Tracking can be more equitable than mixing: Peer effects and college attendance

Marisa Hidalgo-Hidalgo
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Abstract
Parents and policy makers often wonder whether, and how, the choice between a tracked or a mixed educational system affects the efficiency and equity of national educational outcomes. This paper analyzes this question taking into account their impact on educational results at later stages and two main results are found. First, it shows that tracking can be the efficient system in societies where the opportunity cost of college attendance is high or the pre-school achievement distribution is very dispersed. Second, this paper shows that tracking is the most equitable system for students with intermediate levels of human capital required to attend college.

Keywords: Peer Effects, Tracking, Mixing, College attendance gap

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1 Introduction

Parents and policy makers often wonder whether, and how, the choice between a tracked or a mixed educational system affects the efficiency and equity of national educational outcomes. Under tracking, schools are hierarchically organized to accommodate a range of student performance levels, and students are placed in the school that best suits their ability level. By contrast, mixing works by grouping students of differing ability levels within the same school. When comparing these systems, it is critically important to recognize the existence of peer interactions and account for their impact on students’ outcomes.\footnote{There is a large empirical literature on peer effects and still an open debate on the influence of peers on individual educational achievement with some studies finding little or no effects (e.g. Angrist and Lang (2004) and Foster (2006)) and others finding large effects (e.g. Hoxby and Weingarth (2006), Kim et al. (2006) and Ding and Lehrer (2007)).}

This paper analyzes the efficiency and equity of tracking versus mixing, within a theoretical framework, paying special attention to the impact of compulsory school peer effects on college attendance. I define an equitable system as one that gives students equal access to a college education, regardless of their family background. I define an efficient system as one in which the total human capital of the entire cohort is maximized by the end of the educational period (that is, upon college graduation).

To address these issues, a model is introduced with two educational stages: compulsory and college education. Students differ in parental background as well as pre-school achievement levels. Some positive dependence between these two defining variables is assumed, as wealthy parents have more resources to invest in their children, and also tend to be more educated and care more about education, factors that enhance their children’s performance at school. The acquisition of human capital at compulsory level depends on both students’ pre-school achievement and peer group characteristics. The latter also indirectly affects students’ human capital accumulation in college. Two types of tracking are introduced that differ in the degree of elitism or segregation of students of different parental background among the tracks. The degree of elitism of tracking is found to be a central aspect in determining the existence of trade-off between efficiency and equity.

Two main findings result from my analysis. First, I find that maximizing human capital at one level (compulsory) does not immediately imply that human capital is maximized at the end of the whole educational process. The impact of the educ-
ational system prevailing at compulsory level on college attendance rates, together with the properties of the human capital production at compulsory level must also be considered. Second, and more strikingly, I show that tracking may sometimes be more equitable than mixing.

If the government seeks to achieve efficiency, its best option will depend on the properties of the human capital production at compulsory level, on the opportunity cost of college attendance and on the wealth level in the population. Provided that tracking maximizes average human capital at compulsory level (for example, because peer effects and individual achievement are close complements), it will also maximize average human capital across the population if the opportunity cost of college attendance is sufficiently high or the wealth level in the population is sufficiently low. Observe that, in this case, most students with low levels of pre-school achievement are excluded from college under both systems. Therefore, intervention should be focused on those students with high pre-school achievement levels. Choosing tracking over mixing is one way for governments to intervene in favour of these students and maximize college attendance which, in turn, would maximize average human capital across the population.

I also find tracking to be the most equitable system in contexts where the opportunity cost of college attendance is neither too low nor too high. In this case the type of tracking determines the precise set of individuals who enjoy more equality of opportunities. In particular, in the non-elitist tracking (i.e. the one that allows a higher proportion of poor students in the high track) the set of individuals at the margin are those with intermediate levels of pre-school achievement who, under tracking, could join the high track where the peer effect is the strongest. Thus, family background does not critically determine the amount of human capital they can accrue after compulsory education. By contrast, family background matters more under mixing, since peer effects are weaker under this system for this group of students. Therefore, in this case tracking outperforms mixing when it comes to equalizing college attendance opportunities of students. To conclude, non-elitist tracking can be both efficient and equitable if the opportunity cost of college attendance takes intermediate values or the wealth level in the population is low.

There are several theoretical papers related to this. Roemer and Wets (1994)

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2If peer effects matter more for high ability students than for low ability ones, then average human capital at compulsory level will be maximized under tracking since it is the system where high ability students enjoy a stronger peer effect.
and Streufert (2000), for example, focus on other kind of neighborhood effects as important explanatory factors behind individual schooling choices. In Roemer and Wets’ model, individuals form rational beliefs about the return to education when they make schooling decisions. Students estimate the returns to education as the best linear regression of the income against education of the parents in their neighborhood. They find that incorrect beliefs (since the true returns are not linear) tend to cause widespread dispersion of the stationary distribution of talent at any income level. Streufert (2000) proposes a model in which students make inaccurate estimates of the marginal product of effort at school, after observing a biased sample population of adults. Their isolation depresses the level of schooling chosen by underclass youth. Brunello and Giannini (2004) study the efficiency of secondary school design by focusing on the degree of differentiation between vocational and general education. They show that neither a comprehensive nor a stratified system unambiguously outperforms the other in terms of efficiency. Finally, Hidalgo-Hidalgo (2008) compares the academic performance of compulsory school students under tracking and mixing. There are also several empirical papers that explore equality of opportunities of tracked versus mixed education systems. First, Hanushek and Woessmann (2006) find, based on international comparisons of early outcomes, that early tracking increases educational inequality. Brunello and Checchi (2007), who look at later outcomes as employability and earnings, find that tracking reinforces family background effects on labour market outcomes. However these studies suffer from two main limitations, which may explain the difference between their conclusions and my own. On the one hand, Hanushek and Woessmann (2006) only look at compulsory level outcomes, which impedes them from observing the long-term effects of either system. On the other hand, while Brunello and Checchi (2007) analyze later outcomes, they fail to consider the distributional impact of family background on educational outcomes and instead focus solely on mean impacts. My study goes a step further, focusing not only on mean impacts but also on the distributional outcomes of different grouping policies, and determining how those policies may alter the composition of such outcomes by hindering or enhancing the college attendance possibilities for individuals.

This paper complements the existing literature by incorporating an optimal second

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3Role model effects are a good example here. According to this model, characteristics of older group members may influence the behaviour of other individual members of that group. See Durlauf (2004) for an in-depth analysis of these types of models.
stage of education (college), with educational achievement in the compulsory stage being an input to it. This effect that has been neglected in the literature although some empirical studies have shown that the quality of students’ peers at school can influence their college attendance and performance rates (see Betts and Morell (1999) and Hahn et al. (2008)).\textsuperscript{4} Hence, the effects of tracking versus mixing on college choice and ultimate educational achievement are considered in the paper. Finally, this paper complements the theoretical literature on tracking by explicitly discussing the notion of equality of opportunities.

The rest of the paper is organized as follows. Section 2 describes the model and discusses the main features of human capital distribution under the two education systems at compulsory level. Section 3 compares the two systems under efficiency and equity and comments on the potential for a trade-off between both criteria. Section 4 concludes.

2 Model

2.1 Individuals

I consider an economy in which individuals live for two periods. Individuals in each generation differ in two aspects: their family background and their pre-school achievement, $\theta_0$ where $\theta_0 \in [0,1]$. To make the model tractable, I assume that family background takes only two values, that is, individuals can have either poor or rich parents with probabilities $1 - \lambda$ and $\lambda$, respectively.\textsuperscript{5} Population size is 1. I denote by $G_b(\theta_0)$ the C.D.F. (cumulative distribution function) of $\theta_0$ conditional on having family background $b$, where $b = p, r$ for poor and rich parents respectively. To capture the possibility that some level of positive dependence exists between parental background and pre-school achievement, I assume that the density functions of $\theta_0$ for rich and poor parents, denoted by $g_b(\theta_0)$ for $b = r, p$ respectively, cross only once:

Assumption 1 (A.1): There exits a unique $\theta_0^*$ such that $g_p(\theta_0) > (<) g_r(\theta_0)$ for $\theta_0 < (>) \theta_0^*$.

\textsuperscript{4}See Schofield (1995) for a discussion of the possible determinants of the impact of tracking on college attendance from a sociological point of view.

\textsuperscript{5}Alternatively we could interpret the two parent types as black or white, natives or immigrants, etc.
This assumption implies that, conditional on having a pre-school achievement higher (lower) than $\theta_0^*$, the probability of having rich parents is higher (lower) than the probability of having poor parents. This implies not only that the C.D.F. of $\theta_0$ for rich parents dominates the one for poor ones (and thus the means are ordered between both groups) but also that the medians are ordered.\(^6\) This is not a strong assumption. For example, consider the case where, similar to reality, $\theta_0$ follows a lognormal distribution within poor and rich individuals and the variance is the same in both distributions but the mean of $\theta_0$ is higher for rich than for poor individuals. Figure 1 below represents the C.D.F. of $\theta_0$ both for rich and poor students and in the population in black, light-grey and grey line respectively. It also depicts the value $\theta_0^*$. In Section 2.3 below we will see the role of $\theta_0^*$ in the definition of the different tracking systems.

