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Structural Progression Measures for Dual Income Tax Systems*

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Abstract

The structural progression of an income tax schedule measures how liabilities change with changes in the income being taxed. This paper extends the measurement of structural progression to a pure-form dual income tax (DIT) system, which combines progressive taxation of labour income with proportional taxation of income from capital at a lower rate. Firm links are obtained between structural progression and revenue responsiveness for a DIT, and we demonstrate how structural progression measures can aid in redistributive analysis, using Nordic data to highlight problems which can stem from pre-tax distributional changes. We conclude with an assessment of the new theoretical and empirical work that is now required, much of which will be data driven.

Keywords: personal income tax, dual income tax, structural progression.

JEL Classification: D31, D63, H23.

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1. Introduction

At the heart of income taxation in the modern world is the so-called "progressive principle" according to which tax is levied at an increasing average rate on a well-defined base. There are several ways to measure the degree of progression of an income tax schedule, dating back to Musgrave and Thin (1948). Two familiar structural measures, liability and residual progression, have been linked in the landmark papers of Jakobsson (1976) and Kakwani (1977), which have become classics, to overall distributional effects of the income tax. Hutton and Lambert (1979) began a corresponding analysis for average rate progression, which they linked to the responsiveness of income tax revenue to income growth. In this paper, we extend the measurement of average rate progression, liability progression and residual progression to a so-called "pure form" dual income tax (henceforth DIT), which combines progressive taxation of labour and transfer income with proportional taxation of income from capital at a lower level equal to the corporate income tax rate (Sørensen, 1994; Nielsen and Sørensen, 1997).[1]

For the constituent schedules of a DIT, we can draw on the classical work to conclude that the labour income tax decreases inequality in the distribution of labour income, and has revenue which is elastic to income growth, while the capital income tax, being proportional, has no effect on inequality in the distribution of capital income and has unit revenue elasticity. But these income sources are pooled, of course, typically with capital income being the more unequally distributed and also more concentrated at the top of the overall income distribution, making it perilous to predict *overall* inequality effects from structural properties. Some progress has nevertheless been made, in terms of the Gini coefficient (Kristjánsson, 2012) and the Lorenz dominance criterion (Lambert and Thoresen, 2012).

In this paper, we aim to push things further along. In Section 2, we give a brief overview of existing theoretical literature concerning the structural progression of income taxes. In Section 3, we carefully examine structural aspects of a pure form DIT schedule *per se*. In Section 4, we make some initial links between a DIT's structural progression measures, its revenue responsiveness and its redistributive effects. In our concluding Section 5, we consider the new work that could now ensue for DITs, and we briefly sketch out some issues of particular interest.

2. The income tax progression literature: a very brief review

The main two measures of structural progression for an income tax schedule are liability and residual progression, with average rate progression being a runner-up. Formal definitions will be given in the next section. The results already cited, which link increased liability and residual progression with enhanced distributional effects, and increased average rate progression with enhanced responsiveness of revenue to equiproportionate pre-tax income growth, are however contingent upon there being a

[1] The idea for a DIT originated in Denmark in 1985, and DIT systems were introduced in the other Nordic countries, and also in Iceland, in the late eighties and early nineties. A DIT system can now be found in Spain, and a DIT has been suggested as a candidate for a future new tax system in the UK in a contribution to the Mirrlees Review. See Sørensen (2005), Owen (2006), Genser and Reutter (2007), Griffith *et al.* (2010) and Calonge and Tejada (2011) on all of this.

fixed and common distribution of pre-tax income for all schedules being compared. Hayes *et al.* (1995) get around this drawback in the case of residual progression by conducting comparisons for different income tax schedules at common percentile points in the relevant pre-tax income distributions, whilst Dardanoni and Lambert (2002) introduce a transplant-and-compare procedure to make such comparisons, which is normatively sound and equivalent to the Hayes *et al.* procedure if income distributions are isoelastically linked.

In Hemming and Keen (1983), the core structural progression results are reformulated for "normalized tax schedules," standardizing all schedules under consideration to generate the same revenue. In Lambert (1984), links between structural progression and revenue responsiveness to non-equiproportionate income growth patterns are explored; see also Lambert, 1989, chapter 8, for an expanded treatment. The effect of changes in structural progression on labour supply are examined for a representative individual in Hemming (1980) and for a distribution of individuals by wage rate in Sandmo (1983). In Pfähler (1984), the distributional and revenue responsiveness consequences of progression-neutral tax changes are examined, along with effects on voter preferences. In Lambert and Pfähler (1992), the influence on post-tax income distribution of a range of specified changes in pre-tax income distribution are determined, conditional upon structural progression characteristics of the income tax schedule in force. In Keen et al. (2000) the core structural progression results of Jakobsson and Kakwani are extended to cater for the presence of allowances, deductions and credits in the tax code, and links from changing these, for a given tax rate structure, to inequality consequences are obtained. Ebert and Lambert (1999) determine structural progression characteristics for combined income tax and cash benefit systems, whilst in Ebert and Lambert (2004) an "equal progression among equals" criterion is articulated and shown to be fruitful.

