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Abstract

The analysis of growth patterns is a branch of research of great practical importance: the identification of factors that influence changes in the income distribution making them pro-poor or pro-rich can be a crucial element in the process of designing a social policy. It seems natural that analyzing the problem of the relative situation of the poorer with respect to the richer involves – for a given definition of the poorer and the richer – a comparison of the situation of both groups. It is not, however, exactly reflected in actual methodologies aiming at identifying growth patterns. The paper is aimed at proposing a method of identification of a growth pattern by analyzing the direct relation between income (or some other measure of wealth) of the poorer and of the richer. At this end the basic idea of Zenga's inequality index is applied. Proposed relative affluence measures allude to the intuitive concept of the proportion of two averages: upper and lower – with respect to a given quantile of the income distribution. In this sense it directly refers to the relation poorer-richer and can be intuitively understood, even by non-specialists. In this paper relative affluence measures are applied to the analysis of growth patterns in Poland and the focus is on situation of the poor. The proposed measures can nevertheless be also applied to the analysis of the relative situation of the rich. In such a case the proposed measures would be measures of relative affluence in the full sense of these words.

Keywords: growth pattern, income distribution, pro-poor growth, relative affluence
JEL classification: J31, C46

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Introduction

Assisting the poor is one of the important objectives of contemporary, developed states. Depending on the leading political orientation, more or less attention is devoted to this issue and there is also no consensus on how big the group benefiting from a public aid should be. A rationale for such a situation is that fighting against poverty and equalizing the income distribution is very expensive and suspected of reducing the competitiveness and efficiency of the economy[1]. In this context, stable, positive economic growth, improving the situation (absolute and relative) of all the poor in the society could be seen as a dream of every government. That is why the problem of growth pattern became so popular: a permanent, high rate pro-poor growth would improve situation of the whole population and in particular of the poor and this would prevent from a social discontent.

Over the last decade several analyses have been performed, aiming at identifying growth patterns (see for instance [Kakwani and Pernia, 2000], [Dollar and Kraay, 2002], [Kraay, 2006], [Son and Kakwani, 2008], [Deutsch and Silber, 2011]). Among the proposed methodologies two main streams can be distinguished: the absolute and relative approaches (cf. [Ravallion, 2004], [Duclos, 2009]). Roughly speaking, they distinguish between absolute and relative growth of the income of the poor. Without going into the details of these concepts, absolute analyses seem to be more appropriate for low-income countries (analogously as in case of poverty analysis), where the issue of subsistence is the crucial problem. For developed countries absolute growth in the income of the poor (even in real terms) is usually not sufficient as the poor expect a decrease of their "distance" from the rich.

It seems natural that analyzing the problem of the relative situation of the poorer with respect to the richer involves – for a given definition of the poorer and the richer – a comparison of the incomes of both groups. The direct comparison is not, however, exactly reflected in actual methodologies aiming at identifying growth patterns. Some (for example the Growth Incidence Curve, cf. [Ravallion, Chen, 2003]) analyze income changes at given positions along the income distribution,

[1] Hence the famous equity-efficiency trade off. There are, however, no unambiguous indications on the optimal involvement of the state in the economy.

while others relate the situation of the poor to that of the entire population (measures associated with the Lorenz curve – for example [Son, 2004], [Son and Kakwani, 2008], [Duclos, 2009]).

This paper is aimed at proposing a slightly different view of the problem of identification of growth patterns by analyzing the direct relation between income of the poorer and the richer. At this end the basic ideas of Zenga's inequality index and Zenga's curve are applied (cf. [Zenga, 2006]). On this basis new, relative indicators of growth patterns are constructed.

The paper is organized as follows. In the next section discrete forms of the proposed measures are given. After that, properties of these new measures are shortly considered. The fourth part is devoted to an empirical analysis, based on the new measures. The last section concludes.

