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**Kakwani decomposition of redistributive effect: Origins, critics and upgrades**

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## Kakwani decomposition of redistributive effect: Origins, critics and upgrades

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### Abstract

The Kakwani decomposition of redistributive effect into vertical and reranking terms is one of the most widely used tools in measurement of income redistribution. This paper describes how the decomposition has emerged, how its proponents managed to expand and upgrade it, and how extensively it has been employed in empirical research. However, the arguments are presented that the decomposition features certain methodological problems and therefore a reinterpretation is called for.

**Keywords:** income redistribution, Kakwani decomposition, reranking, horizontal inequity, progressivity

**JEL classifications:** D63, H22, H23.

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## 1 Introduction

The last three decades have seen a world-wide interest in the *measurement of the redistributive effects of fiscal systems*. The research in this field is underpinned by a wide belief that the state has a major role in the determination of economic inequality in a society. The contention is undeniably proved by various empirical studies. Many researchers in the field of income redistribution posed further natural questions: what are *the contributions of individual taxes and benefits* to the redistributive effect?

The literature on the measurement of the redistributive effect started to grow in the mid-seventies, perhaps half of decade after the seminal works in the closely related field of economic inequality measurement. Although the first famous paper on the redistributive effect was written by Musgrave and Thin (1948) sixty years ago, the works of Jakobsson (1976) and Kakwani (1977a, 1977b) have established the propositions fundamental for all further research. As Lambert (2001) points out: “Their central results are known [...] as *Jakobsson/Kakwani* theorems, and they expose the links between progressive income taxation and concentration curve properties of the distributions of tax and of post-tax income”.

Progressive tax pushes the income Lorenz curve toward the line of equality. The magnitude of this movement, measured as the difference between Gini coefficients of pre-tax and post-tax incomes, is nowadays known as the *redistributive effect*, and the greater the tax progressivity and the average tax rate, the larger the redistributive effect will be. All these notions were explained by Kakwani (1977b). However, one phenomenon remained hidden for some time, until Atkinson (1980) and Plotnick (1981) uncovered it. They noticed that taxation induces another process besides narrowing of income distances – income units reranking, which is simply measured as a difference between the Gini and the concentration index of post-tax income.

The great synthesis of all these concepts arrived in Kakwani (1984) as a decomposition of the redistributive effect into vertical or progressivity and reranking terms. It became and remained one of the most important tools in the income redistribution literature. The popularity of this decomposition rests on its comprehensiveness (capturing different notions of redistributive justice), simplicity and ease of computation as well as its availability for

straightforward policy interpretation (redistributive power can be enhanced if horizontal inequity is reduced).

The current paper is a critical overview of the Kakwani methodology, of the most important methodologies rooted in Kakwani decomposition, and of the empirical research in income redistribution. Some doubt and criticism is cast upon the Kakwani decomposition by other eminent scholars, which has, however, been mostly neglected. Here, the issues are reopened and the paper calls for further investigation.

The paper is organized as follows. Section *two* contains description of how the decomposition and its component indices have emerged. In the *third* section, the most important extensions and upgrades of the decomposition are presented. Examination of empirical research, which verifies that the Kakwani decomposition is a central methodological tool in the field, is in the *fourth* section. The paper ends with conclusion and recommendations for further research.

## **2 Kakwani decomposition of redistributive effect**

### ***The decomposition***

#### *Redistributive effect*

The standard and the most popular measure of the redistributive strength of a fiscal (tax, or benefit) system is called *the redistributive effect (RE)*. It is defined as a difference between Gini coefficients of pre-fiscal (-tax, -benefit) income ( $G_X$ ) and post-fiscal (-tax, -benefit) income ( $G_N$ ), as shown in (1).

$$(1) \quad RE = G_X - G_N$$

Some other indices are also called “redistributive effect”, but should be distinguished from the redistributive effect as defined in (1). Musgrave and Thin (1948) proposed a different index ( $RE^{MT}$ ), which also relates the indices  $G_X$  and  $G_N$ , but in the way shown by (2).

$$(2) \quad RE^{MT} = \frac{1 - G_N}{1 - G_X}$$

Reynolds and Smolensky's (1977) index of the redistributive effect ( $RE^{RS}$ ) is very similar to  $RE$  in (1), but instead of Gini coefficient of post-fiscal income ( $G_N$ ), we find here a concentration coefficient of post-fiscal income ( $D_N^x$ ), as presented in (3).

$$(3) \quad RE^{RS} = G_x - D_N^x$$

Observe the important difference between the Gini ( $G_N$ ) and concentration ( $D_N^x$ ) indices of post-fiscal income.  $G_N$  is obtained for the Lorenz curve of post-fiscal incomes,  $L_N(p)$ , while  $D_N^x$  is calculated for the concentration curve of post-fiscal incomes,  $C_N^x(p)$ . While for  $L_N(p)$  the units are sorted in ascending order of post-fiscal income, for  $C_N^x(p)$  the income units are ordered by *pre*-fiscal income; hence,  $x$  in the superscript of  $C_N^x(p)$ . It is proved (e.g. in Lambert, 2001) that the concentration curve, such as  $C_N^x(p)$ , never lies below the corresponding Lorenz curve, here  $L_N(p)$ , and therefore,  $G_N$  can never be lower than  $D_N^x$ .

*Kakwani decomposition: original vs. modern presentation*

The decomposition of the redistributive effect ( $RE$ ) that is central to this investigation is first presented in Kakwani (1984:159-163) and repeated with minor differences, mostly in notation, in Kakwani (1986:82-86). The aim of the model is to capture two well-known theoretical concepts – horizontal and vertical equity – into a unified measurement framework. The original methodology used a different presentation (and notation) from the one that is usual nowadays, but it will be useful to compare these two. Kakwani expressed the main components in terms of the pre-tax Gini coefficient, as follows. The redistributive effect ( $R$ ) is the difference between the Gini coefficients of pre- and post-tax income, as in (1), but divided by  $G_x$ , as in (4).

$$(4) \quad R = \frac{G_x - G_N}{G_x}$$

The redistributive effect ( $R$ ) is decomposed into the sum of horizontal inequity ( $H$ ) and vertical equity ( $V$ ) terms, as shown in (5).

$$(5) \quad R = H + V$$

The horizontal inequity index ( $H$ ) is a difference between the concentration coefficient of post-tax income ( $D_N^x$ ) and  $G_N$ , normalized by  $G_X$ , as in (6). The vertical equity index ( $V$ ) is equal to Kakwani progressivity index ( $P_T^K$ ) scaled by the average tax rate ( $t^x$ ), and normalized by  $G_X$ , as shown by (7).

$$(6) \quad H = \frac{D_N^x - G_N}{G_X},$$

$$(7) \quad V = \frac{t^x P_T^K}{(1 - t^x) G_X}$$

Modern exposition is different in several respects. The notion “horizontal inequity” is changed into “reranking”, for reasons that will be discussed below. The Kakwani horizontal inequity term ( $H$ ), based on the difference  $D_N^x - G_N$ , is replaced by its negative correspondent,  $G_N - D_N^x$ , and named more conveniently as the Atkinson-Plotnick index of reranking ( $R^{AP}$ ). In addition, the components are presented as “absolute” values of coefficients, and not in terms of relative to  $G_X$ . Thus, what we have in (8) is representing the decomposition of the redistributive effect ( $RE$ ) into Kakwani vertical effect ( $V^K$ ) and Atkinson-Plotnick index of reranking ( $R^{AP}$ ).

$$(8) \quad RE = V^K - R^{AP}$$

The differences between two presentations are shown in Table 1. The Kakwani horizontal inequity term ( $H$ ) can never be positive. On the other side, the Atkinson-Plotnick index of reranking ( $R^{AP}(= -H)$ ) is always non-negative. The minus sign in front of the reranking term, in the modern expression (8), better reflects the common notion that reranking reduces the redistributive effect (more on this, below).