However, there has been almost no analysis or even discussion of the structural progression characteristics of a DIT.[2] As such tax systems seem to be gaining in popularity (recall footnote 1), we judge it important to extend the measurement of structural progression to DITs. Some interesting new issues are raised by this exercise, as we shall see.

3. **Structural progression for a DIT system**

We begin with the case of a solo income tax schedule, where the tax liability is *t(x)* when income is *x*. Assuming differentiability, strict progression of *t(x)* implies and is implied by

$$(1) \qquad \frac{d}{dx}\left[\frac{t(x)}{x}\right] = \frac{xt'(x) - t(x)}{x^2} = \frac{m(x) - a(x)}{x} > 0 \quad \forall x$$

where $a(x) = \dfrac{t(x)}{x}$ and $m(x) = t'(x)$ are respectively the average and marginal rates of tax experienced by an income unit having x (for weak progression, one may replace the

[2] An exception is provided by Calonge and Tejada's (2011) application of Pfähler's (1984) reasoning to the reform of a dual income tax system.

strict inequality in (1) by a weak one; this would allow for e.g. a region of proportional taxation followed by strict progression at higher income levels; for notational simplicity, we shall assume strict progression in everything that follows). Structural progression may be specified in terms of *m(x)* and *a(x)*. Average rate progression measures the rate of increase of the average tax rate along the income scale:

$$ARP(x) = \frac{d}{dx}\left[\frac{t(x)}{x}\right] = a'(x) = \frac{m(x) - a(x)}{x} > 0 \,. \tag{2}$$

Liability progression at x is

$$LP(x) = \frac{m(x)}{a(x)} > 1 \tag{3}$$

that is, it measures the percentage response of tax liability *t(x)* to a small percentage change in income x (as an instantaneous elasticity). The tax schedule displays enhanced liability progression at x if *LP(x)* increases. Similarly, residual progression is

$$RP(x) = \frac{1 - m(x)}{1 - a(x)} < 1 \tag{4}$$

which measures the percentage response of post-tax (residual) income $x – t(x)$ to a small percentage change in x. The tax schedule displays enhanced residual progression at x if *RP(x) decreases*.[3] Note that for a proportional tax, average rate progression is zero, and liability and residual progression are both unity, at all income values.

We now consider structural progression for a DIT. The first complication is that now there are two income components. Let a person's income be $x = x_L + x_D$, where x_L is the labour income component and x_D is the capital income component (we simplify nomenclature, only, by describing the two types of taxed income in this way; transfer income would normally be included with labour income, and individuals' capital incomes may include not only dividends but also, for example, business owners' shares in the after-tax profits and retained earnings of their firms.[4] The tax liability on labour income is of the form $t_L = \tau(x_L)$ where $\tau(\bullet)$ is progressive, and the tax liability on capital income is of the form $t_D = \gamma x_D$, i.e. it is proportionate at a rate we shall call γ. Hence total tax is

$$t = t_L + t_D = \tau(x_L) + \gamma x_D \tag{5}$$

which, note, is *not* in general a function of total income $x = x_L + x_D$, so that equations (1)-(4) cannot be applied directly to the DIT as a whole. But they are relevant for the progressive labour component. Using Greek letters μ and α for the marginal and average rates of $\tau(\bullet)$, for ease of comparability with (1)-(4), the marginal rate $\mu(x_L) = \tau'(x_L)$ exceeds the average rate $\alpha(x_L) = \dfrac{\tau(x_L)}{x_L}$, and the average rate, liability and

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[3] Our expressions in (2)-(4) are for an income unit experiencing a positive tax liability, and we shall continue with this convention. See Keen *et al.* (2000) for the appropriate expressions when the tax function *t(x)* is non-differentiable, e.g. at a tax threshold caused by an allowance, exemption, or discrete step-up in the marginal tax rate. In Lambert (1989), minor changes *ARP(x)* and *RP(x)* and are proposed, namely, *ARP*(x) = xARP(x) = m(x) - a(x)* and *RP*(x) =1/RP(x)* the former in order to make the measure unit free, as the other two are, and the latter so that an increased value counts as an increase in progression at x. We shall not pursue these modifications in what follows for a DIT.

[4] For the latter, see Bø et al.. (2012) in respect of Norway.

residual progression measures are respectively $ARP_L(x_L) = \frac{\mu(x_L) - \alpha(x_L)}{x_L}$, $LP_L(x_L) = \frac{\mu(x_L)}{\alpha(x_L)}$ and $RP_L(x_L) = \frac{1-\mu(x_L)}{1-\alpha(x_L)}$, to adopt an obvious notation.

