Working Paper Series

An inter-temporal relative deprivation index

Lidia Ceriani
Chiara Gigliarano

ECINEQ WP 2011 – 237

ECINEQ 2011 – 237
November 2011

www.ecineq.org

An inter-temporal relative deprivation index*

Lidia Ceriani
Università Bocconi, Milano

Chiara Gigliarano†
Università Politecnica delle Marche, Ancona

Abstract

The paper provides the axiomatic characterization of a new relative deprivation index. The concept of relative deprivation is here extended towards the inter-temporal framework. In fact, if we agree that deprivation is a relative concept, we should also believe that individuals not only take care of their relative position with respect to others, but also of their relative position with respect to their own past. While in the traditional relative deprivation framework the reference group is only other-regarding, in our work we stretch this idea and we also introduce a history-regarding reference group. The new index is illustrated with an application to EU countries.

Keywords: Relative Deprivation, Inter-temporal Measurement, Distribution, Axioms
JEL classification: I32, D31, D63, D71, D81

* _We are extremely grateful to Casilda Lasso de la Vega, Conchita D'Ambrosio, Buhong Zheng, Lars Osberg, Claudio Zoli, the participants at the 19th international meeting of the Society for Social Choice and Welfare, at the 31st General IARIW Conference, at the IV GRASS workshop and at the IV Meeting of ECINEQ for their interesting suggestions and comments. The general disclaimer applies.
† **Contact details:** lidia.ceriani@unibocconi.it .

Introduction

Deprivation has been always considered as an inter-personal concept: it is the feeling an individual experiences when she realizes to be worse-off than someone else in the society (Runciman (1966) and Stouffer et al. (1949)). The term *deprivation* has been used in the literature in a somehow confusing manner. On one side, deprivation is considered as the number of functionings from which a person is excluded (see, for example, Chakravarty and D'Ambrosio (2006)) or as lack of resources, in particular, employment, access to education, childcare, healthcare facilities, and social partecipation (Eurostat (2010)), or lack of income (Subramanian and Majumdar (2002) and Chakraborty et al. (20008)). On the other side, deprivation is used to denote the sense of difference, diversity, or depression, an individual feels by comparing her situation with the desidered one (Runciman (1966), Bossert et al. (2007), Chakravarty (2007), Mukerjee (2001), Yitzhaki (1979)). We follow the latter approach, since the former concept is strictly related to and hardly separable from the idea of multidimensional poverty.

Going back to the overcited definition of deprivation by Runciman (1966), we find out a neglected point: a person is relatively deprived when she realizes not to have something that other persons, "*which may include himself at some previous or expected time*", have (Runciman (1966), p.9). Inspired by Runciman seminal work, we believe that deprivation is also inter-temporal, meaning that some individual's reference group is made not only by the other individuals, but also by her own history.

Note that, even when time has been taken into account in the deprivation literature (Bossert et al. (2011), Bossert et al. (2007)), the reference group has been always other-related. In particular, when determining the deprivation of some individual, Bossert et al. (2011) stress the importance of taking into account, in a two periods framework, the number of individuals overtaking her. On the other hand, Bossert et al. (2007) bring time into the analysis to consider other-regarding deprivation in different times. In particular, they shift attention from deprivation to social exclusion when adding time to the analysis ("*Social Exclusion*" is "*being in a state of deprivation over time*" p.777).

We believe, instead, that time is in fact another dimension of deprivation, which can be explained as restricting some individual reference group to her own past. In this work, therefore, we embed the new concept of history-regarding deprivation: individuals are deprived not only because their outcomes are lower then someone else today, but also because their outcomes are lower than what it was used to be in the past. Each individual current outcome works as reference point which serves as zero point of the value scale. While in traditional relative deprivation framework, this zero point is used to evaluate the relative position only with respect to other individuals, in our work we stretch this idea and we also measure the deviations from that reference point to the individual previous period outcomes.

Therefore, the novelty of the paper is to look at the direction of individuals outcome path, defining as inter-temporal deprived those individuals whose condition is worsening over time. This idea of inter-temporal relative deprivation is strictly linked with the concept of mobility. Here we think of Fields (2007) concept of *Directional Income Movements*, "*which gauges the extent of fluctuation in individuals incomes*" when "*the observer cares not only about the amounts of the income changes* *but also about their direction*" (Fields, 2007, p.3). In particular, we take into account only downward movements. In this sense, our index bridges two streams of the well-being literature: relative deprivation on one hand and mobility on the other. Also, introducing individual comparisons with her own history, the issue of how to discount past positions arises. In fact, when an individual compares her current situation with her past, memory plays an important role. We believe that an individual is less affected by a remote experience then a more recent one. We chose to evaluate income and time separately, allowing a more specific discussion of the role of time and the attitudes that individuals should take to it. Another novelty of the paper is to adapt what is usually stated by time-discounting analysis on future outcomes (Yi et al. (2006)) to the past. Time-discounting results in valuing past outcomes less than the same level in the present or without discounting. The discounting factors will not be *a-priori* imposed, but axiomatically derived. There new ideas are embedded in the new individual iter-temporal relative deprivation index which is characterized by means of axioms in the first part of the paper.

Moreover, as in previous works (Bossert et al. (2007) or Bossert et al. (2011)), we also take into account the persistence in the state of deprivation, by introducing the individual multi-inter-temporal relative deprivation index. This index additively aggregates some individual inter-temporal deprivations over her own past, and therefore depends on (i) comparisons with others today, (ii) comparisons with the individual own history, and (iii) comparisons with others in the past.

Finally, we aggregate the individual multi-inter-temporal measure to obtain an index which allows comparisons between different societies. We apply this new index to a selection of EU countries, stressing the information we can gain from our approach.

The paper is organized as follows: Section 1 provides the characterization of our individual inter-temporal relative deprivation index, and contains the main result; Section 2 aggregates the individual deprivation over time to obtain a multi-inter-temporal relative deprivation index; Section 3 sum up the individual indices in an aggregate inter-temporal relative deprivation index; Section 4 presents the results of the empirical test of the index, based on EU-Silc longitudinal dataset. Appendices 1 and 2 contains the formal derivation of the indices described in Sections 2 and 3.

1 Individual inter-temporal relative deprivation

1.1 Framework

Consider a population N of individuals $i = 1, 2, \dots, n$, $n \in \mathbb{N}$, and $n \geq 3$ over a period of times $T = (t, t-1, \dots, t-\tau, \dots, t-p)$ of length $(p+1)$, where t is today, $\tau = 0, 1, \dots, p$ represents the lag between today and the each past period, and $p \in \mathbb{N}$ is fixed and depends on the data availability or research purposes. Without loss of generality, let us consider $n \geq 3$, and $p \geq 2$.

We are interested in analyzing the inter-temporal relative deprivation of individual i at time t, with respect to some achievement level (for instance income, or consumption). Traditional measures of deprivation (Yitzhaki (1979) and Ebert and Moyes (2000)) involve only achievement comparisons between individual i-th and other individuals $j \neq i$ at a given point in time. We believe, instead, back to the original Ruciman's idea, that individuals compare themselves also with their own past achievements. In the following, we use notation x_{it} to denote individual i-th achievement level at time t; X_t is the vector of achievements of the entire population N at time t and $X_{(i)t} = X_t - \{x_{it}\}$ denotes the vector of achievement of all individuals other than i at time t. Moreover, X_i is the vector of individual i-th achievements over the period of times T, and $X_{i(t)} = X_i - \{x_{it}\}$ denotes the vector of individual i-th past outcomes.

Individual i-th makes comparisons with her reference group R^i. The reference group R^i is made by two sub-sets: $R^i = R^i_O \cup R^i_H$ where $R^i_O = \{j \in (N - \{i\}) : x_{jt} \in X_{(i)t}\}$ and $R^i_H = \{(t - \tau) \in (T - \{t\}) : x_{i\tau} \in X_{i(t)}\}$. We label R^i_O *Other-regarding* reference group and R^i_H *History-regarding* reference group. Note that traditional deprivation measurement (among the others, Yitzhaki (1979), Paul (1991), Ebert and Moyes (2000)) involves only the other-regarding component, while here, in the spirit of the original Runciman (1966)'s work, we consider also the history-regarding one.

