History dependent growth incidence: A characterisation and an application to the economic crisis in Italy

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Abstract

We propose a characterisation of an aggregate measure of growth that takes into account the initial economic conditions of individuals. Our measure is a weighted average of individual income growth with weights that are decreasing with the rank of the individual in the initial income distribution. We apply our theoretical framework to evaluate the growth processes experienced by the Italian population in the last decade and find that the 2008-2010 crisis deteriorated the income growth of the initially poor disproportionately. This is compatible with the dual nature of the Italian labour market.

Keywords: Individual income growth, directional income mobility, pro-poor growth, economic crisis.

JEL Classification: D31, D63, I32.

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1 Introduction

Eventful days, such as the different phases of the recent economic crisis rapidly follow each other. These events motivate a renewed and increasing interest, both among economists and policy makers, in the measurement of growth and its distributional implications. We focus on the question whether, compared to the growth processes before the financial crisis, and after adjusting for differences in mean growths, the financial crisis disproportionately affected the income growth of the initially poor.

So, we take a history dependent perspective, which evaluates a growth process on the basis of individuals’ growth experiences and their position in the initial distribution of income. Such approaches are becoming increasingly popular (Grimm, 2007; Van Kerm, 2009; Bourguignon, 2011; Jenkins and Van Kerm, 2011; Palmisano and Peragine, 2012). Their main tool is the “Non-anonymous Growth Incidence Curve” (see Bourguignon 2011), abbreviated here to “na-GIC”, or, equivalently, the “Mobility Profile”, which plots the growth in mean income achieved by those individuals belonging to the same quantile in the initial distribution of income as a function of their quantile in this initial distribution. The literature cited above provides formal derivations of dominance conditions that can be used to obtain incomplete rankings of growth processes. A first dominance criterion is that one growth process is better than another one if its na-GIC lies above the other’s na-GIC. In practice, such dominance is rather exceptional, as illustrated in panel (a) of Figure 1, which plots the Italian na-GICs for the period before (2004-06, red line) and after (2008-10, blue line) the financial crisis.

Figure 1: panel (a) na-GICs and (b) cumulative na-GICs for Italy.

![Figure 1](image1.png)

Note: authors’ calculations, based on the Italian “Survey on Household Income and Wealth”. Computations of the (cumulative) na-GICs are based on the mean growth obtained by each percentile in the initial income distribution.

Both na-GICs are positive up to the 50th percentile, are around zero up to the 85th percentile and become negative for the initially richest percentiles; the two growth episodes show a similar progressive path. Hence it appears that in both periods the incomes of the initially poorest grow more than those of the initially rich. However, we encounter a major difficulty in the comparison of these two growth processes. No dominance can be established since the two curves intersect very often.

1 The name “Mobility Profile” is due to Van Kerm (2009) and is based on the similarity with the measurement of directional income mobility.

2 This is a reflects the well known regression-to-the-mean phenomenon.
A second, more powerful dominance condition can be obtained when one is willing to attach a
greater weight to the growth rates of the initially poor. In that case, one growth process is better
than another one if its cumulative na-GIC lies above the other’s cumulative na-GIC. Figure 1, panel
(b) plots the cumulative na-GICs for the two growth processes and, in this case, we obtain a clear
ranking: the growth process 2004-06 was unambiguously better than the growth process 2008-10.

There are several issues worth pointing out in the procedure sketched so far, however.

First, dominance between cumulative na-GICs cannot always be established. In the analyses
performed by Bourguignon (2011) and Jenkins and Van Kerm (2011), for instance, dominance
between cumulative na-GICs is the exception rather than the rule.

Second, the computation of na-GICs requires either that the data are arbitrarily partitioned
into quantiles, or a non-trivial estimation of the na-GIC (on the latter, see Jenkins and Van Kerm
(2011). The choice which quantile to use is arbitrary, and dominance is more likely to be obtained
with coarse rather than fine partitioning of the data. Moreover, as individual data are necessary as
the input in the entire procedure, we believe it is preferable to compute na-GICs and cumulative
na-GICs directly from the individual data. This is done in Figure 2.

Figure 2: panel (a) individual na-GICs and (b) cumulative individual na-GICs for Italy.

The individual na-GIC depicts each individual’s growth rate as a function of his relative position
in the initial income distribution. As can be seen in panel (a) of Figure 2, these individual na-GICs
intersect very often. Panel (b) shows that also the cumulative individual na-GICs intersect.

Third, it has to be pointed out that the comparison of (individual) na-GICs and cumulative na-
GICS is heavily influenced by the mean of the individual income growths. One might be interested
in the purely redistributive aspect of the growth processes, i.e., in a comparison of the growth
processes after correcting for differences in mean of individual income growths.

We propose to deal with these issues by providing an index that embodies the intuitions of the
na-GIC, of the history dependent perspective. With the exceptions of Jenkins and Van Kerm (2011)
and Genicot and Ray (2013), there does not exist a synthetic index of history dependent growth.
Our index is a weighted average of individual income growth with weights that are decreasing with
the rank in the initial distribution of income, while Genicot and Ray’s index weights individual
growth rates on the basis of the levels of their initial incomes. Therefore, our index is more directly
related to the na-GIC. Our weights are the weights in the standard single-series Gini (Donaldson
and Weymark, 1980). We show that, like the Jenkins and Van Kerm index, our index is additively
decomposable into a progressivity index, measuring the pure redistributive effect of the growth
In the axiomatic characterisation of our aggregate measure of history dependent growth a first ingredient is the Rank Dependent Monotonicity (RDM) axiom, which says that aggregate growth is an increasing function of individual growths which are ordered on the basis of the initial income of the individual. A second ingredient is the History Dependent Growth Incidence (HDGI) axiom which says that we like redistributions of individual growth in favor of those having a low level of initial income, and are indifferent between growth redistributions among individuals having the same initial level of income. As a result of these two axioms, the evaluation of aggregate growth proceeds as follows: first, compute individual income growths and order these in a vector on the basis of the individuals’ initial income level (from high to low initial income). Next, since the growth distribution between individuals having the same initial income level does not matter, replace their growth by the mean income growth of those having the same initial income level. The final step is the aggregation of these mean income growths.

Hence, to operationalise the framework, we first need a measure of individual income growth. We axiomatise two standard measures. Both satisfy Normalization, Monotonicity and Independence. Normalization (N) and Monotonicity (M) are common properties in the literature: the former implies that the index is equal to 0 if the initial and final level of income are the same; the latter implies that growth is increasing in second period incomes. The Independence condition (IND) is a new property in this literature. It requires that adding a given amount of income to two individuals with the same initial level of income (but possibly different levels of second period incomes) affects their individual growth rates by the same amount. This independence condition is natural in the present context, as, combined with HDGI (irrelevancy of the distribution of growth between those having the same initial income), it implies irrelevancy of the distribution of second period income between those having the same initial income level. Moreover, it enables us to obtain a unifying characterisation for a relative and an absolute measure of individual growth. Standard Scale Invariance (SI), respectively Addition Invariance (AI), are then introduced to obtain the specific functional form of both measures of individual growth: the proportional, respectively absolute difference between final and initial income.

Next, we need to aggregate the initial income averaged growths. We impose the counterparts of the structural axioms used by Demuynck and Van de Gaer (2012) on the present domain, the domain of initial income level based averaged growths, ranked on the basis of initial income. More in particular, we imposes Relative and Translation Invariance (RI and TI), requiring that the aggregate growth ordering of two growth processes is unaffected when, in both processes, all individual growth numbers are multiplied by the same constant or when, in both growth processes, the same constant is added to all individual growth numbers, respectively. We then impose Decomposability with respect to Lowest Initial Income (DLII) which requires that aggregate growth only depends on the aggregate growth of the \(n-1\) group of initially richest and on the growth of the initially poorest. Further imposing Population Invariance (PI), we obtain our aggregate index of history dependent growth.

Two remarks are in order at this stage. First, remark that, from a formal point of view, the contribution of our work to the existing literature is twofold. The first is that we provide a unifying framework for the derivation of an absolute and a relative measure of individual growth. The
second is the aggregation procedure which is similar to the one leading to the single-series Gini of Donaldson and Weymark. Second, remark that the history dependent perspective is different from the pro-poor perspective, which looks at the extent to which poverty declines over time. The main instrument in this literature is the “Growth Incidence Curve”, abbreviated here to “GIC”, which plots the growth in mean income at the same percentile in the income distributions in two consecutive periods as a function of this percentile (Ravallion and Chen, 2003; Son 2004). Here, contrary to the history dependent perspective, incomes of different individuals are used to compute the growth in mean incomes, as those that are at a particular percentile in the initial income distribution are not necessarily at that same percentile in the second period income distribution. Empirically, dominance between GICs is much more frequent than between na-GICs. Moreover, contrary to the history dependent perspective, in case no dominance can be established, a variety of indices for the measurement of pro-poor growth is available (see Gosse et al., 2008; Kakwani and Son, 2008; Kraay, 2006; Kakwani and Pernia, 2000; Essama-Nssah, 2005; Essama-Nssah and Lambert, 2009).

We end the paper with an empirical illustration of our theoretical framework. It is aimed at comparing different consecutive two-year growth processes that took place in Italy from 1998 against the growth process 2008-2010. The focus on 2008-2010 stems from the observation that this is the period during which the first wave of the economic crisis took place.

The paper is organised as follows. In Section 2 we introduce the general notation and present our theoretical results. Section 3 applies the framework to the recent economic crisis in Italy (2008-10). Section 4 concludes.