The overall average tax rate experienced by a person with income components (x_L, x_D) is

(6) $$a = \frac{t_L + t_D}{x_L + x_D} = \theta\alpha(x_L) + (1-\theta)\gamma$$

where $\theta = \frac{x_L}{x_L + x_D}$. The second complication we face is that, in order to quantify the effective marginal tax rate of the DIT as a whole, and thereby assess its progression measures, we need to make an assumption about how a small increase in a person's overall income would be apportioned between the two sources, and thereby, how that small increase would be taxed. We shall make the reasonable assumption that a small change $dx = dx_L + dx_D$ in the total income of a person with income components (x_L, x_D) is shared in the proportions $\theta : 1-\theta$ between sources, i.e. that $dx_L = \theta dx$ and $dx_D = (1-\theta)dx$. Then the increase in that person's overall tax liability is $dt = \tau'(x_L)dx_L + \gamma dx_D = \left[\theta\mu(x_L) + (1-\theta)\gamma\right]dx$, and thus the effective marginal tax rate for the person is:

(7) $$m = \frac{dt}{dx} = \theta\mu(x_L) + (1-\theta)\gamma$$

Equations (6) and (7) can be used to formulate measures of overall (or effective) structural progression for the DIT as a whole. These will be $ARP_{DIT}(x_L, x_D) = \frac{m-a}{x}$, $LP_{DIT}(x_L, x_D) = \frac{m}{a}$ and $RP_{DIT}(x_L, x_D) = \frac{1-m}{1-a}$, obtained as follows. First, subtracting (6) from (7), and dividing by *x*, we arrive at the measure of effective average rate progression of the DIT, which is a particularly straightforward function of the average rate progression of the labour income tax:

(8) $$ARP_{DIT}(x_L, x_D) = \frac{m-a}{x} = \theta\left[\frac{\mu(x_L) - \alpha(x_L)}{x_L} . \frac{x_L}{x}\right] = \theta^2 ARP_L(x_L) \in \left(0, ARP_L(x_L)\right).$$

Second, dividing (7) by (6), we can link the DIT's effective liability progression with that of the labour income tax:

(9) $$LP_{DIT}(x_L, x_D) = \frac{m}{a} = \frac{\theta\mu(x_L) + (1-\theta)\gamma}{\theta\alpha(x_L) + (1-\theta)\gamma} = \frac{\theta\alpha(x_L)LP_L(x_L) + (1-\theta)\gamma}{\theta\alpha(x_L) + (1-\theta)\gamma} \in \left(1, LP_L(x_L)\right)$$

and third, after only a little manipulation of (6) and (7), we find that

(10) $$\frac{1 - RP_{DIT}(x_L, x_D)}{1 - RP_L(x_L)} = \frac{\theta - \theta\alpha(x_L)}{1 - (1-\theta)\gamma - \theta\alpha(x_L)} \Rightarrow RP_{DIT}(x_L, x_D) \in \left(RP_L(x_L), 1\right).$$ [5]

[5] Since $\gamma < 1$, we have $1 - (1-\theta)\gamma > \theta$. Thus the second term in (10) lies between 0 and 1. Hence the result claimed.

We interpret these as *intra-personal* measures, obtained by assuming that each person's income variation (e.g. through time) is shared in the proportions $\theta : 1-\theta$, where θ specifies the mix between labour and capital income sources for the person concerned. Note that if for a given person θ theta falls because capital income rises with no change in labour income, or if θ is lower for one person than another with the same labor income, then all three measures of structural progression are correspondingly lower.

In general, we may suppose a joint frequency density function for income components, say $f(x_L, x_D)$, i.e. that there is a 'scatter' of (x_L, x_D)-values, with a degree of correlation between the two components on which we do not put any restriction: this correlation may be positive, zero or negative. In Calonge and Tejada (2011), a 'perfect alignment' assumption is made, according to which labour income and capital income increase together in cross-section; in Lambert and Thoresen (2012), an alternative 'perfect inverse alignment' assumption is articulated, according to which capital income falls as labour income rises in cross-section. These are instances of perfect positive and negative correlation respectively, but it is an empirical matter to determine the degree of correlation which actually pertains for any given application.