Deprivation arises on pair-wise comparisons between individual i-th and the generic k-th element in her reference group R^i: we denote these comparisons by $\delta_{ik} = \delta_{ik}(x_{it}, x_k; R^i)$, where x_k could either be an element of $X_{(i)t}$ or of $X_{i(t)}$. If k is an individual different from i-th at time t, $\delta_{ik} = \delta_{ij}(x_{it}, x_{jt}, R^i_O)$ denotes an other-regarding comparison. On the other hand, if k refers to individual i-th at some past time $t - \tau$, $\delta_{ik} = \delta_{i\tau}(x_{it}, x_{i\tau}, R^i_H)$ denotes a history-regarding comparison. Comparisons δ_{ik} could be any function that takes positive values if the outcome of the k-th element is greater than individual $i - th$'s:

$$\delta_{ik} \begin{cases} > 0 & \text{iff} \quad x_{it} < x_k \\ = 0 & \text{iff} \quad x_{it} = x_k \\ < 0 & \text{iff} \quad x_{it} > x_k \end{cases} \tag{1}$$

The available information can be summarized in the vector $\Delta_{it} = \left[\Delta_{(i)t}, \Delta_{i(t)}\right]$. $\Delta_{(i)t}$ is the vector of length $n - 1$ of comparisons between individual i-th and all other individuals $j \in R^i_O$, while $\Delta_{i(t)} = [\delta_{it-1}, \ldots, \delta_{t-\tau}, \ldots, \delta_{t-p}]$ is the vector of length p of comparisons between individual i-th and herself in past times $(t - \tau) \in R^i_H$. Characterizing the form of these income comparisons is beyond the scope of this paper; possible example are (i) $\delta_k = x_k - x_{it}$ or (ii) $\delta_k = \frac{x_k - x_{it}}{max(x_k, x_{it})}$. Let us define $\mathcal{D}^{n+p}$ as the class of all vectors Δ_{it}.

We denotes $I_{it}(\Delta_{it}) : \mathcal{D}^{(n+p)} \rightarrow \mathbb{R}^+$ as the inter-temporal relative deprivation of individual i-th at time t. Aim of the first part of this work is to provide the axiomatic characterization of $I_{it}(\Delta_{it})$.

Note that our approach is different from other contributes that added time in the analysis of relative deprivation (Bossert et al. (2007) or Chakravarty and D'Ambrosio (2006)) which we define *multi-temporal*. In the multi-temporal approach to relative deprivation the reference group is always other-regarding. In our *inter-temporal* approach, instead, we bring time into the analysis in order to enlarge the reference group, by adding the other-regarding component.

1.2 Characterization

The first two axioms are merely technical conditions. Continuity states that if any comparison slightly changes, the index does not jump. While Normalization states that the index is lower-bounded at zero.

Axiom 1.1 (Continuity)**.** *$I_{it}(\Delta_{it})$ is continuous on $\mathcal{D}^{n+p}$.*

Axiom 1.2 (Normalization)**.** *For any $\Delta_{it} \in \mathcal{D}^{n+p}$, $I_{it}(\Delta_{it}) \geq 0$ and $I_{it}(\Delta_{it}) = 0$ if and only if $\delta_k \leq 0$ for all $\delta_k \in \Delta_{it}$.*

Deprivation is increasing in the comparisons between individual i-th current achievement and either (i) other individual current achievements or (ii) individual i-th past achievements, as stated in the following axiom.

Axiom 1.3 (Monotonicity)**.** *$I_{it}(\Delta_{it}) \leq I_{it}(\Delta^{'}_{it})$, where $\Delta^{'}_{it}$ is obtained from Δ_{it} by adding $\beta \in \mathbb{R}_+$ to a generic element $\delta_k \in \Delta_{it}$, for any Δ_{it}, $\Delta^{'}_{it} \in \mathcal{D}^{n+p}$.*

The following axiom allows for individual comparisons to provide an independent contribution to the individual inter-temporal relative deprivation.

Axiom 1.4 (Independence)**.** *For any $\Delta_{it} = (\delta_1, \ldots, \delta_k, \ldots, \delta_{n+p})$, $\Delta^{'}_{it} = (\delta^{'}_1, \ldots, \delta^{'}_k, \ldots, \delta^{'}_{n+p})$ $\in \mathcal{D}^{n+p}$, if*

$$I_{it}(\delta_1, \ldots, \delta_k, \ldots, \delta_{n+p}) = I_{it}(\delta^{'}_1, \ldots, \delta^{'}_k, \ldots, \delta^{'}_{n+p})$$

for some $\delta_k = \delta^{'}_k$, then,

$$I_{it}(\delta_1, \ldots, \theta, \ldots, \delta_{n+p}) = I_{it}(\delta^{'}_1, \ldots, \theta, \ldots, \delta^{'}_{n+p})$$

for any $\theta \in \mathbb{R}$.

If deprivation arising from the vector of comparisons Δ_{it} is the same as deprivation arising from $\Delta^{'}_{it}$, then by replacing the identical generic k-th element both in Δ_{it} and in $\Delta^{'}_{it}$ by θ, deprivation still remains the same.

Anonymity restricts the set of information needed to define some individual i-th inter-temporal relative deprivation. In particular, we impose that at time t only comparisons matter (and this is the traditional anonymity axiom as, for example in Ebert and Moyes (2000)). While, for history-regarding comparisons, information about the time-lag between the past and present, is relevant.

Axiom 1.5 (Anonymity)**.** *Given any permutation π of N, $I_{it}(\Delta_{it}) = I_{it}(\Delta_{\pi(i)t})$.*

This is a kind of symmetry applied only to individuals, not to times.

The next axiom states that only positive comparisons affect individual i-th deprivation. This means that some individual deprivation is affected only by achievements which are greater than hers.

Axiom 1.6 (Focus)**.** *For any $\Delta_{it}, \Delta^{'}_{it} \in \mathcal{D}^{n+p}$, $I_{it}(\Delta_{it}) = I_{it}(\Delta^{'}_{it})$, where $\Delta^{'}_{it}$ is obtained from Δ_{it} by replacing a generic $\delta_k \leq 0$ with any $\theta \leq 0$.*

Reference-group Replication states that, if we replicate ζ_1-times the vector of comparisons with respect to others at time t and ζ_2-times the vector of comparisons with respect to individual i-th own past, the individual deprivation remains unchanged. This axiom allows to compare individuals' reference groups (both other-regarding and history-regarding) of different cardinality.

Axiom 1.7 (Reference-group Replication). *For any $\zeta_1, \zeta_2 \in \mathbb{N}$:*

$$\begin{aligned} I_{it}(\Delta_{it}) &= I_{it}(\Delta_{(i)t}, \Delta_{i(t)}) \\ &= I_{it}(\underbrace{\Delta_{(i)t}, \ldots, \Delta_{(i)t}}_{\zeta_1 \, times}, \underbrace{\Delta_{i(t)}, \ldots, \Delta_{i(t)}}_{\zeta_2 \, times}). \end{aligned}$$

The above set of axioms leads to the following proposition.

Proposition 1.1. *An index of individual inter-temporal relative deprivation I_{it} satisfies Continuity, Normalization, Monotonicity, Independence, Anonymity, Focus and Reference-group Replication, if and only if it is equal to:*

$$I_{it}(\Delta_{it}) = g^{-1}\left(\frac{1}{n}\sum_{\delta_{jt}>0} d(\delta_{jt}) + \frac{1}{p}\sum_{\delta_{i\tau}>0} d_\tau(\delta_{i\tau})\right) \tag{2}$$

where g, d and d_τ are continuous, strictly increasing and such that $g(0) = d(0) = d_\tau(0) = 0$.

Proof. By Continuity, Monotonicity and Independence, from Theorem 5.5 in Fishburn (1970) it follows that

$$I_{it}(\Delta_{it}) = g^{-1}\left(\sum_{\delta_k \in \Delta_{it}} d_k(\delta_k)\right)$$

where g and d_k are continuous and strictly increasing, and $k = 1, 2, \ldots, n+p$.
By Normalization, $g(0) = d_k(0) = 0$.
By Focus we can restrict the sum only to positive comparisons:

$$I_{it}(\Delta_{it}) = g^{-1}\left(\sum_{\delta_k>0} d_k(\delta_k)\right)$$

We can split the sum into the two components other-regarding and history-regarding:

$$I_{it}(\Delta_{it}) = g^{-1}\left(\sum_{\delta_{jt}>0} d_k(\delta_{jt}) + \sum_{\delta_{i\tau}>0} d_\tau(\delta_{i\tau})\right)$$

By Anonymity $d_k = d$ for each $k \in R_O^i$:

$$I_{it}(\Delta_{it}) = g^{-1}\left(\sum_{\delta_{jt}>0} d(\delta_{jt}) + \sum_{\delta_{i\tau}>0} d_\tau(\delta_{i\tau})\right).$$

By Reference-group replication:

$$\begin{aligned} I_{it}(\Delta_{it}) &= I_{it}(\Delta_{(i)t}, \Delta_{i(t)}) \\ &= I_{it}(\underbrace{\Delta_{(i)t}, \ldots, \Delta_{(i)t}}_{\zeta_1 \text{times}}, \underbrace{\Delta_{i(t)}, \ldots, \Delta_{i(t)}}_{\zeta_2 \text{times}}) \end{aligned}$$