2 The framework

In this Section we characterise two individual measures of growth and the aggregation of these measures into a societal index of history dependent growth. We follow the major branch in the literature on income mobility measurement, in working with a set of observations of individuals’ incomes in two periods (see, e.g. Fields and Ok, 1999a). It has the main advantage that we use the income data in the way they are reported in panel data sets; we don’t aggregate them into arbitrary quantiles and compute our index directly on the basis of the individual data. We start by defining the notation we will use throughout this paper.

Let \( N = \{1, 2, \ldots, n\} \) be the set of individuals, \( x_i \) be individual \( i \)'s initial (first period) income and \( w_i \) his second period income. As the history dependent growth perspective evaluates a growth process on the basis of individual growth experiences, we have to keep track of which individual got which income in each period and of every individual’s position in the initial distribution of income. Hence we focus on the domain

\[
\hat{D}_n = \{(x_1, \ldots, x_i, \ldots, x_n, w_1, \ldots, w_i, \ldots, w_n) \in \mathbb{R}^{2n}_+ \text{ such that } x_1 \geq \ldots \geq x_n\}.
\]

Our aim is to characterise an index \( G^n(x, w) : \hat{D}_n \rightarrow \mathbb{R} \), where \( G^n \) is a non-constant function that measures aggregate growth, with special case \( G^1 \) measuring the growth experienced by an individual, for which the domain reduces to \( \mathbb{R}^2_+ \). Let the set \( S_i = \{j \in N \text{ such that } x_j = x(i)\} \) contain all individuals that have the \( i \)-th highest level of income \( x(i) \) and \( n_i \) be the cardinality of \( S_i \). The number of different first period incomes is denoted by \( m \).

A first axiom that we impose is a monotonicity axiom, applied to the domain \( \hat{D}_n \). When comparing two growth processes, the growth process \( G^n(x, w) \) has no lower growth than \( G^n(v, z) \)
if all individuals that occupy the same position in x and v experience higher or equal growth in $G^n(x,w)$ than in $G^n(v,z)$. If, moreover, at least one individual experiences higher growth in $G^n(x,w)$ than in $G^n(v,z)$, then aggregate growth has strictly increased.

**RDM** (Rank Dependent Monotonicity): For all $(x,w)$ and $(v,z) \in \hat{D}^n$,

if for all $i \in N$, and $i \neq j : G^1(x_i, w_i) = G^1(v_i, z_i)$,

then $G^n(x,w) \geq G^n(v,z)$ if and only if $G^1(x_j, w_j) \geq G^1(v_j, z_j)$.

This axiom requires that, over the domain $\hat{D}^n$, aggregate growth is a non-decreasing function of individual growths. It also implies that apart from the individual growths, the only thing that matters is the rank order in the initial income distribution, not the income level in this distribution.

We can now formally define the history dependent growth incidence axiom.

**HDGI** (History Dependent Growth Incidence):

For all $(x,w)$ and $(x,z) \in \hat{D}^n$ that are such that for all $i \neq k, l : G^1(x_i, w_i) = G^1(x_k, z_k)$ and there exists a $\Delta > 0$ such that

$$G^1(x_i, z_l) = G^1(x_i, w_i) + \Delta \text{ and } G^1(x_k, z_l) = G^1(x_k, w_k) - \Delta,$$

then

(a) if $x_i = x_k$, then $G^n(x,w) = G^n(x,z)$,

(b) if $x_i \leq x_k$, then $G^n(x,w) \leq G^n(x,z)$.

Part (a) requires that aggregate growth is not sensitive to income Redistributions between individuals that have the same initial income level, and part (b) that aggregate growth does not decrease if growth is redistributed from an initially richer to an initially poorer individual. Moreover, in view of RDM, part (a) implies that if there are several individuals with the same initial income level, only the sum of the their growth rates matters. Hence, we reformulate the domain by replacing individual growth rates $g_i$ by $\bar{g}_i = \frac{1}{n_i} \sum_{j \in S_i} G^1(x_j, w_j)$, the mean income growth of all those having the same initial income as individual $i$. Formally, we work with the domain

$$D^n = \{(\bar{g}_1, \ldots, \bar{g}_n) \in \mathbb{R}^n : x_1 \geq \ldots \geq x_n \text{ and } \bar{g}_i = \frac{1}{n_i} \sum_{j \in S_i} G^1(x_j, w_j)\}.$$

Every income vector $(x,z) \in \hat{D}^n$ has a unique representation in $D^n$, and all relevant information necessary for the history dependent perspective is present in the definition of this domain. We require, for all $(x,z)$ and $(x',z') \in \hat{D}^n$, and their representations $\bar{g}$ and $\bar{g}' \in D^n$, respectively, that there exists a real valued function $G^n$ defined over the domain $\hat{D}^n$ and a real valued non-decreasing function $W^n$ over the domain $D^n$ such that

$G^n(x,z) \geq G^n(x',z')$ if and only if $W^n(\bar{g}) \geq W^n(\bar{g}')$.

We first characterise a measure of individual growth. Next we turn to the aggregation of these individual growth measures. All proofs are gathered in Appendix A.
2.1 Individual growth

We propose a relative and an absolute measure of individual growth. There are good reasons to use either of both measures, a discussion of their pros and cons for the measurement of growth in a history dependent context is outside the scope of this work.\footnote{For a detailed analysis of this issue in the context of income inequality measurement, see Kolm (1976a,b) and Atkinson and Brandolini (2010).}

Three axioms will be used to characterise both a relative and an absolute measure of individual growth. The first is a standard normalization axiom. It requires that a measure of individual growth should be equal to 0 if the individual does not experience any variation in her level of income.

\textbf{N} (Normalization): For all $x \in \mathbb{R}_{++}$: $G^1(x, x) = 0$.

The second is a trivial monotonicity axiom: growth is increasing in second period incomes.

\textbf{M} (Monotonicity): For all $x, w, z \in \mathbb{R}_{++}$: $w > z \Rightarrow G^1(x, w) > G^1(x, z)$.

The third is an independence condition: for individuals having the same initial level of income, increasing second period incomes changes growth by the same amount, no matter what the original second period level of income is.

\textbf{IND} (Independence): For all $x, w, z \in \mathbb{R}_{++}$ and $\theta > 0$:

$$G^1(x, w + \theta) - G^1(x, w) = G^1(x, z + \theta) - G^1(x, z).$$

As a result of this axiom, and since due the HDGI axiom the distribution of growth between those having the same initial income does not matter, we do not care about how second period incomes are distributed between individuals having the same initial income. This axiom will be used to cardinalise both the relative and absolute measure in such a way that they become a linear function of second period incomes. Axiom \textbf{N} provides further restrictions on the cardinalization.

2.1.1 A measure of relative growth

As is standard, measures of relative growth are scale invariant measures: they are not affected by an equiproportional change in the initial and final level of income.

\textbf{SI} (Scale Invariance): For all $\lambda > 0$ and all $x, w \in \mathbb{R}_{++}$:

$$G^1(\lambda x, \lambda w) = G^1(x, w).$$

It is easy to obtain the following Lemma.

\textbf{Lemma 1}: For all $x, v, w, z \in \mathbb{R}_{++}$ the individual growth measure satisfies SI and M if and only if

$$G^1(x, w) > G^1(z, v) \Leftrightarrow \frac{w}{x} > \frac{v}{z}.$$
**Proposition 1**: A growth measure $G_{1R}(x, w)$ satisfies SI, M, N and IND if and only if there exists $\beta > 0$ such that

$$G_{1R}(x, w) = \beta \frac{(w - x)}{x}.$$

Proposition 1 characterises a standard measure of individual growth: the proportional difference between the final and the initial income.

### 2.1.2 A measure of absolute growth

Measures of absolute growth satisfy addition invariance: the value of the function $G^1$ does not change if the same amount of income is added to both initial and final income.

**AI (Addition Invariance):** For all $\theta > 0$ and all $x, w \in \mathbb{R}_{++}$:

$$G^1(x + \theta, w + \theta) = G^1(x, w).$$

It is easy to obtain the following Lemma.

**Lemma 2**: For all $x, v, w, z \in \mathbb{R}_{++}$ the individual growth measure satisfies AI and M if and only if

$$G^1(x, w) > G^1(z, v) \iff w - x > v - z.$$

Lemma 2 says that if we want to order individual growths in an addition invariant and monotonous way, we have to order them on the basis of their differences between second and first period incomes. The axioms N and IND can be used to cardinalise this ordering. This results in the following.

**Proposition 2**: A growth measure $G_{1A}(x, w)$ satisfies AI, M, N and IND if and only if there exists $\alpha > 0$ such that

$$G_{1A}(x, w) = \alpha (w - x).$$

Proposition 2 characterises a standard measure of individual growth: the difference in level between the final and the initial income.

The indices of individual growth obtained in Propositions 1 and 2 have been already introduced in the literature and are widely implemented in empirical works.\(^4\) We provide a unifying framework to derive both indices, using the new independence axiom.

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\(^4\)There exist evident alternatives, like $\log(w) - \log(x)$ or $(w/x)^r$ with $r > 0$, being the relative growth measures present in the directional income mobility measures of Fields and Ok (1999b), and Schluter and Van de Gaer (2011), respectively, or $\exp[c(w - x)]$ with $c > 0$, the absolute growth measure present in another directional mobility measure of Schluter and Van de Gaer (2011).
2.2 From individual to aggregate growth

In this Section we characterise a measure of aggregate growth. In order to do so, recall that our framework builds on the assumption that, thanks to RDM and HDGI, we can work with the domain $D^n$.

