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Equality of Educational Opportunity Employing PISA Data: Taking Both Achievement and Access into Account*

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Abstract

While PISA datasets have been used for measuring inequality of educational opportunity they have important limitations: (i) samples only cover a relatively limited fraction of developing countries' cohorts of 15-year-olds, and (ii) such fractions are not uniform across countries and waves. This casts doubts on the reliability of such measures when used for international and intertemporal comparisons: a milder calculated inequality of opportunity in a given country at a given moment might simply be the artifact of a more restricted and homogeneous sample. Previous attempts of addressing this problem have focused on explicitly reconstructing full samples. Here an alternative path is followed, relying on bidimensional indices, in which equality of opportunity in achievement is the first dimension and equality of opportunity for access to the exam is the second one. We compute the two dimensions and aggregate them using alternative techniques. Employing PISA 2006/2009 data for six Latin-American countries we observe rank reversals when comparing results based upon our indices and those based upon conventional indices of equality of opportunity for achievement. We then generalize our approach allowing for more dimensions and parameterizing the dimensions' weights.

Keywords: equality of opportunity, measurement of inequality of opportunity, multidimensional measures, PISA test scores, Latin America.
JEL Classification: I24, O54.

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1. Introduction

A liberal-egalitarian theory of justice that has been widely discussed in recent years is that of "equality of opportunity" (EOp), popularized among economists by John Roemer, according to which while inequalities due to different circumstances are intolerable, inequalities due to choices made by the individuals are acceptable (Roemer, 1998). Different methodologies have been proposed attempting to translate the theory into measuring procedures in order, for example, to determine how far apart countries or regions stand from an ideal of equality of opportunity in terms of, say, income distribution (e.g., Checchi & Peragine, 2010; Dunnzlauf et al. 2010). Two recent extensive surveys are available documenting the vast literature produced along the last ten years or so (Pignataro, 2012; Ramos & Van de Gaer, 2012).

Measuring inequality of opportunity in the educational sphere has been the focus of recent contributions, which concentrate either on opportunity for access to a given level of studies (e.g., Paes de Barros et al. 2009; Vega et al. 2010), or on opportunity in terms of educational achievement (e.g., Checchi & Peragine, 2005; Ferreira & Gignoux, 2011; Gamboa & Waltenberg, 2012). In this paper we combine both concerns.

Pupils' educational achievement is usually measured by standardized test scores, such as those made available by OECD's *Programme for International Student Assessment* (PISA), which are taken to be proxies for the knowledge and basic skills pupils possess in different areas: reading, mathematics, or sciences. While PISA datasets present many well-known virtues, they are also plagued by some important limitations, particularly in terms of coverage rates of the population of individuals whose age is 15 (the exam's focus). The reasons for not evaluating individuals include: their being not enrolled in schools, their being enrolled in very low grades, logistic difficulties in the application of the test, and school-level exclusions related to pupils' physical or intellectual deficiencies. While in developed countries, the coverage rate is systematically larger than 80% – sometimes approaching 100% –, the samples cover a relatively limited fraction of developing countries' cohorts of 15-year-olds. Moreover such fractions are not uniform across countries and PISA waves (see Figures 1 and 2).

< Figures 1 and 2 around here >

These sample limitations cast doubts on the reliability of indices of inequality of educational opportunities concerning developing countries, let alone international and intertemporal comparisons: a milder calculated inequality of opportunity in a given country (or at a given moment) might simply be the artifact of a more restricted sample – possibly more homogeneous – as compared to that of another country (or another moment).

Previous attempts of addressing this issue have been ingeniously performed by Ferreira & Gignoux (2011), who have tried to explicitly reconstruct a full sample, but have encountered two obstacles. First, the need to handle simultaneously many ancillary national datasets, (possibly) dissimilar in many respects, contrary to PISA datasets themselves, which are designed to be comparable across countries and years. Second, the need to adopt strong assumptions in order to assign scores to missing pupils in the simulated distribution of scores that they construct trying to mimic the actual distribution of scores that would have been observed had pupils representing the whole cohort taken the exam.

Our strategy is of a very different nature. Instead of dealing with less-than-full coverage by attempting to reconstruct a full sample, we take it for granted that it is not possible to obtain a reliable and uncontroversial reconstructed full sample. We prefer to explicitly acknowledge that there are *two different dimensions of opportunity* – access to PISA exams and achievement conditional on access. We then introduce a bidimensional index of equality of opportunity composed of the (conditional-on-access-) achievement dimension and the access (-to-PISA) dimension. For the first dimension, we compute conventional inequality of opportunity in test scores. Following Ferreira & Gignoux (2011), we propose the proportion of the variance in test scores which is explained by variables reflecting pupils' circumstances.[1] As for the second dimension, we employ two different methods, the first of which is based on each country's reported PISA's coverage rate, while the second relies on Paes de Barros et al.'s (2009) Human Opportunity Index (*HOI*).

Needless to say that both dimensions are important for those worried about inequality of opportunities. Yet it seems to us that access to a given advantage (which here means taking part in PISA exam) is even more pressing and crucial than the relative performance obtained by those individuals for which such advantage is accessible (which here stands for their test scores).[2] The prominence of EOp in access with respect to EOp in achievement accentuates the importance of taking into account the former and not restricting the analysis to the latter.

Whatever the relative importance attributed to each dimension of EOp it is necessary to aggregate the dimensions. We do so by means of two different techniques: either using a

[1] Based on Item Response Theory (IRT), answers given by pupils to exam questions are transformed into test scores and arbitrarily standardized to exhibit a given mean and a given standard deviation. As explained in detail by Ferreira & Gignoux (2011), because of this procedure, many usual inequality indices are not ordinally invariant, which led them to recommend the use of the variance to compute inequality of educational achievement, and the fraction of the variance explained by circumstances to compute inequality of opportunity in educational achievement.

[2] Such "hierarchical view" we suggest here which prioritizes EOp in access with respect to EOp in achievement has implications which are taken up in Section 5.

simple multiplicative specification, or turning to fuzzy sets transformations, which are routine in multidimensional poverty analysis when it comes to aggregating variables expressed in different metrics (Cerioli & Zani, 1990; Cheli & Lemmi, 1995; Lemmi & Betti, 2006).

