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Contributions of taxes and benefits to vertical, horizontal and redistributive effects*

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Abstract
This paper develops new decompositions of the redistributive, vertical and horizontal effects of the fiscal system, revealing the contributions of different tax and benefit instruments. This new methodology brings together two widely acknowledged approaches in the study of income inequality and redistribution: Kakwani's (1984) decomposition of redistributive effect into vertical and horizontal effects and Lerman and Yitzhaki's (1985) decomposition of marginal changes in income inequality into contributions of different income components. The roles of taxes and benefits in achieving vertical, horizontal and redistributive effects are evaluated on the post- and pre-fiscal income margins, as well as on other margins between the borderline ones. Applications to hypothetical and real-world data indicate the suitability of decompositions for cross-country comparisons of tax-benefit systems.

Keywords: decomposition, inequality, horizontal inequity, redistribution, taxes, benefits
JEL Classification: D63; H22; H23.

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1. Introduction

The emergence of exhaustive micro-databases on household incomes that are consistently comparable across countries[1] has revived an interest in investigating how fiscal systems comprised of direct taxes and cash-based social benefits achieve redistributive effects.[2] The studies have unanimously proven that taxes and benefits reduce income inequality, and there is considerable consensus that benefits have a stronger impact on overall redistributive effect than taxes.

One often neglected issue, however, is the existence of separate vertical and horizontal aspects of the income redistribution process. The former concerns the ability of a fiscal system to satisfy the principle of vertical equity, which demands that income is transferred from higher to lower income groups. The latter concerns horizontal inequity (HI) – the violation of the principle of horizontal equity or the equal treatment of income equals. In a recent study of sixteen European countries, Kristjánsson (2011: 406) showed that the broadly defined tax-benefit systems decrease income inequality by 40 percent on average. Concurrently, HI induced by these systems is equal to one quarter of the overall inequality reduction. The question naturally posed is: what are the roles of different tax and benefit instruments in the creation of HI?

Kakwani (1984) was the first to decompose redistributive effect – the difference between pre- and post-fiscal income inequality – into a vertical effect and a horizontal effect. The vertical effect is a measure of the potential redistributive effect that would be achieved in the absence of HI. The horizontal effect represents the loss of redistributive effect due to HI.[3]

[1] E.g. European Community Household Panel, Luxembourg Income Study, EUROMOD.
[2] see Heady, Mitrakos and Tsakloglou (2001), Immervoll et al. (2005), Mahler and Jesuit (2006), Fuest, Niehues and Peichl (2010), Immervoll and Richardson (2011), Wang and Caminada (2011).
[3] For K84's extensions and empirical applications, see Ankrom (1993), Aronson, Johnson and Lambert (1994), Duclos (2000), Wagstaff et al. (1999), Duclos, Jalbert and Araar (2003), Urban and

How can one determine the roles of different taxes and benefits in achieving vertical, horizontal and redistributive effects? The answer was first proposed by Lambert (1985), who decomposes the vertical effect into the contributions made by taxes and benefits.[4] Several models were proposed to decompose horizontal effect. The Jenkins' (1988) decomposition model determines the horizontal effects of separate taxes in benefits "in isolation", i.e., as if each tax or benefit instrument is applied solely to pre-fiscal income. Therefore, the part of overall HI caused by the interaction of different instruments remains unexplained. The Duclos' (1993) approach hypothetically adds instruments one by one to pre-fiscal income and measures their contributions to the horizontal effect at each step. All possible sequential orderings of the instruments are analysed, each of them indicating a different set of contributions for each instrument, which makes interpretation difficult. Urban's (2010) decomposition of horizontal effect is based on the relative deprivation approach. The contributions of instruments are to some extent arbitrarily determined, and lacking analytical interpretation.

This paper proposes a new measurement model for decomposing vertical, horizontal and redistributive effects into the contributions of different taxes and benefits. The basis of the model is the aforementioned K84 decomposition of RE. The device for the further decomposition of K84 effects is Lerman and Yitzhaki's (1985) (henceforth LY85) decomposition of the Gini coefficient of the overall income into the contributions of the different income components. The contribution of a given income component depends on the relative marginal change in the overall income inequality induced by a small proportional increase in that component, with the other components held constant.[5]

Starting with actual values of pre-fiscal income, taxes and benefits, we increase each component independently by a small factor to obtain new counterfactual post-fiscal incomes. For each tax and benefit, we measure the induced marginal changes in the vertical,

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Lambert (2008) and Kim and Lambert (2009). For other approaches to the measurement of HI, see Lambert and Ramos (1997), Duclos and Lambert (2000), Auerbach and Hassett (2002).

[4] For empirical applications, see Ankrom (1993), Duclos (1993), Duclos (2000), Urban (2008), Urban (2010), Kim and Lambert (2009), Kristjánsson (2011).

[5] For application of LY85 in empirical research, see Lerman and Yitzhaki (1994), Podder (1993), Heady, Mitrakos and Tsakloglou (2001) and Podder, and Chatterjee (2002).

horizontal and redistributive effects, which then constitute the contributions to overall changes in these effects. Naturally, the contributions depend on the margin on which the changes occur, and in the basic setup, the margin is post-fiscal income. Since post-fiscal income is the result of a complex interplay of all taxes and benefits combined, the contribution of a particular tax or benefit instrument may not reflect our expectations. For example, a tax that is judged as "regressive" from the perspective of pre-tax income may nevertheless positively contribute to the redistributive effect if the system as a whole significantly reduces income inequality.

Therefore, we introduce the estimates of contributions on the pre-fiscal margin. Suppose that all taxes and benefits are zero and that post-fiscal incomes are equal to pre-fiscal ones. Taxes, benefits and post-fiscal income are now increased by some small percentage of their actual values, and the resulting contributions to the change in vertical, horizontal and redistributive effects are measured. We find that in this setup our decomposition of RE coincides with Lambert's (1985) decomposition of the vertical effect.

The paper is structured as follows. Section 2 presents an overview of the basic literature: the classical model of inequality decomposition by income sources first proposed by Rao (1969), followed by the above-mentioned LY85 and K84 decompositions. This gives us the introduction to derivations of new decompositions in section 3. In section 4, we present an overview of the existing measurement proposals. Section 5 demonstrates how the new decompositions solve the given problem. First, we use a hypothetical example that involves twelve units and six tax and benefit instruments. Second, the applicability is demonstrated on real-world data, using as a case study the Croatian fiscal subsystem of personal income tax and means- and non-means tested benefits. The final section concludes.

2. Overview of the basic models

2.1. The S-Gini and S-concentration coefficients

The overall income of income unit i, Y_i, is a sum of K source components, $Z_{i,k}$:

$$Y_i = \sum_{k=1}^{K} Z_{i,k} \tag{1}$$

The models for measuring income inequality and redistribution presented in this work are based on the single-parameter or S-Gini coefficient proposed by Donaldson and Weymark (1980), Yitzhaki (1983), and others. For empirical purposes, the S-Gini coefficient for the overall income, G_Y, can be obtained as follows:

$$G_Y = 1 - \frac{1}{\mu_Y} \sum_{i=1}^{s} Y_i^y \cdot \phi_i^y \cdot \hat{\omega}_i^y \tag{2}$$

where μ_Y is the mean overall income and s is the number of income units, sorted in increasing order of overall income, which is indicated by the letter y in the superscript of the vectors involved. Y_i^y is the overall income of unit i and ϕ_i^y is its frequency weight. The estimate of the quantile of unit i is $\hat{p}_i^y = (2S)^{-1} \sum_{j=1}^{i} (\phi_j^y + \phi_{j-1}^y)$, with $\varphi_0^y = 0$, and $S = \sum_{j=1}^{s} \varphi_j^y$. The rank-dependent weights ω_i^y are obtained as $\hat{\omega}_i^y = (S)^{-1} \nu (1 - \hat{p}_i^y)^{\nu - 1}$, where ν is the ethical parameter determining the shape of the $\hat{\omega}$-curve.[6]

Models also employ the counterpart of the S-Gini coefficient: the S-concentration coefficient. What is the difference between them? We have seen that to build the S-Gini coefficient of variable *A*, the units are sorted in ascending order of the same variable. To obtain the S-concentration coefficient of variable B "with respect" to variable A, the units are also sorted in ascending order of variable A. Thus, the S-concentration coefficient of income source k with respect to overall income is obtained by replacing Y_i^y with $Z_{i,k}^y$ and μ_Y by $\mu_{Z(k)}$ in equation (2):

$$D_{Z(k);Y} = 1 - \frac{1}{\mu_{Z(k)}} \sum_{i=1}^{s} Z_{i,k}^y \cdot \phi_i^y \cdot \hat{\omega}_i^y \tag{3}$$

where $Z_{i,k}^y$ represents the incomes of the income component k sorted in ascending order of the overall income and $\mu_{Z(k)}$ is the mean component income.

