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Abstract
In this paper, a new class of polarization measures is derived axiomatically. The concept of polarization is here identified with the decline of the middle class. In particular, we extend the definition of middle class towards a more realistic framework: the middle class is defined in terms of central interval rather than median income. Then polarization is measured both as the presence of well-separated poles and as the dispersion inside the middle class. This new class of indices can be seen as a generalization of existing measures of income polarization. Also, a new polarization ordering is introduced. The new approach is illustrated with an application to EU countries.

Keywords: axioms, transfers, polarization measurement, income distribution.

JEL Classification: C43, D63.

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1 Introduction

Income polarization measurement is one of the most relevant issues recently raised up in the welfare economics literature beyond poverty and inequality analysis. It is commonly connected with the division of a society into groups as a possible cause of social conflicts (see for example Esteban and Ray (1999) and Chakravarty (2009)).[1]

Monitoring the degree of polarization in a given income distribution typically means measuring not only how poorer are getting the poor but also how richer are getting the rich and hence how distant these two groups are one from the other. The further the two groups are one from the other and at the same time the more cohesive inside they are, the harder it will be communicating and interacting one to the other.

Two strands are distinguished in the literature on income polarization: the first one, going back to Wolfson (1994, 1997) and Foster and Wolfson (2010), is a measure of the shrinking middle class, monitoring how the income distribution spreads out from its center. The second strand, originating from Esteban and Ray (1994), focuses on the rise of separated income groups: polarization increases if the population groups are getting more homogeneous inside and more separate one to the other. These pioneering contributions have been followed by many others, such as Wang and Tsui (2000), Gradín (2000), Chakravarty and Majumder (2001), D´Ambrosio (2001), Zhang and Kanbur (2001), Duclos et al. (2004), Anderson (2004), Esteban et al. (2007), Massari et al. (2009), Chakravarty and D´Ambrosio (2010), Lasso de la Vega et al. (2010), Yitzhaki (2010), Pittau et al. (2010), Permanyer (2012), Silber, Deutsch and Hanoka (2007), Lasso de la Vega et al. (2006), Chakravarty and Maharaj (2011).

In the Wolfson's approach the middle class constitutes a crucial element. By this class a group of people is meant who are close enough in their socio-economic status to be able to cooperate and form a common political will. A strong middle class has a beneficial influence on the society, as it provides a buffer between the extreme tendencies of the lower and upper social classes; see Pressman (2007). Easterly (2001) for example shows that a higher share of income for the middle class is associated with higher growth, more education, better health status and less political instability in the society. In this context, the decline of the middle class in a developed country signifies a threat

[1] This strand of economic literature is strictly linked with the analysis of social polarization and segregation; see, among others, Chakravarty and Maharaj (2011), Chakravarty et al. (2007) and Hutchens (1991).

for economic growth and socio-political stability.

Wolfson (1994, 1997) and the authors who have followed his approach (in particular, Wang and Tsui (2000), Chakravarty and Majumder (2001), Rodríguez and Salas (2003), Chakravarty et al. (2007) and Chakravarty and D´Ambrosio (2010)) define the middle class using the median income as a reference point, considering the middle class as the group of individuals whose income is exactly equal to the median income. The closer the incomes are to the median the less polarized is the distribution, while the presence of two well separated poles at the right and at the left of the median income identifies a highly polarized income distribution.

From our point of view, the middle class should be defined in a more reasonable way. In particular, we propose to enlarge the definition of middle class from the median income to an interval of incomes that are around the median.

There exist some contributions in the economic literature that define the middle class starting from an interval of income values. On one side, a group of authors interested in monitoring the declining middle class in the United States (among which Blackburn and Bloom (1985), Thurow (1987), Horrigan and Haugen (1988), Beach et al. (1997) and Pressman (2007)) identifies the middle class either as the $h\%$ of the population closest to the center of the distribution (with h equal, for example, to 20, 30 or 60) or as the group of individuals whose income belongs to the interval between the $p\%$ and the $q\%$ of the median income, with p and q given (e.g., $p = 60$ and $q = 225$ in Blackburn and Bloom (1985)). These authors then measure the decline of the middle class by looking only inside this central class and monitoring either its size or its income range or share. Throughout this paper we will refer to this group of contributions as the *U.S. shrinking middle class literature*.[2] On the other side, Scheicher (2010) propose to measure the phenomenon of disappearing middle class by looking at the dispersion of the poor and the rich incomes from a middle class defined in terms of median interval. Similarly to the Wolfson's approach, thus, these authors do not take into account how income is distributed within the middle class, focusing only outside this group. Both these approaches neglect some part of the income distribution, losing important information.

Aim of this paper is, therefore, to introduce a new class of polarization indices that measures the decline of the middle class in a more realistic way, by defining the middle class in terms of a median interval of incomes and by monitoring changes in incomes occurring along the entire

[2]For a recent review of this literature, we refer to Atkinson and Brandolini (2011).

income distribution. Our class of polarization indices has the advantage of creating a link between the Wolfson's approach to polarization (that looks outside the median income), on one side, and the U.S. shrinking middle class literature (that looks only inside the middle class), on the other side.

Since "distributional studies have focused mainly on the poor and on the rich, leaving out the middle" (Atkinson and Brandolini (2011), page 3), the original contribution of this paper is also to analyze the entire income distribution.

In particular, we measure polarization by dividing population into three groups (the poor, the middle class and the rich) and monitoring how far the rich and the poor are from the middle class and how cohesive these three groups are internally. Therefore, the measure of polarization is here considered in terms of: (i) distance between the poor and the middle class, (ii) distance between the rich and the middle class, and (iii) dispersion within the middle class.

The choice of the middle class is arbitrary, as remarked also in Beach (1989) and Pressman (2007), since it depends on the choice of the thresholds that divide the middle class from the rest of the population. Thus our class of polarization measures may produce contradictory conclusions at different reasonable choices of the middle class. In order to reduce this arbitrariness, we consider a set of values for defining the middle class, as it is done in the theory of poverty measurement; see, e.g., Zheng (2000). Therefore, comparisons of income distributions may be done that are uniform in a range of alternative middle classes.

In the following, we provide an axiomatic characterization of the new class of indices, based on two main sets of axioms: invariance axioms and polarization axioms. To demonstrate the usefulness of this new class of measures, we then illustrate its properties through an empirical application to the European Survey on Income and Living Conditions (EU-SILC) data for a selection of European countries, namely Austria, Belgium, France, Italy, Norway, Portugal, Spain and Sweden, in the years 2004-2007. This empirical application demonstrates the usefulness of our polarization index, since the new measure reveals additional information beyond the one provided by the polarization indicators existing in the literature.

The paper is organized as follows. In Section 2 we introduce notation and framework. In Section 3 we discuss about properties and axioms desired for a polarization index, and provide a characterization for a new class of polarization measures. Section 4 discusses special cases of the class of polarization indices, while Section 5 introduces a new polarization ordering. In Section 6 we apply the new indices and orderings to real data and in Section 7 we conclude the paper.

2 Framework

In this paper we follow the Wolfson's approach and conceive polarization in terms of declining middle class. Two are the main steps for measuring the disappearing middle class: (i) the definition of middle class and (ii) the measure of its shrinkage.

In the economic literature, there still exists an open debate on how to define the middle class: the notion of middle class appears to be still vague and arbitrary, as no consensus exists among researchers on how to cut income distribution in order to separate the middle from the remaining population; several authors have already pointed out this issue, among which Atkinson and Brandolini (2011), Eisenhauer (2008), Pressman (2007) and Jenkins (1995). In particular, as already discussed in the previous section, Wolfson's index and its generalizations identify the middle class with the median income and measure the distance from this central class, whereas the U.S. shrinking middle class literature defines the middle class as a central interval of values and monitor the income distribution inside this group.

The class of measures proposed in this paper does not identify the middle class in the way proposed by Wolfson (1994, 1997), but it rather follows the definition of the U.S. shrinking middle class literature: we define the middle class as the group of individuals whose income falls within a given range of central values.