In addition to overcoming the limitations of measuring EOp in education with an exclusive focus on the achievement dimension – which *per se* might lead to misestimating inequality of educational opportunity in some countries – some (though not all) of the versions of the index we introduce also present the attractive feature of economizing on data requirements, since they only involve PISA data complemented by descriptive information contained in PISA technical reports.

We illustrate our approach for six Latin-American countries that took part in PISA 2006 and 2009, observing that ranking those six countries according to different versions of the bidimensional index we introduce differs from ranking them according to a conventional index, exclusively focused on achievement-EOp.

We skip a thorough presentation of equality of opportunity theory, as well as the main controversies around measuring issues, since we believe that will be redundant with the available literature, covered by two recent surveys already mentioned in this introductory Section 1 (Pignataro, 2012; Ramos & Van de Gaer, 2012). The remaining of the paper is organized around five further sections. Section 2 is devoted to explaining the original motivation for this study, namely, PISA's coverage rate problem, as well as previous attempts of addressing it. In Section 3 we uncover our approaches to the problem, which amounts to calculating bidimensional indices of equality of educational opportunities, taking into account both achievement in PISA and access to PISA – each of which if taken alone would provide an incomplete picture of the prevalent degree of inequality of educational opportunity. Section 4 contains an illustration for six Latin-American countries that took part in PISA 2006 and 2009, comparing rankings of inequality of educational opportunity for those countries as calculated by the indices we introduce with rankings obtained from a conventional index. We observe rank reversals, suggesting that disregarding the access dimension has consequences. In Section 5 we undertake a generalization of our approach, allowing for more dimensions and parameterizing the dimensions' weights. We conclude in Section 6, pointing out possible future research paths.

2. PISA's coverage rate problem: paths and attempts to circumvent biases

OECD's PISA datasets have been collected every three years, starting in 2000, allowing over-time comparability. The fourth wave, collected in 2009, is the most recent which is available; next year, data concerning 2012 will be released. PISA datasets include test scores of representative samples of students in dozens of countries in three different subjects – mathematics, sciences and reading – as well as detailed information on students' background and schools' personnel and functioning conditions. The fourth wave, for example, contains samples of about 520 thousand students representing around 28 million pupils of more than 70 countries (OECD, 2012: 25).

Two related limitations that affect PISA samples should be mentioned. First, individuals who are enrolled in a very low grade ("grade 6" or below[3]) or who are not enrolled in schools are not assessed by PISA – they are "ineligible". Another set of eligible pupils does not take the exam for logistic or fortuitous reasons (e.g., pupils living in a remote region, or pupils who were sick in the day the exam took place). Finally, local managers of PISA exams might also exclude some pupils for physical or intellectual deficiencies (the accepted cases are carefully detailed in PISA manuals). As a consequence of these exclusions empirical findings based on PISA data should not be taken as valid for cohorts of 15-year-old individuals, but rather as valid for teenagers represented by a sample of pupils who: (i) have stayed in the educational system, (ii) have not repeated too many grades, (iii) being eligible, have actually been evaluated.

The second limitation is a corollary of the first: the proportion of the cohort of 15-year-old individuals which has been excluded is not uniform across countries or in a given country over time. As shown in figures 1 and 2, differences can be substantial – spatially or temporally – casting doubts on the reliability of cross-country comparisons, as well as on over-time evaluations.

As an example, let us reproduce here coverage rates for six Latin-American countries recently studied (Gamboa & Waltenberg, 2012) and to which we too turn to in the illustration provided below. In 2006, the rates are: Argentina (79%), Brazil (55%), Chile (78%), Colombia (60%), Mexico (54%) and Uruguay (69%); in 2009, they are: Argentina (69%), Brazil (63%), Chile (85%), Colombia (59%), Mexico (61%) and Uruguay (63%). These figures reveal that: (i) the coverage rates are not particularly high on average, (ii) although all countries come from the same region, cross-country dispersion is substantial, with a range of around 25 percentage points in both years, (iii) there are important oscillations for given countries across waves (ranging from -10 to +8 percentage points).

Disregarding coverage rates – which are incomplete and variable across countries and over time – might lead to an imprecise estimation of the level of unfair inequalities in some countries, particularly those with smaller coverage rates. For example, if Mexico turns out to

[3] PISA's "grade 6" corresponds with different names in different countries. PISA technical reports (e.g., OECD, 2009 and 2012) provide tables containing country-by-country corresponding labels.

show lower inequality of opportunity in achievement than Argentina in 2006, one might wonder whether such result actually reflects larger unfair educational achievement inequality in the latter than in the former, or whether the result is driven by a more homogeneous sample in Mexico (the country that has the lowest coverage rate in that year) than in Argentina (the one with the highest rate).[4]

It is important to emphasize that the issue of access that we raise here is not a minor technical problem, but instead a crucial one for those worried about widening opportunities for all. In a sense, as expressed by Paes de Barros et al. (2009) and reinforced by Peragine (2010), lack of access to a given advantage (which here means "not being able to take part in PISA exams") is even more primary and serious for those concerned with equality of opportunity than the relative performance obtained by individuals for which such advantage is accessible (that is, their test scores). In other words, while underperforming in PISA might signal future difficulties in an individual's life, not even been eligible to the exam is probably correlated with much more considerable obstacles in the future. Also, while once again both dimensions of inequality of opportunity might pose problems for future generations, the lack of access is arguably more pressing.

To address PISA coverage problems, we view at least three alternative paths, two of which are mentioned in this section. The third path is the one we adopt in this study, and its (longer) explanation is reserved to the next section.

The first path is simply to be cautious when interpreting the results of any study that employs PISA for developing countries. This is the humblest path, but also the riskiest, since many readers (and possibly policymakers) might basically overlook the call for caution, and judge results by their face value. Gamboa & Waltenberg (2012) opt for what here we call "first path", presenting results based on PISA limited samples and emphasizing that caution is necessary in their interpretation. [5]

A second path consists of explicitly reconstructing full samples. Recently Ferreira & Gignoux (2011) have done so for four countries: Brazil, Indonesia, Mexico, and Turkey. Due to the absence of information in PISA samples about non-participant pupils, it is not possible to perform a correction such as Heckman's familiar procedure. Instead, they have turned to

[4] In Colombia, in turn, a considerable dropout rate has been observed during the 1990s, for multiple reasons among which: economic recession (reducing enrollment in private schools); increase in the standards necessary to be promoted from grade to grade; negative externalities caused by the domestic conflicts. This trend was reverted during the last decade mainly as a consequence of multiple public policies designed to reduce demand barriers. The more recent situation potentially reduces the biases of estimations of equity based on PISA or national standardized tests.

