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On the relationship between objective and subjective inequality indices and the natural rate of subjective inequality*

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Abstract

We establish the conditions under which a close functional relationship between objective and subjective inequality measures can be derived. These conditions are satisfied by many of the most important models for the distribution of income that have been proposed in the literature. We illustrate this result looking at the relationship between the Atkinson indices and the Gini coefficient for the lognormal, the Singh-Maddala, and the second kind beta distributions. While in the first case a positive functional relationship exists regardless of the level of inequality aversion, in the other two cases this relationship is observed when the inequality aversion parameter is smaller and greater than one, respectively. Importantly for the natural rate of subjective inequality (NRSI) hypothesis proposed by Lambert et al. (2003), the proportion of countries with aversion to inequality above the unity in the sample used by these authors is above 50 percent for almost every value of the NRSI considered in the analysis. Consequently, regression analysis aimed to evaluate the validity of this hypothesis could simply have identified the functional relationship between inequality measures, especially when the NRSI is set above 0.1.

Keywords: Natural rate subjective inequality, lognormal distribution, Atkinson index.
JEL Classification: D30, O15.

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1 Introduction

The natural rate of subjective inequality (NRSI) hypothesis introduced by Lambert *et al.* (2003) suggests the existence of a world-wide level of subjective inequality. Under this hypothesis, the level of subjective inequality as measured by the Atkinson index (A) is equal to the NRSI (φ), implying that the cross-country variation in objective inequality measures like the Gini index (G) can be accounted by differences in the country-specific inequality aversion parameter (e). To test this hypothesis, the authors regress both the Gini index and the inequality aversion parameter consistent with $\varphi = 0.1$, on a set of covariates including multiple information on socio-economic factors for 96 countries. The results of the regressions suggest that country-specific attributes that have a positive (negative) impact on inequality aversion have a negative (positive) effect on the Gini coefficient.[3] This result, the authors concluded, provides empirical support for the NRSI hypothesis.

Important for the validity of the NRSI, the results of the regression analysis carried out by Lambert *et al.* (2003) could be merely reflecting a functional relationship between the Gini coefficient and the Atkinson index for a given level of inequality aversion.[4] Although this possibility is acknowledged by the authors, they argue that such a relationship is not clear a priori given that these two indices represent different functions of the underlying income distribution. However, in a note recently published in this journal, Harvey (2005) shows using simulation analysis, that an approximate relationship between the Atkinson index and the Gini indeed exists. More concretely, this author finds that when the Signh-Maddala (1976) distribution function is assumed as a model of the distribution of income, a linear association between

[3]Assuming a value for the NRSI φ, the inequality aversion parameter e can be estimated using the expression of the Atkinson index. In particular, Lambert *et.al* (2003) use the formula for partitioned data given by

$$A(e) = 1 - \left[\frac{1}{k} \sum_j (kq_j)^{1-e} \right]^{1/(1-e)}$$

where k is the number of equal-sized groups and q_j is the share of total income held by group j.

[4]If a functional relationship $A_e = f(G, e)$ exists, then under the NRSI hypothesis, any change in a variable x affecting G or e implies that $\frac{\partial f}{\partial G}\frac{dG}{dx} \equiv \frac{\partial f}{\partial}\frac{de}{dx}$. Consequently, in order to satisfy this identity, factors that have a positive (negative) impact on G must have a negative (positive) impact on e.

the two inequality indices is observed regardless of the level of inequality aversion.[5] Further, a close relationship between A and G is likely to exist when lower values of the NRSI φ are chosen. Given that the regression results in Lambert *et al.* (2003) were obtained assuming a value of $\varphi = 0.1$, the existence of the relationship was communicated to the authors, who replicate the original analysis for higher values of φ. They find that their parameter estimates are not sensible to changes in the choice of φ, which allows them to conclude that the NRSI hypothesis is robust to the approximate relationship between A and G found by Harvey (2005).