By Shorrocks (1980), this forces $I_{it}(\Delta_{it})$:

$$I_{it}(\Delta_{it}) = g^{-1}\left(\frac{1}{\zeta_1}\sum_{\delta_{jt}>0} d(\delta_{jt}) + \frac{1}{\zeta_2}\sum_{\delta_{i\tau}>0} d_\tau(\delta_{i\tau})\right) \tag{3}$$

Without loss of generality, we choose $\zeta_1 = n$ and $\zeta_2 = p$. □

We here add a technical condition, in order for the index to be normalized between zero and one, which we label Averaging. For simplicity of exposition we can start from a reduced form of $I_{it}(\Delta_{it})$, which incorporates the results as in Proposition 1.1:

$$I_{it}(\Delta_{it}) = g^{-1}\left(\underbrace{\frac{1}{n}\sum_{\delta_{jt}>0} d(\delta_{jt})}_{D_O} + \underbrace{\frac{1}{p}\sum_{\delta_{i\tau}>0} d_\tau(\delta_{i\tau})}_{D_H}\right) = g^{-1}(D_O, D_H) \tag{4}$$

Where $D_O = \frac{1}{n}\sum_{\delta_{jt}>0} d(\delta_{jt})$ and $D_H = \frac{1}{p}\sum_{\delta_{i\tau}>0} d_\tau(\delta_{i\tau})$. We define D_O as *other-regarding deprivation* and D_H as *history-regarding deprivation.*

Axiom 1.8 (Averaging)**.** *Averaging is obtained by imposing Traslativity and Homogeneity. For any $F : \mathbb{R}^2_+ \to \mathbb{R}_+$*

- *(Traslativity):*$F(D_O + \eta, D_H + \eta) = F(D_O, D_H) + \eta$

  If both other-regarding deprivation and history-regarding deprivation increase, Translativity states that deprivation must increase by the same amount.

- *(Homogeneity): $F(D_O \cdot \zeta, D_H \cdot \zeta) = F(D_O, D_H) \cdot \zeta$, where $\zeta \neq 0$*

  For instance, if both other-regarding derivation and history-regarding deprivation are doubled, deprivation must be doubled as well.

Proposition 1.2. *An individual inter-temporal relative deprivation index defined as in Proposition 1.1 satisfies Averaging if and only if*

$$I_{it}(\Delta_{it}) = (1-\epsilon)\frac{1}{n}\sum_{\delta_{jt}>0} d(\delta_{jt}) + \epsilon\frac{1}{p}\sum_{\delta_{i\tau}>0} d_\tau(\delta_{i\tau}) \tag{5}$$

where $\epsilon \in [0, 1]$.

Proof. By Aczél, 1966 (Theorem 1, pp.234-235) Traslativity and Homogeneity are satisfied if and only if $F(D_O, D_H) = (1-\epsilon)D_O + \epsilon D_H$, for a generic function F. By choosing $F(D_O, D_H) = g^{-1}(D_O, D_H)$, we get the result. □

Parameter ϵ defines the relative importance of the history-regarding deprivation towards the other-regarding deprivation. For instance, if $\epsilon = 0.5$, the index weights equally the two components; while by setting $\epsilon = 0$ we boil down to the traditional definition of deprivation, where only the other-regarding component is valuable. For example, when $\epsilon = 0$, equation (5) is a generalization of the Chakravarty index, Chakravarty, 1997).

1.3 Further refinements

In this section, we restrict the class of inter-temporal deprivation measures by imposing a second set of axioms. We concentrate on two issues: (i) the index sensitivity to transfers; and (ii) time discounting. Following Paul (1991) (and the subsequent works by Chakravarty and Chattopadhyay (1994), Podder (1996) and Esposito (2010)), we impose that individual deprivation should be sensitive to transfers occurring between members of the reference group. In particular, Paul (1991) believes that an individual feels less envious with respect to an increase in the income of a rich person, than to an increase in income of someone *close* to him in the income distribution, where close means *slightly richer*.

In our inter-temporal framework, an achievement may be *close* to individual i-th under two points of view. The first, traditional, one: an achievement not too different than individual i-th's. The second, new, one: an achievement occurring in a period of time not too far in the past. We are concerned in the sensitivity of the index under both connotations. The first subsection (Sensivity) concerns the traditional idea of proximity, i.e. it regards being similar (or diverse) in terms of achievement levels. The second subsection (Time Discounting), instead, concerns the second idea of nearness, i.e. being recent (or remote) in terms of time.

1.3.1 Sensitivity

Let us first introduce some definitions. An Other-regarding regressing transfer is defined as a transfer from a smaller to a larger comparison with respect to others.
Definition 1.1 (Other-regarding regressing transfer). *For any δ_k, $\delta_j \in \Delta_{(i)t}$, such that $\delta_k \geq \delta_j \geq 0$ and for each $\epsilon \in \mathbb{R}_+$, we say that $\Delta'_{(i)t}$ is obtained from $\Delta_{(i)t}$ by means of other-regarding regressive transfer if $\delta_h = \delta'_h$ for each $h \neq (k, j)$, $\delta'_k = \delta_k + \epsilon$, $\delta'_j = \delta_j - \epsilon$.*

A History-regarding regressing transfer is defined as a transfer from a smaller to a larger comparison with respect to individual $i-$th own past.
Definition 1.2 (History-regarding regressing transfer). *For any δ_k, $\delta_j \in \Delta_{i(t)}$, such that $\delta_k \geq \delta_j \geq 0$ and for any $\epsilon \in \mathbb{R}_+$, we say that $\Delta'_{i(t)}$ is obtained from $\Delta_{i(t)}$ by means of history-regarding regressive transfer if $\delta_h = \delta'_h$ for each $h \neq (k, j)$, $\delta'_k = \delta_k + \epsilon$, $\delta'_j = \delta_j - \epsilon$.*
Axiom 1.9 (Close (Far) Transfer Principle). *Let $\Delta'_{(i)t}$ obtained by $\Delta_{(i)t}$ by means of an other-regarding regressive transfer and $\Delta'_{i(t)}$ obtained by $\Delta_{i(t)}$ by means of a history-regarding regressive transfer. Then: $I_{it}(\Delta_{(i)t}, \Delta_{i(t)}) < (>) I_{it}(\Delta'_{(i)t}, \Delta'_{i(t)})$.*

The Far Transfer Principle states that the increase in individual $i-$th deprivation due to a marginal enlargement of a large comparison overcomes the decrease in individual $i-$th deprivation due to a symmetric marginal decrease of a small comparison. On the other hand, according to Close Transfer Principle the reverse happens: the decrease of a small comparison is strong enough to reduce individual $i-$th deprivation. Note that, Paul (1991) allows only for indexes that satisfy Close Transfer Principle, while we take a more general approach in a twofold way. First of all, we consider also transfers occurring in different past periods. Second and more important, we do not constrain *a priori* the researcher belief in evaluating marginal changes. If we assume Far Transfer Principle, individual i-th feels more deprived if she faces an increase in large comparisons. And these comparisons may be with respect to her own past or to other individuals today. A possible

interpretation of the Far Transfer Principle is when the researcher believes that the wealthiest individual in the society, or the highest achievement someone has received in the past, acts as benchmarks toward which an individual aspires: moving further this threshold increases individual i-th feeling of deprivation. On the other hand (Close Transfer Principle), researchers may think that individual i-th is more affected by changes in achievements closer to hers. On the basis of this reasoning, it lays the idea that each individual compares herself with her alike and therefore that any achievement too much higher than individual i-th achievement today has a minor impact on her deprivation.
Figure 1 shows how, the same regressive transfer of size ϵ (from δ_j to δ_k) implies an overall decreasing effect on deprivation assuming the Close Transfer Principle and an overall increasing effect assuming Far Transfer Principle.