The proposed measures divide the population into three non-overlapping income classes: individuals whose income is lower than any income of the middle class, individuals whose income is included in the middle class' income interval, individuals whose income is greater than any income of the middle class. For simplicity, we name the thresholds that separate the middle class from the rest of the population as the poverty line and the richness line, respectively. However, our approach allows also for alternative definitions of these thresholds. After having defined properly the poverty and the richness lines, we denote as *the poor* the individuals below the poverty line, as *the rich* the persons whose income is greater than the richness line, and as *the middle class* the individuals whose income is between these two lines.

Moving to the second step, we propose to measure the shrinkage of the middle class both as (i) the distance between the individuals outside the middle class and this central class and as (ii) the income dispersion within the middle class. We are interested, therefore, in monitoring the distribution of incomes not only outside the middle class but also within this central group, thus exploiting the whole information provided by the income distribution.

The novelty of our polarization measure consists therefore of linking together the approach of the U.S. shrinking middle class literature, which is interested in looking only inside the middle class, and the approach of Wolfson (1994, 1997); Foster and Wolfson (2010) and Wang and Tsui (2000), who focus on the individuals outside the median income.

Similar to our work, Scheicher (2010) and Peichl et al. (2010) have recently proposed a new class of polarization indices based on the same extended definition of middle class as we use here; in particular, their measure is an aggregation of measures of poverty and affluence, thus discarding the middle class' incomes. However, two are the main differences between their approach and our class of measures: (i) we provide an axiomatic characterization of the class of indices and (ii) we monitor the income distribution also inside the middle class.

Let us now introduce some notation.

Denote with z_1 and z_3 the two thresholds that split the population in three groups, the poor, the middle class and the rich; in this paper, we refer to z_1 as the poverty line and to z_3 as the richness line. They could be any thresholds that create non-overlapping groups, such as income quantiles or some percentages of the median or mean income.[3] Let $\mathcal{X}^n = \{\mathbf{x} = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n_+ : 0 \leq x_1 \leq ... \leq x_{n_1} < z_1 \leq x_{n_1+1} \leq ... \leq x_{n_2} \leq z_3 < x_{n_2+1} \leq ... \leq x_n\}$ be the set of n-dimensional vectors of increasingly ordered incomes such that n_1 individuals have income strictly lower the poverty line (the poor), $n_2 - n_1$ have income between the poverty and the richness line (the middle class) and $n - n_2$ have income strictly greater than the richness line (the rich).

Our polarization measure is an aggregation of the individual contributions to polarization; for the poor and the rich the contribution is the gap to the middle class, while for the individuals belonging

[3]There is a huge debate in the income distribution literature about the definition of the poverty and richness thresholds, due to the fact that their definitions are usually arbitrary. Few attempts have been provided in the literature for defining the thresholds endogenously; for example, D´Ambrosio et al. (2000) apply the change point theory to determine endogenously income thresholds and income classes.

to the middle class the contribution is the distance between their income and some central income value. Each individual is therefore endowed with a specific gap between her current income and her specific reference income level. An individual reference income is the value of income that, if replaced to her current income, would eliminate income polarization.

Our analysis will be therefore based on the vector of individual gaps $\mathbf{d} = (d_1(x_1, m_1), d_2(x_2, m_2), ..., d_n(x_n, m_n))$, where d_i is the distance function of individual i that depends on her income x_i and on her reference income m_i. Let $\mathcal{D}^n = \{\mathbf{d} = (d_1(x_1, m_1), d_2(x_2, m_2), \ldots, d_n(x_n, m_n)) \in \mathbb{R}^n_+ : (x_1, x_2, ..., x_n) \in \mathcal{X}^n\}$ be the set of n-dimensional vectors of individual distances. In this paper we consider as reference income for the poor the poverty line z_1, for the rich the richness line z_3, and for an individual belonging to the middle class some central income value z_2, such that $z_1 \leq z_2 \leq z_3$. The central value z_2 may be any centrality parameter that represents the middle class, such as the median or the mean.[4] Therefore, $m_i = z_1$ $\forall i = 1, \ldots, n_1$, $m_i = z_2$ if $i = n_1 + 1, \ldots, n_2$, and $m_i = z_3$ for $i = n_2 + 1, \ldots, n$. Throughout the paper the thresholds z_1, z_2, z_3 will be considered fixed and exogenous. In this way, a poor (a rich) reduces polarization if she replaces her income with the poverty (richness) line and thus enters the middle class, while an individual belonging to the middle class reduces polarization by replacing her income with the median or the mean income and thus increasing the cohesion within the middle class.[5]

3 Characterization of a new class of polarization indices

We follow an axiomatic approach and provide a characterization for the class $\mathcal{P}$ of polarization indices $P : \mathcal{D}^n \rightarrow \mathbb{R}^+$ that satisfy a given set of suitable axioms.

The first group of axioms that we consider (in Section 3.1) is standard in the income distribution analysis and requires the class of indices to be invariant under simple transformations of the data. In Section 3.2 we then discuss axioms that are peculiar to polarization analysis.

[4] Note that in Wang and Tsui (2000) the reference income is the same for all the individuals and it corresponds to the median income.

[5] Our approach, however, is even more flexible, and allows also for alternative definitions of the reference incomes. For example, we may use z_1 and z_3 for defining the non-overlapping groups and then identify z_2 as the common reference income for all groups.

3.1 Invariance axioms

The first axiom states that the polarization measure can be represented by separable individual contributions to polarization.

Axiom 1 (Independence)**.** *For any* $\mathbf{d}, \tilde{\mathbf{d}} \in \mathcal{D}^n$*, if* $P(\mathbf{d}) = P(\tilde{\mathbf{d}})$ *and* $d_i = \tilde{d}_i$ *for some* $i \in \{1, 2, ..., n\}$*, then* $P(d_1, ..., d_{i-1}, \beta, d_{i+1}, ..., d_n) = P(\tilde{d}_1, ..., \tilde{d}_{i-1},\ \beta,\ \tilde{d}_{i+1}, ..., \tilde{d}_n)$ *for any* $\beta \in \mathbb{R}^+$*.*

Independence assumes that if polarization arising from the vector of gaps $\mathbf{d}$ is the same as polarization arising from $\tilde{\mathbf{d}}$, then by replacing the identical generic i-th element both in $\mathbf{d}$ and in $\tilde{\mathbf{d}}$ by β, polarization still remains the same.

The second axiom requires the polarization index to be a continuous and differentiable function almost everywhere.

Axiom 2 (Continuity)**.** P *is a continuous and differentiable function almost everywhere on* $\mathcal{D}^n$*.*

The following axiom states that if at least one individual distance increases, then polarization should not decrease. In particular, polarization is increasing in the distances (i) between a poor and the middle class, (ii) between a rich and the middle class, and (iii) between an individual belonging to the middle class and the median or middle income.

Axiom 3 (Monotonicity)**.** *For any* $\mathbf{d}, \tilde{\mathbf{d}} \in \mathcal{D}^n$*, if* $\tilde{\mathbf{d}}$ *is obtained from* $\mathbf{d}$ *by adding* $\beta \in \mathbb{R}_+$ *to a generic element* d_i *of* $\mathbf{d}$*, for some* $i = 1, 2, ..., n$*, then* $P(\mathbf{d}) \leq P(\tilde{\mathbf{d}})$ *for any* $\beta \in \mathbb{R}^+$*.*

Population Proportionality states that, if we replicate α-times the population (or, equivalently, the vector of distances), polarization should remain unchanged. This axiom allows to compare distributions of different cardinality.

Axiom 4 (Population Proportionality)**.** *For any* $\mathbf{d} \in \mathcal{D}^n$ *and any* $\alpha \in \mathbb{N}_+$*,*

$$P(\mathbf{d}) = P(\underbrace{\mathbf{d}, \mathbf{d}, \ldots, \mathbf{d}}_{\alpha times}). \tag{1}$$

The next axiom is a slight modification of the well-known Anonimity axiom and states that polarization is indifferent to the labeling of individuals within each of the three socio-economic groups: within the poor, within the middle class and within the rich names do not matter, but just the levels of income are important.

