Working Paper Series

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Francisco Azpitarte
Olga Alonso-Villar

ECINEQ WP 2011 – 226

ECINEQ 2011 – 226
October 2011

www.ecineq.org

Ray-invariant intermediate inequality measures: A Lorenz dominance criterion*

Francisco Azpitarte†
University of Melbourne

Olga Alonso-Villar
Universidade de Vigo

Abstract

This paper introduces a new Lorenz dominance criterion that allows ranking income distributions according to ray-invariant intermediate inequality measures. In doing so, it defines α-Lorenz curves by adapting the generalized Lorenz curves to this case. In addition, it provides an empirical illustration of these tools using Australian income data for the period 2001-2008. The results suggest that despite the reduction of relative inequality, inequality increased for most ray-invariant intermediate value judgments.

Keywords: Income distribution; Lorenz dominance; Intermediate inequality indices; Ray-invariance
JEL classification: D63

* Financial support from the Ministerio de Ciencia e Innovación (grant ECO2008-03484-C02-01/ECON and ECO2010-21668-C03-03), Xunta de Galicia (10SEC300023PR), and from FEDER is gratefully acknowledged. We want to thank Coral del Río for helpful comments.

† **Contact details**: Francisco Azpitarte, Melbourne Institute of Applied Economics and Social Research, University of Melbourne, Victoria 3010, Australia (fraz@unimelb.edu.au)

1. Introduction

When comparing two income distributions which differ in their means, researchers have to specify the type of mean-invariance property that they want their inequality indices to satisfy, a matter with respect to which no agreement has been reached. Most of them choose relative indexes, which imply that inequality remains unaffected when all incomes increase/decrease by the same proportion (scale invariance). If these indexes verify the Pigou-Dalton principle of transfers (PD), symmetry (S), and the population principle (PP) they will be consistent with the Lorenz dominance criterion (Foster, 1985). Therefore, if the Lorenz curve of an income distribution lies at no point below that of another and at some point above, the former distribution will have lower inequality than the latter according to any relative inequality index satisfying the above axioms, which makes Lorenz dominance an attractive tool. Other scholars opt, instead, for absolute measures so that inequality remains unaltered if all incomes are augmented/diminished by the same amount (translation invariance). Absolute indexes verifying PD, S, and PP are also consistent with a Lorenz-type dominance criterion, in this case given by "absolute" Lorenz curves (Moyes, 1987).

Following previous ideas put forth by Dalton (1920), several reports on questionnaires indicate that many people believe that an equiproportional increase in all incomes raises income inequality, whereas an equal increment decreases it.[1] Kolm (1976) labeled these measures "centrist" (i.e., intermediate) and considered "rightist" (i.e., relative) and "leftist" (i.e., absolute) measures extreme cases of this more general view. Since then, several intermediate proposals have been made. Some of them lead to iso-inequality contours which are linear (Bossert and Pfingsten, 1990; Seidl and Pfingsten, 1997; Del Río and Ruiz-Castillo, 2000; Chakravarty and Tyagarupananda, 2008; Del Río and Alonso-Villar, 2010), whereas others are non-linear (Krtscha, 1994; Yoshida, 2005; Zheng, 2007).[2]

[1] See Amiel and Cowell (1992), Harrison and Seidl (1994), and Seidl and Theilen (1994), among others. In particular, Ballano and Ruiz-Castillo (1993) find that 27% of individuals support this perception of inequality.
[2] For a discussion on these notions, see Del Río and Alonso-Villar (2008).

In empirical analyses, intermediate measures are very useful when one finds that an income growth is accompanied by a decrease of inequality according to the relative Lorenz criterion together with an increase according to the absolute Lorenz criterion. This circumstance is not unusual since when the mean of an income distribution rises, absolute measures are more demanding than relative. This is so because giving an equal amount of income to every individual leads to a more egalitarian distribution that giving to each of them an amount that keeps the original income shares. Intermediate measures are not only a theoretical refinement of relative and absolute measures, but a helpful tool for applied research since they allow delving deeper in situations like the one we are describing.

However, as opposed to what happens with scale- and translation-invariant measures, in the centrist context there has been almost no discussion regarding the Lorenz dominance that could be defined. An exception is Yoshida (2005), who not only offers an intermediate notion that generalizes the "fair compromise" concept proposed by Krtscha (1994) but also introduces a concept of Lorenz dominance which allows ranking income distributions according to these non-linear intermediate notions. Yet, as far as we know, no proposal has been made for the α-inequality proposed by Seidl and Pfingsten (1997).

