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Abstract

We axiomatically characterize two classes of poverty measures which are sensitive to inequality of opportunity - one a strict subset of the other. The proposed indices are sensitive not only to income shortfalls from the poverty line, but also to differences in opportunities faced by people with different pre-determined characteristics, such as race or family background. Dominance conditions are established for each class of measures, and a sub-family of scalar indices, based on a rank-dependent aggregation of type-specific poverty levels, is also introduced. Using household survey data from eighteen European countries in 2005, we find substantial differences in country rankings based on standard FGT indices and on the new opportunity-sensitive indices. Cross-country differences in opportunity-sensitive poverty are decomposed into a level effect; a distribution effect; and a population composition effect.

Keywords: Inequality of opportunity, poverty measures, poverty comparisons.

JEL Classification: D31, D63, J62.

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1 Introduction

Two foundational contributions to the modern literature on poverty measurement, Sen (1976) and Foster et al. (1984), were motivated at least in part by a concern to introduce poverty indices that were sensitive to inequality. Sen's paper begins by noting that the headcount measure is "...completely insensitive to the distribution of income among the poor" (p.219). Foster et al. introduce a parametric class of measures which, crucially, "satisfy the transfer sensitivity axiom" (p.763).

But whereas sensitivity to inequality of *outcomes* (e.g. income or consumption expenditures) has been widely incorporated as a desirable property for poverty indices, we are aware of no poverty measures that are sensitive to inequality of *opportunity*. This stands in contrast to the influence that the theory of inequality of opportunity, and in particular its formalization by John Roemer (1998), has had on the measurement of inequality more generally.

Since Rawls's (1971) *A Theory of Justice*, a number of moral philosophers and social choice theorists have argued that not all income differences are identical from a normative or ethical perspective. In particular, income differences due to personal choices, or otherwise attributable to personal responsibility, may be judged to be less morally objectionable than differences due to pre-determined attributes - such as gender, race or family background - over which individuals have no control. Based on increasingly sophisticated variants of this basic distinction, a number of authors have argued that egalitarian justice ought to focus less on the space of outcomes, and more on the space of opportunities. See, for instance, Dworkin (1981a, b), Arneson (1989), Cohen (1989), Roemer (1998), Fleurbaey (2008), and Fleurbaey and Maniquet (2011).

Building on that theoretical literature, other authors have suggested ways in which inequality due to pre-determined circumstances can be appropriately quantified, for the purpose of measuring inequality of opportunity in real labor-force or household survey data. See, for example, Bourguignon et al. (2007), Lefranc et al. (2009), Checchi and Peragine (2010) and Ferreira and Gignoux (2011).

Given the close proximity between the literatures on inequality and poverty measurement, it is surprising that the influence of opportunity egalitarianism on inequality measurement has not, to our knowledge, extended to the measurement of poverty. After all, if our assessment of poverty is informed by the extent of inequality in society, and if our judgment of inequality is in turn affected by whether income differences arise from personal responsibility or pre-determined circumstances, then why should our judgment of poverty not be similarly affected by that distinction?

This paper asks whether poverty measures that are sensitive to inequality of opportunity are conceptually meaningful, and empirically relevant.[1] Conceptually, sensitivity (or aversion) to inequality of opportunity implies two fundamental changes to the standard axiomatic approach. First, partitioning the population into types (circumstance-homogeneous groups) can lead to violations of the gen-

[1] The paper is concerned exclusively with uni-dimensional poverty. An analysis of opportunity-sensitive measures of multidimensional poverty is left for future work.

eral anonymity axiom. Second, aversion to inequality between types, but not within them, raises problems for the transfer axiom: transfers within and across types can be valued differently, and a potential conflict arises between outcome-inequality aversion and inequality-of-opportunity aversion.

We propose logically consistent solutions to these challenges by introducing relatively modest modifications to the standard set of axioms in poverty measurement. We show that inequality-of-opportunity aversion and outcome-inequality aversion need not be inconsistent; the tension between them can be resolved, as long as one is given priority over the other. Our axioms allow us to characterize two classes of opportunity-sensitive poverty measures (OSPM) - one a strict subset of the other. Dominance conditions are established for each class. The first condition, based on the the broad class of OSPM, turns out to be a re-interpretation of an existing result in the literature on poverty dominance for heterogeneous populations (Atkinson and Bourguignon 1987, Jenkins and Lambert 1993). This condition involves a type-sequential comparison of the cumulative distribution functions truncated at the poverty line. The second condition, for the narrower class of OSPM, is a new sequential dominance procedure, which involves comparisons of the group speficic headcount ratios and of average incomes among the poor in each type. Moving from partial to complete orderings, we also propose a specific parametric family of OSPM which combines elements from the Sen and Foster et al. frameworks. We call it the opportunity-sensitive Foster, Greer and Thorbecke index, or OS-FGT.

We then apply both the partial-ordering analysis and the OS-FGT index to eighteen countries in the European Union, using the EU-SILC 2005 data. When aversion to inequality of opportunity is incorporated into the assessment of poverty, we are able to uncover aspects not captured when using standard income-poverty methods. The broad OSPM dominance condition yields a large number of unambiguous rankings: 128 of 153 possible pairwise comparisons. The more demanding sufficient conditions we establish for dominance in the narrow OSPM class are satisfied much more seldom. In addition, income poverty rankings and OS-FGT rankings, although positively correlated, frequently differ: whereas the Netherlands has less income poverty than Germany, for example, the reverse is true for opportunity-sensitive poverty. There are a number of such re-rankings, and we explore the extent to which they are caused by each of three key factors: country differences in overall income poverty; differences in population shares across types; and differences in type-specific income distributions among the poor.

The paper proceeds as follows: Section 2 describes the basic set up and notation. Section 3 lists the axioms that define the two classes of opportunity-sensitive poverty measures: the broad OSPM class, and the narrow OSPM class. Section 4 establishes dominance conditions for both classes. Section 5 defines the OS-FGT family of indices, and discusses some of its properties. Section 6 describes the empirical application to eighteen European countries. Section 7 concludes.

2 The basic set up

Society consists of a large number of single-individual households. Each individual h is completely described by a list of characteristics, which can be divided into two different classes: traits that lie beyond the individual's responsibility, which are denoted by a vector of circumstances $\mathbf{c}$, belonging to a finite set $\Omega = \{\mathbf{c}_1, ..., \mathbf{c}_n\}$; and factors for which the individual is responsible, which can be summarized by a scalar variable denoted effort, $e \in \mathbb{R}_+$. There is no luck, nor random components in our model. The advantage variable of interest - which we will refer to as "income" for short - is solely determined by circumstances and efforts, thus generated by a function $g : \Omega \times \mathbb{R}_+ \to \mathbb{R}_+$.

We partition the population into n types, where a *type* $\mathbf{T_i} \in \mathcal{T}$ is the set of individuals whose circumstance vector is $\mathbf{c}_i$. The set of types, $\mathcal{T}$, is an exhaustive partition of the entire population. $\mathcal{T} \in \Im$, where $\Im$ is the set of admissible partitions.

The (equivalized) income of person h in type i is denoted $x_h, h \in \mathbf{T_i}$, and given by $x_h = g(\mathbf{c}_i, e_h)$ .Income is distributed according to the type-specific income distribution $F_i(x)$, with density function $f_i(x)$.[2] The maximum income in the population and in type i are denoted byd x^{max} and x_i^{max} respectively, and the population share of type i is denoted q_i^F. $F(x) = \sum_{i=1}^{n} q_i^F F_i(x)$ is the overall income distribution of the society, defined as the component-mix distribution of all the type distributions. $F(x) \in D$, where D is the set of admissible distributions.

The support of each type-specific income distribution represents the set of outcomes which can be achieved - through the exertion of different degrees of effort - by individuals with the exact same vector of circumstances, $\mathbf{c}_i$. That is to say, the support of the type distribution is an ex-ante representation (that is before effort levels are realized) of the *opportunity set* open to any individual endowed with circumstances $\mathbf{c}_i$. Furthermore, one could interpret the frequency distribution $f_i(x)$ as an ex- ante indication of the probability attached to each outcome. On this basis, a number of authors have used moments, or other attributes, of the type-specific distribution function $F_i(x)$ to evaluate the type's opportunity sets. See, for example, van de Gaer (1993), Bossert (1997) and Ooghe et al (2007). Most commonly, the mean income of type i has been used to value opportunity sets, and therefore to rank types.

For our purposes, it is not essential that types be ranked by their mean incomes. Indeed, because we are concerned with poverty, rather than general welfare, our application will use a ranking based on the type-specific poverty rates instead. But while we are agnostic about the exact criterion used to rank types, our approach does require that *some* ranking is agreed on, so that all types can be ordered in terms of the value of their opportunity sets. We assume, therefore, that at any particular point in time there exists a generally agreed ordering $\succsim$ on the set of types $\mathcal{T}$ so that $\mathbf{T}_{i+1} \succsim \mathbf{T}_i$ for $i \in \{1, ..., n-1\}$. The ordering implies that

[2]Although society consists of discrete individual households, there is a very large number of them. For simplicity, we use continuous notation for income distributions, without loss of generality.

individuals in $\mathbf{T}_i$ face a "worse" opportunity set than those in $\mathbf{T}_{i+1}$.[3]

Identification of the poor is not the primary object of this exercise. In line with standard practice in uni-dimensional poverty measurement, we denote a poverty line $z \in [0, x^{\max}]$ and define the set of poor individuals in each type as $\mathbf{T}_i^z := \{h \in \mathbf{T}_i | x_h \leq z\}$. The poverty line is type-invariant: any differences in household needs across types should be fully accounted for by the equivalence scale used to construct the income variable x. We denote by μ_i^F the mean income of type i, and by $\mu(F_i^z)$ the mean income of the poor in type i in distribution F.

