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**Disparities in Socio-Economic Outcomes:
Some Positive Propositions and their
Normative Implications**

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Disparities in Socio-Economic Outcomes: Some Positive Propositions and their Normative Implications

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Abstract

Demographic disparities between the rates of occurrence of an adverse economic outcome can be observed to be increasing even as general social improvements supposedly lead towards the elimination of the adverse outcome in question. Scanlan (2006) noticed this tendency and developed a 'heuristic rule' to explain it. In this paper, we explore the issue analytically, providing a criterion from stochastic ordering theory under which one of two demographic groups can be considered disadvantaged and the other advantaged, and showing that Scanlan's heuristic obtains as a rigorous finding in such cases. Normative implications and appropriate social policy are discussed.

Keywords: disparity, economic outcome, poverty, mortality rate.

JEL Classification: D63, I13, I31, I32.

1. Introduction

This paper concerns the dynamics of group differences in experiencing adverse and favourable social outcomes. Scanlan (2006) provides a possible explanation, in terms of a ‘heuristic rule’, for why, for example, demographic disparities in mortality rates have, typically (and unhelpfully), been found to be increasing even as general improvements in health, supposedly leading towards the elimination of high mortality rates, have taken place. In this paper, we supplement the heuristic rule with formal analysis, and find that this perverse and stubborn outcome is actually inherent – it is a characteristic of the measurement system being used. Normative implications and appropriate future directions for social policy are discussed.

The structure of the paper is as follows. In Section 2, we briefly explain Scanlan’s empirical finding. In Section 3, we provide a criterion from stochastic ordering theory under which one of two demographic groups can be considered disadvantaged and the other advantaged, and we show that Scanlan’s heuristic obtains as a rigorous finding in such cases. Section 4 briefly considers some open theoretical questions, left by our construction in terms of the ordering of groups by disadvantage and advantage. In Section 5, we consider normative implications and we discuss the sort of broad social policy which, our analysis suggests, should guide society as it progresses.

2. The problem identified heuristically

Scanlan (2006) presents as ‘*heuristic rule X*’, henceforth *HRX*, the proposition that, when two groups differ in their susceptibility to an outcome, in a range of different contexts, the rarer the outcome, the greater the disparity in experiencing the outcome and the smaller the disparity in avoiding the outcome. Would this be so, then society’s progress in eliminating an outcome will lead to an increase in the disparity between rates, and, as Scanlan says, ‘this disparity will be greatest at the point where society verges on the total elimination of the outcome’ (Scanlan 2006, p. 48) – a striking finding, which is strongly suggestive, at first blush, of a flaw in the measurement system being invoked.

Scanlan presents the essential aspects of *HRX* through an example, based on illustrative data. The context is poverty, and the data is such that one group, in this case the blacks, is more susceptible to poverty than another, the whites, in that blacks comprise a larger proportion of each segment of the combined population up to any supposed poverty line. He claims that this situation could be found in many other contexts too, where one group is in some way disadvantaged with respect to another, in fact, in any set of data reflecting a ‘more or less normal distributions of factors’ (ibid., p. 47). *HRX* is validated in Scanlan’s particular example because the ratio of the black rate to the white rate of falling below each putative poverty line increases as that poverty line declines.¹ Hence social improvements across the general population only go to make the apparent problem, as between the disadvantaged and advantaged groups, more acute. Indeed, Scanlan says, the disparity in poverty rates can increase ‘even when the

¹ As a particular scenario, Scanlan posits an ‘across the board change’, or social improvement, such that everyone between a previously set poverty line and 50% of that poverty line, escapes poverty. Following that decline, the disparity in the black/white rates of experiencing poverty increases given Scanlan’s data.

disadvantaged group especially benefits from the decline in poverty' (ibid., p. 48).

3. The problem explored analytically

The context could be poverty, a condition associated with low values of income, or it could be health, where high mortality tends to occur predominantly towards the unhealthy end of a scale measuring people's health stock – or it could be some other attribute. The problem is founded upon the supposition that the adverse outcome (in this case poverty or high mortality) is more prevalent among the members of one demographic group (which we shall call 'disadvantaged') than another (which we shall call 'advantaged').