Finally I assume that the support of $\theta_0$ for both poor and rich individuals is full. Thus, the aggregate C.D.F. of pre-school achievement, denoted by $G(\theta_0)$, can be expressed as:

$$G(\theta_0) = \begin{cases} 
0 & \text{if } \theta_0 < 0 \\
(1 - \lambda)G_p(\theta_0) + \lambda G_r(\theta_0) & \text{if } 0 \leq \theta_0 \leq 1 \\
1 & \text{if } \theta_0 > 1
\end{cases} \quad (1)$$

In the first period of their lives, individuals accumulate human capital. At the beginning of this period, which takes a fraction $1 - \alpha$, where $\alpha \in [0, 1]$, they attend compulsory school, which is free of charge, and they are not allowed to work. During the rest of the first period, $\alpha$, some individuals attend college and some others work as unskilled workers. Those who attend college become skilled workers. Observe that the parameter $\alpha$ can be interpreted as the opportunity cost of investment in human capital, that is the fraction of earnings that would have been received in the absence of the investment.

During the second period of their lives, all individuals have one unit of time and all of them work. Those who attended college are now skilled workers, while those who did not remain as unskilled ones. Each worker receives a wage that is proportional to her level of human capital.

\(^6\)That is, $G_r(\theta_0) \leq G_p(\theta_0)$ for any $\theta_0 \in [0, 1]$ and $G_r(\theta_0) < G_p(\theta_0)$ for some $\theta_0 \in [0, 1]$. See for example Heckman (2006) where he documents the early emergence of skills gap, mainly explained by family environments.
2.2 Production of Human Capital

At compulsory level, students are separated into different groups or classes in terms of their pre-school achievement. For the sake of simplicity, I consider only two groups here. The production of human capital is assumed to depend on two factors: pre-school achievement \((\theta_0)\) and the “peer group” effect that depends on the characteristics of each student’s specific group.\(^7\) These characteristics are summarized by the mean achievement or “peer” effect of group \(j\), denoted by \(\overline{\theta}_0^j\).\(^8\) An individual with pre-school achievement \(\theta_0\) will have human capital \(\theta_1\) upon graduation from compulsory school:

\[
\theta_1 = \Phi(\theta_0, \overline{\theta}_0^j),
\]

(2)

where \(\Phi(\theta_0, \overline{\theta}_0^j)\) is a twice differentiable, increasing and concave function in each argument.

During the second part of the first period, each student decides whether or not to attend college and thereby add to the human capital acquired during compulsory school. I denote this increase by \(\varphi\), which can be interpreted as the productivity of college education. Those who decide to attend college will end up with human capital \(\theta_2\):

\[
\theta_2 = \theta_1(1 + \varphi(\theta_1)).
\]

(3)

As this paper focuses on comparing college attendance outcomes for students in mixed versus tracked compulsory educational systems, I assume that the acquisition of human capital at college is not directly affected by students’ peers at this educational level. Rather, I assume that \(\varphi(\theta_1)\) is an increasing function only of that human capital acquired at compulsory school, and at a decreasing rate \((\varphi_1 > 0, \varphi_{11} < 0)\).

It is important to note that the characteristics of a student’s assigned peer group at compulsory level condition her final level of human capital \(\theta_2\) in two ways. It does so directly (one’s compulsory school peers affect the amount of human capital that one acquires at that level) and also indirectly, since such human capital also

\(^7\)Observe that both individuals’ characteristics, pre-school achievement and parental background, affect her final human capital, but in a different way. Whereas individual pre-school achievement has a direct effect on it (since further human capital build on previous achievement) parental background has an indirect effect through the positive dependence with individual pre-school achievement.

\(^8\)There is an intense debate on the influence of peers on individual educational attainment. However, this assumption is commonly accepted in the literature. See, among others, Bishop (2006), Epple and Romano (1998) and Epple, Newlon and Romano (2002) who also assume that peers affect an individual through the mean of their characteristics.
determines the productivity of higher education and, thus, as we will see below, can influence the student’s decision whether or not to attend college.\(^9\)

### 2.3 Educational Systems at Compulsory Level

This section describes the two contrasting educational systems, mixing and tracking, and analyzes the distribution of human capital at the end of compulsory level under each system.

#### 2.3.1 Mixing

Under mixing, the pre-school achievement distribution is the same in both classrooms and the average pre-school achievement within each classroom, denoted \(\bar{\theta}^m\), coincides with the average pre-school achievement in the population.

Under mixing, \(\theta_1\) will lie in the support \([m, \bar{m}]\) where \(m\) and \(\bar{m}\) denote the level of human capital \(\theta_1\) acquired under mixing by the “worst” (the lowest pre-school achiever) and the “best” (the highest pre-school achiever) student in the population.

Therefore, the C.D.F. of \(\theta_1\) under mixing, denoted \(F_M(\theta_1)\), is:

\[
F_M(\theta_1) = \begin{cases} 
0 & \text{if } \theta_1 < m \\
(1 - \lambda)G^m_p(\theta_1) + \lambda G^m_r(\theta_1) & \text{if } m \leq \theta_1 \leq \bar{m} \\
1 & \text{if } \theta_1 > \bar{m},
\end{cases}
\]

(4)

where \(G^m_b(\theta_1) = G_b(\Phi^{-1}(\theta_1, \bar{\theta}^m))\) for \(b = p, r\) and \(\Phi^{-1}\) denotes the inverse of the human capital production function.

#### 2.3.2 Tracking

Tracking students implies grouping them by their pre-school achievement level. For the sake of simplicity I permit only two tracks and denote by \(\delta \in (0, 1)\) and \((1 - \delta)\) the proportion of students in the low and the high track respectively. In addition, I denote by \(t(\lambda, \delta)\) the threshold level of pre-school achievement used for grouping students into one track or the other. Thus, a student is assigned to the high (low) track when his/her pre-school achievement \(\theta_0\) is above (below) \(t(\lambda, \delta)\). In order to

\(^9\)Betts and Morell (1999) find a direct link between the quality of one’s high-school peer group and one’s college grade point average. Hahn et al. (2008) also find that peers’ at high school level affect the student’s decision regarding college attendance. In Section 4, I discuss the implications of this specification on the different results.
simplify notation, in the sequel I will refer to \( t(\lambda, \delta) \) as \( t \). From (1) the threshold level \( t(\lambda, \delta) \) it is implicitly defined by the condition:

\[
(1 - \lambda)G_p(t) + \lambda G_r(t) = \delta. \tag{5}
\]

Observe that the higher \( \delta \) the more elitist is the tracking system. I propose two types of tracking according to their level of elitism. These alternatives captures how “segregated” are rich and poor students between the low and the high track. Given a proportion of rich individuals in the population, \( \lambda \) we say that tracking is **elitist** if \( G(\theta^*_0) \leq \delta \), which implies that \( t \geq \theta^*_0 \). In this case it is clear that conditional on being assigned to the high track (i.e. \( \theta_0 \geq t \)) the probability of being rich is *always higher* than the probability of being poor. In the opposite case, that is if \( G(\theta^*_0) \geq \delta \) (which implies that \( t \leq \theta^*_0 \)) we say that tracking is **non-elitist**. In this case, conditional on being assigned to the high track (i.e. \( \theta_0 \geq t \)) the probability of being rich *might be lower* than the probability of being poor. Figure 1 above illustrates these notions. Figure 1 (a) depicts the non-elitist tracking and Figure 1 (b) the elitist tracking. Below we will see the role of the degree of elitism of tracking on the comparison between tracking and mixing according to equality of opportunities and in determining the existence of trade-off between efficiency and equity.

I use \( \theta^l_0(\lambda, \delta) \) and \( \theta^h_0(\lambda, \delta) \) to denote average pre-school achievement levels for students in the low and the high tracks, respectively. Thus, given the distributional assumptions on \( \theta_0 \):

\[
\overline{\theta}^l_0(\lambda, \delta) = \frac{1}{\delta} \int_0^t \theta_0 g(\theta_0) d\theta_0, \tag{6}
\]

\[
\overline{\theta}^h_0(\lambda, \delta) = \frac{1}{1 - \delta} \int_t^1 \theta_0 g(\theta_0) d\theta_0 \tag{7}
\]

where \( g(\theta_0) \) denotes the p.d.f. (probability density function) of \( \theta_0 \) in the population. It is straightforward to check that the more elitist the tracking system is (i.e. the higher is \( \delta \)), the higher the average pre-school achievement both in the low and the high track.