Our aim is to propose a poverty measure that is based on the poverty condition of all individuals in the society, while being sensitive to the type to which each person belongs. In the next section we introduce a set of axioms that are used to define two classes of opportunity-sensitive poverty measures. As noted earlier, the narrow OSPM class is a strict subset of the broad OSPM class.

3 Two classes of opportunity-sensitive poverty measures

Our objective in this section is to define a poverty measure $P : D \times \mathbb{R}_+ \times \Im \rightarrow \mathbb{R}_+$, of the form $P(F(x), z, \mathcal{T})$, which is a meaningful opportunity-sensitive poverty measure. To do so, we present a set of desirable axioms to be imposed on the function, including some that are standard in the literature; a new axiom that introduces the property of inequality of opportunity-aversion; and some suitable modifications of other properties. We then use the axioms to define two classes of opportunity-sensitive poverty measures.

The first two axioms are standard in the poverty measurement literature. If an individual sees her income increase, *ceteris paribus*, overall poverty cannot increase.

A1 Monotonicity (MON): *For all $F \in D$, for all $i \in \{1, ..., n\}$, P is non-increasing in x :*

$\frac{\partial P}{\partial x} \leq 0, \forall i \in \{1, ..., n\}, \forall x \in [0, x^{\max}]$

In addition, the level of poverty in a society is independent of the incomes of the non poor (Sen, 1976):

A2 Focus (FOC): *For all $F, G \in D$, for all $i \in \{1, ..., n\}$,*

$$P(F(x), z, \mathcal{T}) = P(G(x), z, \mathcal{T}) \qquad \text{if } F_i(x) = G_i(x), \ \ \forall x \in [0, z], \forall i \in \{1, ..., n\}$$

The next property allow us to express aggregate poverty as the sum of individual poverty functions.

[3] Whereas the ranking *criterion* is in principle permanent, types may of course be re-ranked over time as their values of the ranking variable change. If, for example, $\mathbf{T}_{i+1} \succsim \mathbf{T}_i \Leftrightarrow F_i(z) \geq F_{i+1}(z)$, then any change in the headcount ranking of types over time would be reflected in their ranking by $\succsim$.

A3 Additivity (ADD). *There exist functions* $p_i : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$, *for all types* $i \in \{1, ..., n\}$, *assumed to be twice differentiable (almost everywhere) in* x, *such that*

$$P\left(F(x), z, \mathcal{T}\right) = \sum_{i=1}^{n} q_i^F \int_0^{x_i^{\max}} p_i(x, z) f_i\left(x\right) dx \qquad \textit{for all } F \in D.$$

Here $p_i(x, z)$ is the individual poverty function for a person earning income x in type i. Next, we introduce a suitably modified version of the symmetry, or anonymity, axiom:

A4 Anonymity within types (ANON): *For all* $F, G \in D$, $P\left(F(x), z, \mathcal{T}\right) = P\left(G(x), z, \mathcal{T}\right)$ *whenever, for any* $i \in \{1, ..., n\}$, $G\left(x\right)$ *is obtained from* $F\left(x\right)$ *by permuting incomes* x_h *and* x_k, *where* $h, k \in T_i^z$.

ANON is a partial symmetry axiom (see Cowell, 1980). Anonymity within types implies that, within types, all that matters for the evaluation of poverty is the income accruing to each individual. In other words, the identity of the individuals does not matter within each type.

The next two axioms are intended to incorporate specific versions of the two fundamental principles of the theory of equality of opportunity - compensation and reward - into the poverty measure. In the ex-ante approach, inequality of opportunity is captured by inequality in the value of the opportunity sets people face. If, as discussed above, all individuals in type $\mathbf{T}_i$ share the same opportunity set, and hence the same value of the opportunity set, then there is no inequality of opportunity within types. All inequality of opportunity is between types, and the Compensation Principle requires that those with "worse" circumstances be compensated for them. Since, as discussed earlier, types are ranked by $\succsim$ from "worst" $(i = 1)$ to "best" $(i = n)$, the principle of compensation can be incorporated by the following axiom:

A5 Inequality of opportunity aversion (IOA):

$$\frac{\partial P}{\partial x_h} \left| h \in \mathbf{T}_i^z \leq \frac{\partial P}{\partial x_k} \right| k \in \mathbf{T}_{i+1}^z, \forall i \in \{1, ..., n-1\}, \forall h \in \mathbf{T}_i^z, \forall k \in \mathbf{T}_{i+1}^z$$

IOA requires that an opportunity-sensitive poverty measure fall by more if an extra income unit is given to a poor individual in a "worse" type, than to a poor individual in a "better" type. (And, given FOC, *any* individual in a "better" type.)

IOA is analogous to a transfer principle between types, but it is deliberately silent on transfers within types. How should an OSPM measure respond to changes in inequality within types? One (strict) view is that all inequality within types is due to efforts, and hence of no ethical concern. The following axiom imposes a requirement of no aversion to inequality within types.

A6 Inequality neutrality within types (INW)

$$\frac{\partial^2 P}{\partial x_h^2 \, | h \in \mathbf{T}_i^z} = 0, \forall i \in \{1, ..., n\}, \forall h \in \mathbf{T}_i^z$$

This axiom expresses the Natural Reward Principle - specifically its utilitarian version - whereby income inequalities within types are considered equitable and need not be compensated, because they are the result of differences in effort exerted (Peragine, 2004; Fleurbaey, 2008).

Alternatively, one might wish to allow for a certain degree of aversion to inequality within types. This might be justified if a society's ethical attitudes to inequality combine inequality aversion in the space of opportunities with some residual aversion to outcome inequality, whatever its source. Or it might be motivated by practical considerations: in any empirical application, the full set of circumstances $\mathbf{c}_i$ is unlikely to ever be observed, so that some of the within-type inequality actually reflects differences driven by unobserved circumstances, as well as those caused by differences in relative effort. (See Ferreira and Gignoux, 2011).

In this spirit, the next axiom requires that a progressive Pigou-Dalton transfer within a given type, all else constant, does not increase poverty.

A7 Inequality aversion within types (IAW)

$$\frac{\partial^2 P}{\partial x_h^2 \, | h \in \mathbf{T}_i^z} \geq 0, \forall i \in \{1, ..., n\} \forall h \in \mathbf{T}_i^z$$

INW is clearly a particular case of IAW, so that two classes of *opportunity-sensitive poverty measures* can be defined. The first is the the narrow OSPM class, the members of which satisfy A1-A6. It is denoted by:

$$\mathbf{P}^N := \{P : D \times \mathbb{R}_+ \times \Im \rightarrow \mathbb{R}_+ \mid P \textit{ satisfies MON, FOCUS, ADD, ANON, IOA, INW } \}$$

The second is the broad OSPM class, whose members satisfy A1-A5 and A7. It is denoted by:

$$\mathbf{P}^B := \{P : D \times \mathbb{R}_+ \times \Im \rightarrow \mathbb{R}_+ \mid P \textit{ satisfies MON, FOCUS, ADD, ANON, IOA, IAW } \}$$

Clearly, $\mathbf{P}^N \subset \mathbf{P}^B$.The combination of these axioms lead immediately to two remarks about these classes of measures:

Remark 1 $P \in \mathbf{P}^N$ *if and only if, for all* $F \in D$, $P\left(F(x), z\right) = \sum_{i=1}^{n} q_i^F \int p_i(x) f_i\left(x\right) dx$, *where the functions* $p_i : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$ *satisfy the following conditions:*

P1. $p_i\left(x, z\right) = 0$, $\forall x > z$ and $\forall i \in \{1, ..., n\}$

P2. $p_i\left(x, z\right) \geq 0$, $\forall x \leq z$ and $\forall i \in \{1, ..., n\}$

P3. $\frac{\partial p_i(x,z)}{\partial x} \leq \frac{\partial p_{i+1}(x,z)}{\partial x} \leq 0 \; \forall x \in [0, x^{\max}], \forall i \in \{1, ..., n-1\}$

P4. $\frac{\partial^2 p_i(x,z)}{\partial x^2} = 0, \; \forall x \in [0, x^{\max}], \forall i \in \{1, ..., n\}$

Remark 2 $P \in \mathbf{P}^B$ *if and only if, for all* $F \in D$, $P(F(x), z) = \sum_{i=1}^{n} q_i^F \int p_i(x) f_i(x)\, dx$, *where the functions* $p_i : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$ *satisfy conditions P1 - P3 and P4′:*

p4′. $\frac{\partial^2 p_i(x,z)}{\partial x^2} \geq 0, \; \forall x \in [0, x^{\max}], \forall i \in \{1, ..., n\}$

4 Opportunity-sensitive poverty dominance

In this section, we identify the poverty dominance conditions corresponding to each of the two classes. The poverty dominance condition corresponding to the broad OSPM class, $\mathbf{P}^B$, has already been identified in the literature, although it has a different interpretation in the current context. It is given by Theorem 1.

Theorem 1 *(Jenkins and Lambert (1993), Chambaz and Maurin (1998)) For all distributions* $F(x), G(x) \in D$ *and a poverty line* z, $P(F(x), z) \leq P(G(x), z)) \quad \forall P \in P^B$ *if and only if the following condition is satisfied:*

$$\sum_{i=1}^{j} q_i^G G_i(x) \geq \sum_{i=1}^{j} q_i^F F_i(x), \quad \forall x \leq z, \quad \textit{and} \quad \forall j \in \{1, ..., n\} \tag{1}$$

Proof *See Chambaz and Maurin (1998).*

This dominance condition can be rearranged in order to show how differences in poverty can be decomposed into differences in the income distributions and differences in the population compositions:

$$\sum_{i=1}^{j} q_i^G (G_i(x) - F_i(x)) + \sum_{i=1}^{j} (q_i^G - q_i^F) F_i(x) \geq 0 \tag{2}$$

Theorem 1 establishes a partial ordering in D: when equation 1 is satisfied, and only then, we can say that opportunity-sensitive poverty is lower in distribution F than in G for any poverty measure in the broad class P^B. Since P^N is a strict subset of P^B, this obviously also holds for the OSPM subclass with inequality neutrality within types, which complies with the utilitarian Natural Reward Principle.

Testing the above dominance condition requires comparisons of cumulative distribution functions. For the narrow class of OSPM it is also possible to obtain sufficient dominance conditions that make use of type-specific income means (among the poor), poverty headcounts and population proportions, as established in Theorem 2 below. The dominance conditions in Theorem 2 are less informationally demanding, and they render the exercise of checking the conditions computationally simpler, albeit at the cost of arriving only at sufficient (but not necessary) conditions.

Theorem 2 *For all distributions F(x), G(x)* $\in D$ *and a poverty line* z, $P\left(F(x),z\right) \leq P\left(G(x),z\right)$ $\forall P \in \mathbf{P}^N$ if the following conditions are satisfied:

(*i*) $\sum_{i=1}^{j} q_i^G G_i\left(z\right) \geq \sum_{i=1}^{j} q_i^F F_i\left(z\right), \quad \forall j \in \{1,...,n\};$

(*ii*) $\sum_{i=1}^{j} q_i^F \mu\left(F_i^Z\right) \geq \sum_{i=1}^{j} q_i^G \mu(G_i^z), \quad \forall j \in \{1,...,n\}.$

Proof. *See appendix.*

Theorem 2 states that a sufficient condition for declaring poverty in distribution F lower than poverty in distribution G according to any index in the family P^N, is that the following sequential conditions are satisfied: (i) the weighted proportion of poor (the headcount) is lower in F than in G at each step of the sequential procedure; (ii)the weighted average income among the poor is higher in F than in G, at each step of the sequential procedure.

The sequential procedure in Theorem 2 - starting from the lowest type, adding the second and so on - incorporates our concern for equality of opportunity. The fact that these conditions are not necessary implies that a difference in means can be counterbalanced by a difference in the proportion of poor individuals (given a poverty line). This is crucial in understanding the empirical results presented below: although the conditions in Theorem 2 are less *informationally* demanding, they are nevertheless quite strong. The (sequential) requirement that both population weighted headcounts be lower *and* population-weighted average incomes be higher is likely to make the conditions for Theorem 2 difficult to observe in practice

Note that the distributional conditions characterized respectively in Theorems 1 and 2 are independent. In fact, while the condition in Theorem 1 (Equation 1) clearly implies condition (i) in Theorem 2, condition (ii) in Theorem 2 is *not* implied by (and does not imply) Equation 1.[4] The only case in which Equation 1 implies both conditions (i) and (ii) of Theorem 2 is the case of equal type partitions; that is when $q_i^F = q_i^G$ for all i.

In Section 6 we apply these two sets of dominance conditions to eighteen EU countries. We find that Theorem 1 allows us to rank distributions in 128 of the 153 possible pairwise comparisons, while Theorem 2 yields only 21 instances of dominance. But before turning to the empirical application, we now turn to complete orderings in D, by defining a specific family of scalar OSPM indices, within $\mathbf{P}^B$.

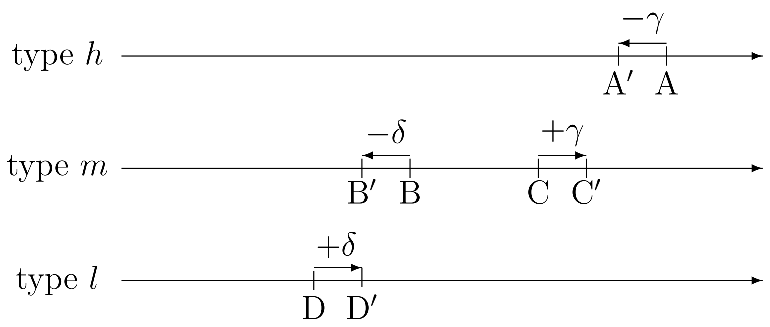
5 From dominance to scalar measures

In this section we focus on a specific family of opportunity-sensitive poverty indices, within the broad class $\mathbf{P}^B$ characterized above. First we note that, in general, there

[4]To see this, consider a case in which the condition of theorem 1 is satisfied: that is, $\sum_{i=1}^{j} q_i^G G_i(x) \geq \sum_{i=1}^{j} q_i^F F_i(x)$, for all $x \leq z$, and for all $j \in 1,...,n$. Now focus on the first step of the sequential procedure (that is $j=1$) : $q_1^G G_1(x) \geq q_1^F F_1(x)$. Consider what happens when $q_1^F \searrow 0$ and $q_1^G \nearrow 1$. In this case condition (i) of theorem 2 is satisfied; on the other hand, looking at condition (ii), we have that: $q_1^F \mu\left(F_1^Z\right) \searrow 0$ and $q_1^G \mu\left(G_1^Z\right) \nearrow \mu\left(G_1^Z\right)$. That is, the condition (ii) of theorem 2 is not satisfied.

is an obvious tension between axioms IOA and IAW. To see this, consider a simple example, with four individuals in three types. Individual A is in the highest type (h), individuals B and C are in the intermediate type (m), while individual D is in the lowest type (l). Suppose B is poorer than C, and imagine positive transfers from A to C and B to D. Both transfers are progressive across types, but they result in an increase in inequality within type m (see Figure 1). This is simply a manifestation of the potential tension between aversion to inequality of opportunity and aversion to inequality of outcomes. If this conflict were irredeemable, the class $\mathbf{P}^B$ would collapse to its narrow subset $\mathbf{P}^N$, where the tension is eliminated by complete neutrality with respect to within-type inequality.

Figure 1: Progressive transfers across types: Conflict between IOA and IAW



That the complement of $\mathbf{P}^N$ in $\mathbf{P}^B$ is not an empty set can be demonstrated by considering a specific example of individual poverty measures. For any $w_i \in \mathbb{R}_+$, $i \in \{1, ..., n\}$, let $p_i(x, z) = w_i p(x, z)$, such that $w_i - w_{i+1} > p(x, z), \forall x$. Then,

$$P\left(F(x), z, \mathcal{T}\right) = \sum_{i=1}^{n} q_i^F \int_0^z w_i p(x, z) f_i\left(x\right) dx,$$

where $p(x, z)$ is weakly convex in x, will always satisfy both IOA and IAW.

A special member of this family of indices is obtained by using the FGT formulation for $p(x) = \left(\frac{z-x}{z}\right)^{\alpha}$, and selecting inverse ranks as type weights: $w_i = \frac{n+1-r(i)}{n}$, where $r(i)$ is an increasing function of the rank of each type, $r : \{1, ..., n\} \to \mathbb{R}_+$, based on the agreed ordering $\mathbf{T}_{i+1} \succsim \mathbf{T}_i$ for $i \in \{1, ..., n-1\}$, introduced in section 2. In particular, let $r(i) = i$, for all $i \in \{1, ..., n\}$ whenever $\mathbf{T}_{i+1} \succ \mathbf{T}_i$, $\exists$ $i \in \{1, ..., n-1\}$, and $r\left(i\right) = 1$, for all $i \in \{1, ..., n\}$ whenever $\mathbf{T}_{i+1} \sim \mathbf{T}_i$ for all $i \in \{1, ..., n-1\}$. In that case, $nw_i = n + 1 - i$, unless the ranking collapses to indifference across all types (in which case all weights collapse to 1).

With that weighting scheme, we obtain the family of *opportunity-sensitive FGT (OS-FGT) measure* $\mathbf{P}_{FGT} \subset \mathbf{P}^B$:

$$P_{FGT}\left(F, z, \mathcal{T}\right) = \frac{1}{n} \sum_{i=1}^{n} q_i^F \left(n + 1 - r(i)\right) \int_0^z \left(\frac{z-x}{z}\right)^{\alpha} f_i\left(x\right) dx \qquad (3)$$

where the parameter α expresses a concern for within-type inequality among the poor, and $\alpha \geq 0$.

