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Abstract

Poverty reduction has emerged as a fundamental objective of development and hence a metric for assessing the aggregate performance of public policy. Declaring a policy outcome pro-poor on the basis of changes in an aggregate indicator may hide more than it reveals about the heterogeneity of impacts underlying the aggregate outcome. This paper demonstrates the use of influence functions to link poverty-focused evaluation functions to individual or household characteristics and perform counterfactual decompositions in order to identify and analyze the endowment and structural effects and their determining factors that ultimately drive pro-poor outcomes. An empirical illustration presents an analysis of the pro-poorness of the growth pattern in Bangladesh in 2000-2010. We find that socioeconomic arrangements in Bangladesh have become more progressive over time.

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1. Introduction

Policy impact analysis entails an assessment of variations in individual and social outcomes attributable to a socioeconomic shock or the implementation of public policy. In essence, this is an exercise in social evaluation. According to Sen (1995) we can learn a great deal about any evaluative approach by considering its *informational basis*, the identification of which involves a distinction between the information required to pass judgment in the chosen approach and that which has no direct evaluative role. This author further identifies two basic components of the informational basis: the *focal space* and the *focal combination*. The former specifies the objects of value or desirable outcomes while the latter provides a rule for combining individual outcomes into an aggregative indicator of the prevailing social state. The focal combination is essentially a social evaluation function (e.g. social welfare function) used to rank states of the world. These considerations, along with the need to understand the determinants of outcomes, suggest that an evaluation framework can be structured around three basic dimensions: (1) the *metric* used to identify desirable outcomes; (2) *attribution* of outcomes to explanatory factors; and (3) *ranking* of social states.

The *policy objective* defines the yardstick by which to assess policy impact. It is commonly accepted that maintaining and improving the living standard of the population is the ultimate goal of public policy and a fundamental expectation of the governed (Sen et al. 1987). The concept of living standard thus plays a crucial role in the specification of the focal space for policy evaluation. The ranking of social states entails the use of a *social evaluation function* (i.e. the focal combination) and a decision rule for choosing socially desirable outcomes. Poverty reduction has emerged as a fundamental social objective of development and hence a metric for assessing the aggregate performance of public policy. In the context of a poverty-focused evaluation, one is interested in whether the distributional changes induced by policy or by the development process in general are "pro-poor" or not?

The *pro-poorness* of a distributional change depends on the chosen value judgments. A variety of standards underlie the concept of pro-poorness. In general, a pro-poor policy leads to a social outcome that is favorable to the poor in some sense. As Duclos (2009) explains,[1] the translation of the term "favorable" can be based on an absolute or a relative standard of evaluation. Let y_0 represent a distribution of outcomes (say living standards) in the absence of the policy under consideration. Also let y_1 stand for the outcome distribution induced by policy implementation. An *absolute pro-poor standard* is a quantity, say α, such that a change in the overall distribution from y_0 to y_1 (e.g. as the result of a policy intervention or the process of economic growth) will be considered pro-poor if the outcomes for the poor all change by at least the amount α. A *relative pro-poor standard* is defined by a factor $(1+\rho)$ to indicate the minimum change in living standards that society would like the poor to experience given the change in the overall distribution. According to this relative standard, a policy that changes the overall distribution from y_0 to y_1 will be considered pro-poor if the outcomes for the poor change by a factor of at least $(1+\rho)$. For instance, if this standard is set to the ratio of the mean of y_1 to that of y_0 then a pro-poor policy should increase the outcomes of the poor in proportion to overall average growth. This echoes the following quote from *The Economist* about the poverty impact of economic growth: "Growth really does help the poor: in fact it raises their incomes as much as it raises the incomes of everybody else" (*The Economist*, May 27, 2000 as cited by Ravallion 2001).

For Ravallion and Chen (2003) and Kray (2006), a distributional change is pro-poor if it involves poverty reduction for some choice of poverty index, P. Osmani (2005) argues that a poverty-reducing change should not be considered automatically pro-poor. He recommends that a policy intervention be considered pro-poor if it achieves an absolute reduction in poverty greater than would occur in a *benchmark* case. Such a benchmark could be a counterfactual or some socially desirable outcome. Essama-Nssah and Lambert (2009) use poverty elasticity to pass pro-poor judgments in a way that is consistent with Osmani's recommendation, using distribution neutrality as the benchmark.

The second issue that one must confront in the formulation of a poverty-focused criterion is whether more weight should be given to the outcome of the poorer of the poor. The construction of a criterion involves aggregation across individuals or households, for

[1] We draw significantly in what follows on issues raised in Duclos (2009).

which a value judgment, e.g. of the degree of inequality or poverty aversion, is typically assumed. Evaluation functions which give more weight to the poorer of the poor are consistent with the Pigou-Dalton principle of transfers based on the idea that less inequality is preferred to more.

Declaring a policy outcome or a pattern of growth pro-poor is the result of an aggregate judgment that may hide more than it reveals about the heterogeneity of impacts underlying the aggregate outcome. Yet, for policymaking and evaluation purposes, there is a need for a deeper understanding of this diversity in policy impacts. Such an understanding could stem from the view that a policy intervention is basically a social arrangement. In other words, an intervention is a mechanism for controlling and coordinating the behavior of concerned socioeconomic agents toward the achievement of the policy objective. In this perspective, an individual outcome is a function of *participation* and *type*, where type is characterized by such things as preferences, capabilities, information and beliefs (Milgrom 2004). In general, the outcome obtained by an individual from participation (in an intervention) is a result of the interaction between opportunities offered by the intervention and the readiness and ability of the individual to take advantage of such opportunities.

Linking individual outcomes explicitly to participation and type provides an opportunity to account for the contributions of each of these factors into changes in the distribution of outcomes from one state of the world (e.g. policy state) to another (e.g. counterfactual state). Differences in outcome distributions therefore reflect, among other things, differences in participation and in the distribution of the characteristics underlying the definition of type. Such characteristics may be observable or not.

The main purpose of this paper is first to demonstrate how influence functions can be used to link a variety of measures of pro-poorness to household (or individual) characteristics, and second to perform counterfactual comparisons in order to identify the composition and structural effects. The structural effect determines the impact of policy viewed as *returns to participation*, whilst the composition effect measures the influence of changes in individual or household characteristics interpreted as *endowments*.[2]

A decomposition method can be viewed as an input-output process that translates variations in a variable of interest into a set of contributory factors known as the *terms of the decomposition*. Basically, such a method is characterized by the outcome model which links the outcome of interest to its determining factors, and by the strategy used to identify and hence estimate the terms of the decomposition. The classic Oaxaca-Blinder method seeks to decompose the overall difference in the unconditional mean of an outcome distribution between two groups or time periods into a component due to changes in the distribution of individual (or household) characteristics (i.e. the composition or endowment effect), and another due to changes in the returns to those endowments (i.e. the price or structural effect). The method relies on the law of iterated expectations to link the unconditional mean outcome to endowments, and on counterfactual comparisons[3] based on *ceteris paribus* variations to identify and estimate those effects.

We focus here on decomposing variations in poverty outcomes. All poverty measures considered here (and in fact all social evaluation functions) can be viewed as real-valued functionals of the relevant outcome distributions. This is our starting point in modeling poverty outcomes. Furthermore, an outcome distribution reflects variation in individual outcomes based on variation in type and participation. These considerations motivate our linking relevant social evaluation functions to individual or household characteristics and performing counterfactual comparisons to identify the composition and structural effects.

The rest of the paper is organized as follows. Section 2 discusses the decomposition framework based on the logic underlying the Oaxaca-Blinder approach, and this is where

[2] The composition and structural effects are also known as endowment and price effects respectively (Bourguignon and Ferreira 2005).

[3] Counterfactual comparisons are commonly used to identify the causal effect of an intervention because *the effect of a cause can be understood only in relation to another cause* (Holland 1986). This is the same idea underlying the economic principle of assessing the return to a resource employed in one activity relative to its *opportunity cost* (i.e. what it would have earned in the next best alternative use). A counterfactual state is the state of the world that, most likely, would have prevailed in the absence of the intervention. The analogy between treatment effect analysis and the decomposition approach used in this paper has been extremely useful for the development of flexible methods of estimating composition and structural effects.

we will encounter influence functions. An influence function is essentially the first-order (directional) derivative of the associated functional. Section 3 applies that framework to understanding the pro-poorness of the pattern of economic growth in Bangladesh in 2000-2010. Concluding remarks are presented in section 4. We find that the distributional change observed over the 2000-2010 period is unambiguously pro-poor. Furthermore, the configuration of the endowment and structural effects suggests that socioeconomic arrangements in Bangladesh have become more progressive over time.

2. A Counterfactual Decomposition Framework

Pro-poor judgments define criteria that might be used to rank social states. For policymaking purposes, it is not enough to declare an outcome pro-poor or not, one is also generally interested in understanding the drivers of the observed outcome. In this section, we present a decomposition framework that can often help with the identification of sources of variation in social outcomes. Even though we are interested in decomposing variations in poverty outcomes, it is instructive to present the analytical framework in more general terms. We thus organize the discussion in terms of decomposing variations in social outcomes represented by social evaluation functions. A decomposition method is characterized by the underlying outcome model and the identification strategy for estimating the terms of the decomposition. We focus on modeling the social outcome and the identification and estimation of the composition and structural effects. We also present some recentered influence functions that one can use in poverty-focused evaluation and explain their roles.

2.1. The Outcome Model

Let θ_t stand for the social outcome of interest at a particular point in time, t. We take t=0 and t=1 subsequently to indicate pre- and post-intervention time points. This is a distributional statistic characterizing the social state represented by the distribution of individual outcomes F_y. This social outcome can be viewed as a functional of F_y, and expressed as follows.

$$\theta_t = \theta\left(F_{y_t}\right); t = 0, 1 \tag{2.1}$$

Individual outcome y_t is a function of type and participation in the life of the community. Ultimately, this outcome depends on endowments, behavior and the circumstances that determine the returns to the endowments from any social transaction. The following equation describes a general relationship between individual outcomes and individual characteristics.

$$y_t = h_t(x_t, \varepsilon_t),\ t = 0, 1 \qquad (2.2)$$

where x_t and ε_t stand for observed and unobserved characteristics respectively. The function $h_t(\cdot)$ represents the outcome structure defining the system that rewards characteristics in any social transaction. Equation (2.2) is a reduced-form representation of behavior and social interaction.