*Table 1: Presentation of Kakwani decomposition*

	<b>Original</b>		<b>Modern</b>
	Redistributive effect		
(4)	$R = \frac{G_X - G_N}{G_X}$	(1)	$RE = G_X - G_N$

Index of progressivity	
(9) $P_T^K = D_T^x - G_X$	
Horizontal / Reranking effect	
(10) $H = \frac{D_N^x - G_N}{G_X}$	(11) $R^{AP} = G_N - D_N^x$
Vertical effect	
(12) $V = \frac{t^x P_T^K}{(1-t^x)G_X}$	(13) $V^K = \frac{t^x P_T^K}{1-t^x} = G_X - D_N^x$
Decomposition of the redistributive effect	
(14) $G_X - G_N = (D_N^x - G_N) + \frac{t^x P_T^K}{1-t^x}$	(15) $G_X - G_N = \frac{t^x P_T^K}{1-t^x} - (G_N - D_N^x)$
(5) $R = H + V$	(16) $G_X - G_N = (G_X - D_N^x) - (G_N - D_N^x)$
	(8) $RE = V^K - R^{AP}$

### *Horizontal inequity or reranking?*

The principle of vertical equity requires that people with larger income pay higher taxes than those with lower income. The standard definition of horizontal equity in taxation requires that people with equal income pay equal taxes. Violation of this principle gives rise to horizontal inequity. Following several other authors, Kakwani identifies horizontal inequity with reranking. However, it seems that by decomposition of the redistributive effect into  $H$  ( $R^{AP}$ ) and  $V$  ( $V^K$ ) we obtain two measures that both deal with unequal treatment of unequals. The following example will prove this contention.

If we want to measure the violation of equality, we must first be able to define equals. People with different ranks are usually not in *equal* positions. For example, person A has income of 100\$ and B has 500\$. Imagine that the fiscal process reversed their incomes, and now A has 500\$ and B only 100\$: reranking occurred. Everybody will agree that inequity has happened, but should we call it *horizontal* inequity? A and B were not in equal positions before the fiscal action: B had five times larger income than A. The followers of Kakwani decomposition have

realized the problem, and today it is common to name the effect reranking instead of horizontal inequity.<sup>1</sup>

### ***The components***

#### *Origins of vertical effect*

The Kakwani index of tax progressivity ( $P_T^K$ ) has emerged as a reaction to certain inadequacies of Musgrave and Thin “index of progressivity” ( $RE^{MT}$ ) and the index of the redistributive effect ( $RE$ ). For Kakwani (1977b), two tax systems with equal elasticity of tax liability to pre-tax income, over the whole income distribution, should be judged as *equally progressive*. As Kakwani shows, the proposed index ( $P_T^K$ ) satisfies this requirement, while the indices  $RE^{MT}$  and  $RE$  fail in this task.

Kakwani (1977a, 1977b) proved the following relationship between the index of progressivity and the redistributive effect, shown in (17).<sup>2</sup>

$$(17) \quad RE = G_x - G_N = \frac{t^x P_T^K}{1 - t^x}$$

According to equation (17), the redistributive effect ( $RE$ ) represents the reduction of inequality of taxation, but it *does not* measure tax progressivity. Two equally progressive tax systems with different average tax rates would therefore be erroneously observed as differently progressive by  $RE$  or  $RE^{MT}$ .

#### *Origins of the reranking effect*

The concept of the reranking effect that appears in the Kakwani decomposition was derived almost simultaneously and independently by two scholars, Atkinson (1980) and Plotnick (1981).<sup>3</sup> Analyzing empirical literature, Atkinson concluded that some studies used the

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<sup>1</sup> Obvious contribution to renaming came from Aronson, Johnson and Lambert (1994) who invented a decomposition of the redistributive effect into vertical, horizontal *and* reranking distinctive effects (see more on this below). Also, see the serious criticism of horizontal inequity by Kaplow (1989; 2000).

<sup>2</sup> Observe that (17) differs from (15): the former equation did not recognize the existence of reranking.

<sup>3</sup> Hence the superscript AP in  $R^{AP}$ , defined in (11). Both works contain “horizontal (in)equity” in their titles. In the previous section we have seen the problem with the term “horizontal”, and why “reranking” is a better choice.



*concentration* coefficient of post-fiscal income ( $D_N^x$ ), which understates the true post-fiscal inequality, as would be measured by the corresponding Gini coefficient of post-fiscal income ( $G_N$ ). Consequently, a measure  $G_x - D_N^x$  (identical to Reynolds-Smolensky index,  $RE^{RS}$ ) overstates the redistributive effect of fiscal system ( $RE^{RS} > RE$ ). Therefore, for Atkinson, one motive for measurement of reranking would be a correct assessment of income redistribution. The other is to estimate the magnitude of mobility along the income scale induced by the fiscal process.

Atkinson explained several ways in which the new concept of reranking could be measured, but left the issue of choosing the best one somewhat open. He was not very inclined toward single index measures, as “any such summary statistic involves assumptions that may be little more than arbitrary”. Use of the transition matrix<sup>4</sup> or Lorenz and concentration curves was considered much more preferable.

Contrary to that, a single index measure of reranking is the most widely used form of presentation. This approach started with Plotnick, whose index is  $R^P = 100\% \cdot R^{AP} / G_N$ . Expressing reranking as a percentage of the post-fiscal Gini coefficient has logical background: the maximum value of  $R^{AP}$  is exactly  $G_N$ .<sup>5</sup> Plotnick also suggested stating  $R^{AP}$  as a percentage of  $RE$ , which has become standard practice in empirical studies.

Plotnick also discusses the issue of the appropriate benchmark ranking of income units. In order to evaluate inequity, we must first establish what is equitable. The pre-fiscal ranking is said to be a natural choice, supported by both theoretical works and common belief, that the ordering of people emerging as a result of activities on the market should not be disturbed in the fiscal process.<sup>6</sup> Of course, we should be able to properly calculate pre-fiscal incomes, i.e. incomes in

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<sup>4</sup> The transition matrix shows complete and detailed transformation of income vector from one ordering to another in steps, where each step involves changing the ranks of two units. For example, before step  $t$ , A has income  $a$  and rank  $v$ ; B has income  $b$  and rank  $w$ . After step  $t$ , A has income  $b$  and rank  $w$ , while B has income  $a$  and rank  $v$ .

<sup>5</sup> In this extreme case, when ranks of all units are changed,  $C_N^x(p)$  lies above the line of absolute equality and is symmetrical to  $L_N(p)$ .

<sup>6</sup> In the last passage, Atkinson (1980:18) briefly mentions several different conceptual views on the overall issue of (re)ranking and horizontal inequity.

the absence of fiscal activities, and this is likely an impossible task. Still, relying on certain fiscal incidence assumptions and existing data we can produce some estimates of fiscal inequities.

*The role of reranking: advice to future developers*

In the view of Atkinson (1980), reranking does not influence overall redistributive effect, *RE* (see also Jenkins (1988), and Duclos (1993), who shared this view). He states: “Changes in the ranking of observations as a result of taxation do not in themselves affect the degree of inequality in the post-tax distribution. They do, however, influence certain ways of representing the redistribution and of calculating summary measures of inequality.” This is a very important statement, with the following messages that can be deduced. Researchers should be aware of the extent of reranking caused by the fiscal system in order to make judgments about the quality of the redistributive process. Reranking is a by-product or a consequence of an income redistribution process; it does not contribute, positively or negatively, to the redistributive effect.

It is interesting that Plotnick (1981) had similar thoughts on the relationship between the measures of reranking and the redistributive effect. In his model “the structure of post-redistribution income inequality is taken as a datum by the measure”. As in Atkinson (1980), this implies that the reranking should not be interpreted as a factor that produces redistribution. Instead, “...given the change in inequality, the measure should tell us how seriously the redistributive activities violated the norms of horizontal equity.” Also, Plotnick warns that the measure of reranking “should not attempt to compare the actual extent of redistribution or change in inequality to some exogenous criterion. Doing so would be an exercise in measuring vertical inequity.”

Thus, both Atkinson and Plotnick, more or less explicitly, avoid setting the reranking effect into any context other than the measurement of “horizontal inequity”. Although aware of the strong connection between the concepts of vertical and horizontal equity, they do not attempt to build a comprehensive model capturing both of them. They implicitly suggest to future users and developers to be cautious about the introduction of a reranking measure into some broader frameworks.

