



Working Paper Series

**Relative Income Changes and an  
Identification of Growth Pattern**

Marek Kośny

**ECINEQ WP 2012 – 268**

## **Relative Income Changes and an Identification of Growth Pattern\***

**Marek Kośny<sup>†</sup>**

*Wrocław University of Economics*

### **Abstract**

The paper proposes a method of identification of a growth pattern by analyzing the direct relation between income (or some other measure of wealth) of the poorer and of the richer. To this end the basic idea underlying Zenga's concept of inequality measurement is applied. The proposed relative income change measures allude to the intuitive concept of the proportion of two averages: upper and lower – with respect to a given quantile of the income distribution. In this sense it directly refers to the relation of the poor and the non-poor.

The relative income change measure is then applied to the analysis of income growth pattern in selected countries, using the data from Luxembourg Income Study Database.

**Keywords:** growth pattern, income distribution, pro-poor growth

**JEL Classification:** D31, C46.

---

\* Revised version of ECINEQ WP 230 (November, 2011).

<sup>†</sup>**Contact details:** Institute of Applied Mathematics, Wrocław University of Economics. ul. Komandorska 118/120, 53-345 Wrocław, POLAND. tel. +48 71 36 80 334, e-mail: [marek.kosny@ue.wroc.pl](mailto:marek.kosny@ue.wroc.pl)

## 1. Introduction

Assisting the poor is one of the important objectives of contemporary, developed states. Depending on the leading political orientation, more or less attention is devoted to this issue. There is also no consensus on how big the group benefiting from a public aid should be. A rationale for such a situation is that fighting against poverty and equalizing the income distribution is considered very expensive and suspected of reducing competitiveness and efficiency of economy<sup>1</sup>. In this context, a stable, positive economic growth, improving the situation (absolute and relative) of all the poor in the society could be seen as a dream of every government. That is why the problem of growth pattern became so popular. A permanent, high rate pro-poor growth would improve the situation of the poor and this would prevent a social discontent.

Over the last decade several analyses have been performed, aiming at identifying growth patterns (see for instance Kakwani and Pernia 2000, Dollar and Kraay 2002, Kraay 2006, Son and Kakwani 2008, Deutsch and Silber 2011). Their results are not equivocal – observed growth patterns differ among countries and are not stable over time. The pattern of changes in income distribution is said to depend on several factors (e.g. regional location, inflation, education, inequality), but also on the way, the pro-poor growth concept is understood. Among the proposed methodologies two main streams can be identified: the absolute and relative approach (cf. Ravallion 2004, Duclos 2009). Roughly speaking, they are distinguished with respect to the point of reference, used for assessment of change in income of the poor: if absolute or relative growth is required for assessing the change pro-poor. This distinction is closely related to the way poverty is measured. When analysis is concentrated on satisfying the basic needs, absolute growth of income would be desired – especially in the case of the poorest countries, where a large proportion of the population receives income below the subsistence minimum (cf. Duclos 2009, p. 38). If relative inequality is of primary concern, an over-proportional growth in income of the poor will be considered pro-poor.

---

<sup>1</sup> Hence the famous equity-efficiency trade off. There are, however, no unambiguous indications on the optimal involvement of the state in the economy.

The actual consequence of absolute or relative pro-poor changes for the distribution of income depends on their precise definition<sup>2</sup>, but the latter approach assumes some reduction in distance between the poor and the rest of the population. In this sense it seems intuitive that analyzing the problem of the relative situation of the poor with respect to the richer involves – for a given definition of poverty – a comparison of incomes (or some other measure of welfare) of both groups. The direct comparison is not, however, exactly reflected in existing methodologies, aiming at identification of growth pattern. As the point of reference for the change in the situation of the poor, there are taken some statistics for the whole population.

In this context, a direct comparison of incomes of the poor and the non-poor, can be a new basis for the assessment of a growth pattern. As the idea of a growth pattern identification is naturally oriented on groups of people<sup>3</sup> (in this case the poor), this approach makes it possible to explicitly define both the group of the poor and the group that is taken as the point of reference. This approach does not necessarily mean lack of interest in the structure (income distribution) inside the group. In fact, the problem of “distribution of poverty” is crucial for the assessment of a growth pattern. Equal treatment of people at every position in income distribution assumes implicitly a “linear” and continuous nature of poverty, but the sense of poverty is not a linear function of a poverty gap. Despite an arbitrary (contractual) character of poverty lines<sup>4</sup>, there exist some thresholds, denoting a radical lowering of the standard of living. And this actual nature of poverty is recognized in the proposed measure of a relative income change by weighting incomes. Such weights are usual in inequality analyses (cf. Atkinson index, generalized entropy indexes, generalized Gini index), but they are

---

<sup>2</sup> If definition of absolute pro-poor growth requires higher absolute income change for the poor than for the non-poor, such a change will be pro-poor also in the relative sense. But if positive change in income is enough, these two assessments may be completely different.

<sup>3</sup> In contrast to the concept of relative deprivation (for analyses on this see, e.g., Bossert and D’Ambrosio 2006, Silber and Verme 2012) this approach is not oriented on individuals. While relative deprivation brings out relation of the individual and the group, the idea discussed in this paper is concerned with relation between groups. And this can be modeled within the theory developed by Temkin (1993). His notion of complaint, analogous to that of relative deprivation, can be aggregated across individuals and in this sense can be used to describe the relation between groups.