Zenga's index and Zenga's curve

Let $\mathbf{x}^k = \left(x_1^k, x_2^k, ..., x_{n_k}^k\right)$ be a vector of non-negative values, representing the distribution of income[2] at time *k*. Then, the lower and the upper mean will be defined as $M^{-k}(p) = \frac{\sum_{i=1}^{\lfloor n_k p \rfloor} x_i^k}{\lfloor n_k p \rfloor}$ and $M^{+k}(p) = \frac{\sum_{i=\lfloor n_k p \rfloor}^{n_k} x_i^k}{n_k - \lfloor n_k p \rfloor + 1}$ respectively, where $\lfloor n_k p \rfloor$ denotes rounding down to the integer closest to $n_k p$ and *p* – quantile of the income distribution ($0 \le p \le 1$). For a given *p*, Zenga's point indexes (cf. [Zenga, 2006]) are defined as

$$I^k(p) = 1 - \frac{M^{-k}(p)}{M^{+k}(p)}.$$

[2] In the next section of the paper income will be used as a measure of affluence. But it could be replaced with expenditure (or the difference between them. See, for example, [Slesnick, 1998]) or any other measure of welfare. On a broader discussion of the application of non-income variables see [Grosse, Harttgen, Klasen, 2008]).

For individual incomes, drawing $I^k(p)$ against *p*, gives Zenga's curve[3] at time *k*. The synthetic inequality index, based on point indexes is defined as:

$$I = \frac{1}{n_k} \sum_{i=1}^{n_k} I^k\left(\frac{i}{n_k}\right).$$

According to Zenga [2006], this inequality index is scale invariant. It decreases in case of translation of the whole distribution by positive value and as a result of Pigou-Dalton transfer. Synthetic inequality index equals to 0 in case of no inequality in the distribution and tends to 1 for a maximum inequality.

Measures of relative affluence

A point measure of relative affluence, denoting changes in distribution of income at a given time will be defined as:

$$RA(p) = I^0(p) - I^1(p) = \frac{\bar{M}^1(p)}{M^1(p)} - \frac{\bar{M}^0(p)}{M^0(p)} \qquad (1)$$

where $I^0(p)$ and $I^1(p)$ denote values of Zenga's point indexes at the beginning and the end of the period respectively.

The values of the point index given by (1) range between -1 and 1. They reflect changes in the relative affluence of the poorer with respect to the richer. The line separating two groups – the poorer and the richer – is set by *p*. For a given *p*, $RA(p)$ indicates the change (expressed in percentage points) in the share of the average income of the poorer (100% · *p* of the population) in relation to the average income of the richer (100% · (1-*p*) of this population). Positive values of $RA(p)$ indicate an improvement in the relative situation of the poorer, negative – decline and zero – proportional or no changes.

[3] Zenga [2006] defines it in the form of a diagram for grouped (weighted) data.

As p is to be chosen freely, a group of the poorer does not necessarily mean the poor – in the extreme case, this group can include everyone except the person with the highest income. The proposed measure is therefore referred to as a measure of relative affluence instead of, for example, relative poverty.

Plotting $RA(p)$ against p gives a relative affluence curve. It allows the analysis of changes in the average income of the poorer with respect to the average income of the richer over the entire distribution (all values of p). In this sense it could be interpreted as a generalization of the Growth Incidence Curve (cf. [Ravallion, Chen, 2003]), giving at a single point information not only on the change in income at a given quantile, but also on the relative change in income of groups below and above this quantile.

The simplest way of obtaining an overall measure, characterizing the pattern of changes in the income distribution, is averaging the point indexes $RA(p)$ up to a given quantile p in the following way (relative affluence index):

$$RAI(p) = \frac{1}{\lfloor n_k p \rfloor} \sum_{i=1}^{\lfloor n_k p \rfloor} RA\left(\frac{i}{n_k}\right) \tag{2}$$

In order to concentrate on the situation of the poorest members of the population, a variant of this measure can be proposed – calculated only for the poor and weighted by the poverty gap. A poverty-adjusted relative affluence index is then given by:

$$PRAI(z) = \frac{1}{\sum_{i=1}^{n_1} \max\left(0, z - x_i^1\right)} \sum_{i=1}^{n_1} \max\left(0, z - x_i^1\right) RA\left(\frac{i}{n_1}\right) \tag{3}$$

where z denotes the poverty line for the income distribution at the end of the analyzed period. $PRAI(z)$ can be calculated assuming that $\sum_{i=1}^{n_1} \max\left(0, z - x_i^1\right) > 0$ – it means that at time 1 (end of the analyzed period) at least one person (household) is poor[4].