### *Advice taken?*

It seems that the message did not reach Kakwani, whose “horizontal inequity” term  $H$  ( $H = R^{AP}$ ) is given a specific interpretation: it measures a reduction of the redistributive effect (“increase in inequality”). New interpretation of the decomposition has later emerged in literature, which goes as in the following passage:

*A progressive fiscal system is “good” because it reduces inequality. In case of progressivity, Kakwani vertical effect ( $V^K$ ) is positive, and the larger it is, the “better”. On the other side, reranking increases inequality. In the presence of reranking, the Atkinson-Plotnick index ( $R^{AP}$ ) is positive and the larger it is, the “worse”. Furthermore,  $V^K$  measures a “potential” redistributive effect, attainable in the absence of reranking. Because of reranking, actual redistribution amounts “only” to  $RE = V^K - R^{AP}$ . If reranking could be somehow eliminated, redistributive effect would be equal to the potential amount:  $RE^{potential} = V^K$ .*

This interpretation contributed significantly to the popularity of Kakwani decomposition among applied researchers and scholars as well. It is straightforward, capturing some of the desired qualities of a fiscal system (achievement of vertical and horizontal equity), and offers deceptively simple advice to policymakers (simply by reduction of horizontal inequity – without additional resources – you can increase the redistributive effect to a certain extent). Although the interpretation based on “potential redistributive effect” was not present in Kakwani (1984; 1986), we may say it was firmly inspired these contributions.

### ***Criticism by Lerman and Yitzhaki and their new framework***

Substantive criticism of the Kakwani decomposition is presented in the article by Lerman and Yitzhaki (1995) (henceforth LY). They develop their own decomposition of the redistributive effect, following, in fact, the philosophy of Kakwani, but arriving at different conclusions about the role of reranking. In their view, reranking positively contributes to the creation of overall inequality reduction, together with another component, “gap narrowing”, which corresponds to vertical effect in Kakwani’s model.

Lerman and Yitzhaki (1995) claim that their method enables a decomposition of the redistributive effect “into two exclusive, exhaustive terms”. As Kakwani, they too regard reranking as an independent source of the redistributive effect. Perhaps, this is most explicitly

stated in the abstract of the article: “...policies may reduce inequality by rearranging rankings as well.”<sup>7</sup> They even claim that Atkinson supported this view, saying that he indicated “that the reranking effect might be important in explaining a proportion of the impact of taxes on inequality.” However, we have already seen that, in fact, Atkinson regarded that reranking could only explain the discrepancy between two measures of the redistributive effect, namely  $RE$  and  $RE^{RS}$ ; he explicitly noted that reranking does not affect the redistributive effect.

Table 2: Comparison of Kakwani and Lerman-Yitzhaki decompositions

Lerman-Yitzhaki	Kakwani
Index of progressivity	
(18) $P_T^{LY} = D_T^n - G_N$	(9) $P_T^K = D_T^x - G_X$
Horizontal / Reranking effect	
(19) $R^{LY} = G_X - D_X^n$	(11) $R^{AP} = G_N - D_N^x$
Vertical effect	
(20) $V^{LY} = \frac{t^n P_T^{LY}}{1-t^n} = D_X^n - G_N$	(13) $V^K = \frac{t^x P_T^K}{1-t^x} = G_X - D_N^x$
Decomposition of the redistributive effect	
(21) $G_X - G_N = \frac{t^n P_T^{LY}}{1-t^n} + (G_X - D_X^n)$	(15) $G_X - G_N = \frac{t^x P_T^K}{1-t^x} - (G_N - D_N^x)$
(22) $G_X - G_N = (D_X^n - G_N) + (G_X - D_X^n)$	(16) $G_X - G_N = (G_X - D_N^x) - (G_N - D_N^x)$
(23) $RE = V^{LY} + R^{LY}$	(8) $RE = V^K - R^{AP}$

Table 2 presents the LY system in comparison with Kakwani’s. The main difference in the two approaches lies in the “reference” income: in the former model, it is the post-fiscal income, whereas in the latter, it is the pre-fiscal income. Observe that the LY index of progressivity ( $P_T^{LY}$ ) and vertical effect ( $V^{LY}$ ), defined in (18) and (23), compare the concentration coefficients of tax ( $D_T^n$ ) and pre-fiscal income ( $D_X^n$ ) with Gini coefficient of *post-*

<sup>7</sup> Other places are: (a) p.46, „one might well care about whether inequality reductions *result* from reranking or from gap-narrowing“. (b) p.46, „In this paper we ask: to what extent is the overall redistributive impact of U.S. taxes and transfers *the result* of rerankings of income versus pure reductions in income gaps (holding rankings constant)?“ (c) p.55, „For taxes, the reranking component amounted to nearly 40 percent of the total reduction in inequality.“

tax income ( $G_N$ ). Here,  $D_T^n$  and  $D_X^n$  are derived from the concentration curves for which the units are sorted in the ascending order of post-fiscal income (hence  $n$  in the superscript), namely  $C_T^n(p)$  and  $C_X^n(p)$ . Also, the average tax rate ( $t^n$ ) is expressed in terms of post-fiscal income. Decomposition of the redistributive effect is shown by (23), and the noticeable difference in comparison with (8) lies in the sign in front of the reranking term – it is positive. Recall that  $R^{AP}$  is non-negative due to its construction; the same is true for  $R^{LY}$  as well.

LY do not say *explicitly* of what would happen with the redistributive effect if reranking disappeared or if it were somehow eliminated. However, their text does imply the interpretation already mentioned in many instances, based on “actual” and “potential” redistributive effect, which goes as follows:

*A progressive fiscal system is “good” because it reduces inequality. In case of progressivity, Lerman-Yitzhaki vertical effect ( $V^{LY}$ ) is positive, and the larger it is, the “better”. Reranking is another factor that decreases inequality. The higher the reranking and Lerman-Yitzhaki index ( $R^{LY}$ ), the “better” it is. Actual redistribution is equal to  $RE = V^{LY} + R^{LY}$ . Therefore, in case that reranking is somehow reduced or eliminated, the redistributive effect would fall below  $RE$ , down to  $RE^{no\_reranking} = V^{LY}$ .*

Why have LY decided to abandon the Kakwani decomposition of the redistributive effect and invent a new one? *First*, they criticize the Kakwani vertical effect: for given redistributive effect,  $V^K$  increases automatically when reranking is increased. *Second*, they suggest that “the after-tax ranking is the correct ranking for calculating progressivity”, because they believe it is a proper ranking in the analysis of marginal changes in the tax system. They illustrate this on an example of the rich and the poor taxpayer whose places on the income scale are reversed due to taxation. This would result in an increase of Kakwani progressivity ( $V^K$ ), that is based on pre-tax rankings: this suggests that *even more* taxation of the “now poor” is desired. However, and unfortunately for the advancement of the field, they do not delve into any deeper technical elaboration of their criticism.

### 3 Upgrades of methodology

#### *Frameworks capturing vertical and horizontal inequity and reranking*

*Model by Aronson, Johnson and Lambert*

We have objected to identifying reranking with horizontal inequity: the latter should be concerned with unequal treatment of equals, while the former considers changing the order on the income scale of the unequals. The issue was reconciled by Aronson, Johnson and Lambert (1994) (henceforth AJL) with much praised methodology that decomposes the redistributive effect into vertical equity, horizontal inequity and reranking effects. The constraint of the model, from empirical an application perspective, is that it works only with true equals – units with identical income  $x$ .

Suppose we can partition the population (sample) into  $J$  groups, such that in each group  $j$  all  $K_j$  units have equal pre-tax income  $x_j$ . Each member  $k$  of group  $j$  has pre-tax income  $x_j$  and post-tax income  $n_{j,k}$ , after paying tax of  $t_{j,k}$ . Average tax paid by the group  $j$  is  $\bar{t}_j$ .

If everybody within each group  $j$  had identical post-tax income, it would mean that the system is horizontally equitable and there is no reranking. In reality, members within groups pay different taxes – this leads to horizontal inequity. It is the inequality of post-tax income within the groups that is the basis for AJL horizontal effect. It is equal to:

$$(24) \quad H^{AJL} = \sum_{j=1}^J \alpha_j \beta_j G_{N,j}$$

where  $\alpha_j$ ,  $\beta_j$  and  $G_{N,j}$  are, respectively, population share, post-tax income share and the Gini coefficient of post-tax income for the group  $j$ .