To close this gap somehow, this paper aims to introduce a new Lorenz dominance criterion which allows comparisons among income distributions according to inequality measures which are ray invariant à la Seidl and Pfingsten (1997). Since the intermediate notions proposed by Del Río and Ruiz-Castillo (2000) and Del Río and Alonso-Villar (2010) can be seen as particular cases of Seidl and Pfingsten (1997), the new dominance criterion is also valid for them. In doing so, this paper adapts the generalized Lorenz curve (Shorrocks, 1983) to our case.

The paper is structured as follows. Section 2 presents the intermediate inequality approach followed in this paper and defines a Lorenz-type curve which gives rise to a dominance criterion consistent with this centrist view. Section 3 offers an empirical illustration of these tools using Australian data for the period 2001-2008. Finally, Section 4 brings the main conclusions.

2. Ray invariance and the Lorenz criterion

In this paper, an inequality index, *I*, is a real function defined on the set income distributions *x*, and satisfying the following basic properties:[3]

a) *Symmetry*: $I(x) = I(x\Pi)$, where Π is a permutation matrix.

b) *Replication invariance*: $I(x) = I(\underbrace{x,...,x}_{k})$, where $(\underbrace{x,...,x}_{k})$ is a k-fold replication of x.

c) *Schur convexity*: $I(Bx) \leq I(x)$ for all bistochastic matrices B that are not permutation matrices.

This index is labeled intermediate or centrist if $I(ax) > I(x)$ and $I(x + b1^n) < I(x)$, for any $a > 1$ and $b \in \mathbb{R}_{++}$ where $1^n \equiv (\underbrace{1,...,1}_{n})$.

Since we focus on symmetric indexes, we can restrict our analysis to the set of all possible ordered income distributions $x_1 \leq x_2 \leq ... \leq x_n$ ($2 \leq n < \infty$), denoted by $D \subset \mathbb{R}^n$.

2.1 The ray invariance of Seidl and Pfingsten (1997)

A centrist inequality attitude can be modeled in various ways, depending on the shape of the set of inequality equivalent income distributions. In what follows, we present the α-inequality concept proposed by Seidl and Pfingsten (1997) (S-P henceforth). According to it, any extra income should be distributed in fixed proportions, given by α, in order to keep inequality unaltered.

Let $\alpha \in D$ be a vector of the simplex (i.e., $\sum_{i=1}^{n} \alpha_i = 1$) that is Lorenz-dominated by vector $\frac{1^n}{n}$. The set of distributions for which α represents an intermediate notion is denoted by $\Gamma(\alpha) = \{x \in D : \alpha \gtrsim_{\mathrm{L}} v_x\}$, where $\gtrsim_{\mathrm{L}}$ denotes (weak) Lorenz dominance and

[3] Note that Schur-convexity implies symmetry and the Pigou-Dalton principle of transfers (Berge, 1963).

$v_x \equiv \left(\frac{x_1}{\sum_{i=1}^{n} x_i}, \ldots, \frac{x_n}{\sum_{i=1}^{n} x_i} \right)$ is the vector of income shares associated with distribution *x*.

Therefore, this inequality notion requires a certain relationship between the direction of the invariance line, given by α, and distribution x if one wants it to represent an intermediate attitude. In other words, α cannot be used for all distributions but only for those which are Lorenz-dominated by it. Conversely, given a distribution $x \in D$ not all α vectors are suitable if one wants them to represent intermediate notions for *x*. Only those included in the set $\Omega(x) = \{ \ \alpha : \alpha \in D, \sum_{i=1}^{n} \alpha_i = 1, \alpha \gtrsim_L x \ \}$ are admissible.

Given a distribution $x \in \Gamma(\alpha)$, the corresponding iso-inequality line is defined by $E_\alpha(x) = \{ y \in D : y = x + \tau\alpha, \ \tau \in \mathbb{R} \}$ (see Figure 1). Note that, on the one hand, any distribution in $E_\alpha(x)$ with $\tau > 0$ is less egalitarian in the Lorenz sense than $x + \tau 1^n$ and more egalitarian than λx when $\lambda > 1$. On the other hand, those distributions in $E_\alpha(x)$ having $\tau < 0$ are more equally distributed than $x + \tau 1^n$ and less equally distributed than λx when $0 < \lambda < 1$.