The structural axioms we use (RDM, RI, TI and D-HII) have been used in the literature, but on different domains. Bossert (1990) used these axioms on the domain of ordered single period income vectors (individual incomes ordered from high to low) to characterise the generalised Gini social evaluation function. Demuynck and Van de gaer (2012) used them on the domain of ordered mobility vectors (individual mobilities ordered from high to low). We translate these structural axioms to the domain $D^n$.

The Relative Invariance axiom says that comparisons between income growth measures remain invariant when all elements of $\bar{g}$ are multiplied by the same constant.

**RI (Relative Invariance)**: For all $\bar{g}$ and $\bar{g}' \in D^n$ and $\lambda > 0$,

\[
\text{if } W^n(\bar{g}) = W^n(\bar{g}'), \text{ then } W^n(\lambda \bar{g}) = W^n(\lambda \bar{g}').
\]

As a result of RDM and RI, the aggregate growth index will be homothetic in $\bar{g}$ on $D^n$.

The next axiom says that comparisons between income growth measures remain invariant when the same constant is added to all elements of $\bar{g}$.

**TI (Translation Invariance)**: For all $\bar{g}$ and $\bar{g}' \in D^n$ and $\lambda > 0$,

\[
\text{if } W^n(\bar{g}) = W^n(\bar{g}'), \text{ then } W^n(\bar{g} + \lambda \cdot \mathbf{1}) = W^n(\bar{g}' + \lambda \cdot \mathbf{1}).
\]

As a result of RDM and TI, the aggregate growth index will be translatable in $\bar{g}$ on $D^n$, meaning that all the iso-aggregate growth curves have the same shape, shifted by a constant $\lambda$ in each direction.

The following axiom says that aggregate growth depends on the aggregate growth measure of the $n - 1$ individuals that are ranked first in $D^n$ and the growth measure of the individual that is ranked last in $D^n$, which is the individual with the lowest initial income level. We can formulate the axiom as follows.

**DLII (Decomposability with respect to the Lowest Initial Income Level)**: For all $n > 1 \in \mathbb{N}$ and all $\bar{g}$ and $\bar{g}' \in D^n$,

\[
\text{if } W^{n-1}((\bar{g}_1, \ldots, \bar{g}_{n-1})) = W^{n-1}((\bar{g}'_1, \ldots, \bar{g}'_{n-1})) \text{ and } \bar{g}_n = \bar{g}'_n, \text{ then } W^n(\bar{g}) = W^n(\bar{g}').
\]

Like in the previous papers where a similar axiom has been used (Bossert (1990), Demuynck and Van de gaer (2012)), its purpose is to separate the contribution of the worst-off from the contribution of the others. In our context, the worst-off is the initially poorest; the $n$-th individual.

The combination of the previous axioms results in the following Lemma.

**Lemma 3**: For all $n \in \mathbb{N}$, an aggregate index of growth $G^n$ satisfies RDM, HDGI (a), RI, TI and DLII if and only if there exist strictly positive coefficients $\gamma_1^n, \gamma_2^n, \ldots, \gamma_n^n$, such that, for all $(x, w) \in \hat{D}^n$ and corresponding $\bar{g} \in \mathbb{R}^n$ with $\bar{g}_i = \frac{1}{n_i} \sum_{j \in S_i} G^1(x_j, w_j), \ldots, \gamma_n^n$, such that, for all $(x, w) \in \hat{D}^n$ and corresponding $\bar{g} \in \mathbb{R}^n$ with $\bar{g}_i = \frac{1}{n_i} \sum_{j \in S_i} G^1(x_j, w_j)$,
\[ G^n(x, w) = \sum_{i=1}^{n} \gamma_i^n \overline{g}_i, \quad \text{with} \quad \sum_{i=1}^{n} \gamma_i^n = 1. \]

The Lemma says that aggregate growth can be written as a weighted average of individuals’ growth, with weights dependent on the individual’s rank in the domain \( D^n \).

Our final axiom is a standard Population Invariance axiom. It states that the measure of aggregate growth is invariant to a \( k \)-fold replication of the same vector of initial and final incomes. This property ensures that we can apply this measure to compare growth processes taking place over distributions with different population sizes.

**PI (Population Invariance):** For all \( n, k \in \mathbb{N} \),

\[ g \in D^n \text{ and } g^k = (\overline{g}_1, \overline{g}_1, \ldots, \overline{g}_n, \overline{g}_n, \ldots, \overline{g}_n, \overline{g}_n) \in D^{kn}, \]

\[ W^n(g) = W^{kn}(g^k). \]

Following Donaldson and Weymark (1980), population invariance allows us to get a functional form for the weights. Formally,

**Proposition 3.** For all \( n \in \mathbb{N} \), an aggregate index of growth \( G^n \) satisfies RDM, RI, TI, DLII, HDGI and PI if and only if there exists a parameter \( \delta \), such that, for all \((x, w) \in D^n \) and corresponding \( g \in R^n \) with \( g_i = \frac{1}{n_i} \sum_{j \in S_i} G^1(x_j, w_j) \),

\[ G^n(x, w) = \sum_{i=1}^{n} \frac{i^\delta - (i-1)^\delta}{n^\delta} g_i \quad \text{and} \quad \delta \geq 1. \]

Our index of history dependent growth attaches to each \( \overline{g}_i \) a weight that is decreasing in the rank of the individual in the initial income distribution. The weights are the standard weights derived for the single-series Gini by Donaldson and Weymark (1980). The parameter \( \delta \) is a sensitivity parameter: for \( \delta = 1 \), everybody’s growth rate gets the same weight; as \( \delta \) increases, the relative weight to the initially poorest increases and the weight to the initially richest decreases; \( \delta = 2 \) gives the standard Gini weights and as \( \delta \) approaches \( \infty \), only the growth rate of the initially poorest matters. If the individual growth measure \( G^{1R}(x, w) \), characterised in Proposition 1, is chosen, we obtain an aggregate relative growth measure, if the individual growth measure \( G^{1A}(x, w) \), characterised in Proposition 2, is chosen, we obtain an aggregate absolute growth measure.

The value of the index derived in Proposition 3 depends on the value of the sensitivity parameter \( \delta \). Abusing notation, we write the index as \( G^n(\delta) \) to make this dependency explicit. The index is sensitive to both the distribution of growth among the individuals and the mean of individual income growths: doubling all individual growths does not affect the distribution of growth, but doubles the value of \( G^n(\delta) \). As advocated by Jenkins and Van Kerm (2011), it is interesting to separate the purely distributive effect of the growth process (the “progressivity aspect”) from the mean income growth experienced by the population as a whole. Since \( G^n(1) \) equals the mean of all individual growths, a natural measure of this progressivity is

\[ \frac{P^n(\delta)}{G^n(1)} = G^n(\delta) - G^n(1). \]
Actual growth processes differ in both the distribution of growth and the overall level of growth. The progressivity index allows us to compare their purely distributive effects.

It is instructive to determine the total weights attached to the mean growth rate associated with each initial income level. To this end, suppose that the individuals ranked between \( l \) and \( k \) share the same initial income level such that for \( l \leq i \leq k \), we have that \( \eta_i = \tilde{g} \). The total weight attached to \( \tilde{g} \) in the sum defined in Proposition 3 equals

\[
\frac{1}{n^\delta} \sum_{i=l}^{k} (i^\delta - (i - 1)^\delta) = \frac{1}{n^\delta} (k^\delta - (l - 1)^\delta).
\]

Hence, after defining \( n_0 = 0 \) and \( g(i) = \frac{1}{n_i} \sum_{j \in S_i} g_j \), the mean growth rate of all sharing the \( i \)-th highest level of initial income, the index can be rewritten as follows:

\[
\sum_{i=1}^{m} \left[ \left( \sum_{j=1}^{i} n_j \right)^\delta - \left( \sum_{j=1}^{i-1} n_j \right)^\delta \right] \frac{n_i}{n^\delta} g(i).
\]

Consider the following first order approximation:

\[
\left[ \frac{\sum_{j=1}^{i-1} n_j}{n} \right]^\delta \approx \left[ \sum_{j=1}^{i} \frac{n_j}{n} \right]^\delta - \delta \left[ \sum_{j=1}^{i} \frac{n_j}{n} \right] \frac{n_i}{n}.
\]

Using this approximation in the expressing preceding it, we find the following approximate value for the index:

\[
\sum_{i=1}^{m} \delta \left[ \sum_{j=1}^{i} \frac{n_j}{n} \right]^{\delta-1} \frac{n_i}{n} g(i),
\]

which is the discrete version of the Jenkins and Van Kerm (2011) index. Hence the Jenkins and Van Kerm index is an approximation of our index. The advantage of our measure, however, is that it works with discrete data, which is the format of all empirical data; it does not require an arbitrary division of the initial income distribution in quantiles and/or its computation does not require the non-trivial estimation of the na-GIC.

Another interesting expression for our index can be derived. Remember that \( x(i) \) is \( i \)-th highest initial income level. Let \( F(\cdot) \) be the cumulative distribution function of initial income. Then the index derived in Proposition 3 can be written as

\[
\sum_{i=1}^{m} \left[ [1 - F(x(i))]^\delta - [1 - F(x(i - 1))]^\delta \right] g(i),
\]

showing that initial income class-averaged growths are weighted by the difference between the fraction of the population with an income level at least equal to the initial income of the class to the power \( \delta \) and the fraction of the population with an income at least equal to the (higher) level of income of the previous class to the power \( \delta \).

