Working Paper Series

A new framework of measuring inequality: Variable equivalence scales and group-specific well-being limits. Sensitivity findings for German personal income distribution 1995-2009

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ECINEQ WP 2011 – 219

ECINEQ 2011 – 219
October 2011

www.ecineq.org

A new framework of measuring inequality: Variable equivalence scales and group-specific well-being limits. Sensitivity findings for German personal income distribution 1995-2009*

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Abstract

The paper examines sensitivity influences on the German personal income distribution in a time-series perspective as well as in a methodically broad manner. The author discusses the following issues: (1) For the first time, (reference) income-dependent, so-called variable equivalence scales are explicitly and extensively applied in a distributional analysis of German data which causes significant increases of income inequality compared with income-independent, constant equivalence scales. (2) Concerning different demarcations of income areas the pattern of income inequality in Germany 1995-2009 is not distinctively changed in the several variants considered. (3) For three alternative inequality indicators out of the class of Generalized Entropy indicators (mean logarithmic deviation, one of Theil's measures of entropy, and normalized coefficient of variation), the patterns of income inequality over time are nearly the same. (4) Regarding current monthly household net income versus yearly household net income of the previous year, different patterns with respect to income inequality occur during the ob-served period of time. Especially in the first decade of the 21st century the corresponding pat-terns differ from each other. In a further step the new approach related to income distribution, which incorporates variable equivalence scales, is applied to socio-demographic stratification to exemplarily demonstrate the power of this new approach. All in all, the analyses of the paper refer to the necessity of a rigorous methodological foundation of distributional studies, especially concerning the selection of a set of (preferably variable) equivalence scales, the choice of the inequality indicator, and – not least – of the income variable.

JEL classification: D30, D31, D60
Keywords: Personal income distribution; equivalence scales; inequality

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* Based on a paper presented at the fourth meeting of the Society for the Study of Economic Inequality (ECINEQ) in Catania (Italy), July 18-20, 2011
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1. Introduction[1]

In general, a lot of possibilities for dispersing results concerning the personal income distribution exist due to methodical settings. This includes – in a technical sense – the choice of the inequality indicator, the income definition (or more general: the definition of the used well-being indicator), the selection of the unit of analysis, the length of accounting periods, and the standardizations in consequence of different household sizes and structures.[2]

In this paper main elements of distributional analyses will be considered: (1) different kinds of equivalence scales, (2) different demarcations of income areas, (3) different inequality indicators, and (4) different income operationalisations.

The corresponding sensitivity analyses refer to a new distributional framework as reference, i. e. to an integrated approach insofar as the complete income area is divided into three areas which can be (approximately) interpreted as social classes. In this context, the income limits of the different areas are fixed by (reference) income-dependent equivalence scales since, according to welfare considerations, this seems more appropriate for distributional purposes than determinations for the whole spectrum of incomes. The income-dependent equivalence scales will be called *variable* equivalence scales – in opposite to income-independent equivalence scales which will be named as *constant* scales.

The paper is organized as follows. After describing the methodical and data framework in Chapter 2, which includes the choice of the inequality indicator as well as the choice of the welfare variable, the issue of equivalence scales in the sense sketched above, and the description of the data base, in Chapter 3 corresponding empirical findings for Germany 1995-2009 are presented. This comprises basic calculations, the differences between variable and constant equivalence scales, different income limits for the three income areas, different inequality indicators, and the confrontation of the concepts of monthly versus yearly household net incomes. Chapter 4 exemplarily uses the findings of Chapter 3, generated by the new approach for measuring (income) inequality, with respect to structural aspects of the German income distribution. Finally, concluding remarks are the topic of Chapter 5.

2. Methodical and data framework

2.1 Inequality indicator

For sensitivity analyses the usage of a general class of inequality indicators is convenient. A very popular class of indicators is the family of Generalized Entropy (GE) measures (in which groups' population shares serve as weighting factors as well as groups' income shares):

$$(1)\quad GE = \frac{1}{(\lambda^2 - \lambda)\cdot n} \cdot \sum_{i=1}^{n}\left[\left(\frac{Y_i}{\mu}\right)^{\lambda} - 1\right] \qquad \text{for } \lambda \neq 0 \wedge \lambda \neq 1;$$

$$GE = \frac{1}{n} \cdot \sum_{i=1}^{n} \ln\left(\frac{\mu}{Y_i}\right) \qquad \text{for } \lambda = 0;$$

[1] The data of this paper rest upon the German Socio-Economic Panel (SOEP) of the German Institute for Economic Research (DIW Berlin). As a reference for the SOEP data base see, e. g., Wagner, Frick, and Schupp 2007.

[2] See e. g. Hussain 2009.

$$GE = \frac{1}{n} \cdot \sum_{i=1}^{n} \left[\frac{Y_i}{\mu} \cdot \ln\left(\frac{Y_i}{\mu} \right) \right] \quad \textit{for } \lambda = 1$$

[GE = Generalized Entropy index, λ = parameter with respect to inequality preferences, n = population size, Y_i = income of person i, μ = mean income].

The parameter λ reflects the social perceptions of inequality. If λ is greater than 0, the upper income area receives a relatively high weight with respect to inequality (increasingly with higher λ-values at that). The opposite is the case if λ is less 0. For λ = 0 the GE measure represents the mean logarithmic deviation, for λ = 1 one of Theil's measures of entropy is the result, and for λ = 2 the GE measure corresponds with the normalized coefficient of variation (= half the square of the coefficient of variation).[3]

In the following I will primarily focus on the normalized coefficient of variation, but, for sensitivity purposes, I will also examine the inequality effects of the two other GE indicators mentioned.

2.2 Welfare variable

The analysis of individual welfare either rests upon individual resources (like income, wealth, or consumption[4]) or upon individual circumstances (concerning nutrition, clothing, habitation, health, education, transportation, communication, legal protection, etc.). Ultimately, both approaches represent individual utility. Despite the fact that the latter approach has received a lot of attention in the recent past – particularly because of Sen's much-noticed capability approach[5] –, I will analyse welfare only on the basis of resources (because large data restrictions exist for an analysis of circumstances). In this context, I will concentrate myself on income inequality since income is a suitable predictor for other welfare categories.[6]

Typically, income analyses are grounded on household net incomes. The reason for this is that this concept includes transfers and tax payments, and thus it represents individual well-being much better than e. g. gross incomes. In order to compare incomes for different household types, the household net incomes must be divided by "normalizing" values called equivalence scales (see the following considerations in Section 2.3). The resulting variable is named as equivalent household net income. Since individuals and not households achieve well-being, the equivalent household net incomes are weighted by the number of persons in each household.[7]

[3] A more comprehensive consideration of the class of GE measures can be found in Shorrocks 1980 or in Mookherjee and Shorrocks 1982.
[4] For an overview about this subject see e. g. Faik 1995, pp. 32-36.
[5] See Sen 1999.
[6] This was even recognized by Townsend, an apologist of a multidimensional welfare concept based on circumstances (see Townsend 1979, p. 253 and pp. 256-262).
[7] Bönke and Schröder 2008 applied an alternative weighting, the so-called needs-related weighting, i. e. weighting of equivalent incomes by equivalence scale values. In my eyes, this alternative weighting is intuitively less plausible than the weighting of incomes by the number of persons (concerning the question of well-being receivers).

2.3 Equivalence scales

2.3.1 General equivalence scale approaches

As mentioned above, an equivalence scale is used in welfare analyses as a "well-being deflator" by dividing (e. g.) household incomes by such scale values. This procedure is necessary because households differ in size and composition from each other. The latter implies that needs may be different between the several household members (e. g.: adults' versus children's needs). In order to capture the sensitivity of the corresponding distributional settings as "purely" as possible, it is meaningful to make use of general equivalence scale formulas.

I will concentrate myself on Buhmann et al.'s very prominent general equivalence scale formula that only depends on household size:

(2) $m_h = S^{\theta} \quad (0 \le \theta \le 1)$.[8]

In Equation (2) the symbols have the following meaning: m_h is the abbreviation for the equivalence scale value of household type h (with respect to the reference household type, in this case a single-person household[9]), S represents household size, and θ is the elasticity of the equivalence scale with regard to household size, and therefore it also reflects the degree of economies of scale. The extreme cases $\theta = 0.0$ and $\theta = 1.0$ correspond with a per-household and with a per-capita scaling of household incomes.

Since the Buhmann et al. formula already encompasses a broad range of possible equivalence scales,[10] the analyses in Section 3 and Figure 1 in Section 2.3.2 focus on the Buhmann et al. approach.[11] That means that this paper primarily investigates socio-demographic differences with respect to household size in order to clarify the relevant causalities.

By the way, concerning the Buhmann et al. scale formula sufficient approximations exist at $\theta = 0.6$ for the well-known "new OECD scale" and at $\theta = 0.8$ for the "old OECD scale".[12]

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[8] See Buhmann et al. 1988, p. 119.

[9] For the dependency of equivalence scales on the chosen reference household type see Ebert and Moyes 2003.

[10] Nevertheless, the Buhmann et al. scales are only some possibilities from a spectrum of a principally infinite number of equivalence scales (at least if it is assumed, for theoretical reasons, that arbitrary small decreases in the scale values are possible).