[6] See Duclos and Araar (2006) for more details.

An important property that always holds between the Gini coefficient of variable A and the S-concentration coefficient of variable B with respect to the variable A is that the former cannot be smaller than the other; see Atkinson (1980). We thus have the following:

$$D_{Z(k);Y} \leq G_Y \tag{4}$$

If we want to calculate the S-Gini coefficient of the *k*-th income source, $G_{Z(k)}$, we must sort income units in increasing order of $Z_{i,k}$ to first obtain the values $\hat{p}_i^{z(k)} = (2S)^{-1}\sum_{j=1}^{i}(\phi_j^{z(k)} + \phi_{j-1}^{z(k)})$ with $\varphi_0^{z(k)} = 0$, and $\hat{\omega}_i^{z(k)} = (S)^{-1}\nu(1-\hat{p}_i^{z(k)})^{\nu-1}$. Then, we proceed as follows:

$$G_{Z(k)} = 1 - \frac{1}{\mu_{Z(k)}}\sum_{i=1}^{s} Z_{i,k}^{z(k)} \cdot \phi_i^{z(k)} \cdot \hat{\omega}_i^{z(k)} \tag{5}$$

2.2. Decompositions of overall income inequality into contributions of income components

The distribution of overall income across the population of income units depends on the distribution of source components as well as their relative share of the overall income. This was first established by Rao (1969; henceforth R69).[7] The S-Gini coefficient of overall income, G_Y, is decomposed as follows:

$$G_Y = \sum_{k=1}^{K} \frac{\mu_{Z(k)}}{\mu_Y} D_{Z(k);Y} \tag{6}$$

The product $(\mu_{Z(k)} / \mu_Y) D_{Z(k);Y}$ is the contribution of the income component k to overall inequality.

Lerman and Yitzhaki (1985; henceforth LY85) have revealed the following property of the S-Gini coefficient of overall income.[8] If the values of the income component k, $Z_{i,k}^{y}$, are increased by a small factor e (say $e = 0.01$) for all income units i, the mean of component k rises to $(1+e)\mu_{Z(k)}$, i.e., by the amount $\dot{\mu}_{Z(k)} = e\mu_{Z(k)}$. Keeping the values of all

[7] See also Kakwani (1977), Pyatt, Chen and Fei (1980) and Shorrocks (1982).
[8] See also Lerman and Yitzhaki (1994), Podder (1993) and Podder and Chatterjee (2002).

other components constant, the resulting change in the S-Gini coefficient of overall income, $\dot{G}_Y^{\partial Z(k)}$, is equal to the following:

$$\dot{G}_Y^{\partial Z(k)} = \frac{\dot{\mu}_{Z(k)}}{\mu_Y}(D_{Z(k);Y} - G_Y) \tag{7}$$

where $\partial Z(k)$ in the superscript of $\dot{G}_Y^{\partial Z(k)}$ indicates that the change is caused by the income component k. By increasing all other income components $k = 1,\dots,K$ separately by the factor e, the other changes in $\dot{G}_Y^{\partial Z(k)}$ are obtained. The sum of these changes is equal to zero:

$$\sum_{k=1}^{K}\dot{G}_Y^{\partial Z(k)} = \sum_{k=1}^{K}\frac{\dot{\mu}_{Z(k)}}{\mu_Y}(D_{Z(k);Y} - G_Y) = 0 \tag{8}$$

Because the factor e is the same for all $\dot{\mu}_{Z(k)}$, we can write:

$$\sum_{k=1}^{K}\frac{\mu_{Z(k)}}{\mu_Y}(D_{Z(k);Y} - G_Y) = 0 \tag{9}$$

The product $(\mu_{Z(k)} / \mu_Y)(D_{Z(k);Y} - G_Y)$ is then the contribution of the *k*-th income component to the overall income inequality on the margin.

2.3. Decompositions of the redistributive effect into vertical and horizontal effects

R69, LY85 and other similar methodologies are not well suited for taking into account all the nuances of the redistributive process. They measure the final or total effect of a given fiscal instrument on income inequality but cannot recognise separate vertical and horizontal effects. Therefore, a set of specifically designated decompositions were derived, to which we now turn.

The post-fiscal income of unit i, N_i, is equal to the pre-fiscal income, X_i, minus all taxes paid, T_i, plus all benefits received, B_i:

$$N_i = X_i - T_i + B_i \tag{10}$$

Following the procedure from section 2.1, we define the S-Gini coefficient of pre-fiscal income as G_X, the S-Gini coefficient of post-fiscal income as G_N, and the S-concentration coefficient of post-fiscal income with respect to pre-fiscal income rankings as $D_{N;X}$.

The redistributive effect (RE) of fiscal system is a change in income inequality while transitioning from pre- to post-fiscal income. It can be measured as the difference between the Gini coefficients of pre-fiscal income and post-fiscal income (G_X and G_N, respectively):

$$\Delta = G_X - G_N \tag{11}$$

However, in calculating RE, some early studies of income redistribution have used post-fiscal incomes sorted in ascending order of pre-fiscal income. In such case, the change of inequality is measured as $\tilde{\Delta} = G_X - D_{N;X}$.

$$\tilde{\Delta} = G_X - D_{N;X} \tag{12}$$

Atkinson (1980) was first to notice that the measure $\tilde{\Delta}$ over-appreciates the magnitude of RE because, as we have seen above, it always holds that $D_{N;X} \leq G_N$. Plotnick (1981) and Kakwani (1984) also reached a similar conclusion. The difference between these two coefficients measures HI induced by changes in income ranks while transitioning from pre- to post-fiscal income distribution. This was henceforth known as the Atkinson-Plotnick-Kakwani index of reranking or RAPK, commonly represented in the following manner:

$$R_{APK} = G_N - D_{N;X} \tag{13}$$

Taking all these elements together, Kakwani (1984) decomposes RE as follows:

$$\Delta = (G_X - D_{N;X}) - (G_N - D_{N;X}) \tag{14}$$

In the first bracket on the right-hand side of (14), we recognise the over-appreciated RE from (12), while in the second bracket, we find RAPK. In Kakwani's (1984) model, the first term represents the progressivity or vertical effect of the fiscal system (henceforth the Kakwani vertical effect or VK):

$$V_K = G_X - D_{N;X} \tag{15}$$

Now we can rewrite K84 as follows:

$$\Delta = V_K - R_{APK} \tag{16}$$

K84 is an attractive analytical tool for several reasons. First, it is based on the most popular inequality index: the Gini coefficient. Second, it measures in a simple way both the vertical equity and horizontal inequity aspects of income redistribution. Third, the empirical results have an apparently simple interpretation for policy purposes.

Namely, VK measures RE of a counterfactual fiscal system in which HI is eliminated. In this system, instead of their actual values, post-fiscal incomes take their expected values conditional on pre-fiscal income; see Urban (2011). As $R_{APK} \geq 0$ by definition, the minus sign in front of it indicates that reranking represents a loss of potential redistribution.

