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On Distributional change, Pro-poor growth and Convergence*

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Abstract

This paper proposes a unified approach to the measurement of distributional change. The framework is used to define indices of inequality, convergence, and pro-poorness of growth and associated equivalent growth rates. A distinction is made between non-anonymous and anonymous measures. The analysis is extended by using the notion of a generalized Gini index. This unified approach is then used to study the link between income and other non-income characteristics, such as education or health. An empirical illustration based on Indian data on infant survival levels in 2001 and 2011 highlights the usefulness of the proposed measures.

Keywords: β-convergence, σ-convergence, Gini index, India, infant mortality, pro-poor growth, relative concentration curve.

JEL Classification: D31, I32, O15.

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1. Introduction

The notions of inequality, convergence, pro-poor growth and income mobility, though related, have typically been considered separately in the literature. Attempts have been made in the last few years to establish formally the relation between some of these notions. For example, Yitzhaki and Wodon (2005) analyze the relationships between growth, inequality, and mobility. They decompose average social welfare over time into components of income growth, σ-convergence and mobility. Jenkins and Van Kerm (2006) formulate a relation between inequality change, pro-poor growth and mobility. They use the generalized Gini class of indices to decompose a change in inequality into components of progressivity of income growth and change in income ranking. O'Neill and Van Kerm (2008) stress the close links that exist between studies of income convergence and those analyzing the progressivity of a tax system and propose a simple algebraic decomposition of σ-convergence into the combined effect of β-convergence and leapfrogging among countries. Nissanov and Silber (2009) use the standard β-convergence regression model and decompose the slope coefficient into components accounting for σ-convergence and a mobility term.

An important contribution of this paper to this literature is that it proposes a unified framework to derive measures of inequality in growth rates, mobility, $\sigma-$ and $\beta-$convergence and the pro-poorness of growth. We show that in all cases the proposed indices amount to comparing the shares in total income received by individual units (persons, households, states or countries) at some original time 0 with the shares in total income received by these individual units at some final time 1. What characterizes each measure is the way these shares are ranked and whether an anonymous or a non-anonymous approach is taken, but the computation algorithm is always the same. The anonymous approach is used in case of cross-sectional data whereas the non-anonymous approach is used when panel data is available and it is possible to track the same unit of observation over multiple years. We provide a simple graphical interpretation of these indices using relative concentration curves (Kakwani, 1980) to compare the cumulative shares at time 0 against the cumulative shares at time 1.

The systematic comparison of two sets of shares is not limited to income shares. It is certainly possible to compare, for example, the shares of individuals at times 0 and 1 in the total number of years of education. One would then compare individual shares in years of education at times 0 and 1, whether the emphasis is on inequality in the individual growth rates in years of education or on the convergence over time in individual years of education. One could also apply our approach to the field of health or to other domains of well-being relevant for comparing changes over time or between geographical areas. In fact the analysis can be even more sophisticated. If individual shares in the total number of years of education at times 0 and 1 are ranked, not by years of education but by individual income level, then we measure to what extent the increase in the number of years of education (assuming there was on average such an increase) was stronger, the lower the original income of the individuals. Such an analysis is implemented on a non-anonymous as well as on an anonymous base. In the former case we verify whether over time there was $\beta-$convergence of educational levels and this convergence is checked with respect to the original income levels. In the latter case (anonymous analysis) we check whether over time there was $\sigma-$ convergence of the educational levels between the various centiles of the original income distribution. Thus the approach proposed in this paper allows us to test, whether there was conditional on income $\beta-$ or $\sigma-$convergence in education levels.

For every proposed index in the framework, we also define an associated "equivalent growth rate". An equivalent growth rate is defined as a weighted average of individual growth rates. In the presence of inequality in growth rates, the equivalent growth rate is smaller than the average growth rate because a penalty is assumed to incur due to the inequality. Such an approach is thus similar to that of Jenkins and Van Kerm (2011) who defined an equally-distributed-equivalent growth rate[1] and Demuynck and Van de Gaer (2012) who characterized a measure of aggregate income growth that gives a greater weight to individuals with lower individual income growth. When checking for $\beta-$convergence, the equivalent growth rate may be higher or smaller than the average growth rate, depending on whether individual growth rates are higher or smaller, the poorer the individuals were at time 0. Such a perspective is comparable to that adopted in a recent paper by Palmisano and Van de Gaer (2013) who derived a characterization of an aggregate measure of growth that takes into account the initial economic conditions of individuals. The measure they proposed is a weighted average of individual income growth with weights that are decreasing with the rank of the individual in the initial income distribution.

We also propose new measures of anonymous and non-anonymous pro-poor growth. Previously, Ravallion and Chen (2003) introduced the concept of Growth Incidence Curve (GIC) while Kakwani and Pernia (2000), Son (2004) and Kakwani and Son (2008a and 2008b) provided several definitions of pro-poor growth. All these studies focused on the anonymous case so that pro-poor growth could be detected on the basis of cross-sections. Grimm (2007) introduced the concept of Individual Growth Incidence Curve (IGIC) and thus applied the analysis of pro-poor growth to the non-anonymous case. Grosse et al. (2008) further extended the analysis of pro-poor growth and defined the notion of Non-Income Growth incidence Curve (NIGIC) which allows examining whether growth in non-income dimensions was pro-poor. The present paper suggests

[1] Jenkins and Van Kerm (2011) define the equally-distributed-equivalent growth rate as "the growth rate which, if received uniformly by each individual, would yield the same evaluation as the observed average growth rate were it also received uniformly". Their approach is derived directly from some social evaluation function while, as will be seen in Section 3, our approach is only indirectly linked to a social evaluation function.

a new definition of pro-poor growth, whether of income or non-income dimensions, which is derived from the unified approach to the analysis of distributional change. We show that our approach is comparable to that of Kakwani and Pernia (2000) or Ravallion and Chen (2003).