In the low track, \( \theta_1 \) lies within the interval \([l, \tilde{l}]\). We denote by \( l \) and \( \tilde{l} \) the human capital \( \theta_1 \) acquired in the low track by the “worst” (lowest pre-school achiever) and the “best” (highest pre-school achiever) student respectively. Likewise, in the high track, \( \theta_1 \) lies within the interval \([h, \tilde{h}]\). We denote by \( h \) and \( \tilde{h} \) the human capital \( \theta_1 \) acquired in the high track by the “worst” (lowest pre-school achiever) and the “best” (highest pre-school achiever) student.
The C.D.F. of $\theta_1$ under tracking, denoted by $F_T(\theta_1)$, is:

$$F_T(\theta_1) = \begin{cases} 0 & \text{if } \theta_1 \leq l \\ (1 - \lambda)G_p^l(\theta_1) + \lambda G_r^l(\theta_1) & \text{if } l \leq \theta_1 \leq \bar{l} \\ \delta & \text{if } \bar{l} \leq \theta_1 \leq h \\ (1 - \lambda)G_p^h(\theta_1) + \lambda G_r^h(\theta_1) & \text{if } h \leq \theta_1 \leq \bar{h} \\ 1 & \text{if } \theta_1 > \bar{h}, \end{cases} \quad (8)$$

where $G^j_b(\theta_1) = G_b(\Phi^{-1}(\theta_1, \bar{\theta}_0^j))$ for $b = p, r$ and $j = l, h$.

2.4 College Attendance

Let us now look at how students decide whether or not to attend college. An important consideration to bear in mind here is the extent to which the choice between a tracked or mixed compulsory school system affects the demand for higher education, by exposing students to stronger or weaker peer effects. I assume that each individual wants to maximize her consumption, equal to her lifetime income, and that the latter is a linear function of her total human capital. Since those who do not attend college work as unskilled workers during a fraction $\alpha$ of the first period and throughout the second period, the lifetime income of any individual in this group can be expressed as $\theta_1(1 + \alpha)$. At the same time, the lifetime income of those individuals who attend college is the skilled wage, that is, the increased level of human capital enjoyed by college graduates $\theta_2 = \theta_1(1 + \varphi(\theta_1))$. Therefore, for all individuals who decide to attend college the following condition must hold:

$$\theta_1(1 + \varphi(\theta_1)) \geq \theta_1(1 + \alpha),$$

or,

$$\varphi(\theta_1) \geq \alpha. \quad (9)$$

This condition determines the minimum level of human capital that students must acquire by the end of their compulsory education, $\widehat{\theta}_1$, if they are to attend college. That is, $\widehat{\theta}_1 \in (0, \bar{h})$ is the value that satisfies Equation (9) with equality.\textsuperscript{10} For any given $\widehat{\theta}_1$, let $\pi_s(\widehat{\theta}_1)$ denote the proportion of individuals attending college under educational system $s$, for $s = M, T$, that is $\pi_s(\widehat{\theta}_1) = 1 - F_s(\widehat{\theta}_1)$. Finally, we can define $\widehat{\theta}_0^M$ as the solution to $\widehat{\theta}_0^M = \Phi^{-1}(\widehat{\theta}_1, \widehat{\theta}_0^M)$. That is, $\widehat{\theta}_0^M$ is the minimum pre-school

\textsuperscript{10}To ensure that $\widehat{\theta}_1$ is interior we will assume that $\varphi(l) < \alpha < \varphi(h)$. 

To
achievement level that individuals must have to go on to college in the mixing education system. Similarly we can define \( \widehat{\theta}_0^T \) as the minimum pre-school achievement level that individuals must have to go on to college in the tracking education system. Note from (2) that if \( \widehat{\theta}_1 \leq \widehat{l} \) then the minimum pre-school achievement level required to attend college under tracking belongs to the low track, \( \widehat{\theta}_0^T = \Phi^{-1}(\widehat{\theta}_1, \widehat{\theta}_0^l) < t \) and thus \( \widehat{\theta}_0^T > \widehat{\theta}_0^M \). Similarly, if \( \widehat{\theta}_1 \geq \widehat{h} \) then the minimum pre-school achievement level required to attend college under tracking belongs to the high track, \( \widehat{\theta}_0^T = \Phi^{-1}(\widehat{\theta}_1, \widehat{\theta}_0^h) > t \) and thus \( \widehat{\theta}_0^T < \widehat{\theta}_0^M \). And finally, for any \( \widehat{\theta}_1 \in (\widehat{l}, \widehat{h}) \) then \( \widehat{\theta}_0^T = t \). The following remark summarizes the relationship between \( \widehat{\theta}_0^M \) and \( \widehat{\theta}_0^T \):

**Remark 1** The relationship between \( \widehat{\theta}_0^M \) and \( \widehat{\theta}_0^T \) depends on the value of \( \widehat{\theta}_1 \):

(i) If \( \widehat{\theta}_1 \leq \widehat{l} \) then \( \widehat{\theta}_0^M < \widehat{\theta}_0^T < t \).

(ii) If \( \widehat{\theta}_1 \geq \widehat{h} \) then \( \widehat{\theta}_0^M > \widehat{\theta}_0^T > t \).

(iii) If \( \widehat{\theta}_1 \in (\widehat{l}, \widehat{h}) \) then \( \widehat{\theta}_0^T = t \) and either \( \widehat{\theta}_0^M > t \) or \( \widehat{\theta}_0^M < t \).

For a given function \( \varphi(\theta_1) \), a rise in \( \alpha \) will increase \( \widehat{\theta}_1 \), meaning that a lower proportion of students will attend college. For a fixed \( \alpha \), an upward shift of \( \varphi(\theta_1) \) implies that a higher proportion of students will attend college.\(^\text{11}\)

Finally, it is crucial to find out the interval where \( \widehat{\theta}_1 \) is placed, since it characterizes the composition of the college student body under each of the two education systems. I define two values for the opportunity cost of college attendance, corresponding to two compositional distributions of the college student body under tracking. Let \( \alpha_T \) denote the opportunity cost such that \( \varphi(\widehat{l}) = \alpha_T \). This value \( \alpha_T \) implies that, when \( \alpha < \alpha_T \), some low track students and all high track students attend college. Let \( \alpha^T \) denote the opportunity cost such \( \varphi(\widehat{h}) = \alpha^T \) and thus, \( \alpha_T < \alpha^T \). This value \( \alpha^T \) implies that, if \( \alpha > \alpha^T \), only some of the high track students attend college.

### 3 Efficiency and Equity: Tracking vs Mixing

Having described the systems of tracking and mixing, we can now address the question posed at the beginning of the paper: which governmental grouping policy best supports the objectives of efficiency and/or equity?

\(^{11}\)A rise in \( \alpha \) can be interpreted as either the result of an increase in the difficulty of college studies or as an increase in the length of time spent at college. An upward shift of \( \varphi(\theta_1) \) can be interpreted as an improvement in the productivity of higher education.
3.1 An Efficient Educational System

For the context at hand, an efficient educational system is the one that maximizes average human capital of the entire cohort by the end of second period. The existing literature has studied the efficient education system by focusing on average human capital at the end of the compulsory schooling phase.\(^\text{12}\) However, maximizing human capital at compulsory level does not immediately imply that human capital is maximized at the end of the whole educational process. To see this let \(E_{2s}(\bar{\theta}_1)\) denote average human capital of the entire individual cohort by the end of second period under educational system \(s\) for \(s = M, T\):

\[
E_{2s}(\bar{\theta}_1) = \int_{\bar{\theta}_{1s}}^{\bar{\theta}_1} \theta_1 f_s(\theta_1) d\theta_1 + \int_{\bar{\theta}_1}^{\bar{\theta}_s} \theta_2 f_s(\theta_1) d\theta_1.
\] (10)

where \(\theta_{1s}\) denotes the human capital \(\theta_1\) acquired by the worst student (lowest pre-school achiever) under education system \(s\), that is, \(\theta_{1M} = \underline{m}\) and \(\theta_{1T} = \underline{l}\), and \(\bar{\theta}_{1s}\) denotes the human capital \(\theta_1\) acquired by the “best” student (the highest pre-school achiever) under education system \(s\), that is, \(\bar{\theta}_{1M} = \overline{m}\) and \(\bar{\theta}_{1T} = \overline{l}\). Let \(E_{1s}\) denote average human capital at the end of the compulsory schooling phase under education system \(s\). Then, by using the p.d.f. of \(\theta_0\) we can rewrite Equation (10):

\[
E_{2s}(\hat{\theta}^*_0) = E_{1s} + \int_{\hat{\theta}_0}^{\overline{\theta}_0} \Phi(\theta_0, \overline{\theta}_0) \varphi(\theta_0, \overline{\theta}_0) g(\theta_0) d\theta_0.
\] (11)

Observe that the second term is the increase in the human capital acquired by those students who choose to attend college, those with \(\theta_0 \geq \hat{\theta}^*_0\). Therefore, maximizing human capital at compulsory level does not guarantee that human capital is maximized at the end of the whole educational process. Instead, college attendance rates might also be considered in this analysis.