[5] Additionally, as a sensitivity analysis the authors report a simple simulation taking the country showing the lowest coverage rate in each year as a baseline, that is, they eliminate observations from other countries, such that the coverage rates for all countries are equal. In order to reward those countries whose coverage rates are higher, the eliminated observations are those concerning pupils with lowest scores.

ancillary databases (i.e. household surveys), which, however, do not contain information on test scores. They have then imposed some assumptions in order to undertake two different kinds of simulation. The first one consists of re-weighting test scores observations in PISA datasets by means of information taken from the ancillary databases on the fraction of different types of individuals in the population.[6] The second one consists of imputing into the dataset pupils who were not evaluated, ascribing to them scores equal to the lowest score obtained by individuals very similar to them – namely, those pertaining to the same "type", where type is defined following Roemer (1998).

The first simulation, which relies on more conventional assumptions, provides results almost equal to the original results, both in terms of inequality of achievement and of opportunities. While such somewhat unexpected finding might offer relief for those employing PISA datasets, it is not excluded that applying the procedure to other countries/years could lead to more substantial changes. The second simulation results in more substantial differences with respect to the naïve calculations. Having said that, the latter technique has a drawback which is particularly important when it comes to undertaking international and intertemporal comparisons, namely, the fact that it requires handling many different country-specific survey datasets, and choosing similar variables in all of them, which might not always be possible or might lead to poor definitions of types. Another disadvantage is that the criterion employed to input pupils and their scores into the dataset is controversial – Ferreira & Gignoux (2011) themselves acknowledge that, stating that their assumptions are "admittedly extreme".

Their pioneering effort deserves to be praised, and we would like to view it as complementary to our approach, not substitute. Yet we believe sample corrections of PISA data still present high costs (especially being data-intensive) and limited benefits (unstable results; small impacts or results based on strong assumptions). For these reasons we tend to favor a third path, which is described below.

3. A bidimensional approach: taking both achievement and access into account

Our strategy is of a very different nature. Instead of dealing with less-than-full coverage by attempting to, so to speak, "reconstruct a full coverage", we take it for granted that it is not possible to obtain a reliable and uncontroversial reconstructed full sample. We prefer to explicitly acknowledge that there are *two different dimensions of opportunity* –

[6] They adapt the methodology proposed by DiNardo et al. (1996).

access (to PISA exams) and achievement (conditional on access), to measure them separately and to aggregate them somehow, as explained below.

1. Dimension 1: inequality of opportunity in achievement. We restrict the calculation of inequality of opportunity in achievement to the available PISA samples, which as stated above represent only a fraction of each country's 15-year-olds. To do so, we employ Ferreira & Gignoux's (2011) regression-based index of inequality of educational opportunity (hereafter: IO_{FG}), which is calculated as the proportion of the variance of test scores that is explained by a set of circumstances, with $0 \le IO_{FG} \le 1$, thus ranging from 0 (perfect equality of opportunity) to 1 (perfect inequality of opportunity). In our approach, again following Ferreira & Gignoux (2011), the set of circumstances includes: mother and father education, father occupation, stock of educational capital, city size and ownership of specific durables.[7] That provides us with a valuable, albeit limited, piece of information upon which we can judge and compare countries' educational systems.[8]
2. Dimension 2: inequality of opportunity in access. We have to take into account the proportion of individuals which are actually represented in a given country's PISA sample, in order to sanction countries according to how far apart they stand from full coverage. To do so, we propose two methods:

   a) The first one consists of simply employing coverage rates available in PISA technical reports, which range from 0 (no coverage) to 1 (full coverage), as the second dimension of our index. Following the notation employed in a related literature[9] we denote the overall coverage rate by $\overline{p}$, with $0 \le \overline{p} \le 1$.

   b) The second one is more involved, but intuitive too. It consists of taking into account not only the overall coverage rate for each country as in (a) above, but also *the coverage rate for different types of a given population*, which might vary across types (e.g., across types defined according to gender and ethnicity). In that we follow Paes de Barros et al. (2009), who compute what they label a "Human Opportunity Index" (*HOI*), as follows: $HOI = \overline{p}.(1-D)$, where: $\overline{p}$ is defined and bounded as above; $0 \le D \le 1$ stands for a dissimilarity index, which aggregates the

[7] A good property of this index is the ease with which it is calculated: it is simply the R-squared of the estimation of scores regressed against the set of circumstances cited above. Another virtue is related to a characteristic of the R-squared: since it never goes down when new variables are added to the regression, we can interpret it in the present context as a lower bound of inequality of opportunity for achievement: if further variables reflecting circumstances could be added (but are unobservable, for example), we can be sure that the index would either remain stable or go up. For a thorough discussion, see Ferreira & Gignoux (2011).

[8] It should be mentioned at this point that although we employ a regression, we are not attempting to establish causality. The exercise undertaken is essentially a static decomposition of inequality (as expressed by the variance) into unfair inequality (the R-squared) and fair inequality ($1-R^2$).

[9] See, for example, Paes de Barros et al. (2009) and Vega et al. (2010).

difference between the average coverage rate $\overline{p}$ and each type's coverage rate weighted by the relative frequency of each type in the population (where type is understood according to Roemer's (1998) meaning), and with $0 \leq HOI \leq 1$. Such index could be viewed as one which expresses opportunity for access, both for the population taken together and for specific groups (types).

The main advantages of method (a) above are its simplicity, the ease with which information can be gathered (available in PISA reports) and the fact that it does not require further datasets, what is important for those willing to make international-intertemporal comparisons. The main advantage of method (b), in turn, is that it is not mute with respect to cross-types differential opportunities.[10]

3. **Aggregating the two dimensions**. The remaining step is to aggregate the two dimensions into one single index, which we generically call the "Bidimensional Index of Equality of educational opportunity" (or *BIE*). We work with two aggregation procedures: a direct multiplicative specification, and the fuzzy sets technique. Since we also have two procedures for calculating the access dimension, we obtain four versions of our index (BIE_V, with V = 1, …, 4). These four procedures are described below in subsections 3.1-3.4.