$\mathbf{P}_{FGT}$ has a number of interesting properties:

$\odot$ $\mathbf{P}_{FGT}$ satisfies both IOA and IAW. This can be seen by noting that, $\forall i \in \{1, ..., n-1\}, \forall h \in \mathbf{T}_i^z, \forall k \in \mathbf{T}_{i+1}^z$,

$$\frac{\partial P}{\partial x_h} \left| h \in \mathbf{T}_i^z \leq \frac{\partial P}{\partial x_k} \right| k \in \mathbf{T}_{i+1}^z$$

$$-\frac{1}{n}\frac{\alpha}{z} q_i^F (n+1-i) \left(\frac{z-x_h}{z}\right)^{\alpha-1} f_i(x) \leq$$

$$-\frac{1}{n}\frac{\alpha}{z} q_{i+1}^F (n-i) \left(\frac{z-x_k}{z}\right)^{\alpha-1} f_{i+1}(x)$$

In words, a marginal income transfer from any poor individual h in type i to any poor individual k of a higher type increases (or at least cannot reduce) poverty. The poverty-increasing effect of the outward transfer from type i is larger in absolute value than the poverty-decreasing effect of the inward transfer of the same amount to any higher type, $i+1$. This is a consequence of the fact that rank differences are integers, while $q^F \leq 1, \frac{\alpha}{z}\left(\frac{z-x}{z}\right)^{\alpha-1} \leq 1$, $\alpha, z \geq 0$.[5]

$\odot$ $\mathbf{P}_{FGT} \in [0, 1)$.

The minimum level is achieved when no individual in the society is considered poor, that is, no one falls below the poverty line z. The maximum possible value of any poverty measure in $\mathbf{P}_{FGT}$ is achieved when all individuals in the society have zero income and $r(i) = 1, \forall i$. This would occur if, for instance, ranks were defined according to the poverty headcount, or type mean incomes, and were equal in case of ties.

$\odot$ When there is perfect equality of opportunity, the poverty status is independent of the group to which the person belongs. Perfect equality of opportunity attains when $F_i(x) = F(x), \forall i$. In this case, $\mathbf{P}_{FGT}$ is given by

$$P_{FGT}(F(x), z, \mathcal{T}) = \frac{Q}{n}\left(n+1-\sum_{i=1}^{n} q_i^F r(i)\right),$$

where $Q = \int_0^z \left(\frac{z-x}{z}\right)^{\alpha} dF(x)$ is the poverty rate common to all types. Since $\mathbf{T}_{i+1} \sim \mathbf{T}_i$ for all $i \in \{1, ..., n-1\}$, $r(i) = 1, \forall i$, and $P_{FGT}(F(x), z, \mathcal{T}) = Q$.

[5] There is also a potential clash between IOA and the standard inequality aversion for the full distribution. It is perfectly possible that, in the above example, $\left(x^h \left| h \in \mathbf{T}_i^z\right.\right) > \left(x_k \left| k \in \mathbf{T}_{i+1}^z\right.\right)$. There exists a non empty set of possible transfers which are both progressive in the standard Pigou-Dalton sense, and opportunity-regressive in the sense of IOA. The maximum reconciliation between the Pigou-Dalton transfer axiom (with no regard to types) and IOA is given by the within-type inequality aversion (IAW) axiom defined above.

The opportunity-sensitive rank-dependent FGT measure collapses to the standard FGT poverty measure.

⊙ If one is strictly neutral with respect to inequality within types, and seeks indices that belong to the narrow OSPM class $\mathbf{P}^N$, then α can be set equal 1 or 0.

The *opportunity-sensitive poverty gap measure* $P_G(F(x,z,\mathcal{T})) \in \mathbf{P}^N$ sets $\alpha = 1$, so that $p(x,z) = \frac{z-x}{z}$ for all $x < z$. Hence,

$$P_G\left(F(x), z, \mathcal{T}\right) = \frac{1}{n}\sum_{i=1}^{n} q_i^F\left(n+1-r(i)\right)\int_0^z \frac{z-x}{z} f_i\left(x\right)dx \tag{4}$$

When α equals zero, $p(x,z) = 1$ for all $x < z$, and we obtain the *opportunity-sensitive poverty headcount measure* defined as

$$P_H\left(F, z, \mathcal{T}\right) = \frac{1}{n}\sum_{i=1}^{n} q_i^F(n+1-r(i))\int_0^z f_i\left(x\right)dx = \frac{1}{n}\sum_{i=1}^{n} q_i^F(n+1-r(i))F_i\left(z\right) \tag{5}$$

Following Roemer (1998), $F_i(z)$ can be interpreted as the degree of effort - expressed in percentile terms - necessary to escape poverty for a person endowed with circumstances $\mathbf{c}_i$. An extreme and interesting case arises when $x_i^{max} < z$: in this case type i is said to be 'poverty-trapped' in the sense that, given circumstances c_i that define $\mathbf{T_i}$, no amount of effort is sufficient to escape poverty.

6 Poverty, inequality and opportunities in the European Union

Introducing a concern for inequality of opportunity into poverty measurement acquires practical importance if the assessment of the evolution of poverty in a country - or poverty comparisons across different regions or countries - differ when the concern is incorporated. This section provides an empirical application of the approach proposed in the paper. Using data for eighteen European Union countries, we show that when aversion to inequality of opportunity is explicitly incorporated into the measurement of poverty we are able to uncover some aspects of poverty not captured when using traditional poverty measures.

We use data from the EU-SILC 2005 round (User Database), which includes a special module on intergenerational transmission of poverty that contains information on socioeconomic background characteristics that can be used to define circumstances.[6] Poverty is assessed on the basis of equivalent disposable household income, expressed in Euros at PPP exchange rates and in 2004 prices (Eurostat, 2008).[7] We restrict the sample to household heads - defined as the individual

[6]The EU-SILC project aims at obtaining internationally comparable information across EU countries. For more information see `http://epp.eurostat.ec.europa.eu/portal/page/portal/microdata/eu_silc`

[7]Equivalent disposable household income is computed as the disposable household income divided by the square root of the household size

with the highest gross earning in the family - and their spouses, with non-negative disposable household incomes.

We use three circumstance variables to define types: gender, parental education and parental occupation when the respondent was between 12 and 16 years old. Parental education is defined as the highest level attained by either of the two parents (when both are available), and is categorized in two groups: *higher* when at least one parent completed upper secondary, and lower otherwise.[8] Parental occupation status is based on the highest ISCO 88 occupation status of the parents, grouped into four categories: highly skilled non-manual (ISCO codes 11-34), lower-skilled non-manual (41-52), skilled manual (61-83), and elementary occupation (91-93). The total population is thus partitioned into 16 types.

From the original sample of 26 countries we exclude Denmark, Norway, Sweden, and the UK due to the high proportion of missing values in parental occupation and parental education variables in those countries (see Nolan, 2012, for a discussion). Iceland, Ireland, Portugal, and Slovenia are also excluded because the sample frequency of some types is too low (fewer than 10 individuals) to compute credible type-mean disposable income and type-mean poverty rates. Therefore, the subsample used in the following exercise includes 18 countries: Austria (AT), Belgium (BE), Cyprus (CY), Czech Republic (CZ), Germany (DE), Estonia (EE), Spain (ES), Finland (FI), France (FR), Greece (GR), Hungary (HU), Italy (IT), Lithuania (LT), Luxembourg (LU), Latvia (LV) Netherlands (NL), Poland (PL), and Slovakia (SK). The resulting sample thus includes a mix of Eastern European, Mediterranean, and North-Western European countries, with substantial variation in income levels and poverty.

As noted in Section 2, our approach requires an agreed ordering $\succsim$ on the set of types $\mathcal{T}$. In the inequality of opportunity literature, types are often ranked according to some evaluation of the opportunity set of individuals belonging to the type. A common criterion is to order types by their mean advantage (in this case, income) level. Since the main focus of this paper is on poverty rather than welfare, we choose to rank types according to the type-specific poverty headcount, $p_i(x) = \int_0^z f_i(x)\,dx = F_i(z)$. Alternative criteria could obviously be applied such as, for example, the type-specific average income among the poor.

6.1 Partial rankings: Testing dominance conditions

Once a criterion for ranking types within countries has been agreed, we can test for poverty dominance across all possible pairs of countries, using both Theorems 1 and 2. We begin with Theorem 1, which is more computationally intensive, but which yields both necessary and sufficient conditions, and refers to the broader OSPM class. Testing for the condition in Equation 1 requires defining a single poverty line, up to which the dominance condition is checked. We follow Decancq et al. (2013) and define an Europe-wide poverty threshold as 60% of the median equivalent household income of the European distribution (of countries included in

[8]Educational levels are defined according to the International Standard Classification of Education (ISCED).

our sample). Individuals are considered poor if their annual equivalent disposable income is below 9,275 Euro at PPP exchange rates, in 2004 prices. This threshold is used for testing the conditions in both Theorems.

Table 1 presents the results from testing the sequential dominance condition in Theorem 1, equation 1. In this table, opportunity-sensitive poverty dominance (OSPM dominance) of country F (G) over country G (F) is indicated by a "<" (">") sign in cell (F, G). Whenever dominance of G over F exists, opportunity-sensitive poverty is *higher* in country F than in country G for *any* poverty measure in the class P^B.

Of the 153 possible pairwise comparisons in our 18-country sample, we find OSPM dominance in 128 pairs (though fewer are significant at 10, 5, and 1% level). It is possible to rank some countries unambiguously against most other countries in the sample. For instance, Latvia is opportunity-sensitive poorer that all other countries, except Lithuania. Lithuania is OS poorer than all other countries, except Estonia, Spain, Latvia and Greece. At the other extreme, Luxembourg is found to be *less* OS poor than all but two countries: Cyprus and France. It is possible to rank any country in our sample against at least half of the other countries. At a more general level, this exercise suggests a pattern among three traditional European social models: with the exception of the Czech Republic, Eastern European countries are all opportunity-sensitive poorer than Mediterranean countries which, in turn, are poorer than North-Western European countries. But within these three groups of countries, dominance is harder to find.