At this point, we need a way to link the social outcome explicitly to individual (or household) characteristics. For this, we rely on the concept of influence function. Basically, the influence function of a functional θ(F) is its first-order directional derivative (Hampel 1974). Let F and M be two distributions and θ(·) a distributional statistic that is qualitatively or infinitesimally robust[4]. When M is close to F, then θ(M) should be close to θ(F). Letting Δ_y be a distribution where the value y occurs with probability 1, now take for M the distribution in which an observation is randomly sampled from distribution F with probability (1-b) or from $\Delta_y$with probability b. Thus

$$M = (1 - b)F + b\Delta_y \qquad (2.3)$$

This mixture distribution and distribution F itself can be made arbitrarily close by choosing b sufficiently small. The influence function of θ at F is

$$IF(y; \theta, F) = \lim_{b\to 0} \frac{\theta(M) - \theta(F)}{b} = \frac{\partial}{\partial b}\theta\left((1 - b)F + b\Delta_y\right)|_{b=0} \qquad (2.4)$$

[4] The concept of influence function arises in the context of robust methods in statistics. Let f(y) be any function of y, and suppose that we do not want this function to change drastically with small changes in y. One way of securing this is to impose *continuity* on this function. To have a measure that is relatively unaffected by small shifts in the underlying distribution we require the associated functional to be *continuous*. In other words, continuity characterizes *qualitative robustness* (Wilcox 2005). Just as continuity is related to the notion of qualitative robustness, *differentiability* is linked *to infinitesimal robustness*. If a function f(y) is differentiable and its derivative is bounded, then small changes in y will not result in large changes in f(y). A search for robust distributional statistics often focuses on functionals with bounded derivatives.

This is essentially the derivative of θ(M) with respect to b evaluated at b = 0. It measures the relative effect of a small perturbation in F on θ(F).

An important property of the influence function is that, in all cases in which the frequency and range of the y-values are bounded, we have $E(IF(y;\theta,F)) = \int_{-\infty}^{\infty} IF(y;\theta)dF(y) = 0$. This fact implies that we can define any social evaluation function for which there is an influence function by the unconditional expectation of the corresponding recentered or rescaled influence function or RIF which equals the functional plus the corresponding influence function:

$$RIF(y;\theta,F) = \theta(F) + IF(y;\ \theta,F) \tag{2.5}$$

The expected value of the RIF is thus equal to the corresponding distributional statistic. In other words, $\theta\left(F_y\right) = E[RIF(y;\ \theta)]$. Equation (2.5) suggests that the RIF represents the leading two terms of a von Mises (1947) linear approximation of the associated functional (Firpo et al. 2009).

By the law of iterated expectations, the social evaluation function, $\theta\left(F_y\right)$, can be written in terms of the conditional expectation of the recentered influence function (given the observable covariates, x). This conditional expectation is known as an RIF regression, which we express as: $E[RIF(y;\theta)|x]$. Thus,

$$\theta(F_y) = \int E[RIF(y;\ \theta)|x]dF(x) \tag{2.6}$$

To assess the impact of covariates on $\theta\left(F_y\right)$, one needs to integrate over the conditional expectation $E[RIF(y;\theta)|x]$. This can be easily done using regression methods as we shall discuss later on.[5]

2.2. The Endowment and Structural Effects

We are interested in decomposing a change in the social outcome defined in equation (2.1) from the base period t=0 to the end period t=1. Let $F_{y_0|t=0}$ stand for the

[5] There is indeed an intimate relationship between regression and the conditional expectation function (CEF). Any random variable y can be decomposed into a component associated with x, $E[y|x]$, and a residual, ε, that is uncorrelated with any function of x. In other words, $y = E[y|x] + \varepsilon$ and $E[x\varepsilon] = 0$. On the basis of this observation, Angrist and Pischke (2009) argue that the CEF is a good summary of the relationship between y and x in the sense that it is the best predictor of y given x in the class of all functions of x. This property stems from the fact that the CEF minimizes the mean squared error of prediction. Furthermore, if the CEF is linear then it is the population regression function since the latter solves the least squares problem at the population level. As it turns out, even when the CEF is nonlinear, regression is still the best linear approximation to it.

outcome distribution observed in the base period and $F_{y_1|t=1}$ the outcome distribution observed in the end period. The overall variation in $\theta(F_y)$ induced by the distributional change from period 0 to period 1 is equal to the following.

$$\Delta_O^{\theta} = \theta\left(F_{y_1|t=1}\right) - \theta\left(F_{y_0|t=0}\right) \tag{2.7}$$

We need to consider the conditions under which we can split this difference into endowment and structural effects along with an estimation procedure to recover these objects from available data.

2.2.1 Identification

Identification concerns restrictions that must be placed on the outcome model in order to recover in a meaningful way the terms of the decomposition. In the context of policy impact evaluation, all identification strategies seek to isolate an independent source of variation in policy and link it to variation in outcome to determine policy impact. This process usually entails a comparison of the observed policy outcome with a counterfactual representing what would have happened in the absence of the policy under evaluation. The construction of the counterfactual state relies on the notion of *ceteris paribus variation.* In other words the only difference between the observed and counterfactual states is policy implementation, everything else is the same. Similarly, decomposition methods in economics rely on this fundamental strategy to identify the terms of the decomposition. In particular, the contribution of a given factor to a distributional change is identified by comparing the outcome distribution observed at the relevant point in time with a counterfactual distribution obtained by changing the factor under consideration while holding all other factors fixed.

The social outcome model discussed above suggests that there are four factors that potentially can account for the distributional change characterized by equation (2.7). The observed distributional change may be due to: (i) Differences in the distribution of observed characteristics, x; (ii) Differences in the distribution of unobserved attributes, ε (see (2.2)); (iii) Differences in the returns to observed characteristics as determined by the pay-off functions h(·); (iv) Differences in the returns to unobserved characteristics as determined by the pay-off functions h(·) in (2.2). Without imposing any separability assumption on the structure of these pay-off functions, we cannot distinguish the contribution of observable characteristics from that of the unobservables. We therefore lump the last two terms into a single one that we refer to as the structural effect, denoted by: Δ_S^θ. For this term to be meaningfully interpreted as structural effect, it must reflect solely differences in the pay-off function. This function must therefore remain stable as the distributions of characteristics (observed and unobserved) change from one period to the next. This would be the case if there were no general equilibrium effects associated with changes in the distribution of characteristics.

Let Δ_X^θ and $\Delta_\varepsilon^\theta$ stand respectively for the contributions of differences in the distributions of x and ε to the overall distributional change Δ_O^θ. Under the assumption of no general equilibrium effects, we can write the overall distributional change as follows: $\Delta_O^\theta = \Delta_S^\theta + \Delta_X^\theta + \Delta_\varepsilon^\theta$. The identification of the contributions of differences in the distribution of observed characteristics hinges on the nature of the joint distribution of x and ε. This identification requires an estimate of a counterfactual outcome indicating what would have been observed if everything was the same for the two periods except the distribution of observable characteristics. Let $y_{0|t=1}$ be the outcome that would have occurred in period 1 had individual characteristics in that period been rewarded according to the pay-off regime of period 0. Let $F_{y_0|t=1}$ stand for the corresponding distribution and $\theta\left(F_{y_0|t=1}\right)$ the associated statistic of interest. The endowment or composition effect is represented by the following expression:

$$\Delta_X^\theta = \left[\theta\left(F_{y_0|t=1}\right) - \theta\left(F_{y_0|t=0}\right)\right] \tag{2.8}$$

For this term to be meaningful and identifiable, it must emerge from a *ceteris paribus* variation of the distribution of observable characteristics. This would be the case if variations in the distribution of observables were not confounded by changes in the distribution of unobservables. It is common to impose the *ignorability* assumption in order to secure the identification of the composition effect when general equilibrium effects have been ruled out. This assumption, also known as conditional independence, translates the idea that the conditional distribution of unobservables (ε) given the observables (x) is the same in both periods. Hence: $\Delta_{\varepsilon}^{\theta}= 0$, and the structural effect is identified by:

$$\Delta_S^{\theta}= \left[\theta\left(F_{y_1|t=1}\right) - \theta\left(F_{y_0|t=0}\right)\right] \tag{2.9}$$

The aggregate decomposition of (2.7) thus boils down to: $\Delta_O^{\theta}= \Delta_S^{\theta} + \Delta_X^{\theta}$. Alternatively, we have:

$$\Delta_O^{\theta}= \left[\theta\left(F_{y_1|t=1}\right) - \theta\left(F_{y_0|t=1}\right)\right] + \left[\theta\left(F_{y_0|t=1}\right) - \theta\left(F_{y_0|t=0}\right)\right] \tag{2.10}$$

This is obtained by subtracting from and adding to the overall distributional change the counterfactual outcome $\theta\left(F_{y_0|t=1}\right)$. Letting P stand for the poverty measure of interest, then equation (2.10) for the observed change in poverty between the base and end periods can be decomposed as follows.

$$\Delta_O^{P}= \left[P\left(F_{y_1|t=1}\right) - P\left(F_{y_0|t=1}\right)\right] + \left[P\left(F_{y_0|t=1}\right) - P\left(F_{y_0|t=0}\right)\right] \tag{2.11}$$

where the first term on the right hand side represents the structural effect and the second the composition effect.

2.2.2 Estimation by RIF Regression

There are both parametric and nonparametric approaches for estimating the terms of the above decomposition. In this paper we follow a parametric approach based on RIF regression. This entails the specification of a regression model on the basis of the conditional expectation of the RIF. This could be a linear or a nonlinear model. In particular, modeling this conditional expectation as a linear function of the observed covariates leads to the following expression:$E[RIF(y;\theta)|x] = x\beta$. The expected value of the linear approximation of the RIF regression is equal to the expected value of the true conditional expectation because the expected value of the approximation error is zero (Firpo et al. 2009).