The basis for the calculation of vertical effect is a vector of taxes that would occur if all members within the group  $j$  paid  $\bar{t}_j$  instead of  $t_{j,k}$ ; call it  $\tilde{T}^x$ . This counterfactual tax is deemed free of horizontal inequity (but, is it free of reranking?). The AJL vertical effect is the Kakwani vertical effect obtained for  $\tilde{T}^x$  in (25), with  $P_{\tilde{T}}^K$  explained in (26), where  $D_{\tilde{N}}^x$  is the concentration coefficient for counterfactual post-income vector  $\tilde{N}^x = X - \tilde{T}^x$ .

$$(25) \quad V^{AJL} = \frac{t^x}{1-t^x} P_{\tilde{T}}^K = G_X - D_{\tilde{N}}^x$$

$$(26) \quad P_{\tilde{T}}^K = D_{\tilde{T}}^x - G_X$$

Some unit  $z$  in group  $j$  may end with post-tax income  $n_{j,z}$ , such that  $n_{j,z} < \max(n_{e,k})$ ,  $k = 1, \dots, K_j$ , and  $e < j$ ; in other words, there may be at least one unit in a lower pre-tax income group  $e$  that has higher post-tax income than the unit  $z$  in group  $j$ . It means that the latter unit is reranked. AJL model enables measurement of reranking, and the corresponding term is  $R^{AJL}$ , obtained as a residual  $R^{AJL} = V^{AJL} - H^{AJL} - RE$ . In the AJL model,  $R^{AJL}$  is identical to  $R^{AP}$ .

Finally, we have the complete decomposition:

$$(27) \quad RE = V^{AJL} - H^{AJL} - R^{AJL}$$

The main constraint of the AJL model is its non-conformability with real empirical data, where one can hardly find pre-tax exact equals. In the first applications, the researchers created artificially those equals, rounding pre-tax incomes of close equals.<sup>8</sup> This procedure has obviously distorted information about individual effects.

Subsequent work by van de Ven, Creedy and Lambert (2001) transformed the original model to avoid the creation of *artificial pre-tax equals*. Instead, close equals groups are used.<sup>9</sup> However, another problem remained unenvisaged by AJL model and this was the issue of whole-group reranking. The recent model of Urban and Lambert (2008) enables decomposition of  $RE$  accounting for all these issues.

AJL set their decomposition in the context of a welfare function, thus giving the indices a normative interpretation. They write:

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<sup>8</sup> For example, if units  $i$ ,  $i+1$  and  $i+2$  have pre-tax incomes of \$101, \$102 and \$103, the researcher may decide to transform all of them into pre-tax income \$102. Now, imagine that all of them paid zero tax; their post-tax incomes are \$101, \$102 and \$103. The AJL model would show the appearance of horizontal inequity, despite the fact that it does not occur.

<sup>9</sup> In the example from the previous footnote, the researcher might form the close pre-tax equals group of units  $i$ ,  $i+1$  and  $i+2$ ; their pre-tax incomes thus remain \$101, \$102 and \$103. Nonetheless, this procedure also requires certain amount of artificial business, which seems to be inevitable when measurement of horizontal inequity is in question.

$$\begin{aligned}
(28) \quad W_{X-T} - W_{X-F} &= \mu^x(1-t^x)RE = \mu^x \left[ t^x P_T^K - (1-t^x)H^{AJL} - (1-t^x)R^{AJL} \right] \\
&= \mu^x(1-t^x)(V^{AJL} - H^{AJL} - R^{AJL})
\end{aligned}$$

Here,  $W_{X-T}$  and  $W_{X-F}$  are respectively social welfares after actual tax and after an equal-yield proportional one;  $\mu^x$  is the mean pre-tax income. The difference  $W_{X-T} - W_{X-F}$  represents a welfare premium due to tax progressiveness. Minuses standing before the horizontal and reranking effects mean that they “provide subtractions from the welfare superiority of the actual tax code over a flat (distributionally neutral) one”.

Lambert (2001) shows how these welfare indices are derived and obtains a corresponding welfare result for the more simple Kakwani (1984) decomposition, which shows how “rerankings detract from the welfare-enhancing property of an otherwise progressive tax system”.

$$\begin{aligned}
(29) \quad W_{X-T} - W_{X-F} &= \mu^x(1-t^x)RE = \mu^x \left[ t^x P_T^K - (1-t^x)R^{AP} \right] \\
&= \mu^x(1-t^x)(V^K - R^{AP})
\end{aligned}$$

We have shown these results serve not only to offer another way of presenting decompositions, but also to discuss the various interpretations of the indexes. It is seen that horizontal and reranking effect are contributing negatively to overall redistributive effect. Does this also mean that  $RE$  would be higher in the absence of horizontal inequity and reranking? The authors think it would, and explicitly confirm it: “If differences in tax treatment... could be eliminated..., the redistributive effect would have been increased, to around  $[100(1+V^{AJL})]$ % of its actual value...” (AJL:p.268). This interpretation of the results has become particularly popular, as it gives an apparently straightforward message which the researcher can send to policy-makers: “Through alignments of taxes and benefits, without additional resources, you can achieve two popular goals at the same time – eliminate horizontal injustices and further reduce post-fiscal inequality”.



*Model by Duclos, Jalbert and Araar*

The model suggested by Duclos, Jalbert and Araar (2003) follows the philosophy of AJL, in which the vertical effect enhances the reduction of inequality, while the horizontal and reranking effects diminish it. The redistributive effect is decomposed as in (30), and in (31) an extended version is presented.

$$(30) \quad RE = V^{DJA} - H^{DJA} - R^{DJA}$$

$$(31) \quad RE = (I_X - I_N) = (I_X - I_N^E) - (I_N^P - I_N^E) - (I_N - I_N^P)$$

This model works with more general inequality indices, unlike AJL, which is based on ordinary Gini coefficients. The pre-tax income social welfare function is defined in the following way:

$$(32) \quad W_X(\varepsilon, \nu) = \int_0^1 U_\varepsilon(X(p))w(p, \nu)dp$$

where  $w(p, \nu)$  is a weighting scheme dependent on the ranks of income units,  $p$ , and a parameter of inequality aversion,  $\nu$ ; in this case we have  $w(p, \nu) = \nu(1-p)^{(\nu-1)}$ ; note that  $\int_0^1 w(p, \nu) = 1$ .  $X(p)$  is the pre-tax income of an income unit with rank  $p$  in the pre-tax income distribution, and  $U_\varepsilon(y)$  is a utility function with  $\varepsilon$  as a parameter of relative risk aversion;  $U_\varepsilon(y) = y^{1-\varepsilon}/(1-\varepsilon)$  for  $\varepsilon \neq 1$  and  $U_\varepsilon(y) = \ln(y)$  for  $\varepsilon = 1$ .

If everybody in society received identical income equal to  $\xi_X(\varepsilon, \nu)$  (the term is called the “equally distributed equivalent”, EDE), welfare would be the same as the actual. Then, by definition:

$$(33) \quad W_X(\varepsilon, \nu) \equiv W_{\xi_X(\varepsilon, \nu)}(\varepsilon, \nu) = \int_0^1 U_\varepsilon(\xi_X(\varepsilon, \nu))w(p, \nu)dp = U_\varepsilon(\xi_X(\varepsilon, \nu))$$

$\xi_X$  is then obtained from  $W_X$  by inversion of the utility function:

$$(34) \quad \xi_X = U_\varepsilon^{-1}(W_X)$$

where  $U_\varepsilon^{-1}(y) = ((1-\varepsilon)y)^{1/(1-\varepsilon)}$  for  $\varepsilon \neq 1$  and  $U_\varepsilon^{-1}(y) = e^y$  for  $\varepsilon = 1$ .

Finally, the index of inequality,  $I_X$ , is given as:

$$(35) \quad I_X = 1 - \frac{\xi_X}{\mu^x}$$

Returning to the model, the decomposition equation consists of several inequality indices, which can all be derived analogously to  $I_X$ . Here,  $I_N$  is simply inequality of post-tax income. In one particular case, when  $\varepsilon = 0$  and  $\nu = 2$ , we obtain  $I_X = G_X$  and  $I_N = G_N$ , the ordinary Gini indexes. On the other side,  $I_N^E$  and  $I_N^P$  are based on counterfactual incomes and utilities.  $I_N^E$  is obtained for the distribution of conditional incomes,  $\bar{N}(p) = \int_0^1 N(q|p) dq$ , which are the expected post-tax incomes of those at rank  $p$  in the distribution of pre-tax income.  $I_N^P$  is obtained for the distribution of expected utilities,  $\bar{U}_\varepsilon(p) = \int_0^1 U_\varepsilon(N(q|p)) dq$ .

