysis of the effect of (marginal) parameter changes in these models may bring new insights.

Our lognormal findings also inform current practice in respect of what has become known as "the simple arithmetic" of poverty and growth analysis.[7] From at least Datt and Ravallion (1992) and Kakwani (1993) onwards, growth and distributional change have been separated and separately assessed in their impact on poverty, typically using fitted Lorenz curves.[8] In Bourguignon (2003),

[6] See for example Table 1 in Ravallion (2001) on this.
[7] See e.g. Epaulard (2003, page 20) for this term.
[8] Kakwani and Pernia (2000, page 6) state that "to understand.. the impact of growth on poverty, one needs to measure separately the impact … of changes in average income and in its distribution. In other words, one needs to decompose the total change ..". See also Epaulard (2003, page 11) and Klasen and Misselhorn (2006, page 23). In Kakwani (2000) an axiomatic

now widely cited, lognormality is used to derive the elasticity of poverty with respect to benchmark growth, and with respect to inequality change, *separately*. Indeed it is true that in a lognormal world g and σ_A are the twin determinants of a growth pattern, $q(x)$ or $GIC(p)$, given the initial inequality level σ_B, but their effects on the incomes of the poor and therefore on poverty are not separable: just look at (3), (5) and (6) to see this.[9] Clearly the simple arithmetic has its limitations.

Finally, we remark that in Duclos (2009), in which pro-poorness is formulated in more abstract terms than any we have used here, the issue of transitivity in a pro-poorness ordering is mooted (p. 53). Adapting Duclos' text slightly, the question in our terminology would be this: "One might ... want to test whether a distributional change from B to A is more pro-poor than a distributional change from B to C ... this could be done by testing whether the movement from C to A is pro-poor ... then, by transitivity, a change from B to A can be considered to be more pro-poor than a change from B to C". Transitivity of pro-poorness is a complex question in general; not least, any adjustment of the poverty line for income growth between scenarios C and A could confound matters. But in our lognormal world, the issue is simple: does inequality fall by more from B to A than from B to C? Provided only that C is a non-destitute society (as we have assumed of B), the move from C to A is pro-poor if $\sigma_A < \sigma_C$, and then indeed inequality falls by more from B to A than from B to C.

In sum, the links between poverty, inequality and growth are unlikely to be so clear-cut in reality as the use of the lognormal model would suggest.

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framework is used to show that appropriately defined pure growth and pure inequality effects will determine the overall effect on poverty as a sum. In Tsui's (1996) axiomatic study of additive growth-equity poverty change decomposition, the equity term refers to distributional change among the poor (only).

[9] This observation echoes Heltberg's (2004, p. 90) warning: "One needs to be careful ….the manner in which growth and inequality interact to shape poverty is not additive".

Appendix

(a) the derivation of (3):

Taking logarithms in (1), and noting that $\ell n(\mu_B) = \theta_B + \frac{1}{2}\sigma_B^2$ and $\ell n(\mu_A) = \theta_A + \frac{1}{2}\sigma_A^2$, we have $\theta_A - \theta_B = \ell n(1+g) + \frac{1}{2}\sigma_B^2 - \frac{1}{2}\sigma_A^2$. Cross-multiplying, rearranging and then taking logs in (2), we have $\ell n\left\{1 + gq(x_B)\right\} = \ell n x_A - \ell n x_B = \theta_A - \theta_B + n(\sigma_A - \sigma_B)$. Combining these two equations, (3) follows immediately.

(b) the displaced lognormal distribution:

Let x again be income and now let k be a number such that $x - k \sim LN(\theta,\sigma)$. Mean income is $k + \exp\left(\theta + \frac{1}{2}\sigma^2\right)$, the coefficient of variation is $\left(1 - \dfrac{k}{\upsilon}\right).\sqrt{\left(e^{\sigma^2} - 1\right)}$ and a typical income is $x = k + \exp\left(\theta + n\sigma\right)$ where $n \sim N(0,1)$. It may be verified that if (θ,σ,υ) changes from (8, 0.5, 1000) to (9, 0.4, 250), the mean more than doubles (from 4378 to 9027), the coefficient of variation falls (from 0.169 to 0.164) and all incomes *x* in the first decile less than double (in fact, this happens for all *n* < -1.179 and also for all *n* > 2.083). So the growth is not pro-poor (Essama and Lambert 2009, theorem 4).