Below I explore which of the two educational system maximizes college attendance rates. As we shall see, it crucially depends on the opportunity cost of college attendance and on the wealth level in the population. To compare tracking and mixing here we have to compare \(F_T(\theta_1)\) and \(F_M(\theta_1)\). Using Equation (4) for \(F_M(\theta_1)\) and

\(^{12}\text{Hidalgo-Hidalgo (2008) finds that tracking maximizes average human capital at compulsory level. In addition, she find that the difference between average human capital under the two systems, decreases with the elasticity of substitution between peer effects and individual pre-school achievement. Arnott and Rowse (1987) and Benabou (1996) find a similar result.}\)
Equation (8) for $F_T(\theta_1)$, and since $G^j_j(\theta_1)$ is decreasing with $\theta_j$ for $j = m, l, h$, we can check that for any $\theta_1 \in (0, \bar{\theta})$, $F_T(\theta_1) - F_M(\theta_1) > 0$ for every $\lambda$, whereas for any $\theta_1 \in (\bar{\theta}, \tilde{\theta})$, $F_T(\theta_1) - F_M(\theta_1) < 0$ for every $\lambda$. That is, neither system dominates the other according to first order stochastic dominance. Thus, I can define $\tilde{\theta}_1 \in (\bar{\theta}, \tilde{\theta})$ as the level of human capital such that $F_T(\tilde{\theta}_1) = F_M(\tilde{\theta}_1)$.\(^\text{13}\) We need this value for Propositions 1 and Corollary 2 below. As a result it is clear that, for any $\theta_1 \in (0, \tilde{\theta}_1)$, $F_T(\theta_1) - F_M(\theta_1) > 0$ for every $\lambda$, whereas for any $\theta_1 \in (\tilde{\theta}_1, \bar{\theta})$, $F_T(\theta_1) - F_M(\theta_1) < 0$. That is, the density function of $\theta_1$ under tracking accumulates more probability in the tails than under mixing, which shows that the distribution of $\theta_1$ under tracking is more dispersed than it is under mixing.

Proposition 1 below provides a first approach regarding which system maximizes college attendance.

**Proposition 1** Mixing (tracking) maximizes college attendance if and only if $\beta^M_0 < (>)t$.

**Proof.** Recall first that $t$ is such that $(1 - \lambda)G_p(t) + \lambda G_r(t) = \delta$. Now, from the definition of $\tilde{\theta}_1$ (see footnote 13) and $G^m_0(\theta_1)$ it can be checked that $\Phi^{-1}(\tilde{\theta}_1, \tilde{\theta}_0^m) = t$. From the fact that $F_M$ always cuts $F_T$ from below, a necessary and sufficient condition to ensure that $\pi_M(\tilde{\theta}_1) < (>) \pi_T(\tilde{\theta}_1)$ is that $\tilde{\theta}_1 < (>) \tilde{\theta}_1$ or, alternatively, $\tilde{\theta}_0^M > (>)t$.

Observe that if $\tilde{\theta}_0^M$ is very low, then there will be a mass university. Mixing maximizes college attendance rates in this situation since it provides with more human capital $\theta_1$ than tracking to all those with low pre-school achievement levels, that is those who might not attend college. However if $\tilde{\theta}_0^M$ is very high, then the college student body will be very limited. Thus, maximum college attendance is better achieved under tracking since it provides with more human capital $\theta_1$ than mixing to all those with high pre-school achievement, that is those who might not attend college in this situation.

As Corollary 2 shows, regardless of the type of tracking, there are two crucial variables that determine whether the minimum pre-school achievement level required to attend college under mixing is above or below the threshold level of $\theta_0$ used to separate students under tracking, and thus which is the system that maximizes college attendance.

\(^\text{13}\)Note that $\tilde{\theta}_1$ is such that $F_M(\tilde{\theta}_1) = \delta$. From Equation (4) $\tilde{\theta}_1$ is implicitly defined as follows: $(1 - \lambda)G^m_p(\tilde{\theta}_1) + \lambda G^m_r(\tilde{\theta}_1) = \delta$.  

\(13\)
Corollary 2 The system that maximizes college attendance depends on the opportunity cost of college attendance as follows:

(i) If $\alpha \leq \alpha_T$, then the system that maximizes college attendance is always mixing.
(ii) If $\alpha_T < \alpha < \alpha^T$, then the system that maximizes college attendance is tracking when $\lambda$ is low and it is mixing when $\lambda$ is high.
(iii) If $\alpha \geq \alpha^T$, then the system that maximizes college attendance is always tracking.

Proof. Recall that since $F_M$ always cuts $F_T$ from below, a necessary and sufficient condition to ensure that $\pi_M(\tilde{\theta}_1) > (\lambda)\pi_T(\tilde{\theta}_1)$ is that $\tilde{\theta}_1 > (\lambda)\tilde{\theta}_1$. If $\alpha \leq \alpha_T$, then we have $\tilde{\theta}_1 > \tilde{\theta}_1$ for all $\lambda$. Now assume that $\alpha \in (\alpha_T, \alpha^T)$ and, thus, $\tilde{\theta}_1 \in (\tilde{l}, \tilde{h})$. For each value of $\tilde{\theta}_1$, there is one value of $\lambda$, denoted by $\lambda$, such that $\tilde{\theta}_1(\lambda) = \tilde{\theta}_1$. Thus, since $\tilde{\theta}_1$ is increasing with $\lambda$ we have that, if $\lambda < \lambda$ then $\tilde{\theta}_1 < \tilde{\theta}_1$ and when $\lambda > \lambda$ then, $\tilde{\theta}_1 > \tilde{\theta}_1$. Finally, if $\alpha \geq \alpha^T$, we have $\tilde{\theta}_1 < \tilde{\theta}_1$ for every $\lambda$. 

Corollary 2 shows that both the opportunity cost of college attendance and the wealth level in the population determine whether the minimum pre-school achievement level under mixing, $\tilde{\theta}_0^M$ is lower or higher than the threshold level of $\theta_0$ used to separate students under tracking, $t$. With respect to the opportunity cost of college attendance recall that a rise in $\alpha$ will increase $\tilde{\theta}_1$ and as a result a lower proportion of individuals will attend college. Regarding the wealth level observe that $\tilde{\theta}_1$ is an increasing function of $\lambda$. When there are few wealthy individuals ($\lambda$ is low), then $F_M$ surpasses $F_T$ for a low value of $\theta_1$. As societal wealth increases, average human capital also rises, and the crossing point $\tilde{\theta}_1$ moves to the right. In other words, the C.D.F. under mixing will fall below the C.D.F. under tracking for a larger interval of values of $\theta_1$.

Figure 2 illustrates Corollary 2. The intuition is as follows. First, regardless of the wealth level in the population, most high pre-school achievers will attend college when the opportunity cost of college attendance is low ($\alpha \leq \alpha_T$). Observe from Remark 1 that in this situation the minimum pre-school achievement level required to attend college under mixing is below the threshold used to separate students under tracking, i.e. $\tilde{\theta}_0^M < t$. Therefore, intervention must be targeted toward those who performed
poorly in pre-school. As the peer effect is stronger for these students under mixing than it is under tracking ($\overline{\theta}_0^m > \overline{\theta}_0^b$), college attendance is maximized by the former.\footnote{A look at the case of Spain during the 1980s may help to clarify this result. Faced with low college attendance rates, the priority of the government at that time was to increase the number of college students. The low opportunity cost of college attendance, together with a compulsory-level educational system based on mixing, yielded an extraordinary increase in the number of college students from the mid-1980s to the mid-1990s (from 744,115 in 1983/84 to 1,508,842 in 1995/96. See Estadística Universitaria (2003)).}

When the opportunity cost is high ($\alpha \geq \alpha^T$), the opposite result is obtained. In this context, most low pre-school achievers are excluded from college under either system. Note from Remark 1 that a high opportunity cost of college attendance induces high minimum pre-school achievement required to attend college, i.e., $\overline{\theta}_0^M > t$. Intervention should target those who performed well during pre-school, as they are the only ones who will potentially attend college. Choosing tracking is the best way to meet this goal as the peer effect is stronger for these students under tracking than it is under mixing ($\overline{\theta}_0^m < \overline{\theta}_0^b$).\footnote{This result may explain the empirical evidence found by Hahn et al. (2008), in their study based on Korean data regarding high school graduates. They conclude that the number of high school graduates who entered top universities (i.e., those universities for which $\overline{\theta}_1$ is high) was higher under a tracking system than it was under a mixing system.}

When the opportunity cost takes an intermediate value, it is the wealth level in the population (as captured by $\lambda$) that ultimately determines which educational system maximizes college attendance. If the wealth level is very low, the case is similar to that where $\alpha$ is high. In this situation, pre-school achievement levels are very low and thus most pre-school achievers will be excluded from college regardless of the educational systems prevailing at compulsory level. Thus, the appropriate choice in order to maximize college attendance consists of the system that enhances the peer effect. If the wealth level is very high, the situation resembles the one where $\alpha$ is low. In this case, the best system is the one that maximizes college attendance rates among low pre-school achievers, as most high pre-school achievers will attend college irrespective of which educational system is selected.

Note that as $\alpha$ increases, the proportion of students for whom mixing is better than tracking, that is, students with low pre-school achievement levels and wealthy parents, decreases. The population must therefore include a proportionally high number of wealthy students, enough to offset the lower human capital acquired by those high pre-school achievers who were also poor (for whom tracking is better than
mixing).