Table 1: Dominance conditions associated with Theorem 1

	AT	BE	CY	CZ	DE	EE	ES	FI	FR	GR	HU	IT	LT	LU	LV	NL	PL	SK
AT	.	$>$																
BE	$<$	.																
CY			.															
CZ	$>$	$>$	$>$	.														
DE	$<$			$<$	.													
EE	$>^{***}$	$>^{***}$	$>^{***}$	$>^{***}$	$>^{***}$	.												
ES	$>$	$>^{**}$	$>^{***}$	$>$	$>$	$<^{***}$	.											
FI			$>$	$<$		$<^{***}$	$<^{***}$	.										
FR		$<$	$<$	$<$	$<$	$<^{***}$	$<^{***}$		.									
GR	$>^{***}$	$>^{***}$	$>^{***}$	$>$	$>^{***}$	$<$	$>$	$>^{***}$	$>^{***}$	.								
HU	$>$	$>$	$>^{***}$	$>^{**}$	$>$	$<$	$>$	$>^{*}$	$>^{***}$	$<$	.							
IT	$>$	$>$	$>^{***}$	$>$	$>$	$<^{***}$		$>^{***}$	$>^{*}$	$<$		.						
LT	$>$	$>^{***}$	$>^{***}$	$>^{*}$	$>^{***}$			$>^{***}$	$>^{***}$		$>$	$>^{**}$	.					
LU	$<$	$<$	$<$	$<$	$<$	$<^{***}$	$<^{***}$	$<$	$<$	$<^{***}$	$<^{*}$	$<^{***}$	$<^{***}$	.				
LV	$>^{***}$	$>^{***}$	$>^{***}$	$>^{***}$	$>^{***}$	$>$	$>^{***}$	$>^{***}$	$>^{***}$	$>$	$>$	$>^{***}$		$>^{***}$	.			
NL				$<$		$<^{***}$	$<^{***}$	$<$	$<$	$<^{***}$	$<^{**}$	$<^{***}$	$<^{***}$	$>$	$<^{***}$	.		
PL	$>^{*}$	$>^{***}$	$>^{***}$	$>^{***}$	$>^{***}$		$>$	$>^{***}$	$>^{***}$		$>$	$>$	$<$	$>^{***}$	$<$	$>^{***}$	.	
SK	$>$	$>^{***}$	$>^{***}$	$>^{**}$	$>^{***}$	$<$		$>^{***}$	$>^{***}$			$>$	$<$	$>^{***}$	$<$	$>^{***}$		.

Source: Authors' calculation from EU-SILC (2005). ">" in row i and column j means that poverty is higher in country i than in country j. The dominance condition for each pair is obtained checking the sequential conditions in Theorem 1 (equation 1). 90%, 95%, 99% confidence intervals are based on the quantile distribution of 200 bootstrap re-sampling with replacement.

Significantly fewer instances of dominance are found when testing the conditions in Theorem 2, as shown in Table 2. Only 21 of the 153 possible pairwise comparisons can be ranked by these conditions, and only two of those are significant at 1% and

Table 2: Dominance conditions associated with Theorem 2

	AT	BE	CY	CZ	DE	EE	ES	FI	FR	GR	HU	IT	LT	LU	LV	NL	PL	SK
AT	.																	
BE		.																
CY			.															
CZ	>	>		.														
DE					.													
EE						.												
ES							.											
FI				<				.										
FR									.									
GR										.								
HU		>					>		>		.							
IT												.						
LT		>						>	>			>*	.					
LU														.				
LV							>								.			
NL				<							<		<			.		
PL		>					>		>***							>	.	
SK		>										>				>		.

Source: Authors' calculation from EU-SILC (2005). ">" in row i and column j means that poverty is higher in country i than in country j. The dominance condition for each is obtained checking the sequential conditions in Theorem 2 . 90%, 95%, 99% confidence intervals are based on the quantile distribution of 200 bootstrap re-sampling with replacement.

10%: France is less OS poor than Poland, and Lithuania is OS poorer than Italy. As expected, by relying on strong sequential requirements for both type-specific poverty incidence *and* mean incomes among the poor, Theorem 2 yields instances of dominance much less frequently than Theorem 1, which relies on weaker, necessary and sufficient conditions.

6.2 Complete rankings: The opportunity-sensitive FGT for Europe

Even when poverty dominance is widespread, as is the case in Table 1 above, one may wish to establish a complete poverty ordering across all pairwise comparisons. This requires imposing further conditions and defining a scalar poverty measure. In this section we employ the OS-FGT family of indices, introduced in Section 5, for the eighteen European countries in our sample. We also compare the resulting ranking of countries with that obtained for the traditional FGT index, which is insensitive to inequality of opportunity among the poor.

Poverty comparisons in the European Union typically rely on country-specific relative poverty lines, defined as 60% of the median of the equivalent disposable household income distribution. Whereas the OSPM dominance exercise reported above required a single poverty line across countries, the same is not true for complete orderings, and in this section we choose to adhere to common European practice, by using 60% of the country median as the poverty line for *each* country. These lines vary from Euro 3,051 (in annual equivalized disposable household income) in Lithuania, to Euro 18,253 in Luxembourg (see Table 4 in the Statistical Annex). As noted above, types are ranked according to their poverty headcount,

$F_i(z)$. As expected, in most countries the most disadvantaged types are those with parents that had little formal education and were employed in elementary occupations when the respondent was young.[9]

Figures 2 and 3 illustrate the main results of the section.[10] Figure *2* plots the country rank (from least poor to poorest) by the opportunity-sensitive poverty headcount measure $P_H(F, z, \mathcal{T})$, on the vertical axis, against the country rank according to the standard poverty headcount $FGT(0)$ in the horizontal axis. Although the correlation between the two is clearly positive, there is also a substantial number of re-rankings. The red line represents the 45 degree line; countries above (below) the line rank higher (lower) in the opportunity-sensitive poverty measure than when the distribution of opportunities is ignored. For instance, Germany (DE) is placed sixth in terms of the poverty headcount but second once we incorporate information about how poverty is distributed among the different types. Conversely, the Netherlands is the least poor country in our sample according to standard $FGT(0)$, but it is placed fourth according to the $P_H(F, z, \mathcal{T})$. A similar re-ranking is found between Mediterranean countries (such as Greece), on the one hand, and Eastern European countries (such as Estonia), on the other.

Figure 3 plots the actual *values* of these measures (as opposed to the ranks).[11] In addition to the re-rankings, and consistent with results from the previous subsection, this graph is suggestive of the existence of three groups of countries. The first group consists of the richer countries in the sample, at the bottom left of the graph, with relatively low values of both standard and opportunity-sensitive poverty headcounts. These are predominantly North-Western European countries, but also include the Czech Republic. The second group includes the Mediterranean countries (Greece, Italy and Spain) with substantially larger poverty headcounts and even higher opportunity-sensitive poverty headcounts, relative to other countries with similar levels of poverty. The third group is composed of Eastern European and former Soviet Union countries with even higher standard headcount poverty, but somewhat lower opportunity-sensitive headcounts.

Where do these cross-country differences in opportunity-sensitive poverty come from? Differences in opportunity-sensitive poverty across distributions arise from three basic sources: differences in overall poverty levels in the population (the level effect); differences in the distribution of povertys across types (the distribution effect); and differences in population shares across types (the population composition effect). For the case of the headcount OSPM, defined in Equation 5 as $P_H(F, z, \mathcal{T}) = \frac{1}{n}\sum_{i=1}^{n} q_i^F (n+1-r(i)) F_i(z)$,the difference between two countries, F and G, can be written as follows:

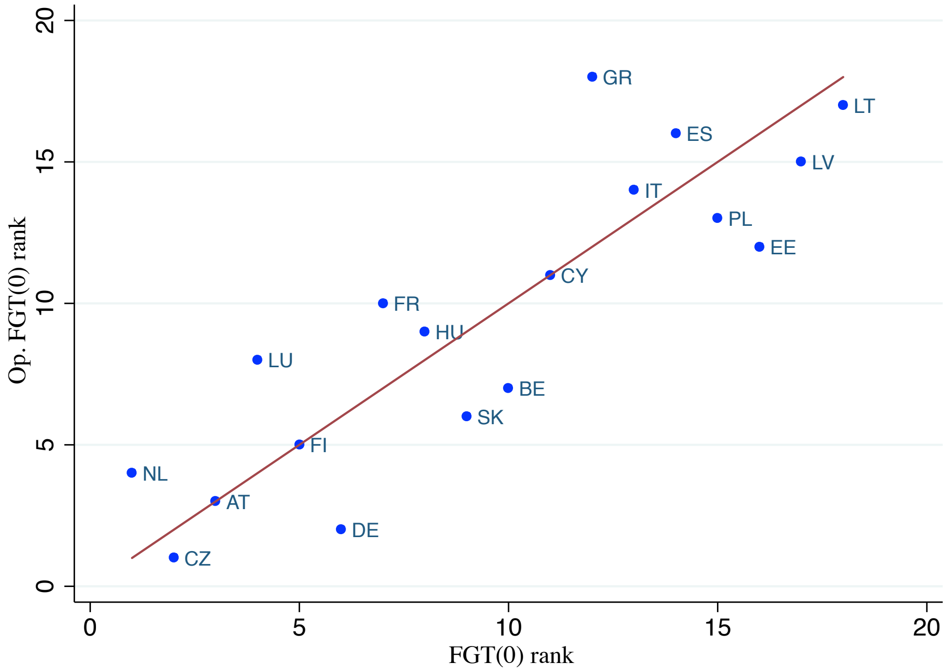
$$P_H^F - P_H^G = \left(P_H^F - P_H^*\right) + \left(P_H^* - P_H^{**}\right) + \left(P_H^{**} - P_H^G\right) \tag{6}$$

[9]Country-specific tables with poverty rates by type are available from the authors upon request.