[11] Test calculations have shown that the results of the Buhmann et al. approach are not substantially altered by some alternative general formulas (see Faik 2009, pp. 16-17). Such an alternative proposal was made by Citro and Michael 1995, p. 161, including a differentiation between the number of adults (A) and of children (C): $m_h = (A + \alpha \cdot C)^{\theta} \quad \left(0 \le \theta \le 1, \ 0 \le \alpha \le 1\right)$ [α = children's needs compared with adults' needs].
Another general proposal is the formula deduced by Faik 2009, p. 9, via an expenditure-based, micro-econometric approach [with ß as sensitivity parameter]: $m_h = 1 + \beta \cdot ln(S) \quad \left(0 \le ß \le \frac{1}{ln(2)}\right)$.

[12] See Faik 2009, p. 8 [weights of the "new OECD scale" for further persons aged 15 years or older: 0.5 and for further persons aged until 14 years: 0.3; weights of the "old OECD scale" for further persons aged 15 years or older: 0.7 and for further persons aged until 14 years: 0.5].

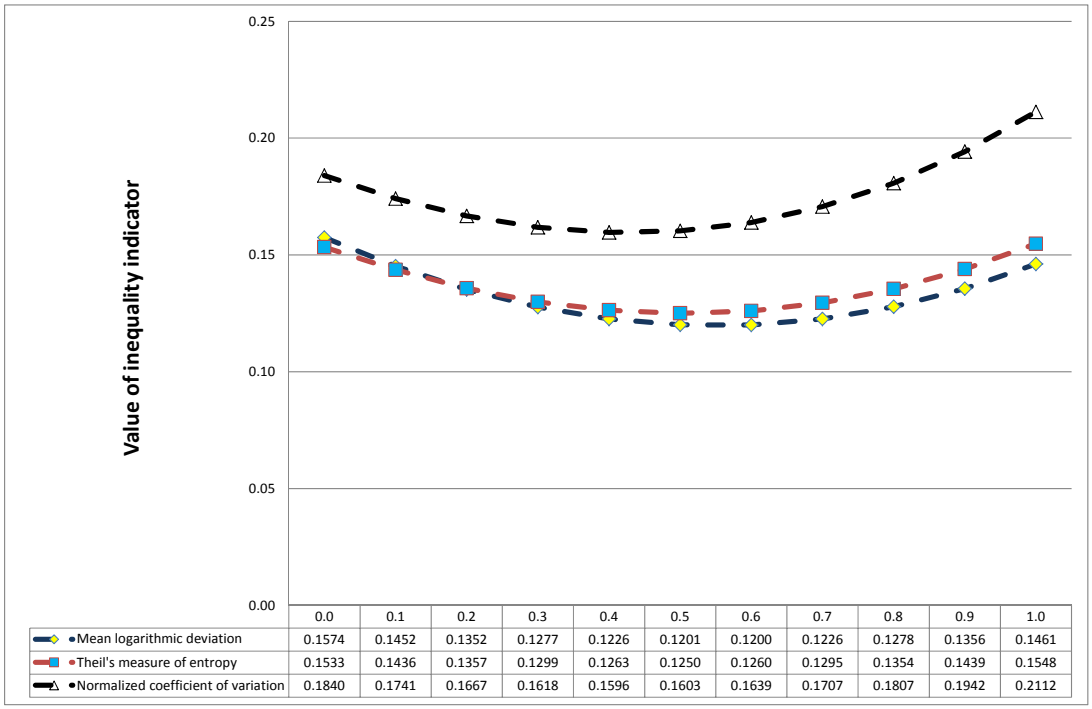
2.3.2 Overall equivalence scales and inequality

When computing the inequality of equivalent incomes, the correlation between household size and household (net) income is important and typically positive. In 2009 (SOEP) a Pearson's correlation coefficient in the amount of +0.393 resulted for Germany as a whole.[13]

Starting with the assumption of greatest economies of scale and thus starting with equivalence scale values in the amount of 1.0 for all household types, subsequently the degree of economies of scale is reduced in increments. This corresponds with higher equivalence scale values for larger households and means a levelling concerning the equivalent household incomes.

Briefly, the measured inequality decreases ("concentration effect"). But the further decrease in the larger household's equivalent incomes leads to an increase in the measured inequality at some point ("re-ranking effect"). As a consequence of this process, a U-shaped curve[14] for the levels of the presented inequality indicators is realistic (as a function of the range of economies of scale; see Figure 1).[15, 16]

Figure 1: Inequality indicators' values for Germany 2009 (SOEP) at different levels of θ (Buhmann et al. formula, equivalent household net income, constant equivalence scales)



	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Mean logarithmic deviation	0.1574	0.1452	0.1352	0.1277	0.1226	0.1201	0.1200	0.1226	0.1278	0.1356	0.1461
Theil's measure of entropy	0.1533	0.1436	0.1357	0.1299	0.1263	0.1250	0.1260	0.1295	0.1354	0.1439	0.1548
Normalized coefficient of variation	0.1840	0.1741	0.1667	0.1618	0.1596	0.1603	0.1639	0.1707	0.1807	0.1942	0.2112

Source: Present author's own calculations

[13] Present author's own calculations.
[14] Typically, the curve is at least J-curved (see e. g. Figini 1998, pp. 8-9).
[15] For a detailed discussion of this issue see especially Cowell and Mercader-Prats 1999, pp. 25-26; see also Coulter, Cowell, and Jenkins 1992, Figini 1998, pp. 7-9, Lancaster, Ray, and Valenzuela 1999, Creedy and Sleeman 2004, and Bönke and Schröder 2008.
[16] If a negative correlation between household size and household income occurs, it is probable that the "inequality curve" will have a positive slope across the whole area or most of the area of scale values, beginning with the "per-household situation" and ending with the "per-capita situation": The relatively low (equivalent) incomes of the larger household sizes, compared with the smaller household sizes, are continuously reduced, and, as a result, the inequality between the different household sizes increases.

2.3.3 Variable equivalence scales and inequality

Since the millennium a lot of studies have discussed aspects of the German personal distribution of equivalent incomes.[17] None of these studies used (reference) income-dependent, variable equivalence scales for distributional purposes, although there are good reasons for basing distributional analyses on such flexible equivalence scales.

I give three examples:

1. It might be argued that in the higher income ranges the reference consumption levels (e. g. concerning accommodation costs) would be fairly high so that a new household member's appearance (e. g. the "adding" of a child) would increase the corresponding costs only slightly, and this would lead to low *relative* costs, i. e. flat equivalence scales for larger households in the upper income range compared with the lower incomes.
2. Prices of commodities can differ from each other across income groups such that members of the upper income classes obtain price advantages.[18]
3. Credit constraints for households in the bottom income range may shift the consumption bundles of these households towards lower expenditure shares of durables which are connected with relatively high economies of scale.[19]

In the context of utility-based, microeconomic estimations of equivalence scales especially two methods for functionalizing an equivalence scale by a reference income level exist: the Barten und the Translating approach.[20]

In Barten's approach[21] it is assumed that higher commodity-specific scale values m_j represent higher household needs for the corresponding commodity compared with the reference household type. Thus, the normalized commodity-specific quantities q_j / m_j (j = 1, 2, …, n) in the direct utility function have the same amount for the different household types:

$$(3)\quad u = u\left[\frac{q_1}{m_1}, \frac{q_2}{m_2}, \ldots, \frac{q_n}{m_n}\right].$$

The socio-demographic standardizations of the Translating approach result from subtractions of socio-demographically functionalized quantity elements l_j from the overall consumption quantities q_j (j = 1, 2, …, n):

$$(4)\quad u = u[\, q_1 - l_1, q_2 - l_2, \ldots, q_n - l_n \,].$$

Unlike Barten's approach, the Translating approach can describe a situation in which the reference household does not buy a special commodity in contrast to other households.[22]

[17] For an overview see Faik 2010, p. 8.
[18] See Schröder 2004, p. 42.
[19] See Koulovatianos, Schröder, and Schmidt 2005, p. 969.
[20] By the way, a synthesis of Barten's and Translating approach stems from Gorman 1976.
[21] See Barten 1964.
[22] See Bradbury 1992, pp.15-16.

Faik (1995), Schröder (2004), and Koulovatianos, Schröder, and Schmidt (2005) estimated variable equivalence scales for Germany.[23] Their results were in accordance with the arguments presented above – in the sense that lower equivalence scale values were computed in the upper income range compared with the bottom income area.[24]

The incorporation of variable equivalence scales into distributional studies is, generally, confronted with the initial problem of separating the upper from the bottom (and the middle) range of equivalent incomes. In order to do this, we might assume a concrete equivalence scale for the whole income range as a starting point which would be a normative decision.[25] To some degree this normative problem can be circumvented by a "decomposition approach" which – for the field of poverty – was outlined in Faik (2011a).

I will apply this approach in the following. In this context, I will assume three income areas: a bottom, a middle, and an upper income area. These income areas will be *separately* generated for each household type so that no *overall* equivalence scale must be specified. My proceeding, which means an orientation of welfare levels only on the behaviour of one's *own* group of households, is based on socio-psychological approaches like Festinger's theory of social comparisons which suggest that people compare themselves with similar people.[26]

This means that people do not have (or do not want to have) complete information on society's entire income situation. Since such welfare comparisons refer to *household* incomes and since households are (very) different with respect to size and composition, it seems to be a Herculean task for each individual to consider all these aspects in the context of his/her well-being rankings. It seems much easier for individuals to compare themselves with household types which are similar to their own type. This implies a kind of bounded rationality.[27] As a consequence and as was already mentioned above, my proceeding is based on an orientation of well-being levels only on the behaviour of one's *own* group of households.[28] A number of empirical findings point towards this direction.[29]

According to the idea of variable equivalence scales, the scale values in the low-income area are highest and those in the upper income area are lowest, i. e.: The income values in the low-income area are divided by higher scale values than the incomes in the middle and in the upper income area (see Figure 2).