We begin with a definition of what it shall mean to characterize one group as disadvantaged and another as advantaged. Let us start with a non-negative variate Y , which we could call well-being², high values of which connote relative freedom from an adverse social outcome and low values of which make that outcome more likely. Let the distribution functions for Y in two demographic groups be $F_D(y)$ and $F_A(y)$. We shall name as Y_D and Y_A the Y -variates in the two groups. A specific relationship between $F_D(y)$ and $F_A(y)$, equivalently between Y_D and Y_A , if it holds, will entitle us to term the two groups disadvantaged (group D) and advantaged (group A), and to rank the latter as higher than the former in a partial ordering \prec among distributions:

Definition 1

$$Y_D \prec Y_A \Leftrightarrow \forall u, P(Y_D > u | a \leq Y_D \leq b) \leq P(Y_A > u | a \leq Y_A \leq b) \text{ whenever } a \leq b.$$

Thinking of Y_D and Y_A as random variables, distributed on $[0, \infty)$ or on some finite sub-interval, and of $P(\cdot)$ as the probability function, this condition posits that, within any tranche $[a, b]$ of well-being, people in group D are less prevalent - 'thinner on the ground' - at the top end of $[a, b]$ than people in group A . It is a strong condition, because we require it to hold for *any* chosen tranche. The condition can equally be expressed in terms of $F_D(y)$ and $F_A(y)$. For $u \in [a, b]$, $Y_D \prec Y_A$ says that $\frac{F_D(b) - F_D(u)}{F_D(b) - F_D(a)} \leq \frac{F_A(b) - F_A(u)}{F_A(b) - F_A(a)}$. We shall use Y_D and Y_A , and $F_D(y)$ and $F_A(y)$, and in fact also the density functions $f_D(y)$ and $f_A(y)$, which are the derivatives of $F_D(y)$ and $F_A(y)$, interchangeably to characterize the groups in subsequent discussion.

The partial ordering \prec of Definition 1 is known as the *likelihood ratio order* in the stochastic orderings literature. This terminology stems from an alternative characterization of \prec in terms of density functions:

Equivalent Definition 2

$$Y_D \prec Y_A \Leftrightarrow \forall y, \frac{f_D(y)}{f_A(y)} \text{ is a decreasing function of } y \text{ over the union of the supports of } Y_D$$

² Other descriptors could include economic benefit, means, good, or 'economic advantage'. In this paper we reserve the descriptor 'advantaged' for one of two groups relative to the other.

and Y_A .

This criterion says that the likelihood of y -values being found among people in group D , relative to being found among people in group A , rises the lower down the scale those y -values occur. It is another sense in which, we could say, people with higher y -values are ‘thinner on the ground’ in group D than in group A .

A third characterization of \prec comes from comparing people’s positions in the distribution of well-being y across the two groups. We could picture a sort of ‘Pen’s Parade’, in which persons in each group are lined up from poorest or unhealthiest (lowest y) to richest or healthiest (highest y), with their heights being made proportional to the level of y that they have, and we could watch how positions vary between members of the two groups who have the same height as each other as the parades pass by. If a height y is selected, $F_D(y)$ and $F_A(y)$ are the positions of persons in group D and group A respectively having that height. It is easy to see that the function $F_A \circ F_D^{-1}$ maps from the position of such a person within group D to the position of a corresponding person within group A . Another characterization of \prec can be given in terms of this mapping:

Equivalent Definition 3

$$Y_D \prec Y_A \Leftrightarrow F_A \circ F_D^{-1} : [0,1] \rightarrow [0,1] \text{ is convex.}$$

See Figure 1, in which the mapping from positions of persons of each given height y in group D to the corresponding positions of persons of height y in group A is displayed when $Y_D \prec Y_A$. People of the lower heights are close to the bottom in group A , but are much more strung out along the y -scale in group D ; and as the height-by-height comparison proceeds, with y rising, the better-off people are compressed at the very top in group D , but are spread out much more in group A ; simply put, taller people are ‘thicker on the ground’ in group A than in group D .

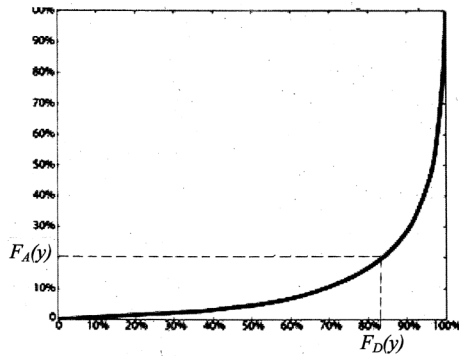


Figure 1: the mapping from positions in D to positions in A

The likelihood ratio order, defined by each of these three equivalent criteria, has a pedigree in the stochastic orderings literature. For background material and appropriate citations to original sources, the reader may refer to chapter 1C of Shaked and

Shanthikumar's (2007) book.³ See also Marshall, Olkin and Arnold (2011). Milgrom (1981) demonstrates the usefulness of the likelihood ratio order in a variety of economic contexts, and points out that, among familiar families of parametric density and probability mass functions, many obey \prec for a shift in an appropriate location parameter (only): he cites the normal, exponential, Poisson, uniform and chi-square distributions - to which one can add the lognormal, which has been used widely for modelling income distributions.