We can then apply Ordinary Least Squares (OLS) to the following equation to obtain estimates of the relevant parameter:

$$RIF(y;\theta,F) = x\beta + \varepsilon \tag{2.12}$$

Applying the standard Oaxaca-Blinder approach,[6] we compute the structural effect

$$\hat{\Delta}_S^{\theta} = \bar{x}_1(\hat{\beta}_1 - \hat{\beta}_0) \quad (2.13)$$

where $\hat{\beta}_t, t = 0, 1$ are the OLS estimates of the coefficients of the observed covariates in (2.12) and $\bar{x}_t, t = 0, 1$ are the sample counterparts of the following expectations: $E(x|t = 0,1)$. The composition effect is

$$\hat{\Delta}_X^{\theta} = (\bar{x}_1 - \bar{x}_0) \cdot \hat{\beta}_0 \quad (2.14)$$

RIF regression thus offers a simple way of establishing a direct link between a social evaluation function and individual (or household) characteristics. This link offers an opportunity to perform both aggregate and detailed decompositions for any evaluation criterion for which one can compute an influence function. This fact makes the extension of the standard Oaxaca-Blinder decomposition to RIF regressions both simple and meaningful. Assuming that the RIF regression model is linear makes it possible to further decompose the endowment and structural effects in terms of the contributions of the relevant covariates[7].

2.3. RIFs for Poverty-Focused Evaluation

We now consider some recentered influence functions we will use in the empirical section for decomposing pro-poorness.[8] Pro-poor judgments are formulated on the basis of variation in poverty outcomes. Potential dominance relations between the initial and end outcome distributions provide a basis for passing *unanimous pro-poor judgments*. Denote the poverty level associated with a distribution F and poverty line z as P(F; z). Thus

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[6] As noted in the introduction, the standard Oaxaca-Blinder method seeks to decompose a change in the unconditional mean, μ_F, of an outcome distribution into a structural and a composition effect. It can be shown that the influence function of the mean is: $IF(y; \mu, F) = y - \mu_F$. Therefore, $RIF(y; \mu, F) = y$ and $E[RIF(y; \mu, F)] = E(y) = \mu_F$. The conditional expectation function underlying RIF regression is: $E[RIF(y; \mu, F)|x] = E(y|x)$. One can apply the law of iterated expectations to this expression to recover the unconditional mean of y. The standard Oaxaca-Blinder method assumes a linear regression model so that the equivalent of equation (2.12) is: $y = x\beta + \varepsilon$. In addition, it is assumed that the conditional expectation of the error term given the observables is equal to zero. One can therefore indentify and estimate the structural and endowment effects by running OLS regression of y on x and using equations (2.13) and (2.14) just ahead. The point here is that the standard Oaxaca-Blinder decomposition is fully consistent with the RIF regression framework.

[7] This can be easily seen by writing the estimate of the endowment effect as: $\hat{\Delta}_X^{\theta} = (\bar{x}_1 - \bar{x}_0)\hat{\beta}_0$. or $\hat{\Delta}_X^{\theta} = \sum_{k=1}^{m}(\bar{x}_{1k} - \bar{x}_{0k})\,\hat{\beta}_{0k}$. The corresponding expression for the structural effect is: $\hat{\Delta}_S^{\theta} = \bar{x}_1(\hat{\beta}_1 - \hat{\beta}_0)$or $\hat{\Delta}_S^{\theta} = (\hat{\beta}_{11} - \hat{\beta}_{01}) + \sum_{k=2}^{m}\bar{x}_{1k}\,(\hat{\beta}_{1k} - \hat{\beta}_{0k})$.

[8] All these functions are specified in Essama-Nssah and Lambert (2012). These authors show how to derive influence functions for most of the distributional statistics used in policy impact analysis.

distribution F_1 *poverty dominates* distribution F_0 if F_1 has less (or no more) poverty than F_0, for a class of poverty measures (Zheng 2000). In particular, the class of additively separable poverty measures is defined by the following expression

$$P(F;z) = \int_0^{m_y} I(y \leq z)\psi(y|z)dF(y) \tag{2.15}$$

where m_y is the maximum observable income, $I(y \leq z)$ is an indicator function which is equal to 1 when its argument is true and zero otherwise, and $\psi(y|z)$ is a convex and decreasing measure of individual contribution to overall poverty.

Let Ψ_M represent the sub-class of additively separable poverty measures satisfying monotonicity.[9] *First-order stochastic dominance* is the necessary and sufficient condition for all members of Ψ_M and all poverty lines in the relevant range to agree on poverty orderings of distributions (Atkinson 1987): given a distributional change from F_0 to F_1, $P(F_1;z) \leq P(F_0;z), \forall P \in \Psi_M, \forall z \in [0,\ z^{max}]$ if and only if $F_1(y) \leq F_0(y), \forall z \in [0, z^{max}]$. Thus, a distributional change from period 0 to period 1 would be considered pro-poor on the basis of the relative standard (1+ρ) if and only if: $F_1((1+\rho)z) \leq F_0(z), \forall z \in [0, z^{max}]$. Poverty comparisons based on this dominance criterion are known as *first-order pro-poor judgments*.

When the density function associated with the relevant cumulative distribution function (CDF) is continuous and strictly positive, the τ^{th} quantile, q_τ, of the distribution is equal to the inverse of that distribution at $\tau \in (0,1)$. First-order pro-poor judgments based on the relative standard (1+ρ) can therefore be equivalently expressed as: $g(\tau) = \frac{\Delta q_\tau}{q_\tau} \geq \rho, \forall \tau \in [0, F_0(z^{max})]$, where $g(\tau)$ rate of change of outcome at the τ^{th} quantile. If income is the outcome variable, then $g(\tau)$ is the growth incidence curve (GIC) ordinate at τ. One can therefore study the contributions of the endowment and structural effects to pro-poorness

[9] Foster, Greer and Thorbecke (2010) distinguish three broad categories of axioms for poverty measures. The first category includes invariance axioms such as symmetry, replication invariance, scale invariance, focus and continuity. All members of this category require that poverty remains unaffected by some change in the outcome variable. In the particular case of the focus axiom, invariance relates to changes in the outcome of the non-poor that do not change their poverty status. The second category represents the following dominance axioms: monotonicity and various version of the transfer axiom. Monotonicity is consistent with the Pareto criterion. In this particular context, it says that, other things being equal, an increase in the living standard of any poor person will reduce poverty. Finally, the third category consists of subgroup axioms such as subgroup consistency which implies that if poverty increases in one subgroup of the population, *ceteris paribus*, overall poverty should increase.

by decomposing the growth incidence curve (GIC) using the following RIF of the τ^{th} quantile of the outcome distribution:

$$RIF(y;\ q_\tau, F) = q_\tau + IF(y;\ q_\tau) = q_\tau + \frac{[\tau - I(y \leq q_\tau)]}{f_y(q_\tau)} \tag{2.16}$$

Using equation (2.16), one can decompose the first 99 percentiles of the outcome distribution of interest and thus construct a decomposition of the GIC into a structural and a composition effect. This RIF decomposition would lead to the following expression for the GIC

$$g(y) = g_S(y) + g_X(y) \tag{2.17}$$

where the first component stands for the structural effect and the second the composition effect. In the case of a linear RIF regression model, these effects can be further decomposed to identify the contributions of the covariates of interest.

When growth is distribution neutral, the rate of income growth at every percentile is equal to the rate of growth of the mean, $\gamma = \frac{\Delta\mu}{\mu}$. This quantity can also be expressed as a weighted sum of points along the GIC as follows: $\gamma = \int_0^{m_y} \frac{y}{\mu} g(y) dF(y)$. Thus, on the basis of equation (2.17), we can decompose the rate of growth into an endowment and a structural effect. Since the level and pattern of growth depend on factor accumulation and productivity, we interpret the endowment effect as an indicator of changes in factor accumulation and the structural effect as an indicator of changes in productivity.

The RIF for the ordinate of the GIC at τ is:$RIF(y; GIC_\tau, F) = \gamma\left(1 + \frac{y}{\mu_F}\right)\eta(q_\tau) + \frac{\gamma[\tau - I(y \leq q_\tau)]}{f_y(q_\tau)}\eta'(q_\tau)$ where $\eta'(q_\tau)$ stands for the first-order derivative of an elasticity function, $\eta(y)$, measuring the responsiveness of (income) y to a 1 percent growth in the overall mean (income). Equivalently, we can express this RIF as follows.

$$RIF(y; GIC_\tau, F) = \left(1 + \frac{y}{\mu_F}\right) g(\tau) + (\tau - I(y \leq q_\tau))g'(\tau) \tag{2.18}$$

This RIF can be used to conduct second-order analysis of changes in the pattern of growth as revealed by changes in the GIC. In other words, equation (2.18) allows us to use RIF regression for the decomposition of observed changes in the pattern of growth.

The first-order dominance condition that underlies first-order pro-poor judgments fails if the curves representing the distribution functions under comparison intersect at least once. This would make the comparison ambiguous to the extent that some poverty measures will rank these distributions differently than others (Ravallion 1994). One possible way out of this ambiguity is to consider *second-order pro-poor judgments*. These are based on second order dominance. Let Ψ_{ST} represent the class of all additively separable poverty measures satisfying the strong transfer axiom. Second-order stochastic dominance is the necessary and sufficient condition for all members of Ψ_{ST} and for all poverty lines in the relevant range to agree on poverty orderings of distributions (see also Atkinson, 1987).

One can test second-order pro-poorness on the basis of the so-called *three I's of poverty* (TIP) *curve* of Jenkins and Lambert (1997). The curve is obtained by partially cumulating individual contributions to overall poverty from the poorest individual to the richest, and normalizing:

$$J_D(\tau; z) = \frac{1}{n}\sum_{i=1}^{h}\left(1 - \frac{y_i}{z}\right) I(y_i \leq z),\ \tau = \frac{h}{n}\ \ \forall h \leq n \tag{2.19}$$

I(·) is an indicator function equal to 1 if the argument is true and zero otherwise.