Importantly, we extend our methodology to a more general setting. Although the inequality and convergence indices mentioned previously are derived from the traditional Gini index, we define additional measures of inequality in growth rates and of convergence, which are derived from the generalization of the Gini index (proposed by Donaldson and Weymark, 1980). Such an extension was proposed by Jenkins and Van Kerm (2006) in their analysis of income mobility, following previous work of Silber (1995) on the derivation of Gini-related measures of distributional change. We show in this paper that similar generalization may also be applied to the analysis of convergence. We also show that the generalized $\beta-$convergence index can be decomposed to measure structural and exchange mobility.

Finally, the unified methodology proposed in this paper, allows the estimation of indices even when the number of observations is limited and available only in aggregate form such as population quintiles or deciles. We provide an empirical example where we analyze infant survival rates in India. We use data at the state level because our approach is particularly useful when the sample size is relatively small. In such a case traditional econometric approaches to convergence analysis cannot be used. We find that between 2001 and 2011, growth in infant survival shares was relatively higher in poorer states.

The paper is organized as follows. In Section 2, we present a unified methodology to the analysis of distributional change and use it to propose measures of inequality in growth rates and convergence, making a distinction between the non-anonymous and the anonymous case. We also define measures of pro-poor growth. In Section 3 we derive generalization of the inequality in growth rates and convergence indices. Section 4 contains an extension of this unified approach to non-income indicators and to the study of the conditional (on income) convergence of these non-income characteristics. Section 5 provides an empirical analysis focusing on infant mortality at the state level in India. Section 6 summarizes the results of our analysis. Proofs of some of the properties of the proposed indices are given in Appendix A, and simple numerical illustrations of the indices are given in Appendix B.

2. A unified framework to analyze distributional change

2.1. Notation

Let x_i and y_i refer to the absolute income of the i^{th} observation and $\bar{x}$ and $\bar{y}$ to the average incomes at times 0 and 1 in a population of n individuals.[2] Define changes in incomes, Δx_i and $\Delta\bar{x}$, as $\Delta x_i = (y_i - x_i)$ and $\Delta\bar{x} = \bar{y} - \bar{x}$. Let $s_i = (x_i/n\bar{x})$ and $w_i = (y_i/n\bar{y}) = (x_i + \Delta x_i)/n(\bar{x} + \Delta\bar{x}))$ refer to the income shares at times 0 and 1. Upon simplification, the difference $(s_i - w_i)$ may be expressed as

$$s_i - w_i = \left(\frac{1}{n\bar{x}}\right)\left(\frac{(x_i\Delta\bar{x} - \bar{x}\Delta x_i)}{(\bar{x}+\Delta\bar{x})}\right) \tag{1}$$

Now define η_i and $\overline{\eta}$ as $\eta_i = (\Delta x_i)/x_i$ and $\overline{\eta} = (\Delta\bar{x}/\bar{x})$, where η_i denotes the growth in income of observation *i* and $\overline{\eta}$ denotes the growth in average income; then (1) can be written as

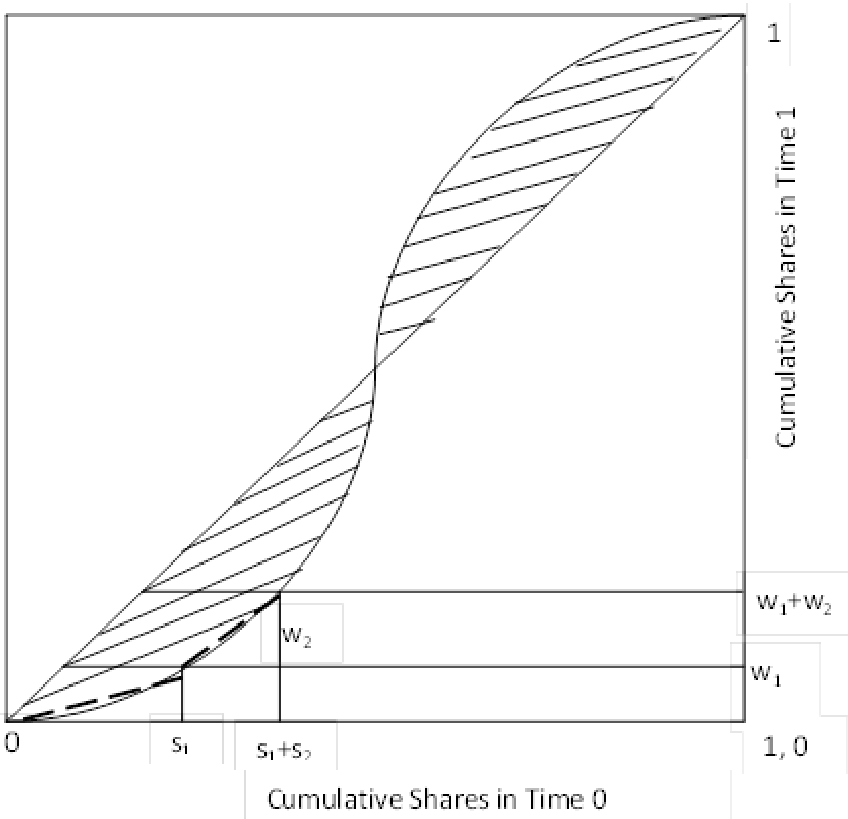
$$s_i - w_i = s_i\frac{(\overline{\eta} - \eta_i)}{1+\overline{\eta}} \tag{2}$$

Let us now plot the cumulative values of the shares s_i and w_i in a one by one square, these shares being ranked according to some criterion. The relative concentration curve obtained is

[2] For the ease of exposition, we refer to *i* as an individual here. However, *i* may represent a population centile, a region, or a country, depending on the application. In the empirical section 5, *i* refers to a state in India.

increasing, starting at point (0, 0) and ending at point (1, 1) but in general it may cross once or more the diagonal (see, Figure 1 below).