(c) the Singh-Maddala distribution:

The Singh-Maddala has cumulative distribution function $F(x) = 1 - \left[1 + \left(\dfrac{x}{b}\right)^a\right]^{-q}$ where *a, b* and *q* are positive. It is unimodal if *a* > 1 and has finite 1st and 2nd moments if *aq* > 2. Inverting the c.d.f., an income x can be specified as $x = b\left[(1-u)^{-1/q} - 1\right]^{1/a}$ where *u* is uniformly distributed on [0,1]. The first and second moments are $b.\Gamma\left(1 + \dfrac{1}{a}\right).\Gamma\left(q - \dfrac{1}{a}\right)\Big/\Gamma(q)$ and $b^2.\Gamma\left(1 + \dfrac{2}{a}\right).\Gamma\left(q - \dfrac{2}{a}\right)\Big/\Gamma(q)$ where $\Gamma(x)$ is the gamma function. In McDonald (1984), parameter values (a,b,q) are fitted to US family nominal income distributions of (1.9652, 18.7288, 2.9388) for 1970, (1.8648, 31.5176, 3.7657) for 1975 and (1.6971, 87.6981, 8.3679) for 1980. Mean income rises by 39% from 1970 to 1975, and by 53% from 1975 to 1980. The coefficient of variation falls between years, but the income growth is not pro-poor: as may be verified, incomes in the first two deciles experience less than 95% of the mean growth during each period.[10]

[10] McDonald reports a rising Gini coefficient between years. This is a useful reminder that when Lorenz curves cross (as here), a distributional change can be deemed inequality-reducing or inequality-increasing according to the index used. For the Singh-Maddala distribution, Lorenz curves cross if and only if *a* and *aq* move in opposite directions (Kleiber 2008, p. 233).

References

Atkinson, A.B. (1970). On the measurement of inequality. *Journal of Economic Theory*, vol. 2, pp. 244-263.

Aitchison, J. and J.A.C. Brown (1957). *The Lognormal Distribution: With Special Reference to Its Uses in Economics*. Cambridge University Press.

Bandourian, R., J. McDonald and R.S. Turley (2003). A comparison of parametric models of income distribution across countries and over time. *Estadistica*, vol. 55, pp. 135–152.

Bourguignon, F. (2003). The growth elasticity of poverty reduction: explaining heterogeneity across countries and time periods. Chapter 1, pp. 3-26, in Eicher, T.S. and S. J. Turnovsky (eds.) *Inequality and Growth: Theory and Policy Implications.* Cambridge MA: MIT Press.

Bresson, F. (2008). A general class of inequality elasticities of poverty. *Journal of Economic Inequality*, forthcoming.

Bresson, F. (2009). On the estimation of growth and inequality elasticities of poverty with grouped data. *Review of Income and Wealth*, vol. 55(2), pp. 266-302.

Cowell, F.A. (1988). Poverty measures, inequality and decomposability. In Bös, D., M. Rose and C. Seidl (eds.) *Welfare and Efficiency in Public Economics*. Heidelberg: Springer Verlag.

Cowell, F.A. (1999). Measurement of inequality. In Atkinson, A.B. and Bourguignon, F. (eds.), *Handbook of Income Distribution*. North-Holland, Amsterdam.

Datt, G. and M. Ravallion (1992). Growth and redistribution components of changes in poverty measures: a decomposition with applications to Brazil and India in the 1980s. *Journal of Development Economics*, vol. 38, pp. 275-295.

Duclos, J.-Y. (2009). What is pro-poor? *Social Choice and Welfare*, vol. 32, pp. 37-58.