[23] For an overview about the corresponding results see Koulovatianos, Schröder, and Schmidt 2005, p. 991. Concerning the estimation of variable equivalence scales see, additionally and among others, Fiegehen, Lansley, and Smith 1977, pp. 105-106, van Hoa 1986, pp. 97-98, Aaberge and Melby 1998, or Donaldson and Pendakur 2003, especially pp. 194-197.

[24] Obviously, the definition of variable equivalence scales used in this paper refers to income areas in the sense of discrete variables, and not to incomes as (quasi-)continuous variables (i. e.: in this sense and on principle unlike Barten's or Translating approach).

[25] See Faik 1995, pp. 286-287.

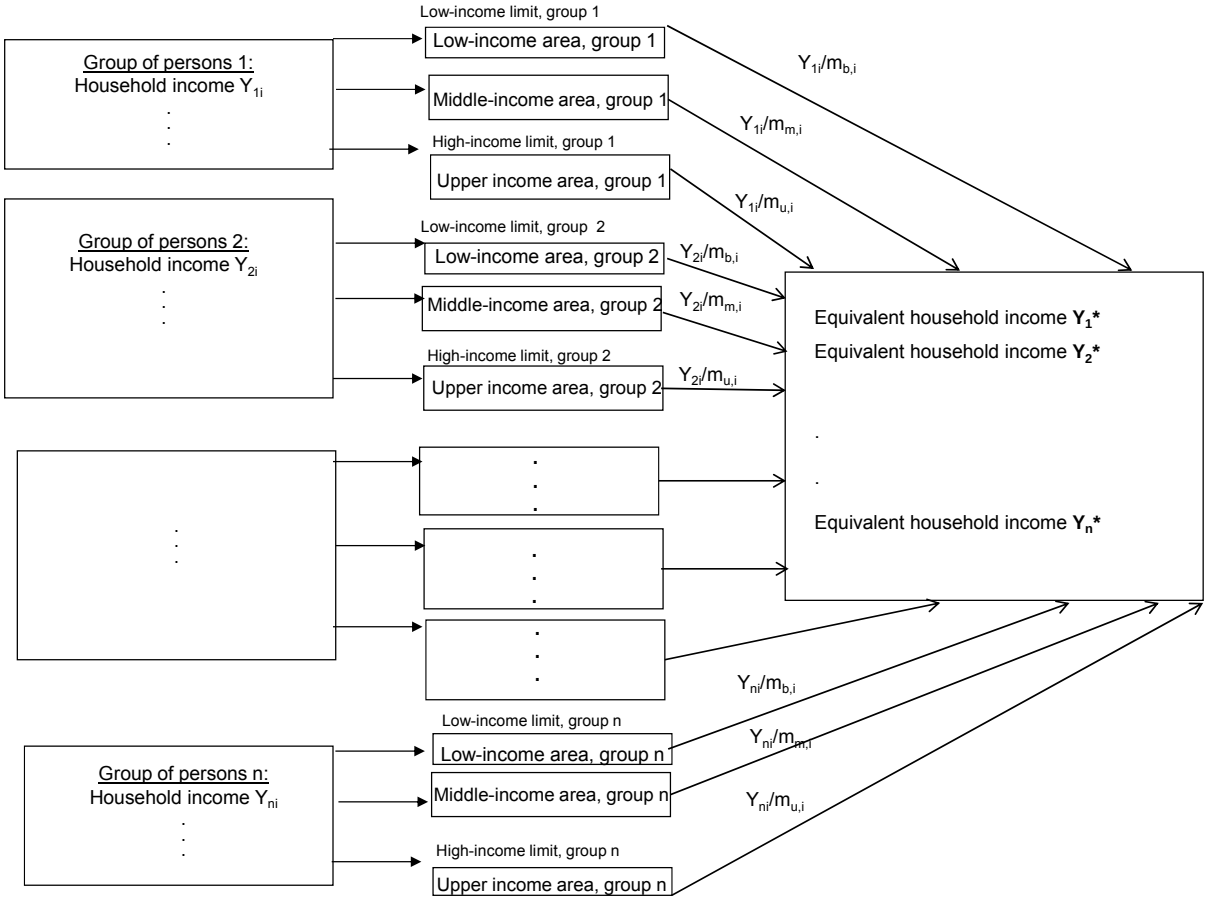
[26] See Festinger 1954.

[27] Concerning this issue see e. g. Simon 1957 or Leibenstein 1976.

[28] With respect to the issue of reference groups in distributional analyses see e. g. Amiel and Cowell 1999, pp. 2-6.

[29] See e. g. Clark and Oswald 1996, or Frey and Stutzer 2002, pp. 88-90.

Figure 2: A decomposition approach for measuring income inequality



Y_{gi}: income of unit of analysis i in group g (g = 1, 2, …, n), m_b = equivalence scale value in the bottom income area, m_m = equivalence scale value in the middle-income area, m_u = equivalence scale value in the upper income area; $m_b > m_m > m_u$; $\mathbf{Y_g}$*: vector of equivalent household incomes within group g (g = 1, 2, …, n)

Source: Present author's own illustration

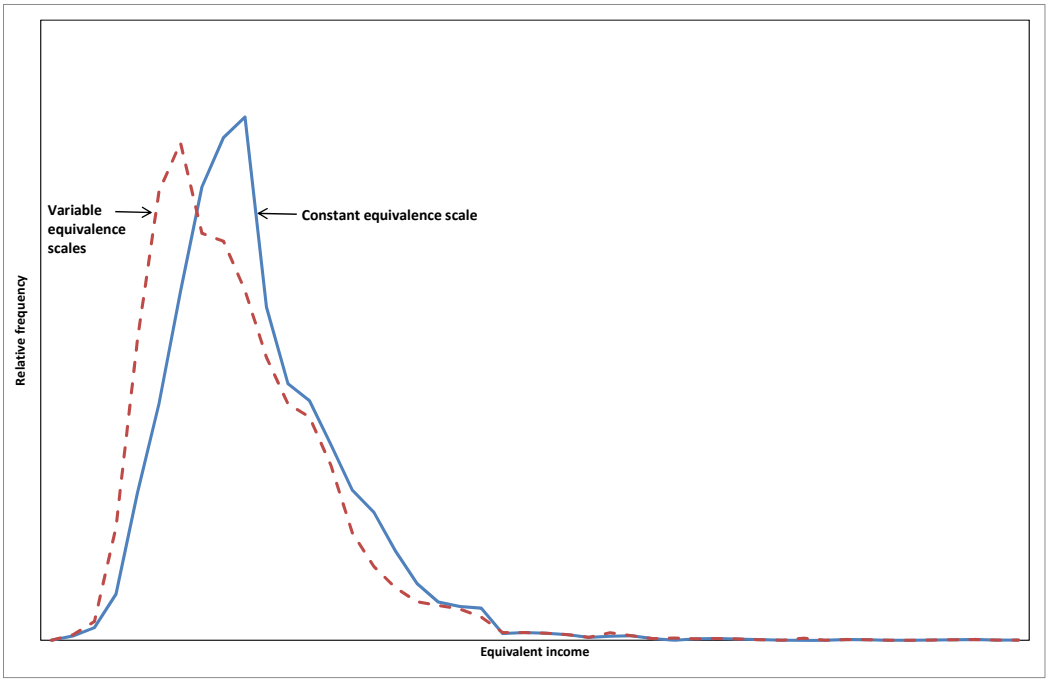
If variable equivalence scales with lower values on the equivalence scale are used for the upper income area, the differences of the equivalent incomes between the bottom and the upper income classes become larger than they would be without using variable equivalence scales. Thus, the measured inequality would increase. In this sense Figures 3a and 3b compare the application of variable equivalence scales with the alternative method which uses income-independent, constant equivalence scales.

In Figure 3a the overall equivalence scale in the income-independent case is set to the same level as in the upper income area in the case with variable equivalence scales. This congruence leads to more inequality because of a more right-skewed income distribution in the latter variant of measuring inequality. The reason for this result is that in the variant with variable equivalence scales the incomes of multi-person households in the lower income area are diminished by higher scale values than in the variant with constant equivalence scales.