Figure 1: A Relative Concentration Curve



Note: The concentration curve above is shown using a smooth line;
it will be piece-wise linear if there is a limited number of observations.

The area *A* lying below this relative concentration curve is expressed as

Area $A = \left(\frac{1}{2}\right)\{s_1 w_1\} + (\frac{1}{2})\left\{\sum_{i=2}^{n} s_i\left[\sum_{j=1}^{i-1} 2w_j + w_i\right]\right\}$ (3)

Similarly the area *B* lying below the diagonal is expressed as

Area $B = \left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)\{\sum_{i=1}^{n} s_i\}\{\sum_{i=1}^{n} s_i\} = \left(\frac{1}{2}\right)\{s_1 s_1\} + (\frac{1}{2})\left\{\sum_{i=2}^{n} s_i\left[\sum_{j=1}^{i-1} 2s_j + s_i\right]\right\}$ (4)

The difference *DIF* between areas *B* and area *A* (shaded area shown in Figure 1) is

DIF =(Area *B* – Area *A*) $= \left(\frac{1}{2}\right)\{s_1(s_1 - w_1)\} + (\frac{1}{2})\left\{\sum_{i=2}^{n} s_i\left[\sum_{j=1}^{i-1} 2\left(s_j - w_j\right) + (s_i - w_i)\right]\right\}$ (5)

2.2. Measure of Distributional Change and Equivalent Growth Rate

Let us now define an index J as being equal to twice the value of the measure *DIF*. J is a measure of "distributional change" since when $s_i = w_i \ \forall i$, J is equal to 0.[3] Combining (2) and (5), we derive

$$J = (2 \times DIF) = \left(\frac{1}{1+\overline{\eta}}\right)\left\{\left[(s_1)^2\left(\overline{\eta} - \eta_1\right)\right] + \left[\sum_{i=2}^{n} s_i\left[\sum_{j=1}^{i-1} 2\left(s_j\left(\overline{\eta} - \eta_j\right)\right) + \left(s_i\left(\overline{\eta} - \eta_i\right)\right)\right]\right]\right\}$$

$$\leftrightarrow J = \sum_{i=1}^{n} s_i\left[s_i + 2\sum_{j>i} s_j\right]\left[\frac{\left(\overline{\eta} - \eta_i\right)}{\left(1+\overline{\eta}\right)}\right] \qquad (6)$$

$$\leftrightarrow J = \sum_{i=1}^{n} \alpha_i \left[\frac{\left(\overline{\eta} - \eta_i\right)}{\left(1+\overline{\eta}\right)}\right] \qquad (7)$$

with

$$\alpha_i = s_i\left[s_i + 2\sum_{j>i} s_j\right] \qquad (8)$$

but since $\left(\overline{\eta} - \eta_i\right) = \left[\left(1+\overline{\eta}\right) - \left(1+\eta_i\right)\right]$, expression (7) may be written as

$$J = \sum_{i=1}^{n} \alpha_i \left[1 - \left(\frac{1+\eta_i}{1+\overline{\eta}}\right)\right] \qquad (9)$$

Since

$$\sum_{i=1}^{n} \alpha_i = 1 \qquad (10)$$

we conclude, combining (9) and (10), that

$$J = 1 - \sum_{i=1}^{n} \alpha_i \left(\frac{1+\eta_i}{1+\overline{\eta}}\right) = 1 - \left(\frac{1+\eta_E}{1+\overline{\eta}}\right) \qquad (11)$$

with

$$\eta_E = \sum_{i=1}^{n} \alpha_i \eta_i \qquad (12)$$

$\overline{\eta}$ refers to the average growth rate observed in the population between times 0 and 1, the indicator η_E refers to the "equivalent growth rate". η_E is a growth rate which is a weighted

[3] See, Cowell (1980 and 1985), and Silber (1995) for more details on this concept.

average of the individual growth rates, the weight of each growth rate being a function of both the income weight s_i of individual i at time 0 and the sum $\left[s_i + 2\sum_{j<i} s_j\right]$ of the income weights of those who at time 0 have an income ranked before that of individual i, according to the ranking criterion selected. The properties of η_E evidently depend on the ranking criterion, that is, on the way the income shares s_i and w_i are ranked.

In order to derive indices of distributional change it is often convenient to express η_E in a matrix form. Let us rewrite (5) as

$$(2 \times DIF) = J = \sum_{i=1}^{n} s_i\left[\sum_{j\le i}\left(s_j - w_j\right) + \sum_{j<i}\left(s_j - w_j\right)\right] \quad (13)$$

$$\leftrightarrow J = \sum_{i=1}^{n} s_i\left\{\left[\left(1 - \sum_{j>i} s_j\right) - \left(1 - \sum_{j>i} w_j\right)\right] + \left[\sum_{j<i} s_j - \sum_{j<i} w_j\right]\right\}$$

$$\leftrightarrow J=\sum_{i=1}^{n} s_i\left\{\left[\sum_{j>i} w_j - \sum_{j<i} w_j\right] - \left[\sum_{j>i} s_j - \sum_{j<i} s_j\right]\right\} \quad (14)$$

$$\leftrightarrow J= (s'Hw) - (s'Hs) \quad (15)$$

where s, w are vectors of respective income shares and H is n x n square matrix whose typical element h_{kl} is equal to 0 if $k = l$, to +1 if $k > l$ and to -1 if $k < l$.[4] Since $(s'Hs) = 0$, we end up with

$$J= (s'Hw) \quad (16)$$

Combining (11) and (15) we conclude that

$$\frac{1+\eta_E}{1+\overline{\eta}} = 1 - \{s'Hw\} = (1 - J)$$

$$\leftrightarrow \eta_E = \overline{\eta}[1 - \{s'Hw\}] - s'HW = \overline{\eta}(1 - J) - J = \overline{\eta} - J(1 + \overline{\eta}) \quad (17)$$

Next we interpret the index J and the corresponding equivalent growth rate η_E under various scenarios, depending on the ranking criterion selected for the shares s_i and w_i. A summary of the proposed indices is given in Table 1.