Finally, Genicot and Ray (2013) propose the following index:

\[
\sum_{i=1}^{n} \frac{x_i^{-\alpha}}{\sum_{j=1}^{n} x_j^{-\alpha}} g_i,
\]

which is the discrete version of the Jenkins and Van Kerm (2011) index. Hence the Jenkins and Van Kerm index is an approximation of our index. The advantage of our measure, however, is that it works with discrete data, which is the format of all empirical data; it does not require an arbitrary division of the initial income distribution in quantiles and/or its computation does not require the non-trivial estimation of the na-GIC.

Another interesting expression for our index can be derived. Remember that \( x(i) \) is \( i \)-th highest initial income level. Let \( F(\cdot) \) be the cumulative distribution function of initial income. Then the index derived in Proposition 3 can be written as

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\sum_{i=1}^{m} \left[ [1 - F(x(i))]^\delta - [1 - F(x(i - 1))]^\delta \right] g(i),
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showing that initial income class-averaged growths are weighted by the difference between the fraction of the population with an income level at least equal to the initial income of the class to the power \( \delta \) and the fraction of the population with an income at least equal to the (higher) level of income of the previous class to the power \( \delta \).

Finally, Genicot and Ray (2013) propose the following index:

\[
\sum_{i=1}^{n} \frac{x_i^{-\alpha}}{\sum_{j=1}^{n} x_j^{-\alpha}} g_i,
\]
with $\alpha > 0$, a parameter determining the weight the measure gives to the initially poor. Observe that, for $\alpha = 0$, we obtain the unweighted average of individual growths (like for our index, when $\delta = 1$), while for $\alpha$ approaching infinity, only the growth rate of the initially poorest matter (like for our index when $\delta$ approaches infinity). In terms of axiomatic properties, this index satisfies HDGI, RI, DLII and PI but not TI nor RDM. Hence, it is also a measure of history dependent growth, but one that satisfies a more classical separability assumption. The disadvantage of the Genicot and Ray index is that it is less directly related to the na-GIC, which only contains information on growth rates and their ranking in the initial income distribution, not on their initial income level.

3 The distributional implications of the crisis in Italy

In this Section we implement our theoretical framework in order to investigate changes in the Italian growth process over the last decade. Using our indices $G^n(\delta)$ and $P^n(\delta)$, we assess the consequences of the recent economic crisis on the Italian growth process from the history dependent perspective.

3.1 The data

Our empirical illustration is based on the panel component of the last seven waves of the Bank of Italy “Survey on Household Income and Wealth” (SHIW). The SHIW is a representative sample of the Italian resident population interviewed every two years. In particular, we consider the 1998, 2000, 2002, 2004, 2006, 2008 and 2010 waves.

The unit of observation is the household, defined as all persons sharing the same dwelling. Our measure of living standard is household net disposable income, which includes all household earnings, transfers, pensions, and capital incomes, net of taxes and social security contributions. Household income is expressed in constant prices of 2010 and then adjusted for differences in household size using the OECD equivalence scale (the square root of household size). In line with the literature, for each wave, we drop the bottom and top 1% in the income distribution from the sample to eliminate the effect of the outliers. Table 1 below reports the yearly growth rate of mean income and the main features of the government in power in each two year period since 1998.

To investigate the distributional impact of the recent economic crisis, we use the growth process 2008-10 as benchmark since the first wave of the crisis took place in Italy in 2008. We compare all previous two-year period growth processes starting from 1998-00 with this benchmark. In the main text, for the sake of brevity, we only provide a detailed report for the comparison with the 2004-06 period, the period immediately preceding the economic crisis. We have chosen this comparison because, apart from the crisis, these adjacent periods are most similar. Moreover, the parties in power were center-rightist in both periods and the Prime Minister (S. Berlusconi) was the same (see Table 1). These periods differ in terms of their growth in mean income and the mean of individual income growths, but our progressivity index eliminates the latter effect. The comparisons of the other periods with the benchmark yield broadly similar results and are briefly discussed at the end of the next Section; the detailed results are reported in Appendix C for completeness.

We use sample weights to compute all estimates. We give each household the sample weight

\begin{footnote}
5Information about sample sizes are reported in Table C.1 in Appendix C.
\end{footnote}

\begin{footnote}
6We use cross-sectional individual sample weights, at time t. As shown by Faiella and Gambacorta (2007) in the case of the SHIW, for the production of longitudinal statistics, there is no unambiguous evidence that the use of longitudinal weights always performs better than cross-sectional weighing in terms of efficiency.
\end{footnote}
Table 1: Growth rate of mean equivalised yearly income, political parties and coalitions in power for each period.

<table>
<thead>
<tr>
<th>Period</th>
<th>Growth rate of mean income</th>
<th>Political Party</th>
<th>Coalition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998-00</td>
<td>0.0230</td>
<td>Democratici di sinistra</td>
<td>L’Ulivo -UDR (L)</td>
</tr>
<tr>
<td>2000-02</td>
<td>0.0159</td>
<td>I Democrati (up2001) / Forza Italia</td>
<td>L’Ulivo (L)/Casa della Liberta (R)</td>
</tr>
<tr>
<td>2002-04</td>
<td>0.0181</td>
<td>Forza Italia</td>
<td>Casa delle Liberta (R)</td>
</tr>
<tr>
<td>2004-06</td>
<td>0.0207</td>
<td>Forza Italia</td>
<td>Casa delle Liberta (R)</td>
</tr>
<tr>
<td>2008-10</td>
<td>-0.0021</td>
<td>Il Popolo della Liberta</td>
<td>PdL MpA LNP (R)</td>
</tr>
</tbody>
</table>


corresponding to the sampling in the first wave of the survey in our analysis (1998). To the households selected into the survey at subsequent waves, we give the sample weight corresponding to the sampling in the wave of their first inclusion into the survey.\(^7\) The standard errors of our estimates are obtained through 1000 bootstrap replications -see appendix B for more details.

### 3.2 Results

In this Section we establish the distributional implications of impressive macroeconomic changes by comparing the growth process 2004-06 against the growth process 2008-10. Remember from panel (b) in Figure 2 that the cumulative individual na-GICs cross. We show that our measure can be used to establish a clear ranking of the growth processes from the history dependent perspective. The numerical values of our measure for both growth processes are reported in Table 2, while the results of comparing them are reported in Table 3.

**Table 2: History dependent growth indices 2004-06 and 2008-10.**

<table>
<thead>
<tr>
<th>(G^n_{04/06}(\delta))</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative</td>
<td>0.0641</td>
<td>0.1128</td>
<td>0.1673</td>
<td>0.2053</td>
<td>0.2366</td>
</tr>
<tr>
<td>Absolute</td>
<td>425.2</td>
<td>1038.5</td>
<td>1335.9</td>
<td>1485.8</td>
<td>1597.6</td>
</tr>
<tr>
<td>(G^n_{08/10}(\delta))</td>
<td>0.0240</td>
<td>0.0523</td>
<td>0.0810</td>
<td>0.0994</td>
<td>0.1136</td>
</tr>
<tr>
<td>Absolute</td>
<td>-41.90</td>
<td>432.2</td>
<td>635.8</td>
<td>716.0</td>
<td>767.6</td>
</tr>
</tbody>
</table>

Note: authors’ calculations, based on the Italian “Survey on Household Income and Wealth”.

**Table 3: Test of the hypothesis \(G^n_{04/06}(\delta) - G^n_{08/10}(\delta) > 0\).**

<table>
<thead>
<tr>
<th>(\delta)</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative</td>
<td>0.0401***</td>
<td>0.0606***</td>
<td>0.0863***</td>
<td>0.1059***</td>
<td>0.1230***</td>
</tr>
<tr>
<td>Absolute</td>
<td>473.9***</td>
<td>606.3***</td>
<td>700.1***</td>
<td>769.8***</td>
<td>830.0***</td>
</tr>
</tbody>
</table>

Note: authors’ calculations, based on the Italian “Survey on Household Income and Wealth”. *** means statistically significant at 99 %, ** statistically significant at 95 % and * statistically significant at 90 %.

\(^7\)See on this Hildebrand et al. (2012) and Jenkins and Van Kerm (2011).
For all values of the sensitivity parameter \( \delta \), our index measures more history dependent growth during the 2004-06 process than during the 2008-10 process. This holds both when a relative and an absolute index is used. The difference is always statistically significant. Hence it can be inferred that the 2004-06 growth episode is better than the 2008-10 episode according to our measure of history dependent (relative and absolute) growth. As the 2004-06 cumulative individual na-GIC curve lies almost everywhere above its 2008-10 counterpart, this information is, to some extent, also present in the cumulative individual na-GICs.

Focusing on the value of the index when \( \delta = 1 \), that is when all individual growth experiences get the same weight such that history dependency is not taken into account, the difference between the indices of the two processes is already substantial. This might imply that the result when history dependency is taken into account (\( \delta > 1 \)), is mostly due to the overall level of growth. In order to investigate this issue, we adopt the solution given at the end of Section 2.3; that is, we compute the progressivity indices \( P^n(\delta) \) to compare the pure distributional effect of both processes.

The results, reported in Table 4, show that, also when the focus is on the pure distributional effect of growth, the 2004-06 growth process remains more desirable from a history dependent perspective than the 2008-10 growth process (although the result is not significant for the absolute index when \( \delta = 2 \), and it is only significant at 90% for the relative index when \( \delta = 2 \) and for the absolute index when \( \delta = 4 \)). Thus, both the overall extent of growth and the pure distributional effect play a role in the history dependent ranking of the growth process 2004-06 above the growth process 2008-10.