### ***Extension to the net fiscal system***

#### *The methodological challenge*

Analysts in the area of income redistribution are naturally interested in capturing the widest possible picture of a fiscal system, given data limitations and theoretical assumptions concerning fiscal incidence. Since the fiscal systems include at least several tax and benefit instruments, it is useful to know how each of them influences the redistribution, and what the interactions between them are.

The Kakwani index of vertical effect can be readily applied to benefits, and the simplest way to estimate the effects of single taxes and benefits is to calculate the redistributive effect of each instrument, one by one, after choosing an appropriate reference income base. However, contributions obtained in such way do not simply add to the total redistributive effect of the net tax system; results from different data sources cannot be combined to get the overall redistribution (not even in case of “fiscal balance”); the interaction of different instruments obfuscates the situation.

*Lambert's approach*

All these problems were envisaged and solved by Lambert (1985; 1988) who decomposed the redistributive effect of the net tax system into parts that explain the contributions of taxes and benefits individually. As (36) demonstrates, the Kakwani vertical effect for the combined system of taxes and benefits ( $V_{T\&B}^K$ ) is a weighted average of vertical effects (or indices of progressivity) obtained for taxes ( $V_T^K$ ) and benefits ( $V_B^K$ ), where the weights are shares of taxes and benefits in pre-fiscal income ( $t^x$  and  $b^x$ ).

$$(36) \quad V_{T\&B}^K = \frac{(1-t^x)V_T^K + (1+b^x)V_B^K}{1-t^x+b^x}$$

$$(37) \quad V_{T\&B}^K = \frac{t^x P_T^K + b^x P_B^K}{1-t^x+b^x}$$

*Table 3: Lambert's decomposition for the net fiscal system*

Auxiliary terms and decompositions	
(38) $V_{T\&B}^K = G_X - D_N^x$	
(39) $P_T^K = D_T^x - G_X$	(40) $P_B^K = G_X - D_B^x$
(41) $V_T^K = G_X - D_{X-T}^x = \frac{t^x P_T^K}{1-t^x}$	(42) $V_B^K = G_X - D_{X+B}^x = \frac{b^x P_B^K}{1+b^x}$

The first important thing exposed by this decomposition is that the vertical effect of a net fiscal system is different from the simple sum of vertical effects of taxes and benefits, i.e.  $V_{T\&B}^K \neq V_T^K + V_B^K$ . This helps us to reveal an interesting interaction: even if the overall tax system is regressive ( $V_T^K < 0$ ), taxes can reinforce the redistributive effect of the net fiscal system, so that  $V_{T\&B}^K > V_B^K$ .<sup>10</sup>

<sup>10</sup> For example, take that  $V_T^K = -0.05$  and  $V_B^K = 0.10$  and assume that  $t^x = b^x = 0.4$ . Then, we have that  $V_{T\&B}^K = 0.6 \cdot (-0.05) + 1.4 \cdot 0.1 = 0.11 > V_B^K$ .

However, this framework does not tell us how different tax-benefit instruments contribute to overall reranking. “This would introduce severe analytical complications”, as Lambert (1985; footnote 2) points out. As we will see shortly, the two other scholars decided to deal with this intricate task.

### *Jenkins’ decomposition*

Following the derivational grounds of Lambert (1985), who dealt with the vertical effect and admittedly ignored reranking, Jenkins (1988) decomposed the overall redistributive effect of the net fiscal system into tax and benefit contributions ( $RE$ ). Equation (43) replicates Jenkins’ formula 10, extending some of the terms.

$$(43) \quad RE = \frac{(1-t^x)(G_X - G_{X-T})}{1-t^x + b^x} + \frac{(1+b^x)(G_X - G_{X+B})}{1-t^x + b^x} + \\ + (D_N^x - G_N) - \frac{(1-t^x)}{1-t^x + b^x}(D_{X-T}^x - G_{X-T}) - \frac{(1+b^x)}{1-t^x + b^x}(D_{X+B}^x - G_{X+B})$$

The last three terms on the right side of (43) represent the reranking effects. The term  $(D_N^x - G_N)$  is simply  $-R^{AP}$  or a measure of the overall reranking induced by a fiscal system consisting of taxes and benefits. The other two terms are the reranking effects that should reflect the contributions of taxes (the fourth term) and benefits (the fourth term). The decomposition (43) fully decomposes the redistributive effect ( $RE$ ). However, the contributions of taxes and benefits to reranking do not add up to total reranking (as measured by  $R^{AP}$ ), and this may be judged as unsatisfactory. One may also pose a question, why in a measurement of reranking due to taxes (benefits) is the term  $D_{X-T}^x - G_{X-T}$  ( $D_{X+B}^x - G_{X+B}$ ) used instead of  $D_{X-T}^x - G_X$  ( $D_{X+B}^x - G_X$ )? Furthermore, the first two terms on the right side of (43) do not represent Kakwani vertical effects of taxes and benefits (as  $V_T^K$  and  $V_B^K$  in (36)), but something else.

### *Duclos’ decomposition*

In another attempt to cope with the difficulties of the reranking effect decomposition was undertaken by Duclos (1993) and the formula (44) shows the result.

$$(44) \quad RE = (G_X - D_N^x) - (G_N - D_N^x) =$$

$$= \frac{\sum_{m=1}^M t_m^x (D_{T_m}^x - G_X)}{1 - \sum_{m=1}^M t_m^x} - \left( \sum_{k=1}^M \left( D_{N, X - \sum_{m=0}^k T_m} - D_{N, X - \sum_{m=0}^{k-1} T_m} \right) \right)$$

In the start, one has to somehow order the tax/benefit variables and call them  $T_1, T_2, \dots, T_M$  (with  $T_0 = 0$ ). Any combination is allowed,<sup>11</sup> leading to a number of final results which are conveniently presented by a tree-root diagram (we will turn to this issue later again). Thus, for the first tax/benefit,  $T_1$  ( $k = 1$ ) the variables in question should be  $X - T_1$  and  $X$ ; for the last tax/benefit,  $T_M$  ( $k = M$ ), the variables are  $N$  and  $N + T_M$ .

The first row of (44) is the already familiar Kakwani decomposition of the redistributive effect into the vertical and reranking effects. The first term in the second row is a decomposition of vertical effect from (36), but applied to  $M$  individual tax or benefit instruments, each with its own concentration coefficient  $D_{T_m}^x$ , and a share in pre-fiscal income  $t_m^x$ .<sup>12</sup> The second term is then a corresponding decomposition of the reranking effect, which is separated into  $M$  differences between specifically defined concentration coefficients.

For all tax/benefit instruments,  $k = 1, \dots, M$ , both concentration coefficients are obtained for (final) post-fiscal income ( $N$ ), as suggested by the subscripts  $N$ . Thus, the same variable is used to determine the contributions of all instruments. What makes the difference between the concentration coefficients is the ordering of units in the construction of the concentration curve. However, this matter is not very clear in the paper.

Duclos (1993:356) says: “ $D_{N, X - \sum_{m=0}^k T_m}$  indicates that the concentration curve used to build  $D$  employs the ranking of units based on  $\sum_{m=0}^k T_m$ ”. What about  $D_{N, X - \sum_{m=0}^{k-1} T_m}$ ? It must be supposed that in this case the ordering variable is  $\sum_{m=0}^{k-1} T_m$ . But, if this is so, for  $k = 1$ , the

<sup>11</sup> For example, if the fiscal system consists of instruments A, B and C, each of these can be  $T_1$ ,  $T_2$  or  $T_3$ , resulting in 6 possible orders.

<sup>12</sup> Duclos (1993) designates both taxes and benefits as *taxes*, using letter  $T$ . This made presentation his more simple, but at a loss of convenience.

ordering is not defined. Also, the sums of taxes/benefits do not seem as intuitive variables for ranking income units. From the overall text, it may rather be guessed that the actual ordering variables are  $X - \sum_{m=0}^k T_m$  and  $X - \sum_{m=0}^{k-1} T_m$ .