[4] Note that the matrix H is in fact the transpose of the matrix G introduced by Silber (1989).

2.3. The non-anonymous case

Let us assume that we have panel data so that we know the incomes of all individuals at times 0 and at time 1. We refer to such a situation as the "non-anonymous case".

2.3.1. Measuring inequality in growth rates

Suppose the shares s_i and w_i are ranked by increasing values of the ratio (w_i/s_i). As a consequence, J is now a measure of the inequality of the individual growth rates, denoted by I_N, where I denotes inequality and the subscript N indicates the non-anonymous case. Given the definition of the matrix H and using (16) it is easy to show that I_N is, in fact, identical to Silber's (1995) income-weighted measure J_{GI} of distributional change.[5] From (11) note that, for a given value of the average growth rate $\overline{\eta}$, the greater the inequality index I_N, the lower the equivalent growth rate η_E which in the present case will be written as $\eta_E(I_N)$.

Properties of I_N and $\eta_E(I_N)$

i) When the growth rates η_i are not all identical, $\eta_E(I_N)$ is always smaller than $\overline{\eta}$ so that $0 < I_N < 1$ (proof in Appendix A). This result holds however only because we assumed that the shares s_i and w_i were ranked by increasing values of the ratio (w_i/s_i), that is, by increasing values of the growth rates η_i.

ii) I_N is invariant to a homothetic change in the incomes between times 0 and 1. [6]

iii) Assume that the only change that occurs between time 0 and time 1 is that two individuals swap their income. The impact of such a change on $I_N$is greater, the greater the income gap

[5] Silber (1995) proposed also a population weighted measure of distributional change which, like the income mobility measures proposed by Chakravarty (1984), takes into account the changes in rank that took place during the period analyzed.

[6] Properties ii) to v) of the inequality index I_N are derived in Proposition 2 in Silber (1995) and hence are only listed here.

between the individuals who swapped their incomes and the lower the value of the lower of the two incomes that are swapped.

iv) If a sum Δ is transferred from individual h to individual k, assuming that the income share s_h is higher than the income share s_k and that there was no change in the ranking of the two individuals, the value of the index I_N is an increasing function of the transfer Δ and of the income of the poorer of the two individuals

v) The index I_N follows Dalton's principle of population (it is invariant to population replications).

2.3.2. Analyzing $\beta-$ convergence of income shares over time

Now suppose the shares s_i and w_i are ranked by increasing values of the shares s_i. In this case, the distributional change index J will be written as C_N where C refers to convergence and the subscript N, as before, refers to the non-anonymous case. This index C_N measures the degree of convergence of the incomes.

Properties of C_N and $\eta_E(C_N)$

i) The equivalent growth rate $\eta_E(C_N)$ may be greater or smaller than $\overline{\eta}$ (proof in Appendix A). If $\eta_E(C_N)$ is greater than $\overline{\eta}$, it means that on average the income of those with low incomes grows at a higher rate than that of those with a high income so that there is convergence of the incomes over time. Such a case corresponds to what is labeled in the literature as $\beta-$ convergence. If however $\eta_E(C_N)$ is smaller than $\overline{\eta}$, there is $\beta-$ divergence.

ii) When the growth rate η_i of an individual i is smaller (higher) than the average growth rate $\overline{\eta}$, the contribution of this individual to the overall distributional change C_N is positive (negative). This follows directly from expression (9).

iii) Since in the present case the relative concentration curve may be above or below the diagonal or even cross several times the diagonal, the index C_N varies between -1 and +1 (proof in Appendix A).

iv) C_N is invariant to a homothetic change in the incomes of the individuals between times 0 and 1. This is evident given that expression (16) is written in terms of income shares.

v) Assume that the only change that occurs between time 0 and time 1 is that two individuals swap their income. The impact of such a change on C_N is negative and greater in absolute value, the greater the income gap between the individuals who swapped their incomes (proof in Appendix A).

vi) If a sum δ is transferred from individual k to individual h, assuming as before that the income share s_k is higher than the income share s_h, the value of the index C_N is negative and its absolute value is an increasing function of the transfer δ. The demonstration is very similar to that given for the swap. We only have to replace Δ with δ.

vii) The index C_N follows Dalton's principle of population.[7]

2.4. The anonymous case

We have hitherto assumed non-anonymity, that is, while comparing the shares s_i (at time 0) and w_i (at time 1) we referred to the same individual i. Suppose we do not have panel data and have only two cross-sections of individuals at times 0 and 1.[8] We are still able to use the tools previously defined if we compare what happened over time to individuals having the same rank

[7] The proof is very similar to that given by Silber (1995) for the index of the distributional change index $J_{GI} = I_N$. The only difference is that the ranking criterion for the income shares is now different.

[8] If, as is generally the case, the number of observations in both cross-sections is different, it is always possible to draw a random sample of the same size n, from each cross-section. Another solution could be to estimate quantile functions for both distributions and then use the obtained values for a given vector of percentiles. We thank an anonymous referee for suggesting this alternative solution.

in the income distributions at times 0 and 1. In this case, we do not look at individual income growth rates but at the growth rate over time of, say, given centiles.