[Figure 1 here]

Proposition 1.3. *An individual deprivation index $I_{it}(\Delta_{it})$ defined as in Proposition 1.2 satisfies Close (Far) Transfer Principle if and only if d and d_τ are strictly concave (convex) on $\mathbb{R}_+$.*

Proof. Close (Far) Transfer Principle is equivalent to assume that $\sum_{\delta_{jt}>0} d(\cdot)$ and $\sum_{\delta_{i\tau}>0} d_\tau(\cdot)$ are Schur-concave (convex), see Kolm (1976), p. 82. By Marshall and Olkin (Marshall and Olkin (1979), theorem C.1.a., p.64) this condition is equivalent to $d(\cdot)$ and $d_\tau(\cdot)$ being strictly concave (convex), for each $\tau = 1, 2, \ldots, p$. □

1.3.2 Time-Discounting

When individual $i-$th compares her current with her past positions, memory plays a major role, analogous to the one played by expectation for future outcomes in the traditional time-discounting analysis. We cannot believe that an individual is affected in the same way by comparing her current position with a recent past or with a remote experience. We believe, instead that the further in the past we look at, the less the individual deprivation is affected.
The following set of axioms is inspired by Ok and Masatlioglu (2007) and al Nowaihi and Dhami (2006) and it concerns the characterization of the history-regarding deprivation. We adapt to our framework what is usually stated by time-discounting analysis on future outcomes.
Recall that $\Delta_{i(t)} = [\delta_{it-1}, \ldots, \delta_{t-\tau}, \ldots, \delta_{t-p}]$ is the vector of length p of comparisons between individual i-th and herself in past times.
Axiom 1.10 (Memory Sensitivity)**.** *For any $\theta, \delta_{i\tau_1},\ \delta_{i\tau_2} \in \Delta_{i(t)}$ and for any $\tau_1 \in \{0, 1, \ldots, p\}$, there exists a $\tau_2 \geq \tau_1$ such that:*

$$I_{it}\left(\Delta_{(i)t}, \underbrace{[\theta, \ldots, \delta_{i\tau_1}, \ldots, \theta, \theta, \theta]}_{\Delta_{i(t)}}\right) \geq I_{it}\left(\Delta_{(i)t}, \underbrace{[\theta, \theta, \theta, \ldots, \delta_{i\tau_2}, \ldots, \theta]}_{\Delta_{i(t)}}\right) \tag{6}$$

This axiom states that there always exists a period of time $(t - \tau_2)$ which is so far in the past that individual $i-$th position in such past period becomes irrelevant for the feeling of deprivation, regardless of the comparison level. In other words, we assume that the flow of time weakens the memories of the past experiences. Note that this axiom is similar to Impatience in al Nowaihi and Dhami (2006) and to Time sensitivity in Ok and Masatlioglu (2007) .

Example 1.1. *Take two scenarios A and B, where* $A = [\Delta_{(i)t}, \Delta^{'}_{i(t)}] = [\Delta_{(i)t}, 10, 0, 0]$*, and* $B = [\Delta_{(i)t}, \Delta_{i(t)}] = [\Delta_{(i)t}, 0, 0, 15]$*. Memory Sensitivity states that deprivation of the latter vector is not necessary higher than deprivation of the former one. In fact, even if in B individual i-th has experienced a higher past comparison than in A (*15 *versus* 10*), it is possible that her memory about* 15 *is weaker because is more remote than her memory about* 10*.*

In other words, Memory Sensitivity states that, regardless of the size of the difference in current and past incomes, it is always possible to find a period which is so far in the past that such difference becomes irrelevant. The next axiom, instead, states that, regardless of how far in the past we look at, it is always possible to find an comparison level which is so high that it is still important in determining the feeling of deprivation.
Axiom 1.11 (Comparison Sensitivity)**.** *For any* θ*,* $\delta_{i\tau_1} \in \Delta_{i(t)}$ *and for any* τ_1*,* $\tau_2 \in \{0, 1, \ldots, p\}$*, such that* $\tau_1 \leq \tau_2$*, there exist* $\delta_{i\tau_2}, \delta^{'}_{i\tau_2} \neq \delta_{i\tau_1}$*, such that:*

$$I_{it}\left(\Delta_{(i)t}, \underbrace{\left[\theta, \theta, \ldots, \delta^{'}_{i\tau_2}, \ldots, \theta\right]}_{\Delta_{i(t)}}\right) \geq \tag{7}$$

$$I_{it}\left(\Delta_{(i)t}, \underbrace{[\theta, \ldots, \delta_{i\tau_1}, \ldots, \theta, \theta]}_{\Delta^{'}_{i(t)}}\right) \geq I_{it}\left(\Delta_{(i)t}, \underbrace{[\theta, \theta, \ldots, \delta_{i\tau_2}, \ldots, \theta]}_{\Delta^{''}_{i(t)}}\right).$$

Even if the past period τ_2 is far enough so that the comparison $\delta_{i\tau_2}$ is negligible with respect to the comparison $\delta_{i\tau_1}$ occurring in a more recent past (as stated in Memory Sensitivity), we can find a comparison level so high ($\delta^{'}_{i\tau_2}$) that memories back to τ_2 are still strong enough to overcome $\delta_{i\tau_1}$.
Example 1.2. *Take three scenarios A, B and C, where* $A = [\Delta_{(i)t}, \Delta_{i(t)}] = [\Delta_{(i)t}, 0, 0, 40]$*,* $B = [\Delta_{(i)t}, \Delta^{'}_{i(t)}] = [\Delta_{(i)t}, 10, 0, 0]$ *and* $C = [\Delta_{(i)t}, \Delta^{''}_{i(t)}] = [\Delta_{(i)t}, 0, 0, 15]$*. Comparison Sensitivity states that deprivation of the latter vector is not necessary higher than deprivation of the second one. In fact, even if in C individual i-th has experienced an higher comparison than in B (*15 *versus* 10*), it is possible that her memory about* 15 *is weaker because she experienced* 15 *in a more remote past than when she experienced* 10*. On the other hand, that past period which is far enough to forget a comparison equal to* 15 *is not far enough to forget about a comparison of* 40*.*

The next axiom states that, if individual i-th is more deprived than individual j-th, then also individual k-th will be more deprived than individual j-th if individual k-th experiences a higher comparison level in a more recent past then individual i-th .
Axiom 1.12 (Time-Comparison Monotonicity)**.** *For any* θ*,* $\delta_{i\tau_1}$*,* $\delta^{'}_{i\tau_2}$*,* $\delta^{''}_{i\tau_3} \in \Delta_{i(t)}$ *and for any* τ_1*,* $\tau_2, \tau_3 \in \{0, 1, \ldots, p\}$*, if* $\tau_2 \leq \tau_3$*, and* $\delta^{'}_{i\tau_2} \geq \delta^{''}_{i\tau_3}$*, then:*

$$\begin{aligned} I_{it}\left(\Delta_{(i)t}, \left[\theta, \theta, \theta, \ldots, \delta^{''}_{i\tau_3}, \ldots, \theta\right]\right) &\geq I_{it}\left(\Delta_{(i)t}, [\theta, \theta, \ldots, \delta_{i\tau_1}, \ldots, \theta, \theta]\right) \\ &\textit{implies} \\ I_{it}\left(\Delta_{(i)t}, \left[\theta, \ldots, \delta^{'}_{i\tau_2}, \ldots, \theta, \theta, \theta\right]\right) &\geq I_{it}\left(\Delta_{(i)t}, [\theta, \theta, \ldots, \delta_{i\tau_1}, \ldots, \theta, \theta]\right). \end{aligned} \tag{8}$$

For sure deprivation stays higher if a higher comparison in a more recent past is recorded.

Axiom 1.13 (Time-Comparison Separability). *For any* $\delta_i, \delta_i^{'}, \delta_i^{''}, \delta_i^{'''} \in \Delta_{i(t)}$ *and for any* $\tau_1 \leq \tau_2 \leq \tau_3 \leq \tau_4$
if

$$\begin{aligned} I_{it}\left(\Delta_{(i)t}, [\theta, \ldots, \delta_{i\tau_1}, \ldots, \theta, \theta, \theta]\right) &= I_{it}\left(\Delta_{(i)t}, \left[\theta, \theta, \ldots, \delta_{i\tau_2}^{'}, \ldots, \theta, \theta\right]\right) \\ I_{it}\left(\Delta_{(i)t}, \left[\theta, \ldots, \delta_{i\tau_1}^{''}, \ldots, \theta, \theta, \theta\right]\right) &= I_{it}\left(\Delta_{(i)t}, \left[\theta, \theta, \ldots, \delta_{i\tau_2}^{'''}, \ldots, \theta, \theta\right]\right) \\ I_{it}\left(\Delta_{(i)t}, [\theta, \theta, \theta \ldots, \delta_{i\tau_3}, \ldots, \theta]\right) &= I_{it}\left(\Delta_{(i)t}, \left[\theta, \theta, \theta, \theta, \ldots, \delta_{i\tau_4}^{'}, \theta\right]\right) \end{aligned} \tag{9}$$

Then

$$I_{it}\left(\Delta_{(i)t}, \left[\theta, \theta, \theta, \ldots, \delta_{i\tau_3}^{''}, \ldots, \theta\right]\right) = I_{it}\left(\Delta_{(i)t}, \left[\theta, \theta, \theta, \theta, \ldots, \delta_{i\tau_4}^{'}, \theta\right]\right).$$

Consider the situation of having one element in the history-regarding reference group: $\delta_{i\tau_1}$, which means that individual i-th experiences a comparison level δ_i at time τ_1.
How much should δ_i increase, in order for deprivation to be unchanged, if the past period when individual $i-$th was better-off is now τ_2 (further in the past than τ_1)? We label the solution of this problem *inflation factor* of moving δ_i from τ_1 to τ_2. If the level of deprivation for having δ_i at τ_1 is the same as having $\delta_i^{'}$ at τ_2, this means that $\delta_i^{'} - \delta_i$ is the *inflation factor* of moving from τ_1 to τ_2. The axiom then states that if, starting from δ_i, the *inflation factor* $(\delta_i^{'} - \delta_i)$ stays constant when moving from τ_1 to τ_2 and from τ_3 to τ_4, and if the *inflation factor* of moving $\delta_i^{''}$ from τ_1 to τ_2 is $(\delta_i^{'''} - \delta_i^{''})$, this *inflation factor* has to remain constant also for a movement from τ_3 to τ_4. Therefore, Time-Comparison Separability states the independence between levels of comparisons and periods of time, and may become clearer looking at figure 2.