In this paper we establish that a close functional relationship between subjective and objective measures of inequality exists under general assumptions on the income distribution. This result includes the Atkinson and the Gini indices on which the NRSI hypothesis is formulated, as well as the generalized entropy (GE) indices introduced by Cowell (1980) and Shorrocks (1980). Thus, Theorem 1 below establishes the conditions under which the functional relationships can be derived. Interestingly, we find that these conditions are satisfied by many of the most important models that have been considered in the literature to describe the distribution of income. More concretely, this group includes distributions with one shape parameter such as the classical Pareto (Arnold, 1983), lognormal (Atkinson and Brown 1957, Sutton, 1997), classical gamma (Salem and Mount, 1974), Weibull (McDonald, 1984), and Fisk (1961) distributions; as well as, non-single shape parameter models like the Singh and Maddala (1976) and the second kind beta distribution, a particular case of the generalized beta distribution of the second kind, which is one of the most important distribution to modeling income and wealth data (McDonald 1984, Sarabia *et al.* 2002, Kleiber and Kotz 2003, Jenkins 2009). In the first case, the functional relationship between the Atkinson index and the Gini coefficient is well-defined for any level of inequality-aversion. For the non-single parameter models, however, the relationship depends on the value of the inequality-aversion parameter. In particular, in the case of the Singh and Maddala (1976) distribution, consistently with the findings reported by Harvey (2005), we find that a close positive relationship between A and G is obtained when low values of e are assumed; for the second kind beta distribution, the functional relationship appears to exist when the inequality-aversion

[5] In this simulation exercise one thousand income vectors with three thousand observations were created. For each vector, they compute G, and A for $e = 0.25, 0.5, 1, 2, 3, 5$ and the limiting case $e \to \infty$. For more details on the exercise see Harvey (2005).

parameter, e, is above 1. Relevant for the NRSI hypothesis, the proportion of countries satisfying this condition in the sample used in Lambert *et al.* (2003) is above 50 percent for almost every value of φ considered in the analysis. This implies that the regression analysis aimed to evaluate the validity of the NRSI could simply have identified the functional relationship between A and G defined in this region of the parameter space, especially when values of φ above 0.1 are chosen.

The rest of the paper is organized as follows. Section 2 presents the main result on the relationship between subjective and objective inequality measures. In Section 3 we use this result to analyze the relationship between the Atkinson and Gini indices and its implications for the NRSI hypothesis. To this purpose, we consider three different parametric models that have been shown to capture the main features of income distributions: the lognormal, the Singh-Maddala, and the second kind beta distributions. Finally, Section 4 briefly concludes the paper.

2 Background and basic result

In this paper, income is taken as a non-negative random variable X, i.e. an income distribution has a cumulative distribution function (cdf) $F(x)$ with support $\mathbb{R}_+ = [0, \infty)$, and F depends on a set of parameters θ. We will write $F(x)$ or $F(x;\theta)$. The α th moment of F is defined as

$$E(X^\alpha) = \int_0^\infty x^\alpha dF(x),$$

where α is any real number, provided previous integral converges.

The Atkinson (1970) family of subjective inequality indices is defined as,

$$A_F(\theta, \epsilon) = 1 - \frac{1}{\mu}\left(\int_0^\infty x^{1-\epsilon} dF(x)\right)^{\frac{1}{1-\epsilon}}, \tag{1}$$

where $\mu = E(X)$ is the first moments of F and $\epsilon > 0$. In the special case $\epsilon = 1$, (1) is the geometric mean of F. The parameter ϵ reflects 'inequality aversion', giving more and more weight to the small incomes as it increases.

The generalized entropy (GE) indices were introduced by Cowell (1980) and Shorrocks (1980) (see also Cowell and Kuga, 1981), and are defined as,

$$GE_F(\theta, \beta) = \frac{1}{\beta(\beta-1)} \int_0^\infty \left[\left(\frac{x}{\mu}\right)^\beta - 1\right] dF(x),$$

where $\beta \in \mathbb{R}\backslash\{0,1\}$. For $\beta = 0, 1$ the generalized entropy coefficients are defined via limiting argument yielding the Theil indices. The parameter β is a sensitivity parameter emphasizing the upper tail for $\beta > 0$ and the lower tail for $\beta < 0$.

The main result on the relationship between subjective and objective inequality measures is given in the following theorem.