<sup>4</sup> Poverty lines are defined arbitrarily – both if they are related to some summary statistics of income in the population (e.g. mean or median) and in case they are defined as absolute values. Subsistence minimum can be given as an example of the latter. According to the definition of this scale, none with lower income should survive. Therefore, accounting for the sense of poverty is given in form of quasi-continuous weights, not defined for thresholds.

usually not applied in an identification of a growth pattern<sup>5</sup>. As these weights are the highest for the poorest, proposed measures relate to the concept of inclusive growth (cf. Ali and Son 2007, p. 12): lack of possibility to participate in subsequent aspects of the life of the society constitutes thresholds<sup>6</sup> discussed above.

Departure from a traditional approach involves some new properties of the measures under consideration. Standard assumptions, attributed to poverty measurement and related fields since work of Sen (1976), does not fully comply with the proposed approach. The main difference concerns sensitivity of new measures to transfers of income (transfer and monotonicity axioms). A direct group comparison, together with weighting incomes of the poor, causes violation of standard axioms. For example, an increase in the income of the poor that are just below the poverty line may cause even a bigger exclusion of the poorest. This mechanism, however, is not covered by standard assumptions, so some departure from them seems to be deliberate.

The paper is organized as follows. As the main concept of the identification of the growth pattern is based on the Zenga's approach to inequality measurement (cf. Zenga 2006), it is shortly presented in Section 2. In Section 3 proposal of the basic measure of the relative income change is presented. Extensions of the proposed measure are characterized in Section 4. Section 5 is devoted to the discussion of properties of the relative income change measures. In Section 6 relative income change measure is applied for assessment of the income growth pattern. Section 7 concludes.

---

<sup>5</sup> Implicitly given weights or parameters denoting “sensitivity to poverty”, are defined in some poverty measures, especially those belonging to the Foster-Greer-Thorbecke class (cf. Foster et al. 1984; for other class see for example Duclos and Gregoire 2002). Through these measures it is also built in some pro-poor growth measures (see, e.g., Kakwani and Son 2008 for PEGR measure), but it is not discussed explicitly.

<sup>6</sup> Analyzing the problem of social exclusion from the point of view of income, all basic aspects of participation in the life of society are available above a given level of income. Below, there are several thresholds, denoting restrictions in taking part in consecutive areas. However, inclusive growth by its definition concerns many other aspects besides the income. It concerns the poverty, understood in the long-term perspective. But, what is important from the point of view of this paper, very popular, direct cause of exclusion is low income (being, of course, consequences of many other factors).

## 2. Zenga's approach

The original concept of Zenga will be applied as a basis for the proposed assessment of distributional changes. Unlike other popular approaches (e.g. based on Lorenz curve or quantiles of the distribution), his method of assumes a direct comparison between groups of people.

Let  $\mathbf{x} = (x_1, x_2, \dots, x_n)$  be an ordered vector of non-negative values  $x_1 \leq x_2 \leq \dots \leq x_n$ , representing the distribution of income<sup>7</sup>. Then, the lower and the upper mean will be defined as

$$\bar{M}(\mathbf{x}, p) = \frac{\sum_{i=1}^{\lfloor n \cdot p \rfloor} x_i}{\lfloor n \cdot p \rfloor} \quad \text{and} \quad M^+(\mathbf{x}, p) = \frac{\sum_{i=\lfloor n \cdot p \rfloor + 1}^n x_i}{n - \lfloor n \cdot p \rfloor + 1} \quad \text{respectively, where } \lfloor n \cdot p \rfloor \text{ denotes rounding}$$

down to the integer closest to  $n \cdot p$  and  $p$  – quantile of the income distribution ( $0 \leq p \leq 1$ ). Because of the concentration on the situation of the poor, in the next of this paper slightly modified definition of the upper mean will be considered:

$$M^+(\mathbf{x}, p) = \frac{\sum_{i=\lfloor n \cdot p \rfloor + 1}^n x_i}{n - \lfloor n \cdot p \rfloor}.$$

This change denotes that both analyzed groups (lower and upper) are separated. In this sense this reflects being or not being the poor, but requires that quantile of the income distribution be lower than 1 ( $0 \leq p < 1$ ).

For a given  $p$ , Zenga's point indexes (cf. Zenga 2006) are defined as:

$$I(\mathbf{x}, p) = 1 - \frac{\bar{M}(\mathbf{x}, p)}{M^+(\mathbf{x}, p)}.$$

---

<sup>7</sup> In the next section of the paper, income will be used as a proxy for welfare. But it could be replaced with expenditure or any other measure of welfare (cf. Slesnick 1998). On a broader discussion on the application of non-income variables see Grosse et al. (2008).

Values of  $I(\mathbf{x}, p)$  vary between 0 and 1. The maximum value denotes a situation of no income in the lower group and minimum – perfect equity of incomes in the population. In the case of unequally distributed positive incomes, values of  $I(\mathbf{x}, p)$  decrease monotonically in the range (0, 1).

Such point indexes are scale invariant and decrease in the case of translation of the whole distribution by a positive value. They also follow transfer principle, but with respect to the formulation differing from that of Pigou-Dalton – definition, provided by Zenga (2006, p. 16) assumes that reduction in inequality is necessary only for transfers between neighbors in the income distribution.

For individual incomes, drawing  $I(\mathbf{x}, p)$  against  $p$ , gives Zenga's curve<sup>8</sup>. As the definition of the poorer (all that are covered by the lower average) does not necessarily mean the poor, this curve represents the relative situation of the poorer with respect to the richer along the whole income distribution.