[Figure 2 here]

The last axiom is a technical condition that allows for a multiplicative relation between the relevance of comparisons and the relevance of time. Without Path-Independence, Time-Comparison Separability alone would impose an additive relation between the relevance of comparisons and the relevance of time. Path-Independence, instead, allows for a multiplicative structure. For a more detailed discussion, we refer to Ok and Masatlioglu (2007).
Axiom 1.14 (Path-Independence). *For any* $\delta_i, \delta_i^{'}, \delta_i^{''}, \delta_i^{'''} \in \Delta_{i(t)}$ *and for any* $\tau_1 \leq \tau_2 \leq \tau_3$
if

$$\begin{aligned} I_{it}\left(\Delta_{(i)t} [\theta, \ldots, \delta_{i\tau_1}, \ldots, \theta, \theta, \theta]\right) &= I_{it}\left(\Delta_{(i)t} \left[\theta, \theta, \ldots, \delta_{i\tau_2}^{'}, \ldots, \theta, \theta\right]\right) \\ I_{it}\left(\Delta_{(i)t}, \left[\theta, \ldots, \delta_{i\tau_1}^{''}, \ldots, \theta, \theta, \theta\right]\right) &= I_{it}\left(\Delta_{(i)t}, \left[\theta, \theta, \ldots, \delta_{i\tau_2}^{'''}, \ldots, \theta, \theta\right]\right) \\ I_{it}\left(\Delta_{(i)t}, \left[\theta, \theta, \ldots, \delta_{i\tau_2}^{'}, \ldots, \theta, \theta\right]\right) &= I_{it}\left(\Delta_{(i)t}, \left[\theta, \theta, \theta, \ldots, \delta_{i\tau_3}^{'''}, \ldots, \theta\right]\right) \end{aligned} \tag{10}$$

Then

$$I_{it}\left(\Delta_{(i)t}, [\theta, \theta, \ldots, \delta_{i\tau_2}, \ldots, \theta, \theta]\right) = I_{it}\left(\Delta_{(i)t}, \left[\theta, \theta, \theta, \ldots, \delta_{i\tau_3}^{''}, \ldots, \theta\right]\right).$$

Figure 3 gives a geometric intuition of Path-Independence.

[Figure 3 here]

The following Proposition 1.4 provides necessary and sufficient conditions for the deprivation index to satisfy the set of time-discounting axioms.

Proposition 1.4. *An individual inter-temporal relative deprivation index $I_{it}(\Delta_{it})$ defined as in Proposition 1.3 satisfies Memory-Sensitivity, Comparison-Sensitivity, Time-Comparison Monotonicity, Time-Comparison Separability and Path-Independence if and only if there exists a continuous function $\omega : \mathbb{N}^2 \rightarrow \mathbb{R}_+$ such that for each $\Delta_{it} \in \mathcal{D}_{it}$:*

$$I_{it}(\Delta_{it}) = (1-\epsilon)\frac{1}{n}\sum_{\delta_{jt}>0} d(\delta_{jt}) + \epsilon\frac{1}{p}\sum_{\delta_{i\tau}>0} \omega_t(\tau)h(\delta_{i\tau}) \tag{11}$$

where $\omega_t(\cdot)$ is decreasing with $\omega_t(\infty) = 0$ and $\omega_t(\tau) = 1/\omega_\tau(t)$ and the function h mantains the same properties as d_τ in Proposition 1.1 and Proposition 1.3

Proof. See Ok and Masatlioglu (2007), theorem 1. □

Proposition 1.4 allows for separating time and comparisons. Note that the history-regarding deprivation has been decomposed into two factors: $\omega_t(\tau)$ which acts as discount factor and tells how the researchers disvalue the past comparisons; and h which is a function of comparisons.

2 Individual *multi*-inter-temporal relative deprivation

Recent literature about relative deprivation, as already mentioned, adds time into the analysis to consider other-regarding deprivation in different times (Bossert et al. (2011), Bossert et al. (2007)). Even in our new inter-temporal framework, we cannot neglect that some individual relative deprivation at a given point in time is the result of the sense of deprivation she felt over all the previous periods.

In this section, therefore, we build the multi-temporal version of our inter-temporal relative deprivation index: the *multi*-inter-temporal relative deprivation index. Individual i-th multi-inter-temporal relative deprivation M_{iT} will be a function of the inter-temporal relative deprivations that affected individual i-th over the period of times $T = (t, t-1, \ldots, t-\tau, \ldots, t-p)$:

$$M_{iT} = M_{iT}(I_{it}, I_{it-1}, \ldots, I_{it-\tau}, \ldots, I_{it-p}), \tag{12}$$

where $I_{it-\tau}$ is individual i-th inter-temporal relative deprivation at time $t-\tau$, $\tau = 0, 1, \ldots, p$.

Notice that this approach completes the concept of inter-temporal relative deprivation by considering the effect of past other-regarding deprivation on the current deprivation status.

We characterize M_{iT} by means of a traditional set of axioms, that we discuss in details in Appendix 1. In particular, we require the index to satisfy Continuity, Independence, Monotonicity, Time-span Proportionality and Transfer Principle. Independence provides the separable contribution of the inter-temporal deprivations $(I_{it}, I_{it-1}, \ldots, I_{it-\tau}, \ldots, I_{it-p})$ on the overall multi-inter-temporal deprivation index. Time-span Proportionality states that multi-inter-temporal deprivation index does not change by replicating the time span $(t, t-1, t-2, \ldots, t-p)$ several times. Transfer principle

imposes the index to put more weight to higher levels of inter-temporal deprivations.
It is easy to prove that these axioms provide necessary and sufficient conditions to characterize the following class of multi-inter-temporal relative deprivation indices:

$$M_{iT} = \frac{1}{p+1} \sum_{\tau=0}^{p} f_\tau \left(I_{it-\tau} \right) \tag{13}$$

where $f_\tau(\cdot)$ is increasing and convex.

3 Aggregate multi-inter-temporal relative deprivation

The last step of the paper is to build an aggregate measure of deprivation which allows to make comparisons between different societies. The aggregate multi-inter-temporal relative deprivation index over the period T, A_N will be a function of multi-inter-temporal relative deprivations M_{iT} of all $i = 1, 2, \ldots, n$ individuals in population N. We proceed in the usual way by imposing few standard axioms which are formally discussed in Appendix 2. We assume that each individual multi-inter-temporal deprivation M_{iT} is independent (Independence), that individuals' names do not matter (Anonymity), that the aggregate index is an increasing function in individual deprivations (Monotonicity) and finally that the most deprived individuals weigh more in the overall deprivation (Transfer Principle).
It is easy to prove these axioms provide necessary and sufficient conditions to characterize the following class of aggregate multi-inter-temporal relative deprivation indices:

$$A_N = \frac{1}{n} \sum_{i=1}^{n} f(M_{iT}) \tag{14}$$

where $f(\cdot)$ is increasing and convex.

4 An Empirical Test

The empirical test is based on EU-Silc (European Union Statistics on Income and Living Conditions) Longitudinal 2007 Dataset. EU-SILC 2007 longitudinal dataset collects comparable longitudinal micro data on households income and living conditions referred to years 2004, 2005, 2006, 2007, for 25 EU member states (excluding Malta) plus Norway and Iceland. As outcome variable we choose the household equivalent disposable income using the modified OECD scale. We then deflate incomes using the harmonized consumer price indices provided by Eurostat (2010), in order to have real incomes, comparable at time 2005, and we clean the dataset dropping all negative incomes. Given the class of indices as defined as in Equation (14), which we write here in extended form:

$$A_N = \frac{1}{n} \sum_{i=1}^{n} f \left(\underbrace{\frac{1}{p+1} \sum_{\tau=0}^{p} f_\tau \left(I_{it-\tau} \right)}_{M_{iT}} \right), \tag{15}$$

we choose the following functional specifications:

$$I_{it-\tau} = (1-\epsilon)\frac{1}{n}\sum_{x_{jt}>x_{it}}\left(\frac{x_{jt}-x_{it}}{x_{jt}}\right)^{\phi} + \epsilon\frac{1}{p}\sum_{x_{i\tau}>x_{it}}\left(\frac{1}{1+\zeta\tau}\right)^{\frac{\eta}{\zeta}}\left(\frac{x_{i\tau}-x_{it}}{x_{i\tau}}\right)^{\psi} \quad (16)$$