In Figure 3b an alternative assumption is made: The equivalence scales in the income-independent case and in the lower income area of the variant with variable equivalence scales shall equal each other. This corresponds with higher equivalent incomes of the multi-

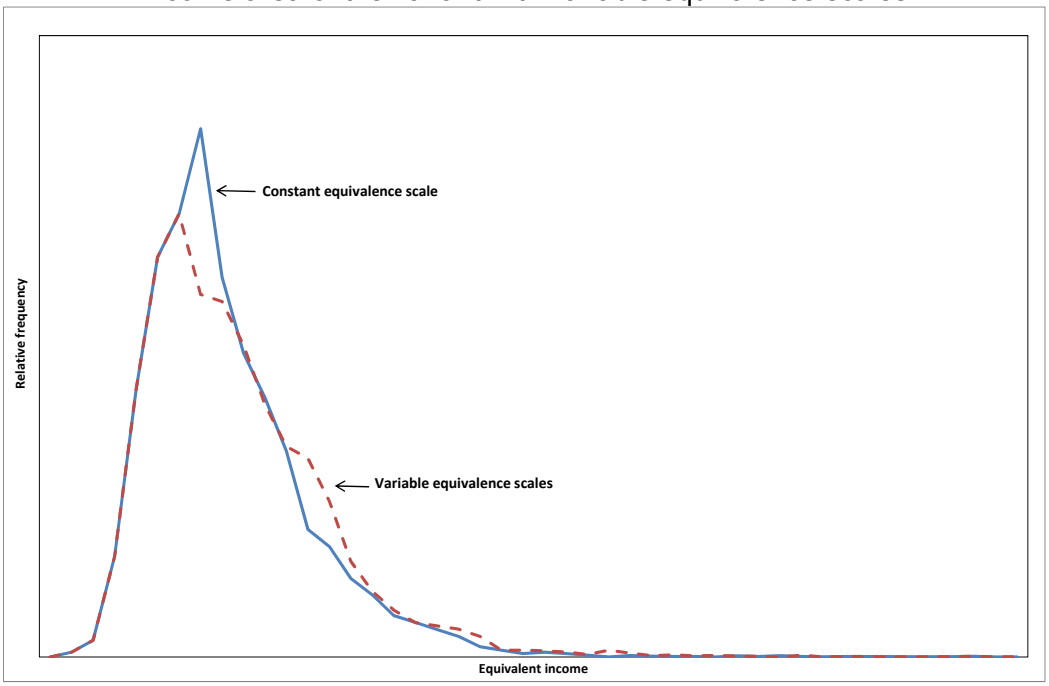
person households within the upper income area in the case with variable equivalence scales and, thus, generates a higher degree of income dispersion.

Figure 3a: Constant versus variable equivalence scales
and their impact on the distribution of income – idealized illustration,
same scale values both in the income-independent case and in the upper income area of the variant with variable equivalence scales



Source: Present author's own illustration

Figure 3b: Constant versus variable equivalence scales
and their impact on the distribution of income – idealized illustration,
same scale values both in the income-independent case and in the lower income area of the variant with variable equivalence scales



Source: Present author's own illustration

2.4 The data base[30]

In this paper I have used data from the German Socio-Economic Panel (SOEP) for the years 1995 to 2009.[31] The SOEP is collected since 1984 in yearly intervals, and it comprises approximately between 5,000 and 15,000 households and currently more than 30,000 persons. The participants of the surveys give detailed information on their incomes, household composition, earnings' and family's biographies, health, life-satisfaction, etc.

Since 1984 a lot of subsamples have been drawn in particular to capture population's dynamics appropriately:

- *Sample A:* German households in the Federal Republic of Germany since 1984,
- *Sample B:* households of foreigners in the Federal Republic of Germany since 1984,
- *Sample C:* private households in eastern Germany (German Democratic Republic) since 1990,
- *Sample D:* households of immigrants in Germany since 1994/1995,
- *Sample E:* complementary sample of households in Germany since 1998,
- *Sample F:* complementary sample of households in Germany since 2000,
- *Sample G:* sample of high-income receivers (households) in Germany since 2002, and
- *Sample H:* complementary sample of households in Germany since 2006.

For distibutional analyses two central income variables are available: Monthly household income of the current year and yearly household income of the previous year. The query for the latter variable is retrospective. In the context of monthly income the respondents are interviewed during one month. Since there has been only a global query concerning monthly household net income until 1995 and because of the fairly overcoming of great economic distortions in eastern Germany in the mid-1990s – i. e., approximately five years after German (re-)unification –, the analyses of this paper start with the year 1995. Because the current income levels are "fresh" in interviewees' memories, information on monthly income seems more precise than that on yearly, retrospective income. Thus, I decided to predominantly use the monthly, current household net income in my analyses instead of the yearly, retrospective household net income.

[30] See Wagner, Frick, and Schupp 2007.
[31] The most recent SOEP – conducted in 2010 – was not available for scientific purposes at the time the paper was written.

3. Empirical sensitivity findings for Germany 1995-2009 on the basis of a new method for measuring inequality

3.1 Basic calculations

As was mentioned in Section 2.3.3, in my following analyses with regard to variable equivalence scales I divide the whole income range into three areas, the bottom, the middle, and the upper income class. According to regression results on the basis of the Functionalized Extended Linear Expenditure System (FELES) in Faik (2011a), in Germany seems to be an empirically based low-income line at 70 percent of single-person households' mean net incomes. For multi-person households,[32] the low-income limits are computed on the basis of the (approximate) old OECD scale, i. e.: on the basis of $\theta = 0.8$ (in the Buhmann et al. formula).

The widespread new OECD scale (i. e.: $\theta = 0.6$) is applied within the upper income area for computing the high-income limits of the multi-person households. In this context the reference high-income line, that for single-person households, is determined – in accordance with other (German) studies[33] – as twice the arithmetic mean of single-person households' net incomes. For the middle-income area, a "medium scale" between old and new OECD scale is used, namely the Buhmann et al. scale with $\theta = 0.7$.

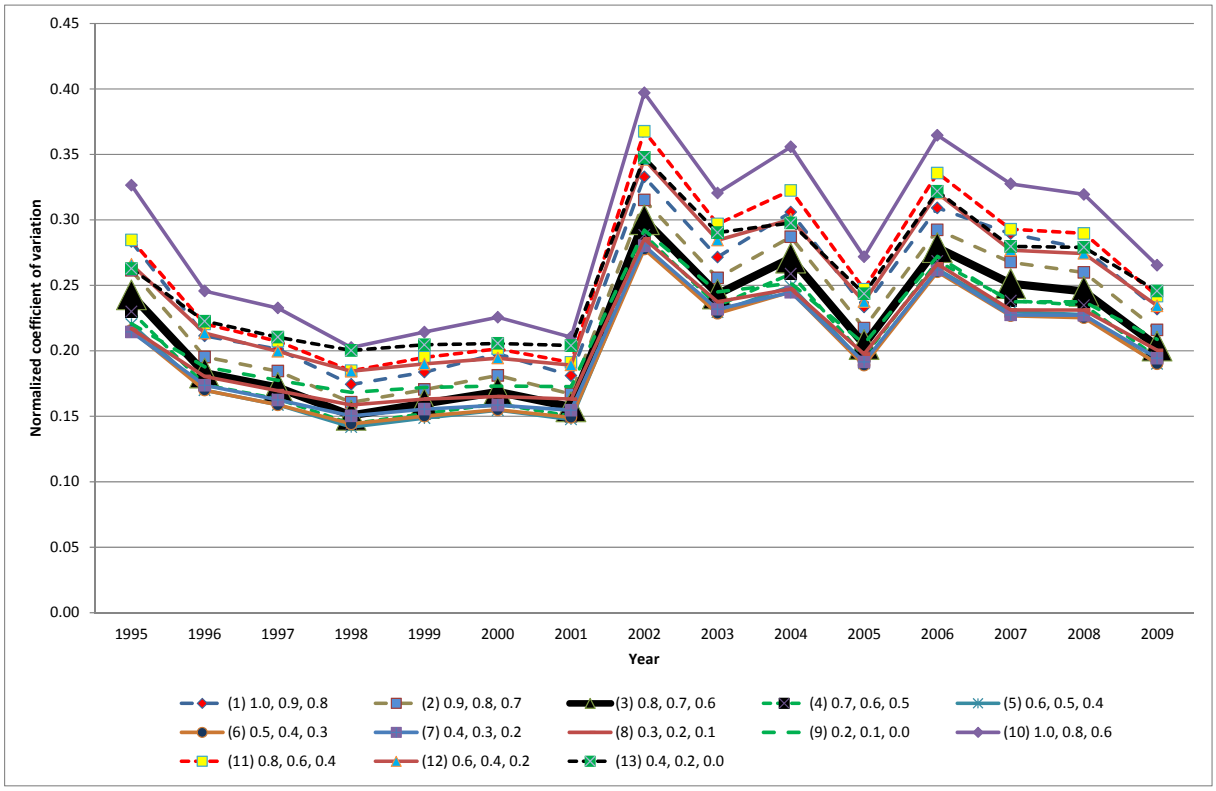
The basic inequality results for Germany 1995-2009, arising from the presented concept, are shown in Figure 4 by the bold line.[34] Between 1995 and 2001 the income inequality in Germany decreased by tendency, and since 2002 the measured income inequality has been at a higher level of the normalized coefficient of variation than before. At least partly this seems to be the result of a sampling effect since for the transition from 2001 SOEP to 2002 SOEP high incomes were captured to a higher degree (by sample G; see Section 2.4). Because of the relatively large high-income sensitivity of the normalized coefficient of variation this sampling effect might explain at least part of the rise of inequality between 2001 and 2002. Moreover, the mentioned increase of income inequality might be partly caused by socio-economic developments in Germany at the beginning of the 21st century, e. g. by the increase of low-paid jobs or by a rise of individual incomes' homogamy (with respect to partner relationships in Germany)[35]. Especially from 2006 to 2009 a tendency towards decreasing income inequality occurred in Germany as a whole. Perhaps (at least partly and by tendency) this was a reflection of the diminished unemployment rates in Germany during this period.

[32] The calculations of the paper are restricted to single- to six-person households since the number of cases for household sizes with seven and more persons is too low for statistical reasons, as can be seen by Table A.1 in the Appendix.
[33] See e. g. Grabka et al. 2007, pp. 60-61.
[34] The bootstrap estimates in Table A.2 in the Appendix give (strong) evidence to the thesis that my estimates of income inequality are statistically significant (at a 95-percent level of significance).
[35] For corresponding empirical evidence – also on the basis of the German Socio-Economic Panel – see Schröder 2011.