2.4.1. Measuring the inequality in growth rates

The approach is the same as in the non-anonymous case. Assuming that the shares s_i and $w_i$refer to a given centile i, if we rank the centiles by increasing ratios (w_i/s_i), the inequality index is written as I_A where the subscript A indicates that we examine the anonymous case. I_A measures the inequality of the growth rates of the various centiles while the equivalent growth rate $\eta_E(I_A)$ is a measure of the growth rates of the various centiles which gives a greater weight to the centiles which have a lower growth rate. By construction, $\eta_E(I_A)$, is smaller than the average growth rate $\overline{\eta}$ of the various centiles. I_A and $\eta_E(I_A)$ have properties similar to those listed previously for I_N and$\eta_E(I_N)$. Note that since we compute growth rates for each centile, I_A will evidently be smaller than I_N, assuming we use panel data to compute anonymous growth rates.

2.4.2. Analyzing σ −convergence of income shares over time

Assume that the shares s_i (at time 0) and w_i (time 1) refer to a given centile i, both sets of centiles being ranked by increasing values of the shares s_i of these centiles at time 0. The index J in (16) is now labeled as C_A and measures the extent of $\sigma-$convergence or divergence. The reason is simple. If C_A is positive (negative), it implies that on average the growth rates of the lower centiles were higher (lower) than those of the higher centiles so that inequality decreased (increase). The equivalent growth rate labeled as $\eta_E(C_A)$ is a weighted average of the growth rates of the various centiles. As in the non-anonymous case, $\eta_E(C_A)$ may be higher or lower than the average growth rate $\overline{\eta}$ of the various centiles. Here again C_A and $\eta_E(C_A)$ share properties with C_N and $\eta_E(C_N)$.

The top panel in Table 1 lists the four indices discussed above. Tables B1 and B2 in Appendix B contain simple illustrations of these measures.

2.5 Defining pro-poor growth

The most popular approach to the analysis of pro-poor growth was proposed by Ravallion and Chen (2003). They defined a growth incidence curve (GIC) and showed that the area under the GIC up to the headcount index is identical to the change in the Watts index times minus 1. They also proved that their measure of the rate of pro-poor growth is equal to the actual growth rate multiplied by the ratio of the actual change in the Watts index to the change that would have been observed with the same growth rate but no change in inequality. A similar perspective was taken by Kakwani and Pernia (2000). Their measure of pro-poor growth could be negative, even if the average growth rate is positive, when there was an important increase in inequality. The definition we propose below also takes into account inequality in the distribution and gives greater weight to the growth rates of the poor, the poorer the individuals are.

2.5.1. Anonymous pro-poor growth

Let us start with the anonymous case which in the literature on pro-poor growth was adopted originally (for example, Kakwani and Pernia, 2000). Assume that a poverty line z has been defined and that, as a consequence, the proportion of poor in the population is (q/n). We define a measure $\eta_E(P_A)$ of the equivalent growth rate among the centiles that were poor at time 0 as

$$\eta_E(P_A) = \left[\left(\sum_{i=1}^{q} \alpha_i \eta_i\right) / \left(\sum_{i=1}^{q} \alpha_i\right)\right] \quad (18)$$

where i refers to a given centile. If $\eta_E(P_A) > \overline{\eta}$, growth has been pro-poor in the anonymous sense, since originally "poor" centiles experienced a higher growth rate.

2.5.2 Non-anonymous pro-poor growth

Similarly, we define a measure of pro-poor growth in the non-anonymous case.

$$\eta_E(P_N) = \left[\left(\sum_{i=1}^{q} \alpha_i \eta_i\right) / \left(\sum_{i=1}^{q} \alpha_i\right)\right] \quad (19)$$

In (19) the subscript i does not refer, as in the anonymous case, to a given centile, but to a given individual whose income is known at times 0 and 1. If $\eta_E(P_N) > \overline{\eta}$, then it implies that non-anonymous growth has been pro-poor. As in the anonymous case, $\eta_E(P_N)$ takes into account the inequality in growth rates among the poor.

Appendix B contains a simple illustration of the pro-poor measures defined below.

Table 1: Summary of Proposed Measures for Income and Non-income Indicators

Data Type	Ranking of shares s_i and w_i	Description	Index	Equivalent Growth Rate	Index	Equivalent Growth Rate
			Traditional Gini		Generalized Gini	
Non-anonymous	Individual shares ranked by increasing values of (w_i/s_i)	Inequality of distribution of income/ non-income dimension	I_N	$\eta_E(I_N)$	$\widetilde{I_N}$	$\widetilde{\eta}_E(\widetilde{I_N})$
Non-anonymous	Individual shares ranked by increasing values of s_i.	β-convergence of income or of a non-income dimension with respect to itself	C_N	$\eta_E(C_N)$	$\widetilde{C_N}$	$\widetilde{\eta}_E(\widetilde{C_N})$
Anonymous	Shares of population centiles ranked by increasing values of (w_i/s_i)	Inequality of distribution of income/ non-income dimension	I_A	$\eta_E(I_A)$	$\widetilde{I_A}$	$\widetilde{\eta}_E(\widetilde{I_A})$
Anonymous	Shares of population centiles ranked by increasing values of s_i	σ-convergence of income or of a non-income dimension with respect to itself	C_A	$\eta_E(C_A)$	$\widetilde{C_A}$	$\widetilde{\eta}_E(\widetilde{C_A})$
Non-anonymous	Individual shares of non-income dimension ranked by increasing values of income shares s_i	β-convergence of non-income dimension with respect to income	C_N^*	$\eta_E(C_N^*)$	$\widetilde{C_N^*}$	$\widetilde{\eta}_E(\widetilde{C_N^*})$
Anonymous	Non-income dimension shares of population centiles ranked by increasing values of income shares s_i	σ-convergence of a non-income dimension with respect to income	C_A^*	$\eta_E(C_A^*)$	$\widetilde{C_A^*}$	$\widetilde{\eta}_E(\widetilde{C_A^*})$

3. Generalized Measures of Convergence and Pro-poor Growth

3.1. Using Generalized Gini Indices

The previous sections have been all based on the idea of extending the use of the traditional Gini index. Jenkins and Van Kerm (2006) have however suggested to measure mobility via the so-called generalized Gini index.[9] One may therefore wonder whether such a generalization can be applied not only to the measurement of inequality and mobility but also to that of convergence and pro-poor growth. Such an extension is in fact quite straightforward. It has been proposed by Deutsch and Silber (2005) in their analysis of normative occupational segregation indices. We summarize their approach, applying it to the measurement of distributional change.