### 3. Measure of relative income change

In accordance to the idea that underlies Zenga's indexes, a point measure of a relative income change, denoting changes in the distribution of income in a given period of time will be defined as:

$$RIC(\mathbf{x}^0, \mathbf{x}^1, p^0, p^1) = I(\mathbf{x}^0, p^0) - I(\mathbf{x}^1, p^1) = \frac{\bar{M}(\mathbf{x}^1, p^1)}{\bar{M}(\mathbf{x}^1, p^1)} - \frac{\bar{M}(\mathbf{x}^0, p^0)}{\bar{M}(\mathbf{x}^0, p^0)} \quad (1)$$

where  $\mathbf{x}^0$  and  $\mathbf{x}^1$  denote distributions of income at the beginning and the end of the period respectively. The line separating two groups – the poorer and the richer – is set by  $p^0$  and  $p^1$  for these two moments. In order to verify pro-pooriness of the distributional change,  $p$  has to denote share of the poor in the population (headcount ratio).

The values of the point index given by (1) range between -1 and 1. They reflect changes in the average income of the poor with respect to the non-poor. For a given  $p$ ,  $RIC$  indicates the change

---

<sup>8</sup> Zenga (2006) defines it in the form of a diagram for grouped (weighted) data.

(expressed in percentage points) in the share of the average income of the poor ( $100\% \cdot p$  of the population) in relation to the average income of the non-poor ( $100\% \cdot (1-p)$  of this population). Positive values of *RIC* indicate an improvement in the relative situation of the poor, negative – decline and zero – proportional or no changes. Being a relative measure, *RIC* holds no information about the absolute situation of both groups. It means that even in the case of decrease in wealth, the change can be considered favorable to the poor – if the decline for the group of the poor is respectively lower.

Properties of the measure can be analyzed according to the proposition of Duclos (2009). He defines a pro-poor evaluation function by formulating several requirements, analogous to those, usually used in an analysis of poverty (cf. Duclos 2009, pp. 41-48). As the proposed measure is relative, a set of requirements concerning relative pro-poorness will be discussed.

**Axiom 1.** Focus on the poor

The distribution of income among the non-poor does not influence the assessment of the changes in the income of the poor:  $RIC(\mathbf{x}^0, \mathbf{x}^1, p^0, p^1) = RIC(\mathbf{x}^0, \mathbf{x}, p^0, p^1)$ , where  $\mathbf{x} = (\min(x_1, z), \dots, \min(x_n, z))$  and  $z$  denotes the poverty line corresponding to the headcount ratio  $p^1$ . To capture general changes in incomes, Duclos allows scaling the vectors by  $(1+g)$ , referred to as “relative standard” (cf. Duclos 2009, p. 40).

This requirement is not met, because reducing incomes of all the non-poor to the poverty line changes the average. However, *RIC* is not sensitive to the distribution within both groups and letting “relative standard” concern only incomes of the non-poor, makes *RIC* compliant with such a modified version of this axiom. This modification reflects the basic idea of the analyzed approach that relates the situation of the poor to that of the non-poor and not the whole population.

**Axiom 2.** Population invariance

The replication of population (vector  $\mathbf{x}^0$  or  $\mathbf{x}^1$ ) does not influence the values of *RIC*, as all sub-group averages remain unchanged.



**Axiom 3. Anonymity**

No information except for the income is taken into account when assessing the distributional change.

Therefore the values of  $RIC$  are the same for permutations of vectors  $\mathbf{x}^0$  and  $\mathbf{x}^1$ .

**Axiom 4. Monotonicity**

The increasing income of any poor in  $\mathbf{x}^1$  by  $\varepsilon > 0$  is considered pro-poor. This assumption is met for  $\varepsilon$  lower than the poverty gap (of the poor whose income is increased) – in such a case this person is still poor after the change. However, in the situation of higher  $\varepsilon$ , an increase in the upper mean or a decrease in the lower mean can cause  $RIC$  to be negative, because the measure is concentrated on the situation of the poor.

**Axiom 5. Neutrality**

No distributional change implies  $RIC(\mathbf{x}, \mathbf{x}, p, p) = 0$ , as all averages do not change with time.

**Axiom 6. Scale invariance**

It is the usual requirement for relative measures: for any positive coefficient of proportionality  $\alpha$ ,  $RIC(\mathbf{x}^0, \mathbf{x}^1, p^0, p^1) = RIC(\mathbf{x}^0, \alpha\mathbf{x}^1, p^0, p^1)$ . Rescaling of all incomes implies a proportional change in the lower and upper average and their quotient remains unchanged.

**Axiom 7. Distribution sensitivity**

Any transfer, changing vector  $\mathbf{x}^0 = (x_1, \dots, x_i, \dots, x_j, \dots, x_n)$  into  $\mathbf{x}^1 = (x_1, \dots, x_i + \varepsilon, \dots, x_j - \varepsilon, \dots, x_n)$  where  $\varepsilon > 0$  and  $x_i + \varepsilon \leq x_j - \varepsilon$ , implies  $RIC(\mathbf{x}^0, \mathbf{x}^1, p^0, p^1) \geq 0$  in the situation where the number of the poor is not decreasing. If such a transfer is made within a lower or upper group, the respective mean will not change and  $RIC(\mathbf{x}^0, \mathbf{x}^1, p^0, p^1) = 0$ . In the case of transfer from the non-poor to the poor, the lower mean will increase and the upper mean decrease (even if the donator is poor after the transfer). It will be then considered pro-poor, implying  $RIC(\mathbf{x}^0, \mathbf{x}^1, p^0, p^1) > 0$ .

The requirements for relative pro-poor evaluation function are then satisfied with reservations about the statement of Axiom 1, Axiom 4 and Axiom 7. A further discussion on the axiomatic structure will be presented in the next two sections.