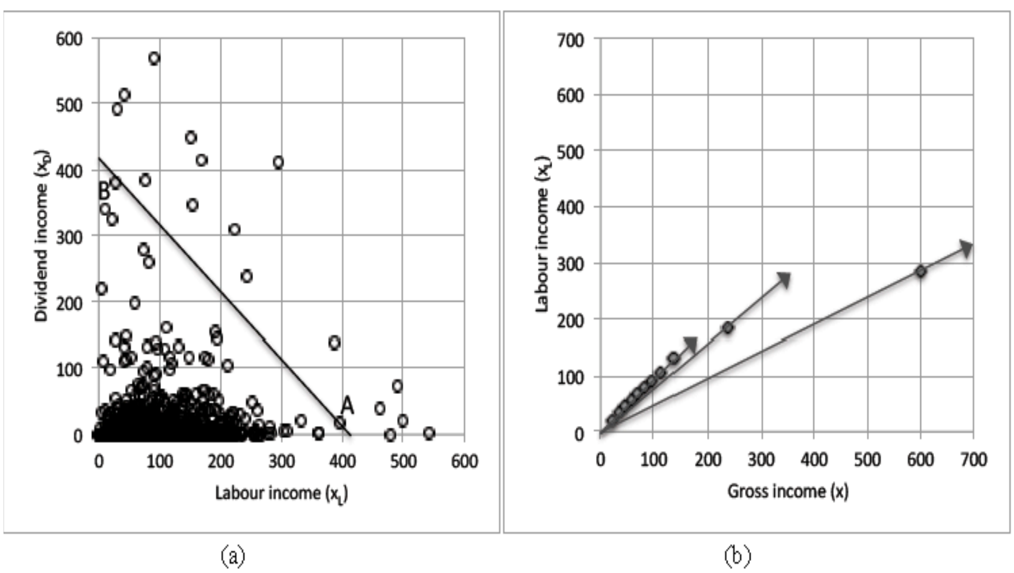

Figure 1: Scatter plots for Icelandic pre tax income (2006).

Annual amounts in thousands of Euros per household. Source: EU-SILC, authors´ calculations.

In Figure 1(a), we show a scatterplot of a sample of Icelandic income components (x_L, x_D) for households in 2006, showing also by means of a downward-sloping 45^{o} line a locus of points with the same total income. Two specific data points, one just above and the other just below this line, are marked as A and B respectively. Because A has a very high labour income, which is taxed more heavily than B's similarly high capital income, A is clearly worse-off after tax than B, a case of what is known as reranking, since the reverse is true before tax. Later, we return to the issue of reranking effects caused by a DIT. Figure 1(b) shows the average labour income x_L and total income x for each decile of this sample, along with a point representing the richest percentile. The arrows in Figure 1(b) show schematically how incomes evolve under our growth-sharing assumption. Each ray from the origin has slope $\theta = \frac{x_L}{x}$ indicating the share of total income accounted for by labour earnings at the point with coordinates (x, x_L); its continuation with an arrow indicates the 'expansion path', i.e. the direction in which income growth is assumed to take place. This conceptualization is particularly appropriate for examining the responsiveness of DIT revenue to the assumed growth pattern of individual or household incomes, which we address next. We shall then turn to redistributive effects, for which we will wish to use structural progression as an interpersonal descriptor, rather than an intrapersonal one. For this, we will have to adduce additional considerations, on the profile of θ across persons and income levels, as we shall see.

A significant finding in this section of the paper is that for each the three structural progression measures, the DIT is less progressive at (x_L, x_D) than the labour income tax schedule is at x_L: this is shown clearly by properties (8), (9) and (10) above.

4. Some overall effects of increases in the DIT's structural progression.

4.1 Revenue growth effects: average rate progression

The effects of some labour income tax reforms on the responsiveness of total DIT revenue to income growth can be determined by an analysis of average rate progression; the intra-personal nature of the effective progression measure $ARP_{DIT}(x_L, x_D)$ makes this quite straightforward. We measure the responsiveness of total DIT revenue to income growth here in two ways, by its elasticity to total income changes, and by its built-in flexibility.

Let $X = X_L + X_D$ and $T = T_L + T_D$ respectively be total income and total income tax. The overall revenue elasticity is $e = \frac{X}{T}.\frac{\partial T}{\partial X}$ and the component revenue elasticities are $e_L = \frac{X_L}{T_L}.\frac{\partial T_L}{\partial X_L}$ and $e_D = \frac{X_D}{T_D}.\frac{\partial T_D}{\partial X_D}$. Assuming that income growth in both components is equiproportionate at the same rate, $e = \frac{X_L + X_D}{T_L + T_D}.\left(\frac{\partial T_L}{\partial X} + \frac{\partial T_D}{\partial X}\right)$ $= \frac{X_L + X_D}{T_L + T_D}.\left(\frac{\partial T_L}{\partial X_L} + \frac{\partial T_D}{\partial X_D}\right)$. Thus we have

(11) $$e = \frac{X_L + X_D}{T_L + T_D}.\left(e_L \frac{T_L}{X_L} + e_D \frac{T_D}{X_D}\right) = g_L e_L + g_D e_D$$

where $g_i = \dfrac{T_i / X_i}{T / X}$ is the ratio of income source i's average tax rate to the overall average tax rate, $i = L,D$. The overall built-in flexibility is $b = \dfrac{\partial T}{\partial X}$, and the component built-in flexibilities are $b_L = \dfrac{\partial T_L}{\partial X_L}$ and $b_D = \dfrac{\partial T_D}{\partial X_D}$, so that $b = b_L + b_D$.