3.1. BIE_1: access as overall coverage rate; aggregation in a simple multiplicative form

An interesting way of dealing with both dimensions – achievement and access –, is to weigh the inverse of IO_{FG} by $\overline{p}$, that is:

$$BIE_1 = \underbrace{\overline{p}}_{access-EOp} \cdot \underbrace{\left(1 - IO_{FG}\right)}_{achievement-EOp} \qquad (3.1)$$

with: $0 < \overline{p} \leq 1$, $0 \leq IO_{FG} < 1$, $0 < BIE_1 \leq 1$.

Clearly, the index is increasing in $\overline{p}$ and decreasing in IO_{FG} as would be desirable. More interesting is to consider some limiting cases for its attributes. First, since IO_{FG} ranges in the interval [0,1), with 0 standing for perfect equality of opportunity in achievement, $(1 - IO_{FG})$ will equal 1 in the case where circumstances are unrelated to outcomes for those pupils who have taken PISA exams. In such limiting case, BIE_1 will depend solely upon the coverage rate: the

[10] The use of HOI – as opposed to Yalonetsky's (2012) recently proposed dissimilarity index – is endorsed both by Yalonetsky himself and in Pignataro's (2012) literature survey whenever the variable of interest is binary with a clear hierarchy, which is the case here: access (= 1) is more desirable than lack-of-access (= 0). Yalonetsky's (2012) index is indicated for multinomial contexts and for situations in which there is no clear advantage of one case over the other.

higher it is, the larger will be the opportunities offered to 15-year-olds of a given country. Conversely, in a case of full coverage, where $\overline{p}$ = 1, the index BIE_1 will depend solely upon inequality of opportunity in achievement. BIE_1 can also be expressed through a penalty, P, on the fact that coverage is less than full: $BIE_1 = \overline{p} \cdot (1 - IO_{FG}) = \overline{p} - \overline{p} \cdot IO_{FG} = \overline{p} - P$.

3.2. *BIE_2*: access as overall coverage rate; aggregation through the fuzzy sets technique

A second way of dealing with the two dimensions is routine in multidimensional poverty analysis. It consists of standardizing observations expressed in each dimension's metric by means of the fuzzy sets technique and then aggregating them additively (with certain weights) to generate BIE_2.

The fuzzy function of a given variable (x_1) from country i will be a linear variation between minimal and maximal values, as follows (Cerioli & Zani, 1990; Cheli & Lemmi, 1995):

$$f(x_{i,1}) = \begin{cases} 0, \; if \;\; x_{i,1} = \min \\ \dfrac{x_{i,1} - \min}{\max - \min}, \;\; if \;\; \min < x_{i,1} < \max \\ 1, \; if \;\; x_{i,1} = \max \end{cases} \tag{3.2}$$

This applies, for example, to the coverage rate variable, $\overline{p}$, where the larger it is, the better it is (i.e., the index should be increasing in $\overline{p}$). A similar function is defined but with inverted minimal and maximal values, for the fuzzy set of another variable (x_2):

$$f(x_{i,2}) = \begin{cases} 0, \; if \;\; x_{i,2} = \max \\ \dfrac{\max - x_{i,2}}{\max - \min}, \;\; if \;\; \max < x_{i,1} < \min \\ 1, \; if \;\; x_{i,2} = \min \end{cases} \tag{3.3}$$

This applies, for example, to the inequality index, IO_{FG}, where the larger it is, the worse it is (i.e., the index should be decreasing in IO_{FG}). The next step is weighing the two dimensions. We postpone a discussion about weights to Section 5 and here, as a first approach, we simply consider the simplest solution of attributing an equal weight to each dimension:

$$BIE_2 = \frac{\overbrace{f(x_1)}^{access-EOp} + \overbrace{f(x_2)}^{achievement-EOp}}{2} \tag{3.4}$$

With respect to BIE_1, a disadvantage of BIE_2 is that the calculated values will depend upon the countries used in the sample, since minimal and maximal values in the sample determine the function. The main advantage of the fuzzy method is that in a more general context (i.e., a multidimensional one) it allows to transform distributions which do not fit the range [0-1] to one that fits, a point to which we come back in Section 5.

3.3. BIE_3: access as *HOI*; aggregation in a simple multiplicative form

A third version of the index relies on *HOI* – which, as argued before, can be viewed as an "opportunity-for-access index" – to express the access dimension. The achievement dimension remains as it was before:

$$BIE_3 = \underbrace{HOI}_{access-EOp} \cdot \underbrace{(1 - IO_{FG})}_{achievement-EOp} \tag{3.5}$$

with: $0 < HOI \leq 1$, $0 \leq IO_{FG} < 1$, $0 < BIE_3 \leq 1$.

And since $HOI = \overline{p}.(1-D)$, we can rewrite (3.5) as:

$$BIE_3 = \underbrace{\underbrace{\overline{p}}_{overall} \cdot \underbrace{(1-D)}_{across-types}}_{access-EOp} \cdot \underbrace{(1 - IO_{FG})}_{achievement-EOp} \tag{3.6}$$

with: $0 < \overline{p} \leq 1$, $0 \leq D < 1$, $0 \leq IO_{FG} < 1$, $0 < BIE_3 \leq 1$.

Now we have an index which is once again decreasing in IO_{FG} (which captures inequality of opportunity in achievement), increasing in $\overline{p}$ (which captures the average access-to-PISA in a given country) but additionally it is also decreasing in *D* (which captures cross-groups inequality of opportunity in access-to-PISA). A broader range of interesting cases emerge, such as those we emphasize below:

a) If a country presents full coverage, we will have $\overline{p} = 1$ but also $D = 0$ (access will be 100% for each type). Then, BIE_1 will depend solely upon inequality of opportunity in achievement. Such case is relevant for advanced countries, where the coverage rate in PISA approaches 100%, such as Switzerland or Canada in 2006 (Figure 1). However, that is not what is observed in most countries, let alone developing countries, but it is implicitly assumed in conventional calculations of inequality of opportunity in achievement.

b) A country could have perfect equality of opportunity in achievement (IO_{FG} = 0), such that BIE_3 would depend exclusively upon access. Such second dimension would depend upon two subdimensions: the overall coverage rate, $\overline{p}$, and the cross-types dissimilarity in coverage rates, D.

c) Two countries could show similar inequality of opportunity in achievement, O_{FG}, as well as similar coverage rates, $\overline{p}$, but could differ in terms of their relative cross-types dissimilarity with respect to access. Such case might apply to pair-wise comparisons of equality of opportunity among countries from a given region.