Plotting  $RIC(\mathbf{x}^0, \mathbf{x}^1, p, p)$  against  $p$  gives a relative income curve. It allows the analysis of changes in the average income of the poorer with respect to the average income of the richer over the entire distribution (all values of  $p$ ) or for  $p$  lower than the share of the poor in population. Such a curve is an auxiliary tool in the assessment of pro-poorness. The analysis of its graph allows the identification of groups that relatively benefit (maximum of this function) or lose (minimum) the most, compared to the group of the richer.

#### **4. Extensions**

##### **4.1. Redefinition of lower or upper group**

The  $RIC$  measure, characterized in the previous section, assumes a specific range of a lower and upper group. However, their definition could be generalized, taking into account the following requirements:

- The minimum income in the lower group cannot be higher than the minimum income in the upper group. And the maximum income in the upper group cannot be lower than the maximum in the lower group.
- Both groups are defined by the lower and upper bound, which ensures that no income within this interval is excluded. Bounds may be given as incomes or quantiles of the income distribution.

These conditions are sufficient for the quotient of averages to be not greater than 1, because both groups come from the same population. The interpretation of the measure based on such quotients is analogous to that of  $RIC$ : it denotes a change in the share of the lower mean in the upper mean, given in percentage points. It allows the analysis of a relative income of two groups as a point

measure or as a curve – for changing (in a given way) the definition of one or both groups. Two examples of such modifications will be considered.

The first option assumes a fixed definition of the upper group. It can include all the non-poor or – what can be more interesting – some of them, for example those with the highest incomes. The results can be presented for a given subgroup of the poor or as a curve for all quantiles below the share of the poor in the population. Such an analysis would be then analogous to the studies based on percentile ratios<sup>9</sup> that are very popular in applied research on poverty and affluence. For groups reduced to one-person at given positions in the income distribution, *RIC* would provide the information about the change in the inverse of the percentile ratio. But the group-oriented extension allows capturing the inequality in the lower and upper tail of the income distribution that is only partially reflected by positional statistics.

The second option is based on simultaneous changes in both subgroups but in a manner different as in the original formulation of the *RIC* curve. For example, according to the concept of relative deprivation, such a measure can be defined for relatively small groups just below and just above the given quantile. Plotting such indexes against  $q$  gives the information about the changes in relative income of groups that can be considered as point of reference<sup>10</sup>. Such an analysis can help in the identification of relative changes within the group of the poor. Some changes – for example in a minimum wage – influence the overall poverty but their actual impact on the situation of subgroups of the poor may be strongly diversified.

These modifications do not significantly influence the properties of the measure. The difference concerns the definition of the group of the poor in Axioms 1, 4 and 7 – the poor has to be replaced with the lower-income subgroup. Therefore, modified measures could be interpreted as pro-

---

<sup>9</sup> For example 90/10, 90/50 and 80/20 that are published as Key Figures in Luxembourg Income Study (<http://www.lisdatacenter.org/>).

<sup>10</sup> Of course, the concept of the reference group is much more complex and involves several characteristics besides the income level. Literature on reference groups and relative deprivation (cf. Podder 1996, Ferrer-i-Carbonell 2005) define reference groups as groups of people with similar characteristics. At the same time these are also groups to which people aspire. The usual definition of such a group (similar education, place of living, household composition and so on) imply a similar level of income. Therefore, in the case of income as the sole characteristic of the person, people with a similar (slightly higher) income seem to build the most probable reference group.

poor evaluation functions only in some special cases. Moreover, Axiom 1 (in the modified form) holds only for the groups that do not overlap.

#### 4.2. Concentration on the poorest

In order to concentrate on the situation of the poorest members of the population, a weighted analogue of the *RIC* measure will be proposed. A new definition of the lower mean is given by

$$\bar{M}(\mathbf{x}, p, \alpha) = \sum_{i=1}^{\lfloor n \cdot p \rfloor} w_i(p, \alpha) x_i. \text{ Weights for incomes at any position in the lower subgroup depend on}$$

$$\text{the poverty gap of the individual at this position. They are defined as } w_i(p, \alpha) = \frac{\left( d_i / \sum_{k=1}^{\lfloor n \cdot p \rfloor} d_k \right)^\alpha}{\sum_{j=1}^{\lfloor n \cdot p \rfloor} \left( d_j / \sum_{k=1}^{\lfloor n \cdot p \rfloor} d_k \right)^\alpha},$$

where  $\mathbf{x}$  denotes the vector of incomes,  $n$  – dimension of this vector,  $p$  – share of the poor in the population ( $0 < p < 1$ ) corresponding to the poverty line  $z$ , and  $d_i = \max(0, z - x_i)$  – poverty gap.

Weights can be calculated assuming that  $\sum_{i=1}^n \max(0, z - x_i) > 0$ , which means that at least one person

(household) is poor. They depend on the parameter  $\alpha \geq 0$  that expresses the attitude towards poverty.

For  $\alpha = 0$  all weights are equal<sup>11</sup>, and the weighted lower mean reduce to the form given in Section 3.

It denotes equal treatment of all the poor and lack of the special interest in the situation of the poorest.

For  $\alpha > 0$  relative weights of the individuals depend on their poverty gap. The weighted relative income change index is then given by:

$$WRIC(\mathbf{x}^0, \mathbf{x}^1, p^0, p^1, \alpha) = \frac{\bar{M}(\mathbf{x}^1, p^1, \alpha)}{\bar{M}(\mathbf{x}^1, p^1)} - \frac{\bar{M}(\mathbf{x}^0, p^0, \alpha)}{\bar{M}(\mathbf{x}^0, p^0)} \quad (2)$$

---

<sup>11</sup> Only individuals with the poverty gap greater than zero can be included in the lower subgroup.