Using Atkinson's (1970) concept of "equally distributed equivalent level of income", Donaldson and Weymark (1980) have defined a generalized Gini index G where

$$G = 1 - \left\{\sum_{i=1}^{n}\left[\left((i^{\gamma} - (i-1)^{\gamma})/n^{\gamma}\right)\left(\frac{x_i}{\bar{x}}\right)\right]\right\} \qquad (20)$$

where x_i is the income of individual i with $x_1 \geq \cdots \geq x_i \geq \cdots \geq x_n$, n is the number of individuals, γ is a parameter measuring the degree of distribution sensitivity ($\gamma > 1$ and the higher γ, the stronger this sensitivity) while $\bar{x}$ is the average income. Donaldson and Weymark (1980) have shown that when $\gamma = 2$, G is equal to Gini's inequality index.

In the case where more than one individual has some income x_i it can easily be shown that expression (20) will be written as

$$G = 1 - \left\{\sum_{i=1}^{n}\left[\left(\left(\left(\sum_{j=1}^{i} n_j\right)^{\gamma}\right) - \left(\left(\sum_{j=1}^{i-1} n_j\right)^{\gamma}\right)\right)\right]\left(\frac{1}{n^{\gamma}}\right)\left(\frac{x_i}{\bar{x}}\right)\right\} \qquad (21)$$

where n_j is the number of individuals with income x_j.

If we now define a coefficient c_i as

[9] See, Yitzhaki, 1983, and Donaldson and Weymark, 1980.

$c_i = [((\sum_{j=1}^{i} n_j)^{\gamma}) - ((\sum_{j=1}^{i-1} n_j)^{\gamma})]/n^{\gamma}$

$$\leftrightarrow c_i = [((\sum_{j=1}^{i} r_j)^{\gamma}) - ((\sum_{j=1}^{i-1} r_j)^{\gamma})] \tag{22}$$

where $r_j = (n_j/n)$ is the relative frequency of income x_j, we can rewrite (20) as

$$G = 1 - \left[\sum_{i=1}^{n} c_i \left(\frac{x_i}{\bar{x}}\right)\right] \tag{23}$$

Let $\tau_i = \left(\frac{r_i x_i}{\bar{x}}\right)$ refer the share of income x_i in total income. The ratio $\left(\frac{x_i}{\bar{x}}\right)$ is expressed as $\left(\frac{\tau_i}{r_i}\right)$ so that (23) will be expressed as

$$G = 1 - \left[\sum_{i=1}^{n} c_i \left(\frac{\tau_i}{r_i}\right)\right] \tag{24}$$

In other words the generalized Gini is a measure transforming a set of "a priori" probabilities r_i (the population shares) into a set of "a posteriori" probabilities τ_i (the income shares) via a set of operators c_i.

3.2. Generalized Inequality Indices

If we now treat as "a priori probabilities" the income shares s_i at time 0 and as "a posteriori probabilities" the set of income shares w_i at time 1, and if we rank these shares by decreasing ratios (w_i/s_i) we obtain, in the non-anonymous case, a generalized measure $\widetilde{I_N}$ of the inequality of individual growth rates, with

$$\widetilde{I_N} = 1 - \left[\sum_{i=1}^{n} c_i \left(\frac{w_i}{s_i}\right)\right] \tag{25}$$

with

$$c_i = [((\sum_{j=1}^{i} s_j)^{\gamma}) - ((\sum_{j=1}^{i-1} s_j)^{\gamma})] \tag{26}$$

It is easy to check that, in the non-anonymous case, when γ=2 and the income shares s_i and w_i are ranked by decreasing ratios (w_i/s_i), $\widetilde{I_N}$ is identical to I_N. Combining (11) and (25) we may derive that

$$\widetilde{I_N} = 1 - \left[\frac{1+\widetilde{\eta}_E(\widetilde{I_N})}{1+\overline{\eta}}\right] \tag{27}$$

where $\widetilde{\eta}_E(\widetilde{I_N})$ refers to the equivalent growth rate when a generalized index of inequality is computed.

Similar results may be derived in the anonymous case. Thus if the income shares s_i and w_i are ranked by decreasing ratios (w_i/s_i), expressions (26) and (27) may be used to derive an anonymous generalized measure $\widetilde{I_A}$ of the inequality of the growth rates of the various centiles and an equivalent growth rate $\widetilde{\eta}_E(\widetilde{I_A})$.