Now let

$$A_L(X_L) = \frac{\partial}{\partial X_L}\left[\frac{T_L}{X_L}\right] = \frac{1}{X_L^2}\left\{X_L \frac{\partial T_L}{\partial X_L} - T_L\right\} \tag{12}$$

be the 'average rate responsiveness' of the labour income tax, corresponding to the value defined by Hutton and Lambert (1979) in the case of a general income tax schedule. In the present context, if N is the number of income units and $f_L(\cdot)$ is the (marginal) frequency density function for labour income components, then $A_L(X_L) = \dfrac{N}{X_L^2}\displaystyle\int_0^\infty x_L^2 ARP_L(x_L) f_L(x_L) dx_L$ (see Hutton and Lambert, 1979, page 378). The labour income tax revenue elasticity is $e_L = 1 + \dfrac{X_L^2}{T_L} A_L(X_L) = 1 + \dfrac{N}{T_L}\displaystyle\int_0^\infty x_L^2 ARP_L(x_L) f_L(x_L) dx_L$; the capital income tax revenue elasticity is simply $e_D = 1$; the overall revenue elasticity is thus $e = g_L e_L + g_D e_D = g_L\left\{1 + \dfrac{N}{T_L}\displaystyle\int_0^\infty x_L^2 ARP_L(x_L) f_L(x_L) dx_L\right\} + g_D$ which, because (8) can be written as $(x_L + x_D)^2 ARP_{DIT}(x_L, x_D) = x_L^2 ARP_L(x_L)$, comes down to

$$e = g_L\left\{1 + \frac{N}{T_L}\int_0^\infty (x_L + x_D)^2 ARP_{DIT}(x_L, x_D) f_L(x_L) dx_L\right\} + g_D \tag{13}$$

in terms of the average rate progression measure for the DIT as a whole. The built-in flexibility of the labour income tax component is $b_L = \dfrac{T_L}{X_L} + X_L A_L(X_L)$ $= \dfrac{T_L}{X_L} + \dfrac{N}{X_L}\displaystyle\int_0^\infty x_L^2 ARP_L(x_L) f_L(x_L) dx_L$ (from (12)); that of the capital income tax component is $b_D = \dfrac{T_D}{X_D} = \gamma$; overall built-in flexibility is thus

$$b = \frac{T_L}{X_L} + \frac{N}{X_L}\int_0^\infty (x_L + x_D)^2 ARP_{DIT}(x_L, x_D) f_L(x_L) dx_L b_L + \gamma . \tag{14}$$

An increase in the average rate progression of the DIT can be achieved only by raising the average rate progression of the labour income tax ((8) shows this). A revenue-neutral increase in the average rate progression of the labour income tax keeps g_L, T_L and g_D fixed, and hence, from (13) and (14), raises both overall revenue elasticity and overall built-in flexibility. A revenue-enhancing increase in the average rate progression of the DIT raises overall built-in flexibility.[6]

4.2 Distributive effects: liability and residual progression

Our assumption on the expansion path for individual or household incomes has been that the share of labour earnings in an income unit's total income, $\theta = \frac{x_L}{x}$ (where, so far, θ is particular to the income unit), is maintained with income growth, i.e. that also $\theta = \frac{dx_L}{dx}$. To use the resulting structural progression measures interpersonally, we shall want the dx_L and the dx to represent the changes which take place from person to person. That is, $\frac{x_L}{x} = \frac{dx_L}{dx}$ for all x and x_L, or, rearranging, $\frac{dx_L}{x_L} = \frac{dx}{x}$ for all x and x_L, which, upon integration, yields $\ell n(x_L) = c + \ell n(x)$ for some constant c, i.e. that $x_L = \theta_0 x$ for all x and x_L, where $\theta_0 = \exp(c)$. We are thus led to a model for interpersonal use in which necessarily the θ we have described in the intra-personal context as particular to the income unit concerned actually needs to be *constant across all income units*.

Setting aside the question of the possible empirical validity of this model for the moment, consider what it implies for the DIT. Total tax is now

(14) $$t = \tau(\theta_0 x) + \gamma(1-\theta_0)x$$

(compare (5)), i.e. this is a total-income-denominated tax, whose liability and residual progression measures enjoy the well-established classical links with distributive properties. The average and marginal tax rates are

(15) $$\frac{t}{x} = \theta_0\alpha(\theta_0 x) + (1-\theta_0)\gamma$$

and

(16) $$\frac{dt}{dx} = \theta_0\mu(\theta_0 x) + (1-\theta_0)\gamma$$

which closely resemble (6) and (7), the latter of which, after all, was derived by treating θ as a constant while x and x_L varied together. The resulting measures of structural progression for the DIT for this model are