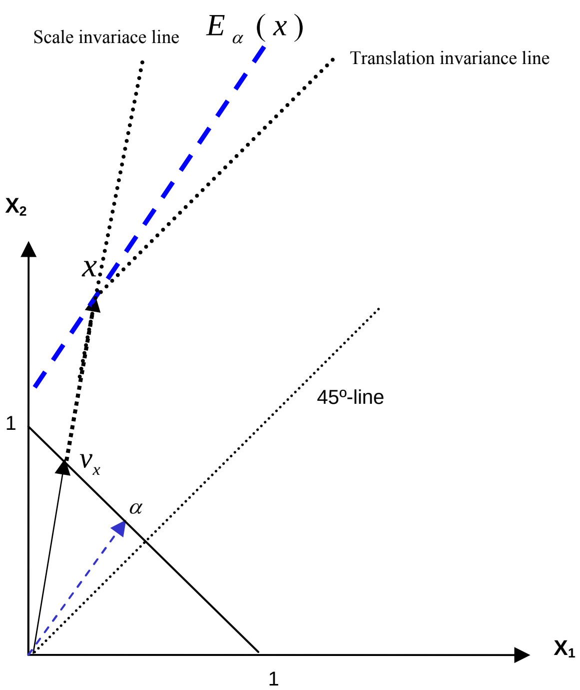

Figure 1. Ray-invariance in Seidl and Pfingsten and scale and translation invariances (n=2).

An intermediate inequality measure, I_α, is labeled ray invariant if $I_\alpha(x) = I_\alpha(x+\tau\alpha)$, where $x \in \Gamma(\alpha)$ and $\tau \in \mathbb{R}$. A binary relation can be then defined as follows:

$$x \succsim_\alpha y :\Leftrightarrow I_\alpha(x) \leq I_\alpha(y),$$

where $x, y \in \Gamma(\alpha)$.

2.2 A new Lorenz dominance criterion

Let $\tilde{x}$ denote a distribution obtained through distribution $x \in \Gamma(\alpha)$ allocating its total income among individuals according to income shares given by α (i.e., $\tilde{x} \equiv \alpha \sum_{i=1}^{n} x_i$, see Figure 2).

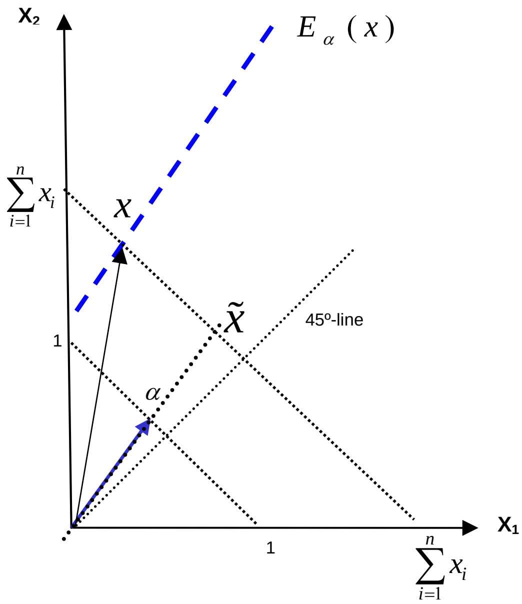

Figure 2. Constructing distribution $\tilde{x}$.

Using $\tilde{x}$, we construct distribution x_α as the ordered vector resulting from the losses and gains experienced by individuals when moving from distribution x to $\tilde{x}$ (i.e., x_α is obtained from $x - \tilde{x}$).

Definition. We define the α-Lorenz curve for distribution $x \in \Gamma(\alpha)$ as the generalized Lorenz curve (Shorrocks, 1983) of distribution x_α:

$$L_\alpha(p, x) = GL(p, x_\alpha),$$

for any $p \in (0,1]$ and adopting the convention $L_\alpha(0,x)=0$. This function is convex with respect to *p*, takes no positive values within the interval $(0,1)$, is equal to zero when *p* is equal to 0 and 1, and reaches a minimum at a point $p^* = \frac{k^*}{n}$, where k^* represents the individual with the smallest loss (i.e., $(x_\alpha)_{k^*} < 0$ and $(x_\alpha)_{k^*+1} \geq 0$, see Figure 3).

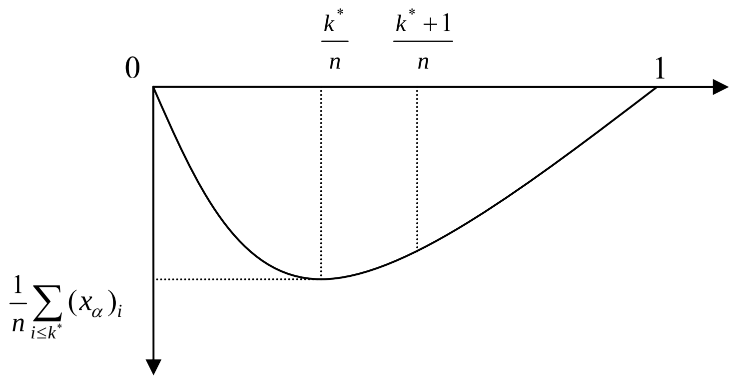

Figure 3. The α-Lorenz curve

Definition. For any $x, y \in \Gamma(\alpha)$ we define an α-Lorenz-dominance criterion as follows:

$$x \succsim_{L\alpha} y :\Leftrightarrow L_\alpha(p,x) \geq L_\alpha(p,y).$$

The binary relation given by $\succsim_{L\alpha}$ allows a partial ordering of income distributions following the α-ray invariance notion proposed by S-P.