Let us show how it would work on a simple example of one tax ( $T_1 = T$ ) and one benefit ( $T_2 = B$ ) instrument ( $M = 2$ ). For  $k = 1$ , the ordering variable for the first concentration coefficient is  $X - \sum_{m=0}^1 T_m = X - T$  and for the second it is  $X - \sum_{m=0}^0 T_m = X$ . For  $k = 2$ , we obtain  $X - \sum_{m=0}^2 T_m = X - T + B = N$  and  $X - \sum_{m=0}^1 T_m = X - T$ , respectively. The decomposition of the reranking effect for this one-tax-one-benefit case is then written as:

$$(45) \quad R^{AP} = G_N - D_N^x = (D_N^{x-t} - D_N^x) + (G_N - D_N^{x-t})$$

### ***Other developments***

#### ***Kakwani-Lambert “new approach”***

The measurement system proposed by Kakwani and Lambert (1998) arises from three axioms of equitable taxation that deal with both horizontal and vertical equity considerations. The first axiom requires „minimal progression“: tax should increase monotonically with respect to income. The second axiom is based on the „progressive principle“, demanding that higher income people are faced with higher tax rates. The third axiom presents the „no reranking“ criterion: the marginal tax rate should not exceed 100 percent. The axioms are designed in such way as to be independent.

The indices  $S_1$ ,  $S_2$  and  $S_3$  are then constructed, whose zero value means that the respective axiom is upheld, or violated if the value is positive:  $S_1 = t^n R_T$ ,  $S_2 = t^n (R_A - R_T)$  and  $S_3 = R_N (= R^{AP})$ , where  $R_T = G_T - D_T^x$ ,  $R_A = G_A - D_A^x$  and  $R_N = G_N - D_N^x$  are obtained for taxes ( $T_i$ ), average taxes ( $A_i = T_i / X_i$ ) and post-tax income ( $N_i = X_i - T_i$ ), respectively. The redistributive effect can now be decomposed, as in (46).

$$(46) \quad RE = t^n (P^K + R_A) - S_1 - S_2 - S_3$$

Recall that the Kakwani decomposition is  $RE = t^n P^K - S_3$ . The term  $t^n(P^K + R_A)$  is a measure of potential redistributive effect that “might be achieved if all inequities could be abolished.” It is analogous to  $V^K$ , which is the potential redistributive effect achievable if reranking could be eliminated. However, as Kakwani and Lambert note, “there is no uniquely well-defined way to abolish axiom violations from a tax system, and thereby to say what maximal value of  $[RE]$  might be achieved”. The decomposition was applied to Australian income tax data, resulting in an estimate of  $RE = 0.0240$ , while  $t^n(P^K + R_A)$  was remarkably high at 0.1382. The authors conclude that  $RE$  could be improved by removal of inequities “without change to the marginal rate structure which governs incentives.”

#### *Approach based on relative deprivation*

The relative deprivation of a person is a sum of the incomes of all people who are richer than that person. Using the concept of relative deprivation as a principle, Duclos (2000) invents the concepts of “fiscal harshness”, “fiscal looseness” and “ill-fortune”, and afterwards combines their measures to reinvent all the terms in the Kakwani decomposition(s).

Kakwani’s index of progressivity ( $P^K$ ) is obtained as a difference between the mean-normalized average fiscal harshness and average relative deprivation in the population. Kakwani’s index of vertical inequity ( $V^K$ ) is a difference between the mean-normalized average relative deprivation and average fiscal looseness. The Atkinson-Plotnick index of reranking is the mean-normalized average of ill-fortune in the population.<sup>13</sup>

## **4 Empirical research in the field of income redistribution**

### *Overview of empirical studies*

Since the 70’s, when the major methodological innovations in the measurement of inequality, progressivity, and the redistributive effect emerged, there was a huge empirical interest in evaluating how fiscal systems affect income distribution. Table 4 presents a summary of studies that measure the redistributive effects of fiscal instruments and overall fiscal systems.

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<sup>13</sup> Loose definitions of the concepts follow. For person  $i$  with pre-fiscal income  $X_i$ , relative deprivation (fiscal harshness; fiscal looseness) is a sum of  $X_j - X_i (T_j - T_i; N_j - N_i)$  for all  $j$  with  $X_j > X_i$ . Ill-fortune occurs for person  $i$ , if  $X_i > X_j$  and  $N_i < N_j$ , and is measured in terms of  $N_j$ .

The aim is not to provide a full review of the research in this field. Instead, we have primarily attempted to illustrate the variety of methodological approaches used in the estimation of the redistributive effects. It can be easily noted that most of the analyses are based on the works of Kakwani (1977b, 1984) – his progressivity index and his decomposition of *RE*.

Another dimension shown in Table 4 is fiscal coverage of the studies. Many of them concentrate on single tax or benefit instruments. However, researchers are typically aware that *all* fiscal activities affect income distribution. Therefore, the studies often cover whole fiscal subsystems – personal taxes, indirect taxes, cash and in-kind benefits, and even complete fiscal systems.<sup>14</sup> Concerning the countries included in the research, the high-income countries like UK, USA, Canada, Australia and Sweden are the most represented. Only a few studies are devoted to low-income countries.

*Table 4: Overview of empirical studies in the field of income redistribution*

<b>Authors</b>	<b>Indices / decompositions used</b>	<b>Countries / Fiscal coverage</b>	<b>Data sources / equivalence scales</b>
Kakwani (1977b)	$P^K$	(a) <i>Australia, Canada, United Kingdom, United States</i> : PIT; (b) <i>Australia, Canada, U.S.</i> : wide range of direct/indirect taxes and public expenditures	(a) Official income-tax statistics, grouped data ITP (b) different, not fully comparable sources
Reynolds, Smolensky (1977)	<i>RE</i>	<i>United States</i> : wide range of direct/indirect taxes and public expenditures	“Survey of Consumer Finances” (1950), “Survey of Consumer Expenditures” (1961), “Current Population Survey” (1970)
Plotnick (1981)	$R^{AP}$	<i>United States</i> : PIT, payroll tax, public cash benefits, food stamps	“Michigan Panel Study of Income Dynamics”
Dilnot, Kay, Norris (1984)	MT	<i>United Kingdom</i> : PIT, SSC, indirect taxes	“Family Expenditure Survey” four groups of households
Berliant, Strauss (1985)	BS, 16 other measures of VE and 1 of HI	<i>United States</i> : PIT	“Statistics of Income Individual Tax Returns”; ITP
Plotnick (1985)	$R^{AP}$ , 4 other measures of HI	<i>United States</i> : PIT, SSC, social benefits	“Current Population Survey”
Kakwani (1986)	K84	<i>Australia</i> : PIT, property taxes, social benefits	“Survey of Consumer Finances and Expenditures”; $E_3(.7,.4)$ , $E_1(.4)$
Nolan (1987)	Tm	<i>United Kingdom</i> : PIT, SSC, social benefits	“Family Expenditure Survey” $E$ : implied by certain benefit programs
Jenkins (1988)	J88	<i>United States</i> : wide range of direct/indirect taxes and public	“Current Population Survey”, other sources

<sup>14</sup> It must be noted that the selection in Table 4 is biased toward the studies covering both taxes and benefits, since the empirical part of the overall research underlying this paper deals with taxes and benefits in Croatia.