3.4. *BIE_4*: access as *HOI*; aggregation through the fuzzy sets technique

Finally, and for the sake of completeness, we mention the version BIE_4, which would standardize *HOI* as in Equation (3.2), and aggregate the dimensions as in Equation (3.4). Pros and cons are those described in Subsection 3.2.

4. An illustration of the methodology: EOp in Latin America

In this section, we provide an illustration of our approach for six Latin-American countries that took part in PISA 2006 and 2009. We compare the rankings of inequality of educational opportunity for those countries as calculated by two versions of the bidimensional index introduced here (BIE_1 and BIE_2)[11] with the ranking obtained from a conventional index that only takes into account inequality of opportunity for achievement. Regarding test scores, we employ PISA's "plausible values" for Mathematics.

We start with results using the first technique (BIE_1) reported in Table 1 and Figure 3.

< Table 1 around here >

Figure 3 clearly reveals that both in 2006 and in 2009 rank reversals are observed when switching from an index of equality of opportunity that focuses exclusively on achievement (1 – IO_{FG}) to a more complete index that encompasses both equality of opportunity in achievement and in access (BIE_1).[12] For example, Argentina is the most opportunity-unequal country in 2006 in terms of achievement, but after taking into account its relatively good coverage rate, it moves to the third position. Chile also moves up, from the third position to the first position. Colombia and Mexico – countries that have low coverage

[11] In a companion paper, we are working in calculating estimates of BIE_3 and BIE_4.
[12] Since samples are not very large, differences in rakings should be handled with caution: the difference between two countries' calculated indices might be statistically insignificant.

rates – do the opposite movement, from first and fourth to third and sixth, respectively. In 2009, Brazil and Chile, as well as Colombia and Mexico exchange positions when we switch from the ranking based on unidimensional equality of opportunity to the one based on a bidimensional equality of opportunity.

< Figure 3 around here >

For presentational purposes, it is useful to observe our results amidst iso-opportunity curves (Barros et al., 2009), which have been plotted in Figure 4 exclusively for PISA 2006. Brazil and Mexico, below the curve "BIE_1=0,4" are to be contrasted to Chile, above the curve "BIE_1=0,5". More interesting is to compare Argentina and Colombia: while the former performs relatively well in the access-EOp dimension and does not fare very well in the achievement-EOp dimension, the latter presents the opposite situation.

< Figure 4 around here >

Results obtained using the second version of the index (BIE_2) appear in Table 2. The same rank reversals are observed. In fact, the correlation between calculated values for BIE_2 and BIE_2 is 0,97, turning the second version redundant at this point.

< Table 2 around here >

5. Generalizing the approach: Further dimensions and varying weights

It is a natural step to generalize our approach, allowing both for more dimensions and for different weights for each dimension. The intuition is that when we use such bidimensional indices, we are in fact:

(i) Defining social welfare functions – or, more precisely in this context: "equality of opportunity functions" – based upon certain attributes, $BIE = f$ (EOp in access, EOp in achievement). However, other attributes could be incorporated. Although we presented BIE_3 as a bidimensional index in which one of the dimensions is subdivided in two subdimensions, we were already hinting in fact on three dimensions: $\overline{p}$, $(1 – D)$, and $(1 - IO_{FG})$. More generally, the index could be *multidimensional*.

(ii) Working with implicit weights and thus *ad hoc* trade-offs between the two (now more) dimensions.

5.1. A multidimensional index

Average scores might be a relevant dimension of equality of educational opportunities. Worried about the quality of education of second-generation immigrants in OECD countries, Kunz (2012) is particularly concerned with those living in Germany, whose average score in reading in PISA 2009 is 474. However such indeed worrying result is in fact much better than the average score in any Latin American country in that year, where the highest average score is Chile's (449). So while second-generation immigrants are in a very bad relative position in the German context, in absolute terms the German schooling system provides more opportunity to that worse-off group for acquiring knowledge and basic skills than Latin-American schooling systems provide to their average pupils. That might be viewed as a dimension of opportunity too: *ceteris paribus*, if country A's average score is higher than country B's, country A provides more opportunities than country B.[13] Those who agree with that view could advocate four dimensions of equality of opportunity: achievement ($1 - IO_{FG}$), overall access $(\overline{p})$, cross-types dissimilarity with respect to access ($1 - D$), and a country's average score, $(\overline{s})$, suitably transformed to fit the (0,1] interval, for example by dividing the average score of a given country by the score of the country presenting the highest average score. We would have the following multidimensional index:

$$MIE_1 = \underbrace{\underbrace{(\overline{p})}_{overall} \cdot \underbrace{(1-D)}_{across-types}}_{access-EOp} \cdot \underbrace{\underbrace{(1-IO_{FG})}_{across-types} \cdot \underbrace{(\overline{s})}_{overall}}_{achievement-EOp} \qquad (3.7)$$

It should be noticed that in Equation 3.7, we have included a country's average score, $(\overline{s})$ as a fourth dimension, but that term can also be interpreted as a subdimension of achievement-EOp: in fact, within the achievement-dimension, $(\overline{s})$ is the analogue of $(\overline{p})$ within the access dimension, in the sense that both indicate the overall educational opportunities available in the country. Similarly the other two terms, ($1 - D$) and ($1 - IO_{FG}$), are analogous since both indicate the way the available opportunities are divided across types.

In table 3, we show BIE_1 multiplied by average scores, rescaled to fit the interval [0,1], both for 2006 and 2009. Mexico and Brazil exchange positions in this new ranking as compared to the BIE_1 ranking due to Brazil's extremely low average scores. The same happens in 2009 between Colombia and Uruguay.

[13] We would like to thank Erwin Ooghe for the suggestion of including average scores as an additional dimension of equality of educational opportunity.

< Table 3 around here >

We would like to add two final comments here. The first is that further dimensions could be added to MIE_1. The second is that although we only presented here as an aggregation method the simple multiplicative form, it is also possible to employ the fuzzy sets approach, a point to which we turn at the end of Section 5.