Lerman and Yitzhaki (1995) (henceforth LY95) have derived a new decomposition of RE similar to K84:

$$\Delta = (D_{X;N} - G_N) + (G_X - D_{X;N}) \tag{17}$$

where $D_{X;N}$ is the S-concentration coefficient of pre-fiscal income with respect to post-fiscal income rankings. The first difference on the right-hand side of (17) is Lerman and Yitzhaki's progressivity or "gap narrowing" effect (henceforth, the Lerman-Yitzhaki vertical effect; VLY):

$$V_{LY} = D_{X;N} - G_N \tag{18}$$

The second term is Lerman and Yitzhaki's horizontal or reranking effect (henceforth, the Lerman-Yitzhaki reranking effect; RLY):

$$R_{LY} = G_X - D_{X;N} \tag{19}$$

The LY95 decomposition of RE can now be rewritten as:

$$\Delta = V_{LY} + R_{LY} \tag{20}$$

As $G_X \geq D_{X;N}$, RLY is never negative, indicating that reranking actually positively contributes to RE. This is exactly the opposite of the finding in K84's decomposition in equation (16), where RAPK measures the loss of potential RE.

2.4. Decompositions of vertical and horizontal effects

K84 and LY95 show the contributions of vertical and horizontal effects to RE. But how do different parts of a fiscal system, including taxes and benefits, act to produce these effects; i.e., what are their contributions to these effects? A breakthrough in this area was achieved by Lambert (1985) (henceforth Lt85), who decomposed VK into the contributions of taxes and benefits as follows:

$$V_K = \frac{\tau}{\eta}(D_{T;X} - G_X) + \frac{\beta}{\eta}(G_X - D_{B;X}) \tag{21}$$

where $D_{T;X}$ and $D_{B;X}$ are the S-concentration coefficients of taxes and benefits, respectively, and with respect to pre-fiscal income; τ, β and $\eta = 1 - \tau + \beta$ are the shares of taxes, benefits and post-fiscal income in pre-fiscal income:

$$\tau = \frac{\mu_T}{\mu_X}; \quad \beta = \frac{\mu_B}{\mu_X}; \quad \eta = \frac{\mu_N}{\mu_X} \tag{22}$$

We will later return to Lt85 several times to show its relationship to new decompositions and to draw new interpretations from it.[9]

3. The model

3.1. Changes on the post-fiscal income margin

Recall equation (10), which states that post-fiscal income is the sum of its components: pre-fiscal income, taxes and benefits, where taxes are the negative income component. Imagine that all pre-fiscal incomes, X_i, are increased by a small percentage e, keeping the values of the taxes and benefits unchanged. Following the LY85 approach from section 2.2 and applying equation (7) to the new variables, we obtain the following change in the S-Gini coefficient of post-fiscal income, $\dot{G}_N^{\partial X}$:

$$\dot{G}_N^{\partial X} = \frac{\dot{\mu}_X}{\mu_N}(D_{X;N} - G_N) \tag{23}$$

where $\dot{\mu}_X$ is the change in the mean pre-fiscal income, μ_N is the mean post-fiscal income, and the symbol ∂X in the superscript of $\dot{G}_N^{\partial X}$ denotes that the change was caused by a change in pre-fiscal income.

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[9] Following the principles of Lt85, Urban (2008) derives the analogous decomposition of VLY.

In this paper, we extend the LY85 analysis by asking the following two questions: how does this imaginary proportional increase in all pre-fiscal incomes affect the S-concentration coefficient of post-fiscal income, and how does it affect the S-Gini coefficient of pre-fiscal income? The change in the S-concentration coefficient of post-fiscal income, $\dot{D}_{N;X}^{\partial X}$, is obtained analogously to equation (23), as follows:

$$\dot{D}_{N;X}^{\partial X} = \frac{\dot{\mu}_X}{\mu_N}(G_X - D_{N;X}) \tag{24}$$

The S-Gini coefficient of pre-fiscal income, however, is unchanged in this process because the incomes of all units are increased by the same factor; thus, we can simply write that $\dot{G}_X^{\partial X} = 0$. Now, if all taxes T_i are increased proportionally, keeping pre-fiscal incomes and benefits intact, the S-Gini coefficient of post-fiscal income would change by $\dot{G}_N^{\partial T}$, while in another process, keeping pre-fiscal incomes and taxes at their original levels, the proportional increase of all benefits B_i would change the S-Gini coefficient of post-fiscal income by a magnitude of $\dot{G}_N^{\partial B}$. The two mentioned changes are obtained analogously to equation (23), and we can now write the full set of equations:

$$\begin{aligned}
\dot{G}_N^{\partial X} &= \frac{\dot{\mu}_X}{\mu_N}(D_{X;N} - G_N) \\
\dot{G}_N^{\partial T} &= \frac{-\dot{\mu}_T}{\mu_N}(D_{T;N} - G_N) \\
\dot{G}_N^{\partial B} &= \frac{\dot{\mu}_B}{\mu_N}(D_{B;N} - G_N)
\end{aligned} \tag{25}$$

where $\dot{\mu}_T$ and $\dot{\mu}_B$ are marginal changes in the mean taxes and benefits, respectively, while $D_{T;N}$ and $D_{B;N}$ are, respectively, the S-concentration coefficients of taxes and benefits with respect to post-fiscal income. The minus sign in front of $\dot{\mu}_T$ signifies that taxes are a "negative source of income". Similarly, changes in the S-concentration coefficient of post-fiscal income are derived analogously to equation (24) and are described by:

$$\begin{aligned}
\dot{D}_{N;X}^{\partial X} &= \frac{\dot{\mu}_X}{\mu_N}(G_X - D_{N;X}) \\
\dot{D}_{N;X}^{\partial T} &= \frac{-\dot{\mu}_T}{\mu_N}(D_{T;X} - D_{N;X}) \\
\dot{D}_{N;X}^{\partial B} &= \frac{\dot{\mu}_B}{\mu_N}(D_{B;X} - D_{N;X})
\end{aligned} \tag{26}$$

We have seen above that the S-Gini coefficient of pre-fiscal income is neither affected by proportional changes in pre-fiscal income nor influenced by changes in taxes and benefits; thus, we can write:

$$\dot{G}_X^{\partial X} = \dot{G}_X^{\partial T} = \dot{G}_X^{\partial B} = 0 \tag{27}$$

If we independently increase all income sources by same the percentage e, as shown in equation (8), the sum of the changes will be equal to zero. Thus, for (25) and (26), respectively, we can write:

$$\dot{G}_N^{\partial X} + \dot{G}_N^{\partial T} + \dot{G}_N^{\partial B} = 0 \tag{28}$$

$$\dot{D}_{N;X}^{\partial X} + \dot{D}_{N;X}^{\partial T} + \dot{D}_{N;X}^{\partial B} = 0 \tag{29}$$

In equations (23) to (27), we have defined the marginal changes in three coefficients spanning the K84 components from equation (14): the S-Gini coefficient of pre- and post-fiscal income and the S-concentration coefficient of post-fiscal income. We are now able to obtain the marginal changes in VK, RAPK and RE that occur due to a proportional increase in different income sources.

Let us begin with RAPK. Recall from equation (13) that $R_{APK} = G_N - D_{N;X}$. Subtracting equation (29) from (28) we obtain:

$$\dot{R}_{APK}^{\partial XTB} = \dot{G}_N^{\partial X} - \dot{D}_{N;X}^{\partial X} = (\dot{D}_{N;X}^{\partial T} - \dot{G}_N^{\partial T}) + (\dot{D}_{N;X}^{\partial B} - \dot{G}_N^{\partial B}) \tag{30}$$

The term $\dot{R}_{APK}^{\partial XTB} = \dot{G}_N^{\partial X} - \dot{D}_{N;X}^{\partial X}$ in equation (30) is the change in RAPK due to independent, equal proportional increases in pre-fiscal incomes, taxes and benefits. The term $\dot{D}_{N;X}^{\partial T} - \dot{G}_N^{\partial T}$ is the marginal contribution of taxes, while $\dot{D}_{N;X}^{\partial B} - \dot{G}_N^{\partial B}$ is the contribution of benefits to a marginal change in RAPK.