Axiom 5 (Within-group Anonymity)**.** *For any* $\mathbf{d}$, $\mathbf{d}^\pi \in \mathcal{D}^n$, *if*

$$\mathbf{d}^\pi = \left(d_{\pi_1(1)}, \ldots, d_{\pi_1(n_1)}, d_{\pi_2(n_1+1)}, \ldots, d_{\pi_2(n_2)}, d_{\pi_3(n_2+1)}, \ldots, d_{\pi_3(n)}\right),$$

where π_1, π_2 *and* π_3 *are permutation functions defined, respectively, on the group of the poor, on the middle class and on the group of the rich, then* $P(\mathbf{d}) = P(\mathbf{d}^\pi)$.

The last axiom in this section imposes a lower bound to the values of the index.

Axiom 6 (Normalization)**.** *For any* $\mathbf{d} \in \mathcal{D}^n$*,(i)* $P(\mathbf{d}) \geq 0$ *and (ii)* $P(\mathbf{d}) = 0$ *if and only if* $x_i = z_2$ *for all* $i = 1, \ldots, n$.

Normalization states that polarization is always non-negative, and the absence of polarization coincides with the egalitarian distribution.

The following proposition provides a characterization for the polarization indices that satisfy the set of invariance axioms introduced above.

Proposition 1. *A polarization index* $P \in \mathcal{P}$ *satisfies Independence, Continuity, Monotonicity, Population Proportionality, Within-group Anonymity and Normalization if and only if*

$$P(\mathbf{d}) = \frac{1}{n}\left\{\sum_{i \in G_1} \tau\left(d_i(x_i, z_1)\right) + \sum_{i \in G_2} \xi\left(d_i(x_i, z_2)\right) + \sum_{i \in G_3} \psi\left(d_i(x_i, z_3)\right)\right\}, \tag{2}$$

where $G_1 = \{i \in \{1, \ldots, n\} : x_i < z_1\}$, $G_2 = \{i \in \{1, \ldots, n\} : z_1 \leq x_i \leq z_3\}$, $G_3 = \{i \in \{1, \ldots, n\} : x_i > z_3\}$, *and* $\tau, \xi, \psi : \mathbb{R}_+ \to \mathbb{R}_+$ *are continuous and differentiable almost everywhere, strictly increasing functions such that* $\xi(0) = 0$.

Proof. The necessary condition is straightforward. We focus on the sufficient condition.
By Continuity and Monotonicity, P is continuous and differentiable a.e. and strictly increasing in $\mathcal{D}^n$. By Independence it follows from Theorem 5.5 in Fishburn (1970) that for $n \geq 3$

$$P(\mathbf{d}) = \phi^{-1}\left(\sum_{i=1}^{n} \phi_i(d_i(x_i, m_i))\right)$$

where the functions ϕ and ϕ_i, for $i = 1, ..., n$ are continuous, differentiable and strictly increasing.
By Normalization, $\xi(0) = 0$ and $\tau, \xi, \psi : \mathbb{R}_+ \to \mathbb{R}_+$.
By Population proportionality:

$$P(\mathbf{d}) = P(\underbrace{\mathbf{d}, \mathbf{d}, \ldots, \mathbf{d}}_{\alpha \text{ times}}), \tag{3}$$

by choosing $\phi^{-1}(\cdot)$ the identity function and by Theorem 4 in Shorrocks (1980), this forces $P(\mathbf{d})$ to be of the form:

$$P(\mathbf{d}) = \frac{1}{\alpha}\sum_{i=1}^{n}\phi_i(d_i(x_i, m_i)). \tag{4}$$

Choosing $\alpha = n$:

$$P(\mathbf{d}) = \frac{1}{n}\sum_{i=1}^{n}\phi_i(d_i(x_i, m_i)). \tag{5}$$

By Within-group Anonymity,

- $\phi_i(d_i(x_i, m_i)) = \tau(d_i(x_i, z_1))$ for each $i \in \{1, ..., n_1\}$ such that $x_i < z_1$,
- $\phi_i(d_i(x_i, m_i)) = \xi(d_i(x_i, z_2))$ for each $i \in \{n_1 + 1, ..., n_2\}$ such that $z_1 \leq x_i \leq z_3$,
- $\phi_i(d_i(x_i, m_i)) = \psi(d_i(x_i, z_3))$ for each $i \in \{n_2 + 1, ..., n\}$ such that $x_i > z_3$,

and therefore

$$P(\mathbf{d}) = \frac{1}{n}\left(\sum_{i=1}^{n_1}\tau(d_i(x_i, z_1)) + \sum_{i=n_1+1}^{n_2}\xi(d_i(x_i, z_2)) + \sum_{i=n_2+1}^{n}\psi(d_i(x_i, z_3))\right). \tag{6}$$

Defining $G_1 = \{i : i = 1, \ldots, n_1\}$, $G_2 = \{i : i = n_1 + 1, \ldots, n_2\}$ and $G_1 = \{i : i = n_2 + 1, \ldots, n\}$ we get expression (2). $\square$

Note that individuals with income exactly equal to the poverty line or to the richness line are considered as part of the middle class. Within each group the index in (2) is continuous, in sense that if any gap slightly varies (without changing the class), the index does not jump. Moreover, the polarization measures characterized in Proposition 1 are equal to zero if and only if (i) none has income below the poverty line, (ii) none has income above the richness line, and (iii) the middle class is totally cohesive, in sense that individuals belonging to the middle class have income equal to the central income z_2. Therefore, the distribution of incomes inside the middle class does matter: if everyone belongs to the middle class, polarization will be zero only if all the individuals have the same income, equal to the central income z_2, while there will be positive polarization if income is uniformly distributed over the middle class.[6]

[6]This property clearly distinguishes our class of polarization indices from the ones proposed in Scheicher (2010).

3.2 Polarization axioms

We now introduce a set of axioms that are specific to polarization measurement and able to outline the main characteristics of this phenomenon. In these axioms we state that polarization is affected by (i) the dispersion of incomes within the group of the poor and within the group of the rich (Axiom 7), (ii) the distance between these two groups (Axioms 8 - 10) and (iii) the cohesion within the middle class (Axiom 11).

The first axiom requires that a clustering of incomes below or above the middle class leads to a distribution at least as polarized as before. This means that if incomes become more similar within the rich and within the poor, then the two groups become more cohesive in terms of socio-economic interests and may develop a stronger influence in taking decisions that affect the entire society.

The particular type of incomes' movement that is required by this axiom is described in the following definition.

Definition 1 (Increased Bipolarity Transfer)**.** *For any* $\mathbf{x} = (x_1, ..., x_n)$, $\tilde{\mathbf{x}} = (\tilde{x}_1, ..., \tilde{x}_n) \in \mathcal{X}^n$ *such that* $z_1 = \tilde{z}_1$ *and* $z_3 = \tilde{z}_3$, $\tilde{\mathbf{x}}$ *is obtained from* $\mathbf{x}$ *by means of an Increased Bipolarity Transfer if and only if there exist* $h, i, k, l \in \{1, 2, \ldots, n\}$ *such that* $\tilde{x}_h = x_h + \epsilon_1$, $\tilde{x}_i = x_i - \epsilon_1$, $\tilde{x}_k = x_k + \epsilon_2$, $\tilde{x}_l = x_l - \epsilon_2$ *with* $\epsilon_1, \epsilon_2 \geq 0$, $\tilde{x}_h \leq \tilde{x}_i$, $x_i < z_1$, $x_k > z_3$, $\tilde{x}_k \leq \tilde{x}_l$ *and* $\tilde{x}_j = x_j$ *for all* $j \neq h, i, k, l$.

An Increased Bipolarity Transfer corresponds to a Pigou-Dalton transfer between two poor individuals h and i or/and between two rich persons k and l (see Figure 1). Note that this type of transfers changes neither the individuals' relative position nor their membership to the original group (a poor person remains poor and a rich person remains rich).