5.2. Parameterized weights

Let us take Equation 3.7 above as a starting point. There, we had implicit exponentials of 1 for all the four terms: that is, each attribute was equally weighted in such Cobb-Douglas function. But we could very well imagine that different persons value differently each dimension of equality of opportunity, so that a more appropriate way of presenting the index would be in a general form, such as the one that follows:

$$MIE = (\overline{p})^{\alpha}.(1-D)^{\beta}\cdot\left(1-IO_{FG}\right)^{\chi}.(\overline{s})^{\delta} \qquad (3.8)$$

where: α, β, γ and δ are (normative) weights, all of them nonnegative and possibly normalized to sum 1.

As a first example of the relevance of explicitly parameterizing the weights, it could be the case that for some observers average scores might have nothing to do with equality of opportunity. Based on Equation 3.8, that would simply mean they assume δ=0.

As a second example, remember that in a previous section of this paper we have expressed our view that while both access and achievement are important, access is more crucial an issue. Using a simplified version of Equation 3.8 (in which β = δ = 0), the hierarchical view advocated there could be reflected, for example in assuming α > γ. In Figure 4a, we have plotted iso-opportunity curves with β = δ = 0 and α > γ. According to the specific parameters chosen (α = ¾ and γ=1/4), we now observe that Uruguay and Argentina which were in different iso-opportunity curves when we had α = γ (Figure 4a), now share the same iso-opportunity curve, a result which is driven by the now larger weight attributed to the access dimension, in which Argentina performs very well in comparison to other Latin American countries.

Finally, we could also turn to fuzzy sets transformations in order to write a general index with i-weights, which we now denote *MIE'*$_i$: which has the advantage of allowing a less *ad hoc* transformation of the average scores than the one commented on above.[14]

$$MIE'_i = \frac{\overbrace{\underbrace{\alpha \cdot f(x_1)}^{overall} + \overbrace{\beta \cdot f(x_2)}^{across-types}}^{access-EOp} + \overbrace{\overbrace{\chi \cdot f(x_3)}^{across-types} + \overbrace{\delta \cdot f(x_4)}^{overall}}^{achievement-EOp}}{\alpha + \beta + \chi + \delta} \qquad (3.8)$$

This could be viewed as the most complete and general of all those discussed here.

6. Final remarks

The measurement of inequality of opportunity in the educational sphere has been the focus of recent contributions, which concentrate either on opportunity for access to a given level of studies, or on opportunity in terms of educational achievement. In this paper, we combine both concerns, as a way of addressing important limitations of PISA datasets regarding developing countries' coverage rates, which cast doubts on the reliability of previously calculated levels of equality of opportunity.

Instead of trying to explicitly reconstruct a full sample for each country as previously attempted in the literature, our strategy consists of calculating a bidimensional index, in which conventional equality of opportunity in test scores represents one dimension (the *achievement*-conditional-on-participating dimension) while the second dimension reflects PISA's coverage rate (or the *access*-to-PISA dimension). The method we propose could attenuate biases affecting inequality of educational opportunity indices that overlook coverage rates, which stem from the fact that many young individuals abandon the educational system in early years of their lives, either temporarily or for good – in either case, becoming ineligible to PISA exams.

It is important to notice that, while motivated by PISA's problem, our method's usefulness is not restricted to that particular test scores database; it is applicable, for example, to national datasets presenting similar problems, and possibly to noneducational spheres.

For a number of reasons – including the message given to policymakers concerned by equality of educational opportunity calculations – it does not seem reasonable to simply ignore pupils not represented by PISA samples, in particular those who are out of school or who are enrolled but attending a very low grade. These individuals face indeed even more primary

[14] There is also the technical advantage of allowing extreme cases such as zero coverage rate or average scores, as well as D=1 and IO_{FG}=1, which would lead the index in a multiplicative form to trivially collapse to zero.

forms of inequality of opportunity, and should not be disregarded. Through the illustrative exercise we undertake employing Latin-American countries that took part in PISA 2006 and 2009, we confirm our initial intuition that taking into consideration only inequality of educational opportunity in terms of achievement led to biased results: ranking the six countries according to the bidimensional index we propose differs from ranking them according to a conventional index.

The index we propose could be extended to account for more dimensions. The cost of adding more details would be paid in terms of reduced parsimony, since we would need to turn to national datasets.

There are a few possibilities of extension. For example, estimating confidence intervals for the calculated levels of equality of educational opportunity. Or testing the sensitivity of our estimations to the set of circumstances included into the regression-based estimations of inequality of opportunity for achievement. Including *HOI* in what we presented as BIE_3 and BIE_4 is another possible extension, on which we are working right now. Finally, it would be interesting to decompose the contribution to (in)equality of educational opportunities (or to variations of EOp over time) of different dimensions and subdimensions.

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Figure 1 – PISA 2006: coverage rate

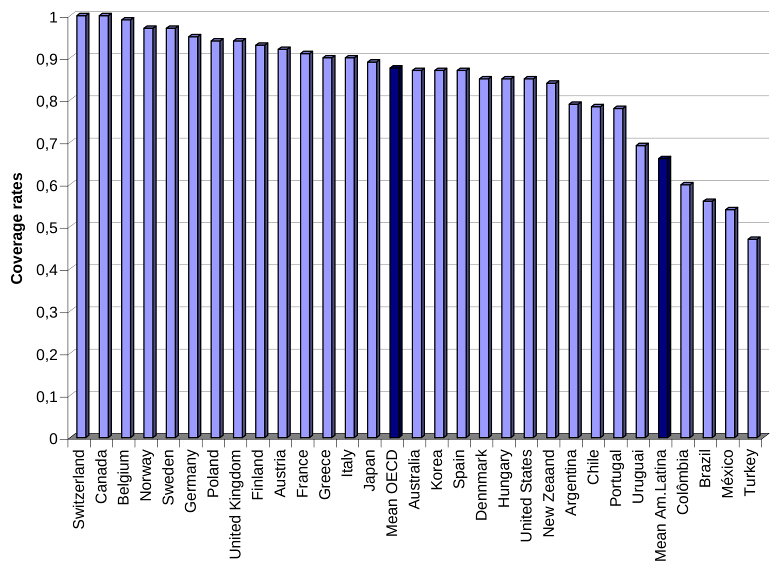
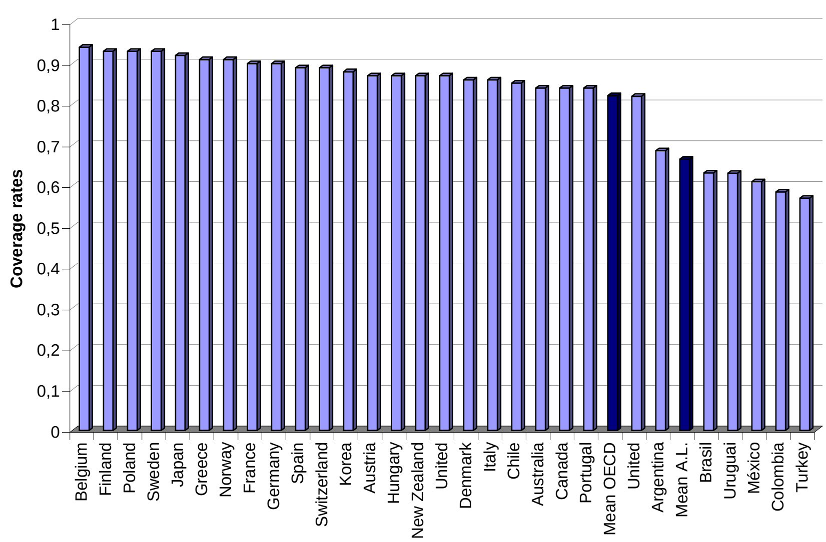