$$M_{iT} = \frac{1}{p+1}\sum_{\tau=0}^{p}\left(\frac{1}{1+\zeta\tau}\right)^{\frac{\eta}{\zeta}} I^{\alpha}_{it-\tau} \quad (17)$$

$$A_N = \frac{1}{n}\sum_{i=1}^{n} M^{\beta}_{iT} \quad (18)$$

Parameter $\epsilon \in [0,1]$ in equation (16) regulates the relative weight given to other-regarding and history-regarding deprivations. If $\epsilon = 0$ index $I_{it-\tau}$ boils down to a traditional other-regarding measure of deprivation and M_{iT} will be an index of social exclusion as defined in Bossert et al. (2007). The higher the value of ϵ, the larger the weight given to history-regarding deprivation in determining the overall deprivation status of some individual. If $\epsilon = 1$ individual deprivation is only history-related. In the empirical application we present results for three possible values of $\epsilon = \{0, \frac{1}{2}, 1\}$.
Parameters $\phi, \psi \in \mathbb{R}_+$ controls for the individual inter-temporal deprivation index to satisfy Close Transfer Principle (if $\phi, \psi \in (0,1)$) or Far Transfer Principle (if $\phi, \psi > 1$). Recall that in the first case we evaluate more the increase of smaller comparison levels while, in the second case, we give more weight to marginal increases in larger comparison levels. We impose $\phi = \psi = \{\frac{1}{2}, 2\}$.
Parameters ζ and η are chosen from Yi et al. (2006) as: $\zeta = 0.0006$, and $\eta = 1$.
Finally, parameters α and β are tasked to regulate the importance of the more deprived past periods or the more deprived individuals respectively, and in the application we set $\alpha = \beta = 2$.
Table 1 shows the values of different functional specification of the aggregate multi-inter-temporal relative deprivation index for a sample of european countries.

Table 1: Aggregate Deprivation $A_{\epsilon,\phi=\psi}$

	$A_{0,\frac{1}{2}}$	$A_{\frac{1}{2},\frac{1}{2}}$	$A_{1,\frac{1}{2}}$	$A_{0,2}$	$A_{\frac{1}{2},2}$	$A_{1,2}$
AT	0.4872	0.0820	0.0112	0.0374	0.0065	0.0008
BE	0.5392	0.0801	0.0088	0.0328	0.0045	0.0004
ES	0.6165	0.1056	0.0141	0.0714	0.0129	0.0016
FR	0.5508	0.0830	0.0099	0.0318	0.0047	0.0007
IT	0.7734	0.1115	0.0110	0.0935	0.0125	0.0009
NO	0.6919	0.0946	0.0093	0.0676	0.0121	0.0017
PT	0.6382	0.0897	0.0089	0.0607	0.0064	0.0004
SE	0.5449	0.0689	0.0051	0.0249	0.0040	0.0006

Source: our elaboration on EU-Silc 2007 Longitudinal Dataset

Table 2 shows instead the rank-position of each country for the different choices of the index' parameters, where 1 stands for the most deprived country and 8 for the least deprived. Sweden and Belgium are are among the least deprived countries according to any choice of parameters. Italy and Spain are, instead, always among the most deprived countries regardless of the specification of the parameters. Nevertheless, some differences appears: while Italy looks less deprived the more weight is given to the history-regarding deprivation ($\epsilon = 1$), Spain becomes more deprived when

Table 2: Aggregate Deprivation $A_{\epsilon,\phi=\psi}$, Ranks (1-most deprived)

	$A_{0,\frac{1}{2}}$	$A_{\frac{1}{2},\frac{1}{2}}$	$A_{1,\frac{1}{2}}$	$A_{0,2}$	$A_{\frac{1}{2},2}$	$A_{1,2}$
AT	8	6	2	5	4	4
BE	7	7	7	6	7	8
ES	4	2	1	2	1	2
FR	5	5	4	7	6	5
IT	1	1	3	1	2	3
NO	2	3	5	3	3	1
PT	3	4	6	4	5	7
SE	6	8	8	8	8	6

Source: our elaboration on EU-Silc 2007 Longitudinal Dataset

also the history-regarding component is taken into account ($\epsilon = \frac{1}{2}, 1$). Norway ends to the top of the deprivation scale if we concentrate on the history-regarding deprivation ($\epsilon = 1$) and we allow the index to satisfy Far Transfer Principle ($A_{1,2}$): some individuals must have experienced a harsh drop in their income. France and Portugal shows the opposite situation, since we see a consistent increase in their deprivation when the index satisfies Close Transfer Principle ($\psi = \phi = \frac{1}{2}$): therefore it seems that the large majority of deprived individuals face small comparisons with their reference group. Moreover, in Portugal, the history-regarding deprivation ($\epsilon = 1$) is very small: we should therefore conclude that the portuguese are not worsening their economic situation over time (either they are growing or they are staying constant).

5 Conclusion

In this paper we provide the axiomatic characterization of a new class of deprivation indices which is based on a more general and complete idea of deprivation: people compare themselves not only with other individuals but also with their own past history. The index is obtained in three steps. First, and this is the true novelty of the paper, we provide an individual inter-temporal relative deprivation measure. Differently from the traditional definition of relative deprivation, the new concept of inter-temporal relative deprivation is the result of comparisons made by some individual not only with respect to an other-regarding reference group, but also to an history-regarding reference group, defined as the individual achievements in previous times. Since memory plays an important role when an individual makes comparisons with her own history, the issue of how to discount past positions arises. In fact, an individual is surely less affected by a remote experience then a more recent one. We therefore choose to evaluate income and time separately, taking advantage of the literature about time-discounting in order to obtain a flexible weighting system. Second, we axiomatically derive the multi-temporal extension of our inter-temporal relative deprivation index, to incorporate in the analysis also the persistence in the status of deprivation. This allows to take into account also other-regarding deprivation at previous point in time. Finally, we aggregate over all the population, in order to obtain an index which allows for comparisons between different societies. The empirical exercise shows how the new index contributes to disentangle the different faces of relative deprivation.

Future research may explore two additional directions. On one hand, the idea of history-regarding deprivation may be extended towards the future, as in the seminal suggestion by Runciman (an individual reference group "*may include himself at some previous or expected time*", (Runciman (1966), p.9). On the other hand, the traditional idea of other-regarding deprivation may be refined by means of weighting system that allows different subgroups to have a different impact on some individual relative deprivation.

References

ACZÉL, J. (1966). *Lectures on functional equations and their applications.* Academic Press, New York.

AL NOWAIHI, A. and DHAMI, S. (2006). A note on the Loewenstein–Prelec theory of intertemporal choice. *Mathematical Social Sciences* **52**, 99–108.

BOSSERT, W., CHAKRAVARTY, S. R. and D'AMBROSIO, C. (2011). Poverty and Time. *Journal of Economic Inequality* ***Forthcoming***.

BOSSERT, W., D'AMBROSIO, C. and PERAGINE, V. (2007). Deprivation and Social Exclusion. *Economica* **74**, 777–803.

CHAKRABORTY, A., PATTANAIK, P. K. and XU, Y. (20008). On the mean of squared deprivation gaps. *Economic Theory* **34**, 181–187.

CHAKRAVARTY, S. R. (1997). Relative deprivation and satisfaction orderings. *Keio Economic Studies* **34**.

CHAKRAVARTY, S. R. (2007). A deprivation-based axiomatic characterization of the absolute Bonferroni index of inequality. *Journal of Economic Inequality* **5**, 339–351.

CHAKRAVARTY, S. R. and CHATTOPADHYAY, N. (1994). An ethical index of relative deprivation. *Research on Economic Inequality* **5**, 231–240.

CHAKRAVARTY, S. R. and D'AMBROSIO, C. (2006). The Measurement of Social Exclusion. *Review of Income and Wealth* **52**, 377–398.

EBERT, U. and MOYES, P. (2000). An axiomatic characterization of Yitzhaki's index of individual deprivation. *Economics Letters* **68**, 263–270.

ESPOSITO, L. (2010). Upper boundedness for the measurement of relative deprivation. *Review of Income and Wealth* **56**, 632–639.

EUROSTAT (2010). *Combating poverty and social exclusion. A statistical portrait of the European Union 2010.* Eurostat Statistical Books. European Union.

FIELDS, G. S. (2007). Income Mobility. *Cornell University ILR Collection Working Papers* .

FISHBURN, P. (1970). *Utility Theory for Decision Making.* Wiley, New York.

KOLM, S.-C. (1976). Unequal Inequalities II. *Journal of Economic Theory* **13**, 82–111.

MARSHALL, A. W. and OLKIN, I. (1979). *Inequalities: theory of majorization and its applications.* Academic Press, San Diego.