Relative pro-poor evaluation functions that are distribution-sensitive will declare a distributional change pro-poor if and only if the initial poverty gap index based on the poverty line z is larger than the posterior poverty gap index based on the poverty line (1+ ρ)z for all poverty lines (z) in the relevant range (Ravallion 1994). The poverty gap index associated with a poverty line z is defined by the following expression: $PG(z) = \frac{1}{n}\sum_{i=1}^{n} z^{-1}\,(z - y_i) I(y_i \leq z)$. In terms of the TIP curve, second-order relative pro-poor judgments rely on the following condition: [10]

$$J_{D1}(\tau;\ (1+\rho)z^{max}) \leq J_{D0}(\tau; z^{max}),\ \forall \tau \in [0,1] \tag{2.20}$$

One can base an RIF decomposition of the TIP curve on the following function

$$RIF(y; TIP_\tau, F) = I(z \leq q_\tau)A + (1 - I(z \leq q_\tau))B \tag{2.21}$$

[10] In terms of the poverty gap index, second-order relative pro-poor judgments are based on the following condition: $PG_1\big((1+\rho)z\big) \leq PG_0(z), \forall z \in [0, z^{max}]$. This condition is equivalent to: $\int_0^z F_1\ \big((1+\rho)y\big)dy \leq \int_0^z F_0(y)dy$. It is the second-order analog of the condition underlying first-order pro-poor judgments; first-order pro-poorness implies second-order pro-poorness, but not the other way around.

where the first component is: $A = \left(1 - \frac{y}{z}\right) I(y \leq z)$. It depends on whether or not $y \leq z$. The second component, $B = \left[I(y \leq q_\tau)\left(\tau + (1-\tau)\frac{q_\tau}{z} - \frac{y}{z}\right) + \left(1 - I(y \leq q_\tau)\right)\tau(1 - \frac{q_\tau}{z})\right]$, depends on the level of y relative to the τ^{th} quantile q_τ.

A dominance relation yields only a *partial ordering* between the initial and the posterior outcome distributions. One way of proceeding is not to insist on unanimity, but to compare distributions on the basis of value judgments underlying a specific poverty measure. This approach leads to *complete ranking* of alternative outcome distributions. For the class of measures defined by (2.15), these value judgments are encoded in the poverty contribution functions, $\psi(y|z)$. Recentered influence functions associated with that class of poverty measures can be used to decompose variations in poverty outcomes on the basis of equation (2.11). These functions take the following form:

$$RIF(y; P, F) = I(y \leq z)\psi(y|z) \tag{2.22}$$

For the class of additively separable poverty measures, a change in poverty over time can be written as a weighted sum of points along the GIC up to the poverty line (Essama-Nssah and Lambert 2009). Specifically, we have:

$$\Delta_O^P = \int_0^z y\psi'(y|z)g(y)dF(y) \tag{2.23}$$

Such variations in poverty inherit the decomposability of the GIC. Equation (2.23) is therefore equivalent to the following.

$$\Delta_O^P = \int_0^z y\psi'(y|z)g_S(y)dF(y) + \int_0^z y\psi'(y|z)g_X(y)dF(y) \tag{2.24}$$

The first term on the right hand side of (2.24) is the structural effect (Δ_S^P) and the second is the composition effect, Δ_X^P.

3. Empirical Considerations

Policy analysis in general can be viewed as a process designed to provide evidence to answer questions that decision makers care about. In the context of evidence-based decision making, policymakers are interested in knowing what works, what does not and why. Addressing these questions entails describing what happened in a particular context and looking for factors that might explain the observed outcomes. In this section, using consumption expenditures in Bangladesh in 2000-2010, we show that the pattern of growth has been unambiguously pro-poor. We then apply the counterfactual decomposition framework described above to identify factors that might explain the observed pattern of growth and its poverty implications.

3.1. A Profile of Growth, Inequality and Poverty

Table 3.1 presents a summary of the distribution of *per capita* expenditure based on the 2000, 2005 and 2010 rounds of the Household Income and Expenditure Survey (HIES) conducted by the Bangladesh Bureau of Statistics (BBS). This is a multi-module survey that is representative not only at the national level, but also for rural and urban areas as well as for divisions in the country.[11] There are 7,440 observations for 2000, and 1,080 for 2005 and 12,240 for 2010. The summary information in table 3.1 includes, for each round, mean *per capita* expenditure in nominal terms and the decile distribution of *per capita* expenditure across households. This information shows a steady increase in the average living standard and a stable distribution of consumption expenditure. A closer examination of the information contained in table 3.1 reveals that, over the 2000-2010 period, the increase in the shares of expenditure going to the first eight deciles ranged from 0.08 percent to 0.19 percent. The shares of the 9th and 10th deciles fell by .28 and .78 percent respectively.

Table 3.1. Distribution of per capita Expenditure in Bangladesh, 2000-2010

Year	Mean	Lowest Decile	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
2000	876.84	3.73	4.90	5.67	6.44	7.32	8.29	9.55	11.41	14.83	27.46
2005	1230.59	3.85	4.91	5.71	6.47	7.29	8.30	9.56	11.39	14.52	27.73
2010	2447.27	3.85	5.00	5.83	6.63	7.49	8.48	9.73	11.49	14.55	26.68

Source: Authors' calculations based on the 2000, 2005 and 2010 rounds of the Household Income and Expenditure Survey (HIES).

The growth, inequality and poverty implications of these distributional changes are presented in table 3.2. In real terms, *per capita* expenditure grew by about 2.2 percent on

[11] There are six divisions in Bangladesh, namely: Barisal, Chittagong, Dhaka, Khulna, Rajshahi and Sylhet.

average between 2000 and 2005, and by about 1.4 between 2005 and 2010. The average annual growth rate over the entire 2000-2010 period stands at about 1.8 percent (not shown in the table). Stability in the distribution of consumption expenditure is confirmed by the fact that the Gini coefficient stayed between 34 and 32 percent over the entire period. Steady growth in real *per capita* expenditure combined with a stable distribution and a slight decrease in inequality led to a significant reduction in poverty between 2000 and 2010. The results in table 3.2 indicate that poverty incidence fell continuously by about 17.4 percentage points over the entire period under consideration. Between 2000 and 2005, poverty incidence fell by about 9 percentage points. The rest of the poverty measures presented in the same table show similar trends.

Table 3.2. Growth, Inequality and Poverty in Bangladesh, 2000-2010

	2000	2005	2010
Average Annual Growth Rate	-	2.21	1.37
Gini	34.44	33.20	32.13
Headcount	48.86	40.00	31.51
Poverty Gap	12.79	8.99	6.54
Squared Poverty Gap	4.57	2.88	2.00
Watts	16.10	11.0	7.92

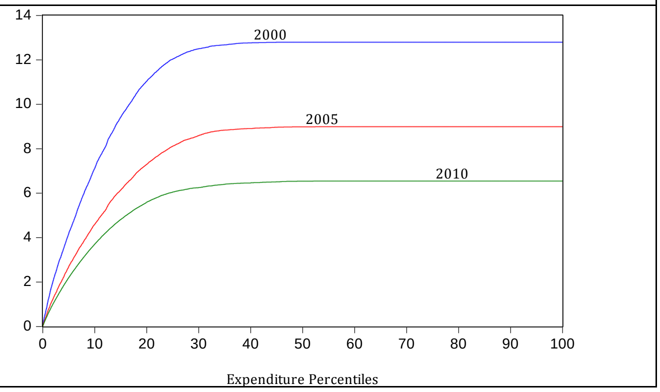
Source: Authors' calculations

This impressive reduction in poverty is clearly demonstrated by figure 3.1 summarizing the evolution of poverty over time on the basis of TIP curves associated with poverty measures that are members of the Foster-Greer-Thorbecke (FGT) family. The available evidence thus points overwhelmingly to the conclusion that distributional changes associated with economic growth in Bangladesh over the 2000-2010 period have led to a significant reduction in poverty. The question now is: To what extent have these distributional changes been pro-poor?

Pro-poorness is in the eye of the beholder to the extent that it depends on the chosen value judgments. For sure, the distributional changes observed in Bangladesh over the last decade are pro-poor in the sense of Ravallion and Chen (2003) and Kray (2006) since they all induced poverty reduction for some members of the additively separable poverty measures. In particular, the dominance relation among the TIP curves in figure 3.1

implies second-order pro-poorness in the sense that all poverty measures which are members of Ψ_{ST} agree that poverty fell between 2000 and 2005, and again between 2005 and 2010. We will see later on that the growth incidence curves for 2000-2010 indicate first-order pro-poorness.

Figure 3.1 A Picture of Poverty in Bangladesh, 2000-2010



Source: Authors' calculations

Table 3.3. Measures of Pro-Poorness of Economic Growth in Bangladesh, 2000-2010

	Headcount	Poverty Gap	Squared Poverty Gap	Watts
	Additive Measure of Pro-Poorness			
2000-2010	0.10	0.06	0.03	0.08
2000-2005	0.01	0.03	0.02	0.04
2005-2010	0.29	0.08	0.03	0.10
	Ratio Measure of Pro-Poorness			
2000-2010	1.14	1.16	1.15	1.15
2000-2005	1.01	1.07	1.10	1.08
2005-2010	1.37	1.27	1.19	1.25

Source: Authors' calculations

To further characterize the extent of pro-poorness of the observed distributional changes, we follow Osmani (2005)'s recommendation that a distributional change be considered pro-poor if it induces an absolute reduction in poverty greater than would occur in a benchmark case. In particular we consider a distributional change pro-poor if it reduces poverty more than would a distribution neutral change. Using available data, we compute an additive and a ratio measure of pro-poorness(Essama-Nssah and Lambert 2009, Kakwani and Pernia 2000). The results are presented in table 3.3 for the headcount, the poverty gap, the squared poverty gap and the Watts index. A distributional change is considered pro-poor if the additive measure is positive or the ratio measure greater than one. The results in table 3.3 show that the underlying distributional changes are pro-poor in the sense of Osmani (2005) given distribution neutrality as benchmark case.

3.2. Accounting for Changes in Observed Outcomes

What is driving the steady decline in poverty observed in Bangladesh in the last decade? Any answer to this question based on counterfactual decomposition depends on how one chooses to model poverty. We consider first changes in aggregate poverty. We then try to account for the heterogeneity of impacts underlying aggregate outcomes.