Figure 1: Increased Bipolarity Transfer

We can now state the first axiom of this section, called *Increased Bipolarity Principle*, according to which a Pigou-Dalton transfer between two poor individuals or between two rich persons should not decrease polarization.

Axiom 7 (Increased Bipolarity Principle). *For any* $\mathbf{x}, \tilde{\mathbf{x}} \in \mathcal{X}^n$, $\mathbf{x} \neq \tilde{\mathbf{x}}$ *, if* $z_1 = \tilde{z}_1$ *and* $z_3 = \tilde{z}_3$ *and if* $\tilde{\mathbf{x}}$ *has been obtained from* $\mathbf{x}$ *by means of an Increased Bipolarity Transfer, then* $P(\tilde{\mathbf{d}}) \geq P(\mathbf{d})$*, where* $\tilde{\mathbf{d}} = (d_1(\tilde{x}_1, \tilde{m}_1),\ d_2(\tilde{x}_2, \tilde{m}_2), ...,\ d_n(\tilde{x}_n, \tilde{m}_n))$ *and* $\mathbf{d} = (d_1(x_1, m_1),\ d_2(x_2, m_2),\ ..., d_n(x_n, m_n))$*.*

This axiom is similar to the *non-decreasing bipolarity* axiom introduced in Wolfson (1994, 1997), to the *Increased Bipolarity* in Wang and Tsui (2000), to the *Non-decreasing Bipolarity* in Chakravarty and Majumder (2001), to the *Within-Group Clustering* in Bossert and Schworm (2008) and to the *Increasing bipolarity outside* $[\pi, \rho]$ in Scheicher (2010).

The following proposition provides a characterization of the indices that satisfy the Increased Bipolarity Principle.

Proposition 2. *The polarization index* $P \in \mathcal{P}$ *defined in expression (2) satisfies the* Increased Bipolarity Principle *if and only if*

$$\begin{aligned} \tfrac{\partial \tau(d_i(x_i, z_1))}{\partial x_i} &\geq \tfrac{\partial \tau(d_j(x_j, z_1))}{\partial x_j}\ \forall x_i \leq x_j < z_1 \\ &\textit{and} \\ \tfrac{\partial \psi(d_i(x_i, z_3))}{\partial x_i} &\geq \tfrac{\partial \psi(d_j(x_j, z_3))}{\partial x_j}\ \forall z_3 < x_i \leq x_j. \end{aligned}$$

Proof. Following Marshall and Olkin (1979), the polarization index P defined in (2) satisfies the Increased Bipolarity Principle if and only if $\sum_i \tau(d_i(x_i, z_1))$ and $\sum_i \psi(d_i(x_i, z_3))$ are Schur-concave functions. By Theorem $C.1.a$ in Marshall and Olkin (1979) page 64, this means that $\tau(d_i(\cdot))$ and $\psi(d_i(\cdot))$ are concave functions. This condition can be expressed in terms of derivatives, as in (7), since τ and ψ are differentiable by the Continuity axiom. □

The condition for functions τ and ψ to be concave implies that the polarization index is more affected by income changes occurring closer to the middle class than to income movements happening further from the central group.

We now move to the second basic axiom in the polarization literature, which states that a movement of incomes from the tails towards the central values of the income distribution does not increase polarization. We first provide a definition of the kind of income transfers that characterize this second axiom.

Definition 2 (Weak Contraction towards the Middle Class Transfer)**.** *For any* $\mathbf{x} = (x_1, ..., x_n), \tilde{\mathbf{x}} = (\tilde{x}_1, ..., \tilde{x}_n) \in \mathcal{X}^n$ *such that* $z_1 = \tilde{z}_1$ *and* $z_3 = \tilde{z}_3$*,* $\tilde{\mathbf{x}}$ *is obtained from* $\mathbf{x}$ *by means of a Weak Contraction towards the Middle Class Transfer if and only if there exist* $k, l \in \{1, 2, ..., n\}$ *such that* $\tilde{x}_k = x_k + \epsilon < z_1$ *and* $\tilde{x}_l = x_l - \epsilon > z_3$ *with* $\epsilon \geq 0$*, and* $\tilde{x}_j = x_j$ *for all* $j \neq \{k, l\}$*.*

The transfer described in Definition 2 is a slight modification of the *mean-preserving contraction* discussed by Rothschild and Stiglitz (1970) and of the *transfers about* θ introduced in Mosler and Muliere (1996). It consists of moving some positive income amount from an individual above the richness line to an individual below the poverty line, as shown in Figure 2. The middle class is not affected by this kind of transfers. Note that this transfer modifies neither the relative order of the two individuals nor their position above the richness line and and below the poverty line. Since no individuals cross the lines, this means that no mobility across social classes is allowed.

Figure 2: Weak Contraction towards the Middle Class Transfer

Axiom 8 (Weak Contraction towards the Middle Class Principle)**.** *For any* $\mathbf{x} \neq \tilde{\mathbf{x}} \in \mathcal{X}^n$*, if* $z_1 = \tilde{z}_1$ *and* $z_3 = \tilde{z}_3$ *and if* $\tilde{\mathbf{x}}$ *has been obtained from* $\mathbf{x}$ *by means of a Weak Contraction towards the Middle Class, then* $P(\tilde{\mathbf{d}}) \leq P(\mathbf{d})$*.*

This axiom states that if there is a progressive transfer from a rich person to a poor person, without allowing them to cross the relative thresholds, then polarization should not increase. Analogously, one can say that a regressive transfer from a poor to a rich should not decrease polarization. This axiom is similar to the *non-decreasing spread* axiom introduced in Wolfson (1994, 1997), to the *Increased Spread* axiom in Wang and Tsui (2000), to the *Non-decreasing Spread* in Chakravarty and Majumder (2001), to the *Between-Group Spread* in Bossert and Schworm (2008) and to the *Increasing spread outside* $[\pi, \rho]$ in Scheicher (2010).[7]

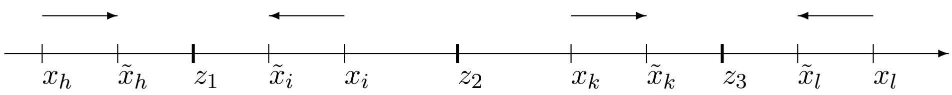
[7]Interestingly, Amiel et al. (2010), in investigating whether people's perceptions of income polarization are con-

Note that the Weak Contraction towards the Middle Class Principle is always satisfied by the class of polarization indices introduced in (2).

We now introduce an interesting modification of the Weak Contraction towards the Middle Class Transfer, which allows for progressive transfers between the middle class, on one side, and either the poor or the rich, on the other side. Differently from the Weak Contraction towards the Middle Class Transfer, now also the middle class is involved. This new kind of transfers is illustrated in Figure 3 and its rigorous definition is the following.

Definition 3 (Strong Contraction towards the Middle Class Transfer). *For any* $\mathbf{x} = (x_1, ..., x_n), \tilde{\mathbf{x}} = (\tilde{x}_1, ..., \tilde{x}_n) \in \mathcal{X}^n$ *such that* $z_1 = \tilde{z}_1$, $z_2 = \tilde{z}_2$ *and* $z_3 = \tilde{z}_3$, $\tilde{\mathbf{x}}$ *is obtained from* $\mathbf{x}$ *by means of a Strong Contraction towards the Middle Class Transfer if and only if there exist* $h, i \in \{1, 2, ..., n\}$ *such that* $\tilde{x}_h = x_h + \epsilon_1 < z_1$, $\tilde{x}_i = x_i - \epsilon_1 > z_1$ *and* $x_i < z_2$ *and there exist* $k, l \in \{1, 2, ..., n\}$ *such that* $\tilde{x}_k = x_k + \epsilon_2 < z_3$, $\tilde{x}_l = x_l - \epsilon_2 > z_3$ *and* $x_k > z_2$, *and* $\tilde{x}_j = x_j$ *for all* $j \neq \{h, i, k, l\}$ *and* $\epsilon_1, \epsilon_2 \geq 0$.