(17) $$LP_{DIT}(x) = \frac{\theta_0\mu(\theta_0 x) + (1-\theta_0)\gamma}{\theta_0\alpha(\theta_0 x) + (1-\theta_0)\gamma} = \frac{\theta_0\alpha(\theta_0 x)LP_L(\theta_0 x) + (1-\theta_0)\gamma}{\theta_0\alpha(\theta_0 x) + (1-\theta_0)\gamma} \in \left(1, LP_L(\theta_0 x)\right)$$

and

(18) $$\frac{1-RP_{DIT}(x)}{1-RP_L(\theta_0 x)} = \frac{\theta_0 - \theta_0\alpha(\theta_0 x)}{1-(1-\theta_0)\gamma - \theta_0\alpha(\theta_0 x)} \Rightarrow RP_{DIT}(x) \in \left(RP_L(\theta_0 x), 1\right)$$

and one can readily connect changes in the liability and/or residual progression of the

[6] An example of a revenue-*reducing* increase in the average rate progression of the labour income tax would be the introduction of a lump-sum tax credit (see Hutton and Lambert, 1979, page 379), but the effect of this on overall revenue elasticity and built-in flexibility depends on relative magnitudes and cannot be signed a priori: in particular g_L falls and g_D rises. Raising the tax rate γ on capital income, all else fixed, would increase g_D and lower g_L. The effect on revenue elasticity is indeterminate, but built-in flexibility would be increased.

labour income tax with changes in the liability and/or residual progression of the DIT using these equations and thereby, using the standard theory, with overall distributive effects.

Namely, a reform which increases the liability progression of the labour income tax and also increases all labour income tax liabilities raises the liability progression of the DIT, and thereby the disproportionality in the overall tax burden. And conversely, a reform which decreases the liability progression of the labour income tax and also decreases all labour income tax liabilities lowers the liability progression of the DIT and thereby the disproportionality in the overall tax burden. The results for residual progression are slightly different (recall that an increase/decrease in an RP-measure connotes a reduction/enhancement of progression). A reform which augments the residual progression of the labour income tax and decreases all labour income tax liabilities augments the residual progression of the DIT, thereby enhancing overall redistributive effect; whilst a tax reform which diminishes the residual progression of the labour income tax and increases all labour income tax liabilities diminishes the residual progression of the DIT and thereby the overall redistributive effect of the DIT.

These are very standard results, which do not extend our knowledge of the distributive effects of DIT reforms significantly because, after all, the model in which they derive is merely a variant of the classical model in which tax is a well-defined function of total income.[7]

Moreover, the question of empirical validity clearly needs to be addressed for this model. In Figure 2, we show values of θ by decile, and for the top percentile, in four Nordic countries all of which have DITs. With the exception of Iceland, θ is approximately constant at about $\theta \approx 0.98$ across all deciles but the last, in fact falling slightly for the higher deciles. The top 10% and especially the top 1% have much larger shares of their incomes coming from capital.[8]

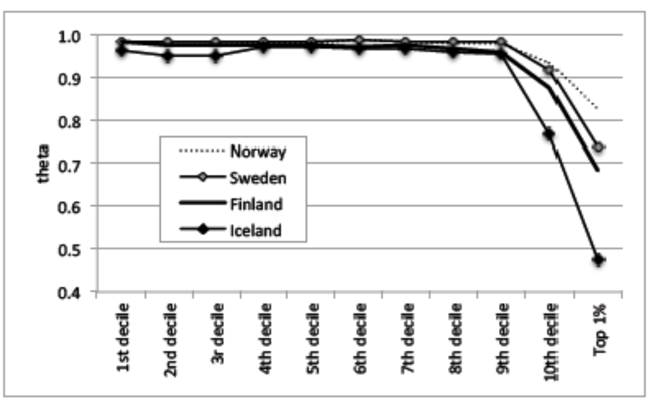

Figure 2: Profile of θ by decile and for top 1% in four Nordic countries.
Deciles are defined by households (2006). Source: EU-SILC, authors´ calculations.

[7] Laborda (2006) considered this model, and showed that the residual and liability progression of this DIT are less than those of the labour income tax contained within it (see his Proposition 1), as we have shown to be true for our general model in Section 3.

[8] See Atkinson and Piketty (2010) for trends within the top 1%, down to the top 0.01%, in some Nordic countries.

A model in which θ is a decreasing and weakly concave function of total income x might thus be more appropriate for Scandinavian DITs, and such a model has in fact also been considered by Laborda (2006).[9] The tax liability in the DIT of this model is still a well-defined function of total income, raising no new issues of principle for us, but there is an interesting twist. Writing tax liability as

(19) $$t = \tau(\theta(x)x) + \gamma(1-\theta(x))x$$

the average rate $\frac{t}{x}$ is

(20) $$\frac{t}{x} = \theta(x)\alpha\big(\theta(x)x\big) + \big(1-\theta(x)\big)\gamma$$

(this as in (15), but with θ_0 replaced by $\theta(x)$), and the marginal rate becomes