MUKERJEE, D. (2001). Measuring multidimensional deprivation. *Mathematical Social Sciences* **42**, 233–251.

OK, E. A. and MASATLIOGLU, Y. (2007). A theory of (relative) discounting. *Journal of Economic Theory* **137**, 214–245.

Paul, S. (1991). An index of relative deprivation. *Economics Letters* **36**, 337–341.

Podder, N. (1996). Relative Deprivation, envy and economic inequality. *Kyklos* **3**, 353–376.

Runciman, W. (1966). *Relative deprivation and social justice.* Routledge, London, UK.

Shorrocks, A. F. (1980). The class of additively decomposable inequality measures. *Econometrica* **48**, 613–625.

Stouffer, S., Suchman, E., De Vinney, L., Star, S. and Williams, R. (1949). *The American Soldier: adjustments during army life 1.* Princeton University Press, Princeton.

Subramanian, S. and Majumdar, M. (2002). On measuring deprivation adjusted for group disparities. *Social Choice and Welfare* **19**, 265–280.

Yi, R., Gatchalian, K. M. and Bickel, W. K. (2006). Discounting of past outcomes. *Experimental and Clinical Psychopharmacology* **14**, 311–317.

Yitzhaki, S. (1979). Relative deprivation and the Gini coefficient. *Quarterly Journal of Economics* **32**, 321–324.

Appendix 1: Characterization of the individual multi-inter-temporal relative deprivation index

Aim of this section is to characterize the individual multi-inter-temporal relative deprivation index: $M_{iT} = M_{iT}\left(I_{it}, I_{it-1}, \ldots, I_{it-\tau}, \ldots, I_{it-p}\right)$. Let $\mathcal{I}^{p+1}$ be the class of all vectors of inter-temporal relative deprivation indices $\mathbf{I}_{iT} = (I_{it}, I_{it-1}, \ldots, I_{it-\tau}, \ldots, I_{it-p})$ of length $p+1$.

Axiom A1. 1 (Continuity)**.** *$M_{iT}(\mathbf{I}_{iT})$ is continuous on $\mathcal{I}^{p+1}$.*

Axiom A1. 2 (Monotonicity)**.** *For any $\mathbf{I}_{iT}, \mathbf{I}_{iT}^{'} \in \mathcal{I}^{p+1}$, if $\mathbf{I}_{iT}^{'}$ is obtained from $\mathbf{I}_{iT}$ by adding $\beta \in \mathbb{R}_+$ to a generic element $I_{it-\tau} \in \mathbf{I}_{iT}$, then $M_{iT}\left(\mathbf{I}_{iT}\right) \leq M_{iT}(\mathbf{I}_{iT}^{'})$.*

Axiom A1. 3 (Independence)**.** *For any $\mathbf{I}_{iT} = (I_{it}, I_{it-1}, \ldots, I_{it-\tau}, \ldots, I_{it-p})$, and for any $\mathbf{I}_{iT}^{'} = (I_{it}^{'}, I_{it-1}^{'}, \ldots, I_{it-\tau}^{'}, \ldots, I_{it-p}^{'})$, $\mathbf{I}_{iT}, \mathbf{I}_{iT}^{'} \in \mathcal{I}^{p+1}$, if*

$$\begin{aligned} M_{iT}(I_{it}, I_{it-1}, \ldots, I_{it-\tau}, \ldots, I_{it-p}) &= M_{iT}(I_{it}^{'}, I_{it-1}^{'}, \ldots, I_{it-\tau}^{'}, \ldots, I_{it-p}^{'}) \\ \textit{and } I_{it-\tau} &= I_{it-\tau}^{'}, \textit{ then} \\ M_{iT}(I_{it}, I_{it-1}, \ldots, \theta, \ldots, I_{it-p}) &= M_{iT}(I_{it}^{'}, I_{it-1}^{'}, \ldots, \theta, \ldots, I_{it-p}^{'}) \\ \textit{for any } \theta \in \mathbb{R}_+. & \end{aligned}$$

Axiom A1. 4 (Time-Span-Proportionality)**.** *For any $\zeta \in \mathbb{N}$: $M_{iT}(\mathbf{I}_{iT}) = M_{iT}(\underbrace{\mathbf{I}_{iT}, \ldots, \mathbf{I}_{iT}}_{\zeta times})$.*

Axiom A1. 5 (Transfer Principle)**.** *For any $\mathbf{I}_{iT}, \mathbf{I}_{iT}^{'} \in \mathcal{I}^{p+1}$, such that $\mathbf{I}_{iT} = (I_{it}, I_{it-1}, \ldots, I_{it-\tau_1}, \ldots, I_{it-\tau_2}, \ldots, I_{it-p})$, $\mathbf{I}_{iT}^{'} = (I_{it}, I_{it-1}, \ldots, I_{it-\tau_1} - \epsilon, \ldots, I_{it-\tau_2} + \epsilon, \ldots, I_{it-p})$, and $I_{it-\tau_1} \geq I_{it-\tau_2}$, then: $M_{iT}(\mathbf{I}_{iT}) \leq M_{iT}(\mathbf{I}_{iT}^{'})$, for any $\epsilon > 0$.*

Proposition 5.1. *An index of individual multi-inter-temporal relative deprivation M_{iT} satisfies Continuity, Monotonicity, Independence, Time-Span-Proportionality and Transfer Principle if and only if it is equal to:*

$$M_{iT} = \frac{1}{p+1} \sum_{\tau=0}^{p} f_\tau\left(I_{it-\tau}\right) \tag{19}$$

where $f_\tau : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ are increasing and convex functions, for any $\tau = 0, 1, \ldots, p$.

Proof. By Continuity, Monotonicity and Independence from Theorem 5.5 in Fishburn (1970), it follows that:

$$M_{iT} = g^{-1}\left(\sum_{\tau=0}^{p} f_\tau\left(I_{it-\tau}\right)\right) \tag{20}$$

where g and f_τ are continuous and strictly increasing for any $\tau = 0, 1, \ldots, p$. By Time-Span-Proportionality:

$$M_{iT}(\mathbf{I}_{iT}) = M_{iT}(\underbrace{\mathbf{I}_{iT}, \ldots, \mathbf{I}_{iT}}_{\zeta times})$$

By Shorrocks (1980) this forces the index to become:

$$M_{iT} = g^{-1}\left(\frac{1}{\zeta}\sum_{\tau=0}^{p} f_\tau\left(I_{it-\tau}\right)\right)$$

without loss of generality, we choose $\zeta = p + 1$ and g^{-1} as the identity function.
By Kolm (1976) and Marshall and Olkin (1979), Tranfer Principle insures that $f_\tau(\cdot)$ are convex. $\square$

Appendix 2: Characterization of the aggregate multi-inter-temporal relative deprivation

Let us define an aggregate multi-inter-temporal relative deprivation measure as a function $A_N = A_N(M_{1T}, M_{2T}, \ldots, M_{iT}, \ldots, M_{nT})$. Let $\mathcal{M}^n$ be the class of all vectors of individual multi-inter-temporal relative deprivation indices $\mathbf{M_N} = (M_{1T}, M_{2T}, \ldots, M_{iT}, \ldots, M_{nT})$ of length n.

Axiom A2. 1 (Continuity)**.** *$A_N(\mathbf{M}_N)$ is continuous on $\mathcal{M}^n$.*

Axiom A2. 2 (Monotonicity)**.** *For any $\mathbf{M}_N$, $\mathbf{M}_N^{'} \in \mathcal{M}^n$, if $\mathbf{M}_N^{'}$ is obtained from $\mathbf{M}_N$ by adding $\beta \in \mathbb{R}_+$ to a generic element $M_{iT} \in \mathbf{M}_N$, then $A_N(\mathbf{M}_N) \leq A_N(\mathbf{M}_N^{'})$.*

Axiom A2. 3 (Independence)**.** *For any $\mathbf{M_N} = (M_{1T}, M_{2T}, \ldots, M_{iT}, \ldots, M_{nT})$, and for any $\mathbf{M_N^{'}} = (M_{1T}^{'}, M_{2T}^{'}, \ldots, M_{iT}^{'}, \ldots, M_{nT}^{'})$, $\mathbf{M_N}, \mathbf{M_N^{'}} \in \mathcal{M}^n$, if*

$$\begin{aligned} A_N(M_{1T}, M_{2T}, \ldots, M_{iT}, \ldots, M_{nT}) &= A_N(M_{1T}^{'}, M_{2T}^{'}, \ldots, M_{iT}^{'}, \ldots, M_{nT}^{'}) \\ \text{and } M_{iT} &= M_{iT}^{'}, \text{ then} \\ A_N(M_{1T}, M_{2T}, \ldots, \theta, \ldots, M_{nT}) &= A_N(M_{1T}^{'}, M_{2T}^{'}, \ldots, \theta, \ldots, M_{nT}^{'}) \\ \text{for any } \theta \in \mathbb{R}_+. \end{aligned}$$