Finally, some conclusions may be drawn as to which of two systems maximizes average human capital by the end of the second period, $E_{2m}(\hat{\theta}_1)$. In particular, the following Proposition shows that if tracking maximizes both average human capital at compulsory level and college attendance rates, then it is the efficient system:

**Proposition 3** Let $\pi_T(\hat{\theta}_1) > \pi_M(\hat{\theta}_1)$. If $E_{1T} > E_{1M}$ then $E_{2T}(\hat{\theta}_1) > E_{2M}(\hat{\theta}_1)$.

**Proof.** Let $\pi_T(\hat{\theta}_1) > \pi_M(\hat{\theta}_1)$ and then, from Proposition 1 we have that $\tilde{\theta}_0^T > t$. From Remark 1 we know that then $\tilde{\theta}_0^M > \tilde{\theta}_0^T$ where $\tilde{\theta}_0^T > t$. Thus, from Equation (11) the average human capital under tracking is:

$$E_{2T}(\hat{\theta}_1) = E_{1T} + \int_{\tilde{\theta}_0^T}^{\tilde{\theta}_0^M} \Delta(\theta_0, \tilde{\theta}_0^T) g(\theta_0) d\theta_0 + \int_{\tilde{\theta}_0^M}^{1} \Delta(\theta_0, \tilde{\theta}_0^M) g(\theta_0) d\theta_0,$$

and under mixing:

$$E_{2M}(\hat{\theta}_1) = E_{1M} + \int_{\tilde{\theta}_0^M}^{1} \Delta(\theta_0, \tilde{\theta}_0^M) g(\theta_0) d\theta_0,$$

where $\Delta(\theta_0, \tilde{\theta}_0^j) = \Phi(\theta_0, \tilde{\theta}_0^j) \varphi(\theta_0, \tilde{\theta}_0^j)$ for $j = m, h$. Thus it is clear that since $\Delta(\theta_0, \tilde{\theta}_0^T) > \Delta(\theta_0, \tilde{\theta}_0^M) > 0$ then $E_{1T} > E_{1M}$ is a sufficient condition to guarantee $E_{2T}(\hat{\theta}_1) > E_{2M}(\hat{\theta}_1)$. \[\square\]

The intuition is as follows. Assume that the opportunity cost of college attendance is very high, then from Corollary 2 we know that tracking is the system that guarantees maximum college attendance. In particular, those individuals with pre-school achievement $\theta_0 \in (\tilde{\theta}_0^T, \tilde{\theta}_0^M)$ would attend college only under tracking. In addition, since the peer effect that these students enjoy under tracking is stronger than the one they enjoy under mixing, for them the increase in human capital experienced after college is higher under tracking than under mixing (i.e. $\int_{\tilde{\theta}_0^T}^{\tilde{\theta}_0^M} \Delta(\theta_0, \tilde{\theta}_0^T) g(\theta_0) d\theta_0 > \int_{\tilde{\theta}_0^M}^{1} \Delta(\theta_0, \tilde{\theta}_0^M) g(\theta_0) d\theta_0$). As a result, if tracking maximizes average human capital at compulsory level then it also maximizes average human capital at the end of the whole educational process.

In all other cases, the final result regarding which system maximizes average human capital across the population depends on which of the two determining factors,
or the total increase in human capital acquired by those individuals who choose to attend college dominates the other. For example, consider the case where average human capital at compulsory level is higher under mixing than it is under tracking, and mixing maximizes college attendance. However, in this case the increase in human capital experienced by those individuals with pre-school achievement \( \theta_0 \geq t \) (and thus, who would attend college regardless of the education system) is higher under tracking than it is under mixing (i.e. \( \int \Delta(\theta_0, \theta_0^M)g(\theta_0)d\theta_0 > \int \Delta(\theta_0, \theta_0^T)g(\theta_0)d\theta_0 \)). Thus, if the increase in human capital acquired by those students who would attend college only under mixing (i.e. those with \( \theta_0 \in (\hat{\theta}_0^M, \hat{\theta}_0^T) \)) is higher than the lost in human capital experienced by those who would attend college regardless of the education system then mixing is the efficient system.

If the government seeks to achieve efficiency, its best option will depend on the properties of the human capital production at compulsory level, the opportunity cost of college attendance and the wealth level in the population. If we assume that \( \theta_0 \) and \( \theta_0^T \) are close complements then \( E_{1T} > E_{1M} \). In this case tracking will also maximize average human capital across the population if the opportunity cost of college attendance is sufficiently high or the wealth level in the population is sufficiently low.

### 3.2 Equality of Opportunities

Most governments care about equity issues in a broad sense, although there seems to be no single, widely-accepted definition of equity. To circumvent this problem, and for our purposes, I propose that an equal-opportunity policy should aim to equalize college entrance probabilities among students of differing family backgrounds but similar pre-school achievement.

For any given minimum level of human capital required to attend college \( \hat{\theta}_1 \), let \( \pi_{b,s}(\hat{\theta}_1) \) denote the probability of college attendance among students with parental background \( b \), for \( b = p, r \) under education system \( s \) for \( s = M, T \). Thus:

\[
\pi_{b,s}(\hat{\theta}_1) = 1 - F_s(\hat{\theta}_1 | b) = 1 - G^j_b(\hat{\theta}_1),
\]

(14)

where \( F_s(\hat{\theta}_1 | b) \) denotes the C.D.F. of \( \theta_1 \) under education system \( s \) conditional on having parental background \( b \), and \( G^j_b(\hat{\theta}_1) = G_b(\Phi^{-1}(\hat{\theta}_1, \hat{\theta}_0^j)) \) for \( j = m, l, h \).

Thus, equality of opportunities implies that \( \pi_{p,s}(\hat{\theta}_1) \) should equal \( \pi_{r,s}(\hat{\theta}_1) \) and the grouping criteria is proposed as a policy instrument for guaranteeing it. Specifically,
I suggest that governments choose the education system under which the college attendance gap between rich and poor students is the narrowest. The college attendance gap $\hat{\pi}_s(\hat{\theta}_1)$ under education system $s$, for $s = M, T$, is defined as the difference between the college attendance probabilities for rich and poor students:

$$\hat{\pi}_s(\hat{\theta}_1) = \pi_{r,s}(\hat{\theta}_1) - \pi_{p,s}(\hat{\theta}_1).$$

(15)

In the sequel, and in order to consider realistic situations, attention is restricted to the interior case where a proper subset of each type, rich and poor, attend college under either regime, i.e. $\hat{\theta}_1 \in (\underline{m}, \bar{m})$. Thus, from Equations (4) and (15), the attendance gap under mixing is:

$$\hat{\pi}_M(\hat{\theta}_1) = G^m_p(\hat{\theta}_1) - G^m_r(\hat{\theta}_1),$$

(16)

and from Equations (8) and (15), the attendance gap under tracking is:

$$\hat{\pi}_T(\hat{\theta}_1) = \begin{cases} 
G^t_p(\hat{\theta}_1) - G^t_r(\hat{\theta}_1) & \text{if } m \leq \theta_1 \leq \bar{t} \\
G_p(t) - G_r(t) & \text{if } \bar{t} \leq \theta_1 \leq \bar{h} \\
G^h_p(\hat{\theta}_1) - G^h_r(\hat{\theta}_1) & \text{if } \bar{h} \leq \theta_1 \leq \bar{m},
\end{cases}$$

(17)

Therefore, the comparison between $\hat{\pi}_M(\hat{\theta}_1)$ and $\hat{\pi}_T(\hat{\theta}_1)$ depends on the minimum level of human capital required to attend college, $\hat{\theta}_1$ which determines a minimum pre-school achievement level required to attend college, under tracking $\hat{\theta}_0^T$ and under mixing $\hat{\theta}_0^M$. From the definition of $\hat{\theta}_0^T$, $\hat{\theta}_0^M$ and $G^t_p(\hat{\theta}_1)$ the college attendance gap under education system $s$ is:

$$\hat{\pi}(\hat{\theta}_0^s) = G_p(\hat{\theta}_0^s) - G_r(\hat{\theta}_0^s),$$

(18)

for any $\hat{\theta}_0^s \in [0, 1]$ and $s = M, T$. The next Lemma shows that $\hat{\pi}(\hat{\theta}_0^s)$ is maximized for $\hat{\theta}_0^s = \theta_0^s$. This result on $\hat{\pi}(\hat{\theta}_0^s)$ is useful in the analysis that follows.

**Lemma 4** The college attendance gap under both systems $\hat{\pi}(\hat{\theta}_0^s)$ is maximized for $\hat{\theta}_0^s = \theta_0^s$.

**Proof.** From Equation (18) we have that $\frac{\partial \hat{\pi}(\hat{\theta}_0^s)}{\partial \theta_0^s} = 0$ if and only if $g_p(\hat{\theta}_0^s) - g_r(\hat{\theta}_0^s) = 0$, that is, if $\hat{\theta}_0^s = \theta_0^s$. From (A.1) it is clear that $\frac{\partial \hat{\pi}(\hat{\theta}_0^s)}{\partial \theta_0^s} \geq (\leq)0$ for any $\hat{\theta}_0^s \leq (\geq) \theta_0^s$. ■

As follows from (A.1), conditional of having a pre-school achievement level in the interval $[0, \theta_0^s]$ the probability of having poor parents is higher than the probability of having rich parents. The reverse occurs for any pre-school achievement level in
the interval \((\theta_0^*, 1]\). Thus, it is clear that the college attendance gap, that is, the difference between the cumulative probabilities of pre-school achievement for poor and rich students, must be the highest for students with \(\theta_0 = \theta_0^*\).