[4] In case of no poverty, the index (2) can be calculated.

Both measures given by equations (2) and (3) take values varying between -1 and 1. A negative (positive) value denotes anti-poor (pro-poor) pattern of changes in the income distribution,

Description of measures

The basic property of the proposed measures is their strictly relative character. Being based on a relation of average incomes, it captures relative changes in income distribution, but gives no information on the absolute level of income. As a consequence, an increase in the income of every person in the population can be associated with negative values of the proposed measure, indicating thus an anti-poor pattern of observed growth.

All proposed measures are scale invariant – changing the scale does not influence their values. A translation of the entire distribution by a positive value results in positive values of measures, indicating a pro-poor change. (An equal decrease in all incomes denotes an anti-poor change). The impact of a Pigou-Dalton transfer depends on the position of those involved in the transfers. A transfer to the group of the poorer from the group of the richer is obviously pro-poor and results in positive values of relative affluence. But transfers within the groups of the poorer or the richer do not affect the values of the proposed measures.

As mentioned previously, curves and indexes of relative affluence, given by expressions (1) – (3) take values in the interval $[-1;1]$. Negative values indicate an anti-poor change in the income distribution – a decrease in the average income of the poorer with respect to that of the richer. Positive values denote pro-poor changes, when relative situation of the poorer is improving.

The detailed interpretation of the obtained values depends on the index used. Values of the point indexes, given by expression (1), used to construct the relative affluence curve, denote changes in the relative affluence of individuals in the bottom part of the distribution (distinguished by quantile p) between the original and final periods. This change is given in percentage points.

Two examples of possible changes in the income distribution are presented in Table 1. The first row of the table gives the incomes at the beginning of the period while the other rows represent

variants (A and B) of the distribution at the end of the period. The corresponding relative affluence curves are plotted in Figure 1.

Table 1. Hypothetic income distribution

Initial distribution $(\mathbf{x}^0)$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Final distribution A $(\mathbf{x}^1_A)$	4	9	10	10	11	12	14	16	18	20	21	22	23	24	25	26	27	28	29	30
Final distribution B $(\mathbf{x}^1_B)$	0.5	1	1	3	4	5	6	7	8	10	10	11	13	15	15	15	15	15	15	15

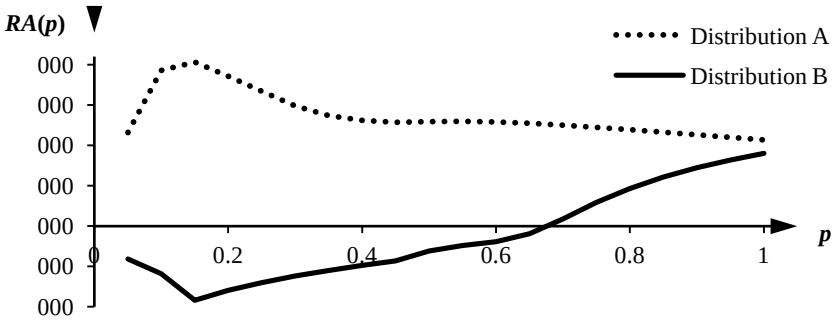

Figure 1. Relative affluence curves for hypothetic income distributions

The analysis of relative affluence curves enables one to identify groups in the worst and in the best situation. The global maximum for the final distribution A at point $p^* = 0.15$ shows that at this percentile one observes the highest improvement of the income of the poorer people in the population. Similarly the global minimum for the final distribution B at point $p^{**} = 0.15$ indicates that at this percentile one observes the highest relative loss of the poorer. A monotonic relative affluence curve would show that extreme values of gains and losses are observed for the poorest and the richest persons in the population. In Figure 1 the curves are not monotonic, but the minimum (final distribution A) and maximum (final distribution B) changes are observed for $p = 1$. Moreover one has

almost the same values for both distribution at $p = 1$, irrespectively of the fact that the highest income in the final distribution A was 2 times higher than that in the final distribution B. This clearly indicates the dependence of single point values on the entire income distribution.