Theorem 1 *Let X be a non-negative random variable, which represents an income distribution, with cdf $F(x; \underline{\theta}, \lambda)$, where $\underline{\theta}$ is a vector shape of parameter of dimension k and λ a scale parameter. Let $\mathcal{I} = \mathcal{I}_F(\underline{\theta})$ a scale-invariant inequality index, and assume that $\mathcal{I}$ is continuous and differentiable around a point $(\theta_1^*, \ldots, \theta_k^*)$.*

Assume that exists a parameter θ_i such that $\mathcal{I}$ is a monotone function of θ_i and denote $\theta_i = \mathcal{I}_F^{-1}(\underline{\theta}_{-i})$ the inverse of $\mathcal{I}$ with respect θ_i, where $\underline{\theta}_{-i}$ denotes the vector θ with the ith coordinate deleted.

a) Assume that $E(X)$ is finite and let $A_F(\underline{\theta}, \epsilon)$ the corresponding Atkinson index. Then, we have next functional relationship between $A_F(\underline{\theta}, \epsilon)$ and $\mathcal{I}$,

$$\tilde{A}_F(\mathcal{I}, \underline{\theta}_{-i}, \epsilon) = A_F(\mathcal{I}_F^{-1}(\underline{\theta}_{-i}), \underline{\theta}_{-i}, \epsilon). \quad (2)$$

b) Assume now $E(X^\beta)$ is finite and let $GE_F(\underline{\theta}, \beta)$ the corresponding GE index. Then, we have next functional relationship between the $GE_F(\underline{\theta}, \beta)$ and $\mathcal{I}$,

$$\widetilde{GE}_F(\mathcal{I}, \underline{\theta}_{-i}\beta) = GE_F(\mathcal{I}_F^{-1}(\underline{\theta}_{-i}), \underline{\theta}_{-i}, \beta). \quad (3)$$

Proof: Part a) The Atkinson measures of inequality $A_F(\underline{\theta}, \epsilon)$ exist for any distribution with a finite mean (Dagum, 1979). The index $\mathcal{I}$ is scale-invariant, and only depends on θ; now, because $\mathcal{I}$ is a monotone function of θ_i we can write,

$$\mathcal{I} = \mathcal{I}_F(\underline{\theta}) \iff \theta_i = \mathcal{I}_F^{-1}(\underline{\theta}_{-i}),$$

and substituting in $A_F(\underline{\theta}, \epsilon)$ we obtain (2). Part b) and result (3) is similar, except now we need the existence of $E(X^\beta)$. ∎

Theorem 1 states the conditions under which a functional relationship between subjective inequality measures, such as Atkinson's or entropy indices, and any scale invariant inequality index can be derived. Importantly, many of the most important parametric models that have been proposed in the income distribution literature satisfy the conditions of Theorem 1. This group includes single shape parameter models (case $k = 1$) such as the classical Pareto (Arnold, 1983), lognormal (Aitchison and Brown 1957, Sutton, 1997), classical gamma (Salem and Mount, 1974), Weibull (McDonald, 1984), and Fisk (1961) distributions.[6] Remarkably, however, the conditions of Theorem 1 are also met in the case of non-single shape parameter models where subjective and objective inequality measures may lead to different inequality orderings. Thus, as we show in the next section, a close functional relationship between the Atkinson index and the Gini coefficient can be derived for the Singh and Maddala (1976) model used by Harvey (2005) in the simulation analysis. Furthermore, this result also holds for the second kind beta distribution, a particular case of the generalized beta distribution of the second kind, which is one of the most important distribution to modeling income and wealth data (McDonald 1984, Sarabia *et al.* 2002, Kleiber and Kotz 2003, Jenkins 2009).

3 Subjective and objective inequality measures and the NRSI

In this section, we analyze the relationship between objective and subjective inequality indices implied by Theorem 1 and its implications for the NRSI hypothesis. We derive the functional relationships between inequality measures for three different parametric models that have been shown to capture the main features of income distributions, namely: the lognormal, the second kind beta, and the Singh-Maddala distributions. Given that the NRSI hypothesis has been exclusively discussed in terms of the Atkinson and Gini indices, we focus the discussion on these two inequality measures.