3.2 Variations of income limits for multi-person households

Alternative θ-values for the three income areas alter the inequality level but not the ranking of the different years which is also demonstrated in Figure 4. Higher differences of θ-values between the income areas – cases (10) – (13) compared with cases (1) – (9) in Figure 4 – increase income inequality by tendency which appears plausible (against the background of the considerations in Section 2.3.3).

Figure 4: Variable equivalence scales and income inequality in Germany 1995-2009 on the basis of the normalized coefficient of variation (monthly equivalent household net income, different θ-values of the Buhmann et al. scale within three income areas)



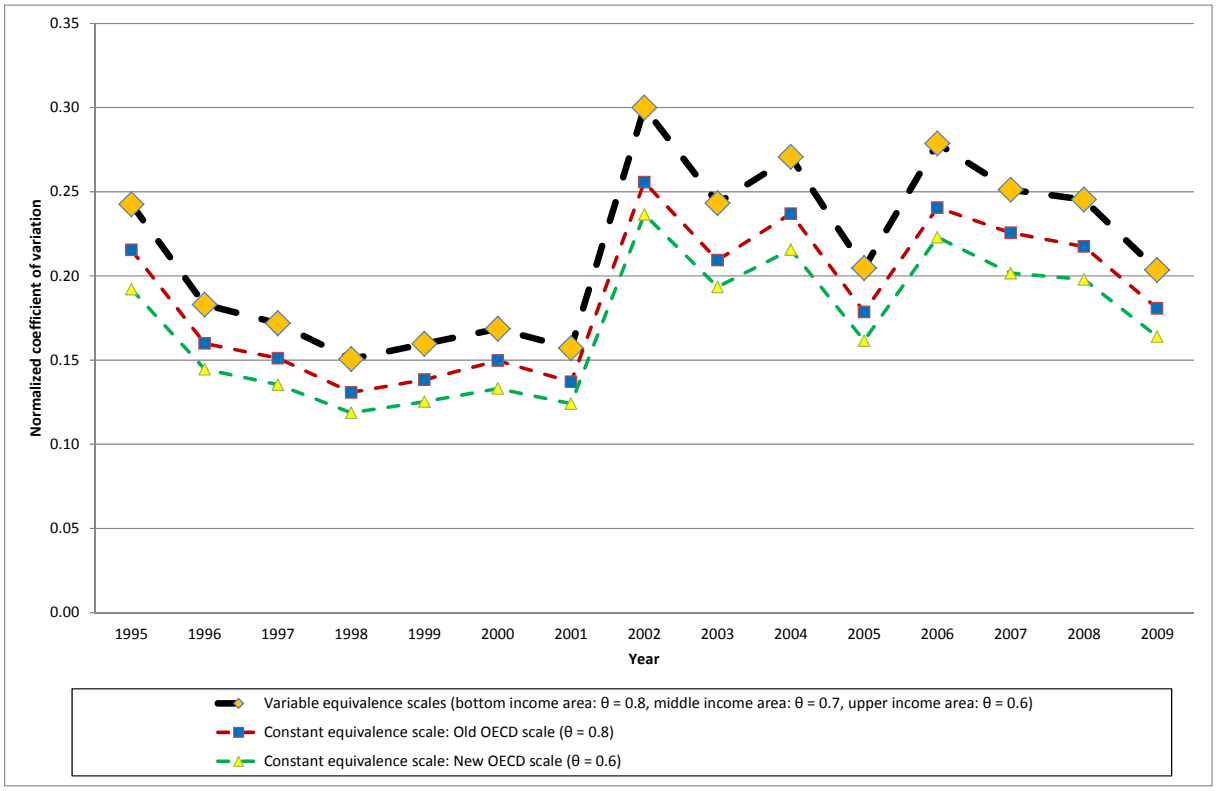
In all cases: first θ-value in legend: representing low-income area, second θ-value in legend: representing middle-income area, third θ-value in legend: representing high-income area

Source: Present author's own calculations

3.3 Variable versus constant equivalence scales

For the cases with constant equivalence scales, Figure 5 shows the same pattern of income inequality as in the case with variable equivalence scales but at lower inequality levels. Referring to the case with variable equivalence scales, the measured value differences of the normalized coefficients of variation are within the ranges of 11 to 17 percent (compared to the case with a constant equivalence scale and θ = 0.8) and of 24 to 27 percent (compared to the case with a constant equivalence scale and θ = 0.6).

Figure 5: Variable and constant equivalence scales in Germany as a whole 1995-2009
SOEP on the basis of the normalized coefficient of variation
(Buhmann et al. scale, monthly equivalent household net income)



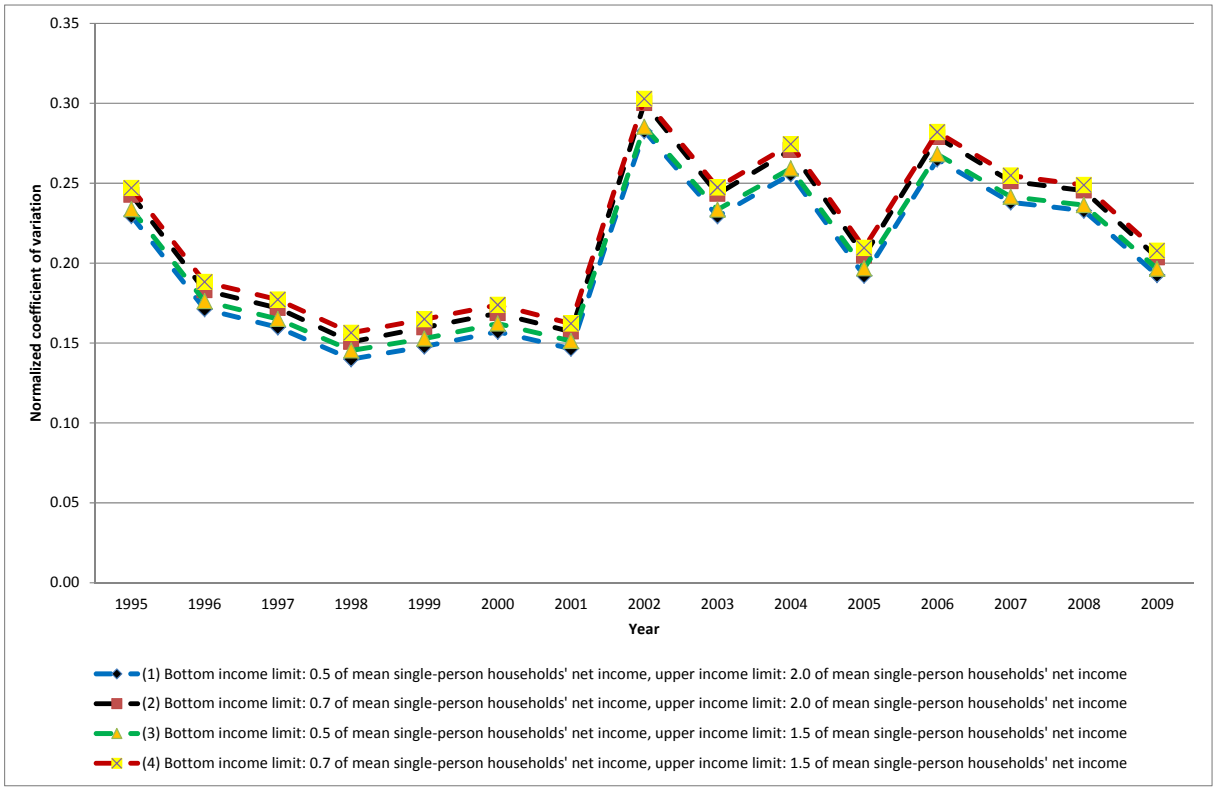
Source: Present author's own calculations

3.4 Different reference income limits

Figure 6 illustrates the consequences of different reference limits for the three income areas in the sense of different fractions or multiples of single-person households' mean equivalent incomes. Once more, for the θ-values in all cases of Figure 6 the value 0.8 is assumed for the bottom income area, 0.7 for the middle-income area, and 0.6 for the upper income area.

As a rule, it becomes obvious that larger differences in reference income limits between the upper and the bottom income area cause lower income inequality than smaller corresponding differences. This simply is because of a broader middle class with identical θ-values (in the amount of θ = 0.7) in the cases first mentioned. Thus, in these cases a broader section of the entire income distribution is scaled by an identical equivalence scale than in the cases with smaller differences between the upper and the bottom income area.