3.3. Generalized Convergence Indices

From expressions (26) and (27) one can also derive generalized expression of measures of convergence. If we rank the income shares s_i and w_i by decreasing values of the original shares s_i, we obtain, in the non-anonymous case, a generalized measure $\widetilde{C_N}$ of $\beta-$convergence (convergence over time of the various income shares).

$$\widetilde{C_N} = 1 - \left[\sum_{i=1}^{n} d_i \left(\frac{w_i}{s_i}\right)\right] \tag{28}$$

with

$$d_i = \left[\left(\left(\sum_{j=1}^{i} s_j\right)^{\gamma}\right) - \left(\left(\sum_{j=1}^{i-1} s_j\right)^{\gamma}\right)\right] \tag{29}$$

It is also possible to derive an equivalent growth rate $\widetilde{\eta}_E(\widetilde{C_N})$ when a generalized measure $\widetilde{C_N}$ of convergence is computed. Expressions (28) and (29) may be used to derive an anonymous generalized measure $\widetilde{C_A}$ of $\sigma-$convergence (convergence of the income shares of the various centiles). It is important to stress that although expressions (25) and (26) on one hand, and (28) and (29) on the other hand, are very similar, they are not identical as the ranking criterion is different. We can also derive an equivalent growth rate $\widetilde{\eta}_E(\widetilde{C_A})$ in the anonymous case, assuming a generalized distributional change index $\widetilde{C_A}$ is computed. Thus the parameter γ allows us to define generalized measures of the inequality of individual growth rates as well as of convergence.[10]

The generalized indices are listed alongside with the indices based on traditional Gini indices in the top panel of Table 1. Tables B3 and B4 in Appendix B present a numerical illustration of the generalized indices for different values of γ.

3.4 Decomposition of the generalized convergence index

Combining (27) and (28) we can express

$$\widetilde{C_N} = \left\{1 - \left[\sum_{i=1}^{n} c_i \left(\frac{w_i}{s_i}\right)\right]\right\} + \left\{\left[\sum_{i=1}^{n} c_i \left(\frac{w_i}{s_i}\right)\right] - \left[\sum_{i=1}^{n} d_i \left(\frac{w_i}{s_i}\right)\right]\right\}$$

$$\leftrightarrow \widetilde{C_N} = \widetilde{I_N} + \left[\sum_{i=1}^{n}(c_i - d_i)\left(\frac{w_i}{s_i}\right)\right] \quad (30)$$

Since $\widetilde{I_N}$ is always positive and since in the case of convergence we know that $\widetilde{C_N} < 0$, there are two conditions to observe convergence:

i) the difference $\left[\sum_{i=1}^{n}(c_i - d_i)\left(\frac{w_i}{s_i}\right)\right]$ must be negative

ii) $\left|\left[\sum_{i=1}^{n}(c_i - d_i)\left(\frac{w_i}{s_i}\right)\right]\right| > \widetilde{I_N}$

The first condition implies that we should observe that when c_i is small, d_i is high. But c_i is small for high growth rates and d_i is high for low incomes. The first condition shows then clearly

[10] We could also derive generalized measures of pro-poor growth. The corresponding expressions may be obtained upon request from the authors.

that to observe convergence, the low incomes (at time 0) should have high growth rates, which implies, as expected, in the case where $\gamma = 2$, that the relative concentration curve should be above the diagonal for low incomes.

Note that $\widetilde{I_N}$ in fact is a measure of structural mobility, since it measures the inequality in the individual growth rates. The second expression on the R.H.S. of (30) is a measure of the extent of re-ranking of individual shares which is observed when individual growth rates are ranked by decreasing ratios of these growth rates rather than by decreasing values of the original income shares. Hence it is a measure of exchange mobility. Similar decompositions may evidently be derived in the anonymous case.[11]

Using again the data of Table B1, Table B5 in Appendix B gives in the non-anonymous case, the contribution of what was labeled previously structural and exchange mobility. Given that in our numerical example (see, Table B1) the poorest individual has an extremely high growth rate (his/her income increases from 5 to 100), we should not be surprised to observe that the higher the value of the parameter γ, the greater the relative importance of exchange mobility.

4. Measures of Distributional Change for Non-Income Indicators

So far we have measured inequality in growth rates as well as convergence over time, in incomes. A similar analysis can naturally be extended to study variations over time in other types of variables, such as educational levels or some measures of health. We do not repeat the corresponding expressions since the only change to be implemented is to replace income growth by, say, growth in years of education (see the top panel of Table 1 for a summary of indices).

[11] Jännti and Jenkins (2013) provide a thorough review of the issues related to the measurement of income mobility.

4.1. Convergence in Non-Income Indicators with respect to Income

In the previous analysis we considered two ranking criteria, in both the non-anonymous and the anonymous case. The first one classified the shares s_i and w_i according to the value of the ratio$\left(\frac{w_i}{s_i}\right)$. The second one classified these two sets of shares according to the values of the original shares s_i. Suppose we look at the growth rates in individual years of education but we classify these growth rates according to the incomes of the individual. Then convergence indices for education will be measured as a function of income. For example, we can define, in the non-anonymous case, a measure C_N^* of convergence of individual levels of education as a function of individual incomes (hence the subscript *). If C_N^* is negative, then it implies that the lower the original income, the higher the growth rate in educational levels. The expression for C_N^* would be identical to that used to define C_N, the latter being computed on the basis of expression (16). The only difference is that, first the shares s_i and w_i refer now to educational and not to income shares, second, the ranking criterion is not that of the original educational shares but that of the original income shares[12]. Using the generalized Gini index, we can also compute a generalized version of the convergence index and denote it by $\widetilde{C_N^*}$. We can also derive similar indices C_A^* and $\widetilde{C_A^*}$ for the anonymous case.

The bottom panel of Table 1 summarizes the indices proposed in this section.[13]

4.2. Convergence conditional on income

[12] Dawkins (2004, 2006) had somehow a similar idea when, extending the use of Silber's (1989) G-matrix, he proposed a Gini-related formulation of an index of residential segregation.

[13] We can also compute equivalent growth rate of education by restricting the analysis to those defined as income poor. For instance, we can check whether the growth in education was pro-poor, that is, in favor of those who originally had a low income by computing a weighted sum of the growth rates in educational levels $\eta_E^*(P_N)$. This sum would be limited to those individuals considered as poor at time 0.