Recalling equation (22), we write the following equalities:

$$\frac{\dot{\mu}_X}{\mu_N} = \frac{e}{\eta}, \qquad \frac{\dot{\mu}_T}{\mu_N} = \frac{e\tau}{\eta}, \qquad \frac{\dot{\mu}_B}{\mu_N} = \frac{e\beta}{\eta} \tag{31}$$

Using equations (25), (26) and (31), and dividing by e, we can rewrite equation (30), as follows:

$$\begin{aligned} &\frac{1}{\eta}\left[(D_{X;N} - G_N) - (G_X - D_{N;X})\right] = \\ &= \frac{-\tau}{\eta}\left[(D_{T;X} - D_{N;X}) - (D_{T;N} - G_N)\right] + \frac{\beta}{\eta}\left[(D_{B;X} - D_{N;X}) - (D_{B;N} - G_N)\right] \end{aligned} \tag{32}$$

Multiplying by $-\eta$, and after rearrangement, we obtain:

$$\begin{aligned} &(G_X - D_{N;X}) - (D_{X;N} - G_N) = \\ &= \tau\left[(D_{T;X} - D_{T;N}) + (G_N - D_{N;X})\right] + \beta\left[(D_{B;N} - D_{B;X}) - (G_N - D_{N;X})\right] \end{aligned} \tag{33}$$

The right-hand side of equation (33) presents the contributions of taxes and benefits to the marginal change in RAPK, expressed in terms of S-Gini and S-concentration coefficients. Notice that on the left-hand side we do not find the original expression for RAPK from equation (13), but a different equation. Rearranging this term, we obtain:

$$\begin{matrix} (G_X - D_{N;X}) & - & (D_{X;N} - G_N) & = & (G_N - D_{N;X}) & + & (G_X - D_{X;N}) \\ V_K & - & V_{LY} & = & R_{APK} & + & R_{LY} \end{matrix} \tag{34}$$

From equation (34), we find that the sum of the contributions of taxes and benefits to the marginal change in RAPK is equal to the difference between VK and VLY, or the sum of RAPK and RLY (see their definitions in equations (15), (18), (13) and (19), respectively).

The next step is to decompose VK, which is, according to equation (15), equal to $V_K = G_X - D_{N;X}$. Proceeding in the same way as we did for RAPK, we subtract equation (29) from (27), to obtain:

$$\dot{V}_K^{\partial XTB} = -\dot{D}_{N;X}^{\partial X} = \dot{D}_{N;X}^{\partial T} + \dot{D}_{N;X}^{\partial B} \tag{35}$$

Because pre-fiscal inequality remains constant, as shown in equation (27), the change in the vertical effect is explained only by changes in the concentration coefficients of post-fiscal income, where $-\dot{D}_{N;X}^{\partial X}$ represents the overall change and $\dot{D}_{N;X}^{\partial T}$ and $\dot{D}_{N;X}^{\partial B}$, respectively, show the contributions of taxes and benefits. Expanding equation (35) using equations (26) and (31), we obtain:

$$-\frac{1}{\eta}(G_X - D_{N;X}) = \frac{-\tau}{\eta}(D_{T;X} - D_{N;X}) + \frac{\beta}{\eta}(D_{B;X} - D_{N;X}) \tag{36}$$

Multiplying by $-\eta$, we obtain:

$$G_X - D_{N;X} = \tau(D_{T;X} - D_{N;X}) + \beta(D_{N;X} - D_{B;X}) \tag{37}$$

Equation (37) presents the decomposition of VK on the post-fiscal margin. The term on the left-hand side coincides with the original expression for VK from equation (15).

Finally, we can identify the contributions of taxes and benefits to RE, which take into account both the vertical and reranking aspects of each tax or benefit instrument. From equation (11), we know that $\Delta = G_X - G_N$, and therefore, subtracting (28) from (27), we obtain:

$$\dot{\Delta}^{\partial XTB} = -\dot{G}_N^{\partial X} = \dot{G}_N^{\partial T} + \dot{G}_N^{\partial B} \tag{38}$$

As the pre-fiscal income inequality does not change, the decomposition of RE depends only on changes in post-fiscal income inequality, as equation (38) indicates. Proceeding in the same way as above, we derive:

$$D_{X;N} - G_N = \tau(D_{T;N} - G_N) + \beta(G_N - D_{B;N}) \tag{39}$$

where $\tau(D_{T;N} - G_N)$ and $\beta(G_N - D_{B;N})$ are the contributions of taxes and benefits to RE, respectively. We could obtain the same results by subtracting equation (33) from (37), and this would reflect the notion from equation (14), which says that RE is the difference between VK and RAPK. Similarly, as in equation (33), the left-hand side of (39) does not represent the original RE from equation (14), but VLY from equation (18).

Thus, the equations (33), (37) and (39) show the contributions of taxes and benefits to RAPK, VK and RE on the post-fiscal margin. They are expressed in terms of various S-Gini and concentration coefficients and component shares. We must be aware that these are decompositions of “marginal changes” (divided by e), not of “overall indices”. Recalling that $R_{APK} \geq 0$ and $R_{LY} \geq 0$, the following inequality can be derived:

$$\frac{R_{APK} + R_{LY}}{\Delta - R_{LY}} > \frac{R_{APK}}{\Delta} \tag{40}$$

It shows that the share of the marginal change of RAPK, equal to $R_{APK} + R_{LY}$ from equation (34), in the marginal change of RE, obtained from equation (39) as $V_{LY} = \Delta - R_{LY}$, is greater than the share of RAPK in RE obtained with respect to the "overall indices".

3.2. Changes on the pre-fiscal income margin

In the previous section, we analysed changes in inequality as a result of changes in the components of post-fiscal income – pre-fiscal income, taxes and benefits. From an economic point of view, this analysis seems quite natural and has intuitive and practical significance. In this section, we will treat pre-fiscal income as an overall income, whose components are post-fiscal income, taxes and benefits. We rewrite equation (10) as:

$$X_i = N_i + T_i - B_i \tag{41}$$

Thus, we analyse the changes in different coefficients caused by small proportional changes in post-fiscal income, taxes and benefits. The changes in the S-Gini coefficient of pre-fiscal income are obtained as follows:

$$\begin{aligned} \dot{G}_X^N &= \frac{\dot{\mu}_N}{\mu_X}(D_{N;X} - G_X) \\ \dot{G}_X^T &= \frac{\dot{\mu}_T}{\mu_X}(D_{T;X} - G_X) \\ \dot{G}_X^B &= \frac{-\dot{\mu}_B}{\mu_X}(D_{B;X} - G_X) \end{aligned} \tag{42}$$

The S-Gini and S-concentration coefficients of post-fiscal income do not change as a consequence of proportional changes in post-fiscal income, taxes and benefits:

$$\dot{G}_N^N = \dot{G}_N^T = \dot{G}_N^B = 0 \tag{43}$$

$$\dot{D}_{N;X}^N = \dot{D}_{N;X}^T = \dot{D}_{N;X}^B = 0 \tag{44}$$

The sum of changes in equation (42) is equal to zero:

$$\dot{G}_X^N + \dot{G}_X^T + \dot{G}_X^B = 0 \tag{45}$$

From equations (43) and (44), we can easily conclude that the change in RAPK, defined in equation (13) as $R_{APK} = G_N - D_{N;X}$, is zero because neither the Gini nor the concentration coefficient of post-fiscal income changes. What is the change in VK on the pre-fiscal margin? Recalling again from equation (15) that $V_K = G_X - D_{N;X}$, we subtract equation (44) from (45). Because the terms in equation (44) are all zero, we obtain, after rearrangement:

$$\dot{G}_X^N = -\dot{G}_X^T - \dot{G}_X^B \tag{46}$$

In terms of the Gini and concentration coefficients from equation (42), the decomposition in equation (46) is rewritten as:

$$G_X - D_{N;X} = \frac{\tau}{\eta}(D_{T;X} - G_X) + \frac{\beta}{\eta}(G_X - D_{B;X}) \tag{47}$$

Equation (47) is identical to the already familiar Lt85 decomposition of VK presented in equation (21), which decomposes VK on the pre-fiscal margin. Furthermore, it also presents the decomposition of RE on the pre-fiscal margin as RAPK remains unchanged. We can now contrast equation (47) to (37), the decomposition of VK on the pre- and post-fiscal margins. The two equations are quite similar, with one important difference: while in equation (37), the concentration indices of taxes and benefits, $D_{T;X}$ and $D_{B;X}$, are compared to $D_{N;X}$, in equation (47), they are compared to G_X. Therefore, the two decompositions may obtain quite different estimates of the contributions as G_X ($D_{N;X}$) reflects the levels and rankings of incomes before (after) the fiscal process takes place.