Figure 6: Different (reference) borders of income areas, variable equivalence scales, and their consequences for income inequality in Germany 1995-2009 SOEP on the basis of the normalized coefficient of variation (Buhmann et al. scale, monthly equivalent household net income)



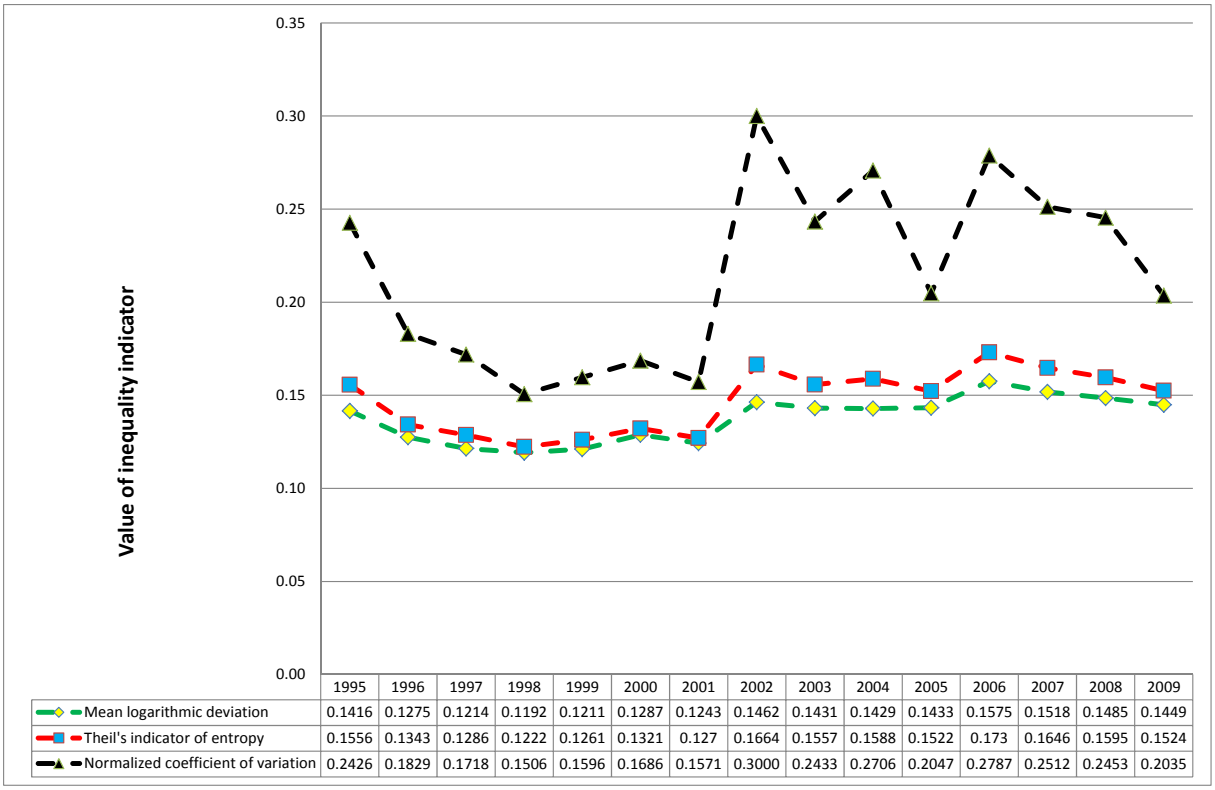
Source: Present author's own calculations

3.5 Different inequality measures

Figure 7 shows the inequality consequences of different inequality measures (mean logarithmic deviation, Theil's measure of entropy applied in this paper, and normalized coefficient of variation) for Germany 1995-2009. As was mentioned in Section 2.1, these three inequality indicators disagree with respect to their sensitivity on changes within different income regions. Compared with the largely high-income sensitive normalized coefficient of variation, the mean logarithmic deviation and Theil's measure of entropy used here – both not as sensitive to changes in high-income regions as the normalized coefficient of variation – reveal a rather smoothed "inequality curve" over time.

Especially – and expectedly – this is true for the transition between 2001 and 2002, i. e.: during the period of time in which the SOEP was filled up by high-income receivers. For 1995-1998 and for 2006-2009, the tendencies towards decreasing income inequality stated above are (also) confirmed by applying the alternative inequality indicators mean logarithmic deviation and Theil's measure of entropy.

Figure 7: Different inequality indicators, variable equivalence scales,
and their consequences for income inequality in Germany 1995-2009 SOEP
(Buhmann et al. scale, monthly equivalent household net income)



	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Mean logarithmic deviation	0.1416	0.1275	0.1214	0.1192	0.1211	0.1287	0.1243	0.1462	0.1431	0.1429	0.1433	0.1575	0.1518	0.1485	0.1449
Theil's indicator of entropy	0.1556	0.1343	0.1286	0.1222	0.1261	0.1321	0.127	0.1664	0.1557	0.1588	0.1522	0.173	0.1646	0.1595	0.1524
Normalized coefficient of variation	0.2426	0.1829	0.1718	0.1506	0.1596	0.1686	0.1571	0.3000	0.2433	0.2706	0.2047	0.2787	0.2512	0.2453	0.2035

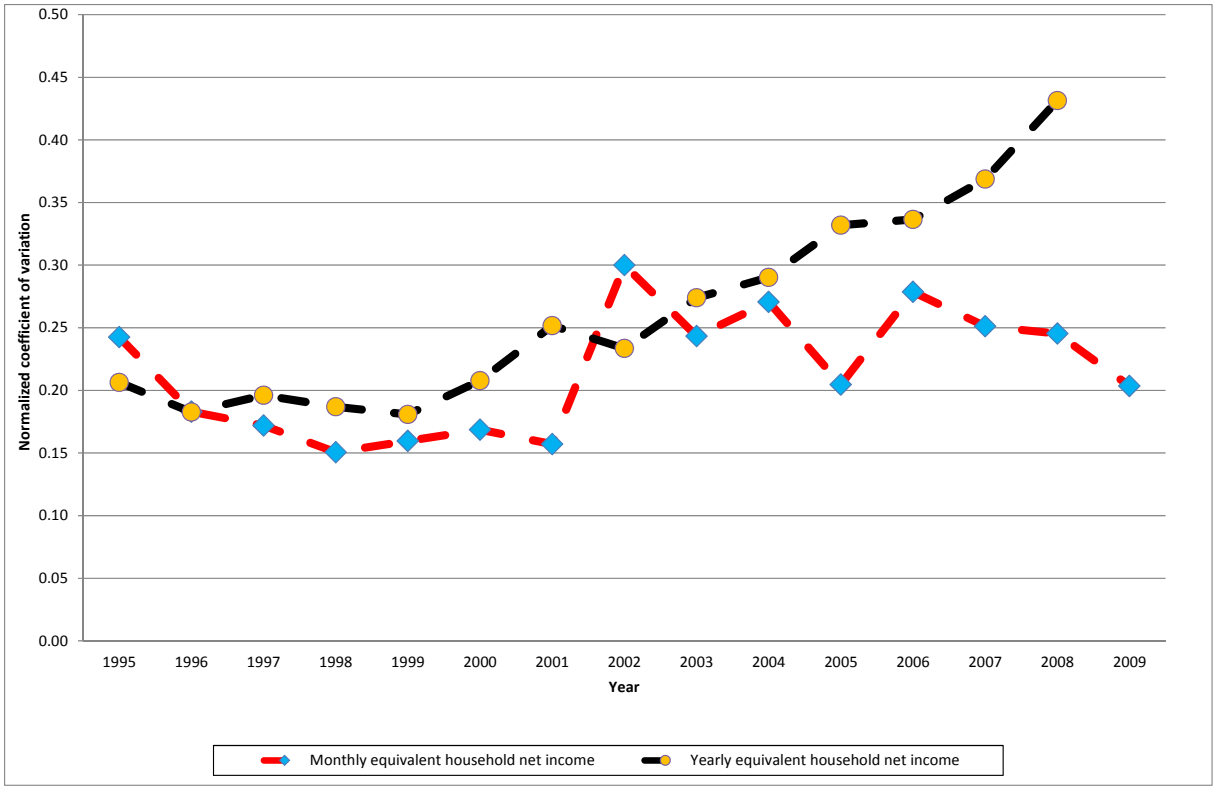
Source: Present author's own calculations

3.6 Current monthly income versus yearly income of the previous year[36]

Qualitatively speaking, substantial inequality differences between the variant with current monthly incomes and the variant with yearly incomes of the previous year exist – especially in the periods since the millennium (see Figure 8). While the concept of yearly income has indicated an increase of income inequality since the beginning of the new century, the reference on monthly incomes has shown a tendency towards diminishing income inequality at least since 2006.

Partly these divergences depend on methodical differences: E. g., the concept of monthly income does not include special payments like Christmas bonuses in contrast to yearly household net incomes. Another methodical difference is the embedding of socio-demographic characteristics: While in the case with monthly incomes socio-demographic characteristics belong to the same period of time as the variable "income", in the other case both variables differ from each other by one year with respect to chronological reference.[37]

Figure 8: Different income definitions, variable equivalence scales, and their consequences for income inequality in Germany 1995-2009 SOEP on the basis of the normalized coefficient of variation (Buhmann et al. scale)



Source: Present author's own calculations

[36] Concerning this issue, principally, see e. g. Cantó, Gradin, and del Rio 2006.

[37] By the way, in contrast to the above findings present author's own poverty calculations on the basis of the headcount ratio reveal similar patterns over time for both income concepts in Germany 1995-2009 (see Faik 2011b, p. 15).

4. Applications of the new method for measuring inequality

Analysing the structure of the several income areas is one "natural" application of the alternative approach for measuring inequality I have presented in this paper. In this sense and exemplarily, Section 4.1 deals with the results of binary logistical regressions for Germany 2009. Section 4.2 is written from a longitudinal perspective by considering transition matrices for the years 2004 to 2009 (with respect to Germany as a whole).