Changes in Aggregate Poverty

Table 3.4 Shapley Decomposition of Changes in Poverty in Bangladesh into Size and Redistribution Effects (2000-2010)

	Overall	Size	Redistribution
Headcount	-0.173	-0.152	-0.021
Poverty Gap	-0.062	-0.055	-0.007
Squared Poverty Gap	-0.026	-0.023	-0.003
Watts	-0.082	-0.072	-0.010
	2000-2005		
Headcount	-0.089	-0.087	-0.002
Poverty Gap	-0.038	-0.035	-0.003
Squared Poverty Gap	-0.017	-0.015	-0.002
Watts	-0.051	-0.047	-0.004
	2005-2010		
Headcount	-0.085	-0.066	-0.019
Poverty Gap	-0.024	-0.020	-0.004
Squared Poverty Gap	-0.009	-0.008	-0.001
Watts	-0.031	-0.026	-0.005

Source: Authors' calculations

The starting point of most decomposition methods is to consider a poverty measure as a functional of the underlying distribution of living standards which is fully characterized by its mean and the degree of inequality. Thus changes in poverty can be seen as driven by changes in these same factors. In particular, one can decompose changes in poverty in terms of two components. The size effect is linked to changes in the mean of the underlying outcome distribution while the redistribution effect is associated with changes in relative inequality.[12] Table 3.4 shows the results of a Shapley decomposition of changes in aggregate poverty in Bangladesh over the 2000-2010 period.[13] Because the size and redistribution effects are negative for all poverty measures considered, we conclude that both effects contributed to the observed poverty reduction. However, in absolute value, the size effect is much greater than the redistribution effect. Therefore, the observed poverty reduction was driven mostly by the increase in *per capita* expenditure.

Table 3.5 RIF Regression Decomposition of Changes in Poverty in Bangladesh into Endowment and Structural Effects

(2000-2010)

		2000-2010			
		Linear Model		**Nonlinear Model**	
	Overall	**Endowment**	**Structure**	**Endowment**	**Structure**
Headcount	-0.173	-0.109	-0.064	-0.109	-0.064
Poverty Gap	-0.062	-0.039	-0.024	-0.041	-0.022
Squared Poverty Gap	-0.026	-0.016	-0.010	-0.012	-0.014
Watts	-0.082	-0.050	-0.031	-0.050	-0.032
		2000-2005			
Headcount	-0.089	-0.085	-0.003	-0.081	-0.008
Poverty Gap	-0.038	-0.032	-0.006	-0.029	-0.009
Squared Poverty Gap	-0.017	-0.014	-0.003	-0.007	-0.010
Watts	-0.051	-0.043	-0.008	-0.034	-0.017
		2005-2010			
Headcount	-0.085	-0.036	-0.049	-0.035	-0.050
Poverty Gap	-0.024	-0.007	-0.018	-0.013	-0.012
Squared Poverty Gap	-0.009	-0.002	-0.007	-0.001	-0.008
Watts	-0.031	-0.008	-0.023	-0.013	-0.018

Source: Authors' calculations

Decomposing changes in aggregate poverty into a size and a redistribution effect provides limited information for policymaking since it is hard to target aggregate statistics such as the mean of a distribution or a measure of its inequality with policy instruments.

[12] See Essama-Nssah (2012) for a detailed discussion.

[13] The Shapley value, a solution to a cooperative game with transferable utility, provides a formula for dividing a joint cost or a jointly produced output among claimants on the basis of individual contribution to the formation of total cost or the production of a surplus. According to Moulin (2003) this formula can also be viewed as an interpretation of the reward principle of distributive justice. The Shapley decomposition respects the following value judgments: (i) Symmetry or anonymity: the share assigned to any factor does not depend on its label or the way it is listed; (ii) adding up: all shares must add up to the total; (iii) the share of each factor is taken to be its (first round) marginal impact.

We therefore focus on decompositions based on the notion that the distribution of living standards and the associated poverty outcomes are determined by individual endowments and returns to those endowments. Table 3.5 presents the results of such a decomposition for members of the FGT family of poverty measures and the Watts index based on both linear and nonlinear specifications of the RIF regression model. All specifications include the same set of household characteristics as explanatory variables. We consider four broad categories of household characteristics: (1) *Demographics* (age of head, whether head is female, whether head is married, whether head is non Muslim, number of children including infants, and the number of adults); (2) *Household assets* (level of education of head, land ownership, number of non-farm enterprises, electricity, safe latrine, remittances); (3) Main *occupation* of head of household (farmer, agricultural laborer, self employed, salaried worker); (4) *Location* (area/division of residence).[14]

Table A1 in the appendix presents a sample of RIF regression results for the headcount and Watts measure of poverty. These results are for the year 2000. However, the same pattern holds for 2005 and 2010. Among the demographic variables, poverty tends to fall as the age of the household head increases. As expected, there is a positive association between poverty and household size. Both the number of children and that of adults have positive coefficients in the RIF regression results for all specifications and years. Among the remaining variables, the following clearly have a positive and statistically significant effect on poverty reduction across models and years: the level of education of head, land ownership, the number of non-farm enterprises, electricity, safe latrine, and remittances. The results for the main occupation and location do vary somewhat across models and years. However, among the occupations considered, self and salaried employment tend to be associated with poverty reduction (relative to employment in agriculture, the reference occupation) across models and years. In the case of geographical location, the results show that residence in Chittagong or Sylhet is associated with poverty reduction compared to residence to Dhaka Division.

[14] Many of the variables are dummies or categorical. We dropped categories in such a way that the reference household is landless, with no electricity or safe latrine and does not receive remittances. It resides in the rural part of Dhaka and is headed by a male with no education who is Muslim, not married and works in agriculture.

The decomposition results presented in table 3.5 confirm that both the endowment and the structural effect contribute to the reduction in poverty observed over the 2000-2010 period. This claim is based on the fact that both effects have a negative sign over 2000-2010 and over each sub-period considered. However, the relative contribution of each factor depends on the poverty measure, the specification of the RIF regression model as well as the period under consideration. Under linear specification, the contribution of the endowment effect dominates that of the structural effect in 2000-2010 for all poverty measures considered in this study. Within the two sub-periods, the endowment effect dominates the structural effect in 2000-2005 and the opposite in true in 2005-2010. The pattern is similar for the nonlinear specification of the RIF regression, except that for the squared poverty gap, the structural effect is greater that the composition effect in 2000-2010. In 2005-2010, the structural effect dominates the composition effect for all poverty measures except the poverty gap.

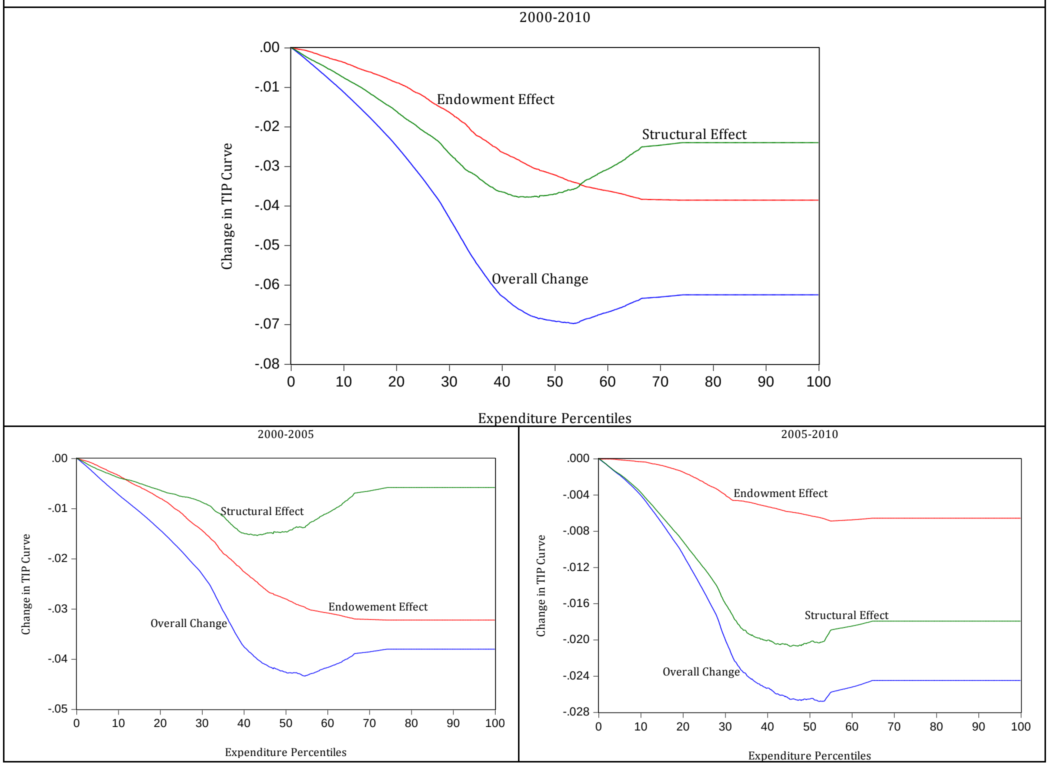
Table 3.6. Contribution (in %) of Endowment and Structural Effects to Changes in Poverty Outcomes in Bangladesh, 2000-2010

		2000-2010			
		Linear Model		**Nonlinear Model**	
	Overall	**Endowment**	**Structure**	**Endowment**	**Structure**
Headcount	100.00	63.01	36.99	63.01	36.99
Poverty Gap	100.00	62.90	37.10	66.13	33.87
Squared Poverty Gap	100.00	61.54	38.46	46.15	53.85
Watts	100.00	60.98	39.02	60.98	39.02
		2000-2005			
Headcount	100.00	95.51	4.49	91.01	8.99
Poverty Gap	100.00	84.21	15.79	76.32	23.68
Squared Poverty Gap	100.00	82.35	17.65	41.18	58.82
Watts	100.00	84.31	15.69	66.67	33.33
		2005-2010			
Headcount	100.00	42.35	57.65	41.18	58.82
Poverty Gap	100.00	29.17	70.83	54.17	45.83
Squared Poverty Gap	100.00	22.22	77.78	11.11	88.89
Watts	100.00	25.81	74.19	41.94	58.06

Source: Authors' calculations

The change in the pattern of the contribution of endowment and structural effects to poverty reduction as described above is confirmed by the decomposition of shifts in the TIP curve presented in figure 3.2. Each panel of that figure show three curves: the overall change in the TIP curve, the endowment and the structural effect. All three curves lie below the horizontal axis (through zero). This means that both the endowment and the structural effects contribute to poverty reduction as measured by members of the FGT family. The distance between the horizontal axis and a given curve indicates the magnitude of the effect. The further the curve is from the horizontal axis, the more the associated effect reduces poverty. The 2000-2005 panel of the figure thus indicates that the endowment effect reduces poverty more than the structural effect over the relevant range. In the 2005-2010 panel, it is the structural effect that dominates the endowment effect. The fact that neither effect dominates the other over the entire 2000-2010 period is revealed by the curve for the structural effect crossing that for the endowment effect in the top panel (2000-2010).