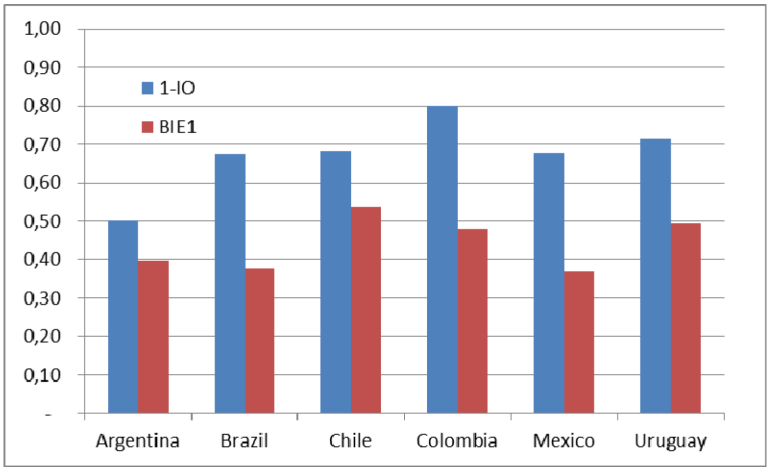
Figure 2 – PISA 2009: coverage rate



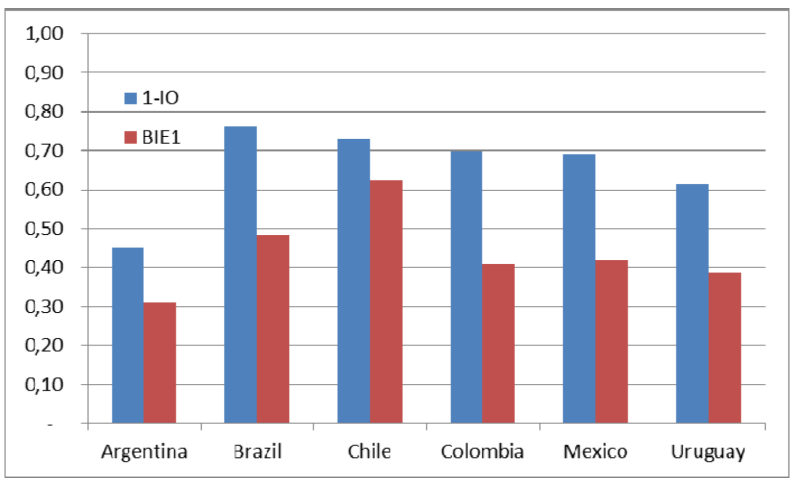
Source: PISA (OECD)

Figure 3. Rankings: equality of opportunity in achievement (1-*IO*) *versus* equality of opportunity in both achievement and access (BIE_1)

a. 2006

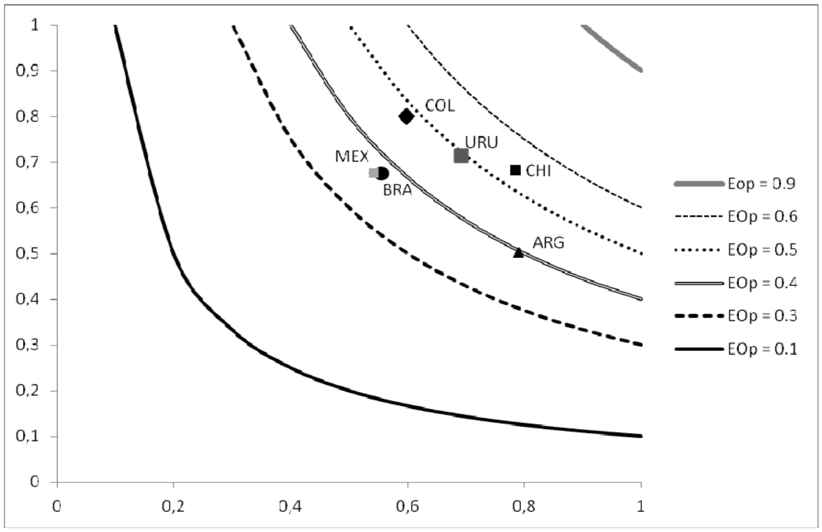


b. 2009



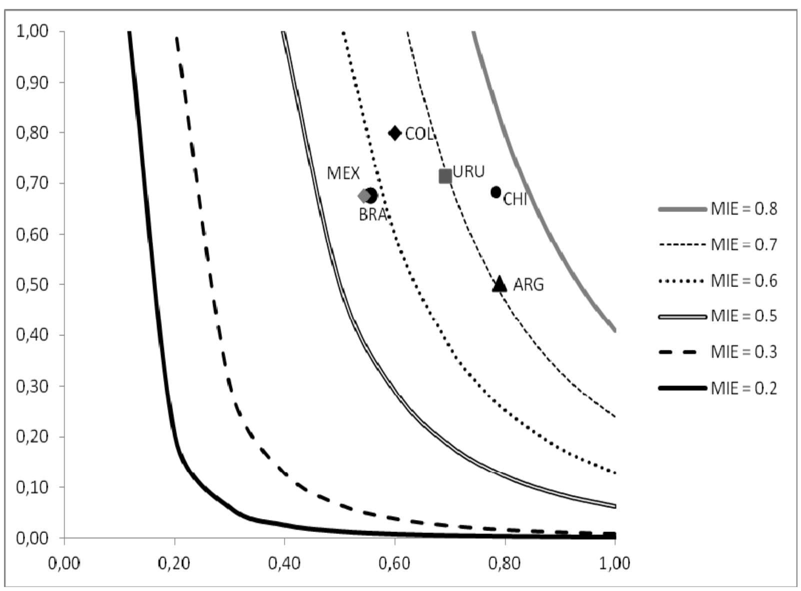
Source: PISA datasets (2006 and 2009)

Figure 4a. Iso-opportunity curves and EOp in Latin-American countries employing (BIE_1).



5b. Iso-opportunity curves and EOp in Latin-American countries employing (*MIE*)

(β = δ = 0 and α > γ)



Source: PISA 2006. α =3/4 and γ =1/4

Table 1 – Unidimensional and bidimensional indices of (in)equality of educational opportunity – 2006-2009

Country	Panel A: 2006			
	Coverage rate in PISA, $\overline{p}$	Inequality of opportunity in achievement, IO_{FG}	**UNIDIMENSIONAL equality of opportunity in achievement, (1-IO_{FG})**	**BIDIMENSIONAL equality of opportunity (achievement and access) BIE_1**
Argentina	0,7902	0,4974	**0,5026**	**0,3971**
Brazil	0,5551	0,3247	**0,6753**	**0,3748**
Chile	0,7841	0,3175	**0,6825**	**0,5351**
Colombia	0,5988	0,1994	**0,8006**	**0,4795**
Mexico	0,5423	0,3239	**0,6761**	**0,3667**
Uruguay	0,6917	0,2858	**0,7142**	**0,4940**

Country	Panel B: 2009			
	Coverage rate in PISA, $\overline{p}$	Inequality of opportunity in achievement, IO_{FG}	**UNIDIMENSIONAL equality of opportunity in achievement, (1-IO_{FG})**	**BIDIMENSIONAL equality of opportunity (achievement and access) BIE_1**
Argentina	0,6858	0,5503	**0,4497**	**0,3084**
Brazil	0,6319	0,2381	**0,7619**	**0,4814**
Chile	0,8525	0,2687	**0,7313**	**0,6234**
Colombia	0,5849	0,3026	**0,6974**	**0,4080**
Mexico	0,6067	0,3080	**0,6920**	**0,4198**
Uruguay	0,6314	0,3870	**0,6130**	**0,3871**

Note: Estimations using PISA 2006 and 2009.

Table 2 – Bidimensional indices of inequality of educational opportunity using fuzzy sets technique (*BIE_2*) – 2006-2009

Country	Panel A: 2006				
	Coverage rate ($\overline{p}$, or x_1)	Fuzzy function f(x_1)	**UNIDIMENSIONAL equality of opportunity in achievement, (*IO_{FG}*, or x_2)**	Fuzzy function f(x_2)	**BIDIMENSIONAL equality of opportunity (achievement and access) *BIE_2***
Argentina	0,7902	1,0000	**0,4974**	0,0000	**0,5000**
Brazil	0,5551	0,0513	**0,3247**	0,5794	**0,3153**
Chile	0,7841	0,9753	**0,3175**	0,6035	**0,7894**
Colombia	0,5988	0,2279	**0,1994**	1,0000	**0,6140**
Mexico	0,5423	0,0000	**0,3239**	0,5823	**0,2911**
Uruguay	0,6917	0,6025	**0,2858**	0,7100	**0,6562**
Minimum	0,5423		0,1994		
Maximum	0,7902		0,4974		

Country	Panel B: 2009				
	Coverage rate ($\overline{p}$, or x_1)	Fuzzy function f(x_1)	**UNIDIMENSIONAL equality of opportunity in achievement, (*IO_{FG}*, or x_2)**	Fuzzy function f(x_2)	**BIDIMENSIONAL equality of opportunity (achievement and access) *BIE_2***
Argentina	0,6858	0,3768	**0,5503**	0,0000	**0,1884**
Brazil	0,6319	0,1754	**0,2381**	1,0000	**0,5877**
Chile	0,8525	1,0000	**0,2687**	0,9020	**0,9510**
Colombia	0,5849	0,0000	**0,3026**	0,7935	**0,3967**
Mexico	0,6067	0,0813	**0,3080**	0,7762	**0,4287**
Uruguay	0,6314	0,1737	**0,3870**	0,5232	**0,3484**
Minimum	0,5849		0,2381		
Maximum	0,8525		0,5503		

Note: Estimations using PISA 2006 and 2009.

Table 3 – A multidimensional index: Taking average scores into account– 2006-2009

Country	Panel A: 2006					
	Coverage rate in PISA, $\overline{p}$	Inequality of opportunity in achievement, IO_{FG}	BIDIMENSIONAL equality of opportunity (achievement and access) BIE_1	Average score	Score as a fraction of the maximum possible score (700)	**MULTIDIMENSIONAL equality of opportunity index**
Argentina	0,7902	0,4974	0,3971	381	0,54	**0,22**
Brazil	0,5551	0,3247	0,3748	370	0,53	**0,20**
Chile	0,7841	0,3175	0,5351	411	0,59	**0,31**
Colombia	0,5988	0,1994	0,4795	370	0,53	**0,25**
Mexico	0,5423	0,3239	0,3667	406	0,58	**0,21**
Uruguay	0,6917	0,2858	0,4940	427	0,61	**0,30**

Country	Panel B: 2009					
	Coverage rate in PISA, $\overline{p}$	Inequality of opportunity in achievement, IO_{FG}	BIDIMENSIONAL equality of opportunity (achievement and access) BIE_1	Average score	Score as a fraction of the maximum possible score (700)	**MULTIDIMENSIONAL equality of opportunity index**
Argentina	0,6858	0,5503	0,3084	388	0,55	**0,17**
Brazil	0,6319	0,2381	0,4814	386	0,55	**0,27**
Chile	0,8525	0,2687	0,6234	421	0,60	**0,37**
Colombia	0,5849	0,3026	0,4080	381	0,54	**0,22**
Mexico	0,6067	0,3080	0,4198	419	0,60	**0,25**
Uruguay	0,6314	0,3870	0,3871	427	0,61	**0,24**

Note: Estimations using PISA 2006 and 2009.