4.1 Binary logistical regressions for Germany 2009

In order to cover the significance of socio-demographic influences on income inequality a small binary logistical regression's model is estimated. Exemplarily, these regression results also refer to the year 2009. As can be seen in Table 1, small households – in the definition of not more than two persons – have significant parameter values in all three areas with a negative algebraic sign in the low-income area and with positive signs in the middle- and in the high-income area which partly represents high well-being levels of "double income – no kids" couples.

Moreover, Table 1 reveals significantly higher levels of well-being for Germans, for male household members, for persons living in western Germany, for married persons, and for very qualified persons compared with their corresponding reference groups.

Comparing old household members ("60 years and older") and young household members ("until 29 years") with the reference (dummy) group "30-59 years", highlights that young and older persons have higher likelihoods for being within the low-income area and lower likelihoods for being located within the middle- and within the high-income area.

Last but not least, the parameter of the variable "unemployed" is strongly positive in the low-income area and strongly negative in the middle- and in the high-income area. This indicates – on average – a relatively low well-being level for unemployed persons in Germany 2009.

Table 1: Binary logistical regression's parameters due to different income areas in Germany 2009 (SOEP) based on the decomposition approach

Covariates (0/1 dummies) and statistical information	Low-income area (dependent variable: "being a member of this income area", 0/1 dummy)	Middle-income area (dependent variable: "being a member of this income area", 0/1 dummy)	High-income area (dependent variable: "being a member of this income area", 0/1 dummy)
Absolute term	+1.276***	-0.859***	-5.715***
Living in western Germany	-0.606***	+0.274***	+1.453***
Male household member	-0.125***	+0.026***	+0.056
German household member	-1.115***	+0.950***	+0.509***
Person living in a small household (not more than two persons)	-0.647***	+0.311***	+1.120***
Until 29 years	+0.426***	-0.438***	-0.120
60 years and older	+0.239***	-0.107***	-0.361***
Unemployed household member[1)]	+1.965***	-1.677***	-2.323***
Married person	-0.067*	-0.004	+0.364***
Non-qualified person[2)]	+1.090***	-0.927***	-1.049**
Very qualified person[3)]	-1.409***	+0.352***	+1.798***
Number of observations (dependent dummy = 1)	9,649 persons	14,339 persons	1,571 persons
Nagelkerke's coefficient of determination	0.187	0.085	0.198

*: significant at 10-percent level; **: significant at 5-percent level; ***: significant at 1-percent level

[1)] unemployed and non-working, [2)] no school-leaving qualification achieved, [3)] university degree (or the like) achieved

Source: Present author's own calculations

4.2 Transition matrices for Germany 2004-2009

So far cross-sectional findings were presented. Additionally, Section 4.2 has a longitudinal perspective in order to cover income dynamics by dealing with year-to-year transitions between 2004 and 2009 in Germany (see Table 2).

The following five income classes are distinguished concerning income limits and equivalence scales (expressed by Buhmann et al.'s θ):[38]

- "poverty area":]0 €; 0.5 times mean of single-person households' net incomes[and θ = 0.80,
- "low-income area": [0.5 times mean of single-person households' net incomes; 0.7 times mean of single-person households' net incomes[and θ = 0.75,
- "middle-income area": [0.7 times mean of single-person households' net incomes; 1.5 times mean of single-person households' net incomes[and θ = 0.70,
- "wealthiness area": [1.5 times mean of single-person households' net incomes; 2.0 times mean of single-person households' net incomes[and θ = 0.65,
- "richness area": [2.0 times mean of single-person households' net incomes; +∞ €[and θ = 0.60.

Table 2: Transition matrices in Germany 2004/05-2008/09 SOEP based on the decomposition approach (monthly equivalent household net incomes)

Well-being position in period t	Well-being position in period t+1				
	PA	LIA	MIA	WA	RA
2004/2005:					
PA	73.8 %	20.3 %	5.7 %	0.1 %	0.2 %
LIA	18.1 %	59.3 %	22.2 %	0.3 %	0.1 %
MIA	3.2 %	12.1 %	80.0 %	3.6 %	1.2 %
WA	0.8 %	0.8 %	38.0 %	47.2 %	13.2 %
RA	1.3 %	2.7 %	13.9 %	17.8 %	64.3 %
2005/2006:					
PA	74.1 %	18.8 %	5.7 %	0.5 %	0.8 %
LIA	18.6 %	54.8 %	25.7 %	0.6 %	0.3 %
MIA	2.9 %	9.0 %	82.9 %	4.0 %	1.1 %
WA	0.4 %	2.0 %	30.7 %	51.1 %	15.8 %
RA	0.5 %	1.4 %	16.0 %	16.1 %	66.1 %
2006/2007:					
PA	69.5 %	21.5 %	8.5 %	0.3 %	0.2 %
LIA	12.5 %	56.7 %	30.2 %	0.4 %	0.2 %
MIA	2.3 %	8.6 %	84.6 %	3.6 %	1.0 %
WA	0.9 %	1.5 %	32.0 %	50.4 %	15.2 %
RA	2.7 %	1.4 %	12.0 %	15.7 %	68.0 %
2007/2008:					
PA	69.2 %	22.9 %	7.7 %	0.0 %	0.2 %
LIA	15.4 %	59.1 %	25.2 %	0.2 %	0.1 %
MIA	2.4 %	8.1 %	84.8 %	3.9 %	0.8 %
WA	0.3 %	1.5 %	28.4 %	51.7 %	18.1 %
RA	1.0 %	0.1 %	11.4 %	15.5 %	72.0 %
2008/2009:					
PA	74.2 %	16.0 %	9.0 %	0.5 %	0.3 %
LIA	15.3 %	60.1 %	24.3 %	0.3 %	0.0 %
MIA	2.3 %	9.6 %	83.2 %	3.9 %	1.0 %
WA	0.2 %	1.6 %	33.1 %	51.6 %	13.4 %
RA	0.4 %	0.5 %	10.6 %	20.0 %	68.5 %

t = 2004, 2005, 2006, 2007, 2008; t+1 = 2005, 2006, 2007, 2008, 2009; PA: poverty area, LIA: low-income area, MIA: middle-income area, WA: wealthiness area, RA: richness area

Source: Present author's own calculations

[38] The extension from three to five income areas was made to investigate transitions in more detail.

As can be seen by Table 2, there is only small dynamics in the sense of movements from bottom income areas towards upper income areas. For instance, between 2008 and 2009 only about ten percent of persons moved upwards from the poverty area into the middle-income area or higher. Furthermore, the shares of the stayers within the several income classes are relatively high in the border areas of poverty (69.2 % – 74.2 %) and richness (64.3 % – 72.0 %) as well as in the middle-class (80.0 % – 84.8 %). In the groups "low-income area (LIA)" (54.8 % – 60.1 %) and "wealthiness area (WA)" (47.2 % – 51.7 %) the shares of the stayers are lower but nevertheless considerable.

5. Concluding remarks

The findings of the paper revealed the sensitivity of distributional results due to different methodical settings.

My analyses yielded the following conclusions:

1. For a spectrum of equivalence scales I obtained a U-shaped curve for the measured inequality as a function of the degree of economies of scale (on the basis of an income-independent, therefore constant equivalence scale formula). In this context, many popular equivalence scales like the new or the old OECD scale could be captured by appropriate settings of parameters.
2. In the case with variable equivalence scales the measured inequality was substantially higher than in the case with income-independent equivalence scales. The higher the differences between scale values within the bottom income area and within the upper income areas were, the higher income inequality was.
3. A narrowing of (reference) income limits between the upper and the bottom income area led to increases in income inequality (within the methodical framework presented here).
4. For Germany the usage of three alternative inequality indicators showed – on the basis of monthly equivalent household net incomes – tendencies for inequality decreases at least between 1995 and 1998 and between 2006 and 2009 (but a – partly artificial – notable inequality increase from 2001 to 2002).
5. With respect to the periods since 2006 the variable yearly equivalent household net income indicated an opposite tendency. But it should be kept in mind that a) in the data base there is a temporal incongruence between non-income characteristics (e. g. household size) and yearly income, and in this context b) yearly income values are prompted retrospective so that (possibly) they are not as concise as monthly incomes of the current period.

Socio-demographic examples were added to the afore-mentioned sensitivity analyses which were also based on the new approach for measuring income inequality (with variable equivalence scales). E. g., binary logistical regression's estimations revealed different (and in most cases significantly different) likelihoods for the several social groups for being located within the three income areas considered in this paper (low-income, middle-income, and high-income area).

Because of the normativity of the measurement of income inequality sensitivity analyses are helpful to structure judgements in this field of research. In this context, there are good reasons for the usage of variable equivalence scales. Such welfare elements should be applied in distributional studies, as was done in this paper, for the first time in an extensive manner empirically for Germany.[39] In future research the concept presented here might be refined e. g. in the direction of (a good deal) more than three income areas and towards socio-demographic specifications in greater depth.

[39] With respect to a rather cursory application see Faik 2010. Another field of application for variable equivalence scales might be, e. g., the socio-demographic functionalization of tax systems.