Figure 3.2 Decomposition of Shifts in the TIP Curve for Bangladesh, 2000-2010

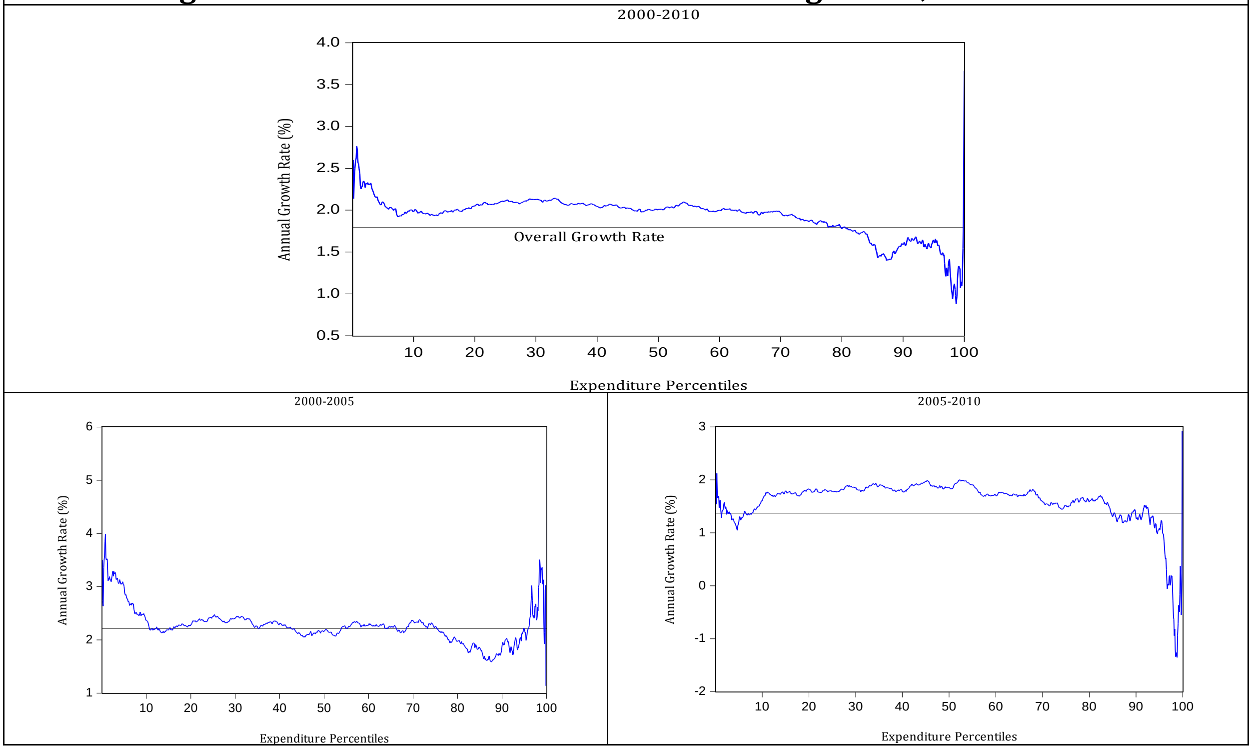


Source: Authors' calculations

Growth Incidence

Pro-poorness provides a characterization of a distributional change. In the case of economic growth, this change can be represented by the corresponding GIC, which also provides a basis for first-order pro-poor judgments. We now consider how the endowment and structural effects vary along the GIC.

Figure 3.3. Growth Incidence Curves for Bangladesh, 2000-2010

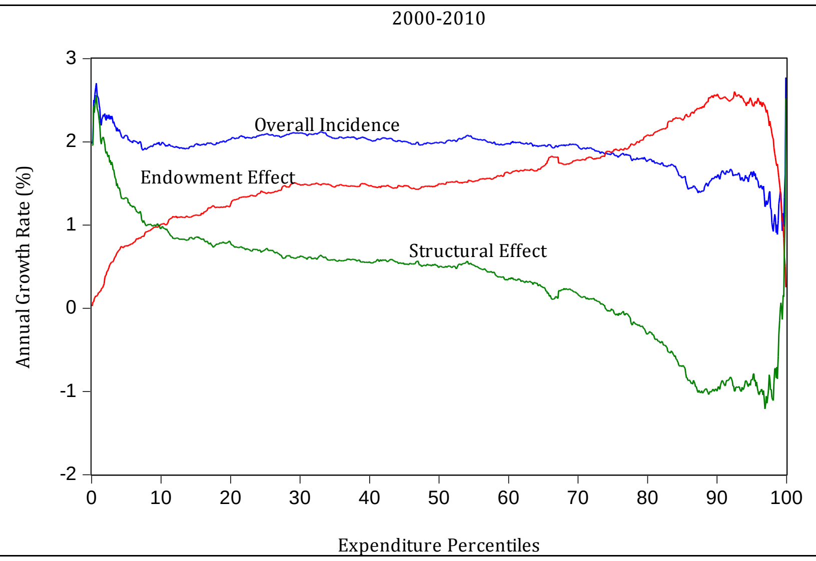


Source: Authors' calculations

Figure 3.3 presents three curves showing the incidence of economic growth in Bangladesh over the 2000-2010 decade. The top panel shows the GIC for the overall period, 2000-2010. This curve is greater than zero for every expenditure percentile. The distribution of *per capita* consumption expenditure in 2010 therefore dominates the distribution in 2000. All poverty measures which are members of the Ψ_M class will show that poverty fell over that period for all poverty lines in the relevant range. Economic growth in Bangladesh has also been pro-poor to the first-order on the basis of the relative standard of pro-poorness $(1+\gamma)$, where γ is the average annual growth rate of *per capita* expenditure. This conclusion is based on the fact that the rate of growth at every percentile up to the 78th is greater than γ.

The GIC for 2000-2005 also indicates that the distribution of per capita expenditure for 2005 dominates that of 2000 to the first order. However, this first-order dominance relation does not hold for the relative standard of pro-poorness (1+γ) since the GIC crosses the average annual growth rate several times between to 10th and the 75th percentile. The configuration of this GIC also indicates that economic growth benefited the people at both extremes of the distribution (those below the 10th percentile and above the 75th percentile) more than it benefited those in the middle. While inequality fell among the very poor, it did increase among the very rich. The situation in 2005-2010 is almost the opposite of that in 2000-2005. Except for a small dip below average growth rate between the 3rd and the 6th percentile, the growth rate of *per capita* expenditure between 2005 and 2010 was above average for all percentiles up to the 85th.

Figure 3.4. Decomposition of Growth Incidence in Bangladesh, 2000-2010

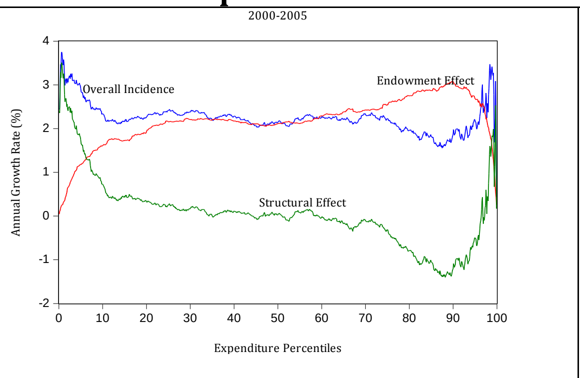


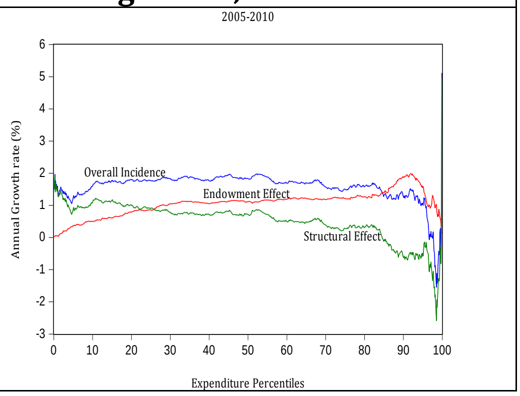
Source: Authors' calculations

To further understand the effect of changes in the distribution of household characteristics and that of returns to those characteristics, we decompose the various GICs into their endowment and structural effects. The results are presented in figures 3.4 and 3.5. For the overall period, 2000-2010, figure 3.4 shows that the structural effect is downward sloping and crosses the endowment effect from above at the 10th percentile. Past this point the endowment effect dominates the structural effect and the gap between the two becomes wider and wider as we move to the upper part of the distribution. The structural effect turns negative from the 75th percentile on. These observations imply that it is the composition effect that keeps the GIC above zero while the structural effect accounts for the slightly declining slope of the GIC and hence the modest fall in relative inequality observed over the period. The relationship between the endowment and structural effect depicted in figure 3.4 also explains our earlier finding that, compared to the structural effect, the endowment effect accounts for a larger share of the variation in poverty outcomes over the 2000-2010 period.

Looking at the sub-periods, figure 3.5 shows that the pattern of the endowment and structural effects in 2000-2005 is similar to the one found in 2000-2010. However, the structural effect turns negative sooner (around the 45th percentile) than it does in the 2000-2010 period. This pattern also supports our finding that the composition effect plays a larger role than the structural effect in determining poverty outcomes over this sub-period. In the case of the second sub-period, 2005-2010, the structural effect dominates the endowment effect on a wider range than in the two previous cases. It crosses the endowment effect from above at the 26th percentile. Even though the endowment effect dominates the structural effect beyond this point, the gap between the two curves is much narrower than it is for the overall period or the 2000-2005 sub-period. Focusing on the segment of the distribution representing the poor in 2005 (i.e. up to the 40th percentile), the structural effect dominates the endowment effect over 65 percent of that truncated range and the distance between the two curves is greater to the left of the point of intersection than it is between that point and the 40th percentile. This is why we found earlier that the structural effect accounts for more of the variation in poverty in 2005-2010 than the endowment effect.

Figure 3.5. Decomposition of Growth Incidence in Bangladesh, 2000-2005-2010

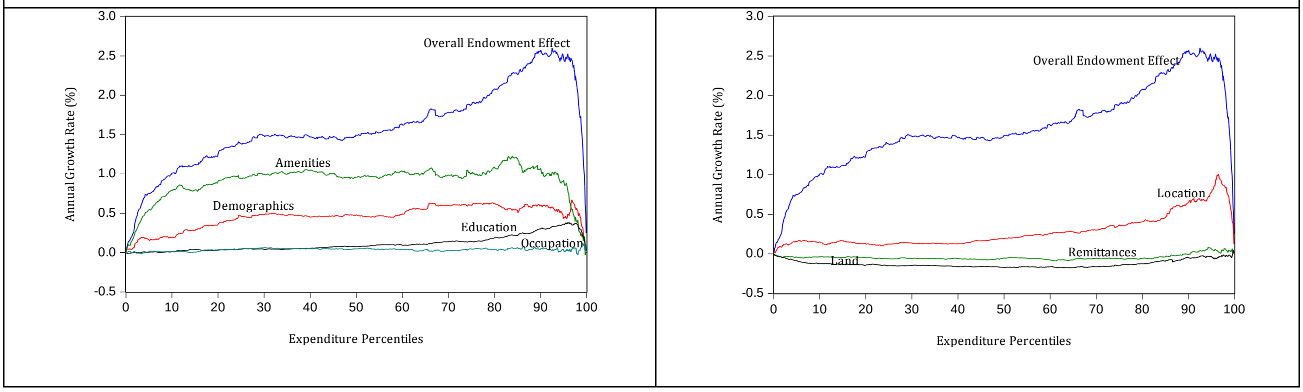




Source: Authors' calculations

The study of economic growth is anchored on two basic ideas: *accumulation* and *productivity*.[15] Growth accounting is an exercise designed to identify the key drivers of economic growth by decomposing the growth of output into two components: one attributable to changes in factors of production such as physical and human capital, and a residual not related to changes in input levels. This residual is interpreted as the rate of change in *total factor productivity* (TFP). Thinking of the living standard of an individual or a household as an outcome of participation in the life of society (subject to type and circumstances that determine the returns to type from any social interaction) clearly establishes an analogy between growth accounting and the counterfactual decomposition of the GIC considered here. On the basis of this analogy, we link the endowment effect to accumulation and we take the structural effect to be an indicator of productivity in socioeconomic interaction. Thus, the fact that the structural effect is inequality reducing over the entire period (while the endowment effect tends to increase inequality) and accounts for a larger share of the poverty reduction observed in the second half of the decade suggests that socioeconomic arrangements in Bangladesh have become more progressive over time.

Figure 3.6. Sources of Variation in the Endowment Effect of Growth in Bangladesh, 2000-2010



Source: Authors' calculations

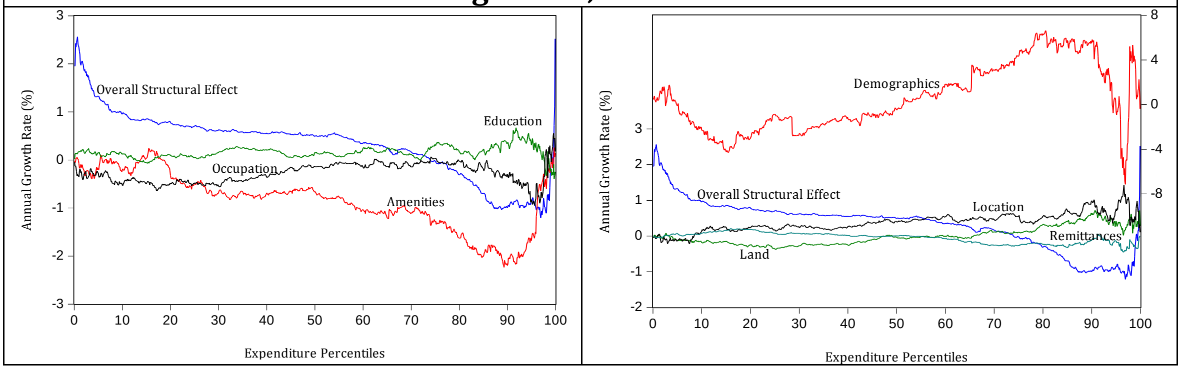
What drives the endowment and structural effects? To answer this question, we focus on the overall period 2000-2010 and further disaggregate these two components of

[15] Aggregative growth models are usually built around two key relationships: (i) a production function showing how different combinations of inputs translate into levels of output, (ii) an equation describing the accumulation of reproducible factors of production as a function of gross investment and replacement requirement (depreciation).

growth incidence on the basis of sets of covariates used earlier in various regression models. Figure 3.6 shows the sources of variation in the endowment effect over the period under consideration. The left panel of the figure compares the overall endowment effect with the contributions of amenities (i.e. electricity and safe latrine), demographics, education and occupation. The right panel compares the total effect with components related to location, remittances and land. These results show that over the 2000-2010 period, the endowment effect is driven mostly by amenities, demographics and location.

Figure 3.7 presents our decomposition of the structural effect over the 2000-2010 period. The comparisons made in both panels of the figure parallel those made for the composition effect. It can be seen from the left panel that the shape of the overall structural effect follows closely that of the component associated with amenities. On the right panel, the structural effect associated with demographics has the shape of the inverted overall structural effect. The configuration of the curves presented in this figure suggests that amenities and household demographics are the main drivers of the overall structural effect and they pull in opposite directions.

Figure 3.7. Sources of Variation in the Structural Effect of Growth in Bangladesh, 2000-2010



Source: Authors' calculations

The prominent role played by amenities and household demographics in accounting for variation in the endowment and structural effects (and hence for poverty reduction) is consistent with the findings by Zaman et al. (2012). These authors use the standard Oaxaca-Blinder decomposition to analyze changes in poverty outcomes in Bangladesh in 2000-2005. They find that the observed reduction in poverty over that period was accompanied by improved access to electricity and sanitation. They also note a fall in the dependency ratio within households.

4. Concluding Remarks

Poverty reduction has emerged as an important objective of socioeconomic development and therefore a metric for evaluating the performance of public policy. Policy evaluation can be viewed as a process designed to provide evidence to answer questions that decision-makers and other stakeholders care about. In particular, policymakers are interested in knowing not only what works and what does not, but what explains the observed outcome. It is therefore not enough to declare a policy-induced distributional change pro-poor without attempting to identify factors that might explain that outcome. This paper demonstrates how to use influence functions and counterfactual decomposition to identify and estimate factors that might account for variation in poverty outcomes.

A key step in explaining an outcome involves establishing an association between that outcome and possible explanatory factors. Pro-poorness is a characterization of a change in the distribution of living standards on the basis of value judgments that define the chosen standard of evaluation. A distribution is fully determined by its mean and the degree of inequality. Pro-poorness can therefore be seen as driven by these factors. However, it is hard to target distributional statistics such as the mean or a measure of inequality with policy instruments. The living standard of an individual (or household) is a pay-off from participation in the life of society subject to individual endowments and the circumstances that determine the returns to those endowments from social interaction. The paper relies on the concept of influence function and the conditional expectation function to establish a parametric relationship between the relevant distributional statistics and household characteristics. Given a set of covariates representing those characteristics, the conditional expectation of the RIF of a distributional statistic captures the essence of that relationship.

The relationship between a distributional statistic (such as a poverty measure) and household characteristics offers an opportunity to identify the sources of variation in that statistic or in the underlying distribution in terms of the endowment and structural effects.

The fundamental identification strategy relies on the notion of *ceteris paribus* variation and counterfactual comparison. The endowment effect is identifiable when changes in the distribution of observable characteristics have no general equilibrium effects and are not confounded by changes in the distribution of unobservables (ignorability). These conditions facilitate the estimation of the counterfactual outcome distribution that would have been observed in the end period had observable characteristics in that period been rewarded according to the pay-off regime of the initial period. A comparison of this counterfactual with the outcome distribution observed in the end period yields the structural effect. Comparing the same counterfactual with the distribution observed in the initial period produces the endowment effect.

An application of this analytical framework to consumption expenditure data for Bangladesh for the period 2000-2010 shows that the distributional change observed over that period is unambiguously pro-poor. Poverty fell continuously and significantly over the entire period. All members of the class of additively separable poverty measures Ψ_M satisfying monotonicity and those of the class Ψ_{ST} that respect the strong transfer assumption support this conclusion for a wide range of poverty lines. Furthermore, both additive and ratio measures of pro-poorness indicate that the observed reduction in poverty is greater than what would have occurred under distribution neutrality.

A counterfactual decomposition of changes in poverty outcomes over the 2000-2010 period based on the Shapley method reveals that both the size and redistribution effects contribute to the observed poverty reduction. However, the size effect dominates the redistribution effect over the entire period. Furthermore, the Oaxaca-Blinder decomposition of changes in aggregate poverty and the underlying distributional changes shows that both the endowment and structural effects contribute to poverty reduction. The contribution of the endowment effect is greater than that of the structural effect over 2000-2005 while the structural effect more important that the endowment effect over 2005-2010. These findings are robust across poverty measure and RIF regression models. Since the structural effect is inequality reducing and linked to circumstances that determine returns to endowments in social interaction, the switch in the relative importance of the two effects from the first sub-period to the second suggests that socioeconomic arrangements in Bangladesh have become more progressive over time. However, when the overall distributional change is considered, the endowment effect dominates the structural effect. A detailed decomposition further reveals that these effects are driven mostly by demographics, amenities, location and education.

We conclude by pointing out that, even though they are derived from counterfactual comparisons along with assumptions used to identify the causal effect of an assigned intervention, the decomposition results discussed in this paper do not provide a causal explanation of the observed outcomes. The logic of causal inference entails establishing a plausible association between the outcome of interest and the explanatory factor, and ruling out alternative explanations (i.e. confounders) of that association. Furthermore, causal explanation clarifies the mechanisms that bring the outcome about. The approach followed here relies on associational inference based on the conditional expectation function which is a reduced form of the underlying causal mechanism. Given that the living standard of an individual is an outcome of her participation in the life of society that depends on endowments, behavior and the circumstances that determine the returns to these endowments from any social interaction, a causal explanation of pro-poorness must rest on a full structural model of individual behavior and social interaction.

While the decomposition method used in this study does not provide a causal explanation of pro-poorness, it can help quantify in a descriptive sense the contribution of various factors to changes in poverty or distributional outcomes. Such an accounting exercise identifies factors that are quantitatively important and therefore deserve more attention either for further analysis or for policy targeting. On the basis of these considerations, Fortin et al. (2011) suggest a two-step approach to analyzing distributional changes whereby the standard decomposition method discussed in this paper would be applied first to identify the main forces driving the observed changes. Then, counterfactual decompositions based on a structural model would be used to explain the results from the first step.