4. Comparison with other approaches

4.1. Silber's decomposition of income inequality

Silber (1993) and Araar (2006) decompose net income inequality into two separate groups of contributions from different income sources. Applied to post-fiscal income as obtained in equation (10), the decomposition can be rewritten as follows:

$$\begin{aligned} G_N = &\left[\frac{1}{\eta}G_X + \frac{1}{\eta}(D_{X;N} - G_X)\right] - \left[\frac{\tau}{\eta}G_T + \frac{\tau}{\eta}(D_{T;N} - G_T)\right] + \\ &+ \left[\frac{\beta}{\eta}G_B + \frac{\beta}{\eta}(D_{B;N} - G_B)\right] \end{aligned} \tag{48}$$

The terms $(1 / \eta)G_X$, $(\tau / \eta)G_T$ and $(\beta / \eta)G_B$ are inequality components, while $(1 / \eta)(D_{X;N} - G_X)$, $(\tau / \eta)(D_{T;N} - G_T)$ and $(\beta / \eta)(D_{B;N} - G_B)$ are reranking or, in Silber's terminology, "permutation" components. After rearranging equation (48) in the spirit of decomposing RE, we obtain the following version of the formula:

$$\begin{aligned} G_X - G_N &= \left[\tau(G_T - G_N) + \beta(G_N - G_B)\right] + \\ &+\left[(G_X - D_{X;N}) + \tau(D_{T;N} - G_T) + \beta(G_B - D_{B;N})\right] \end{aligned} \tag{49}$$

On the right-hand side of equation (49), the terms in the first bracket might measure the contributions of instruments to RE, while the terms in the second bracket could be the contributions to reranking. Indeed, we recognise the first term, $G_X - D_{X;N}$, as RLY. The obvious problem concerning the reranking of terms is that the contribution of taxes will always be either zero or negative because it always holds that $D_{T;N} \leq G_T$. Therefore, the decomposition equation (49) is not a good candidate for decomposing RE.

If we sum up the corresponding contributions of taxes and benefits from the first and second bracket, we obtain the following equation:

$$G_X - G_N = \left[\tau(D_{T;N} - G_N) + \beta(G_N - D_{B;N})\right] + (G_X - D_{X;N}) \tag{50}$$

Equation (50) represents the LY decomposition of RE where in the first bracket, we find the decomposition of VLY as in equation (39), while the second term is simply RLY.

If we start from the notion that pre-fiscal income is the sum of its components ($X_i = N_i + T_i - B_i$), equation (48) is written as:

$$G_X = \left[\eta G_N + \eta(D_{N;X} - G_N)\right] + \left[\tau G_T + \tau(D_{T;X} - G_T)\right] - \left[\beta G_B + \beta(D_{B;X} - G_B)\right] \tag{51}$$

After rearrangement, we obtain the following decomposition of RE:

$$\begin{aligned} G_X - G_N &= \left[\frac{\tau}{\eta}(G_T - G_X) + \frac{\beta}{\eta}(G_X - G_B)\right] - \\ &-\left[(G_N - D_{N;X}) + \frac{\tau}{\eta}(G_T - D_{T;X}) + \frac{\beta}{\eta}(D_{B;X} - G_B)\right] \end{aligned} \tag{52}$$

The reranking terms are now subtracted from the inequality terms, as was performed in K84, and we notice the appearance of RAPK in the second bracket. However, a problem arises with the contribution of benefits, which should always be zero or negative as $D_{B;X} \leq G_B$.

Rearranging equation (52) in the same way as we did equation (49), we obtain:

$$G_X - G_N = \left[\frac{\tau}{\eta}(D_{T;X} - G_X) + \frac{\beta}{\eta}(G_X - D_{B;X}) \right] - (G_N - D_{N;X}) \tag{53}$$

This is the K84 decomposition of RE, where the VK is further decomposed into the contributions of taxes and benefits, as shown by equations (21) and (47).

4.2. Jenkins' decomposition of the redistributive effect

Jenkins (1988) decomposed RE with the aim of showing the contributions of taxes and benefits to both vertical and reranking effects. The following equation replicates his formula 10, extending some terms:

$$\begin{aligned} G_X - G_N &= \left[\frac{1-\tau}{\eta}(G_X - G_{X-T}) + \frac{1+\beta}{\eta}(G_X - G_{X+B}) \right] + \\ &+ (D_{N;X} - G_N) - \frac{1-\tau}{\eta}(D_{X-T;X} - G_{X-T}) - \frac{1+\beta}{\eta}(D_{X+B;X} - G_{X+B}) \end{aligned} \tag{54}$$

Before analysing the equation, we slightly rearrange it, to obtain:

$$\begin{aligned} G_X - G_N &= \left[\frac{1-\tau}{\eta}(G_X - G_{X-T}) + \frac{1+\beta}{\eta}(G_X - G_{X+B}) \right] - \\ &- (G_N - D_{N;X}) + \frac{1-\tau}{\eta}(G_{X-T} - D_{X-T;X}) + \frac{1+\beta}{\eta}(G_{X+B} - D_{X+B;X}) \end{aligned} \tag{55}$$

The last three terms on the right-hand side of equation (55) represent reranking effects. The first term is equal to R_{APK} and has a minus sign as in the K84 model. The other two terms are both positive and "added back". They measure the contributions of taxes and benefits to reranking in isolation, i.e., as if each instrument is separately applied to prefiscal income. However, the final reranking effect is achieved through the interaction of all instruments, an aspect that remains unexplained by equation (55).

Summing up the corresponding tax and benefit terms in equation (55), we obtain:

$$G_X - G_N = \left[\frac{1-\tau}{\eta}(G_X - D_{X-T;X}) + \frac{1+\beta}{\eta}(G_X - D_{X+B;X}) \right] - (G_N - D_{N;X}) \tag{56}$$

In equation (56), the terms in the brackets are Kakwani's (1977) vertical effects of taxes and benefits, equal to:

$$\begin{aligned} G_X - D_{X-T;X} &= \tau \,/\, (1-\tau)(D_{T;X} - G_X) \\ G_X - D_{X+B;X} &= \beta \,/\, (1+\beta)(G_X - D_{B;X}) \end{aligned} \tag{57}$$

Substituting these terms into equation (56), we again obtain equation (53).

5. Application

5.1. Hypothetical examples

In the following two examples, we analyse hypothetical populations of twelve units each, paying three kinds of taxes (T1, T2, T3) and receiving three sorts of benefits (B1, B2, B3). In the first setup (Tables 1 and 2), we create taxes and benefits such that no reranking is present to fully concentrate on relative contributions to the vertical effect. In the second setup (Tables 3 and 4), we introduce reranking into the system and analyse contributions to redistributive, reranking and vertical effects.

Table 1 contains the values of pre-fiscal income, all taxes and benefits, and post-fiscal income for the first setup. The values of T1 (T2, T3) rise proportionally (over-, under-proportionately) with pre-fiscal income. B1 is equal for all units. The amounts of B2 fall along with pre-fiscal income and are equal to zero for units with the highest pre-fiscal income. B3 is proportional to pre-fiscal income.

Table 1. Hypothetical population 1 (no-reranking setup)

unit	X	T1	T2	T3	B1	B2	B3	N
1	10	3.0	0.4	2	30	36	2.50	73.10
2	15	4.5	1.2	3	30	35	3.75	75.05
3	20	6.0	2.4	4	30	34	5.00	76.60
4	25	7.5	4.0	5	30	33	6.25	77.75
5	30	9.0	6.0	6	30	32	7.50	78.50
6	40	12.0	9.6	7	30	31	10.00	82.40
7	50	15.0	14.0	8	30	30	12.50	85.50
8	80	24.0	25.6	9	30	20	20.00	91.40
9	120	36.0	43.4	10	30	10	30.00	100.60
10	180	54.0	72.0	11	30	0	45.00	118.00
11	250	75.0	110.0	12	30	0	62.50	145.50
12	380	114.0	182.4	13	30	0	95.00	195.60

Because the post-fiscal ranks are equal to the pre-fiscal ranks, the concentration coefficients of taxes and benefits with respect to post-fiscal income are equal to those obtained with respect to pre-fiscal income (Table 2; rows 2 and 3). RAPK is equal to zero, while VK is equal to RE=0.3828. Thus, taxes and benefits decrease income inequality by almost 70%, indicating that a large degree of redistribution is taking place.