Figure 3: Strong Contraction towards the Middle Class Transfer



This kind of transfer is a novelty in the polarization literature, as it concerns interactions among all the population subgroups. Two are the main trends within this new transfer: on one side, the Strong Contraction towards the Middle Class Transfer induces an increase in the income dispersion within the middle class, while, on the other side, it brings about a reduction in the distances of the poor and the rich from the middle class. These income movements create opposite effects on our polarization index, and the researcher has to decide which one of the two trends is more relevant. Here, we allow for two alternative beliefs of the researcher in evaluating the effects on polarization of the Strong Contraction towards the Middle Class Transfer. If he puts more weight to the reduction in the distances from the middle class, then the following Strong Contraction towards the Middle Class Principle will be satisfied.

sistent with the Wolfson's key axioms, found more support for the Non-decreasing Spread axiom than for the Non-decreasing Bipolarity axiom.

Axiom 9 (Strong Contraction towards the Middle Class Principle)**.** *For any* $\mathbf{x} \neq \tilde{\mathbf{x}} \in \mathcal{X}^n$*, if* $z_1 = \tilde{z}_1$*,* $z_2 = \tilde{z}_2$ *and* $z_3 = \tilde{z}_3$ *and if* $\tilde{\mathbf{x}}$ *has been obtained from* $\mathbf{x}$ *by means of a Strong Contraction towards the Middle Class, then* $P(\tilde{\mathbf{d}}) \leq P(\mathbf{d})$*.*

The Strong Contraction towards the Middle Class Principle states that, if there is a progressive transfer from a person belonging to the middle class (individual i in Figure 3) in favor of a poor person (individual h), without allowing them to cross the poverty line, then polarization should not increase. Analogously, a progressive transfer from a rich person (individual l in Figure 3) to a person belonging to the middle class (individual k), without allowing them to cross the richness line, should not increase polarization. The rationale behind this axiom is that the researcher gives more importance to income movements that make individuals outside the middle class closer to this central group, rather than to income changes that weaken the cohesion within the middle class.

The following Strong Spread from the Middle Class Principle puts instead more emphasis on the increase in income dispersion within the middle class due the Strong Contraction towards the Middle Class Transfer; therefore, polarization should not decrease.

Axiom 10 (Strong Spread from the Middle Class Principle)**.** *For any* $\mathbf{x} \neq \tilde{\mathbf{x}} \in \mathcal{X}^n$*, if* $z_1 = \tilde{z}_1$*,* $z_2 = \tilde{z}_2$ *and* $z_3 = \tilde{z}_3$ *and if* $\tilde{\mathbf{x}}$ *has been obtained from* $\mathbf{x}$ *by means of a Strong Contraction towards the Middle Class, then* $P(\tilde{\mathbf{d}}) \geq P(\mathbf{d})$*.*

Note that Axioms 9 and 10 preclude any form of mobility among the socio-economic groups: the transfers introduced in Definition 3 do not allow individuals to change their original group.

The following proposition provides a sufficient and necessary condition for polarization index P to satisfy the Strong Contraction towards (Strong Spread from) the Middle Class Principle.

Proposition 3. *a) Polarization index* P *defined in (2) satisfies the* Strong Contraction towards the Middle Class Principle *if and only if* $\forall\, \mathbf{x} \in \mathcal{X}^n$

$$
\begin{aligned}
min_{x_i<z_1} \quad \frac{\partial\tau(d_i(x_i,z_1))}{\partial x_i} \;&\geq\; max_{z_1\leq x_j<z_2}\frac{\partial\xi(d_j(x_j,z_2))}{\partial x_j} \\
&and \\
max_{z_2\leq x_i<z_3} \quad \frac{\partial\xi(d_i(x_i,z_2))}{\partial x_i} \;&\leq\; min_{x_j>z_3}\frac{\partial\psi(d_j(x_j,z_3))}{\partial x_j}.
\end{aligned}
\tag{7}
$$

b) Polarization index P *defined in (2) satisfies, instead, the* Strong Spread from the Middle

Class Principle *if and only if* $\forall\, \mathbf{x} \in \mathcal{X}^n$

$$max_{x_i<z_1} \quad \frac{\partial \tau(d_i(x_i,z_1))}{\partial x_i} \leq \; min_{z_1 \leq x_j < z_2} \frac{\partial \xi(d_j(x_j,z_2))}{\partial x_j}$$

and

$$min_{z_2 \leq x_i < z_3} \quad \frac{\partial \xi(d_i(x_i,z_2))}{\partial x_i} \geq \; max_{x_j > z_3} \frac{\partial \psi(d_j(x_j,z_3))}{\partial x_j}. \tag{8}$$

Proof. We show the proof for point a). Let $\tilde{\mathbf{x}}$ be an income vector obtained from $\mathbf{x}$ through a Strong Contraction towards the Middle Class Transfer. From Theorem 4 in Mosler and Muliere (1996), P satisfies the Strong Contraction towards the Middle Class Principle if and only if

$$min_{x_h<z_1} \quad \frac{\partial P(\mathbf{d})}{\partial x_h} \geq \; max_{z_1 \leq x_i < z_2} \frac{\partial P(\mathbf{d})}{\partial x_i}$$

and

$$max_{z_2 \leq x_k < z_3} \quad \frac{\partial P(\mathbf{d})}{\partial x_k} \leq \; min_{x_l > z_3} \frac{P(\mathbf{d})}{\partial x_l}.$$

By Marshall and Olkin (1979) this is true if and only if

$$\frac{\partial \tau(d_h(x_h, z_1))}{\partial x_h} \geq \frac{\partial \xi(d_i(x_i, z_2))}{\partial x_i}, \text{ for all}\, x_h < z_1, z_1 \; \leq x_i < z_2$$

and

$$\frac{\partial \xi(d_k(x_k, z_2))}{\partial x_k} \leq \frac{\partial \psi(d_l(x_l, z_3))}{\partial x_l}, \text{ for all}\, z_2 \leq x_k \leq z_3, x_l > z_3.$$

This means that P satisfies the *Strong Contraction towards the Middle Class Principle* if and only if (7) holds. The proof for point b) can be derived straightforwardly. □

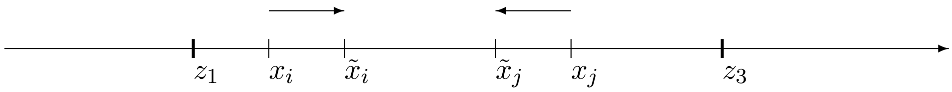
We now move to the last axiom of this section, which is a novelty in the polarization literature: when measuring polarization we do not focus only on the poor and the rich, but we also measure the extent of cohesion inside this central group.[8]

Definition 4 (Within Middle Class Transfer)**.** *For any* $\mathbf{x} = (x_1, ..., x_n), \tilde{\mathbf{x}} = (\tilde{x}_1, ..., \tilde{x}_n) \in \mathcal{X}^n$ *such that* $z_1 = \tilde{z}_1$ *and* $z_3 = \tilde{z}_3$*,* $\tilde{\mathbf{x}}$ *is obtained from* $\mathbf{x}$ *by means of a Within Middle Class Transfer if and only if* $\tilde{x}_i = x_i + \epsilon$*,* $\tilde{x}_j = x_j - \epsilon$ *with* $z_1 \leq x_i$*,* $x_j \leq z_3$ *and* $\tilde{x}_i \leq \tilde{x}_j$ *and* $\epsilon \geq 0$*.*

This is a Pigou-Dalton transfer within the middle class. See Figure 4 for a graphical illustration. This kind of transfer does not modify the relative position of the two individuals. Note also that the transfer may occur between individuals whose incomes are (i) both smaller than the centrality parameter z_2, (ii) both greater than z_2, or (iii) on opposite sides of z_2.

[8] This axiom clearly distinguishes our polarization measure from the index proposed in Scheicher (2010).