In addition, from Lemma 4 it is straightforward that the college attendance gap under tracking \(\hat{\pi}^T(\theta_0^*)\) and mixing \(\hat{\pi}^M(\theta_0^*)\) crucially depends on whether the pre-school achievement level required to attend college under mixing \(\hat{\theta}_0^M\) is higher or lower than the pre-school achievement level required to attend college under tracking \(\hat{\theta}_0^T\). Recall from Remark 1 that this relationship depends on whether \(\hat{\theta}_1\) lies in the low or in the high track.

Using Lemma 4 and Remark 1 we can now compare the college attendance gap under tracking and mixing for any \(\hat{\theta}_1\). In addition Proposition 1 above shows that the proportion of college students under mixing is higher (lower) than under tracking if the minimum level of human capital required to attend college \(\hat{\theta}_1\) is below (above) \(\hat{\theta}_1\).

What we study here is the impact of \(\hat{\theta}_1\) being below or above \(\hat{\theta}_1\) on the mass of rich and poor students attending college, which in turn determines the attendance gap under either system. Observe from (16) and (17) above that if the minimum level of human capital required to attend college is such that the proportion of college students under both systems coincides, that is, if \(\hat{\theta}_1 = \hat{\theta}_1\), then the attendance gap under both systems coincides too:

\[
\hat{\pi}^M(\hat{\theta}_1) - \hat{\pi}^T(\hat{\theta}_1) = 0 \quad (19)
\]

In addition to \(\hat{\theta}_1\) there might be some other level of human capital required to attend college, denoted by \(\theta'_1\) such that the college attendance gap under both education systems coincides. Proposition 5 below shows that whether or not \(\theta'_1\) differs from \(\hat{\theta}_1\) and the precise interval where it belongs to depends on the type of tracking. Therefore this aspect determines for which precise set of values of \(\hat{\theta}_1\) tracking is the most equitable system. For expositional purposes I define two values for the opportunity cost of college attendance, corresponding to two compositional distributions of the college student body. Let \(\alpha = \min\{\phi(\hat{\theta}_1), \phi(\theta'_1)\}\) and \(\alpha_T = \max\{\phi(\hat{\theta}_1), \phi(\theta'_1)\}\). Notice that if the opportunity cost is equal to \(\alpha\) or \(\alpha_T\), then the attendance gap under both educational systems coincides. However, the underlying student composition body under tracking differs between them. If \(\alpha = \alpha\), then some low track students and all high track students attend college under tracking (since \(\alpha < \alpha_T\)), whereas if

\[16\text{Recall that } \hat{\theta}_1 \in (l, h) \text{ and thus from Equation (17) } \hat{\pi}^T(\hat{\theta}_1) = G_p(t) - G_r(t). \text{ In addition, } \hat{\theta}_1 = \Phi(t, \theta_0^m) \text{ and thus from Equation (16) } \hat{\pi}^M(\hat{\theta}_1) = G_p(t) - G_r(t).\]
\( \alpha = \bar{\pi} \), then all of the high track students and none of the low track ones attend college (since \( \bar{\pi} > \alpha_T \)). Proposition 5 below shows that the opportunity cost of college attendance is a crucial variable in determining the most equitable system.

**Proposition 5** The most equitable system depends on the opportunity cost of college attendance:

(i) If \( \alpha \in [\alpha, \bar{\pi}] \) then tracking is the most equitable system.

(ii) If \( \alpha < \underline{\alpha} \) or \( \alpha > \bar{\pi} \) then mixing is the most equitable system.

**Proof.** (1) Let tracking be elitist and then \( t \geq \theta_0^* \). First it can be checked that for any \( \tilde{\theta}_1 \geq \underline{\theta} \) then \( \tilde{\pi}(\tilde{\theta}_1) > \tilde{\pi}_M(\tilde{\theta}_1) \). Observe that it implies that \( \tilde{\theta}_0^M > \tilde{\theta}_0^T \). Now \( \tilde{\theta}_0^T > \tilde{\theta}_0^* \) which, from Lemma 4 implies that \( \tilde{\rho}(\tilde{\theta}_0^T) > \tilde{\rho}(\tilde{\theta}_0^M) \). Second, if \( \tilde{\theta}_1 \leq \Phi(\theta_0^*, \bar{\theta}_0^M) \) then \( \tilde{\pi}_T(\tilde{\theta}_1) > \tilde{\pi}_M(\tilde{\theta}_1) \). Observe that it implies that \( \tilde{\theta}_0^T > \theta_0^* \). Since \( t \geq \theta_0^* \) this implies that \( \tilde{\theta}_0^T < t \). From Remark 1 we have that in this case \( \tilde{\theta}_0^M < \tilde{\theta}_0^T \) which, from Lemma 4 implies that \( \tilde{\pi}_T(\tilde{\theta}_1) < \tilde{\pi}_M(\tilde{\theta}_1) \). Then it can be checked that that \( \theta_1' \in (\Phi(\theta_0^*, \bar{\theta}_0^M), \Phi(\theta_0^*, \bar{\theta}_0^M)) \) and thus \( \theta_1' < \tilde{\theta}_1 \). Suppose now that \( \Phi(\theta_0^*, \bar{\theta}_0^M) > \underline{\theta} \) and then if \( \tilde{\theta}_1 \in (\Phi(\theta_0^*, \bar{\theta}_0^M), \underline{\theta}) \) it is clear from Remark 1 that \( \theta_0^* < \tilde{\theta}_0^M < \tilde{\theta}_0^T < t \) which, from Lemma 4 implies that \( \tilde{\pi}_T(\tilde{\theta}_1) < \tilde{\pi}_M(\tilde{\theta}_1) \). Then it can be checked that that \( \theta_1' \in (\Phi(\theta_0^*, \bar{\theta}_0^M), \Phi(\theta_0^*, \bar{\theta}_0^M)) \) and thus \( \theta_1' < \tilde{\theta}_1 \). Therefore in the elitist tracking \( \underline{\alpha} = \varphi(\theta_1') \) and \( \bar{\pi} = \varphi(\tilde{\theta}_1) \).

(2) Let tracking be non-elitist and then \( t < \theta_0^* \). First it can be checked that for any \( \tilde{\theta}_1 \leq \underline{\theta} \) then \( \tilde{\pi}_T(\tilde{\theta}_1) > \tilde{\pi}_M(\tilde{\theta}_1) \). Observe that it implies that \( \tilde{\theta}_0^T < t \) which, from Remark 1, implies that \( \tilde{\theta}_0^M < \tilde{\theta}_0^T \). Since \( t < \theta_0^* \) then \( \tilde{\theta}_0^T < \theta_0^* \) which, from Lemma 4 implies that \( \tilde{\rho}(\tilde{\theta}_0^T) > \tilde{\rho}(\tilde{\theta}_0^M) \). Second, if \( \tilde{\theta}_1 \geq \Phi(\theta_0^*, \bar{\theta}_0^M) \) then \( \tilde{\pi}_T(\tilde{\theta}_1) > \tilde{\pi}_M(\tilde{\theta}_1) \). Observe that it implies that \( \tilde{\theta}_0^T > \theta_0^* \). Since \( t < \theta_0^* \) this implies that \( \tilde{\theta}_0^T > t \). From Remark 1 we have that in this case \( \tilde{\theta}_0^M > \tilde{\theta}_0^T \) which, from Lemma 4 implies that \( \tilde{\pi}(\tilde{\theta}_0^T) > \tilde{\pi}(\tilde{\theta}_0^M) \). Now suppose that \( \underline{\theta} < \Phi(\theta_0^*, \bar{\theta}_0^M) \) and then if \( \tilde{\theta}_1 \in (\underline{\theta}, \Phi(\theta_0^*, \bar{\theta}_0^M)) \) it is clear from Remark 1 that \( t < \tilde{\theta}_0^T < \tilde{\theta}_0^M < \theta_0^* \) which, from Lemma 4 implies that \( \tilde{\pi}_T(\tilde{\theta}_1) < \tilde{\pi}_M(\tilde{\theta}_1) \). Then it can be checked that that \( \theta_1' \in (\Phi(\theta_0^*, \bar{\theta}_0^M), \Phi(\theta_0^*, \bar{\theta}_0^M)) \) and thus \( \theta_1' > \tilde{\theta}_1 \). Suppose now that \( \underline{\theta} > \Phi(\theta_0^*, \bar{\theta}_0^M) \) and then if \( \tilde{\theta}_1 \in (\Phi(\theta_0^*, \bar{\theta}_0^M), \underline{\theta}) \) it is clear from Remark 1 that \( \tilde{\theta}_0^T = t < \theta_0^* < \tilde{\theta}_0^M \) and thus from Lemma 4 we might have that \( \tilde{\pi}_T(\tilde{\theta}_1) < \tilde{\pi}_M(\tilde{\theta}_1) \) for any \( \tilde{\theta}_1 \in (\Phi(\theta_0^*, \bar{\theta}_0^M), \underline{\theta}) \) or only for some \( \tilde{\theta}_1 \) in this interval. In both cases it can
be checked again that \( \theta_1' \in (\Phi(\theta_0', \theta_0^m), \Phi(\theta_0', \theta_0^h)) \) and thus \( \theta_1' > \theta_1 \). Therefore in the non-elitist tracking \( \alpha = \varphi(\theta_1) \) and \( \bar{\alpha} = \varphi(\theta_1') \). ■

Several comments can be made. The first one is that there is no system for which the college attendance gap is always the narrowest, for any minimum level of human capital required to attend college \( \hat{\theta}_1 \). Instead the comparison between \( \hat{\pi}_M(\hat{\theta}_1) \) and \( \hat{\pi}_T(\hat{\theta}_1) \) depends on the particular value of \( \hat{\theta}_1 \) or on the opportunity cost of college attendance, \( \alpha \). Second, regardless of the type of tracking, if the opportunity cost of college attendance is either too low or too high mixing is the most equitable system. The intuition could be as follows. For each \( \hat{\theta}_1 \), we must focus on the individuals at the margin, that is on those whose decision whether or not to attend college depends critically on their family background and may be shaped by the educational system prevailing. If the opportunity cost of college attendance is low, in particular \( \alpha < \bar{\alpha} \) (which implies that the minimum level of human capital required to attend college \( \hat{\theta}_1 \) is very low) then the group of individuals at the margin are those students with low levels of \( \theta_0 \) who under tracking are placed in the low track. Since the peer effect is stronger under mixing than it is in the low track, then the family background plays also a less important role there. Thus, the attendance gap will be narrower under mixing than it is under tracking. Similarly, if \( \alpha \) takes very high values, particularly \( \alpha > \bar{\alpha} \), then the set of individuals at the margin is comprised of those with very high levels of \( \theta_0 \). Therefore the set of individuals at the margin is comprised of those with very high levels of \( \theta_0 \). Recall that advanced students experience a higher peer effect under tracking than they do under mixing and that the mean of \( \theta_0 \) conditional on having poor parents is lower than the mean of \( \theta_0 \) conditional on having rich parents. The difference in \( \theta_1 \) for rich versus poor students will be higher under tracking than under mixing, therefore, due to the complementarity between peer effects and individual achievement.

The third lesson we extract is that if \( \alpha \) takes intermediate values, then the type of tracking (elitist or non-elitist) is the only aspect that determines whether there is room for tracking to be the most equitable system and for which precise set of values of \( \hat{\theta}_1 \) it is so. Suppose first that tracking is non-elitist. Then the group of individuals at the margin would be placed into the high track where the peer effect is the strongest, and family background will not critically condition the total human capital that each student may acquire in compulsory school. By contrast, under mixing since the peer effect is lower than it is in the high track, family background
has a higher relative weight. Thus, the attendance gap will be higher under mixing than it is under tracking. Finally, assume that tracking is elitist. Then, the group of individuals at the margin would be placed in the low track. Recall from the definition of elitist tracking that, in this case conditional on being assigned to the low track (i.e. $\theta_0 \leq t$) the probability of being rich might be higher than the probability of being poor. In addition consider that these students experience a higher peer effect under mixing than they do under tracking and that the mean of $\theta_0$ conditional on having poor parents is lower than the mean of $\theta_0$ conditional on having rich parents. The difference in $\theta_1$ for rich versus poor students will be higher under mixing than under tracking, therefore, again due to the complementarity between peer effects and individual achievement. In the numerical example below we perform a detailed analysis of both types of tracking: elitist versus non-elitist.

### 3.2.1 A numerical example: elitist vs non-elitist tracking

In order to illustrate the results on the comparison between $\hat{\pi}_M(\hat{\theta}_1)$ and $\hat{\pi}_T(\hat{\theta}_1)$ I present and discuss numerical simulations so that the results will provide suggestive evidence about the education system that better achieves equality of opportunities.

To get closed-form solutions, the model in Hidalgo-Hidalgo (2008) is adopted and extended to consider college attendance. Thus, regarding the distribution of pre-school achievement, I assume that $G_r(\theta_0) = \theta_0$ and $G_p(\theta_0) = \theta_0^\gamma$ where $\gamma \in (0, 1]$. That is, the lower is $\gamma$, the higher the gap in pre-school achievement between poor and rich students.$^{17}$

With respect to the production of human capital at compulsory level $\Phi(\theta_0, \bar{\theta}_0^j)$, I assume that it is a CES of the two inputs, $\theta_0$ and $\bar{\theta}_0^j$: $^{18}$

$$\Phi(\theta_0, \bar{\theta}_0^j) = A(\rho(\theta_0)^\beta + (1 - \rho)(\bar{\theta}_0^j)^\beta)^{1/\beta};$$

where $A > 1$, $\rho \in [0, 1]$ and $\beta \in (-\infty, 1]$. The parameter $\rho$ captures the weight of pre-school achievement on $\theta_1$. Observe that, for $\beta$ sufficiently low, both $\theta_0$ and $\bar{\theta}_0^j$ have a high level of complementarity and as $\beta$ tends to 1, the two factors become perfect substitutes.

$^{17}$Observe from (15) and the definition of $G_r(\theta_0)$ and $G_p(\theta_0)$ above that $\partial \hat{\pi}_s(\hat{\theta}_1)/\partial \gamma < 0$ for $s = M, T$. That is, for both educational systems and as expected, the lower the pre-school achievement gap, the lower the attendance gap between the rich and the poor.

$^{18}$See Hidalgo-Hidalgo (2008) for a detailed discussion of the properties of this education production function and how it captures the main empirical evidence.
The college attendance gap under both educational system are now compared for
two societies that differ in the level of elitism of their tracking systems, captured by
parameter $\delta$.

To perform this numerical exercise, I need to look for reasonable values of the
parameters. However, scant empirical evidence exists on some crucial parameters,
which must be kept in mind when interpreting the results. This exercise should not
be taken as a full-fledged calibration exercise, since the model is too abstract to be
calibrated properly. We need common values for $\lambda, \rho, \beta$ and $\gamma$ for both and two levels
of $\delta$ for each society. Table 1 shows the selected parameter values. This selection is
briefly explained below.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>$3/4$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$5/6$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$3/4$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$0.8$</td>
</tr>
<tr>
<td>$A$</td>
<td>$2$</td>
</tr>
<tr>
<td>$\delta$ (non-elitist)</td>
<td>$0.11$</td>
</tr>
<tr>
<td>$\delta$ (elitist)</td>
<td>$0.75$</td>
</tr>
</tbody>
</table>

As families, rather than schools, are mainly responsible for the inequalities in
school performance (see the Coleman Report and more recent works such as those by
Heckman (2006) and references therein), it seems appropriate to assign a high value
to $\rho$. In particular, I fix $\rho = 3/4$. The value for $\lambda$ is drawn from OECD data. Recall
that $\lambda$ captures the proportion of rich individuals in the population. According to the
OECD questionnaire on household income distribution (2002), the proportion of in-
dividuals considered as poor averaged 10%, with an increasing tendency (see OECD
(2002)). Hence I set $\lambda = 5/6$, which also seems appropriate if we, alternatively,
interpret $\lambda$ as the proportion of natives in the population.20 There is no empirical
evidence on the degree of complementarity/substitutability between peer effects and

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19In most studies on poverty, the poor are all individuals living in households with income below
the poverty line, which is fixed at 50% of national median adjusted income. See Collado and Iturbe-
Ormaetxe (2008) for an in-depth analysis of this topic.

20Immigrants accounted for just under 12% of the total population in OECD countries in 2006
(see OECD (2008)).
individual pre-school achievement. However, recent empirical evidence (see Ding and Lehrer (2007) and Kim et al. (2006)) suggests that both factors are close comple-ments. Thus, I set $\beta = 3/4$ which corresponds to an elasticity of substitution between $\theta_0$ and $\tilde{\theta}_0$ equal to 4. The value of $\gamma$ is also drawn from OECD data. Specifically, I assume $\gamma = 0.8$, which means that the mean pre-school achievement level for poor students represents 89% of the achievement level for rich students (see PISA 2003 Report).  

The elitism parameter $\delta$ for the non-elitist society is chosen so that the proportion of students in the low track matches the average proportion of 15-year-olds enrolled in programmes that give access to either vocational studies or to the labour market in OECD countries in 2006 (11.3%). However, the parameter $\delta$ for the elitist society is simply chosen high enough to contrast the non-elitist case.

Figure 3 depicts $\hat{\pi}_M(\hat{\theta}_1)$ and $\hat{\pi}_T(\hat{\theta}_1)$ for both societies. Here $\hat{\pi}_M(\hat{\theta}_1)$ is shown as a solid line and $\hat{\pi}_T(\hat{\theta}_1)$ as a dashed line.  

Here Figure 3 (The college attendance gap)

Figure 3 confirms the results in Proposition 5 regarding which system minimizes the attendance gap. In particular, in both societies $\hat{\pi}_T(\hat{\theta}_1) > \hat{\pi}_M(\hat{\theta}_1)$ if $\hat{\theta}_1$ is sufficiently low or high whereas $\hat{\pi}_T(\hat{\theta}_1) < \hat{\pi}_M(\hat{\theta}_1)$ if $\hat{\theta}_1$ takes intermediate values. In addition, in the non-elitist tracking the set of students for which tracking is more equitable than mixing are in the high track whereas in the elitist tracking the set of students for which tracking is more equitable than mixing are in the low track. It is immediate to check that the difference between the college attendance gap under tracking and mixing for students with intermediate levels of $\hat{\theta}_1$ can be significant (up to 3.4% in the non-elitist case and to 13.6% in the elitist case).

To conclude, contrary to the general belief that equality of opportunities is best achieved under mixing, I find that tracking may be a more effective means of achieving that goal in some cases. In effect, my study suggests that switching from tracking to mixing will not automatically further each student’s access to equal opportunities. Indeed, it will actively work against that goal for students with intermediate levels of $\hat{\theta}_1$.

\footnote{According to the PISA 2003 results for OECD countries, the mean in math performance among immigrants represents about 90%, on average, of the mean in math performance for native students.}

\footnote{In addition it also matches the corresponding proportion of 15-year-olds enrolled in programmes that give access to either vocational studies or to the labour market for countries like some Serbia (75.7%) and is close to some EU countries like The Netherlands (55%).}
This result is quite surprising and contrasts with the main conclusions heretofore reported in the empirical literature on the subject, including studies by Hanushek and Woessmann (2006) and Brunello and Checchi (2007) (see my comments above).

Finally, this result is consistent with observed stylized facts. Some European countries are experiencing a decline in college attendance and an increase in college drop-out rates. The most notable example is Spain where, even without explicitly mentioning it, the most recent education reform during the 1990s (known as LOGSE), entailed a mixing grouping system. There, both of the effects commented above are particularly strong among those students with intermediate levels of achievement, who might have graduated from college under a different grouping policy.23

3.3 On the trade-off between efficiency and equity

Finally I would like to comment on the possibility of a trade-off between efficiency and equity. If we assume complementarity between peer effects and individual preschool achievement, then \( E_{1T} > E_{1M} \) and from Propositions 3 and 5 the presence of trade-offs between efficiency and equity will depend solely on the opportunity cost of college attendance and the societal wealth.

First, if the opportunity cost of college attendance is low (which implies that \( \theta_1 \) is low), then mixing might achieve both efficiency and equity. In this case, Corollary 2 implies that the probability that students, of any parental background, attend college is always higher under mixing than under tracking. Proposition 5 also shows mixing to be the most equitable system. Yet mixing does not maximize the average human capital obtained at the compulsory level (under the assumption of complementarity between peer effects and individual achievement). However, if the increase in the level of human capital brought about by college attendance is high enough to compensate for the lower average human capital achieved at the compulsory level, then mixing will maximize average human capital across the population and will be both the efficient system and the one that better guarantees equality of opportunities among students.

23LOGSE is the Spanish abbreviation for Ley de Ordenación General del Sistema Educativo. Among other things, this reform raised the compulsory schooling age to 16 and softened the requirements for grade advancement. Today we observe a decrease in the entry rates into tertiary-type A programmes from 47% in 2000 to 43% in 2006 (see Education at a Glance 2008). In addition, the proportion of college students among those in the corresponding age group whose parents hold a secondary education degree dropped from 55% in 1998 to 45% in 2007 (see EPA 2008 and Albert (2008)).
Figure 4 displays these results. It represents the values $\pi_{r,s}(\hat{\theta}_1)$ and $\pi_{p,s}(\hat{\theta}_1)$ for the non-elitist and elitist societies. Here, $\pi_{b,M}(\hat{\theta}_1)$ is shown as a solid line and $\pi_{b,T}(\hat{\theta}_1)$ as a dashed line for both poor (in black) and rich (in grey).

Second, observe that tracking can be both efficient and equitable in some situations. For example, if the opportunity cost of college attendance takes intermediate values and wealth level in the population is low, then whether tracking is also efficient in this interval depends on the type of tracking. If it is non-elitist, then tracking guarantees higher college attendance rates (for both rich and poor students) than mixing. However, if tracking is elitist, then college attendance rates (for both rich and poor students) are lower under tracking than under mixing. Figure 4 illustrates this result. Note that, in the non-elitist case the interval where $\hat{\theta}_1$ might lie partially corresponds to students in the high track. Observe that the probability of college attendance, for students of any parental background is always higher under tracking than under mixing. However, in the elitist case this interval of $\hat{\theta}_1$ partially corresponds to students in the low track. Thus, the probability of college attendance, for students of any parental background is always higher under mixing than under tracking. To conclude, provided that tracking is non-elitist it will not only be the efficient system but also the most equitable one according to Propositions 3 and 5.

Finally, a trade-off between efficiency and equity will clearly take place when the opportunity cost of college attendance is high. According to Proposition 3, tracking is the efficient system in this case, since it maximizes both the average human capital at compulsory level and college attendance. Yet we from Proposition 5 above we know that, for this case, mixing does better than tracking at guaranteeing equality of opportunities among students. In all other cases we can not conclude that there is not a trade-off between efficiency and equity.

4 Discussion and final Comments

This paper analyzes public intervention in education when the government, taking into account the existence of peer effects at compulsory education level and its impact at college level, has to decide how to group compulsory school students. Two different education systems (tracking and mixing) are examined. Efficiency and equity are assumed to be two central governmental concerns.
Conventional wisdom suggests that equality of opportunities is best guaranteed under mixing. My study shows that this is not necessarily the case, and that the impact on educational results at later stages (i.e. college attendance rates) must be taken into account when weighing the pros and cons of either educational system. In this context, I find tracking to be the most equitable system for students with intermediate levels of human capital required to attend college. The degree of elitism of tracking is found to be a central aspect in determining the existence of trade-off between efficiency and equity. In particular, if tracking is non-elitist then it is not only the most equitable system and also the most efficient one if the opportunity cost of college attendance takes intermediate values.

The paper abstracts from variation in schooling public expenditure in compulsory and college levels, previously considered by Arnott and Rowse (1987), Benabou (1996) and Epple and Romano (1998) among others. Observe that the analysis of efficiency might not be reversed by considering for example the direct cost of providing college education. Higher college attendance rates imply higher total cost of college education, however this cost will always be lower than its associated benefits (otherwise it makes no sense to promote college attendance). In addition, abstracting from this concern enables me to isolate the role of compulsory school peer effects on college attendance, which has not been considered in the prior literature.

The paper allows for some extensions. An important one is the introduction of prices which are omitted in this paper under the assumption of free education in both levels. This would imply modelling parental income explicitly and could enable second-best analysis to be introduced in the comparison between tracking and mixing. The crucial assumption in this analysis would be on the degree of complementarity/substitutability between parental income and peer group effects. In addition, we might consider the effect on wages of the number of college graduates. I think that introducing this assumption would not change qualitatively the main results of the paper. Note that, this would make the opportunity cost of college attendance an endogenous variable, $\alpha(\pi_s)$. Thus, the equilibrium proportion of college students under education system $s$, $\pi_s$, would be given by $\pi_s = 1 - F_s(\varphi^{-1}(\alpha(\pi_s)))$. If we just consider the conventional supply effect on the skilled wage, it can be checked that $\pi_s$ exists and it is unique. Finally, it might also be interesting to compare the two education systems in a dynamic setup, or to consider alternative governmental criteria with respect to equity. In this regard, we might assume that the government wishes to maximize the probability of college attendance among only the worst-off
individuals in the population, assuming that by “worst-off” we mean children of poor parents. The results in this case would be quite similar to the ones found above (see also Hidalgo-Hidalgo (2005)): when the opportunity cost of college attendance is low, college attendance is maximized under mixing, and the reverse is true when the opportunity cost of college attendance is high.

Finally, I believe my results on compulsory school peers’ impact on college outcomes are of value and seem relevant to several key issues currently under debate among economists of education. In addition these theoretical results yield two hypothesis to be tested empirically: the impact of grouping policies on the deceleration in college entry rates recently observed in some European countries (see Education at a Glance (2008) and Hahn et al. (2008)) and the distributional impact of these grouping policies on students with different background.
References


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Figure 1: Distribution of $\theta_0$ and types of tracking
Figure 2: College attendance under both systems
Non-Elitist Tracking

Elitist Tracking

Note: \( \hat{\pi}_u(\hat{\theta}_l) \) \( \hat{\pi}_T(\hat{\theta}_l) \)

Figure 3: The attendance gap
Non-Elitist Tracking

Elitist Tracking

Note: 

- $\pi_{p,M}(\hat{\theta}_1)$ 
- $\pi_{p,T}(\hat{\theta}_1)$ 
- $\pi_{r,M}(\hat{\theta}_1)$ 
- $\pi_{r,T}(\hat{\theta}_1)$ 

Figure 4: College attendance for poor and rich students