Table 4: Test of the hypothesis \( P^n_{04/06}(\delta) > P^n_{08/10}(\delta) \).

<table>
<thead>
<tr>
<th>( \delta )</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative</td>
<td>TRUE*</td>
<td>TRUE**</td>
<td>TRUE**</td>
<td>TRUE***</td>
</tr>
<tr>
<td>Absolute</td>
<td>TRUE</td>
<td>TRUE*</td>
<td>TRUE**</td>
<td>TRUE**</td>
</tr>
</tbody>
</table>

Note: authors’ calculations, based on the Italian “Survey on Household Income and Wealth”. *** means statistically significant at 99 %, ** statistically significant at 95 % and * statistically significant at 90 %.

In order to put the distributional implications of the crisis into further perspective, it is interesting to describe briefly the results of the other comparisons. In fact, notice that according to our family of indices also the 1998-00 episode outperforms the 2008-10 episode. The difference in the value of their corresponding indices of history dependent growth is impressive. Moreover, the 1998-00 process also performs better than the 2008-10 process when only distributional aspects are taken into consideration; this turns out to be statistically significant for all values of \( \delta \) (see Tables C.3, C.4 and C.5 in Appendix C.2). Similar conclusions can be drawn from the comparison 2008-10 versus 2000-02 and 2002-04. The period of the crisis performs always worse than the growth episodes in 2000-02 and 2002-04. The superior performance of the 2000-02 and the 2002-04 over the 2008-10 growth process is statistically significant for every value of \( \delta \) with both a relative and an absolute measure of individual growth (see Table C.7 in Appendix C.3 and Table C.10 in Appendix C.4). Most importantly, when the pure redistributive aspect of growth is considered, the ranking remains the same and is usually statistically significant.  

The following Table gives the mean of individual growth rates, \( G(1) \) and the values of the

8For the comparisons 2000-02 versus 2008-10 the difference in progressivity is not significant when an absolute measure with \( \delta = 2, 4 \) or 6 is chosen.
Comparing the $G(1)$ column of Table 5 with the growth rate of mean incomes reported in Table 1, we see that the 2004-06 process, ranked second best in on the basis of the growth in mean income, ranks third best on the basis of the mean of individual income growths. The growth rate of mean income can be written as a weighted average of individual income growth rates, with weights equal to the share of the individual in the initial distribution of income. As such, it attaches a larger weight to the growth experienced by the initially richest, while the mean of individual income growths weights all individual income growths equally. The fact that the 2004-06 process drops one position in the ranking is indicative of it being less in favor of the initially poor.

In Table 5, it is striking that the ranking of the processes based on the progressivity indices, irrespective of the value for $\delta$, is almost the same as the ranking of the processes based on the mean of individual growth rates $G(1)$. The only difference is that the purely distributive ranking of the 2004-06 process is worse than its ranking based on $G(1)$: again it drops one position in the ranking and becomes worse than the 2000-02 process, confirming the poor distribution of growth from a history dependent perspective during the 2004-06 process.

When looking into the composition of the 10% initially poor over occupational category of the household head, we find an explanation for the close relationship between the purely redistributive effect of history dependent growth and the mean of individual growth rates.

Table 6 reports the share of different occupational categories in the poorest 10% relative to their share in the population at large; numbers larger (smaller) than 10 indicate that this category is over- (under-) represented in the initially poor. Evidently, only one category is overrepresented:

9The results are very similar when the absolute progressivity index is used.
the not employed. As a result, what happens to the income growth of the initially poor will be very sensitive to whether these not employed manage to find a job or not. This, of course depends on the aggregate performance of the economy. In times of recession, the mean of individual growths will be low, it will be difficult to find a job, fewer not employed will find a job, such that their incomes increase less and the purely redistributive effect of the growth process will be smaller. The opposite happens in times of sustained economic growth. Observe that the dual nature of the Italian labour market could be responsible for this phenomenon. At the one hand, there are many low paid jobs with fixed contracts and (mostly young) workers that receive very limited unemployment insurance, and, at the other hand permanent, well paid jobs whose (typically older) workers are covered by unemployment insurance. As a result, young people, having lost their job and not being covered by unemployment insurance, end up in the group of initially poor. They can only escape their precarious situation by finding a fixed term low paid job. Especially these kinds of jobs are destroyed during recessions and created during booms. The link between macro-economic activity and the fate of the initially poor is further strengthened by what happens to low paid blue collar workers in times of recession. As many work in the unprotected segment of the labour market, many of them loose their job, and, as they are insufficiently covered by unemployment insurance, this adversely affects their incomes, which further depresses the income growth of the initially poor. These two phenomena that link macroeconomic activity to the incomes of the initially poor not employed and blue collar workers can be clearly seen in Table 7.

Table 7: Mean of individual income growth for each occupational category of those individuals in the poorest 10% in each initial period.

<table>
<thead>
<tr>
<th>Year</th>
<th>Blue-collar</th>
<th>White-collar</th>
<th>Executive / Manager</th>
<th>Entrepreneur / member of professions</th>
<th>Self-employed</th>
<th>Retired</th>
<th>Not employed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998-00</td>
<td>60.8</td>
<td>46.3</td>
<td>na</td>
<td>58.6</td>
<td>65.4</td>
<td>40.8</td>
<td>65.2</td>
</tr>
<tr>
<td>2000-02</td>
<td>26.9</td>
<td>9.2</td>
<td>14.9</td>
<td>73.3</td>
<td>47.4</td>
<td>25.3</td>
<td>45.2</td>
</tr>
<tr>
<td>2002-04</td>
<td>24.2</td>
<td>30.6</td>
<td>na</td>
<td>84.7</td>
<td>72.3</td>
<td>18.5</td>
<td>52.3</td>
</tr>
<tr>
<td>2004-06</td>
<td>32.4</td>
<td>21.4</td>
<td>-5.43</td>
<td>82.5</td>
<td>65.9</td>
<td>18.8</td>
<td>30.6</td>
</tr>
<tr>
<td>2008-10</td>
<td>3.9</td>
<td>17.9</td>
<td>na</td>
<td>62.2</td>
<td>43.7</td>
<td>10.2</td>
<td>22.8</td>
</tr>
</tbody>
</table>

Note: authors' calculations, based on the Italian “Survey on Household Income and Wealth”; “na” means “not applicable” as there is nobody from this category in the bottom 10% of the initial income distribution. Occupational category refers to the household head.

When reading Table 7, one has to keep in mind that the numbers in the poorest 10% of the initial period is very small for the categories “white-collar”, “Executive / Manager”, “Entrepreneur / member of professions” and “Self-employed”, implying great variability of the numbers for these categories (small sample sizes), and that what happens to their incomes is of little consequence to explain what happens to the initially poor.

The close association between the mean of individual income growths and the history dependent progressivity effect of the growth process does not mean that policy is irrelevant.

To start with, the liberalization of the Italian labour market, in 1997-98 through the so-called “Treu law” (law 196/1997), expanded in 2000 (see Ichino et al. 2005 and Isfol 2001) is to some extent responsible for the dual nature of the Italian labour market. The Treu-law regulated some forms of flexible work such as apprenticeship and internship. Most importantly it legalised the supply of temporary workers by the Temporary Work Agencies (TWA), which were forbidden until 10See Table C.2 in Appendix C.
then (due to a law introduced in 1960). Hence they increased wage flexibility and job turnover giving more job opportunities during periods of consistent positive growth (from 1998 to 2006) but, during slowdown and recessions (2008-10), workers with atypical job contracts have become more likely to be fired and cannot benefit from social protection.  

Moreover, the poor performance of the 2004-06 growth process is probably due to the regressive tax reform implemented by the government in 2005 (see, e.g. Baldini and Pacifico (2009)). The good performance of the 1998-00 process is not only due to the high level of macroeconomic growth but was also helped by the improvement in means-testing of benefits and the introduction of new means tested family and maternity allowances (see, e.g., Baldini et al (2002)).

Finally, the impact of the great recession has been tempered for the initially poor by the existing social protection schemes (see, e.g. Baldini and Ciani (2011)). In addition, the government introduced an additional means-tested family benefit\textsuperscript{11} and the social card\textsuperscript{14}. Table 7 makes clear, however, that these measures were insufficient to prevent a serious drop in the income growth of initially poor blue-collar workers. Again, this is due to the dual labour market: those low skilled blue collar workers with temporary contracts that lost their job and were not covered by unemployment insurance faced a serious decline in their income which adversely affected the income growth of the initially poor.

We conclude our analysis by addressing an issue related to the variation in households composition. It might be argued that the results of our analysis, about the pattern of the individual income growth and the values taken by our history dependent growth indices, are sensitive to changes in household composition between the initial (first) period and the second period. In order to investigate whether this is the case, we recalculated our estimates using the subsample of households which do not change composition over the period considered. The results of this check suggest that our substantive conclusions are robust to variations in households’ composition (see Appendix D).

4 Conclusions

The size of the recent economic crisis begs the question of the distributional impact of the crisis. More in particular, we want to know whether the crisis is affecting more the initially poor or the initially rich. This is a history dependent approach since it takes into account individuals’ initial economic conditions.