3.1 Lognormal income population

Let X be a lognormal distribution with probability density function (pdf) given by,

$$f(x;\mu,\sigma) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{\log x - \mu}{\sigma}\right)^2\right\}, \quad x > 0,$$

[6]The order equivalence between inequality indices is consistent with the non-crossing Lorenz curves implied by all these parametric models.

where $\mu \in \mathbb{R}$ and $\sigma > 0$. An income distribution with lognormal distribution will be represented by $X \sim \mathcal{LN}(\mu, \sigma^2)$. In this case, the Atkinson's measure of inequality (1) reduces to (see Cowell, 2011),

$$A(\sigma, \epsilon) = 1 - \exp\left(-\epsilon\sigma^2/2\right), \quad \epsilon > 0. \tag{4}$$

where ϵ reflects the aversion to inequality. The Gini index of the lognormal distribution is given by,

$$G = 2\Phi\left(\frac{\sigma}{\sqrt{2}}\right) - 1, \tag{5}$$

where $\Phi(\cdot)$ represents the normal cumulative distribution function (cdf). Therefore, the conditions of Theorem 1 are satisfied and a simple relationship between the Gini index and the Atkinson indices can be easily derived. Thus, if we solve (5) for σ we obtain,

$$\sigma = \sqrt{2}\Phi^{-1}\left(\frac{G+1}{2}\right). \tag{6}$$

Finally, substituting (6) in (4) we obtain the expression,

$$A(G, \epsilon) = 1 - \exp\left\{-\epsilon\left[\Phi^{-1}\left(\frac{G+1}{2}\right)\right]^2\right\}, \tag{7}$$

which relates Gini and Atkinson indices. Figure 1 plots the relationship between these two indices for different values of the aversion parameter. The figure highlights a positive association for these two measures regardless of the level of inequality aversion. The fact that these two indices give rise to the same ordinal ranking of distributions is consistent with the fact that Lorenz curves derived from this model do not intersect. However, it is worth noting that in a lognormal world under the NRSI hypothesis, countries with lower inequality aversion will present larger levels objective inequality as measured by the Gini index. Consequently, the empirical evidence supporting the NRSI hypothesis provided in Lambert *et al.* (2003) could be accounted by the relationship outlined in (7).

3.2 The Singh-Maddala income distribution

Let X be a Singh-Maddala distribution with cdf,

$$F(x; a, q, \sigma) = 1 - \frac{1}{\left[1 + \left(\frac{x}{\sigma}\right)^a\right]^q}, \quad x > 0, \tag{8}$$

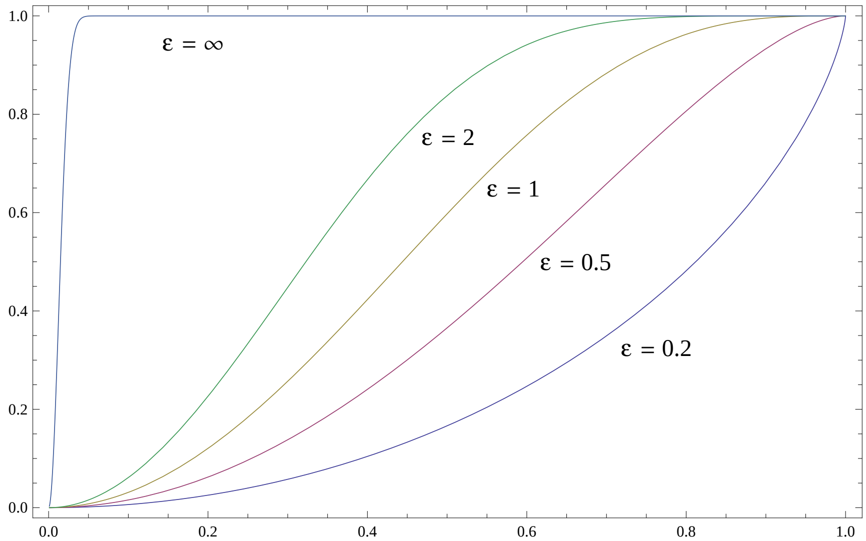

Figure 1: Plot of the relationship between the Gini index and the Atkinson inequality measures for ϵ=0.2; 0.5; 1; 2 and ∞ for the lognormal income distribution.