(21) $$\frac{dt}{dx} = \{\theta + x\theta'\}\mu(\theta x) + (1-\theta-x\theta')\gamma$$

which is smaller than the value in (16), for which θ is held constant at $\theta = \theta_0$, by the amount $x|\theta'|\big(\mu(x_L)-\gamma\big)$, because the increase in θ reduces the proportion of total income x which is subjected to the marginal tax rate $\mu(x_L)$ for labour income and raises the proportion which is subjected to the marginal tax rate γ on capital income (which, for a pure form DIT, is lower). We may call $x|\theta'|\big(\mu(x_L)-\gamma\big)$ the "indirect effect" of the income increase, and think of it as an anti-progressive element which is built into the DIT, in that, as overall incomes increase along the income scale, there is a tax break coming from the change in income composition towards the lower-taxed capital component. The links from changes in the structural progression of the labour income tax to changes in the structural progression of the DIT are moderated by the magnitude of this opposing effect,[10] but the connections from the structural progression of the DIT to overall distributive effects are the standard ones of the classical literature and will not be articulated here.

5. Discussion and ways forward

In the pure-form DIT model, which combines a moderate flat tax on capital income with a progressive labour income tax, there are additional instruments for the policymaker, as compared to the more straightforward case of a comprehensive income tax with a unified base. Discretionary change in tax parameters is often designed to achieve limited objectives such as more revenue, tax simplification or a change in the balance of taxation. An important constraint upon such discretionary change is likely to be that the distributional consequences be controlled or minimized.

We have looked closely at the revenue responsiveness consequences of income growth under a DIT system, and more briefly at the ways in which our structural progression measures could be applied in distributive analysis. We have found that the

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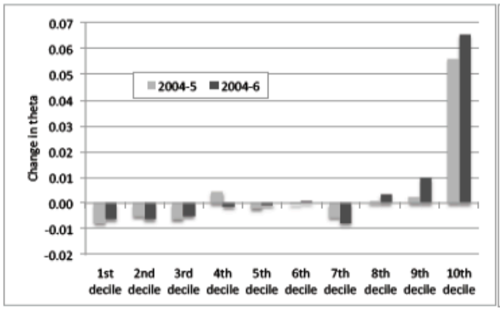
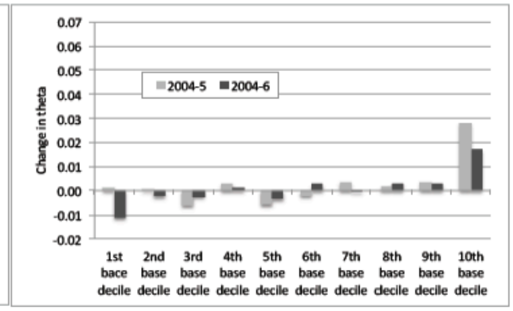
[9] Laborda's assumption is that capital gain income is an increasing and weakly convex function of wage income, which translates into the model we consider here.

[10] Laborda's (2006) Propositions 2 and 3 explore these effects.

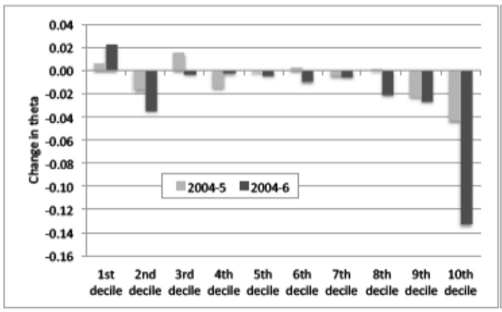
most appropriate model for distributive analysis is in a sense data-driven, in that the profile of θ comes very much into the picture; income distribution could change, e.g. between years, and even with no change in the DIT system itself, new modeling could be called for. In Figure 3, we consider how the profile of θ has changed between years in the four Nordic countries previously considered. In part (a) for each country, we plot changes in θ across deciles, and we see that the major changes have been at the very top: most of the magnitudes are changing by about one percentage point per year or per two years only. Between years, mobility in pre-tax income positions may also be at play, causing income units to move into different deciles. Such mobility effects are subsumed in the part (a) values, and hence these do not necessarily measure changes in θ which have actually been experienced by households, rather they track changes in θ-values from one year's decile occupants to the occupants of the same decile in the subsequent year. In part (b) for each country, we have used panel data to plot changes in θ *for deciles of the base-year distribution*: these are purely intra-personal changes in θ, tracking actual growth experiences of households. As can be seen, the picture changes. In some cases, 'more seems to be going on' in the panel values, except at the very top (Iceland, Norway), and in others the comparison is less clear (Finland, Sweden). Future modeling in this respect will need to be quite careful, if additional distributive findings for DITs are to be gained.