Table 2. Results for hypothetical population 1 (no-reranking setup)

	total	T1	T2	T3	B1	B2	B3	X	N
The share in PFI		0.3000	0.3925	0.0750	0.3000	0.2175	0.2500	1.0000	1.0000
Gini / concentr. coeff. (X)		0.5514	0.6597	0.2648	0.0000	-0.3515	0.5514	0.5514	0.1686
Gini / concentr. coeff. (N)		0.5514	0.6597	0.2648	0.0000	-0.3515	0.5514	0.5514	0.1686
RE	0.3828								
VK	0.3828								
RAPK	0.0000								
VK (37)	0.3828	0.1148	0.1928	0.0072	0.0506	0.1131	-0.0957		
%		30.0	50.4	1.9	13.2	29.5	-25.0		
VK or RE (47)	0.3828	0.0000	0.0425	-0.0215	0.1654	0.1964	0.0000		
%		0.0	11.1	-5.6	43.2	51.3	0.0		
Decomposition (6)	0.1686	0.1654	0.2589	0.0199	0.0000	0.0765	-0.1378	0.5514	
%		43.2	67.6	5.2	0.0	20.0	-36.0		

The contributions of taxes and benefits to VK are obtained on post- and pre-fiscal margins by equations (37) and (47), respectively. The first observation is that the two approaches reflect large discrepancies. In general, equation (37) shows a larger relative contribution from taxes, while the opposite is true for equation (47). On the one hand, according to equation (47), the contributions of T1 and B3 are zero because both of these instruments are proportional to the pre-fiscal income. On the other hand, equation (37) says that T1 significantly contributes to the vertical effect (30%), while B3 has negative contribution (–25%).

To understand the differences, we must remember that the overall system reduces inequality: $D_{N;X} = G_N < G_X$. In such a context, equation (37) tends to ascribe a larger influence to taxes and a smaller influence to benefits than equation (47) because $D_{T;X} - D_{N;X} > D_{T;X} - G_X$ and $D_{N;X} - D_{B;X} < G_X - D_{B;X}$. For T1 and B3, we have that $D_{T;X} = D_{B;X} = G_X > D_{N;X}$. On the post-fiscal margin, the contribution of T1 is positive as

$D_{T;X} > D_{N;X}$, while the contribution of B3 is negative because $D_{N;X} - D_{B;X} < 0$. On the pre-fiscal margin, both instruments have contributions equal to zero as $D_{T;X} - G_X = G_X - D_{B;X} = 0$.

The classical decomposition in equation (6) values the contributions of benefits even less than does equation (37), and only B2 reduces inequality as it is the only instrument whose $D_{B;X}$ is negative. The negative contribution of B3 is larger than in equation (37) (–36%), while B1 is ascribed a zero contribution. The last result is well recognised as a failure of equation (6) to appropriately indicate the role of benefits such as B1, namely, equal transfers to everyone should be interpreted as decreasing inequality.

The second system is presented in Table 3. It is a replica of the first setup except for several changes in T1 and B1 that introduce reranking. In T1, we have slightly reshuffled the amounts paid by different people. For example, units 11 and 12 paid \$75 and \$114 of T1, respectively, in the first setup, while they pay \$114 and \$75, respectively, in the second step. In B1, the benefit is increased to \$50 for some units, but it is decreased to \$10 for several others. For both T1 and B1, the mean values remain the same. The consequence of these changes is a significant reranking in the transition from pre- to post-fiscal income. For example, observe unit 7 with a post-fiscal income of \$44.50. From a pre-fiscal rank of 7, it falls to a post-fiscal rank of 1, i.e., the poorest post-fiscal income unit.

Table 3. Hypothetical population 2 (reranking setup)

p	X	T1	T2	T3	B1	B2	B3	N
0.042	10	3.0	0.4	2	30	36	2.50	73.10
0.125	15	4.5	1.2	3	50	35	3.75	95.05
0.208	20	12.0	2.4	4	10	34	5.00	50.60
0.292	25	7.5	4.0	5	30	33	6.25	77.75
0.375	30	9.0	6.0	6	30	32	7.50	78.50
0.458	40	6.0	9.6	7	50	31	10.00	108.40
0.542	50	36.0	14.0	8	10	30	12.50	44.50
0.625	80	24.0	25.6	9	30	20	20.00	91.40
0.708	120	15.0	43.4	10	50	10	30.00	141.60
0.792	180	54.0	72.0	11	10	0	45.00	98.00
0.875	250	114.0	110.0	12	50	0	62.50	126.50
0.958	380	75.0	182.4	13	10	0	95.00	214.60

As Table 4 shows, VK is roughly the same as in the first setup (0.3802), but RE has significantly fallen (0.3237), the difference between them being equal to RAPK (0.0565). The contributions to VK according to equation (37) have not changed significantly for instruments other than T1 and B1. However, while the contribution of T1 to VK is now lower (26.4%), it has increased for B1 (16.4%).

Before analysing the setup that includes the reranking, we discuss the contributions to RE as generated by equations (39) and (47). According to equation (39), the impact of T1 (10.5%) and B1 (12.5%) is smaller than their impact in VK, which agrees with the prediction made by the K84 decomposition method that reranking diminishes RE. T2 (56.1%) and B2 (47.3%), in contrast, increase in influence. The contribution of B3 (−25%) remains unchanged, while the role of T3 stays low but becomes negative (−1.4%).

The results for equation (47) tell a somewhat different story. Comparing the concentration coefficients of the taxes and benefits in Tables 2 and 4 reveals that only for T1 and B1 is there a difference: they are both lower. However, due to the way the contributions are obtained, this implies that the contribution of T1 falls (−3.6%) while the contribution of B1 (46.4%) increases.

Table 4. Results for hypothetical population 2 (reranking setup)

	total	T1	T2	T3	B1	B2	B3	X	N
The share in PFI		0.3000	0.3925	0.0750	0.3000	0.2175	0.2500	1.0000	1.0000
Gini / concentr. coeff. (X)		0.5056	0.6597	0.2648	-0.0370	-0.3515	0.5514	0.5514	0.1712
Gini / concentr. coeff. (N)		0.3104	0.5644	0.1833	0.1296	-0.2838	0.4632	0.4632	0.2277
RE	0.3237								
VK	0.3802								
RAPK	0.0565								
VK (37)	0.3802	0.1003	0.1917	0.0070	0.0625	0.1137	-0.0950		
%		26.4	50.4	1.8	16.4	29.9	-25.0		
RAPK (33)	0.1447	0.0755	0.0596	0.0104	0.0330	0.0024	-0.0362		
%		52.2	41.2	7.2	22.8	1.7	-25.0		
RE (39)	0.2355	0.0248	0.1322	-0.0033	0.0294	0.1113	-0.0589		
%		10.5	56.1	-1.4	12.5	47.3	-25.0		
RE or VK (47)	0.3802	-0.0138	0.0425	-0.0215	0.1765	0.1964	0.0000		
%		-3.6	11.2	-5.7	46.4	51.7	0.0		
Decomposition (6)	0.1686	0.1654	0.2589	0.0199	0.0000	0.0765	-0.1378	0.5514	
%		43.2	67.6	5.2	0.0	20.0	-36.0		

Explaining the contributions to RAPK is more difficult because the instruments that do not induce reranking per se, such as T2, have quite a large share, while the contribution of B3 is negative, signifying that the instrument in fact reduces the marginal change of reranking. Again, the contribution of each instrument depends on the interaction between all instruments in producing the post-fiscal income. The decomposition equation (33) tells us that the share of a tax (benefit) depends on the difference $D_{T;X} - D_{T;N}$ ($D_{B;N} - D_{B;X}$) and the total amount of reranking, $G_N - D_{N;X}$. Here, $D_{T;N}$ and $D_{B;N}$ depend on post-fiscal rankings, and the final results may seem counterintuitive.