The properties enunciated above give specific senses to the notion that the people in group D are disadvantaged relative to those in group A , and they form our characterization. Groups D and A which satisfy $Y_D \prec Y_A$ will be termed 'disadvantaged' and 'advantaged' respectively, for the remainder of the paper. The condition noticed by Scanlan (2006), and articulated by him as HRX , is in fact rendered inevitable when the groups satisfy this disadvantaged/advantaged ranking condition.

Theorem 1

$$Y_D \prec Y_A \Rightarrow \frac{F_D(y)}{F_A(y)} > 1 \text{ and } \frac{1-F_D(y)}{1-F_A(y)} < 1 \text{ and both are decreasing in } y$$

It means that the rarer the outcome, in the sense that one is led to look for it at lower and lower levels of well-being y , the greater the disparity $\frac{F_D(y)}{F_A(y)}$ between the rates

of experiencing the outcome; and the smaller the disparity $\frac{1-F_D(y)}{1-F_A(y)}$ between the rates of

avoiding the outcome (the latter, because as y increases, $\frac{1-F_D(y)}{1-F_A(y)}$ increases towards 1,

the value at which the rates become the same). In Scanlan's example, HRX is seen to be operating because the ratio of the black rate to the white rate of falling below each putative poverty line increases as that poverty line declines – and conversely, the black rate of escaping poverty approaches the white rate, from below. As Theorem 1 makes clear, these are inevitabilities if the disadvantaged and advantaged groups are ordered as $Y_D \prec Y_A$. Indeed, social improvements across the general population go to make the perceived problem, as between the groups, more acute in such cases.

Scanlan also remarks in respect of HRX that the disparity in poverty rates can increase 'even when the disadvantaged group especially benefits from the decline in poverty'. One can think of a tax and benefit policy, where a positive benefit for the poorest is tapered down to zero against income and then becomes a tax, so that the incomes of those lower down each distribution are boosted at the expense of those higher up. We could model this by a transformation $y \rightarrow \varphi(y)$ where $\varphi(0) > 0$ and

³ For the reader's convenience and ease of transcription, \prec is denoted \leq_r in this book, and the random variables and distributions corresponding to our Y_D , Y_A and $F_D(y)$, $F_A(y)$ are denoted X , Y and $F(t)$, $G(t)$ respectively.

$0 < \varphi'(y) < 1 \quad \forall y$, as illustrated in Figure 2.

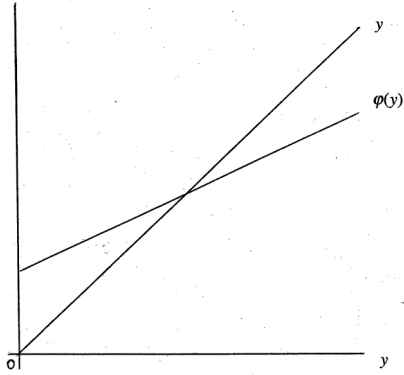


Figure 2: a tax and benefit policy φ

The introduction of such a policy does not overturn the *HRX* property, rather, this property persists in the distributions which pertain after income support has been applied. That is, the disadvantaged/advantaged characterization of the groups remains intact, post-policy - although this is not to deny that the policy mitigates, even if it does not eliminate, the extent of inter-group disparity.

Theorem 2

If φ is a strictly increasing function of y and $Y_D \prec Y_A$, then $\varphi(Y_D) \prec \varphi(Y_A)$

A more interesting question is, what sort of tax and benefit policy could reverse a specific disparity relationship between groups D and A , where the disparities are measured for a fixed poverty line $y = z$ which is unaffected by the policy? This could deliver a one-time reduction in the relative disadvantaged/advantaged rate of occurrence of the unfavourable outcome, even though, as Theorem 2 makes clear, the problem will rear its head again for subsequent general social improvements. Let the distribution functions for $\varphi(Y)$ in the two groups be F_D^φ and F_A^φ . We have just asked, could we obtain

$\frac{F_D^\varphi(z)}{F_A^\varphi(z)} < \frac{F_D(z)}{F_A(z)}$ from an appropriate policy? Note that $F_D^\varphi = F_D \circ \varphi^{-1}$ and $F_A^\varphi = F_A \circ \varphi^{-1}$.

Hence $\frac{F_D^\varphi(z)}{F_A^\varphi(z)} = \frac{F_D(\varphi^{-1}(z))}{F_A(\varphi^{-1}(z))}$, which, from Theorem 1, is less than $\frac{F_D(z)}{F_A(z)}$ if and only if

$\varphi^{-1}(z) > z$, that is, if and only if $\varphi(z) < z$: the policy must be such that a person at the poverty line z in either group is a taxpayer – and so therefore, by continuity, are some who lie below the poverty threshold.⁴

⁴ Notice, however, that if one allowed for the possibility of $\varphi'(y) \geq 0 \quad \forall y$, then inter-group disparity could be altogether eliminated by having $\varphi(y)$ be perfectly horizontal at \bar{y} , the mean. Provided that society is not destitute, i.e. provided that the poverty line is below the mean (Cowell, 1988, p.159), no poor person would pay tax in this case.

This surely surprising result only goes to reinforce Scanlan's (2006) basic message - that measuring social progress should not be attempted solely in terms of an improvement in the relative rate of occurrence of undesired outcomes as between a disadvantaged and an advantaged group. *HRX*, or its analytical equivalent as expressed in Theorem 1, is built into the measurement system, whenever the disadvantaged/advantaged dichotomy between two groups is as we have portrayed it.

4. Relationship with other stochastic orders

The property that $\frac{F_D(y)}{F_A(y)}$ is decreasing in y defines the hazard rate order: let us write $F_D \prec_{hr} F_A \Leftrightarrow \frac{F_D(y)}{F_A(y)}$ is decreasing in y . That $\frac{1-F_D(y)}{1-F_A(y)}$ is decreasing in y defines the reverse hazard rate order: let us write $F_D \prec_{rh} F_A \Leftrightarrow \frac{1-F_D(y)}{1-F_A(y)}$. These orders are used in reliability theory. The ordering $F_D(y) > F_A(y) \forall y$ also features in Theorem 1, and is known as first degree stochastic dominance: let us write $F_D \prec_{fsd} F_A \Leftrightarrow F_D(y) > F_A(y) \forall y$. Theorem 1 could have been expressed differently, as $Y_D \prec Y_A \Rightarrow Y_D \prec_{hr} Y_A \ \& \ Y_D \prec_{rh} Y_A \Rightarrow Y_D \prec_{fsd} Y_A$: see Shaked and Shanthikumar (2007, p. 43). First-degree stochastic dominance might seem a natural condition to suppose as holding between the well-being distributions of a disadvantaged and an advantaged group, but it is significantly weaker than the likelihood ratio ordering we have advocated.⁵ However, the likelihood ratio order is itself stronger than we actually need to obtain *HRX* : from the preceding discussion, *HRX* only requires the conjunction of $Y_D \prec_{hr} Y_A$ and $Y_D \prec_{rh} Y_A$.⁶ There is meat here for a deeper analysis of the role of stochastic orderings in characterizing the dichotomy between distributions of well-being in terms of disadvantage and advantage.

5. Normative issues and appropriate social policy

It seems to be an implicit or background assumption in Scanlan (2006) that it would be socially unacceptable for a government to give direct support to members of a disadvantaged group *only*. The obvious policy, of focusing support on that group only, could be challenged as an instance of reverse discrimination, leaving members of the

⁵ $F_D \prec F_A$ demands that the curve in Figure 2 be convex, and running from (0,0) to (1,1), whilst $F_D \prec_{fsd} F_A$ merely demands that this curve lie below the 45° line running from (0,0) to (1,1). In fact, from Definition 3, $Y_D \prec Y_R$ is equivalent to $\{Y_D | a \leq Y_D \leq b\} \prec_{fsd} \{Y_A | a \leq Y_A \leq b\}$ whenever $a \leq b$, which is clearly significantly stronger than $F_D \prec_{fsd} F_A$.

⁶ If X is uniform on $\{1,2,3,4\}$ and Y is defined by $P(Y=1) = 0.1$, $P(Y=2) = 0.3$, $P(Y=3) = 0.2$, $P(Y=4) = 0.4$ then, as Shaked and Shanthikumar (2007, p. 43) note, $X \prec_{hr} Y \ \& \ X \prec_{rh} Y$ but it is not the case that $X \prec Y$.

non-disadvantaged group to their fate in terms of the undesired outcome, to which they are also vulnerable. In this view, and as Scanlan says, ‘progress in almost every area of human well-being is a matter of increasingly restricting adverse outcomes to the point where only the most susceptible segments of the population experience the outcome’ (ibid., p. 48). The case against preferential discrimination is not, however, unproblematically obvious. Part of the difficulty with ‘reverse’ discrimination resides in the very nomenclature employed for the policy: the phenomenon becomes altogether less objectionable when it is referred to as ‘compensatory’ discrimination. This is no mere matter of playing with words. ‘Compensatory’ discrimination is a notion that arises naturally when the claim it supports is what Dworkin (1977) calls ‘the right to treatment as an equal’, rather than ‘the right to equal treatment’. The former, for Dworkin (ibid., p.227) is the right to be treated ‘with the same respect and concern as anyone else’, while the latter is the right to ‘an equal distribution of some opportunity or resource or burden’. Dworkin believes the first right to be fundamental, and the second right to be contingent on, and derivative from, the first. Substantive equality, in this perspective, can only be secured from an unequal – not an equal – treatment of unequals. Compensatory discrimination would indeed leave the non-disadvantaged group to their fate, but that is only because of the given prior fact that the fate of the non-disadvantaged group has been kinder to it than the fate of the disadvantaged group has been to the latter.

This, however, does not negate the fact that policies to increasingly restrict adverse outcomes do, indeed, lead to increases in the disparity between the rates at which the outcome is experienced in the two groups. Indeed, it is a consequence of $Y_D \prec Y_A$ that $\frac{F_D(y)}{F_A(y)}$ becomes maximal as $y \rightarrow 0$, and so does $\frac{1 - F_D(y)}{1 - F_A(y)}$, where $[0, y]$ is the range to which the adverse outcome is confined. The former measure may become unboundedly large, whilst the latter one, which compares disparity rates in *avoiding* unfavourable outcomes, approaches unity from below. Scanlan’s insightful reasoning notes that these twin properties will hold in any context in which *HRX* operates. Theorem 1 establishes that these properties are in fact inevitable whenever disadvantaged and advantaged groups are ordered by \prec . Comparisons made in terms of the adverse outcome will indicate that disparities are increasing, whilst comparisons made in terms of the favourable outcome will provide the opposite conclusion. This could lead to the inference that these disparity measures are unsuited to their intended purpose.

We take a somewhat wider perspective here, arguing that the practice of focusing exclusively on between-group disparity, and indeed of confining that focus lower and lower down the distribution of well-being as society progresses, without any consideration of improvements in the aggregate/average level of well-being, or indeed of inequality generally between the advantaged and disadvantaged groups, is not useful. In general, ‘Rawlsian’ approaches of focusing exclusively on the worst-off section of a population are compatible with what some commentators have called ‘a dictatorship of the weakest, or poorest, or most disadvantaged’. This is also why analysts of welfare, inequality and poverty such as Shorrocks (2004) have found ‘Rawls-type’ demands of equity to be extreme, and somewhat seriously overriding of the claims of efficiency, unlike a more relaxed and yet equalitarian requirement such as is encompassed in Sen’s

(1973) ‘weak equity axiom’ which insists only that an optimal distribution of resources between two individuals should offer a larger share to the uniformly more disadvantaged person. These concerns, as it happens, are congruent with a social policy that promotes compensatory discrimination – not least because an assertion of equality through this route favours the idea of helping those that have lagged behind rather than the idea of hindering those that are ahead: such an approach, in particular, resists the achievement of equality by resort to what Parfit (1997) calls ‘levelling down’.

What we suggest, in brief, is a more plural approach to social assessment, in which the overall ‘goodness’ of any state of affairs is seen to depend on both the average level of well-being and on the inequality of its distribution, not just on one or other of these two aspects – and certainly not on that ‘lower tail statistic’ which is at the heart of *HRX*. But this does not warrant a reconsideration of the way in which ‘unambiguous disadvantage’ is customarily defined. Rather, we need to take into account mean levels as well as relative differences in outcomes *across the entire spectrum of society*.

This does not mean throwing out the baby along with the bath water. Indeed, a wider discussion of appropriate social policy can be framed in terms of the measurement of relative disparities, as we shall now see. Let us define the *disparity curve*, for the purposes of this discussion, as $\frac{f_D(y)}{f_A(y)}$, $y \in \mathbb{R}^+$: see Figure 3. In this figure, the horizontal axis measures levels of individual well-being y ; the sort of cut-off value, at which Scanlan applies *HRX*, might be given as a poverty line, below which deprivation is higher and of greater social concern, but disparities in the unfavourable outcome are found at higher levels of y too, though of a lesser intensity.

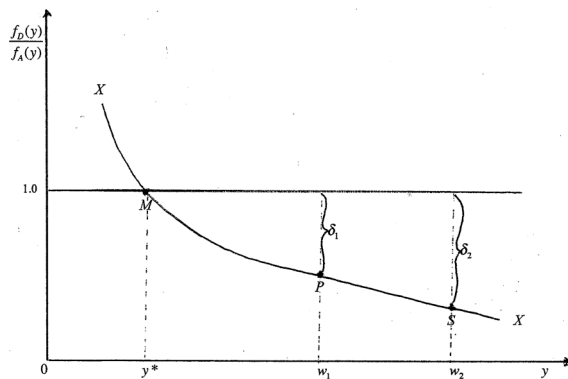


Figure 3: a disparity curve

In a typical ‘social state’, let the mean level of well-being be $\bar{y} = w_1$. This value is marked on the horizontal axis in Figure 3. The value δ_1 , also marked, can serve as a proxy for inter-group inequality in this social state, measuring how far the relative disparity between groups is away from unity *at the mean*. Thus, let the point marked as *P* in Figure 3 represent this social state.

The value y^* , also marked in Figure 3, denotes a ‘threshold’ level of individual well-being at which relative frequencies are the same in both groups: $f_D(y^*) = f_A(y^*)$. Were society to reach the point *M*, with average well-being y^* , inter-group inequality would

effectively cease to exist. But of course this happy situation could only occur at a very low level of average well-being – and this would be found appealing only by a commentator who gives strict lexical priority to equality over efficiency.

Suppose society finds itself at point P in Figure 3, and wishes to improve its mean level of well-being, from w_1 to w_2 , say. This would involve a movement from point P to point S . The gain in efficiency would unfortunately be accompanied by an increase in inter-group inequality, from δ_1 to δ_2 . One is inclined to ask: what sort of decision-maker would resist ‘progress’ such as this? Presumably only one who holds the value judgment that the virtue of equality is to be lexicographically prized above all other values, including that of society’s mean level of well-being. But for such a decision-maker, the movement from P to *any* point to the right of P on the disparity curve is not perceived as ‘progress’!

Scanlan’s *HRX* bears upon a supposed concern that rates of occurrence of an adverse economic outcome can be observed to be increasing *even as society progresses*. Then it is misdirected, surely, to focus only on the diminishing tails of the distributions of well-being in disadvantaged and advantaged groups to the exclusion of all other factors. One should be expected to entertain the possibility of a tradeoff between the claims of distribution and size. For example, one could conceivably find merit in a real-valued measure G of the ‘goodness’ of a social state, such that G is increasing in w and declining in δ ; a trivial example is furnished by the formulation $G = w/\delta$. Under these circumstances, there would be nothing objectionable about increasing the level of w even if δ should rise into the bargain, as long as the value of G too increases.

The disparity curve can indeed be a relevant instrument in the discussion and framing of social policy.⁷ *HRX* does not invalidate the use of the disparity curve – only the use to which it has been put by some commentators, the use which Scanlan’s *HRX*, supported by our analytics earlier in this paper, decisively rejects as being misguided. At the same time, one should not think that the slope of the disparity curve is immutable. Most egalitarians, it seems to us, would insist precisely on the requirement that social policy should be directed toward flattening the disparity curve. Now consider Figure 4, which tells us that if, starting from the point P , social policy has succeeded in transforming the disparity curve XX to the curve $X'X'$, then all points on the line segment QR are unambiguously ethically superior to the point P . Through a sequence of increasingly flattened disparity curves one could, presumably, engineer the society from the point P along a path such as that marked, to a point T , which is both *on* the unit line and to the right of P . From T , any further movement rightward would constitute an unambiguously ethically superior move: for it is precisely along the unit line, that increases in well-being will be unaccompanied by increases in disparity.

⁷ Its ingredients, which are the relative frequency curves $f_D(y)$ and $f_A(y)$, feature centrally in the class of segregation measures proposed by Duncan and Duncan (1957), and used since in modified form to capture occupational discrimination by gender and by race. In that literature, differences in the distributions of the respective groups, as well as differences in mean earnings between the respective groups, are both highly relevant; see Wolff (1976, p. 152).

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