where σ is a scale parameter and $a, q > 0$ are shape parameters. This distribution was introduced by Singh and Maddala (1976) for modeling income distribution and has received an important attention in the literature of income distributions (see Kleiber and Kotz, 2003, Chapter 6). The moment of order k is,

$$\mu(k; a, q) = E(X^k) = \sigma^k \frac{\Gamma(1 + k/a)\Gamma(q - k/a)}{\Gamma(q)}, \quad -a < k < aq,$$

and the Atkinson inequality index ($q > \max\{1/a, \epsilon/a\}$),

$$A(a, q, \epsilon) = 1 - \frac{\mu(\epsilon, a, q)}{\mu(1, a, q)}. \tag{9}$$

The Gini index corresponding to (8) is,

$$G(a, q) = 1 - \frac{\Gamma(q)\Gamma(2q - 1/a)}{\Gamma(q - 1/a)\Gamma(2q))},$$

which is a decreasing function in a, and in consequence Theorem 1 holds. On the other hand,

$$\frac{\partial A}{\partial G} = \frac{\partial A}{\partial a} : \frac{\partial G}{\partial a}. \tag{10}$$

Using the Atkinson inequality measure formula given by (9) we have,

$$\frac{\partial A(p,q,\epsilon)}{\partial a} = \frac{\Gamma(q-\frac{\epsilon}{a})\Gamma(1+\frac{\epsilon}{a})[\epsilon\psi(1+\frac{\epsilon}{a}) - \psi(1+\frac{1}{a}) + \psi(q-\frac{1}{a}) - \epsilon\psi(q-\frac{\epsilon}{a})]}{a^2\Gamma(1+\frac{1}{a})\Gamma(q-\frac{1}{a})}. \tag{11}$$

Finally, since $\frac{\partial A}{\partial a} < 0$ and using (10) and (11) we have,

$$\frac{\partial A(p,q,\epsilon)}{\partial G} > 0 \iff \epsilon < 1,$$

assuming $q > \max\{1/a, \epsilon/a\}$. Consequently, under certain conditions, a functional relationship between the A and G showing a positive relationship between the two indices can be derived. This result is consistent with the simulation results reported in Harvey (2005). In fact, this author finds that in the case of the Singh-Maddala distribution, there exists a close association between these two indices for values of the aversion parameter lower than unity, with the value of the correlation coefficient being close to one.[7] Importantly for the NRSI hypothesis, the regression analysis in Lambert *et al.* (2003) was carried out assuming a value of φ equal to 0.1, which yields values of ϵ lower than one for more than 90 percent of the countries used in their analysis, which implies that their results could be reflecting the relationship between A and G described above. However, robustness analysis performed by the authors after the finding documented in Harvey (2005), suggest that the regression results do not alter when the number of observations with values of ϵ lower than one is reduced by considering larger values of φ.[8]

[7]Table 1 in Harvey (2005) reports the correlation for values of e equal to $0.25, 0.5, 1, 2, 3, 5$, and for the Rawlsian case. In the first three cases the correlation coefficient is above 0.995, and it significantly reduces for values of the $e \geq 2$.

[8]According to the figures in Table A.2. reported in Lambert *et al.* (2003), the number of observations with values of ϵ larger than one increases with the value of φ. Thus, when φ is set equal to 0.4, only 6 out of 96 countries show an aversion parameter lower than one.

3.3 The second kind beta income distribution

Let X be second kind beta income distribution with pdf,

$$f(x;p,q,\sigma)=\frac{x^{p-1}}{\sigma^p B(p,q)\left(1+\frac{x}{\sigma}\right)^{p+q}},\quad x>0 \tag{12}$$

where $p,q>0$ are shape parameters and $\sigma>0$ is a scale parameter and $B(p,q)=\frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$ denotes the beta function and $\Gamma(x)=\int_0^\infty t^{x-1}\exp(-t)dt$ the gamma function. The second kind beta distribution is a particular case of the generalized beta distribution of the second kind, which is one of the most important distribution to modeling income and wealth data (McDonald, 1984; Slottje, 1987; Kleiber and Kotz, 2003; Jenkins, 2009). The usual moment of order k of (12) is given by,

$$\mu(k;p,q)=E(X^k)=\sigma^k\frac{\Gamma(p+k)\Gamma(q-k)}{\Gamma(p)\Gamma(q)},\quad -p<k<q.$$

and then the Atkinson inequality index is given by ($q>\max\{1,\epsilon\}$),

$$A(p,q,\epsilon)=1-\frac{\mu(\epsilon,p,q)}{\mu(1,p,q)}=1-\frac{\Gamma(p+\epsilon)\Gamma(q-\epsilon)}{\Gamma(p+1)\Gamma(q-1)}. \tag{13}$$

The Gini index of (12) is

$$G(p,q)=\frac{2B(2p,2q-1)}{pB^2(p,q)}.$$

Because G is a decreasing function in p and q (see Kleiber and Kotz, 2003, page 193), then Theorem 1 holds. Consequently, there exits a function $u(\cdot,\cdot)$ such that $p=u(G,q)$ and we can find a functional relationship linking the Atkinson and Gini indices.

On the other hand, taking partial derivatives we have,

$$\frac{\partial A}{\partial G}=\frac{\partial A}{\partial p}:\frac{\partial G}{\partial p}. \tag{14}$$

From (13) we have,

$$\frac{\partial A(p,q,\epsilon)}{\partial p}=\frac{\Gamma(p+\epsilon)\Gamma(q-\epsilon)[\psi(p+1)-\psi(p+\epsilon)]}{\Gamma(p+1)\Gamma(q-1)}, \tag{15}$$

where $\psi(z) = d\Gamma(z)/dz$ is the digamma function. Since $\frac{\partial G}{\partial p} < 0$ and using (15) and (14) we get:

$$\frac{\partial A(p, q, \epsilon)}{\partial G} > 0 \iff \epsilon > 1, \tag{16}$$

assuming $q > \max\{1, \epsilon\}$. Therefore, a positive functional relationship between A and G can be derived. Differently to the case of the Singh-Maddala distribution, however, condition (16) suggests that this relationship holds only for values of the aversion parameter larger than one. As regards the NRSI hypothesis, according to the figures in Table A.2. included in Lambert *et al.* (2003), the number of countries satisfying this condition is significantly high for almost every value of φ considered.[9] In fact, the proportion of countries in the sample with $\epsilon > 1$ is above 50 percent for all the values of φ except when φ is below 0.15, where this rate is less 30 percent. Consequently, if a second kind beta function is assumed as a model of the distribution of income, regression results aimed to check the validity of the NRSI hypothesis could simply have identified the functional relationship between A and G that exists under this model, especially when the value of φ is set above 0.15.

4 Conclusions

The possibility that the results from the regression analysis carried out by Lambert *et al.* (2003) to validate the NRSI hypothesis could be merely reflecting a functional relationship between the Gini and Atkinson indices is acknowledged by the authors. They argue, however, that such a relationship is not clear a priori given that these two indices represent different functions of the underlying income distribution. In this paper we establish that a close functional relationship between subjective and objective measures of inequality exists under general assumptions on the income distribution. Importantly, we find that these conditions are satisfied by many of the most important single and non-single shape parametric models for income distribution that have been considered in the literature.

We illustrate this result looking at the functional relationships between inequality measures for three different parametric models: the lognormal,

[9] This table provides estimates for values of φ equal to $0.1, 0.15, 0.2, 0.25, 0.30, 0.35$, and 0.40.

the Singh-Maddala, and the second kind beta distributions. In the case of single shape parameter models such as the lognormal distribution, the functional relationship between the Atkinson index and the Gini coefficient is well-defined for any level of inequality-aversion. For the Singh and Maddala (1976) distribution, we find that a close positive relationship between I_e and G is obtained for values of the inequality-aversion parameter e, smaller than one; in the case of second kind beta distribution, the functional relationship appears to exist when $e > 1$. According to the figures reported in Lambert *et al.* (2003), the distribution of countries by the aversion parameter varies with the particular choice of the NRSI. Importantly for the NRSI hypothesis, the proportion of countries with $e > 1$ is above 50 percent for most of the values of φ considered in the analysis, being above 90 percent when φ is set equal to 0.4. Consequently, if the second kind beta function is assumed as a model of the distribution of income, empirical analysis carried out to evaluate the validity of the NRSI could simply have identified the functional relationship between A and G defined in this region of the parameter space, especially when φ is set above 0.15.

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