Authors	Indices / decompositions used	Countries / Fiscal coverage	Data sources / equivalence scales
		expenditures	
Norregaard (1990)	$P_T^K$	17 OECD countries: PIT, SSC	OECD data base (decile groups); ITP
Ankrom (1993)	K84; L85	Sweden, United States, United Kingdom: PIT, SSC, property taxes, other direct taxes, indirect taxes, social benefits	Household budget surveys; $E_1(.54)$
Duclos (1993)	K84, D93	United Kingdom: PIT, SSC, social benefits	“Family Expenditure Survey” and author’s own tax-benefit model
Aronson, Johnson, Lambert (1994)	AJL	United Kingdom: PIT	“Family Expenditure Survey” and Tax-benefit model of the Institute for Fiscal Studies; $E_2([0,1], [0,1])$
Lerman, Yitzhaki (1994)	LY94	United States: PIT, SSC, social benefits	“Current Population Survey”; $E_1(1)$
Bishop, Chow, Formby (1995)	LC	Australia, Canada, Sweden, West Germany, United States, United Kingdom: PIT, payroll taxes, other direct taxes	“Luxembourg Income Study”
Lerman, Yitzhaki (1995)	LY95	United States: PIT, SSC, property taxes, social benefits	“Current Population Survey”
Jänni (1997)	S82	Canada, the Netherlands, Sweden, United Kingdom, United States: PIT, SSC, social benefits	“Luxembourg Income Study”; $E$ : implied by the U.S. poverty line
Ervik (1998)	RE	8 high-income countries: PIT, SSC, social benefits	“Luxembourg Income Study”; $E_1(.5)$
Kakwani, Lambert (1998)	KL	Australia: PIT	“Household Expenditure Survey”; $E_4(1, 2, 4, 7, 1)$
Aronson, Lambert, Trippeer (1999)	AJL	United States: PIT	“Statistics of Income Panel of Individual Returns”; $E_2(.5, .5)$
Fellman, Jänni, Lambert (1999)	$V^K$	Finland: personal taxes and social benefits	Household Budget Survey; $E_3(.7, .5)$
van Doorslaer et al. (1999)	AJL	12 OECD countries: various sources of financing health expenditures: taxes, social and private insurance, direct payments	Household budget surveys; $E_2(.5, .5)$
Wagstaff et al. (1999a)	AJL	12 OECD countries: PIT	Household budget surveys; $E_2(.5, .5)$
Wagstaff et al. (1999b)	$P_T^K$	12 OECD countries: various sources of financing health expenditures: taxes, social and private insurance, direct payments	Household budget surveys; $E_2(.5, .5)$
Duclos (2000)	K84; L85	Canada: personal taxes and social benefits	“Survey of Consumer Finances”; $E_3(.7, .5)$
Förster (2000)	LC, S82	21 OECD countries: PIT, SSC, social benefits	national household budget surveys and tax administration

Authors	Indices / decompositions used	Countries / Fiscal coverage	Data sources / equivalence scales
			databases; $E_1(.5)$
Duclos, Lambert (2000)	DL	Canada: personal taxes and social benefits	“Survey of Consumer Finances”; $E_3(.7,.5)$
Iyer, Seetharaman (2000)	AJL	United States: PIT	“Statistics of Income Panel of Individual Returns”; ITP
Creedy, van de Ven (2001)	VCL	Australia: personal taxes and social benefits	Simulated incomes over the life cycle $E_2([0,1],[0,1])$
Dardanoni, Lambert (2001)	DL01	United Kingdom, Israel, Canada: cash benefits and direct taxes	“Family Expenditure Survey” and EBORTAX (for UK), “Family Expenditure Survey” (Israel), “Survey of Consumer Finances” (Canada); $E_2(.5,.5)$
Decoster, Van Camp (2001)	$P^K, V^K$	Belgium: PIT and indirect taxes	Administrative data (IPCAL) with microsimulated taxes; household budget survey (ASTER); $E_3(.7,.5)$
Heady, Mitrakos, Tsakloglou (2001)	PCF	13 EU countries: social benefits, public pensions	“European Community Household Panel”; $E_3(.5,.3)$
Smith (2001)	$P^K$	Australia: PIT	Aggregated data from annual statistics (1917-1997); ITP
van de Ven, Creedy, Lambert (2001)	VCL	Australia: personal taxes and social benefits	“Income Distribution Survey”
Wagstaff, van Doorslaer (2001)	$PL(P^K)$	15 OECD countries: PIT	OECD data base (decile groups); ITP
Creedy (2002)	VCL	Australia: Goods and Services Tax	“Household Expenditure Survey”; $E_2(\theta, \alpha)$ , wide range of $\theta$ - and $\alpha$ -values
Dardanoni, Lambert (2002)	DL02	see Dardanoni, Lambert (2001)	see Dardanoni, Lambert (2001)
Duclos, Jalbert, Araar (2003)	DJA	Canada: PIT, SSC, social benefits	“Survey of Consumer Finances”; $E_3(.7,.5), E_2(.5,.5)$
Thoresen (2004)	$P^K$	Norway: PIT	“Income Distribution Survey”; $E_1(.5)$
Verbist (2004)	$PL(P^K)$	EU-15 countries: PIT, SSC paid by employees, other income taxes	EUROMOD data; $E_3(.5,.3)$
Dyck (2005)	RSA	Canada: overall fiscal system	“Social Policy Simulation Database and Model” census family groups
Hyun, Lim (2005)	AJL	South Korea: PIT	Administrative data – microsimulated taxes;

Authors	Indices / decompositions used	Countries / Fiscal coverage	Data sources / equivalence scales
			$E_2(.5,.5)$
Immervoll et al. (2005)	$RE$	<i>EU-15 countries</i> : PITs, SSC paid by employees, other income taxes, social benefits, public pensions	EUROMOD data; $E_3(.5,.3)$
Johannes, Akwi, Anzah (2006)	$RE, P^K$	<i>Cameroon</i> : wide range of direct and indirect taxes, expenditures for health and education	“ECAM2” (household budget survey), 2001; additional government sources
Mahler, Jesuit (2006)	$RE$	<i>13 high-income countries</i> : PIT, SSC, social benefits	“Luxembourg Income Study”; $E_1(.5)$
Urban (2006)	$PL(V^K)$	<i>Croatia</i> : PIT	Administrative tax data; ITP
Cissé, Luchini, Moatti (2007)	$P^K$	<i>Abidjan</i> (Ivory Coast), <i>Bamako</i> (Mali), <i>Conakry</i> (Guinea) and <i>Dakar</i> (Mali): health care payments	questionnaires
Čok, Urban (2007)	UL	<i>Slovenia and Croatia</i> : PIT and SSC	Administrative tax data; ITP
Creedy et al. (2008)	$RE, P^K$	<i>New Zealand</i> : PIT, Family Tax Credit, unemployment benefit, superannuation, Working for Families, accommodation supplement	“Household Expenditure Survey” $E_2(.7,.6)$
Urban (2008)	K84; LY95; L85	<i>Croatia</i> : PIT, SSC, social benefits, public pensions	“APK” (household budget survey); $E_3(.5,.3)$
Urban, Lambert (2008)	UL	<i>Croatia</i> : PIT	Administrative tax data; ITP
Kim, Lambert (2009)	UL; L85	<i>United States</i> : PIT, Earned Income Tax Credit, property tax, payroll tax, social benefits	“Current Population Survey” and “Annual Social and Economic Supplement”; $E_2(.5,.5)$
Lambert, Thoresen (2009)	BD, KJ; DL01, DL02; DL	<i>Norway</i> : PIT	“Income Distribution Survey”; $E_7(\cdot)$
Zaidi (2009)	$RE$	<i>Slovakia, Slovenia, Poland, Czech R., Estonia, Lithuania, Hungary, Latvia</i> : social benefits, PIT, SSC, taxes on wealth	“EU-SILC” databases; $E_3(.5,.3)$

Notes:

(a) The term “social benefits”, if not specified differently, denotes a wide range of cash and near-cash direct transfers from government to the households (near-cash transfers are in-kind benefits whose values are easily determined).

(b) ITP = unit of observation is individual tax-payer

(c) The meaning of the terms like  $E_1(.5)$  etc., is explained in the following section.

## Classification of studies

### By equivalence scales

The third class of information presented in Table 5 relates to data sources and equivalence scales. Almost all studies deal with household budget survey data, while administrative data are used only in the studies of PIT progressivity. Household data require some form of aggregation / averaging at the household level – equivalence scales are used for this purpose. The two most frequent types of scales are: the so-called “Cutler and Katz” scale (denoted with  $E_2$ ) and the “OECD” scale (denoted with  $E_3$ ). Both of them take into account economies of scale and recognize the difference between children and adults.

Table 5: Equivalence scales

$E_1(\sigma)$	$m = (n)^\sigma$	power / root scale
$E_2(\theta, \alpha)$	$m = (n_a + \theta n_c)^\alpha$	“Cutler and Katz” scale
$E_3(\delta, \varepsilon)$	$m = 1 + \delta(n_a - 1) + \varepsilon n_c$	“OECD” scale
$E_4(\eta, \varepsilon_1, \varepsilon_2, \varepsilon_3, \phi)$	$m = (\eta n_a + \varepsilon_1 n_{c1} + \varepsilon_2 n_{c2} + \varepsilon_3 n_{c3}) + \phi n_w$	

Notes:  $m$  = equivalent adults;  $n$  = adults and children;  $n_a$  = adults;  $n_c$  = children;  $n_{c1}/n_{c2}/n_{c3}$  = children aged 0-5/6-14/15-17 years;  $n_w$  = working adults.