The poverty adjusted relative affluence index, given in expression (3) stresses the situation of those who are the poorest. In other words here the focus of the analysis is on the poor in the population (identification of the status – poor or not poor – is made on the basis of the income distribution at the end of the period). The interpretation of this index is similar to that given in (2) but the relative affluence is not a per person average but a per poverty gap average.

The values of the relative affluence index (2) for the poor (assuming a relative poverty line at the level of 60% of median income), are equal to 0.169 and -0.067 for distributions A and B respectively. It means that the ratio of the average income of the poor over that of the non-poor rose by 16.9 percentage points in the first case and fell by 6.8 percentage points in the second case. When we assign higher weights to people with higher poverty gap (poverty-adjusted relative affluence index), we get 0.155 and -0.066 respectively.

Note that the indices proposed in this paper strongly depend on outliers. Using positional measures (e.g. median instead of mean based measure) would, however, completely change the key characteristics of the proposed measures. Therefore, if there is a high asymmetry in the data as it sometimes happens for income distributions, it might be better to work with truncated distributions[5] (see the empirical example in the next section).

Application

An illustration of the use of the proposed measures will now be given and it is based on Polish data. The biggest, officially available data set concerning living conditions, income and expenditures is the Household Budget Survey, conducted yearly by the Polish Central Statistical

[5] Another rationale for working with truncated data is that data on income distribution mostly come from surveys and the reliability of such data in both tails (especially in the upper tail) of the income distribution is rather low.

Office. This survey was first conducted in the 1950s, long before the political and economic transition that started in 1989. However, because of methodological discrepancies, relatively consistent data are available only since 1998.

In the period under analysis (1998-2008), between 31000 and 37000 households were surveyed each year. These data are representative for Poland as a whole. The reliability of these data – especially as far as income and expenditures are concerned – is, however, an open question (cf. [Ravallion, 2004, p. 8]). This is especially true for the richest; incomes in the upper tail of the income distribution strongly depend on the sample selection and the willingness to respond to a survey. And, as mentioned previously, this part of the sample, taking into account the high asymmetry of income distributions, could significantly influence measures based on the arithmetic mean.

Relative affluence curves for the period 1998-2008 are presented on Figure 2. Because of the characteristic of the data and the sensitivity for the values in the upper tail, two curves are plotted: first, for the entire sample and second – when 1% of the highest values are deleted. Both curves are calculated for the distributions of equivalent income (a modified OECD scale 0.5/0.3 has been used). Note that the construction of the relative affluence curve does not require adjusting the data (e.g. with the inflation index).

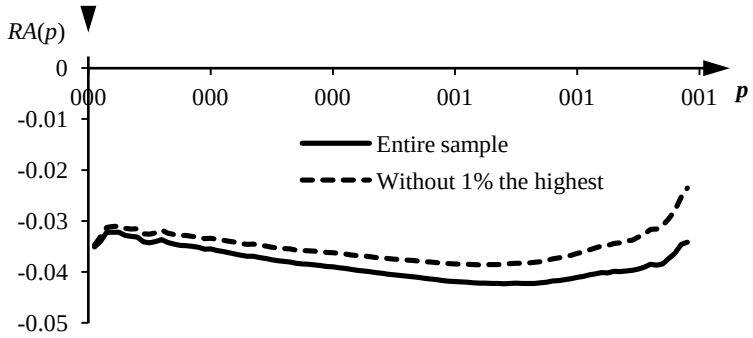

Figure 2. Relative affluence curve for Poland, 1998-2008

The graph of the relative affluence curve shows an anti-poor pattern of growth. The minimum for the entire sample is achieved for $p = 0.68$. This means that up to this point the relative situation of the poorer is worsening and that the situation of the bottom two thirds of the population in

relation to the highest one third is the least favorable. As expected, there are quite big differences between the entire and truncated samples at the upper tail, and removing the highest values influenced the whole curve.

The analysis of year-to-year changes in relative affluence suggests that the observed overall, anti-poor effect of distributional changes, is however not a constant property of growth in Poland. A detailed analysis suggests distinguishing between two subperiods – up to 2004 and after that (in 2004 Poland accessed the European Union; from this year the unemployment rate started to decrease significantly and the Polish GDP grew much faster than earlier). The relative affluence curves for both subperiods are plotted in Figures 3 and 4.