The parameter  $\alpha$  can be interpreted as the sensitivity to poverty<sup>12</sup>. For  $\alpha = 1$  weights are proportional to the poverty gap, calculated for the individual at a given position. The increasing value of  $\alpha$  makes *WRIC* more concerned with the situation of the poorest. For  $\alpha$  tending to infinity, only the situation of the poorest individual is recognized, which could be interpreted in the context of the original position of Rawls (1971).

As the values of the weighted lower mean are the highest for  $\alpha = 0$ , quotients of means for the beginning and the end of the period take values from the interval (0, 1). Thus, values of *WRIC* range between -1 and 1. The interpretation of *WRIC* is the same as in the case of *RIC* and denotes a change in the quotient of mean incomes of the poor and the non-poor, given in percentage points. The negative (positive) values denote anti-poor (pro-poor) changes in the income distribution.

Weighting incomes of the poor is justified in the nature of poverty. It cannot be treated as proportional to the poverty gap. The income below a certain level significantly limits or even precludes from participation in a social life. Even a lower income may negatively influence a health condition by lack of healthcare and proper food. In an extreme situation it can lead to death. Social welfare institutions constrain severity of poverty (especially in developed countries), but it does not change the essence of the problem. Characterized thresholds, however, cannot be directly reflected in the assessment of distributional changes because of their contractual nature. In this context, the most reasonable way to proceed seems to be an application of an appropriate weighting scheme: the reduction of an unweighted, aggregated poverty gap (deficit) seems neither to be a sufficient nor necessary condition for poverty reduction. While the increase in the income of the poorest is unconditionally considered pro-poor, in the case of other distributional changes the answer does not need to be so obvious. For example, the poorest can feel more deprived as a result of the increase in the income of “the richer poor” (who are still poor after the change). Moreover, such a change in the distribution of income in some situations may cause an absolute worsening of the situation of the poorest: growth of the purchasing power of the poor as a whole can result in the increase in prices of

---

<sup>12</sup> On the ground of welfare and inequality analyses, it denotes the distributional judgment parameter (cf. Lambert 1993, p. 115).

some basic goods. Such mechanisms, however, are not necessarily reflected in usual axiomatic structures. A more detailed discussion on this problem will be presented in Section 5.

When it comes to the compliance of the *WRIC* measure with axioms characterized in Section 3, it has properties analogous to those of *RIC*, except for “transfer” axioms (4 and 7). *WRIC* is focused on the poor – in the modified sense, when “relative standard” concern only incomes of the non-poor. It is also compliant with requirements of population invariance (Axiom 2), anonymity (Axiom 3), neutrality (Axiom 5) and scale invariance (Axiom 6) – weights in the lower mean do not change properties in these fields.

The largest discrepancies concern the impact of transfers. For  $\alpha > 0$  increase in the income of the poor, whose income is just below the poverty line may reduce the value of the lower mean – because higher incomes are accompanied with lower weights. In this way *WRIC* does not comply with Axiom 4 and Axiom 7 – even with modifications, characterized in Section 3.

## 5. Discussion

The problem of an axiomatic structure for measures of changes in the income distribution requires an answer to some fundamental questions. From the point of view of changes in the situation of the poor over the time, especially important is the identification of the group under analysis. The first option is to assess changes in the income for the share<sup>13</sup> of the population that is set arbitrarily (it can be, for example, equal to the initial headcount ratio). The second possibility is to compare the situation of the group that was poor at the beginning of the analyzed period, to the group that was poor at the end of this period. In the first situation the analysis is not necessarily concerned with the situation of the poor. In the latter, some overall statistics can be misleading, because the population of the poor has changed.

The proposed approach to assess of distributional changes, based on the direct comparison between subgroups of the population can be applied to both types of analyses. In Sections 3 and 4, *RIC*

---

<sup>13</sup> Or even each person individually. In such a case, the analysis usually involves multidimensional characteristic of individuals.

and *WRIC* were presented in the form reflecting changes in the headcount ratio. But they can be easily switched to a fixed share by setting  $p^0 = p^1$ . Of course, such a modification implies changes in axiomatic properties.

The *RIC* measure, given by formula (1) is monotonic in the sense of Axiom 4 only for transfers within the subgroups (the poor or the non-poor). If an individual escapes poverty as a result of a transfer, it is not necessarily considered pro-poor. Thus, in the case of a separate treatment of the poor at the beginning and the end of the period, the situation of all those that are still poor may become worse. The interpretation of such a change is, however, intuitive. Those remaining poor after the change can be in a relatively worse situation. Of course, from the point of view of the population as a whole, the situation has improved. And this is reflected in the fixed-share analysis. If *RIC* is calculated for  $p^1 = p^0$ , such a transfer will always increase the lower mean. However, within the considered framework, the final assessment will also depend on the change in the situation of the non-poor. If the transfer was enormously high, the overall evaluation can be anti-poor, because of the increase in the upper mean.

For *WRIC* the situation is more complex. For the reasons characterized in Section 4, the increase in the income of the poor can be considered anti-poor. Capturing this property requires a weaker definition of monotonicity, assuming that the lower is the income of the recipient, the higher evaluation of this transfer should be.

**Axiom 4'. Relative monotonicity**

Let  $\mathbf{x}^0 = (x_1, \dots, x_i, \dots, x_j, \dots, x_n)$ ,  $\mathbf{x}' = (x_1, \dots, x_i + \varepsilon, \dots, x_j, \dots, x_n)$  and  $\mathbf{x}'' = (x_1, \dots, x_i, \dots, x_j + \varepsilon, \dots, x_n)$  denote vectors of income. Then relative monotonicity denote  $WRIC(\mathbf{x}^0, \mathbf{x}', p^0, p^1, \alpha) \geq WRIC(\mathbf{x}^0, \mathbf{x}'', p^0, p^1, \alpha)$  for  $\varepsilon \geq 0$ ,  $x_j < z$ .