### By methodologies used

In this part we present the classification of empirical studies from Table 4 according to the methodology employed. Table 6 is divided into two major parts, distinguishing two groups of approaches: (a) those based on Kakwani (1977b), and (b) other approaches. Indices and decompositions rooted in Kakwani (1977b) were used 49 times, while other methodologies in the measurement of the redistributive effects were employed in 15 papers. It was already mentioned that the selection of studies is not exhaustive and is biased toward the research capturing both taxes and benefits (see footnote 14), but anyway, the ratio of 3:1 evidences that Kakwani’s methodologies pervade the field.

Many studies have estimated only one of the main indices. Thus, the index of the redistributive effect ( $RE$ ), Kakwani index of progressivity ( $P^K$ ), Kakwani index of vertical effect ( $V^K$ ) and Atkinson-Plotnick index of reranking ( $R^{AP}$ ) are used in 6, 9, 2 and 2 studies, respectively. Among the various decompositions of these indices, we have to mention Kakwani (1984), used in 5 studies. This may seem a small number given the importance devoted to the

decomposition in this chapter. However, we must remember another decomposition which originates from Kakwani's, and this is one of the most popular: Aronson, Johnson and Lambert (1994). It served as a tool in 6 studies, and if we add studies based on adaptations of this decomposition (VCL and UL) we arrive at the number of 12.

What about the studies that attempted to reveal the contributions of individual taxes and benefits to the redistributive effect? Three methodologies were developed to cope with this task, as we have seen above. Lambert (1985) attracted most attention, with 5 studies employing it, while approaches by Jenkins (1988) and Duclos (1993) were not adopted in later studies.

*Table 6: Overview of methodologies used in measurement of income redistribution*

Symbol / Abbrev.	Approach	Studies using the methodology
<i>(a) Approaches based on Kakwani (1977b)</i>		
$RE$	the redistributive effect (a difference between Gini coefficients of pre- and post-TB income)	Reynolds, Smolensky (1977), Ervik (1998), Immervoll et al. (2005), Johannes, Akwi, Anzah (2006), Mahler, Jesuit (2006), Creedy et al. (2008) [6]
$P^K$	Kakwani (1977b) index of progressivity	Kakwani (1977b), Norregaard (1990), Wagstaff et al. (1999b), Decoster, Van Camp (2001), Smith (2001), Thoresen (2004), Johannes, Akwi, Anzah (2006), Cissé, Luchini, Moatti (2007), Creedy et al. (2008) [9]
$V^K$	Kakwani (1977b) index of vertical effect	Fellman, Jäntti, Lambert (1999), Decoster, Van Camp (2001) [2]
$R^{AP}$	Atkinson (1980) / Plotnick (1981) index of reranking	Plotnick (1981), Plotnick (1985) [2]
K84	Kakwani (1984) decomposition of the redistributive effect	Kakwani (1986), Ankrom (1993), Duclos (1993), Duclos (2000), Urban (2008) [5]
L85	Lambert (1985) decomposition of vertical effect for the net fiscal system	Ankrom (1993), Duclos (1993), Duclos (2000), Urban (2008), Kim, Lambert (2009) [5]
PL	Pfähler (1990) / Lambert (2001) decomposition of $P^K$ and $V^K$	Wagstaff, van Doorslaer (2001), Verbist (2004), Urban (2006) [3]
LY95	Lerman and Yitzhaki (1995) decomposition of the redistributive effect	Lerman, Yitzhaki (1995), Urban (2008) [2]
KL	Kakwani, Lambert (1998) decomposition of the redistributive effect	Kakwani, Lambert (1998)

Symbol / Abbrev.	Approach	Studies using the methodology
AJL	Aronson, Johnson, Lambert (1994) decomposition of the redistributive effect, as originally applied	Aronson, Johnson, Lambert (1994), Aronson, Lambert, Trippeer (1999), van Doorslaer et al. (1999), Wagstaff et al. (1999a), Iyer, Seetharaman (2000), Hyun, Lim (2005) [6]
VCL	van de Ven, Creedy, Lambert (2001) adaptation of AJL decomposition	van de Ven, Creedy, Lambert (2001), Creedy, van de Ven (2001), Creedy (2002) [3]
UL	Urban, Lambert (2005, 2008) adaptation of AJL decomposition	Čok, Urban (2007), Urban, Lambert (2008) Kim, Lambert (2009) [3]
D93	Duclos (1993) decomposition of vertical and reranking effect for the net fiscal system	Duclos (1993)
J88	Jenkins (1988) decomposition of vertical and reranking effect for the net fiscal system	Jenkins (1988)
<i>(b) Other approaches</i>		
$R^{MT}$	Musgrave, Thin (1948) index of the redistributive effect	Dilnot, Kay, Norris (1984)
BS	Berliant, Strauss (1984) measures of vertical and horizontal inequity	Berliant, Strauss (1985)
DJA	Duclos, Jalbert, Araar (2003) decomposition of the change in inequality	Duclos, Jalbert, Araar (2003)
DL	Duclos, Lambert (2000) measurement of horizontal and vertical inequity	Duclos, Lambert (2000)
DL01	Dardanoni, Lambert (2001) horizontal inequity comparisons	Dardanoni, Lambert (2001)
DL02	Dardanoni, Lambert (2002) progressivity comparisons	Dardanoni, Lambert (2002)
LC	Lorenz curves comparisons	Bishop, Chow, Formby (1995), Förster (2000) [2]
LY94	Lerman, Yitzhaki (1994) decomposition of Gini coefficient	Lerman, Yitzhaki (1994)
PCF	Pyatt, Chen, Fei (1980) decomposition of Gini coefficient	Heady, Mitrakos, Tsakloglou (2001)
RSA	Baum (1987) relative share adjustment index	Dyck (2005)
S82	Shorrocks (1982) decompositions	Jännti (1997), Förster (2000) [2]
Tm	Atkinson (1980) analysis of reranking using transition matrices	Atkinson (1980), Nolan (1987) [2]
KJ	King (1983) / Jenkins (1994) “no reranking procedure”	Lambert, Thoresen (2009)
BD	Blackorby, Donaldson (1984) index of progressivity	Lambert, Thoresen (2009)

Note: A number in the square brackets appearing in the third column shows total number of studies in each subgroup.

## 5 Conclusion

Kakwani’s (1984) decomposition of redistributive effect into vertical and reranking indices became the cornerstone for much research on income redistribution. This is evidenced by

the huge number of empirical studies employing it, and plenty of extensions and upgrades provided by its supporters. This paper describes the origins of the Kakwani decomposition and of its components. It reviews other different methodologies in decomposing redistributive effect, rooted in the Kakwani decomposition. Finally, an overview of empirical research is presented accounting for about fifty ~~of~~ different studies.

Careful study of the origins of the Kakwani decomposition, and of the different concepts and indices it brought together into one framework, has revealed contrasting opinions of different scholars on the possibility, meaning and interpretation of the decomposition. Atkinson (1980) and Plotnick (1981), who invented the reranking term, implicitly suggested to future users and developers to be cautious about introduction of it into more comprehensive frameworks. Nonetheless, the Kakwani decomposition is exactly such a model capturing vertical equity and reranking.

For Lerman and Yitzhaki (1995) one of the weaknesses of the Kakwani approach is the use of pre-tax income rankings. More reranking is regarded by these authors as favourable from the policy maker's perspective, because the increase in the Atkinson-Plotnick reranking index ( $R^{AP}$ ) automatically increases the Kakwani vertical effect ( $V^K$ ). On the other side, for both Kakwani (1984) and Lerman and Yitzhaki (1995), reranking has an active role in the determination of the magnitude of redistributive effect, and this is completely opposite to the views of Atkinson (1980) and Plotnick (1980). Kakwani's contributions have inspired many followers to claim that elimination of reranking would increase or decrease the redistributive effect.

Thus, one of the main purposes of this paper is to call for further research which should examine these problems more thoroughly. Such research has already begun, resulting in the paper by Urban (2009), which suggests the ways of proper interpretation and reinterpretation of existing indices.

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