Figure 4: Within Middle Class Transfer



A Pigou-Dalton transfer within the middle class reduces the within-group income inequality, and therefore, it increases the feeling of cohesion in this social group. The more cohesive the middle class is the more stable the whole society is (see e.g. Easterly (2001) and Pressman (2007)): for this reason, we want polarization not to increase, as stated in the following axiom.

Axiom 11 (Middle Class Cohesion)**.** *For any* $\mathbf{x} \neq \tilde{\mathbf{x}}$*, if* $z_1 = \tilde{z}_1$*,* $z_3 = \tilde{z}_3$ *and if* $\tilde{\mathbf{x}}$ *has been obtained from* $\mathbf{x}$ *by means of a Within Middle Class Transfer, then* $P(\tilde{\mathbf{d}}) \leq P(\mathbf{d})$*.*

The following proposition provides a necessary and sufficient condition for polarization indices to satisfy the Middle Class Cohesion axiom.

Proposition 4. *The polarization index P in (2) satisfies the* Middle Class Cohesion *if and only if*

$$\frac{\partial \xi(d_i(x_i, z_2))}{\partial x_i} \leq \frac{\partial \xi(d_j(x_j, z_2)}{\partial x_j}$$

for all $z_1 \leq x_i \leq x_j \leq z_3$*.*

Proof. See Marshall and Olkin (1979) Theorem A.3. page 56 or Castagnoli and Muliere (1990). □

As a consequence of Proposition 4, function $\xi(d_i(\cdot))$ has to be strictly increasing and convex. This means that polarization is more affected by income changes occurring closer to the middle class' borders z_1 and z_3 than to income movements happening close to the central threshold z_2.

Finally, we conclude this section by providing a characterization for the class of polarization measures that satisfy the polarization axioms introduced in this section.

Proposition 5. *A polarization index P defined in (2) satisfies Increased Bipolarity Principle, Strong Contraction towards (Strong Spread from) the Middle Class Principle, Middle Class Cohesion if and only if*

$$P(\mathbf{d}) = \frac{1}{n}\left\{\sum_{i \in G_1} \tau\left(d_i(x_i, z_1)\right) + \sum_{i \in G_2} \xi\left(d_i(x_i, z_2)\right) + \sum_{i \in G_3} \psi\left(d_i(x_i, z_3)\right)\right\}, \tag{9}$$

where τ and ψ are concave functions and ξ is a convex function such that $min_{x_i<z_1}\frac{\partial\tau(d_i(x_i,z_1))}{\partial x_i} \geq max_{z_1\leq x_j<z_2}\frac{\partial\xi(d_j(x_j,z_2))}{\partial x_j}$ *and* $max_{z_2\leq x_i<z_3}\frac{\partial\xi(d_i(x_i,z_2))}{\partial x_i} \leq min_{x_j>z_3}\frac{\partial\psi(d_j(x_j,z_3))}{\partial x_j}$
$\left(max_{x_i<z_1}\frac{\partial\tau(d_i(x_i,z_1))}{\partial x_i} \leq min_{z_1\leq x_j<z_2}\frac{\partial\xi(d_j(x_j,z_2))}{\partial x_j} \text{ and } min_{z_2\leq x_i<z_3}\frac{\partial\xi(d_i(x_i,z_2))}{\partial x_i} \geq max_{x_j>z_3}\frac{\partial\psi(d_j(x_j,z_3))}{\partial x_j}\right)$.

Proof. See proofs of Propositions 2, 3 and 4. □

Therefore, the effect of the polarization axioms discussed in this section is to characterize properly the functions τ, ξ and ψ that weigh the individual contributions to polarization.

4 Special cases of polarization indices

Special cases of the class of polarization indices introduced in the previous section can be obtained by choosing particular distance functions d_i. For example, if we consider $d_i(x_i, m_i) = |x_i - m_i|$ or $d_i(x_i, m_i) = \frac{|x_i-m_i|}{m_i}$, then the polarization indices become the following:

$$P_1(\mathbf{d}) = \frac{1}{n}\left\{\sum_{i\in G_1}\tau\left(|x_i-z_1|\right)+\sum_{i\in G_2}\xi\left(|x_i-z_2|\right)+\sum_{i\in G_3}\psi\left(|x_i-z_3|\right)\right\} \quad (10)$$

$$P_2(\mathbf{d}) = \frac{1}{n}\left\{\sum_{i\in G_1}\tau\left(\left|\frac{x_i-z_1}{z_1}\right|\right)+\sum_{i\in G_2}\xi\left(\left|\frac{x_i-z_2}{z_2}\right|\right)+\sum_{i\in G_3}\psi\left(\left|\frac{x_i-z_3}{z_3}\right|\right)\right\} \quad (11)$$

for some functions τ, ξ, ψ characterized in Propositions 1 and 5.

Measure P_1 in (10) is an *absolute* polarization measure, as it remains unchanged when adding an equal amount to all incomes, while P_2 is a *relative* polarization measure, since it is not affected by equiproportionate variations in all incomes.[9]

Since the thresholds z_1, z_2, z_3 are exogenously given, we may choose the particular case of $z_1 = z_2 = z_3$, thus defining the middle class as the set of individuals whose income is exactly equal to the median income. In this case, our polarization measure boils down to traditional polarization indices based on the Wolfson's approach; in particular, for $z_1 = z_2 = z_3 = me$, where me is the median income, and $\tau(\cdot) = \psi(\cdot) = f(\cdot)$, polarization measures P_1 and P_2 in (10) and (11) coincide

[9]Note that Chakravarty and D´Ambrosio (2010) propose a new class of polarization indices that are intermediate between the absolute and the relative measures.

with the polarization measures proposed by Wang and Tsui (2000):

$$\begin{aligned} P_1^{WT} &= \frac{1}{n}\sum_{i=1}^{n} f(|x_i - me|) \\ P_2^{WT} &= \frac{1}{n}\sum_{i=1}^{n} f\left(\left|\frac{x_i - me}{me}\right|\right) \end{aligned}$$

where f is a continuous, strictly increasing and strictly concave function. Note that Wolfson's measure is a special case of the Wang and Tsui's class of measures (see Wang and Tsui (2000)).

5 A polarization partial ordering

The particular values of the thresholds that define the middle class have to be chosen exogenously and different choices can order income distributions, in terms of polarization, in different ways.

For that reason, in their study of the declining middle class, Beach et al. (1997) and Scheicher (2010) define more than one middle class, inducing a sensitivity analysis, and propose a semi-ordering of distributions that unanimously respects the various definitions of middle class. Analogously, in the poverty measurement literature, poverty orderings are proposed to check whether poverty comparisons are robust over ranges of poverty lines; see, e.g., Foster and Shorrocks (1988) and Zheng (2000).

Here, we follow the same idea, proposing at first a class of polarization measures in (9), which depend on the particular values of z_1 and z_3. We then consider not only a single middle class $[z_1, z_3]$, but rather a set of reasonable middle classes, in order to reduce the subjectivity due to the choice of these thresholds. We can, therefore, check the robustness of polarization comparisons and order different distributions, according to the following dominance criterion.

Definition 5. *Consider an interval of poverty lines $[z_1^{min}, z_1^{max}]$ and an interval of richness lines $[z_3^{min}, z_3^{max}]$, with $z_1^{max} \leq z_3^{min}$. For any $\mathbf{x} \neq \tilde{\mathbf{x}} \in \mathcal{X}^n$ we say that income distribution $\mathbf{x}$ is more polarized than income distribution $\tilde{\mathbf{x}}$, in symbols $\mathbf{x} \succ_P \tilde{\mathbf{x}}$, if the inequality $P(\mathbf{d}) \geq P(\tilde{\mathbf{d}})$ holds for all the middle classes $[z_1, z_3]$ such that $[z_1^{max}, z_3^{min}] \subset [z_1, z_3] \subset [z_1^{min}, z_3^{max}]$.*

The ordering in Definition 5 is partial, in sense that not all income distributions may satisfy an unanimous agreement on polarization rankings for a set of poverty lines and richness lines. If we observe an unambiguous ranking for all possible thresholds, then the comparison of distributions may be considered robust to the choice of values of the parameters.