Appendix

Table A.1: Unweighted number of households in Germany 1995-2009 SOEP due to household size

Year	1 person	2 persons	3 persons	4 persons	5 persons	6 persons	7 persons	8 persons and more	Sum
1995	1,443	2,121	1,431	1,250	392	99	36	20	6,792
1996	1,466	2,138	1,378	1,215	366	99	36	16	6,714
1997	1,442	2,194	1,332	1,182	364	85	33	17	6,649
1998	1,735	2,478	1,441	1,258	341	100	34	13	7,400
1999	1,692	2,470	1,356	1,193	340	94	31	11	7,187
2000	3,260	4,336	2,195	1,958	615	151	39	18	12,572
2001	2,943	3,999	1,990	1,788	579	130	39	13	11,481
2002	2,970	4,440	2,115	1,911	611	138	37	14	12,236
2003	2,912	4,238	1,961	1,750	557	122	34	14	11,588
2004	2,864	4,214	1,905	1,691	524	112	28	11	11,349
2005	2,897	4,105	1,815	1,583	494	103	27	8	11,032
2006	3,247	4,523	1,926	1,600	483	105	23	12	11,919
2007	3,100	4,273	1,926	1,600	483	105	23	12	11,262
2008	2,986	4,117	1,696	1,353	388	87	18	8	10,653
2009	3,153	4,352	1,709	1,405	391	91	20	6	11,127

Source: Present author's own calculations

Table A.2: Bootstrap estimations for mean equivalent household net income (unweighted samples), standard deviation of equivalent household net incomes, and normalized coefficients of variations in Germany 1995-2009 SOEP (95-percent confidence intervals)

Year	Mean (sample)	Mean (bootstrap, below)	Mean (bootstrap, above)	Standard deviation (sample)	Standard deviation (bootstrap, below)	Standard deviation (bootstrap, above)	NCV (sample)	NCV (bootstrap, below)	NCV (bootstrap, above)
1995	12,124	12,018	12,233	7,781	7,190	8,424	0.2060	0.2457	0.1727
1996	12,653	12,546	12,764	7,216	6,810	7,680	0.1626	0.1873	0.1423
1997	12,808	12,702	12,922	7,049	6,644	7,503	0.1515	0.1745	0.1322
1998	13,046	12,940	13,149	6,833	6,596	7,064	0.1372	0.1490	0.1258
1999	13,492	13,373	13,596	7,181	6,932	7,452	0.1417	0.1552	0.1300
2000	14,099	14,011	14,191	7,938	7,673	8,241	0.1585	0.1730	0.1462
2001	14,519	14,428	14,624	8,012	7,739	8,318	0.1522	0.1662	0.1400
2002	18,066	17,878	18,260	17,796	15,632	20,063	0.4852	0.6297	0.3664
2003	17,585	17,424	17,758	14,909	13,310	16,626	0.3594	0.4552	0.2809
2004	17,806	17,626	17,989	15,738	13,517	18,061	0.3906	0.5250	0.2823
2005	17,662	17,499	17,816	12,740	12,066	13,423	0.2601	0.2942	0.2293
2006	17,809	17,628	17,996	15,299	13,123	17,771	0.3690	0.5081	0.2659
2007	18,253	18,086	18,425	13,313	12,527	14,381	0.2660	0.3162	0.2311
2008	18,607	18,427	18,791	14,762	12,610	17,524	0.3147	0.4522	0.2251
2009	18,694	18,539	18,861	13,064	12,516	13,598	0.2442	0.2690	0.2202

NCV: Normalized coefficient of variation

Source: Present author's own calculations

References

Aaberge, R., and I. Melby (1998): The Sensitivity of Income Inequality to Choice of Equivalence Scales. In: Review of Income and Wealth, 44 (4), pp. 565-569.

Amiel, Y., and F. A. Cowell (1999): Thinking about Inequality, Cambridge.

Barten, A. B. (1964): Family Composition, Prices and Expenditure Patterns. In: Hart, P. E., G. Mills, and J. K. Whitaker (eds.): Econometric Analysis for National Economic Planning, London, pp. 277-297.

Bönke, T., and C. Schröder (2008): Equivalence Scales Reconsidered – an Empirical Investigation, ECINEQ – Society for the Study of Economic Inequality, Working Paper Series, ECINEQ WP 2008-102, Palma de Mallorca.

Bradbury, B. (1992): Measuring the Cost of Children, SPRC Discussion Papers, No. 32, University of New South Wales, Kensington (Australia).

Buhmann, B., et al. (1988): Income, Well-Being, Poverty, and Equivalence Scales: Sensitivity Estimates Across Ten Countries Using the LIS Data base. In: Review of Income and Wealth, 34, pp. 115-142.

Cantó, O., C. Gradin, and C. del Rio (2006): Poverty Statics and Dynamics: Does the Accounting Period Matter? In: International Journal of Social Welfare, 15, pp. 209-218.

Citro, C. F., and R. T. Michael (1995): Measuring Poverty – A New Approach, Washington (D. C.).

Clark, A. E., and A. J. Oswald (1996): Satisfaction and Comparison Income. In: Journal of Public Economics, 61, pp. 359-381.

Coulter, F. A. E., F. A. Cowell, and S. P. Jenkins (1992): Equivalence Scale Relativities and the Extent of Inequality and Poverty. In: Economic Journal, 102, pp. 1067-1082.

Cowell, F., and M. Mercader-Prats (1999): Equivalence Scales and Inequality, Distributional Analysis Research Programme, No. DARP 27, London.

Creedy, J., and C. Sleeman (2004): Adult Equivalence Scales, Inequality and Poverty in New Zealand, New Zealand Treasury Working Paper 04-21, Wellington.

Donaldson, D., and K. Pendakur (2003): Equivalent-expenditure functions and expenditure-dependent equivalence scales. In: Journal of Public Economics, 88, pp. 175-208.

Ebert, U., and P. Moyes (2003): Equivalence Scales Reconsidered. In: Econometrica, 70, pp. 319-343.

Faik, J. (1995): Äquivalenzskalen. Theoretische Erörterung, empirische Herleitung und verteilungsbezogene Anwendung für die Bundesrepublik Deutschland, Berlin.

Faik, J. (2009): Is the Overall German Personal Income Distribution Constant or Variable over Time? Cross-Section Analyses for Germany 1969-2003, FaMa Discussion Paper #1-2009, Frankfurt (Main).

Faik, J. (2010): The Influence of Income-Dependent Equivalence Scales on Income Inequality – An Empirical Analysis for Germany since 1993, FaMa Discussion Paper #6-2010, Frankfurt (Main).

Faik, J. (2011a): Der Zerlegungs-Ansatz – ein alternativer Vorschlag zur Messung von Armut. In: AStA – Wirtschafts- und Sozialstatistisches Archiv, 4, pp. 293-315.

Faik, J. (2011b): A Behaviouristic Approach for Measuring Poverty: The Decomposition Approach – Empirical Illustrations for Germany 1995-2009, SOEPpapers on Multidisciplinary Panel Data Research # 383, Berlin.

Festinger, L. (1954): A Theory of Social Comparison Processes. In: Human Relations, 7, pp. 117-140.

Fiegehen, G. C., P. S. Lansley, and A. D. Smith (1977): Poverty and Progress in Britain 1953-73, Cambridge.

Figini, P. (1998): Inequality Measures, Equivalence Scales and Adjustment for Household Size and Composition, Trinity Economic Paper Series, Technical Paper No. 98-8, Dublin.

Frey, B. S., and A. Stutzer (2002): Happiness & Economics. How the Economy and Institutions Affect Human Well-Being, Oxford.

Gorman, W. M. (1976): Tricks with Utility Functions. In: Artis, M. J., and A. V. Nobay (eds.): Essays in Economic Analysis, Cambridge, pp. 211-243.

Grabka, M. M., et al. (2007): Integrierte Analyse der Einkommens- und Vermögensverteilung, Final Report for the German Federal Ministry for Labour and Social Affairs, Berlin et al.

Hussain, M. A. (2009): The Sensitivity of Income Polarization. Time, Length of Accounting Periods, Equivalence Scales, and Income Definitions. In: Journal of Economic Inequality, 7, pp. 207-223.

Koulovatianos, C., C. Schröder, and U. Schmidt (2005): On the Income Dependence of Equivalence Scales. In: Journal of Public Economics, 89, pp. 967-996.

Lancaster, G., R. Ray, and M. R. Valenzuela (1999): A Cross-Country Study of Equivalence Scales and Expenditure Inequality on Unit Record Household Budget Data. In: Review of Income and Wealth, 45, pp. 455-482.

Leibenstein, H. (1976): Beyond Economic Man. A New Foundation for Microeconomics, Cambridge (Mass.) and London.

Mookherjee, D., and A. F. Shorrocks (1982): A Decomposition Analysis of the Trend in UK Income Inequality. In: Economic Journal, 92, pp. 886-902.

Schröder, C. (2004): Variable Income Equivalence Scales. An Empirical Approach, Heidelberg and New York.

Schröder, Ch. (2011): Einkommensungleichheit und Homogamie. In: iw-trends, 38, pp. 1-13.

Sen, A. (1999): Development as Freedom, New York.

Shorrocks, A. F. (1980): The Class of Additively Decomposable Inequality Measures. In: Econometrica, 48, pp. 613-625.

Simon, H. A. (1957): Models of Man. Social and Rational. Mathematical Essays on Rational Human Behavior in a Social Setting, New York and London.

Townsend, P. (1979): Poverty in the United Kingdom. A Survey of Household Resources and Standards of Living, London.

van Hoa, T. (1986): Measuring Equivalence Scales. A New System-Wide Method. In: Economics Letters, 20, pp. 95-99.

Wagner, G. G., J. R. Frick, and J. Schupp (2007): The German Socio-Economic Panel Study (SOEP) – Scope, Evolution and Enhancements. In: Schmollers Jahrbuch, 127, No. 1, pp. 139-169.