Epaulard, A., 2003. Macroeconomic Performance and Poverty Reduction. *Working Paper No 03/72,* International Monetary Fund.

Essama-Nssah, B. and P.J. Lambert (2009). Measuring pro-poorness: a unifying approach with new results. *Review of Income and Wealth,* vol. 55, pp. 752-778.

Foster, J.E. and M. Szekely (2000). How good is growth? *Asian Development Review*, vol. 18, pp. 59-73.

Gottschalk, P. and S. Danziger (1985). A framework for evaluating the effects of economic growth and transfers on poverty. *American Economic Review*, vol. 75, pp. 153-161.

Grimm, M. (2007). Removing the anonymity axiom in assessing pro-poor growth. *Journal of Economic Inequality*, vol. 5, pp. 179–197.

Harrison, A.J. (1981). Earnings by size: a tale of two distributions. *Review of Economic Studies,* vol. 48, pp. 621-631.

Heltberg, R. (2004). The growth elasticity of poverty. Chapter 4, pages 81-91, in A. Shorrocks and R. van der Hoeven (eds.), *Growth, Inequality and Poverty*. Oxford University Press for UNU/WIDER.

Jenkins, S.P. and P. Van Kerm (2006). Trends in income inequality, pro-poor income growth, and income mobility. *Oxford Economic Papers,* vol. 58, pp. 531-548.

Kakwani, N.C. (1993). Poverty and economic growth with application to Côte d'Ivoire. *Review of Income and Wealth*, vol. 39, pp. 121-139.

Kakwani, N.C. (2000). On measuring growth and inequality components of poverty with application to Thailand. *Journal of Quantitative Economics*, vol. 16(1), pp. 67-80.

Kakwani, N.C., S. Khandker and H.H. Son (2004). Pro-poor growth: concepts and measurement with country case studies. *Working Paper Number 2004-1*, International Poverty Center, Brasil.

Kakwani, N.C. and E.M. Pernia (2000). What is pro-poor growth? *Asian Development Review,* vol. 18, pp. 1-16.

Kalwij, A., Verschoor, A., 2006. Globalisation and poverty trends across regions. The role of variation in the income and inequality elasticities of poverty. In Nissanke, M. and E. Thorbecke (eds.), *The Impact of Globalization on the World's Poor*. Palgrave Macmillan.

Kalwij, A. and A. Verschoor (2007). Not by growth alone: the role of the distribution of income in regional diversity in poverty reduction. *European Economic Review,* vol. 51 pp. 805–829

Klasen, S. (2008). Economic growth and poverty reduction: measurement issues using income and non-income indicators. *World Development,* vol. 36(3), pp. 420–445.

Klasen, S. and M. Misselhorn (2006). Determinants of the growth semi-elasticity of poverty reduction. *No. 15, Proceedings of the 2006 German Development Economics Conference*, Berlin.

Kleiber, C. (2008). The Lorenz curve in economics and econometrics. Chapter 13, pp. 225-242, in Betti, G. and A. Lemmi (eds.) *Advances on Income Inequality and Concentration Measures.* London: Routledge.

Kraay, A. (2006). When is growth pro-poor? Evidence from a panel of countries. *Journal of Development Economics,* vol. 80, pp. 198– 227

López, J.H. and L. Servén (2006). A normal relationship? Poverty, growth, and inequality. *Policy Research Working Paper No. 3814*, World Bank.

McDonald, J.B. (1984). Some generalized functions for the size distribution of income. *Econometrica*, vol. 52, pp. 647-663.

Osmani, S. (2005). Defining pro-poor growth. *One Pager Number 9*, International Poverty Center, Brasil.

Ravallion, M. (2001). Growth, inequality and poverty: looking beyond averages. *World Development*, vol. 29, pp. 1803-1815.

Ravallion, M. and S. Chen (2003). Measuring pro-poor growth. *Economics Letters*, vol. 78, pp. 93-99.

Tsui, K.-Y. (1996). Growth-equity decomposition of a change in poverty: an axiomatic approach. *Economics Letters*, vol. 50, pp. 417-423.