6 An empirical application

We now apply our new class of measures to real data and compare it with the traditional polarization indices proposed by Wang and Tsui (2000), based on the median income as reference point, in order to illustrate the main differences.

The empirical test is based on the EU-SILC (European Union Statistics on Income and Living Conditions) dataset for the years between 2004 and 2007 and for the following countries: Austria, Belgium, France, Italy, Norway, Portugal, Spain and Sweden. The EU-SILC dataset collects comparable cross-sectional micro data on households income and living conditions in European countries.

The unit of analysis is the individual and for each person we consider the equivalent per-capita total net income, which is the household's total disposable income divided by the equivalent household size according to the modified OECD scale. The disposable income is defined as the market income minus direct taxes and social contributions plus cash benefits, including pensions. For each country we deflate incomes using the harmonized consumer price index provided by Eurostat, in order to have real incomes, comparable at time 2005, and we clean the dataset dropping all negative incomes. As thresholds we choose country-specific poverty and richness lines, defined, respectively, as the 60% and the 200% of the median equivalent income over the 4 years for each country.[10] In each country the thresholds are kept fixed over time, thus allowing comparisons within countries that are consistent with the axioms introduced in Section 3.

Using the Wang and Tsui's index we find out (see Table 1) that from year 2004 to year 2007 income polarization has increased in all the selected countries, except for Austria and France, where polarization has slightly reduced.

We now want to explore how our new class of polarization measures can enrich the picture. For this empirical application we choose, as special case of the class of polarization measures introduced in expression (9), the following:

[10] The choice of the richness line as twice the median income is a common practice; for more discussion we refer, in particular, to Peichl et al. (2010) and Medeiros (2006).

$$P_{\alpha,\beta}(\mathbf{d}) = \frac{1}{n}\left\{\sum_{i\in G_1}\left(\frac{z_1-x_i}{z_1}\right)^{\alpha}+\sum_{i\in G_2}\left(\left|\frac{x_i-z_2}{z_2}\right|\right)^{\beta}+\sum_{i\in G_3}\left(g\left(\frac{x_i-z_3}{z_3}\right)\right)^{\alpha}\right\} \tag{12}$$

with $\alpha \in (0,1)$, $\beta > 1$, and $g(\cdot)$ is any strictly increasing and concave function of the distances d_i with $lim_{d_i\to\infty} g(d_i) = 1$. The particular choices in (12) for thresholds and distance functions ensure the gaps d_i of the poor and the middle class to be bounded between 0 and 1, while, for the rich, the distances $d_i = \frac{x_i-z_3}{z_3}$ are only lower-bounded at 0. The role of function $g(\cdot)$ is therefore to make the gaps d_i of the rich bounded within the unit interval; in particular, we choose $g(d_i) = \left(1 - \frac{1}{1+d_i}\right)$ as in Peichl et al. (2010).

Table 1 shows the values of the polarization index in (12) for the following different choices of α and β: $(\alpha;\beta) = (0.1;1.1);(0.5;2);(0.9;3)$.

For each country the new measures register polarization trends over time that are analogous to what emerges from the Wang and Tsui's measure. Nevertheless, some differences appears. If we give more importance to income changes occurring close to the middle class' borders ($\alpha = 0.9$ and $\beta = 3$) than to changes happening far from the middle class' borders ($\alpha = 0.1$ and $\beta = 1.1$), then polarization in Norway and in Sweden increases at a much higher rate. This means that the increase in income polarization in these countries is mainly due to changes in the incomes of the poor and of the rich individuals who are settled close to the middle class. This is confirmed also by looking at Figure 5 and Table 2.

Figure 5 shows the marginal contribution to the polarization index $P_{\alpha,\beta}$, for $\alpha = \beta = 0$, of each of the three population groups, the poor, the middle class and the rich; therefore, Figure 5 shows the proportions of the poor, the middle and the rich in each country. We note that Sweden, Norway and Belgium are the countries with the most populated middle class, while Italy, Spain and Portugal have the highest proportion of poor individuals, and Portugal has the highest percentage of rich people.

Table 2 shows, instead, the decomposition of the polarization index $P_{\alpha,\beta}$, for $\alpha = 0.9$ and $\beta = 3$, into the three main groups of contributions to polarization. In particular, we note that in Norway the increase in polarization over the recent years is mainly due to a consistent increase in the middle class dispersion and to an increase in the distance between the poor and the middle class;

in Sweden polarization reduces because the increase both in the gaps of the rich and in the middle class dispersion more than compensate the reduction of the distances of the poor. On the other side, in France and Austria polarization reduces because the gaps between the poor and the middle class decrease consistently over time.

We also check for the polarization ordering introduced in Section 5. For each country, we propose a set of alternative middle classes defined in terms of a set of values for the poverty line and for the richness line. In particular, following Scheicher (2010), we choose $z_1 = 0.4 + 0.02 \cdot i$ and $z_3 = 2.4 - 0.02 \cdot i$ times the median income, with $i = 0, 1, 2, \ldots, 20$. Thus, in this application the polarization ordering introduced in Definition 5 is based on the following sequence of nested middle classes: $[0.4, 2.4] \supset [0.42, 2.38] \supset \ldots \supset [0.8, 1.6]$ times the median income.

For each country, we check whether a polarization ranking between income distribution in 2004 and income distribution in 2007 is uniform in this range of values for z_1 and z_3. Figures 6, 7 and 8 plot the difference between polarization registered in the year 2004 and in the year 2007 over the set of values of $i = 0, ..., 20$, for each of the three polarization measures considered, P_1, P_2, P_3. Results show that we are able to rank income distributions in terms of polarization over the entire sequence of nested middle classes for almost all the countries. The countries that do not show an unanimous ranking are Sweden, Norway and Belgium , as their curves cross the horizontal line in correspondence to high values of i, i.e. for narrower middle classes. Uniform ranking is instead guaranteed for the remaining countries.

7 Concluding remarks

In this paper, we have proposed a new class of income polarization measures. In contrast to the traditional polarization measures based on the Wolfson's approach, we have defined the middle class in a broader and more realistic way, based on median income intervals rather than on the median income. The class of indices has been characterized through a set of axioms, some of which are based on modifications and extensions of the Pigou-Dalton transfers. The proposed class of measures is a generalization of existing polarization indices. The empirical application has demonstrated the usefulness of our polarization index in revealing additional information beyond the one provided by the traditional polarization measures; thus we suggest using it in addition to the traditional measures for a more comprehensive analysis of polarization.

Since the new polarization index is based on thresholds chosen a priori, different choices of these lines can order income distributions, in terms of polarization, in different ways. In order to reduce the analysis sensitivity to the choice of the thresholds, we have followed the approach proposed in the poverty measurement literature and, in particular, in Zheng (2000) and considered a set of reasonable values for each threshold.

The approach proposed in this paper may be extended to a multidimensional context for measuring polarization of multivariate distributions; see in particular, Gigliarano and Mosler (2009), Scheicher (2010), Anderson (2010).

Further research may also extend the polarization analysis towards the issue of inter-classes mobility, by introducing transfers that allow individuals to change group, similarly to the *transfers next to the thresholds* introduced in Mosler and Muliere (1996).

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Table 1: A comparison of polarization measures ($P_1 = P_{\alpha=.1,\beta=1.1}, P_2 = P_{\alpha=.5,\beta=2}, P_3 = P_{\alpha=.9,\beta=3}$, WT=Wang and Tsui's measure, %Diff= percentage change of the index between year 2004 and year 2007).