5.2. The "evolution" of contributions to VK and RAPK

Thus far, we have estimated the contributions to vertical, redistributive and horizontal effects on two margins, post- and pre-fiscal. The roles of different taxes and benefits in achieving RE and VK in our hypothetical case were significantly different depending on the reference point chosen. We have seen in section 4.2 that the contributions to a marginal change in RAPK can be estimated only at the post-fiscal margin.

In this section, we present the results of the exercise where we estimate the contributions of taxes and benefits to VK and RAPK on many different margins situated between the two borderline margins. Keeping pre-fiscal incomes at their actual values throughout the procedure and starting with the actual values of the taxes and benefits, we decrease these by the same proportion in cycles, each time recalculating post-fiscal income and the corresponding values of VK and RAPK. By the end of the procedure, the taxes and benefits of all income units are zero, while post-fiscal incomes are equal to pre-fiscal incomes.

In the beginning (the zero-th cycle), we have that $T_{i,0} = T_i$, and $B_{i,0} = B_i$. In each of the following cycles, we decrease the values of the taxes and benefits by a certain percentage, namely by 2%. The tax of unit i in cycle s is equal to $T_{i,s} = (1 - s / 50)T_i$, and the benefit is equal to $B_{i,s} = (1 - s / 50)B_i$. The post-fiscal income in cycle s is equal to $N_{i,s} = X_i - T_{i,s} + B_{i,s}$. In each cycle, we decompose the changes of VK and RAPK into the contributions of taxes and benefits as follows:

$$\dot{V}_{K,s} = (V_{K,s-1} - V_{K,s}^{T}) + (V_{K,s-1} - V_{K,s}^{B}) \quad (58)$$

$$\dot{R}_{APK,s} = (\dot{R}_{APK,s-1} - \dot{R}_{APK,s}^{T}) + (\dot{R}_{APK,s-1} - \dot{R}_{APK,s}^{B}) \quad (59)$$

$$\dot{\Delta}_{s} = (\dot{\Delta}_{s-1} - \dot{\Delta}_{s}^{T}) + (\dot{\Delta}_{s-1} - \dot{\Delta}_{s}^{B}) \quad (60)$$

where $V_{K,s-1}$ ($\dot{R}_{APK,s-1}$, $\dot{\Delta}_{s-1}$), $V_{K,s}^{T}$ ($\dot{R}_{APK,s}^{T}$, $\dot{\Delta}_{s}^{T}$) and $V_{K,s}^{B}$ ($\dot{R}_{APK,s}^{B}$, $\dot{\Delta}_{s}^{B}$) are the values of VK (RAPK, RE) obtained for $N_{i,s-1}$, $N_{i,s}^{T} = X_i - T_{i,s} + B_{i,s-1}$ and $N_{i,s}^{B} = X_i - T_{i,s-1} + B_{i,s}$, respectively.

Figure 1. Contributions to marginal changes in RAPK

a) Contributions *b) Percentage contributions*

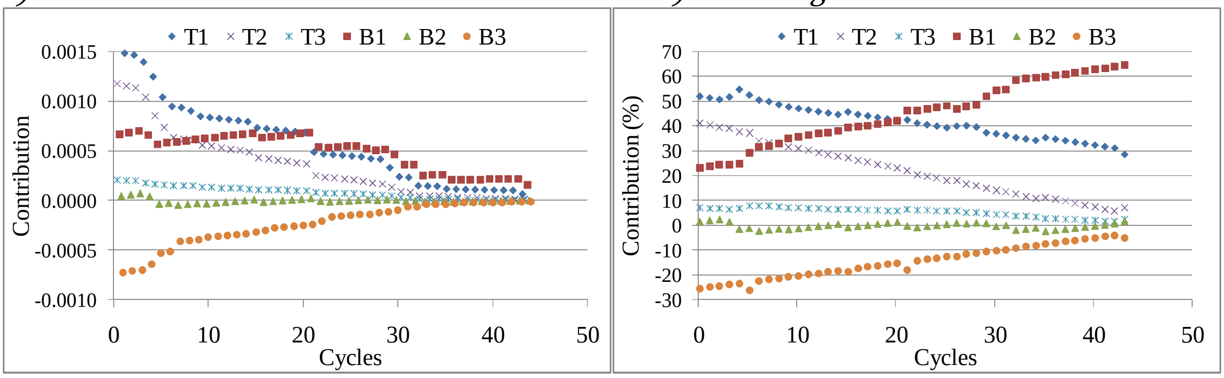


Figure 1 plots the contributions of taxes and benefits to marginal changes in RAPK. In panel (a), we see the values obtained by equation (59), and in panel (b), they are expressed as percentages. The total sum of the contributions of all instruments in all cycles is equal to RAPK. If we move in the opposite direction, from the right to the left side of panel (a), we observe the evolution of RAPK in the hypothetical scenario where the values of the taxes and benefits increase from zero to their actual values in proportional steps. RAPK is zero in the beginning, and the contributions slightly increase to their final values on the post-fiscal margin. The percentage contributions on the post-fiscal margin are shown in Table 4. However, on the pre-fiscal margin, we see a different picture; the contributions of T2, T3, B2 and B3 all converge to zero while the whole change is attributed to B1 (circa 65%) and T1 (approximately 30%). These are exactly the instruments intentionally designed to induce reranking, and if we look from the pre-fiscal perspective, these results confirm our intuitive expectations regarding the role played by taxes and benefits in the creation of reranking.

Figure 2. Percentage contributions to marginal changes in VK and RE

a) VK *b) RE*

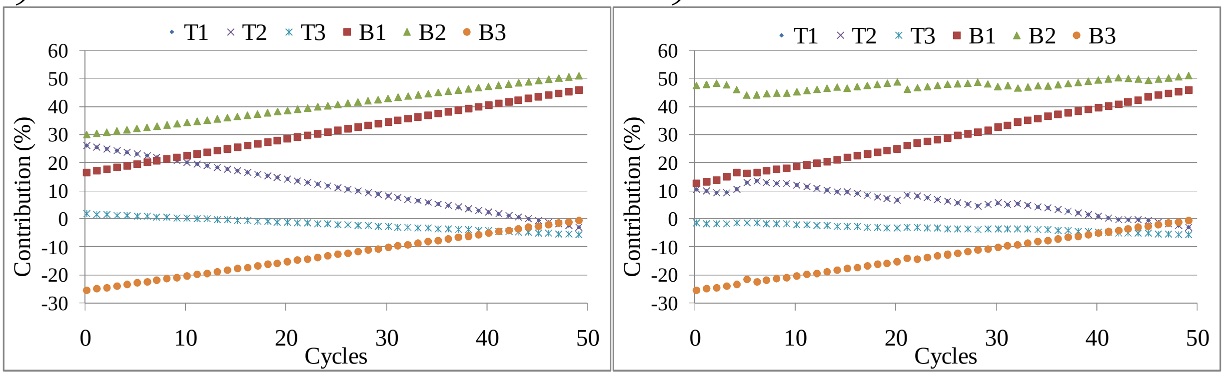


Figure 2 presents the percentage contributions to marginal changes in VK and RE. For both measures, the contributions on the pre-fiscal margin converge to values obtained by equation (47). The contributions made by benefits steadily increase as we move from a post- to pre-fiscal income margin while the opposite is true for taxes.

5.3. Empirical example

To further illustrate the applicability of the decompositions derived above, we use them to estimate the contributions of taxes and benefits to VK, RAPK and RE in Croatia. The analysed fiscal subsystem consists of personal income tax (T_i^1) and various social benefits categorised as means-tested benefits (B_i^1) and non-means-tested benefits (B_i^2). The former group includes basic support allowance and child allowance. The latter group consists of unemployment benefits, sick-leave benefits, maternity and layette supplements, and supports for the rehabilitation and employment of people with disabilities. Pre-fiscal income (X_i) is the sum of market incomes, pensions, and the value of gifts and household production for its own use.

The data come from the 2008 Household Budget Survey (HBS) collected by the Croatian Bureau of Statistics (CBS). The basic income unit is a household, but we divide the total household pre-fiscal income, taxes, benefits and post-fiscal incomes by the factors obtained using the “modified OECD scale”, which gives 1 point for the first adult member, 0.5 point for each additional adult member, and 0.3 points for each child.