Because of higher weights attributed to lower incomes, *WRIC* complies with Axiom 4'. For the formulation given by (2), requirement of Axiom 4' is met if the transfer does not change the headcount ratio ( $x_j + \varepsilon < z$ ). For  $p^1 = p^0$  this additional condition is not needed.

The second requirement, closely related to monotonicity, concerns sensitivity to transfers. There exist several formulations of this axiom, referred to as Pigou-Dalton principle in welfare economics literature. Cowell and Ebert (2004) argue that Dalton's generalization of Pigou's idea is not so obvious and justified by people's perception of inequality. And they define "progressive transfer" as the transfer that does not alter the original ranking of incomes. Even a weaker condition is formulated by Zenga (2006) – he analyses only transfers between neighbors in the income distribution. In this context, the definition provided by Duclos (2009) is relatively strong. As given in Section 3, it requires that almost any transfer from the richer to the poorer (even changing the sequence of individuals in the income distribution) has to be considered pro-poor. This requirement is met for the *RIC* measure given by (1) only for transfers within subgroups of the poor and the non-poor, but if  $p^1$  is assumed to be equal to  $p^0$  – for all transfers from the richer to the poorer. For *WRIC*, the requirement of distribution sensitivity is met for transfers made within subgroups. For the lower mean it results from the compliance with Axiom 4'. As the upper mean is not weighted, any transfer within this group does not influence an average. For transfers between groups, only the weaker condition, analogous to 4', is met.

**Axiom 7'. Relative distribution sensitivity**

Let  $\mathbf{x}^0 = (x_1, \dots, x_i, \dots, x_j, \dots, x_n)$ ,  $\mathbf{x}' = (x_1, \dots, x_i + \varepsilon, \dots, x_j, \dots, x_k - \varepsilon, \dots, x_n)$  and  $\mathbf{x}'' = (x_1, \dots, x_i, \dots, x_j + \varepsilon, \dots, x_k - \varepsilon, \dots, x_n)$  denote vectors of income. Then relative monotonicity denote  $WRIC(\mathbf{x}^0, \mathbf{x}', p^0, p^1, \alpha) \geq WRIC(\mathbf{x}^0, \mathbf{x}'', p^0, p^1, \alpha)$  for  $\varepsilon \geq 0$ .

Axioms 4' and 7' as weaker formulations of their analogs, simplify understanding of specific consequences of applying weights. Sometimes it is not possible to assess the change as pro-poor or anti-poor (it depends on the sensitivity to poverty, expressed by  $\alpha$ ), but changes can be compared in terms of their "pro-poorness".

Besides vulnerability to transfers, significant characteristic influencing properties of *RIC* and *WRIC* measures is the method of understanding poverty. As argued earlier, poverty is the phenomenon related to the specific population and the non-poor are the point of reference for the poor in



experiencing and evaluating of poverty. This justifies the modification of Axiom 1, presented in Section 3: the assessment of the situation of the poor is conditioned on the average situation of the non-poor<sup>14</sup>. Measures *RIC* and *WRIC* are compliant with Axiom 1 (in the modified version) if they are calculated for  $p^0$  and  $p^1$  denoting the share of the poor in both moments. For  $p^1 = p^0$ , measures are not necessarily focused on the poor.

The existing methods of identification of growth pattern take the whole population as a point of reference. It could be easily seen on the examples of the Growth Income Curve and the aggregated measure proposed by Ravallion and Chen (2003) – both methods assess changes in the situation of the poor, but as a point of reference take the average growth in income of all population members<sup>15</sup>. In this context, a redefinition of the reference group for the poor in the assessment of distribution changes is the main contribution of both measures. On the contrary to the existing measures, *RIC* and *WRIC* are not concentrated on the relation of the poor to the whole population, but to the group of the non-poor. It is important in the case of significant changes among the poor and minor (or lack of) changes among the non-poor, where the results of a direct comparison are much clearer and intuitively interpretable.

A wide range of possible adjustments allows *RIC* and *WRIC* to comply with the different practical needs. They enable a definition of subgroups and their change over the time, the choice of the reference point for each subgroup of the poor and the use of weights that are aimed at differentiation of the situation of the poor on the ethical basis. The weights allow for sensitivity to poverty – from lack of interest in distributional issues to the concentration on the situation of the poorest individual. Moreover, analyzing the *RIC* curve it is possible to identify groups in relatively the best and the worst situation.

---

<sup>14</sup> The non-poor are considered as a group and internal distribution within this group does not matter.

<sup>15</sup> Taking the change in the whole income distribution as a benchmark in the pro-poor assessment is explicitly suggested by Duclos (2009) in the definition of absolute and relative standards.

## 6. Application

The proposed methods of analysis will now be used to assess the pattern of growth for some countries that are covered by the Luxembourg Income Study Database (2012). The available data generally come from national household budget surveys (except for Denmark, where the data are taken from the tax register). The harmonization of national datasets is completed before making the data available. However, the full harmonization is not possible because of the differences between the analyzed countries (fiscal systems, education etc.). Such discrepancies should not, however, significantly influence the presented results because the basic comparison between the poor and the non-poor are made within the same dataset (for a given country and year).