Country	Year	P_1	P_2	P_3	WT	Country	Year	P_1	P_2	P_3	WT
Austria	2004	0.35	0.17	0.10	0.55	Belgium	2004	0.36	0.18	0.10	0.55
	2005	0.35	0.18	0.10	0.56		2005	0.36	0.18	0.10	0.56
	2006	0.34	0.17	0.10	0.55		2006	0.37	0.19	0.11	0.57
	2007	0.33	0.16	0.10	0.54		2007	0.36	0.18	0.11	0.56
	%Diff	-4.5%	-5.3%	-6.0%	-1.6%		%Diff	2.5%	3.6%	4.2%	2.0%
France	2004	0.38	0.19	0.12	0.58	Italy	2004	0.43	0.23	0.14	0.62
	2005	0.39	0.20	0.12	0.59		2005	0.43	0.23	0.15	0.62
	2006	0.37	0.19	0.11	0.58		2006	0.43	0.24	0.15	0.62
	2007	0.38	0.19	0.11	0.58		2007	0.44	0.24	0.15	0.63
	%Diff	-1.7%	-2.4%	-3.0%	-0.8%		%Diff	3.0%	5.1%	6.3%	1.7%
Norway	2004	0.31	0.15	0.08	0.52	Portugal	2004	0.46	0.26	0.16	0.67
	2005	0.32	0.15	0.09	0.52		2005	0.47	0.26	0.17	0.69
	2006	0.32	0.16	0.09	0.54		2006	0.47	0.26	0.17	0.69
	2007	0.33	0.17	0.10	0.53		2007	0.47	0.27	0.17	0.70
	%Diff	5.7%	14.1%	21.1%	3.5%		%Diff	2.5%	3.3%	3.4%	4.5%
Spain	2004	0.43	0.23	0.14	0.61	Sweden	2004	0.31	0.14	0.08	0.51
	2005	0.45	0.25	0.16	0.63		2005	0.31	0.14	0.08	0.51
	2006	0.44	0.24	0.15	0.62		2006	0.31	0.15	0.09	0.52
	2007	0.44	0.24	0.15	0.63		2007	0.33	0.17	0.10	0.55
	%Diff	3.1%	5.9%	8.6%	3.7%		%Diff	7.7%	15.1%	21.4%	7.2%

Source: own elaboration on EU-SILC 2007 dataset.

Figure 5: Decomposition of the polarization index $P_{\alpha,\beta}$, with $\alpha = \beta = 0$.

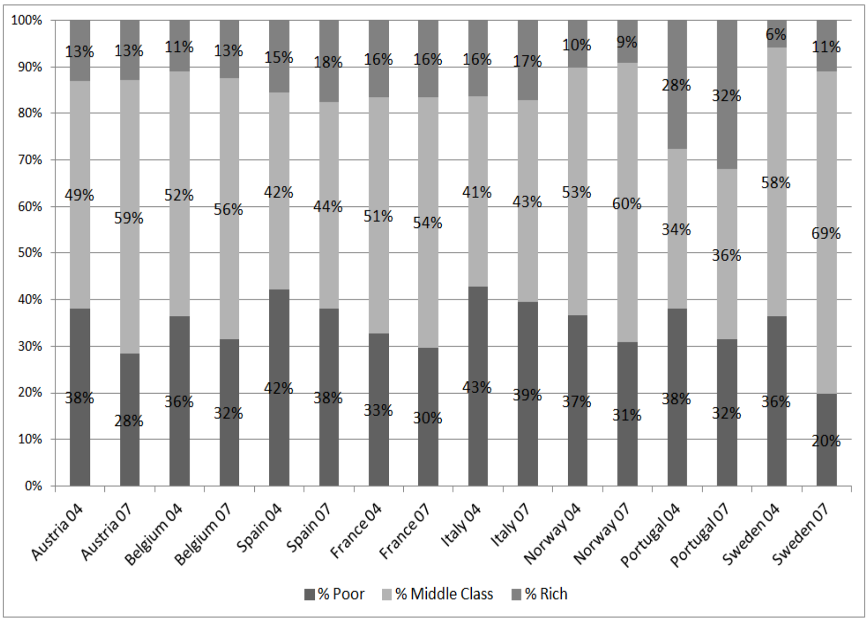

Table 2: Decomposition of the polarization index $P_{\alpha,\beta}$, with $\alpha = 0.9$ and $\beta = 3$.

Country	Year	Poor	Middle Class	Rich
Austria	2004	42%	46%	12%
	2007	28%	59%	13%
Belgium	2004	39%	50%	11%
	2007	32%	56%	12%
France	2004	33%	51%	16%
	2007	29%	55%	16%
Italy	2004	46%	38%	16%
	2007	41%	42%	17%
Norway	2004	40%	49%	11%
	2007	34%	58%	8%
Portugal	2004	39%	33%	28%
	2007	31%	36%	33%
Spain	2004	44%	41%	15%
	2007	40%	44%	16%
Sweden	2004	40%	55%	5%
	2007	20%	70%	10%

Source: own elaboration on EU-SILC 2007 dataset.

Figure 6: Difference in polarization, 2007 minus 2004, for measure $P_{\alpha,\beta}$ ($\alpha = 0.1$, $\beta = 1.1$)

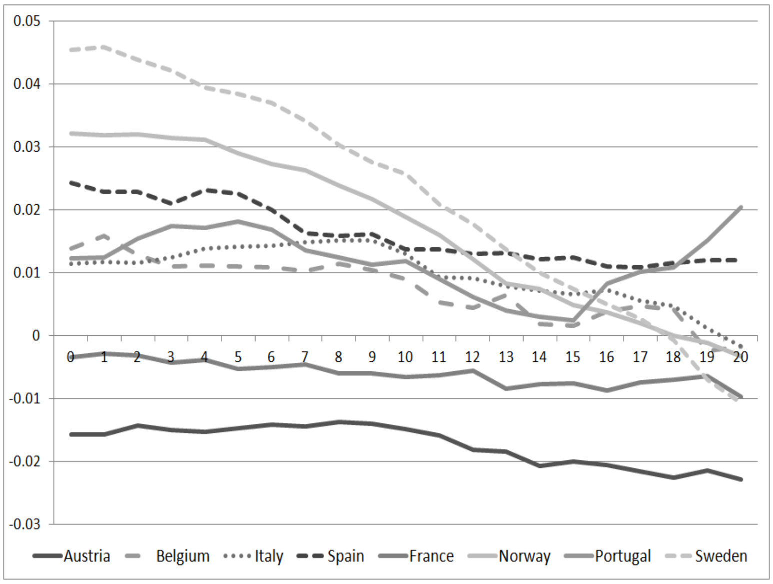

Figure 7: Difference in polarization, 2007 minus 2004, for measure $P_{\alpha,\beta}$ ($\alpha = 0.5$, $\beta = 2$)

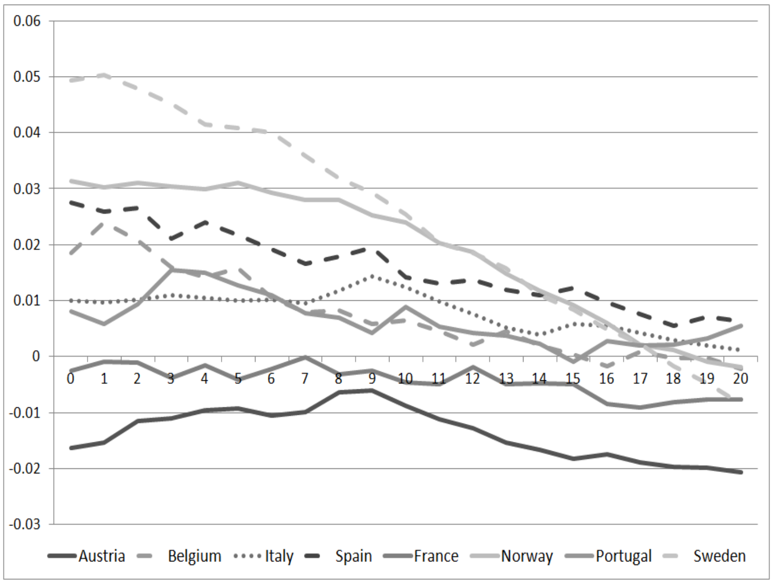

Figure 8: Difference in polarization, 2007 minus 2004, for measure $P_{\alpha,\beta}$ ($\alpha = 0.9$, $\beta = 3$)