Endorsing this perspective, we provide a characterisation of an index of history dependent growth. The crucial steps in the characterisation are the definition of the domain, which allows to keep track of individuals’ position in the initial income distribution and the history dependent growth axiom, which prefers redistributions of growth to the initially poorest and is indifferent between growth redistributions between individuals having the same initial income. The resulting index of history dependent growth is expressed as a weighted average of the mean income growth associated with each initial income level, with weights that decrease with the rank of these individuals in the initial distribution of income. Our index turns out to be closely related to the mobility

\textsuperscript{11}See Jappelli and Pistaferi (2009) for a detailed analysis.
\textsuperscript{12}These include unemployment insurance for the better segment of the dual labour market and the redundancy fund (“cassa integrazione guadagni”), which protects worker’s incomes and jobs in case of a temporary crisis of their firm. The latter scheme was extended during the recession.
\textsuperscript{13}una tantum: a one-off monetary benefit for low-income households.
\textsuperscript{14}A means tested (based on household income and wealth) voucher for general expenditures for households with elderly people (over 65) or with at least one child younger than 3 years of age. Due to the age restrictions and the limited amount (40 Euros per month) the social card did not do a lot to protect most of the initially poor.
measure of Jenkins and Van Kerm (2011), but is easier to compute, and, like their index, it is additively decomposable into a pure distributive effect and the mean of individual income growth.

We have shown the applicability of our framework with an empirical application in which we describe effects of the economic crisis on the Italian population. Individual na-GICs cross frequently, making it impossible to obtain clear conclusions about the ranking of growth processes. Our measure allows us to obtain also in such situations a clear ranking. Concerning the impact of the economic crisis on Italian households, we find that the growth process during the crisis is worse from a history dependent perspective than any of the preceding growth processes, even when we correct for the differences in the mean of individual income growth and only consider the way growth is distributed. This is compatible with the dual nature of the Italian labour market, with very limited or no unemployment insurance for workers on temporary, fixed term contracts. As a result, during the crisis, low paid blue collar workers are laid off and experience a substantial income fall, and for the non employed, finding a job becomes very difficult, such that fewer see their incomes increase. Both these phenomena adversely affect the income growth of the initially poor during recessions.

References


A Proofs

A.1 Proof of Lemma 1 and Proposition 1

Proof of Lemma 1. The proof is simple: first apply SI, define the function $\hat{G}^1 (x) = G^1 (1, x)$ and then apply M, to get

$$G^1 (x, w) > G^1 (z, v) \iff G^1 \left(1, \frac{w}{x}\right) > G^1 \left(1, \frac{z}{v}\right) \iff \hat{G}^1 \left(\frac{w}{x}\right) > \hat{G}^1 \left(\frac{z}{v}\right) \iff \frac{w}{x} > \frac{v}{z},$$

as stated in the Lemma. □

Proof of Proposition 1. From IND and the definition of the function $\hat{G}^1 (x)$, we get

$$\hat{G}^1 \left(\frac{w + \theta}{x}\right) - \hat{G}^1 \left(\frac{w}{x}\right) = \hat{G}^1 \left(\frac{z + \theta}{x}\right) - \hat{G}^1 \left(\frac{z}{x}\right).$$

With a trivial redefinition of variables, this becomes that for all $a, b$ and $c \in \mathbb{R^+}$,

$$\hat{G}^1 (a + c) - \hat{G}^1 (a) = \hat{G}^1 (b + c) - \hat{G}^1 (b),$$

which implies that the function $\hat{G}^1$ must be linear: $\hat{G}^1 (x) = \alpha + \beta x$, such that

$$G^1 (x, w) = \hat{G}^1 \left(\frac{w}{x}\right) = \alpha + \beta \frac{w}{x}.$$  

Due to N, we get $\alpha = -\beta$, and from M, $\beta > 0$ from which the result follows. □

A.2 Proof of Lemma 2 and Proposition 2

Proof of Lemma 2. In order to prove the Lemma, we distinguish 4 cases.

(i) If $w > x$ and $v > z$, apply AI, define the function $\tilde{G}^1 (x) = G^1 (0, x)$ and then apply M, to get

$$G^1 (x, w) > G^1 (z, v) \iff G^1 (0, w - x) > G^1 (0, v - z) \iff \tilde{G}^1 (w - x) > \tilde{G}^1 (v - z) \iff w - x > v - z.$$  

(ii) If $w < x$ and $v < z$, apply AI, define the function $\tilde{G}^1 (x) = G^1 (x, 0)$ and then apply M, to get

$$G^1 (x, w) > G^1 (z, v) \iff G^1 (x - w, 0) > G^1 (z - v, 0) \iff \tilde{G}^1 (x - w) > \tilde{G}^1 (z - v) \iff w - x > v - z.$$  

(iii) If $w > x$ and $v < z$, then $G^1 (x, w) > G^1 (z, v)$ for every growth measure satisfying AI and M. This follows from M, AI and M again, which yields

$$G^1 (x, w) > G^1 (x, x) = G^1 (z, z) > G^1 (z, v).$$  

(iv) If $w < x$ and $v > z$, then $G^1 (x, w) > G^1 (z, v)$ can never hold for any growth measure satisfying AI and M. This follows from M, AI and M again, which yields

$$G^1 (x, w) < G^1 (x, x) = G^1 (z, z) < G^1 (z, v).$$
Cases (ii) and (iv) hold automatically for every growth ordering satisfying AI and M, and therefore these cases have no bite. The lemma follows since it holds for all $x, v, w, z \in \mathbb{R}^+$. □

**Proof of Proposition 2.** We only prove the case where $(w > x$ and $z > x)$. From IND and the definition of the function $\tilde{G}^1(x)$, we get

$$\tilde{G}^1(w + \theta - x) - \tilde{G}^1(w - x) = \tilde{G}^1(z + \theta - x) - \tilde{G}^1(z - x).$$

With a trivial redefinition of variables, this becomes that for all $a, b$ and $c \in \mathbb{R}^+$,

$$\tilde{G}^1(a + b) - \tilde{G}^1(a) = \tilde{G}^1(b + c) - \tilde{G}^1(b),$$

which implies that the function $\tilde{G}^1$ must be linear: $\tilde{G}^1(x) = \alpha + \beta x$, such that

$$G^1(x, w) = \tilde{G}(w - x) = \alpha + \beta(w - x).$$

Due to N, we get $\alpha = 0$, and from M, $\beta > 0$ from which the result follows.

The case where $\theta$ is such that both $x > w + \theta$ and $x > z + \theta$ can be developed similarly to show that the function $\tilde{G}^1$ is equal to $G^{1,A}$. □

**A.3 Proof of Lemma 3 and Proposition 3**

The following proof is a straightforward adaptation from Demuynck and Van de gaer (2012). Under RDM, the characterisation of the index of aggregate growth boils down to determine the properties of the function $W^n(g)$ which is defined on the domain $\mathcal{D}^n$.

For each $n \in \mathbb{N}$, consider the function $\varepsilon^n(\mathfrak{g}): \mathcal{D}^n \rightarrow \mathbb{R}$ such that:

$$W^n(\mathfrak{g}) = W^n(\varepsilon^n(\mathfrak{g}) \cdot 1).$$

The function $\varepsilon^n(\mathfrak{g})$ is similar to the equally distributed equivalent income that is well known from the literature on inequality measurement see Atkinson (1970). It is the amount of individual growth, which if distributed equally to everyone would render aggregate growth equal to the case where the individual growth vector is equal to $\mathfrak{g}$. The ‘greater than or equal to’ ordering implied by $\varepsilon^n$ coincides with the ordering derived from $W^n$. This follows from axiom RDM, which implies monotonicity of the function $W^n$, such that

$$W^n(\mathfrak{g}) \geq W^n(\mathfrak{g}') \iff W^n(\varepsilon^n(\mathfrak{g}) \cdot 1) \geq W^n(\varepsilon^n(\mathfrak{g}') \cdot 1) \iff \varepsilon^n(\mathfrak{g}) \geq \varepsilon^n(\mathfrak{g}').$$

We proceed by deriving the implications of the axioms for the function $\varepsilon^n(\mathfrak{g})$. Observe that for all $g \in \mathbb{R}$:

$$W^n(g \cdot 1) = W^n(\varepsilon^n(g \cdot 1) \cdot 1).$$

This implies that

$$\varepsilon^n(g \cdot 1) = g,$$  \hspace{1cm} (AIG)

for all values of $g$ and $n$. 