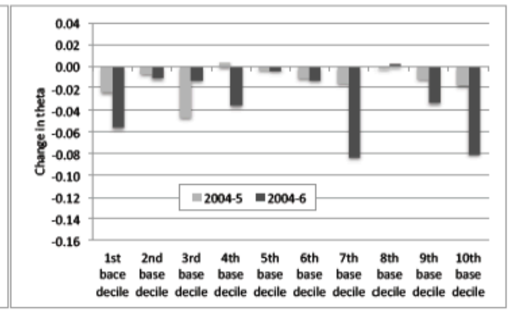
(a) Finland – Cross-Section



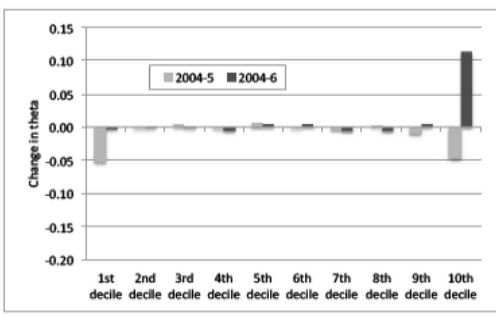
(b) Finland – Panel



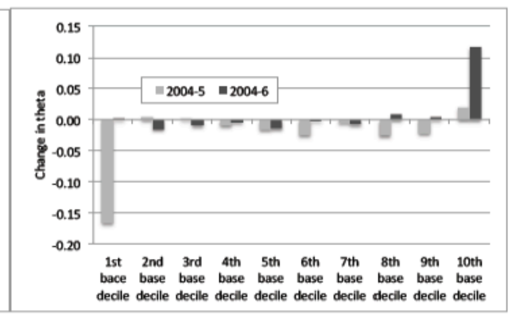
(a) Iceland – Cross-Section



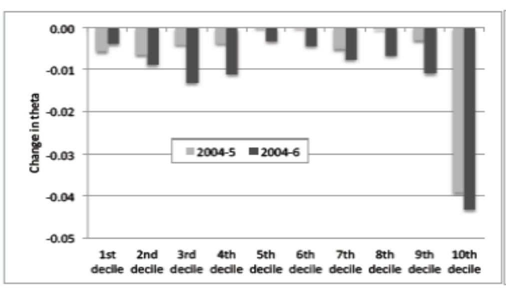
(b) Iceland – Panel



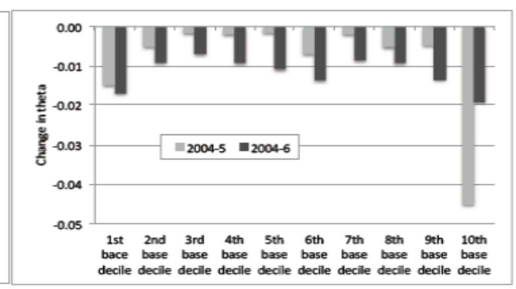
(a) Norway – Cross-Section



(b) Norway – Panel



(a) Sweden – Cross-Section



(b) Sweden – Panel

Figure 3: Changes in θ by decile in cross-section and using panel data.

Deciles are defined by households. Source: EU-SILC, authors´s calculations.

Reranking poses a further problem for the evaluation of progression in a DIT. In our modelling, we first fixed θ for each income unit, to yield intra-personal measures of effective progression, and we went on to explore fixing θ across all income units, finally letting θ be defined as a function of total income *x.* In neither of these two special cases does the DIT induce reranking. In real world however, as we have seen just above, θ is not a straightforward function of x and its values differ substantially across income units, even though on average θ decreases with income (see figures 1 and 2 on this). Within the labour income tax we typically have some degree of reranking, for example caused by income units' differing access to allowances and deductions. It is by no means obvious a priori whether this would be re-inforced or indeed offset by the additional reranking that a DIT can induce given similarly-placed income units' differing labour and capital income components. Nothing can be said in general, meaning that reranking essentially comes down to an empirical question, although a Gini coefficient and concentration index methodology is available to process the relevant empirical information: see Kristjánsson (2012) for this.

Additional questions, not germane to the case of a solo income tax schedule with a unified base, can hopefully also be addressed in terms of the structural progression measures we have developed here. First, is there a notion of "the right" or "an acceptable" degree of progression for a DIT, politically – in these days of widening gaps between rich and poor (OECD, 2011) and growing concern with the morality of capital markets (Atkinson, 2009)? Second, must the capital income component necessarily be proportional, implying a leniency towards the taxation of capital, and what would be the global effects of changing that? And third, what would constitute the "right degree of harmonization" between the labour and capital tax rates in order to control or eliminate tax-base shifting (under which e.g. small business owners may elect to pay themselves shareholder income in place of managerial wages in order to reduce their taxes)?[11]

Questions like these can hopefully be addressed now that we have framework for measuring and assessing the structural progression characteristics of a DIT system. It is plain that much work remains to be done, both theoretical and empirical. This paper constitutes only a beginning.

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[11] See Thoresen and Alstadsæter (2010) and Pirttilä and Selin (2011) on this, with respect to the Norwegian and Finnish experiences. Note that if a small business owner receives managerial wages in period 1, and then switches to shareholder income in period 2, the value of θ for that individual would experience a discontinuous reduction between periods, complicating an intertemporal analysis.

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