[10]To economize on space, we present only the results from the opportunity-sensitive poverty headcount, but similar conclusions are obtained when computing the measure for $\alpha = 1, 2$. Results using opportunity-sensitive poverty gap and severity measures are available from the authors upon request.

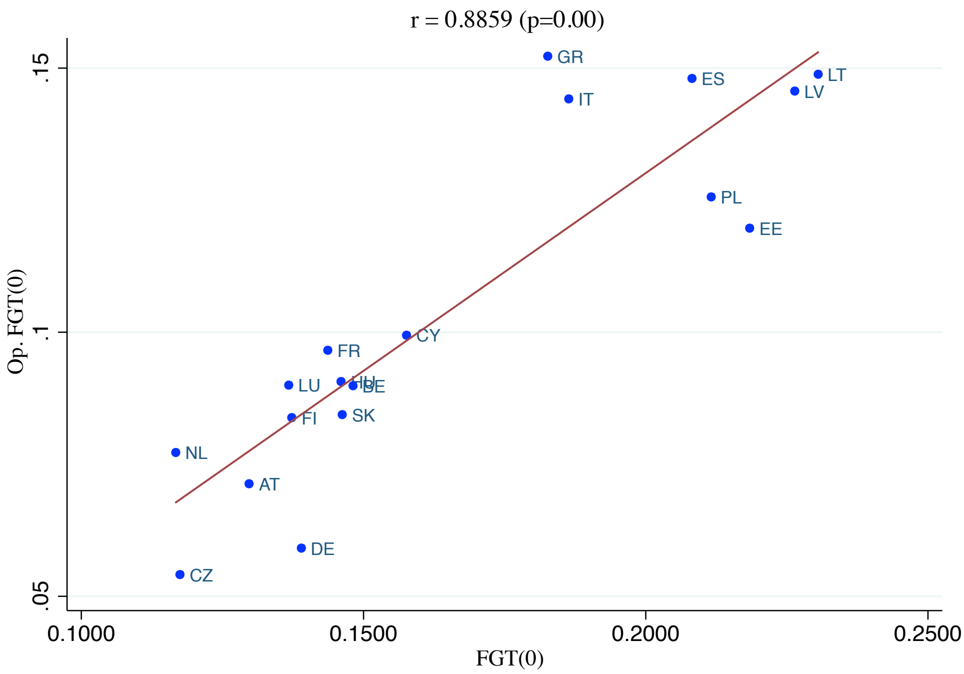
[11]All values of $P_H(F, z, \mathcal{T})$, with standard errors, are in the Statistical Annex, figure 4.

Figure 2: Poverty headcount and opportunity-sensitive poverty headcount. Rankings



Source: Authors' calculation from EU-SILC (2005)

Figure 3: Poverty headcount and opportunity-sensitive poverty headcount. Levels



Source: Authors' calculation from EU-SILC (2005) where the two counterfactual poverty incidence measures are given by:

$$P_H^{*} = \frac{1}{n}\sum_{i=1}^{n} q_i^F(n+1-r(i))F_i(z)\,\frac{G(z)}{F(z)} \tag{7}$$

$$P_H^{**} = \frac{1}{n}\sum_{i=1}^{n} q_i^F(n+1-r(i))G_i(z) \tag{8}$$

Analogous decompositions can straightforwardly be written for $\alpha > 0$, by replacing $F_i(z)$, with $\int_0^z \left(\frac{z-x}{z}\right)^\alpha f_i(x)\,dx$. Notice that the first term in brackets on the right-hand side of Equation 6 denotes the *level effect*, which is obtained by scaling up poverty incidence across types in country F by a common factor, so as to replicate country G's standard poverty incidence, while preserving the cross-type population composition and poverty distribution of country F. The second term corresponds to the *distribution effect*: with country G's overall poverty level, it moves from the distribution of poverty across types observed in country F to that of country G.[12] Finally, the third term comprises the *population composition effect*: it incoporates into the counterfactual distribution the population shares of types in country G.

The decomposition specified in (6) is exact, but it is also path-dependent: the value of each of the three effects would vary if the counterfactual indices were differently specified, corresponding to different "paths" for the decomposition. One might have changed the population shares first, for example, then the relative poverty incidences across types (given the overall poverty level in country F), and only shifted to the overall poverty in G in the last step. That such different paths yield different results for the decomposition, and that no path is in any sense more correct than another, is well understood (see, e.g. diNardo, Fortin and Lemieux 1996). So is the solution to the problem, which consists of taking the Shapley-value of each effect across all possible paths of the decomposition (see Shorrocks, 2013). This "average of decomposition paths" is now commonly known as the Shapley-Shorrocks decomposition. With three effects, there are six (3!) possible paths.

Table 3 reports the Shapley-Shorrocks decomposition of the differences in the opportunity-sensitive poverty headcount measure for three pairs of countries: The Netherlands and Lithuania; the Czech Republic and the Netherlands; and Luxemboug and France. These pairs were chosen to exemplify countries ranked very far apart by both $FGT(0)$ and C (NL vs. LT); countries ranked differently by the two measures (CZ and NL); and countries ranked similarly by the two indices (FR and LU). The entries listed in the Table are the Shapley values for the column effects across all possibly paths of the decomposition given by (6).

Results indicate that differences in opportunity-sensitive poverty arise from different sources in each case. Differences between the Nethelands and Lithuania -

[12]Since types are ranked by $F_i(z)$, this step includes any necessary re-ranking of types between countries F and G: when the distribution of poverty across types that prevails in country G - $G_i(z)$ - is imported into the counterfactual poverty index, the inverse-rank weights associated with each type adjust accordingly.

Table 3: Changes in opportunity-sensitive headcount

Countries	$\Delta opFGT0$	*population composition effect*	*distribution effect*	*level effect*
NL-LT	-0.0716272	-0.0160272	0.0157706	-0.0713705
CZ-NL	-0.0231285	-0.0101637	-0.0133965	0.0004316
LU-FR	-0.0065687	-0.0200333	0.0181715	-0.0047069

Source: Authors' calculation from EU-SILC (2005). Poverty thresholds are defined as the country-specific relative poverty lines in 2004 PPP Euro.

ranked at the two extremes of the $FGT(0)$ range in our sample - are unsurprisingly driven by the level effect, with the distribution and population composition effects largely offsetting each other. The Czech Republic is less OS poor than the Netherlands both because the distribution of poverty across types is more favourable, and because poorer types are less populous there. The level effect, as implied by the reverse ranking, goes in the opposite direction. Differences between France and Luxembourg are interesting: although the overall difference in $FGT(0)$ is very small, this actually reflects two relatively large but mutually offsetting effects: The distribution of poverty is more concentrated in poorer types in Luxembourg, but the poorest types are less populous there than in France.

7 Conclusion

The last decade and a half has seen growing interest in inequality of opportunity among economists. A number of different approaches to its formal measurement have been proposed, and applications to both developing and developed countries now abound. From a normative perspective, there is a coalescing consensus that equality of opportunity is the appropriate "currency for egalitarian justice " (Cohen, 1989). From a positive perspective, there is some evidence that inequality of opportunity is more closely (and negatively) associated with future economic performance than inequality of outcomes (Marrero and Rodriguez, 2013).

Yet, although a concern with inequality among the poor has been central to poverty measurement at least since the mid-1970s, sensitivity to inequality of opportunity has hitherto not been introduced into formal poverty measurement (so far as we are aware). This may reflect, at least in part, the fact that measures of inequality of opportunity explicitly depend on personal characteristics other than income, thereby clashing with the standard anonymity axiom. Similarly, most perspectives on inequality of opportunity would treat transfers within and between types differently, requiring adjustments to the transfer axiom.

In this paper, we have sought to address these challenges and to axiomatically derive a class of opportunity-sensitive poverty measures (OSPM). A broad OSPM class was defined, which satisfies the standard axioms of monotonicity, focus and additivity, as well new axioms of within-type anonymity, inequality of opportunity aversion, and (weak) inequality aversion within types. A narrow OSPM subclass was also defined, for the case when weak inequality aversion within types is replaced by a more stringent axiom of inequality neutrality within types.

We then identify poverty dominance conditions corresponding to each of the two classes. For the broad OSPM class we rely on a reinterpretation of the earlier results by Jenkins and Lambert (1993), and Chambaz and Maurin (1998). A separate, original, sufficient condition is identified for dominance in the narrow OSPM class.

We also consider complete poverty orderings by proposing a specific parametric family of indices, which belongs to the broad OSPM class. This measure is essentially a transformation of the seminal Foster et al. (1984) 'FGT' class, where rank-dependent weights are attached to different types. These inverse rank weights are in the spirit of Sen (1976), but are applied here to groups (types) rather than to individuals, and are thus consistent with decomposability. They also allow for an elegant resolution of the tension between inequality of opportunity aversion and inequality aversion within types. Like the traditional FGT, this family of indices ranges between zero and one. Under equality of opportunity, each member of the family converges to the corresponding standard FGT index.

In an application to poverty comparisons across eighteen European countries, we find that the broad-OSPM dominance conditions are satisfied rather often: there are 128 instances of dominance, out of 153 possible pairwise comparisons. The more stringent sufficient conditions for narrow-OSPM dominance proposed in Theorem 2 hold much less frequently, suggesting that their computational simplicity exacts a high cost in practice. Broadly, three groups of countries emerge from the dominance comparisons: Eastern European countries (other than the Czech Republic) tend to be most opportunity-poor, and are dominated by most other countries. Mediterranean countries (such as Greece, Italy and Spain) tend to dominate the Eastern European countries, but are dominated by the third group, namely North-Western Europe.

The existence of these three broad country groupings is confirmed by computing the scalar OS-FGT index. This index is positively correlated with the standard FGT index, but a large number of re-rankings is observed, and some are substantial. Germany, for example, ranks as the sixth least-poor country in terms of the standard headcount measure (poorer than the Netherlands), but second least-poor in terms of the opportunity-senstive headcount (less poor than the Netherlands). Cross-country differences in the OS-FGT index are driven by three factors: differences in overall poverty levels in the population (the level effect); differences in the distribution of poverty across types (the distribution effect); and differences in population shares across types (the population composition effect).

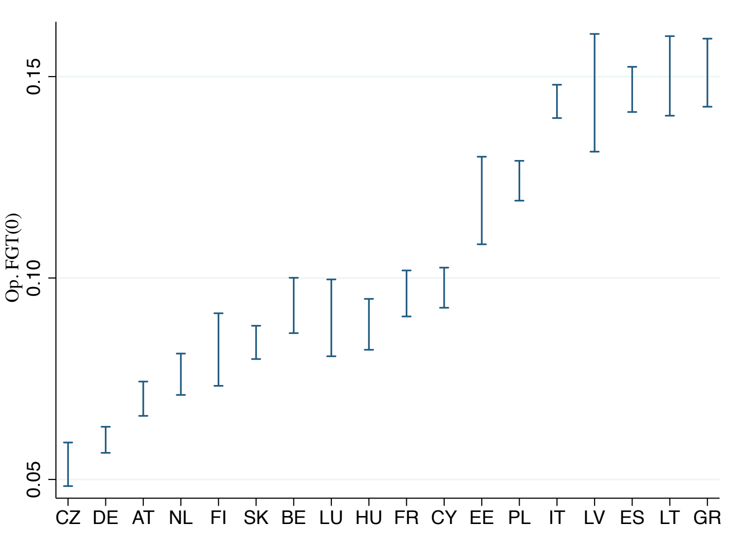
We hope that both the dominance conditions we have identified for a broad class of opportunity-sensitive poverty measures and the specific, rank-dependent FGT index we have proposed may be useful to empirical researchers and practitioners interested in characterizing the nature of poverty in different settings, or in monitoring changes in poverty over time. The measures should be of particular interest to societies averse both to income poverty and to unequal opportunities.

8 Statistical Annex

Table 4 contains descriptive statistics for the eighteen country samples used in Section 6, including EU-SILC sample sizes, average incomes, relative poverty lines (60% of the median of equivalent household income distributions), FGT (0, 1, 2), the mean logarithmic deviation ($E(0)$) of incomes, and the between-type share of $E(0)$, as a measure of inequality of opportunity.

Figure 4 graphically depicts the 95% bootstrapped confidence intervals around P_H for each country in the sample.

Figure 4: 95% confidence intervals for P_H



Source: Authors' calculation from EU-SILC (2005)

country	*sample*	*average income*	*poverty threshold*	*FGT(0)*	*FGT(1)*	*FGT(2)*	*total inequality*	*Ineq of Opp*
Austria	5,650	22,380	12,080.47	0.1298	0.0328	0.0150	0.1270	2.75
Belgium	4,697	22,230.25	11,713.56	0.1481	0.0360	0.0144	0.1674	10.23
Cyprus	4,486	20,521.00	10,800.54	0.1577	0.0398	0.0160	0.1468	4.87
Czech Rep.	5,098	10,601.02	5,559.589	0.1175	0.0287	0.0108	0.1260	6.05
Estonia	4,779	7,557.72	3,843.491	0.2185	0.0757	0.0396	0.2192	7.46
Finland	7,241	19,604.28	10,450.91	0.1374	0.0317	0.0122	0.1380	3.81
France	10,830	19,730.18	10,525.69	0.1437	0.0307	0.0110	0.1248	4.58
Germany	13,152	21,003.09	11,091.51	0.1390	0.0376	0.0164	0.1455	1.51
Greece	6,050	15,835.91	8,018.675	0.1827	0.0570	0.0286	0.1880	7.05
Hungary	7,293	7,855.96	4,054.386	0.1461	0.0359	0.0137	0.1512	8.23
Italy	22,328	20,312.67	10,520.81	0.1864	0.0569	0.0287	0.1900	6.83
Latvia	3,931	6,593.19	3,239.392	0.2264	0.0847	0.0484	0.2611	9.08
Lithuania	5,185	6,226.57	3,051.889	0.2305	0.0851	0.0471	0.2479	6.96
Luxembourg	4,608	33,996.06	18,253	0.1368	0.0347	0.0137	0.1247	9.57
Netherlands	5,281	21,110.64	11,400.55	0.1167	0.0294	0.0136	0.1210	2.22
Poland	19,127	7,724.96	3,785.902	0.2116	0.0745	0.0401	0.2483	7.35
Slovakia	6,191	7,213.88	3,930.293	0.1463	0.0413	0.0180	0.1295	2.23
Spain	13,968	16,931.32	8,964.699	0.2082	0.0695	0.0365	0.1925	6.54

Table 4: Descriptive Statistics

Source: Authors' calculation from EU-SILC (2005). Poverty thresholds are defined as the country-specific relative poverty lines in 2004 PPP Euro. Total inequality is the mean logarithmic deviation of incomes, IOp is ex-ante inequality of opportunity calculated non parametrically.

Appendix

Proof - Theorem 2

$P(F(x), z) \geq P(G(x), z) \iff$

$$\Delta P = \sum_{i=1}^{j} q_i^F \int_0^z p_i(x) f_i(x) dx - \sum_{i=1}^{j} q_i^G \int_0^z p_i(x) g_i(x) dx \geq 0$$

Integrating by parts: $\int v du = uv - \int u dv$: $v = p_i(x)$, $u = F_i(x)$ from which:

$$\int_0^z p_i(x) f_i(x) = [F_i(x) p_i(x)]_0^z - \int_0^z F_i(x) p_i'(x) dx$$

ΔP becomes:

$$\Delta P = \sum_{i=1}^{j} q_i^F \left([F_i(x) p_i(x)]_0^z - \int_0^z F_i(x) p_i'(x) dx \right) - \sum_{i=1}^{j} q_i^G \left([G_i(x) p_i(x)]_0^z - \int_0^z G_i(x) p_i'(x) dx \right)$$

If $F_i(0) = 0$, then $[F_i(x)p_i(x)]_0^z = [G_i(x)p_i(x)]_0^z = 0$, as $p_i(z) = 0$. Hence

$$\Delta P = \sum_{i=1}^{j} q_i^F \left(- \int_0^z F_i(x) p_i'(x) dx \right) - \sum_{i=1}^{j} q_i^G \left(- \int_0^z G_i(x) p_i'(x) dx \right)$$

$$\Delta P = \sum_{i=1}^{j} q_i^G \int_0^z G_i(x) p_i'(x) dx - \sum_{i=1}^{j} q_i^F \int_0^z F_i(x) p_i'(x) dx$$

Integrating again by parts: $\int u dv = uv - \int duv$, $u = p_i'(x)$ and $v = \int^x F_i(y)$

$$\Delta P = \sum_{i=1}^{j} q_i^G \left([p_i'(x)]_0^Z \int_0^Z G_i(x) - \int_0^z p_i''(x) \int^x G_i(y) dy \right)$$
$$- \sum_{i=1}^{j} q_i^F \left([p_i'(x)]_0^Z \int_0^Z F_i(x) - \int_0^z p_i''(x) \int^x F_i(y) dy \right)$$

that is

$$\Delta P = \sum_{i=1}^{j} [p_i'(x)]_0^Z \left(q_i^G \int_0^Z G_i(x) - q_i^F \int_0^Z F_i(x) \right) + \int_0^z p_i''(x) \int^x \left(q_i^F F_i(y) - q_i^G G_i(y) \right) dy$$

Assuming $p_i''(x) = 0$ the second term disappears.
Now, $\int_0^Z F_i(x) = zF_i(z) - \mu(F_i^z)$,
where $\mu(F_i^Z)$ is the mean of the distribution F_i truncated at z.

This comes from the fact that:

$$\mu\left(F_i^z\right) = \int_0^z x f(x) dx$$

integrating by parts one obtains

$$\mu\left(F_i^z\right) = [xF(x)]_0^z - \int_0^z F(x)dx = zH - \int_0^z F(x)dx$$

.