Assume we have data for two periods on both the income and educational level of individuals. We can compute an index C_N of the convergence over time in individual educational levels. We can also compute, as mentioned previously, a measure C_N^* of the convergence of individual levels of education as a function of individual incomes. The difference between C_N and C_N^* measures the conditional (on income) β-convergence. Given that a negative index is a sign of pro-poorness, we can infer, if this difference is negative (positive), that the growth rates in individual educational levels were generally higher for individuals having low (high) values of the other determinants (income excluded) of these growth rates. Similarly in the anonymous case we can find the difference between C_A and C_A^* and check whether the growth rates in the educational levels of the various centiles were generally higher, the lower the level of the other non-income determinants of educational levels.

5. An empirical illustration

5.1 Data

In this section we compute some of the proposed indices to analyze infant mortality rates in India, a key indicator in the Millennium Development Goals (MDGs). We use data at the state level because our approach is particularly useful when the number of observations is relatively small.[14] Reducing child mortality is one of the eight MDGs; the goal being to reduce by two thirds, between 1990 and 2015, the under-five mortality rate in member countries of the United Nations. Compared to the rest of the world, child mortality rates in India have been significantly higher, primarily because of high infant mortality rates.[15] In 1990, the infant mortality rate in

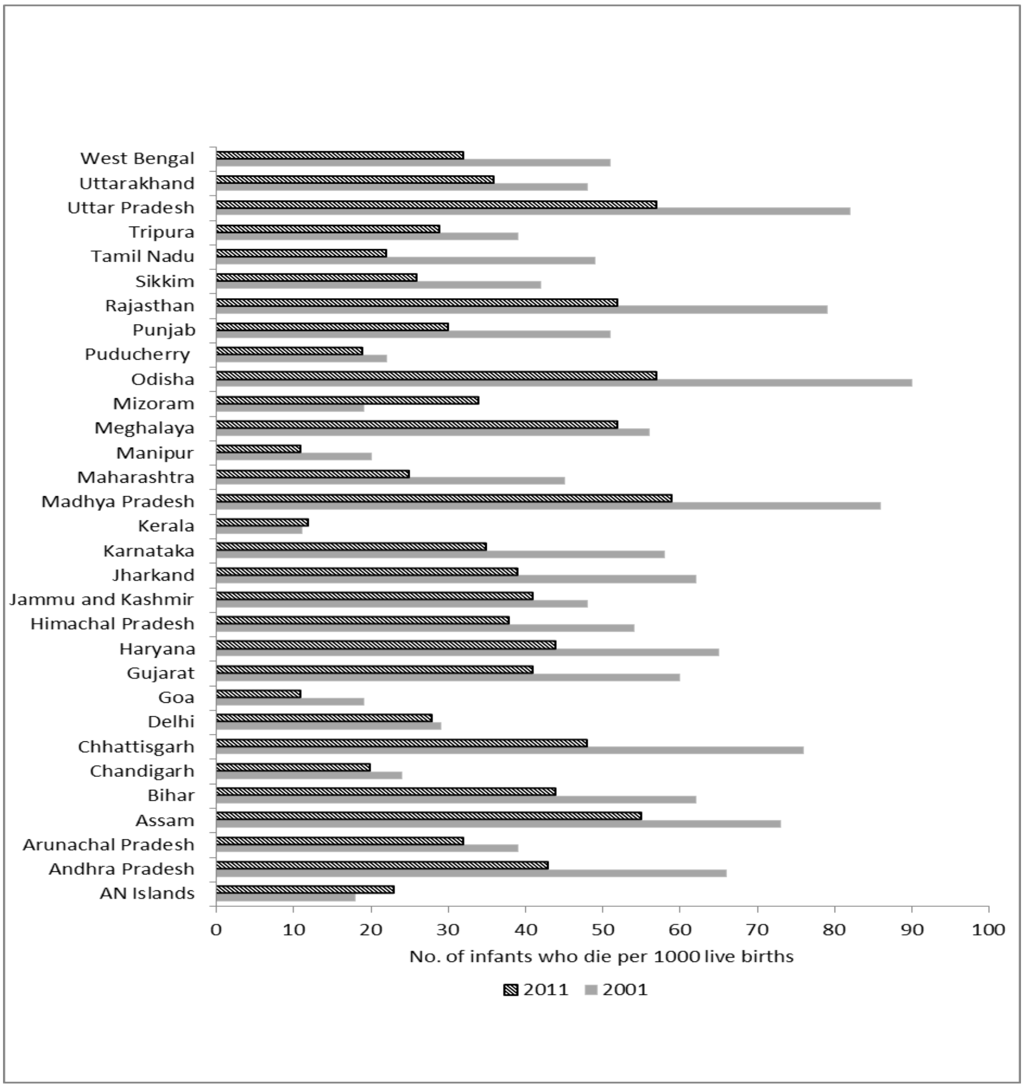
[14] See, Harrtgen and Klasen (2011), for an interesting approach to the measurement of survival, and more generally human development, at the household level.

[15] The infant mortality rate measures the number of children (aged less than one year) who die per 1000 live birth.

India was 80 per 1000 and it declined to about 44 per 1000 in 2011. India needs to reduce the rate further to 27 per 1000 by 2015, in order to achieve the MDG.

There exists significant regional variation in infant mortality rates as seen in Figure 2. For example, in 2011, infant mortality rate was the lowest (11 per 1000) in Goa and highest (59 per 1000) in Madhya Pradesh. Only a handful of states such as, Goa, Kerala and Tamil Nadu in the south and Manipur and Sikkim in the east had rates lower than the target (27 per 1000). On the other hand, many states (Assam, Madhya Pradesh, Meghalaya, Odisha, Tamil Nadu, