Redistributive effects in a dual income tax system

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Abstract

Equity issues of the dual income tax have been left aside in the field of economics. Since a dual income tax needs different modelling than a comprehensive one this paper offers firstly a quantitative framework to measure redistributive effects; it turns out that this involves both direct and indirect effects. The effects of horizontal inequity and re-ranking are also incorporated into the decomposition analysis. The approach is applicable using available income and tax statistics. Secondly, partial effects of changes in tax parameters are presented; they are channelled through the direct and indirect effects; and are not always straightforward.

Keywords: redistributive effect, progressivity, dual income tax.
JEL Classification: H23, D63.

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1. Introduction

In the 1980´s and 1990´s the Nordic countries adopted dual income tax systems, which combine progressive taxation of labour income with proportional taxation of capital income. The *raison d'être* for this innovation was concern for efficiency (Sørensen 1994, Nielsen and Sørensen 1997, Boadway 2005). Subsequent concerns have been for the amount of revenue raised, the distortion created, and aspects of equity and fairness. Concern for equity has gained much attention in the literature for a comprehensive income tax, both theoretically and empirically (see for example Lambert 2001, Wagstaff et. al. 1999), but there is no theoretical analysis that we know of in existing literature of the equity and/or inequality effects of a dual income tax system.

In this paper, a quantitative framework is devised for analyzing the contributions of different parameters in a dual income tax system to the overall redistributive effect of the system. The problem comes, of course, in merging the (known) separate effects of the two taxes, on the distributions of labour and capital incomes respectively, into an overall effect on the joint distribution. By selecting the Gini coefficient as inequality measure, we are able to combine more-or-less familiar results on inequality decomposition across income sources (Shorrocks 1982, 1983), on redistributive effect (Kakwani 1977, Pfähler 1990) and on horizontal inequity and reranking (Aronson and Lambert 1994), to gain insight into the workings of a dual income tax system.

The structure of the paper is as follows. In Section 2, we briefly outline relevant results from the measurement literature in respect of a comprehensive income tax. Our new approach for decomposition of the redistributive effect of a dual income tax is presented in Section 3. In Section 4, a numerical example is given, to illustrate the workings of a dual income tax system and indicate the potential of the method in empirical applications. Section 5 investigates how changes in a dual income tax system may affect redistribution. Section 6 concludes with an evaluation of the contribution of the paper and discussion of ways forward.

2. Progressivity and redistributive effect for a comprehensive income tax: a brief review

A comprehensive income tax is progressive if the average tax rate rises with income, proportional (flat) if average tax rate is constant and regressive if average tax rate decreases with income. Inequality is reduced by application of a progressive tax, stays the same after application of a flat tax, and is increased by application of a regressive tax (Fellman 1976, Jakobsson 1976). Kakwani (1977) measures progressivity by twice the area between the Lorenz curve of pre-tax income, L_x, and the concentration curve of tax liabilities, L_t, formally:

$$(1) \qquad \Pi^K = 2 \int_0^1 [L_x(p) - L_t(p)] \, dp = C_t - G_x$$

where C_t is the concentration coefficient of tax liabilities and G_x is the Gini coefficient of pre-tax income. Redistributive effect is correspondingly measured by twice the area between the Lorenz curve of pre-tax income and the concentration curve of post-tax (net) income:

$$（2）\qquad \Pi^{RS} = 2 \int_0^1 [L_x^N(p) - L_x(p)]\, dp = G_x - C_x^N$$

where L_x^N is the concentration curve for post-tax income with respect to the pre-tax ranking of income units, and C_x^N is the concentration coefficient. Π^{RS} is known as the Reynolds-Smolensky index, after Reynolds and Smolensky (1977). Kakwani shows that:

$$（3）\qquad \Pi^{RS} = \frac{g}{1-g} \Pi^K$$

where g is the fraction of all income taken in tax, or the 'total tax ratio'. Equation (3) decomposes redistributive effect multiplicatively into tax level and progressivity contributions.

Pfähler (1990) extends this methodology to determine the components of progressivity and redistributive effect that are attributable to features of the tax code, namely, to allowances and deductions, and to the rate structure. Pfähler's Kakwani measure for the allowance, viewed as a subtraction from gross income in the determination of taxable income, is:

$$（4）\qquad \Pi_A^K = 2 \int_0^1 [L_x(p) - L_A(p)]\, dp = C_A - G_x$$

where L_A is the concentration curve and C_A is the concentration coefficient for the allowance,[1] and his Reynolds-Smolensky index is:

$$（5）\qquad \Pi_A^{RS} = 2 \int_0^1 [L_{x-A}(p) - L_x(p)]\, dp = G_x - C_{x-A}$$

where L_{x-A} is the concentration curve and C_{x-A} the concentration coefficient for income net of the allowance (taxable income in the absence of income-related deductions). Redistributive effect decomposes into level and progressivity components as:

$$（6）\qquad \Pi_A^{RS} = \frac{\gamma}{1-\gamma} \Pi_A^K$$

where γ is the average rate of allowance, $\gamma = \bar{A}/\bar{x}$.[2] Pfähler's Kakwani and Reynold-Smolensky indexes for the rate structure *per se* are these:

$$（7）\qquad \Pi_R^K = 2 \int_0^1 \left[L_y(p) - L_t(p)\right] dp = C_t - C_y$$

$$（8）\qquad \Pi_R^{RS} = 2 \int_0^1 \left[L_{y-t}(p) - L_y(p)\right] dp = C_y - C_{y-t}$$

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[1] Allowances are generally lump sum amounts, hence $C_A = 0$ and $\Pi_A^K < 0$.

[2] The same can be done for income-related tax deductions, if present. The equations are similar: see Pfähler (1990) for details.

where y is taxable income, $y = x - A$ in the absence of income-related deductions in the tax code, C_y is the concentration curve and L_y the Lorenz curve for taxable income. These values are also linked:

$$(9) \qquad \Pi_R^{RS} = \frac{g_R}{1 - g_R} \Pi_R^K$$

where g_R is the total tax ratio of the rate structure, or $g_R = \bar{t}/\bar{y}$. The bottom line is Pfähler's decomposition:

$$(10) \qquad \Pi^{RS} = \frac{g}{1 - g} \left\{ \Pi_R^K - \frac{\gamma}{1 - \gamma} \Pi_A^K \right\}$$

Equation (10) decomposes redistributive effect multiplicatively into components representing the features of the tax code as well as tax level and progressivity contributions.

The horizontal and reranking effects of a comprehensive income tax are dealt with in Aronson and Lambert (1994), who offer a technique to measure vertical redistribution, horizontal inequity and reranking effects commensurately using Gini-based measures. These authors demonstrate that the Reynolds-Smolensky index for any tax can be decomposed, as:

$$(11) \qquad \Pi^{RS} = V - H - R = \frac{g}{1 - g} \Pi^K - H - R$$

where V measures the redistribution that would have occurred if equals had been treated equally (the so-called 'vertical' contribution to the overall redistributive effect), H measures horizontal inequity, as a loss of redistribution effect due to the unequal treatment of exact equals (i.e when individuals with the same gross income or pre-tax living standard do not pay same tax), and R measures the additional effect of reranking (if any) caused by the tax system since post tax incomes are often differently ranked than pre tax incomes.

3. Decomposing redistribution for a dual income tax: the new approach

For a typical dual income tax, income from labour is taxed progressively and income from capital is taxed proportionally. The overall inequality effect of such a tax system can be positive or negative, depending on the component tax functions themselves and also upon the component income sub-distributions to which the taxes are applied, and how they combine into the joint distribution. To see that inequality may be increased, one only needs to consider scenarios where there is complete equality overall before tax, but people have different combinations of labour and capital income: in such scenarios, except in very special circumstances, a dual income tax system inevitably introduces inequality where there was none.[3] To be quite general, we need to capture the redistributive effects which are due to the composition of income as well as those stemming from the *ceteris paribus* actions of the component income tax functions on the relevant component distributions.

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[3] Let $t_L(.)$ and $t_K(.)$ be the component tax functions The tax liability of a person having x in labour income and y in capital income, where $x > y$, would not be the same as the tax liability of a person having y in labour income and x in capital income unless $t_L(x) - t_L(y) = t_K(x) - t_K(y)$. Therefore, given a dual income tax system $\{t_L(.), t_K(.)\}$, scenarios can be found in which overall inequality is increased (from zero) by application of the tax, unless the restriction $t_L(x) - t_L(y) = t_K(x) - t_K(y)$ holds for all x and all $y > x$, in which case the component taxes must be proportional and have a common rate.

The key is to decompose the Gini coefficient of gross income, G_x, and concentration coefficient of net income, C_x^N, into components expressing the characteristics of the source distributions, and then "rebuild" the overall measures from information (from Section 2 of this paper) about the component taxes. We therefore need a way to write a Gini or concentration coefficient in the form $\sum_k S_k$ where S_k is a contribution coming from income component k (and $k = 2$ here, though in general it may be larger). That is, we need to apply a *decomposition rule*. Shorrocks (1982, 1983) debates several forms of decomposition rule, and not only for Gini and concentration coefficients. The perfect one for us is the so-called 'natural rule' in which $S_k = \alpha_k C_k$, where α_k is the share of the kth income component in total income and C_k is the concentration index of the kth income component with respect to the ranking of income units by their overall incomes.[4]

Let α_L and α_K be the shares of labour and capital income in overall gross income, let g_L and g_K be the respective total tax ratios, or $g_L = \bar{t}_L / \overline{x_L}$ and $g_K = \bar{t}_K / \overline{x_K}$, and g the overall total tax ratio, $g = \bar{t} / \bar{x}$. Then the shares of labour and capital in overall *net* income are:

(12) $\qquad \alpha_L^N = \alpha_L \left(\dfrac{1 - g_L}{1 - g} \right)$

and

(13) $\qquad \alpha_K^N = \alpha_K \left(\dfrac{1 - g_K}{1 - g} \right)$

The natural decomposition rule tells us that

(14) $\qquad G_x = \alpha_L C_L + \alpha_K C_K$

(15) $\qquad C_x^N = \alpha_L^N C_L^N + \alpha_K^N C_K^N$

where C_L and C_K are the concentration indices of gross labour and capital income and C_L^N and C_K^N are the concentration indices of net labour and capital income.

The overall Reynolds-Smolensky index, $\Pi^{RS} = G_x - C_x^N$, can now be written, using equations (14) and (15), as

(16) $\qquad \Pi^{RS} = [\alpha_L C_L - \alpha_L^N C_L^N] + [\alpha_K C_K - \alpha_K^N C_K^N]$

or as

(17) $\qquad \Pi^{RS} = \alpha_L [C_L - C_L^N] + \alpha_K [C_K - C_K^N] - [\varepsilon_L C_L^N + \varepsilon_K C_K^N]$

where ε_L and ε_K are such that $\alpha_L^N = \alpha_L + \varepsilon_L$ and $\alpha_K^N = \alpha_K + \varepsilon_K$ and, of course, $\varepsilon_L + \varepsilon_K = 0$. (17) can also be written as

(18) $\qquad \Pi^{RS} = \alpha_L \Pi_L^{RS} + \alpha_K \Pi_K^{RS} - [\varepsilon_L C_L^N + \varepsilon_K C_K^N]$

equivalently, applying the Kakwani (1977) methodology, as

(19) $\qquad \Pi^{RS} = \alpha_L \dfrac{g_L}{1 - g_L} \Pi_L^K + \alpha_K \dfrac{g_K}{1 - g_K} \Pi_K^K - [\varepsilon_L C_L^N + \varepsilon_K C_K^N]$

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[4] In particular, Shorrocks developed a decomposition rule, independent of the choice of inequality measure, which is such that the contribution of different components to overall inequality is independent of the index chosen. In empirical research, some scholars have chosen to use this alternative decomposition rule (see e.g. Jäntti 1997, Jenkins 1995), while others have adopted the natural rule (see e.g. Brandolini and Smeeding 2009, Kakwani 1986, Pyatt et. al. 1980).

in which $\varepsilon_L < 0 < \varepsilon_K$ if $g_K < g < g_K$, $\varepsilon_K < 0 < \varepsilon_L$ if $g_L < g < g_K$ and, $\varepsilon_K = \varepsilon_K = 0$ if $g_K = g_K = g$.[5]

When $g_K \neq g_L$, the overall redistributive effect thus has indirect as well as direct effects, the indirect effects stemming purely from differences in tax levels. If $g_L > g_K$, as typically, the indirect effects causes an increase in the contribution of the labour income tax to overall redistribution, but a decrease the contribution of the capital income tax. The net indirect effect is positive, and hence increases Π^{RS}, if $\varepsilon_L C_L^N > -\varepsilon_K C_K^N$. Since $\varepsilon_L = -\varepsilon_K$, the condition becomes $C_L^N > C_K^N$. Normally though, the condition does not hold (Fräßdorf et. al. 2010, Jäntti 1997, Kakwani 1986).

To summarize, equation (19) decomposes the Reynold-Smolensky index of overall redistributive effect into three components. First, the direct redistributive effect of (progressive) labour taxation on the labour income distribution; second, the same for the capital income tax; and third, the effect of the difference in tax levels (if any) between the income components, the indirect effect. This third term also depends on how the component income distributions 'fit together'.[6]

If the component taxes within a dual income tax system do not themselves treat pre-tax equals equally (although this would be unusual, given that the one is typically proportional and the other linear above a lump-sum allowance), then additional terms involving reranking and horizontal inequity could be incorporated into (18), along the lines of Aronson and Lambert (1994). Briefly, this involves an expansion in (19):

$$(20) \qquad \Pi^{RS} = \alpha_L \left[\frac{g_L}{1 - g_L} \Pi_L^K\right] - \alpha_L^N[H_L + R_L] + \alpha_K \left[\frac{g_K}{1 - g_K} \Pi_K^K\right] - \alpha_K^N[H_K + R_K]$$
$$- [\varepsilon_L C_L^N + \varepsilon_K C_K^N]$$

where H_L and H_K are the respective horizontal inequities for labour and capital income and R_L and R_K are the reranking effects. Implementation of the method shown in equation (20) is not straightforward, involving the construction of pre-tax close equals groups with an optimal bandwidth, and details will be omitted here. See Urban and Lambert (2008) for a very full treatment and discussion.[7]

Finally, observing that it is usual for the component tax on capital incomes to be proportional, so that $C_K = C_K^N$ and $\Pi_K^K = 0$ in (19) and (20), and for the labour income tax to be linear above a lump-sum allowance (i.e. progressive), the Pfähler (1990) decomposition may now be applied. Setting aside the unequal treatment effects shown in (20), our simplified representation of (19) becomes:

$$(21) \qquad \Pi^{RS} = \alpha_L \left[\frac{g_L}{1 - g_L} \left\{\Pi_R^K - \frac{\gamma}{1 - \gamma} \Pi_A^K\right\}\right] - [\varepsilon_L C_L^N + \varepsilon_K C_K^N]$$

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[5] From (12) and (13), we have $\varepsilon_L = \alpha_L \left(\frac{g - g_L}{1 - g}\right)$ and $\varepsilon_K = \alpha_K \left(\frac{g - g_K}{1 - g}\right)$.

[6] A word of caution is in order. The terms in (19), and therefore those in (21), are not independent. For example, changes in the capital income distribution may affect the ordering of persons by their total incomes, and thereby affect C_L^N and C_K^N (etc.).

[7] See also Duclos and Araar (2006) where an alternative methodology using kernel density estimation is sketched, and also the free software for distributive analysis named *DAD,* which has been developed by these authors, is described and could be used to evaluate all components of our decompositions

(19)-(21) provide a convenient and succinct quantitative framework to analyze contributions to the overall redistributive effect of a dual income tax system.[8]

4. Numerical example

In order to illustrate the decomposition formula represented in equation (21) here a numerical example is shown for a hypothetical distribution of income. Table 1 shows the income and taxation of 10 individuals. They have income from capital and labour. In the example, capital income is highly skewed towards the upper tail of the income distribution. Capital income is taxed proportionally, with a marginal and average tax rate of 10%. Labour income is taxed progressively with a 40% marginal tax rate on taxable income below 10 and 50% for taxable income above 10. The tax threshold, i.e. the allowance, is 7.5. Since, in this example, individuals below the tax threshold do not gain the entire allowance, $C_A \neq 0$.[9]

[TABLE 1 ABOUT HERE]

Table 2 shows redistribution for the hypothetical income distribution decomposed as in equation (21). Note that there is no horizontal inequity or reranking in this simple example. The table therefore shows the exact decomposition of the Reynold-Smolensky index of redistribution and demonstrates that labour income tax decreases inequality while capital income tax has no effect on income inequality.

[TABLE 2 ABOUT HERE]

While the direct effects of the tax system reduce the Gini coefficient by 0,0302 points, the indirect effects increase it by 0,0170. The net effect is a decrease of the Gini by 0,0132. The indirect effects of the tax system increase the Gini coefficient because capital income is taxed at a lower rate than labour income and is also concentrated at the top of the overall income distribution.

5. Effects on overall redistribution of changes in the dual income tax

Equation (21), or an appropriately expanded version of it (see footnote 8), can be used to determine the effects of changes in the dual income tax system on redistribution. The changes which we consider are threefold. First, we consider changes in the tax level (i.e. in one of g_R, γ and g_K). Second, we investigate a change in one of the progressivities Π_L^K and Π_K^K. Third, we show the effects of changes in the composition of gross income. The results to follow were inaccessible prior to the development of our methodology.

The effect of an increase of the allowance can be found by differentiating equation (21) with respect to γ, and observing that $\partial \varepsilon_L / \partial \gamma = - \partial \varepsilon_K / \partial \gamma$:

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[8] Additional terms can be inserted into (21) if the labour income tax has income-related deductions and/or if the capital income tax is progressive.

[9] The tax system and distribution of income shown in table 1 have similarities with the Icelandic income distribution in the years before the financial crisis. Financial earnings in Iceland have, however, decreased substantially following the financial crisis in Iceland.

$$(22) \quad \frac{\partial \Pi^{RS}}{\partial \gamma} = \left[\underbrace{\frac{\partial \left(\frac{g_L}{1-g_L} \right)}{\partial \gamma}}_{\div} \left\{ \underbrace{\Pi_R^K}_{normally+} - \underbrace{\frac{\gamma}{1-\gamma}}_{+} \underbrace{\Pi_A^K}_{normaly \div} \right\} \right]$$

$$- \underbrace{\left[\underbrace{\frac{g_L}{1-g_L}}_{+} \underbrace{\frac{\partial \left(\frac{\gamma}{1-\gamma} \right)}{\partial \gamma}}_{+} \underbrace{\Pi_A^K}_{normally \div} \right]}_{\div} + \underbrace{\left[\underbrace{\frac{\partial \varepsilon_L}{\partial \gamma}}_{+} \underbrace{[C_K^N - C_L^N]}_{normally+} \right]}_{+} = ?$$

This shows that an increase in the allowance has ambiguous effects on redistribution. The direct effects are ambiguous while the indirect effect enhances redistribution if the distribution of capital income is more unequal than of labour income. The result that increase in the tax allowance has ambiguous effects on Π^{RS} is in line with Lambert (1985). Lambert showed that increasing the allowance beyond a certain point would become counterproductive for raising Π^{RS}.[10] The indirect effects do though increase the likelihood of an increase in the allowance to increase Π^{RS}.

The effect of an increase in the level of capital income taxation is given by:

$$(23) \quad \frac{\partial \Pi^{RS}}{\partial g_K} = \underbrace{\left[\underbrace{\alpha_K}_{+} \underbrace{\frac{\partial \left(\frac{g_K}{1-g_K} \right)}{\partial g_K}}_{normally\ 0} \underbrace{\Pi_K^K}_{+} \right]}_{0} - \underbrace{\left[\underbrace{\frac{\partial \varepsilon_L}{\partial g_K}}_{\div} \underbrace{[C_K^N - C_L^N]}_{normally+} \right]}_{\div} > 0$$

If the taxation of capital income is proportional, an increase in its rate has no direct effects on the level of redistribution. The net indirect effect is positive if capital income is more unequally distributed than labour income (as is typical): decreasing the after tax share of capital income enhances the redistribution Π^{RS} of the dual income tax system.

The effect of an increase in the rate of the labour income tax, without altering its distribution, is given by:

$$(24) \quad \frac{\partial \Pi^{RS}}{\partial g_R} = \underbrace{\left[\underbrace{\alpha_L}_{+} \underbrace{\frac{\partial \left(\frac{g_L}{1-g_L} \right)}{\partial g_R}}_{normally+} \underbrace{\Pi_R^K}_{+} \right]}_{+} + \underbrace{\left[\underbrace{\frac{\partial \varepsilon_L}{\partial g_R}}_{\div} \underbrace{[C_K^N - C_L^N]}_{normally+} \right]}_{\div} = ?$$

whose sign is ambiguous if capital income is distributed more unequally than labour income.

The effect of increasing the progressivity of the rate structure can also be found by differentiating in (21):

$$(25) \quad \frac{\partial \Pi^{RS}}{\partial \Pi_R^K} = \underbrace{\left[\underbrace{\alpha_L}_{+} \underbrace{\frac{g_L}{1-g_L}}_{+} \underbrace{\Pi_R^{K'}}_{+} \right]}_{+} - \underbrace{\left[\underbrace{\varepsilon_L}_{+} \underbrace{\frac{\partial C_L^N}{\partial \Pi_R^K}}_{\div} \right]}_{\div} > 0$$

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[10] Lambert´s results are based on a linear income tax, the result should though hold for a tax system with a progressive marginal rate structure.

Redistributive effect is enhanced through both direct and indirect effects. Exactly the same holds for the effect of an increase in the progressivity of the capital income tax (obtained by differentiating an expanded version of (21), in which Π_K^K appears, and then setting $\Pi_K^K = 0$). Introducing progression into the capital income tax component of the dual system would increase redistribution through both direct and indirect effects.

Finally, we consider the effect of an increase in the share α_K of capital income in gross income, by differentiating in the expanded version of (18) and again setting $\Pi_K^K = 0$:

$$(26) \qquad \frac{\partial \Pi^{RS}}{\partial \alpha_R} = \left[\underbrace{\underbrace{\alpha_K{'}}_{+} \underbrace{[\Pi_K^{RS} - \Pi_L^{RS}]}_{normally \div}}_{\div} \right] - \left[\underbrace{\underbrace{\frac{\partial \varepsilon_k}{\partial \alpha_k}}_{normally+} \underbrace{[C_K^N - C_L^N]}_{normally+}}_{+} \right] < 0$$

If the share of capital income increases, all else equal,[11] the redistributive effect of the tax system will diminish. The opposite is true for labour income: overall redistributive effect increases if α_L increases.

6. Discussion

A dual income tax system typically combines progressive taxation on labour (and transfer) income with a proportional tax on capital income. Theoretically, the adoption of such an income tax system has mostly been argued using efficiency reasoning, while distributive reasoning has been left aside. New methodology is required to assess redistributive effects in a dual income system. We have presented a new approach here, involving both direct and indirect effects on overall redistribution, which is applicable using available income and tax statistics. The effects of horizontal inequities and reranking can also be accommodated in the new approach. Implications and outcomes have been illustrated using a hypothetical income distribution, where capital income was very unequally distributed. We also investigated partial effects of changes in individual tax parameters on overall redistribution, which are not always straightforward.

Our approach depends specifically on the choice of the Gini coefficient as inequality measure, and on the choice of a particular decomposition rule, the so-called 'natural' one, for the overall Gini coefficient across income sources. One could of course use other decomposition rules, albeit with a significant loss of tractability; and/or design equivalent methodology using the Atkinson index or extended Gini coefficient (for example) to measure inequality.

It is plain that much work remains to be done, both theoretical and empirical, on the redistributional effects of a dual income tax. This paper presented a methodology for measurement purposes. Other issues, for example concerning changes in market incomes, need also to be explored. No empirical study has to our knowledge yet been done trying to assess the distributional effects of a dual income tax.

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[11] Note however that C_K and C_L may also change if α_L and α_L change, as the ordering of persons by their overall gross income may vary. This possibly second-order effect (if α_L and α_L change by marginal amounts) is not shown in (26) and would be difficult to quantify.

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Table 1: A hypothetical income distribution.

	x	x_K	x_L	t	t_K	t_L	A	$y = x\text{-}A$	$x\text{-}t$
1	7	0	7	0.0	0.0	0.0	7.0	0.0	7.0
2	9	0	9	0.6	0.0	0.6	7.5	1.5	8.4
3	9	0	9	0.6	0.0	0.6	7.5	1.5	8.4
4	10	0	10	1.0	0.0	1.0	7.5	2.5	9.0
5	15	1	14	2.7	0.1	2.6	7.5	6.5	12.3
6	18	2	16	3.6	0.2	3.4	7.5	8.5	14.4
7	20	3	17	4.1	0.3	3.8	7.5	9.5	15.9
8	25	4	21	6.2	0.4	5.8	7.5	13.5	18.9
9	30	5	25	8.3	0.5	7.8	7.5	17.5	21.8
10	60	54	6	5.4	5.4	0.0	6.0	0.0	54.6
Sum	**203**	**69**	**134**	**32.4**	**6.9**	**26.0**	**73**	**61**	**171**
Gini / concentration	**0.363**	**0.799**	**0.139**	**0.432**	**0.799**	**0.333**	**-0.012**	**0.320**	**0.350**

Table 2: Redistribution decomposed for the hypothetical distribution.

Direct effects			**0.0302**
Labour income taxation $(\alpha_L \Pi_L^{RS})$		0.0302	
Rate structure $\left(\alpha_L \frac{g_L}{1-g_L}\Pi_R^K\right)$		0,0021	
Progressivity (Π_R^K)	0.0137		
Average tax rate $\left(\frac{g_L}{1-g_L}\right)$	0.2350		
Labour income share (α_L)	0.6601		
Allowance $\left(-\alpha\frac{g_L}{1-g_L}\frac{\gamma}{1-\gamma}\Pi_A^K\right)$		0.0281	
Progressivity (Π_A^K)	-0.1511		
Average tax rate $\left(\frac{g_L}{1-g_L}\frac{\gamma}{1-\gamma}\right)$	0.2813		
Labour income share (α_L)	0.6601		
Capital income taxation $(\alpha_K \Pi_K^{RS})$		0.0000	
Progressivity (Π_K^K)	0.0000		
Average tax rate $\left(\frac{g_K}{1-g_K}\right)$	0.1111		
Capital income share (α_K)	0.3399		
Indirect effects			**-0.0170**
From labour income $(\varepsilon_L C_L^N)$	-0.0022		
From capital income $(\varepsilon_K C_K^N)$	0.0193		
Redistribution (Π^{RS})			**0.0132**