22
The implication of axiom \( RI \) is that the function \( \varepsilon^n(g) \) becomes homogeneous of degree one. Indeed, from the definition of \( \varepsilon^n(g) \), we have that \( W^n(g) = W^n(\varepsilon^n(g) \cdot 1) \) such that, by \( RI \)

\[
W^n(\lambda g) = W^n(\lambda \varepsilon^n(g) \cdot 1).
\]

From the definition of \( \varepsilon^n(g) \) we also have that,

\[
W^n(\lambda g) = W^n(\varepsilon^n(\lambda g) \cdot 1).
\]

Combining the last two equalities, we get that \( W^n(\lambda \varepsilon^n(g) \cdot 1) = W^n(\lambda \varepsilon^n(g) \cdot 1) \), from which since \( W^n \) is monotonic

\[
\lambda \varepsilon^n(g) = \varepsilon^n(\lambda g). \tag{ARI}
\]

Next observe that axiom \( TI \) imposes that \( \varepsilon^n(g) \) is independent of origin. From the definition of \( \varepsilon^n(g) \), we have \( W^n(g) = W^n(\varepsilon^n(g) \cdot 1) \), such that, by \( TI \),

\[
W^n(g + \lambda \cdot 1) = W^n(\varepsilon^n(g) \cdot 1 + \lambda \cdot 1) = W^n((\varepsilon^n(g) + \lambda) \cdot 1).
\]

At the same time, from the definition of \( \varepsilon^n(g) \),

\[
W^n(g + \lambda \cdot 1) = W^n(\varepsilon^n(g + \lambda \cdot 1) \cdot 1),
\]

such that the combination of the last two equations yields, because of the monotonicity of \( W^n \),

\[
\varepsilon^n(g + \lambda \cdot 1) = \varepsilon^n(g) + \lambda. \tag{ATI}
\]

Together, axioms \( RDM, RI \) and \( TI \) impose a very specific functional for \( \varepsilon^2(\overline{g}_1, \overline{g}_2) \): for populations of two individuals, aggregate growth can be written as a weighted sum of individual growth. Formally:

**Lemma A1** The function \( W^2 \) satisfies \( RDM, RI \) and \( TI \), if and only if there exist numbers \( \gamma_1^2 \) and \( \gamma_2^2 \in [0, 1] \), such that:

\[
\gamma_1^2 + \gamma_2^2 = 1 \text{ and } \varepsilon^2(\overline{g}_1, \overline{g}_2) = \gamma_1^2 \overline{g}_1 + \gamma_2^2 \overline{g}_2.
\]

**Proof.** Consider \( \overline{g} = (\overline{g}_1, \overline{g}_2) \) and assume wlog that \( \overline{g}_1 \geq \overline{g}_2 \) then, using first (ATI) and then (ARI),

\[
\varepsilon^2(\overline{g}_1, \overline{g}_2) = \varepsilon^2(\overline{g}_1 - \overline{g}_2, 0) + \overline{g}_2 = \varepsilon^2(1, 0)(\overline{g}_1 - \overline{g}_2) + \overline{g}_2 = \varepsilon^2(1, 0)\overline{g}_1 + (1 - \varepsilon^2(1, 0))\overline{g}_2.
\]

Now, let \( \gamma_1^2 = \varepsilon^2(1, 0) \) and set \( \gamma_2^2 = (1 - \varepsilon^2(1, 0)) \). By \( RDM \) and Equation (AIG),

\[
0 = \varepsilon^2(0, 0) \leq \varepsilon^2(1, 0) \leq \varepsilon^2(1, 1) = 1.
\]

Hence, both \( \gamma_1^2 \) and \( \gamma_2^2 \) are positive and sum to 1. \( \square \)

Using axiom \( DHII \) together with \( RDM, RI \) and \( TI \), we can derive the following partial result:
**Lemma 3** The function $G^n$ satisfies RDM, RI, TI and DHII, if and only if there exist positive numbers $\gamma_1^n, \ldots, \gamma_n^n$ summing to one, such that:

$$\varepsilon^n(\mathbf{g}) = \sum_{i=1}^{n} \gamma_i^n g_i.$$  

*Proof.* Observe that axiom DHII allows the existence of a two placed function $L^n$ such that:

$$\varepsilon^n(\mathbf{g}) = L^n(\varepsilon^{n-1}(\mathbf{g}_1, \ldots, \mathbf{g}_{n-1}), \mathbf{g}_n).$$  

(AD-HM)

The proof of the lemma is by induction. Lemma 3 gives the proof for $n = 2$. Now, assume that it holds up to $n - 1$ and let us show that the result holds for $n$. Then:

$$\begin{align*}
\varepsilon^n(\mathbf{g}) &= L^n(\varepsilon^{n-1}(\mathbf{g}_1, \ldots, \mathbf{g}_{n-1}), \mathbf{g}_n) \\
&= L^n(\varepsilon^{n-1}(\mathbf{g}_1 - \mathbf{g}_n, \ldots, \mathbf{g}_{n-1} - \mathbf{g}_n), 0) + \mathbf{g}_n \\
&= L^n(\varepsilon^{n-1}(1, 0), \varepsilon^{n-1}(\mathbf{g}_1 - \mathbf{g}_n, \ldots, \mathbf{g}_{n-1} - \mathbf{g}_n)) + \mathbf{g}_n \\
&= L^n(1, 0) \cdot (\varepsilon^{n-1}(\mathbf{g}_1, \ldots, \mathbf{g}_{n-1}) - \mathbf{g}_n) + \mathbf{g}_n \\
&= L^n(1, 0) \cdot \varepsilon^{n-1}(\mathbf{g}_1, \ldots, \mathbf{g}_{n-1}) + (1 - L^n(1, 0)) \mathbf{g}_n.
\end{align*}$$

Now, substituting $\varepsilon^{n-1}(\mathbf{g}_1, \ldots, \mathbf{g}_{n-1}) = \sum_{i=1}^{n-1} \gamma_i^{n-1} \mathbf{g}_i$ and defining for $i < n$,

$$\gamma_i^n = \left(\frac{i \delta - (i - 1) \delta}{n \delta}\right),$$

and for $i = n$,

$$\gamma_n^n = (1 - L^n(1, 0)),$$

we derive the expression:

$$\varepsilon^n(\mathbf{g}) = \sum_{i=1}^{n} \gamma_i^n g_i.$$  

It is easy to see that $\sum_{i=1}^{n} \gamma_i^n = 1$ and that all terms are positive. □

Axiom PI allows us to determine the functional form of the coefficients $\gamma_i$. Indeed, theorems 1 and 2 of Donaldson and Weymark (1980) show that PI imposes that there exist a $\delta \in \mathbb{R}_{++}$ such that for all $i \in \mathbb{N}$,

$$\gamma_i^n = (i^\delta - (i - 1)^\delta)/n^\delta.$$  

Hence, the function $G^n$ satisfies RDM, TI, RI, DHII and PI if and only if there exists a number $\delta$ such that:

$$\varepsilon^n(\mathbf{g}) = \sum_{i=1}^{n} \left(\frac{i^\delta - (i - 1)^\delta}{n^\delta}\right) g_i.$$  

(INDEX)

This completes the proof of Proposition 3.
B Bootstrap Procedure

To take into account the dependence structure of our observations, we use the non-parametric block bootstrap procedure described by Cameron and Trivedi (2010). That is, we stratify the full sample 1998-2010 such that each stratum contains those individuals that appear in exactly the same waves. The resampling then takes place within each stratum, since the observations within that stratum can be considered as independent. In this procedure, the bootstrap samples are obtained by implementing the *bsweight* stata routine proposed by Kolenikov (2010), which takes the stratification of data into account.

Let $Y^b_t$ be the $b$-th bootstrap replication of the full SHIW 1998-2010 sample, with $b = 1, ..., B$ and $B = 1000$. Let then $S_t(Y^b)$ be the replication $b$ subsample for period $(t, t + 2)$, with $t = \{1998, 2000, 2002, 2004, 2008\}$. All our indices and their differences are estimated on each replicate subsample $S_t(Y^b)$ and we denote it by $\hat{\beta}_t^b = \beta(S_t(Y^b))$.

The standard error of the statistic $\hat{\beta}_t^b$ is obtained as:

$$\hat{\sigma} = \sqrt{\frac{\sum_{b=1}^{B} \left(\hat{\beta}_t^b - \bar{\beta}_t\right)^2}{B - 1}}$$

where $\bar{\beta}_t = \sum_{b=1}^{B} \hat{\beta}_t^b / B$.

The lower and upper confidence bounds are the $B * \alpha / 2$-th and $B * (1 - \alpha / 2) - th$ ordered elements, respectively. For $B = 1000$ and $\alpha = 5\%$ these are the 25th and 975th ordered elements of the empirical distribution $F(\hat{\beta}_t)$.

We are aware that the quality of our estimates depends on strong assumptions. However, as will become clear in the discussion of our results, the ranking of growth processes obtained through our indices appear rather reliable for the illustrative purpose of the exercise.
C Empirical Appendix

C.1 Descriptive statistics

Table C.1: Sample description and the initial income level frequencies for the periods 1998-00, 2000-02, 2002-04, 2004-06 and 2008-10.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample size</td>
<td>3740</td>
<td>3504</td>
<td>3491</td>
<td>3848</td>
<td>4510</td>
</tr>
<tr>
<td>Total number of</td>
<td>3661</td>
<td>3400</td>
<td>3393</td>
<td>3722</td>
<td>4385</td>
</tr>
<tr>
<td>initial income</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>levels</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Individuals per</td>
<td>3596</td>
<td>3316</td>
<td>3317</td>
<td>3633</td>
<td>4298</td>
</tr>
<tr>
<td>initial income</td>
<td>2</td>
<td>53</td>
<td>70</td>
<td>70</td>
<td>63</td>
</tr>
<tr>
<td>level 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>10</td>
<td>13</td>
<td>19</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
| Note: The top part of the table reports the total number of observations and the number of distinct initial income levels for each growth process considered in this paper. The bottom part reports the frequency of each initial income level for each growth process.

Table C.2: Composition by occupational status of the poorest 10% in each initial period.