The newest data in this database are available for so-called Wave 6 (around 2004; Wave 7's data is under harmonization). As a point of reference the data from Wave 5 (around 2000) have been taken<sup>16</sup>. The GDP growth in the period under consideration was mediocre – both in the advanced economies and in the whole world (cf. World Economic Outlook Database 2011). The burst of the “dot-com bubble” resulted in the economic slowdown after the rapid growth in the late 1990s.

The analysis is based on the disposable income<sup>17</sup>. To identify the poor in the population, three definitions of poverty line have been applied, referred to as POOR40, POOR50 and POOR60. They are set at 40, 50 and 60 percent of the median equivalent income respectively, while the equivalence scale is defined as square root of the number of household members<sup>18</sup>.

The headcount ratio estimates strongly differed between the analyzed countries. For POOR40 the observed poverty rate was the lowest in Luxembourg (1.3% in year 2000) and the highest

---

<sup>16</sup> With the additional assumption that minimum 4-year distance between the beginning and the end of the analyzed period is required. As only datasets available in the Web Tabulator were taken into account, the final set of countries under consideration included Austria, Canada, Denmark, Finland, Greece, Hungary, Israel, Italy, Luxembourg, Mexico, Norway, Poland, Taiwan (China), Spain, Sweden, Switzerland, the United Kingdom and the United States.

<sup>17</sup> Variable *dpi* denotes net income (gross income minus income taxes and mandatory payroll taxes *GI-(PAYROLL+V11)* for gross datasets and net income *GINET* for net datasets) per household. Each household is assigned the person weight in order to adjust for the structure of the population and the number of individuals in the household. The detailed description of the data and the precise definitions of variables (components and structure) are provided by LIS Data Center on website of the project <http://www.lisdatacenter.org>.

<sup>18</sup> Such poverty line definitions are built in data analyses by LIS Data Center because the only information required is a number of individuals in the household. As considered earlier, the choice of poverty line is subjective in its nature.

in Mexico (15.4% in year 2000). For POOR50 corresponding values were equal to 5.4% (Finland, year 2000) and 21.5% (Mexico, year 2000), while for POOR60 12.0% (Sweden, year 2004) and 28.1% (Mexico, year 2000). Because of such large differences in poverty extent, basic formulation of the proposed measure (*RIC*) will be used in this analysis<sup>19</sup>. Values of *RIC* are given in Table 1.

**Table 1. Relative income changes**

Country	Period	Relative income change (in percentage points)		
		POOR40	POOR50	POOR60
Austria	2000-2004	-0.13	-0.67	-0.18
Canada	2000-2004	1.41	0.48	0.19
Danmark	2000-2004	-2.55	-2.40	-1.25
Finland	2000-2004	4.70	1.41	-0.50
Greece	2000-2004	-1.46	-0.11	0.59
Hungary	1999-2005	-0.22	-1.51	-1.83
Israel	2001-2005	-0.42	-2.64	-3.01
Italy	2000-2004	-0.29	-0.13	0.19
Luxembourg	2000-2004	-3.68	-2.63	-3.30
Mexico	2000-2004	0.54	1.39	2.23
Norway	2000-2004	1.74	-1.14	-1.74
Poland	1999-2004	-0.94	-2.45	-3.27
Taiwan (China)	2000-2005	-1.26	-1.59	-1.96
Spain	2000-2004	0.14	0.48	0.77
Sweden	2000-2005	2.31	3.84	3.77
Switzerland	2000-2004	-5.02	-2.62	-0.26
United Kingdom	1999-2004	3.28	1.44	1.27
United States	2000-2004	-1.00	-0.94	-0.77

As could be observed, the dominant pattern of growth was anti-poor. In 10 out of 18 countries negative values of *RIC* have appeared for all three definitions of the poverty line. There are, however, differences in the relative situation of three groups of the poor with respect to the non-poor. The lowest value indicates the group whose relative situation has worsened most. In the case of Israel, relatively the highest decline in income of the poor with respect to the non-poor can be observed for the POOR60 poverty line, while the lowest – for POOR40. It denotes the highest decline in the relative income of the richest among the poor. The opposite pattern of anti-poor changes is observed

<sup>19</sup> The use of weighted measure *WRIC* is strongly advised in comparison of countries with similar level of wealth or in the case of analysis over time for one country. However, in the case of the set of so diversified countries, the weights may not reflect the actual situation of the poor.

for Switzerland. In this case a typical anti-poor change is observed and the relative loss decreases with an increase in the income level.

On the other hand, the most favorable for the poor is the situation of the highest positive changes in the poorest group. Such a pattern can be observed for the United Kingdom and Canada, indicating a typically pro-poor pattern of changes in the income distribution.

All point indexes used in this analysis, denoting the share of income of the poor in the income of the non-poor, were calculated for the groups that are considered poor at a given moment of time. As discussed earlier, it can result in the decline of relative income of the poor that is accompanied by reduction in the headcount ratio or – on the opposite – an increase in the relative income of the poor together with the increase in the headcount ratio. The first happened in the case of Austria and the latter – in the case of Canada. It can be, of course, avoided by calculating *RIC* for the same share of the population at the beginning and the end of the analyzed period. But analyzing these two aspects jointly indicates a complex nature of changes in the situation of the poor.

## **7. Conclusion**

The analysis of growth patterns is a branch of research of great practical importance: identifying pattern of changes in the income distribution can be a crucial element in the process of designing a social policy. Therefore, it is not sufficient that the assessment of changes comply with some formal requirements, but it has to reflect the actual nature of this phenomenon in the way it is perceived by individuals.