Table 5 presents the estimates of the S-concentration coefficients of taxes and benefits later used to calculate the indicators for income redistribution. They are obtained for three values of the ethical parameter ν : 1.5, 2 and 3.

Table 5. The shares in income, Gini and concentration coefficients

	$D_{T1;X}$	$D_{T1;N}$	$D_{B1;X}$	$D_{B1;N}$	$D_{B2;X}$	$D_{B2;N}$
$\nu = 1.5$	0.5719	0.5604	-0.2782	-0.2016	-0.1491	0.0043
$\nu = 2$	0.7636	0.7513	-0.505	-0.3465	-0.264	0.0279
$\nu = 3$	0.9015	0.8915	-0.8746	-0.5535	-0.4329	0.0872
	τ_1		β_1		β_2	
	0.0626		0.0119		0.0127	

Table 6 presents the results of the K84 decomposition and the contributions of taxes and benefits. When $\nu = 2$, RE is equal to 0.0428, which means that taxes and benefits decrease income inequality by 12.5%. RAPK is equal to 0.0031 or 7.3% of RE. Thus, both RE and RAPK are modest compared to the inequality of pre-fiscal incomes. We observe that on the post-fiscal margin, the share of the overall marginal change in RAPK (0.0065) in the overall marginal change in RE (0.0394) is equal to 16.4%, which is significantly higher than the above-mentioned share of 7.3%, predicted by the rule from equation (40).

The contributions to VK obtained on the post- and pre-fiscal income margin, using equations (37) and (47), are relatively similar, with the observed difference being that the latter approach ascribes relatively greater importance to benefits. While T1 is the greatest contributor to VK, B2 is the most responsible for the creation of reranking, followed by B1 and T1.

Table 6. The results for the real-world taxes and benefits

	$\nu = 1.5$			$\nu = 2$			$\nu = 3$		
	value	%	$	value	%	$	value	%	$
G_X	0.2280			0.3422			0.4655		
G_N	0.1960			0.2994			0.4143		
$D_{N;X}$	0.1943			0.2963			0.4086		

$D_{N;X}$	0.1943			0.2963			0.4086		
RE	0.0319			0.0428			0.0512		
VK	0.0336			0.0459			0.0568		
RAPK	0.0017			0.0031			0.0056		
VK (37)	0.0336			0.0459			0.0568		
T1	0.0236	70.3	1.00	0.0293	63.7	1.00	0.0309	54.3	1.00
B1	0.0056	16.8	1.25	0.0096	20.8	1.71	0.0153	27.0	2.60
B2	0.0043	12.9	0.91	0.0071	15.4	1.20	0.0107	18.7	1.71
RAPK (33)	0.0036			0.0065			0.0113		
T1	0.0008	22.7	1.00	0.0010	14.9	1.00	0.0010	8.7	1.00
B1	0.0009	24.6	5.67	0.0019	28.6	10.09	0.0038	33.4	20.11
B2	0.0019	52.8	11.50	0.0037	56.5	18.75	0.0065	57.8	32.80
RE (39)	0.0300			0.0394			0.0456		
T1	0.0228	76.1	1.00	0.0283	71.7	1.00	0.0299	65.6	1.00
B1	0.0047	15.8	1.09	0.0077	19.6	1.43	0.0116	25.4	2.03
B2	0.0024	8.1	0.53	0.0034	8.7	0.60	0.0041	9.1	0.69
VK or RE (47)	0.0336			0.0459			0.0568		
T1	0.0224	66.6	1.00	0.0274	59.7	1.00	0.0284	49.9	1.00
B1	0.0063	18.7	1.47	0.0105	22.9	2.01	0.0166	29.3	3.07
B2	0.0050	14.8	1.10	0.0080	17.4	1.44	0.0118	20.8	2.06
Decomposition (6)	0.1960			0.2994			0.4143		
X	0.2350			0.3523			0.4780		
T1	-0.0365	93.7	1.0000	-0.0489	92.6	1.0000	-0.0580	91.0	1.0000
B1	-0.0025	6.4	0.3598	-0.0043	8.1	0.4612	-0.0069	10.8	0.6209
B2	0.0001	-0.1	-0.0077	0.0004	-0.7	-0.0371	0.0011	-1.8	-0.0979

Still, these interpretations do not account for differences in the relative shares of different instruments in the overall income. In our example, T1's share is approximately 5 times larger than the share of B1. Following Lerman and Yitzhaki (1994), we divide the contributions of each instrument by its share in the overall income (τ_1, β_1 and β_2), and then, for easier comparison, we divide these ratios for all three instruments by the value obtained for T1. The results are shown in the "$" columns of Table 6.

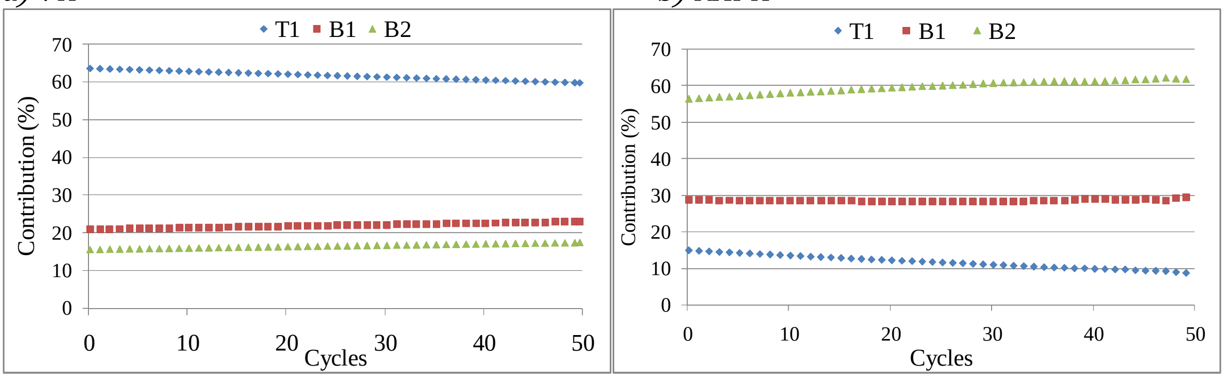
This procedure casts a different light on the contributions of the three analysed instruments. Concentrating again on the results for $\nu = 2$, B2 creates 19 times more reranking than T1. B2's impact on the vertical effect, however, is only approximately 1.7 times larger than T1's.

For $\nu = 1.5$, the contributions of T1 to VK and RE compared to B1 and B2 are even larger, and this is also true for their contribution to RAPK. The opposite is true for $\nu = 3$, where the contributions of the benefits are closer to that of T1. The benefits have a greater impact on the income of households at the bottom of the income distribution while PIT more strongly affects those with higher incomes. The value of $\nu = 3$ gives a much greater relative importance to poor households than $\nu = 1.5$, hence the difference in the results.

Finally, to check the robustness of the results for VK and RAPK, we undertake the same exercise as above for the hypothetical example. Figure 3 indicates the steady decline of T1's contribution to VK, from 63.7% to 59.7%, and the mild increase of B1 and B2's contributions. The contribution of B2 to RAPK steadily increases from 56.5% on the post-fiscal margin to 61.9% on the pre-fiscal margin; B1's role remains constant while T1's contribution falls below 10% on the pre-fiscal margin.

Figure 3. Contributions to marginal changes in VK and RAPK

a) VK *b) RAPK*



6. Conclusion

This study offers new methodology to evaluate the contributions of different tax and benefit instruments to vertical, horizontal and redistributive effects in the overall fiscal system under consideration. The background, consisting of Kakwani's (1984) decomposition of the redistributive effect and Lerman and Yitzhaki's (1985) decomposition of income inequality, is widely recognised by researchers. In our paper, these two approaches are amalgamated to obtain new decomposition methods.

These formulas evaluate the contributions of taxes and benefits on different margins: pre- and post-fiscal income. The contribution of each instrument depends on the chosen margin, and the conjecture is that differences will be larger for greater vertical, horizontal and redistributive effects in the overall system.

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