<table>
<thead>
<tr>
<th></th>
<th>Blue-collar</th>
<th>White-collar</th>
<th>Executive / Manager</th>
<th>Entrepreneur / member of professions</th>
<th>Self-employed</th>
<th>Retired</th>
<th>Not employed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998</td>
<td>16.0</td>
<td>4.00</td>
<td>0</td>
<td>2.33</td>
<td>10.3</td>
<td>36.2</td>
<td>31.2</td>
</tr>
<tr>
<td>2000</td>
<td>18.0</td>
<td>1.58</td>
<td>0.31</td>
<td>1.90</td>
<td>9.46</td>
<td>38.3</td>
<td>30.4</td>
</tr>
<tr>
<td>2002</td>
<td>19.5</td>
<td>3.34</td>
<td>0</td>
<td>1.52</td>
<td>6.08</td>
<td>40.7</td>
<td>28.9</td>
</tr>
<tr>
<td>2004</td>
<td>18.2</td>
<td>5.71</td>
<td>0.27</td>
<td>1.09</td>
<td>5.98</td>
<td>42.1</td>
<td>26.6</td>
</tr>
<tr>
<td>2008</td>
<td>21.5</td>
<td>4.68</td>
<td>0</td>
<td>1.10</td>
<td>4.97</td>
<td>40.8</td>
<td>26.2</td>
</tr>
</tbody>
</table>

Note: Authors’ calculations, based on the Italian “Survey on Household Income and Wealth”. Occupational category refers to the household head.

Figure C.1: panel (a) individual na-GICs and (b) cumulative individual na-GICs for Italy.

Table C.3: History dependent growth indices 1998-00 and 2008-10.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G^{n}_{98/00}(\delta) ) Relative</td>
<td>0.0963</td>
<td>0.1734</td>
<td>0.2721</td>
<td>0.3441</td>
<td>0.4026</td>
</tr>
<tr>
<td></td>
<td>432.0</td>
<td>1139.9</td>
<td>1516.3</td>
<td>1695.5</td>
<td>1811.9</td>
</tr>
<tr>
<td>( G^{n}_{08/10}(\delta) ) Relative</td>
<td>0.0240</td>
<td>0.0523</td>
<td>0.0810</td>
<td>0.0994</td>
<td>0.1136</td>
</tr>
<tr>
<td></td>
<td>-41.90</td>
<td>432.2</td>
<td>635.8</td>
<td>716.0</td>
<td>767.6</td>
</tr>
</tbody>
</table>

Table C.4: Test of the hypothesis \( G^{n}_{98/00}(\delta) - G^{n}_{08/10}(\delta) > 0 \).

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative</td>
<td>0.0724***</td>
<td>0.1211***</td>
<td>0.1911***</td>
<td>0.2448***</td>
<td>0.2889***</td>
</tr>
<tr>
<td>Absolute</td>
<td>467.1***</td>
<td>707.7***</td>
<td>880.5***</td>
<td>979.5***</td>
<td>1044.3***</td>
</tr>
</tbody>
</table>

Table C.5: Test of the hypothesis \( P^{n}_{98/00}(\delta) > P^{n}_{08/10}(\delta) \).

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative</td>
<td>TRUE***</td>
<td>TRUE***</td>
<td>TRUE***</td>
<td>TRUE***</td>
</tr>
<tr>
<td>Absolute</td>
<td>TRUE**</td>
<td>TRUE***</td>
<td>TRUE***</td>
<td>TRUE***</td>
</tr>
</tbody>
</table>
C.3 Comparison: 2000-2002 versus 2008-2010

Figure C.2: panel (a) individual na-GICs and (b) cumulative individual na-GICs for Italy.


<table>
<thead>
<tr>
<th>δ</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>G_{00/02} (δ) Relative</td>
<td>0.0612</td>
<td>0.1104</td>
<td>0.1660</td>
<td>0.2061</td>
<td>0.2400</td>
</tr>
<tr>
<td>G_{00/02} (δ) Absolute</td>
<td>305.7</td>
<td>891.6</td>
<td>1161.8</td>
<td>1286.7</td>
<td>1382.8</td>
</tr>
<tr>
<td>G_{08/10} (δ) Relative</td>
<td>0.0240</td>
<td>0.0523</td>
<td>0.0810</td>
<td>0.0994</td>
<td>0.1136</td>
</tr>
<tr>
<td>G_{08/10} (δ) Absolute</td>
<td>-41.90</td>
<td>432.2</td>
<td>635.8</td>
<td>716.0</td>
<td>767.6</td>
</tr>
</tbody>
</table>

Table C.7: Test of the hypothesis \( G_{00/02} (δ) - G_{08/10} (δ) > 0 \).

<table>
<thead>
<tr>
<th>δ</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative</td>
<td>0.0372***</td>
<td>0.0582***</td>
<td>0.0850***</td>
<td>0.1067***</td>
<td>0.1264***</td>
</tr>
<tr>
<td>Absolute</td>
<td>347.6***</td>
<td>459.4***</td>
<td>526.0***</td>
<td>570.7***</td>
<td>615.1***</td>
</tr>
</tbody>
</table>

Table C.8: Test of the hypothesis \( P_{00/02} (δ) > P_{08/10} (δ) \).

<table>
<thead>
<tr>
<th>δ</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative</td>
<td>TRUE*</td>
<td>TRUE**</td>
<td>TRUE***</td>
<td>TRUE***</td>
</tr>
<tr>
<td>Absolute</td>
<td>TRUE</td>
<td>TRUE</td>
<td>TRUE</td>
<td>TRUE*</td>
</tr>
</tbody>
</table>
C.4 Comparison: 2002-2004 versus 2008-2010

Figure C.3: panel (a) individual na-GICs and (b) cumulative individual na-GICs for Italy.


<table>
<thead>
<tr>
<th></th>
<th>δ 1</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{02/04}^{n} (\delta)$ Relative</td>
<td>0.0692</td>
<td>0.1268</td>
<td>0.1997</td>
<td>0.2533</td>
<td>0.2971</td>
</tr>
<tr>
<td></td>
<td>Absolute</td>
<td>378.4</td>
<td>1021.2</td>
<td>1382.8</td>
<td>1572.3</td>
</tr>
<tr>
<td>$G_{08/10}^{n} (\delta)$ Relative</td>
<td>0.0240</td>
<td>0.0523</td>
<td>0.0810</td>
<td>0.0994</td>
<td>0.1136</td>
</tr>
<tr>
<td></td>
<td>Absolute</td>
<td>-41.90</td>
<td>432.2</td>
<td>635.8</td>
<td>716.0</td>
</tr>
</tbody>
</table>

Note: authors’ calculations, based on the Italian “Survey on Household Income and Wealth”.

Table C.10: Test of the hypothesis $G_{02/04}^{n} (\delta) - G_{08/10}^{n} (\delta) > 0$.

<table>
<thead>
<tr>
<th></th>
<th>δ 1</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative</td>
<td>0.0486***</td>
<td>0.0713***</td>
<td>0.1023***</td>
<td>0.1266***</td>
<td>0.1475***</td>
</tr>
<tr>
<td>Absolute</td>
<td>572.8***</td>
<td>656.5***</td>
<td>705.9***</td>
<td>747.3***</td>
<td>783.1***</td>
</tr>
</tbody>
</table>

Note: authors’ calculations, based on the Italian “Survey on Household Income and Wealth”. *** (** *) [ (*)] means statistically significant at 99 (95) [90]%.

Table C.11: Test of the hypothesis $P_{02/04} (\delta) > P_{08/10} (\delta)$.

<table>
<thead>
<tr>
<th></th>
<th>δ 2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative</td>
<td>TRUE*</td>
<td>TRUE***</td>
<td>TRUE***</td>
<td>TRUE***</td>
</tr>
<tr>
<td>Absolute</td>
<td>TRUE*</td>
<td>TRUE**</td>
<td>TRUE***</td>
<td>TRUE***</td>
</tr>
</tbody>
</table>

Note: authors’ calculations, based on the Italian “Survey on Household Income and Wealth”. *** (** *) [ (*)] means statistically significant at 99 (95) [90]%.

Figure D.1: panel (a) individual na-GICs and (b) cumulative individual na-GICs for Italy.

Table D.1: History dependent growth indices 2002-04 and 2008-10.

<table>
<thead>
<tr>
<th></th>
<th>Relative</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{04/06}^\delta$</td>
<td></td>
<td>0.0675</td>
<td>0.1108</td>
<td>0.1577</td>
<td>0.1897</td>
<td>0.2153</td>
</tr>
<tr>
<td>$G_{08/10}^\delta$</td>
<td>Absolute</td>
<td>560.1</td>
<td>1088.2</td>
<td>1314.4</td>
<td>1436.7</td>
<td>1527.3</td>
</tr>
</tbody>
</table>

Table D.2: Test of the hypothesis $G_{04/06}^\delta > G_{08/10}^\delta$.

<table>
<thead>
<tr>
<th></th>
<th>Relative</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{04/06}^\delta$</td>
<td></td>
<td>0.0397***</td>
<td>0.0557***</td>
<td>0.0752***</td>
<td>0.0894***</td>
<td>0.1008***</td>
</tr>
<tr>
<td>$G_{08/10}^\delta$</td>
<td>Absolute</td>
<td>507.4***</td>
<td>601.9***</td>
<td>652.6***</td>
<td>702.6***</td>
<td>743.6***</td>
</tr>
</tbody>
</table>

Table D.3: Test of the hypothesis $P_{04/06}^\delta > P_{08/10}^\delta$.

<table>
<thead>
<tr>
<th></th>
<th>Relative</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{04/06}^\delta$</td>
<td>Absolute</td>
<td>TRUE</td>
<td>TRUE**</td>
<td>TRUE**</td>
<td>TRUE**</td>
</tr>
<tr>
<td>$P_{08/10}^\delta$</td>
<td></td>
<td>TRUE</td>
<td>TRUE</td>
<td>TRUE</td>
<td>TRUE</td>
</tr>
</tbody>
</table>

Note: authors’ calculations, based on the Italian “Survey on Household Income and Wealth”. *** (**) [*] means statistically significant at 99 (95) [90] %.