Appendix

Table A1. Sample RIF Regression Results for the Headcount and Watts Measure of Poverty

Eq Name:	LRIFHDEQ_2000	LRIFWTSEQ_2000	NLRIFHDEQ_2000	NLRIFWTSEQ_2000
Dep. Var:	RIFHD	RIFWTS	RIFHD	RIFWTS
Constant	0.751447	0.323694	0.788271	0.187909
	(0.0573)**	(0.0276)**	(0.3228)*	(0.0475)**
Age of Head	-0.011469	-0.004348	-0.069039	-0.012549
	(0.0022)**	(0.0011)**	(0.0136)**	(0.0020)**
Age of Head Squared	0.000112	0.000046	0.000703	0.000131
	(0.0000)**	(0.0000)**	(0.0001)**	(0.0000)**
Head Female	0.010054	0.017178	0.253591	0.074209
	(0.0279)	(0.0134)	(0.1490)	(0.0222)**
Head Married	0.002406	-0.012141	-0.138909	-0.018984
	(0.0239)	(0.0115)	(0.1308)	(0.0198)
Head Non Muslim	0.020378	-0.003892	-0.211454	-0.038511
	(0.0166)	(0.0080)	(0.1080)	(0.0159)*
Number of Children	0.154822	0.066930	1.089490	0.185210
	(0.0062)**	(0.0030)**	(0.0565)**	(0.0083)**
Number of Children Squared	-0.011851	-0.004295	-0.083672	-0.013262
	(0.0007)**	(0.0003)**	(0.0097)**	(0.0014)**
Number of Adults	0.045483	0.009531	0.524299	0.074076
	(0.0085)**	(0.0041)*	(0.0809)**	(0.0119)**
Number of Adults Squared	-0.004129	-0.001150	-0.049479	-0.007114
	(0.0007)**	(0.0003)**	(0.0092)**	(0.0014)**
Level of Education of Head				
Below Class 5	-0.097290	-0.041579	-0.622063	-0.110442
	(0.0214)**	(0.0103)**	(0.1371)**	(0.0210)**
Class 5	-0.135917	-0.043264	-0.679666	-0.095342
	(0.0160)**	(0.0077)**	(0.0981)**	(0.0149)**
Class 6 to 9	-0.135354	-0.038571	-0.900310	-0.140708
	(0.0148)**	(0.0071)**	(0.0914)**	(0.0143)**
Higher	-0.182102	-0.049310	-1.645292	-0.271924
	(0.0169)**	(0.0081)**	(0.1208)**	(0.0188)**
Land Ownership				
0.05 to 0.49 acres	-0.030244	-0.049918	-0.505521	-0.097098
	(0.0162)	(0.0078)**	(0.0993)**	(0.0142)**
0.5 to 1.5 acres	-0.111647	-0.088451	-0.924604	-0.168970
	(0.0145)**	(0.0070)**	(0.0963)**	(0.0141)**
1.5 to 2.5 acres	-0.221918	-0.128584	-1.415882	-0.257033
	(0.0183)**	(0.0088)**	(0.1374)**	(0.0210)**
2.5 acres or more	-0.331037	-0.163084	-2.297900	-0.402713
	(0.0166)**	(0.0080)**	(0.1552)**	(0.0237)**
Main Occupation of Head				
Self Employed	-0.033333	-0.040146	-0.288376	-0.051816

	(0.0178)	(0.0086)**	(0.1237)*	(0.0182)**
Salaried	-0.092511	-0.053974	-0.560045	-0.103484
	(0.0175)**	(0.0084)**	(0.1097)**	(0.0164)**
Non Ag. Laborer	-0.011170	-0.035015	-0.137868	-0.047170
	(0.0165)	(0.0079)**	(0.0966)	(0.0132)**
None	-0.039147	-0.030361	-0.217584	-0.020912
	(0.0190)*	(0.0091)**	(0.1228)	(0.0180)
Number of Non Farm Enterprises	-0.067266	-0.018102	-0.389899	-0.062153
	(0.0112)**	(0.0054)**	(0.0894)**	(0.0131)**
Electricity	-0.210160	-0.092726	-1.249018	-0.238425
	(0.0126)**	(0.0061)**	(0.0837)**	(0.0127)**
Safe Latrine	-0.142353	-0.080091	-0.694810	-0.126782
	(0.0111)**	(0.0053)**	(0.0674)**	(0.0099)**
Remittances (Domestic)	-0.073482	-0.041785	-0.417183	-0.074519
	(0.0125)**	(0.0060)**	(0.0766)**	(0.0115)**
Remittances (Abroad)	-0.164473	-0.056485	-1.010806	-0.180032
	(0.0162)**	(0.0078)**	(0.1227)**	(0.0190)**
Urban	0.029294	0.018770	0.353265	0.089930
	(0.0172)	(0.0083)*	(0.0859)**	(0.0124)**
Barisal	0.053505	0.001974	0.154516	-0.001823
	(0.0224)*	(0.0108)	(0.1128)	(0.0163)
Chittagong	-0.052114	-0.053095	-0.550415	-0.121966
	(0.0139)**	(0.0067)**	(0.0919)**	(0.0135)**
Khulna	-0.045232	-0.055372	0.031931	-0.028398
	(0.0166)**	(0.0080)**	(0.0988)	(0.0145)
Rajshahi	0.005297	0.000893	0.166527	0.022445
	(0.0138)	(0.0066)	(0.0839)*	(0.0119)
Sylhet	-0.131331	-0.098330	-0.948666	-0.188404
	(0.0200)**	(0.0096)**	(0.1435)**	(0.0215)**
Observations:	7416	7416	7416	7416
R-squared:	0.3531	0.3321	0.3093	NA
F-statistic:	125.9326	114.7136	NA	NA

Sources: Authors' calculations (standard errors in parentheses)

Table A2. Results of OLS Regression of Log Expenditure on Household Characteristics, 2000-2010

Eq Name:	**EQOLS_2000**	**EQOLS_2005**	**EQOLS_2010**
Dep. Var:	**LRPCEXP**	**LRPCEXP**	**LRPCEXP**
Constant	6.498949	6.524323	6.539555
	(0.0515)**	(0.0471)**	(0.0444)**
Age of Head	0.010686	0.014624	0.011790
	(0.0020)**	(0.0018)**	(0.0016)**
Age of Head Squared	-0.000092	-0.000126	-0.000098

	(0.0000)**	(0.0000)**	(0.0000)**
Head Female	-0.030098	0.035057	0.004965
	(0.0251)	(0.0222)	(0.0194)
Head Married	-0.003060	0.046084	0.007843
	(0.0215)	(0.0201)*	(0.0182)
Head Non Muslim	-0.052617	-0.094199	-0.066170
	(0.0149)**	(0.0122)**	(0.0115)**
Number of Children	-0.155340	-0.157146	-0.133070
	(0.0056)**	(0.0061)**	(0.0059)**
Number of Children Squared	0.010383	0.012098	0.008671
	(0.0006)**	(0.0008)**	(0.0009)**
Number of Adults	-0.050726	-0.051131	-0.048010
	(0.0076)**	(0.0066)**	(0.0081)**
Number of Adults Squared	0.004295	0.004167	0.002747
	(0.0007)**	(0.0006)**	(0.0008)**
Level of Education of Head			
Below Class 5	0.068114	0.103123	0.072369
	(0.0193)**	(0.0175)**	(0.0150)**
Class 5	0.110739	0.104269	0.095334
	(0.0144)**	(0.0129)**	(0.0124)**
Class 6 to 9	0.140450	0.178835	0.138628
	(0.0133)**	(0.0116)**	(0.0108)**
Higher	0.304235	0.392688	0.414208
	(0.0151)**	(0.0129)**	(0.0113)**
Land Ownership			
0.05 to 0.49 acres	0.040250	0.055763	0.032775
	(0.0145)**	(0.0120)**	(0.0108)**
0.5 to 1.5 acres	0.105138	0.145384	0.122956
	(0.0130)**	(0.0112)**	(0.0106)**
1.5 to 2.5 acres	0.209652	0.240434	0.167247

	(0.0164)**	(0.0146)**	(0.0155)**
2.5 acres or more	0.328952	0.351520	0.346330
	(0.0150)**	(0.0139)**	(0.0145)**
Main Occupation of Head			
Self Employed	0.047104	0.072276	0.048400
	(0.0160)**	(0.0137)**	(0.0137)**
Salaried	0.079009	-0.021442	0.006722
	(0.0157)**	(0.0133)	(0.0127)
Non Agricultural Laborer	0.026269	-0.030982	-0.078898
	(0.0148)	(0.0133)*	(0.0123)**
None	0.060503	0.070785	0.044907
	(0.0170)**	(0.0145)**	(0.0140)**
Number of Non-Farm Enterprises	0.080306	0.071261	0.078091
	(0.0100)**	(0.0086)**	(0.0084)**
Electricity	0.249868	0.165873	0.147989
	(0.0114)**	(0.0095)**	(0.0086)**
Safe Latrine	0.146355	0.095126	0.111096
	(0.0100)**	(0.0100)**	(0.0092)**
Remittances (Domestic)	0.076373	0.038625	0.041733
	(0.0113)**	(0.0098)**	(0.0121)**
Remittances (Abroad)	0.206073	0.212081	0.181813
	(0.0145)**	(0.0131)**	(0.0125)**
Urban	0.274196	0.245177	0.380729
	(0.0154)**	(0.0114)**	(0.0100)**
Barisal	0.017565	-0.169567	-0.055712
	(0.0201)	(0.0189)**	(0.0176)**
Chittagong	0.173990	0.027325	0.190771
	(0.0125)**	(0.0112)*	(0.0098)**
Khulna	-0.052507	-0.264937	-0.076515
	(0.0150)**	(0.0142)**	(0.0128)**
Rajshahi	-0.130717	-0.223598	0.021050
	(0.0124)**	(0.0112)**	(0.0118)
Sylhet	0.166989	0.069114	0.088652
	(0.0180)**	(0.0162)**	(0.0172)**

Observations:	7416	10074	12240
R-squared:	0.5340	0.5505	0.5181
F-statistic:	264.3587	384.3480	410.0513

Source: Authors' calculations (standard errors in parentheses)

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