Axiom A2. 4 (Anonimity)**.** *Given any permutation π of N, $A_N(\mathbf{M}_N) = A_N(M_{\pi(1)T}, M_{\pi(2)T}, \ldots, M_{\pi(i)T}, \ldots, M_{\pi(n)T})$.*

Axiom A2. 5 (Population Proportionality)**.** *For any $\zeta \in \mathbb{N}$: $A_N(\mathbf{M}_N) = A_N(\underbrace{\mathbf{M}_N, \ldots, \mathbf{M}_N}_{\zeta times})$.*

Axiom A2. 6 (Transfer Principle)**.** *For any $\mathbf{M}_N, \mathbf{M}_N^{'} \in \mathcal{M}^n$, such that $\mathbf{M}_N = (M_{1T}, M_{2T}, \ldots, M_{iT}, \ldots, M_{jT}, \ldots, M_{nT})$, $\mathbf{M}_N^{'} = (M_{1T}, M_{2T}, \ldots, M_{iT} - \epsilon, \ldots, M_{jT} + \epsilon, \ldots, M_{nT})$, and $M_{iT} \geq M_{jT}$, then: $A_N(\mathbf{M}_N) \leq A_N(\mathbf{M}_N^{'})$, for any $\epsilon > 0$.*

Proposition 5.2. *An index of aggregate multi-inter-temporal relative deprivation A_N satisfies Continuity, Independence, Monotonicity, Anonimity, Population Proportionality and Transfer Principle if and only if it is equal to:*

$$A_N = \frac{1}{n} \sum_{i=1}^{n} f\left(M_{iT}\right) \tag{21}$$

where $f : \mathbb{R}_+ \to \mathbb{R}_+$ is increasing and convex.

Proof. By Continuity, Monotonicity and Independence from Theorem 5.5 in Fishburn (1970), it follows that:

$$A_N = g^{-1}\left(\sum_{i=1}^{n} f_i\left(M_{iT}\right)\right) \tag{22}$$

where g and f_i are continuous and strictly increasing for any $i = 1, \ldots, n$. By Population Proportionality:

$$A_N(\mathbf{M}_N) = A_N(\underbrace{\mathbf{M}_N, \ldots, \mathbf{M}_N}_{\zeta times})$$

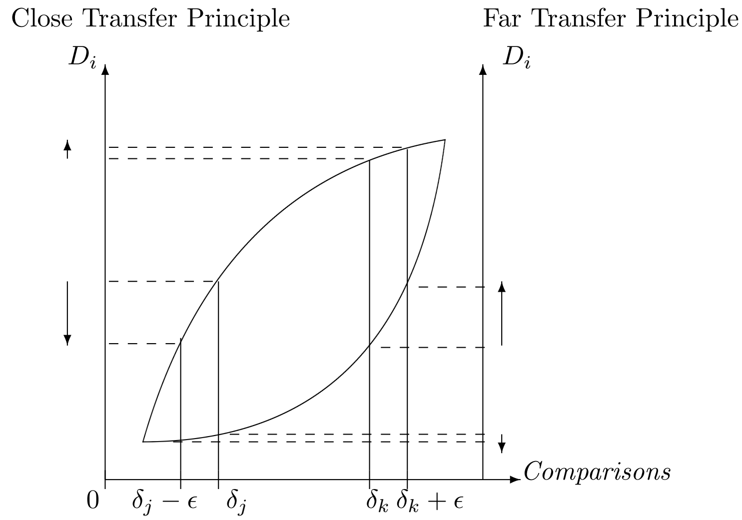
By Shorrocks (1980) this forces the index to become:

$$A_N = g^{-1}\left(\frac{1}{\zeta}\sum_{i=1}^{n} f_i\left(M_{iT}\right)\right)$$

without loss of generality, we choose $\zeta = n$ and g^{-1} as the identity function.
By Anonimity $f_i = f$ for each $i = 1, 2, \ldots, n$.
By Kolm (1976) and Marshall and Olkin (1979), Tranfer Principle insures that $f(\cdot)$ is convex. $\square$

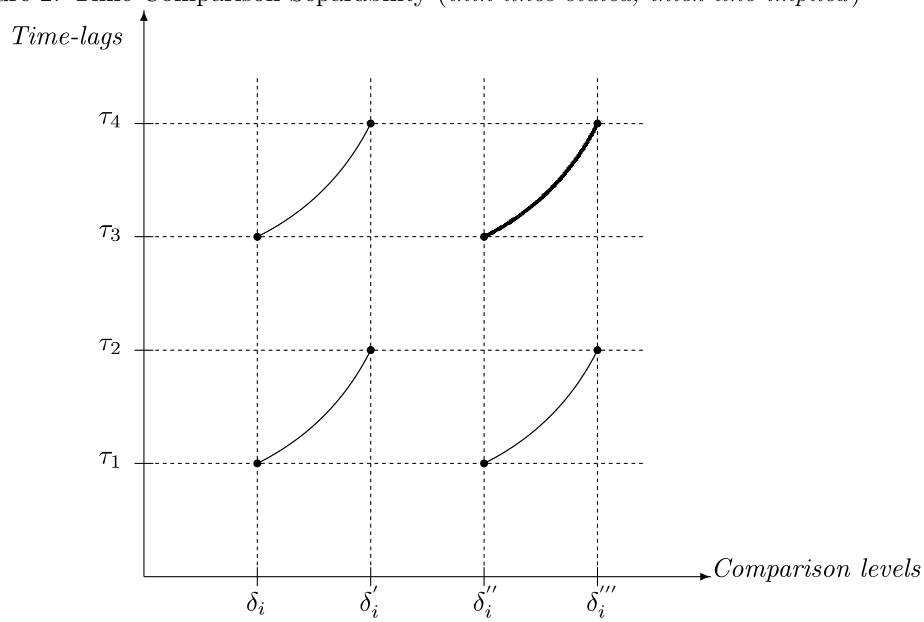
Annex 1: Figures

Figure 1: I_{it} satisfying Close(Far) Transfer Principle



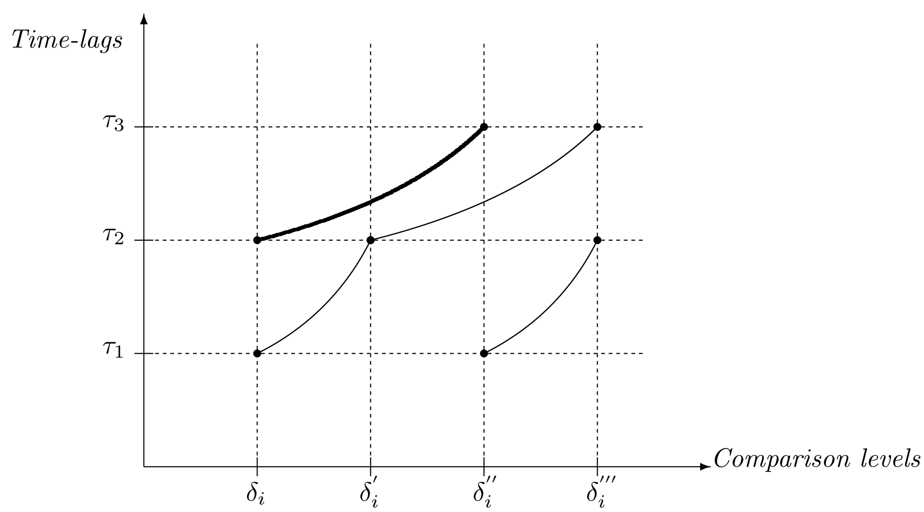
On the left-hand vertical axes we measure individual inter-temporal relative deprivation index satisfying the Close Transfer Principle, while on the right-hand vertical axes we measure individual inter-temporal relative deprivation index satisfying the Far Transfer Principle. The same regressive transfer of size ϵ (from δ_j to δ_k) implies an overall decreasing effect on deprivation assuming the Close Transfer Principle and an overall increasing effect assuming Far Transfer Principle.

Figure 2: Time-Comparison Separability (*thin lines stated, thick line implied*)



On the vertical axes we measure time lags with respect to time t, such that τ_1 *is a more recent past than* τ_2 *and so forth. On the horizontal axes we measure comparison levels. For instance, the point* $(\delta_i,\ \tau_1)$ *corresponds to individual i-th experiencing a comparison level* δ_i *at time* $t - \tau_1$.

Figure 3: Path-Independence (*thin lines stated, thick line implied*)



On the vertical axes we measure time lags with respect to time t, such that τ_1 *is a more recent past than* τ_2 *and so forth. On the horizontal axes we measure comparison levels. For instance, the point* $(\delta_i,\ \tau_1)$ *corresponds to individual i-th experiencing a comparison level* δ_i *at time* $t - \tau_1$.