The proposed relative income measures are based on the idea underlying Zenga's inequality index. The essential element of this approach is the direct comparison of income for subgroups in the population: for pro-poor assessment, the groups are made up of the poor and the non-poor. Such a direct comparison reflects the general idea of a reference group – of course in a restricted way because

the basis of differentiation between groups is income<sup>20</sup> (or another measure of welfare). Introducing weights within the group of the poor, allows taking into account the non-linear relationship between the poverty gap and the actual situation of the poor. The consequence of such a construction is that the proposed measures do not comply with all the axioms usually postulated in the literature. The modified axiomatic structure, however, reflects the nature of changes in the income distribution.

The very important feature of both measures is their intuitive interpretation. They reflect changes in the relative situation of the poor and the non-poor, given as a change in the share of the average income of the lower-income group in the average income of the higher-income group. Thus, positive values denote a pro-poor change and negative – anti-poor. Additionally, the maximum and minimum of the relative income curve indicate subgroups that are the most and the least favored by the change.

As can be seen on the basis of the empirical analysis presented in Section 6, prevailing pattern of changes is anti-poor. Results are, however, strongly diversified between the countries in the period under analysis – with respect to both direction and scale of changes. This observation seems to be in accordance with previous findings on pro-poor growth, presented so far in the literature, where no unique pattern of growth has been identified.

The presented analysis points to certain limitations of reasoning, underlying usually applied methods of an analysis of distribution changes. The lack of distinction between the treatment of the poorest and the richer among the poor implies too little interest in the situation of the socially and economically excluded. In this context, a mutual exchange of methods applied in the analyses on the inclusive growth and the pro-poor growth seem to be the direction of the evolution in these fields.

---

<sup>20</sup> Proposed measures can be generalized on the other definitions of groups (for example those discussed in Section 4.1), allowing for better reflecting the idea of reference groups.

## Acknowledgements

I would like to thank Jacques Silber for very helpful comments and suggestions, and the Polish National Science Centre for the financial support. I take full responsibility for any remaining errors.

## References

- Ali, I., Son, H. (2007), Measuring Inclusive Growth, *Asian Development Review*, 24, 11-31.
- Bossert, W., D'Ambrosio, C. (2006), Reference groups and individual deprivation, *Economics Letters*, 90, 421–426.
- Cowell, F., Ebert, U. (2004), Complaints and inequality, *Social Choice and Welfare*, 23, 71–89.
- Deutsch, J., Silber, J. (2011), On Various Ways of Measuring Pro-Poor Growth. *Economics: The Open-Access, Open-Assessment E-Journal*, 5, 2011-13 (<http://dx.doi.org/10.5018/economics-ejournal.ja.2011-13>, 10.10.2011).
- Dollar, D., Kraay, A. (2002), Growth is Good for the Poor, *Journal of Economic Growth*, 7, 195-225.
- Duclos, J.-Y. (2009), What is “Pro-Poor”?, *Social Choice and Welfare*, 32, 37-58.
- Duclos, J.-Y., Gregoire, P. (2002), Absolute and Relative Deprivation and the Measurement of Poverty, *Review of Income and Wealth*, 48, 471-492.
- Ferrer-i-Carbonell, A. (2005), Income and well-being: an empirical analysis of the comparison income effect, *Journal of Public Economics*, 89, 997– 1019.
- Foster, J., Greer, J., Thorbecke, E. (1984), A Class of Decomposable Poverty Measures, *Econometrica*, 52, 761–767.
- Grosse, M., Harttgen, K., Klasen, S. (2008), Measuring Pro-Poor Growth in Non-Income Dimensions, *World Development*, 36(6), 1021-1047.
- Kakwani, N., Pernia, E. (2000), What is Pro-poor Growth?, *Asian Development Review*, 18(1), 1-22.
- Kakwani, N., Son, H. (2008), Poverty Equivalent Growth Rate, *Review of Income and Wealth*, 54, 643-655.
- Kraay, A. (2006), When is growth pro-poor? Evidence from a panel of countries, *Journal of Development Economics*, 80(1), 198-227.
- Lambert, P. (1993), *The Distribution and Redistribution of Income – A Mathematical Analysis*, Manchester University Press, Manchester.

- LIS Web Tabulator* (2012), <http://www.lisdatacenter.org>, Luxembourg, LIS.
- Podder, N. (1996), Relative Deprivation, Envy and Economic Inequality, *Kyklos*, 49, 353-376.
- Ravallion, M. (2004), Pro-Poor Growth: A Primer, *World Bank Policy Research Working Paper*, 3242.
- Ravallion, M., Chen S. (2003), Measuring pro-poor growth, *Economics Letters*, 78, 93-99.
- Rawls, J. (1971), *A Theory of Justice*, Harvard University Press, Cambridge.
- Sen, A. (1976), Poverty: An Ordinal Approach to Measurement, *Econometrica*, 44, 219-232.
- Silber, J., Verme, P. (2012), Relative Deprivation, Reference Groups and the Assessment of Standard of Living, *World Bank Policy Research Working Paper*, 5930.
- Slesnick, D. (1998), Empirical Approaches to the Measurement of Welfare, *Journal of Economic Literature*, 36, 2108-2165.
- Son, H., Kakwani, N. (2008), Global Estimates of Pro-poor Growth, *World Development*, 36(6), 1048-1066.
- Temkin, L. (1993), *Inequality*, Oxford University Press, Oxford.
- World Economic Outlook Database* (2011), <http://www.imf.org/external/ns/cs.aspx?id=28>, Washington D.C., International Monetary Fund.
- Zenga, M. (2007), Inequality curve and inequality index based on the ratios between lower and upper arithmetic means, *Statistica & Applicazioni*, 1, 3-27.