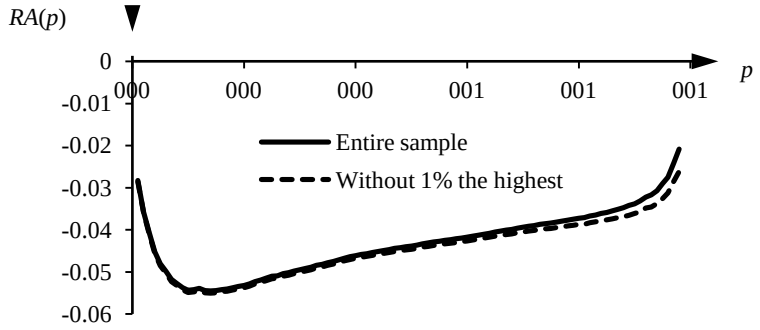

Figure 3. Relative affluence curve for Poland, 1998-2004

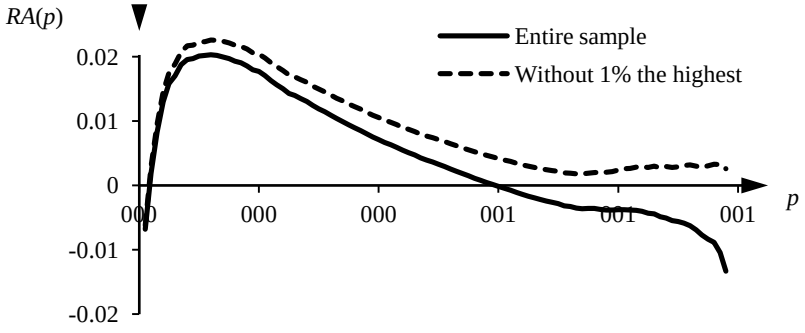

Figure 4. Relative affluence curve for Poland, 2004-2008

In both Figures 3 and 4 extreme values of the relative affluence curves are observed for $p \approx 0.15$, what approximately reflects the extent of relative poverty in Poland. In the first period 15% of the poorest were those who lose the most while during the second period the opposite was true. Differences observed in the upper tail of the distributions in Figure 4 result from the unexpectedly high income of the richest in 2008 (this increase is much higher than increase in average income).

Table 2. Relative affluence indexes for Poland, 1998-2008

	RAI(*p*)			*PRAI*(*z*)		
	1998-2008	1998-2004	2004-2008	1998-2008	1998-2004	2004-2008
Entire sample	-0.034	-0.049	0.015	-0.033	-0.044	0.010
Without 1% the highest	-0.032	-0.050	0.017	-0.032	-0.045	0.011

Values of relative affluence indexes are given in Table 2. Both unweighted *RAI*(*p*) and poverty adjusted *PRAI*(*z*) were calculated for the poor – people with an equivalent income lower than 60% of the median of the equivalent income in the population. The values of all the indices are reflected in the graphs of corresponding curves. They indicate a pro-poor growth pattern between 2004 and 2008 and an anti-poor pattern between 1998 and 2004. The distributional change over the entire period was also anti-poor. Deleting 1% of the highest incomes does not influence significantly the values of the indices. Generally, during the first period the ratio of the average income of the poorer over that of the richer decreased by about 4-5 percentage points. During the second period it increased by about 1%

Conclusion

The analysis of growth patterns is a branch of research of a great practical importance: identifying factors that influence changes in the income distribution making them pro-poor can be a

crucial element in the process of designing a social policy. Therefore indicators of growth pattern have to meet some requirements. In addition to obeying reasonable axioms, the indices need also to be easily interpreted.

Proposed relative affluence measures are based on the idea underlying the Zenga's inequality index. Therefore the measures reflect changes in the relative situation of the poor and the rich: they allow to specify, if a relation between incomes below and above given level has changed in a pro-poor or anti-poor manner. Construction based on the Zenga's index also allows to ensure the desired properties of the measures.

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