Proposition. *The ranking given by an α-Lorenz curve is consistent with that of any index satisfying symmetry, replication invariance, Schur-convexity, and invariance along α-rays. Namely, if $x, y \in \Gamma(\alpha)$, $x \succsim_\alpha y \Leftrightarrow x \succsim_{L\alpha} y$.*

Proof

Firstly, we prove that for any $x, y \in \Gamma(\alpha)$, $x \succsim_\alpha y \Rightarrow x \succsim_{L\alpha} y$.

From $x \succsim_\alpha y$ it follows that $I_\alpha(x) \leq I_\alpha(y)$ for any α-invariant intermediate inequality index. Since I_α satisfies symmetry and is α-invariant, we have that $I_\alpha(x) = I_\alpha(x_\alpha) \leq I_\alpha(y_\alpha) = I_\alpha(y)$. Using Theorem 1 in Dasgupta et al. (1973), the Schur-convexity of I_α implies that $(x_\alpha)_1 + ... + (x_\alpha)_k \geq (y_\alpha)_1 + ... + (y_\alpha)_k \;\; \forall k \leq n$ and

$(x_\alpha)_1 + ... + (x_\alpha)_n = (y_\alpha)_1 + ... + (y_\alpha)_n$. Consequently, $GL(p, x_\alpha) \geq GL(p, y_\alpha)$. In other words, $L_\alpha(p, x) \geq L_\alpha(p, y)$.

Secondly, we prove that for any $x, y \in \Gamma(\alpha)$, $x \succsim_{L\alpha} y \Rightarrow x \succsim_\alpha y$.

If $L_\alpha(p, x) \geq L_\alpha(p, y)$, then $\frac{1}{n}\left[(x_\alpha)_1 + ... + (x_\alpha)_k\right] \geq \frac{1}{n}\left[(y_\alpha)_1 + ... + (y_\alpha)_k\right] \ \forall k \leq n$ (with strict inequality in the case *k*=*n*). From the aforementioned theorem, it follows that the value of any Schur-convex function evaluated at x_α is lower than at y_α. Consequently, $I_\alpha(x_\alpha) < I_\alpha(y_\alpha)$. Since I_α is α-invariant $I_\alpha(x) = I_\alpha(x_\alpha)$ and $I_\alpha(y_\alpha) = I_\alpha(y)$. Therefore, $x \succsim_\alpha y$. □

2.3 Interpreting α according to Del Río and Ruiz-Castillo (2000)

The ray invariance concept proposed by S-P has no clear economic interpretation—which makes it difficult its use in empirical analyses—and violates the horizontal equity axiom since individuals who have the same income level initially may be treated differently (Zoli, 2003). To solve these problems, Del Río and Ruiz-Castillo (2000) offered a ray-invariant notion that can be considered as a special case of the former. According to their proposal, inequality depends on two parameters, instead of one: the income shares in the distribution of reference that gives rise to the rightist and leftist views, denoted by simplex vector v, and $\pi \in [0,\ 1]$, which is used to define a convex combination between them.[4] Once v and π are fixed, it is possible to calculate the n-dimensional simplex vector $\alpha = \pi v + (1-\pi)\frac{1^n}{n}$ that defines the direction of the ray. The economic meaning of this invariance notion is simple: when total income increases, inequality remains unchanged if π100% of the income surplus is allocated preserving income shares in the distribution of reference and $(1-\pi)$100% is distributed in equal absolute amounts. In other words, α can be interpreted as a convex combination of the value judgments behind the distribution of reference, v, and those behind the equalitarian distribution, $\frac{1^n}{n}$ (see Figure 4). If we chose a value of π close to 1, the notion represents value judgments rather rightist, while if π is close to 0, the inequality attitude is rather leftist, as compared to the distribution of reference.

[4] Vector v is assumed to represent an ordered distribution.

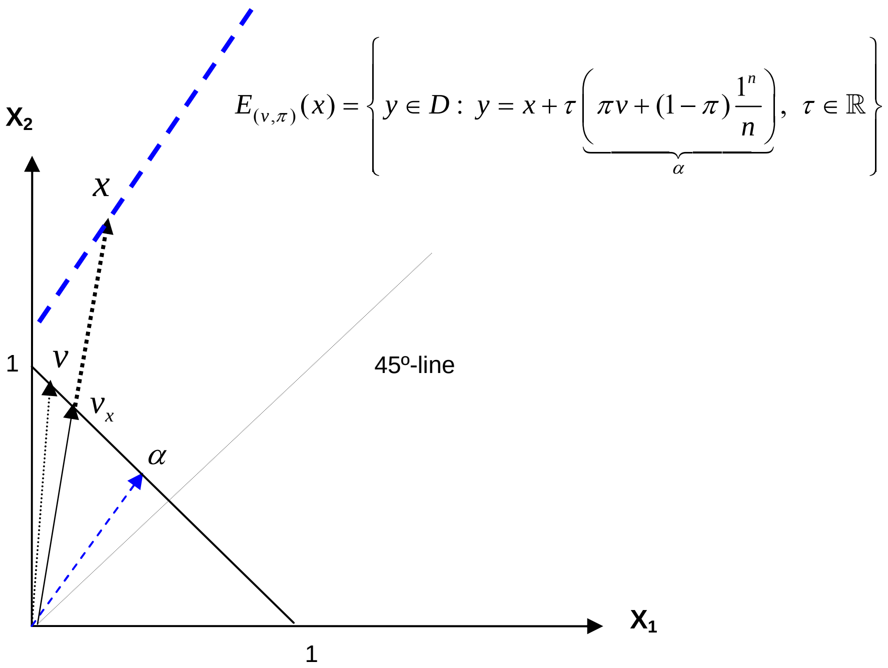

Figure 4. Invariance in Del Río and Ruiz-Castillo (2000) ($n = 2,\ \pi = 0.25$).

The distribution of reference, v, plays a very important role in this approach. Note that vector v does not necessarily have to coincide with vector v_x (as shown in Figure 4). However, in comparing distribution x and distribution y (which can be assumed to have a higher mean without loss of generality), vector v could be chosen as the income shares of x, i.e., $v = v_x$. By using this benchmark, together with the parameter π reflecting the inequality-invariance value judgments of society, it would be possible to determine whether y has a lower inequality than the distribution reached if π 100% of the income gap had been distributed according to income shares in x and $(1-\pi)$ 100% in equal amounts among individuals. Note that, in doing so, the same vector of reference ($v = v_x$) has to be used for both distributions x and y (which makes this centrist notion path independent, as explained in Del Río and Alonso-Villar, 2010). It would not be possible to use $v = v_x$ for measuring the inequality level corresponding to x, while using $v = v_y$ in the case of distribution y, since that would imply that different inequality attitudes would be used for each distribution. In other words, once v and π are chosen, they cannot be changed: the same intermediate notion must be used when comparing any two income distributions. Therefore, when studying the evolution of an economy over time, this approach allows the possibility of taking into account the starting point.[5]

3. An Illustration: Recent Evolution of Income Inequality in Australia

In this section, we provide an empirical illustration of the new intermediate dominance criterion using Australian income data for the period 2001-2008. In this time, Australia experienced high and sustainable economic and employment growth, the real economy growing on average by more than 3.5 per cent every year (ABS 2011). As will be shown, the evolution of the income distribution in Australia along this period provides a suitable case for an intermediate inequality analysis.

The data used in our analysis come from the first and eighth waves of the Household, Income and Labour Dynamics in Australia (HILDA) Survey conducted by the Melbourne Institute of Applied Economics and Social Research to analyze the change in income inequality that took place in Australia during that period. We look at changes in the distribution of annual private income before taxes and transfers from the public sector. This income variable is defined as the sum of market income including labour income in the form of wages and salaries, capital income from businesses, investments, and private pensions plus the value of all non-market private transfers received by any household member. Differences in non-income needs across households of different size and composition are taking into account in the analysis, so household income is converted into household equivalent income using an equivalence scale. Thus, we use the parametric family of equivalence scales introduced by Buhmann *et al.* (1988) defined as:

$$e(s,\Theta) = s^{\Theta},$$

where s is the size of the household and Θ is the elasticity of the scale rate. Adjusted income values are computed by dividing household income by scale factor s^{Θ}, with different values of Θ being used to check the robustness of the results. All income values correspond to real values expressed in constant 2008 Australian dollars derived

[5] This approach was used by Del Río and Ruiz-Castillo (2001) to compare income distributions in Spain between 1980 and 1990.

using Consumer Price Index figures provided by the Australian Bureau of Statistics. Finally, all the estimates are computed using the population weights reported in HILDA to obtain population rather than sample estimates.

Table 1 summarizes the main changes in the distribution of household income that took place between 2001 and 2008. In this period, households in Australia witnessed a general increase in their incomes, with the mean household income growing more than 1,163 dollars every year, equivalent to an annual rate of growth of 2.7 per cent. This growth, however, was not uniform across the whole distribution. Thus, the first column in Table 1 suggests that the *absolute* income gains among the top three quintiles were larger than those experienced by the bottom positions, whose mean income grew less than the overall mean. On the contrary, the largest *relative* income growth was for the poorest three quintiles, which grew above the population average, with the growth rate declining as me move up in the distribution. This growth pattern is consistent with the increase in the income share of the three bottom quintiles and the corresponding decline of the share accumulated by the two top quintiles (columns 3 and 4).

Table 1
Annual Changes in Mean Income and Income Shares by Income Quintiles
in Australia between 2001 and 2008

Quintile	Absolute change (AUD $)	Relative change (%)	Income Shares (%) 2001	2008
1	178.99	19.33	0.26	0.74
2	929.13	5.71	6.92	8.48
3	1,039.46	2.87	16.75	16.96
4	1,237.54	2.22	26.39	25.53
5	2,465.89	2.34	49.68	48.29
Total	1,163.84	2.71	100	100

Note: Equivalent incomes computed assuming a value for Θ equal to 0.5.

These changes in the income distribution have important implications in terms of inequality. The fact that the absolute difference between top and bottom positions widened suggests an increase in absolute inequality. On the contrary, the convergence in the income shares of the groups points toward a reduction in relative inequality. These findings are corroborated in Figure 5, which shows the absolute and relative Lorenz curves for the 2001 and 2008 income distributions. In the relative case (Figure 5.a), the Lorenz curve for 2008 dominates the initial one, which implies that these two distributions will be unambiguously ranked by the class of scale invariant inequality indexes (Foster, 1985). In contrast, the absolute Lorenz curve for 2001 lies above that of 2008 at every point (Figure 5.b), so that any inequality measure verifying the translation invariance property would indicate an increase in the level of absolute inequality during this period.[6]

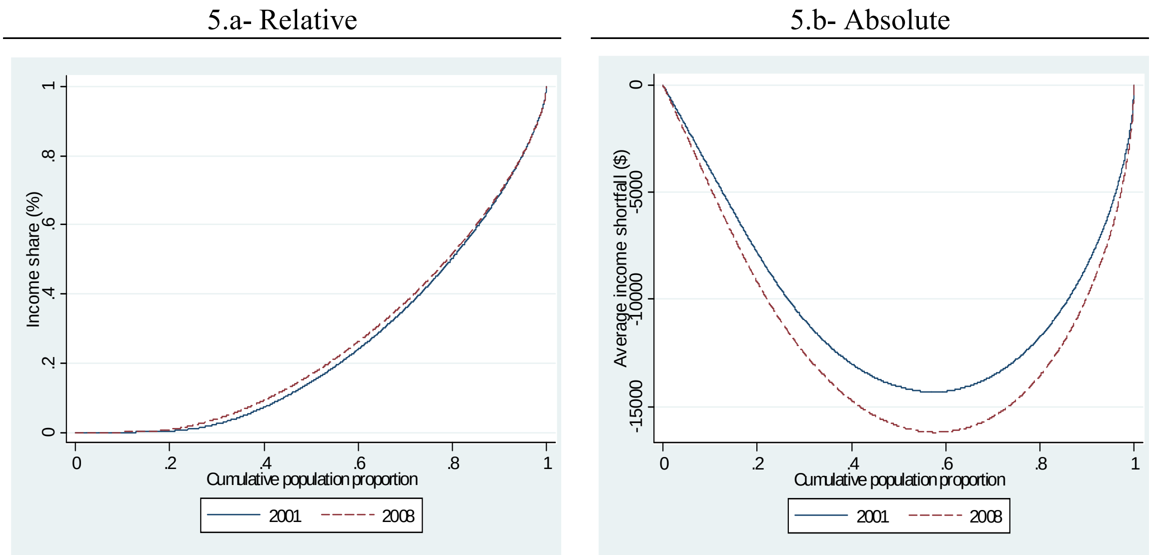

Figure 5. Absolute and relative Lorenz curves for Australia, years 2001 and 2008 ($\Theta = 0.5$).

It is important to note that the relative and absolute dominance criteria do not inform us about the changes in inequality for intermediate notions of inequality lying between the "rightist" and "leftist" extreme cases. To deal with this issue, we now use our α-Lorenz curves and the corresponding dominance criterion to compare the 2001 and 2008 distributions adopting centrist notions of inequality. This methodology allows us to search for unambiguous rankings among the classes of intermediate inequality

6 The dominance results presented in Figure 5 are not sensitive to the choice of the equivalence scale parameter. Thus, for any value of $\Theta \in [0,1]$, we find that absolute (relative) inequality in Australia increased (decreased) between 2001and 2008.

measures. For that purpose, we use the class of inequality indexes consistent with the ray-invariant notion proposed by Del Río and Ruiz-Castillo (2000).

Let x and y denote the income distributions in Australia in 2001 and 2008. Also, let v_x, v_y, and $\frac{1^n}{n}$ be the vectors of the simplex associated to the x, y, and egalitarian distributions, respectively. Taking the initial distribution as the distribution of reference, we consider inequality-invariance value judgments, α, of the form

$$\alpha = \pi v_x + (1-\pi)\frac{1^n}{n},$$

where $\pi \in [0, 1]$. If $\pi = 0$, then α becomes the absolute ray-invariant notion, whereas $\pi = 1$ leads to the relative ray. Further, for a particular α to represent a centrist attitude we must have that α Lorenz-dominates both v_x and v_y. This condition holds for $\pi = 0$ but it is not satisfied in the case π is set equal to 1 since, as aforementioned, v_x is Lorenz-dominated by v_y.[7]

We denote by π^M the maximum value of π for which a valid intermediate notion can be derived. For a given vector α, the ordered distributions x_α and y_α defined in Section 2.2 have to be constructed in order to check for the existence of α-Lorenz dominance. When x and y come from populations of different sizes, this requires the use of replications of the initial distributions in order to ensure that the vectors are conformable. Notice that this does not impose any limitation on the validity of the analysis since inequality rankings are unaffected by population replication. In fact, for the general class of inequality indexes satisfying the replication invariance principle considered in this paper, the original and replica distributions exhibit exactly the same level of inequality independently of the mean-invariance notion. Furthermore, for any value of π, the invariance value judgment given by α is equivalent to the one obtained for the replicated population since both α vectors have the same Lorenz curve. This means that in order to keep inequality unaltered, both notions require an analogous allocation of the income growth among individuals.

[7] This comes from the fact that x is Lorenz-dominated by y.

As an example, Figure 6 shows the α-Lorenz curves for 2001 and 2008 for an equivalence scale parameter Θ of 0.5 and values of π equal to 0.25, 0.5, 0.75, and 0.9. As the figure shows, it is possible to derive unambiguous rankings of the initial and the final distributions for some centrist notions of inequality. Thus, when π is set equal to 0.25 or 0.5, the curve for 2001 α-Lorenz dominates that of 2008 and, therefore, it follows from our proposition that any inequality index consistent with these centrist attitudes would conclude that income inequality increased during this period. Interestingly, this result does not hold anymore when centrist views that are closer to the relative notion of inequality are considered. For both π equal to 0.75 and 0.9, the α-Lorenz curves for 2001 and 2008 cross multiple times, which implies that no unambiguous ranking of these two distributions can be derived for these attitudes towards inequality

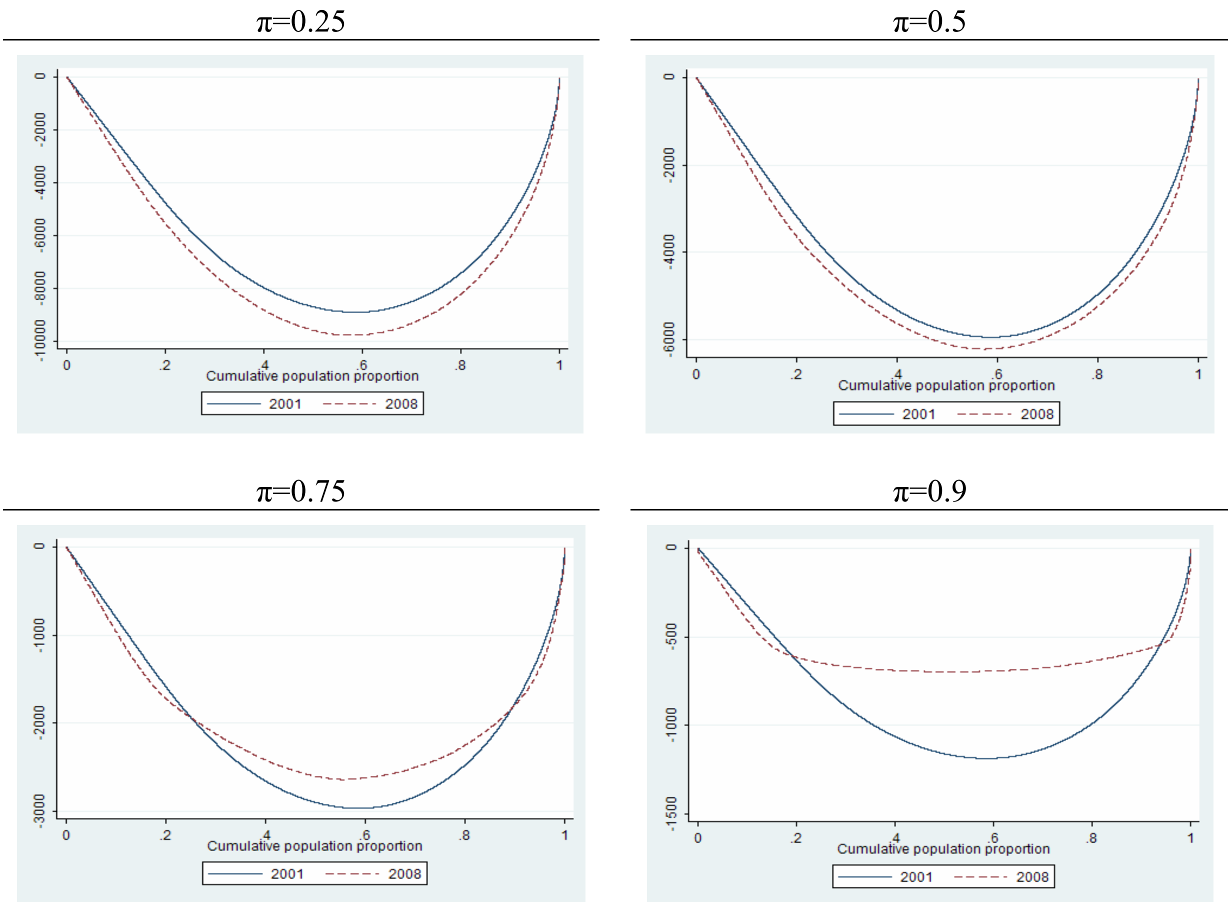

Figure 6. α-Lorenz curves for Australia, years 2001 and 2008 (Θ = 0.5).

Table 5 summarizes the main results of the intermediate inequality analysis for different values of the equivalence scale parameter Θ. For each case, the value of π^M and three sets of centrist notions expressed in terms of π are provided: the values according to which income inequality increased during the period as the distribution in 2001 dominates that in 2008; those π's for which these two distributions cannot be unambiguously ranked; and the set of notions for which one could claim income inequality declined over the period 2001-2008.

We find that the initial and the final distributions are comparable for most of the centrist notions that can be constructed as a convex combination of the vectors v_x and $\frac{1^n}{n}$. Thus, π^M is around 0.9 for all the equivalence scales considered.

Remarkably, our results suggest that income inequality in Australia increased for most of the intermediate value judgments. In fact, inequality increased for all those centrist attitudes that require the equal distribution of at least 31-40 per cent, depending on the value of parameter Θ, of the income gains in order to keep inequality unchanged. Thus, for example, when $\Theta = 0.5$, inequality increases if using centrist views according to which at most 65 per cent of the income growth is distributed across individuals according to income shares in 2001 (Table 2, column 2) and, consequently, at least 35 per cent of the growth is allocated in equal amounts.

On the other hand, for $\Theta = 0.5$, the initial and the final distributions cannot be unambiguously ranked when using invariant notions according to which inequality is maintained if the proportion of the income growth that is equally distributed among individuals ranges between 6 and 35 per cent (Table 2, column 3). For the remaining equivalence scale values, the lower and upper limits of the interval are 9-13 and 31-40 per cent, respectively.

Finally, for none of the values of π for which a valid centrist notion can be defined we can claim income inequality in Australia actually declined between 2001 and 2008.

Table 2
Intermediate Inequality in Australia: 2001 – 2008

		Values of π according to which inequality		
Θ	π^M	Increased ($x \gtrsim_\alpha y$)	Equivalent	Reduced ($y \gtrsim_\alpha x$)
0	0.88	[0,0.69]	(0.69, 0.88]	-
0.25	0.91	[0,0.64]	(0.64, 0.91]	-
0.5	0.94	[0,0.65]	(0.65, 0.94]	-
0.75	0.92	[0,0.62]	(0.62, 0.92]	-
1	0.87	[0,0.60]	(0.60, 0.87]	-

4. Conclusions

This paper has introduced a new dominance criterion that allows ranking income distributions according to the centrist inequality notion proposed by Seidl and Pfingsten (1997). In doing so, α-Lorenz curves, which are related to the generalized Lorenz curves (Shorrocks, 1983), are defined. Our proposal allows finding out those cases in which one distribution has higher inequality than another not only according to a particular α-index but according to all those α-indexes consistent with our dominance criterion (as also happens with the relative and absolute Lorenz dominance criteria and the indexes verifying the scale invariance and the translation invariance axioms, respectively).

To illustrate the usefulness of these tools, the evolution of income inequality in Australia between 2001 and 2008 has been analyzed. The results show that even though relative inequality decreased during the period, according to most ray-invariance centrist views of inequality, inequality increased.

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