From which:

$$\Delta P = \sum_{i=1}^{j} p_i'(z) \left[q_i^G \left(zG_i\left(z\right) - \mu\left(G_i^z\right)\right) - q_i^F \left(zF_i\left(z\right) - \mu\left(F_i^z\right)\right)\right]$$

As $p_i'(x) \leq p_{i+1}'(x)$ for all $i = 1, ..., n-1$ (see property 3 of Remark 1) we can apply Abel lemma, thus obtaining that $\Delta P \geq 0$ if and only if

$$\sum_{i=1}^{j} \left[q_i^G \left(zG_i\left(z\right) - \mu\left(G_i^z\right)\right) - q_i^F \left(zF_i\left(z\right) - \mu\left(F_i^z\right)\right)\right] \leq 0$$

This can be written in the following way:

$$\sum_{i=1}^{j} z \left(q_i^F F_i\left(z\right) - q_i^G G_i\left(z\right)\right) + \sum_{i=1}^{j} \left(q_i^G \mu\left(G_i^z\right) - q_i^F \mu\left(F_i^z\right)\right) \geq 0 \tag{9}$$

Alternatively, adding and subtracting $q_i^F \mu(G_i^z)$ and $zq_i^F G(z)$, as

$$\sum_{i=1}^{j} z \left(q_i^F F_i\left(z\right) - q_i^G G_i\left(z\right) - zq_i^F G_i(z) + zq_i^F G_i(z)\right) +$$

$$\sum_{i=1}^{j} \left(q_i^G \mu\left(G_i^z\right) - q_i^F \mu(G_i^z) - q_i^F \mu\left(F_i^z\right) + q_i^F \mu(G_i^z)\right) \geq 0$$

$$(q_i^F - q_i^G)(zG_i(z) - \mu(G_i^z) + zq_i^F(F_i(z) - G_i(z)) + q^F(\mu(G_i^z) - \mu(F_i^z)) \geq 0 \tag{10}$$

We obtain a decomposition of the difference in responsibility sensitive poverty in three terms: the differences in population shares, in headcount poverty ratios, and in average incomes of the poor. The sign of the contribution of each term is positive as they are multiplied by a positive number:

$$zG_i(z) - \mu(G_i^z) = \int_0^z Gxdx \geq 0$$

From (9) we obtain that sufficient conditions for $\Delta P \geq 0$ are:

(i) $\sum_{i=1}^{j} q_i^F F_i\left(z\right) \geq \sum_{i=1}^{j} q_i^G G_i\left(z\right), \quad \forall j \in \{1, ..., n\}.$

(ii) $\sum_{i=1}^{j} q_i^F \mu(F_i^z) \leq \sum_{i=1}^{j} q_i^G \mu(G_i^z), \quad \forall j \in \{1, ..., n\}$.

From (10) we obtain that sufficient conditions for $\Delta P \geq 0$ are:

(i) $\sum_{i=1}^{j} \mu(F_i^z) \leq \sum_{i=1}^{j} \mu(G_i^z), \quad \forall j \in \{1, ..., n\}$.

(ii) $\sum_{i=1}^{j} F_i(z) \geq \sum_{i=1}^{j} G_i(z), \quad \forall j \in \{1, ..., n\}$.

(iii) $\sum_{i=1}^{j} q_i^F \geq \sum_{i=1}^{j} q_i^G, \quad \forall j \in \{1, ..., n\}$.

Remark 1.2

If $F_i(0) \neq 0$

$$\Delta P = \sum_{i=1}^{j} q_i^F \left([F_i(x)p_i(x)]_0^z - \int_0^z F_i(x)p_i'(x)dx \right) - \sum_{i=1}^{j} q_i^G \left([G_i(x)p_i(x)]_0^z - \int_0^z G_i(x)p_i'(x)dx \right)$$

$$\Delta P = \sum_{i=1}^{j} \left[q_i^G G_i(0)p_i(0) - q_i^F F_i(0)p_i(0) \right] + \left[\sum_{i=1}^{j} q_i^G \int_0^z G_i(x)p_i'(x)dx - \sum_{i=1}^{j} q_i^F \int_0^z F_i(x)p_i'(x)dx \right]$$

we know the sufficient conditions for the second term to be positive, the first term adds a new condition:

$$\sum_{i=1}^{j} \left[q_i^G G_i(0)p_i(0) - q_i^F F_i(0)p_i(0) \right] \geq 0$$

$$\sum_{i=1}^{j} \left[q_i^G (G_i(0) - F_i(0)) + (q_i^G - q_i^F) F_i(0) \right] \geq 0$$

$$\sum_{i=1}^{j} G_i(0) \geq \sum_{i=1}^{j} F_i(0) \tag{11}$$

Where $F_i(0)$, $G_i(0)$ are the proportions of the individuals in the type i with no income. The sum of the these proportions at each step $i \leq j = 1, ..., n$ must be larger in G than in F.

References

[1] Arneson R. (1989). "Equality of opportunity for welfare", *Philosophical Studies*, 56: 77-93.

[2] Atkinson A. B, F. Bourguignon (1987) "Income distirbtuions and differences in needs", in Feiwel, G. R. (ed.) *Arrow and the Foundations of the Theory of Economic Policy*. Macmillan. London.

[3] Bossert, W. (1997), "Opportunity sets and individual well-being" *Social Choice and Welfare* 14: 97-112.

[4] Bourguignon, F., F.H.G. Ferreira and M. Menendez (2007), "Inequality of opportunity in Brazil", *Review of Income and Wealth* 53: 585-618.

[5] Chambaz C., E. Maurin (1998) "Atkinson and Bourguignon's dominance criteria: Extended and applied to the measurement of poverty in France", *Review of Income and Wealth*, 44(4): 497-513.

[6] Checchi D., V. Peragine (2010) "Inequality of opportunity in Italy", *Journal of Economic Inequality*, 8(4): 429-450.

[7] Cohen G. A. (1989) "On the currency of egalitarian justice", *Ethics*, 99: 906-944.

[8] Decancq K., Goedemè T., Van den Bosch K., Vanhille J., (2013). "The Evolution of Poverty in the European Union: Concepts, Measurement and Data," *ImPRovE Working Papers* 13-01, Herman Deleeck Centre for Social Policy, University of Antwerp.

[9] DiNardo, J., N. Fortin and T. Lemieux (1996), "Labor market institutions and the distribution of wages, 1973-1992: A semi-parametric approach", *Econometrica* 64 (5): 1001-1044.

[10] Dworkin R. (1981a) "What is equality? Part 1. Equality of welfare", *Philosophy and Public Affairs*, 10 (3): 185-246.

[11] Dworkin R. (1981b) "What is equality? Part 2. Equality of resources". *Philosophy and Public Affairs*, 10 (4): 283-345.

[12] Elster J. and J.E. Romer (1993). *Interpersonal Comparisons of Well-Being (Studies in Rationality and Social Change)*, Cambridge University Press, 1993.

[13] Eurostat (2008) "The Social Situation in the European Union", European Commission Directorate-General for Employment, Social Affairs and Equal Opportunities, April 2008.

[14] Ferreira F. H. G., J. Gignoux (2011). "The measurement of inequality of opportunity: Theory and an application to Latin America" , *Review of Income and Wealth*, 57(4): 622-657.

[15] Fleurbaey M. (2007). "Poverty as a form of oppression", in *Freedom from poverty as a human right: who owes what to the very poor?*, Thomas Winfried and Menko Pogge and Thomas Pogge eds., Unesco, pp 133-155.

[16] Fleurbaey M. (2008). *Fairness, Responsibility and Welfare*, Oxford University Press. Oxford.

[17] Fleurbaey M. and V. Peragine (2013). "Ex-ante versus ex-post equality of opportunity". *Economica* 8(317): 118-30.

[18] Fleurbaey M. and F. Maniquet (2011). *A theory of fairness and social welfare*, Cambridge University Press.

[19] Foster J., J. Greer and E. Thorbecke (1984). "A class of decomposable poverty measures", *Econometrica*, 52: 761-776.

[20] Jenkins S. and P.J. Lambert (1993). "Ranking income distributions when needs differ". *Review of Income and Wealth*, 39(4): 337-356.

[21] LeFranc, Arnaud, Nicolas Pistolesi, and Alain Trannoy. (2009). "Equality of opportunity and luck: Definitions and testable conditions, with an application to income in France." *Journal of Public Economics* 93(11-12): 1189-1207.

[22] Nolan B. (2012), "Analysing Intergenerational Influences on Income Poverty and Economic Vulnerability with EU-SILC", *Gini Discussion Paper* N. 46 May 2012, Growing Inequality's Impact.

[23] Ooghe, E., E. Schokkaert and D. van de Gaer (2007), "Equality of opportunity versus equality of opportunity sets", *Social Choice and Welfare* 28: 209-230.

[24] Peragine V. (2004), "Ranking income distributions according to equality of opportunity", *Journal of Economic Inequality*, 2(1): 11-30.

[25] Rawls J. (1971), *A Theory of Justice*, Harvard University Press, Cambridge, MA.

[26] Roemer, J. E. (1998) *Equality of Opportunity*. Harvard University Press, Cambridge, MA.

[27] Sen, A.K. (1976), "Poverty: An ordinal approach to measurement", *Econometrica*, 44(2): 219-31.

[28] Shorrocks, A. F. (2013), "Decomposition procedures for distributional analysis: a unified approach based on the Shapley value", *Journal of Economic Inequality*, 11: 99-126.

[29] van de Gaer, D. (1993), *Equality of opportunity and investment in human capital* (KULeuven, Leuven).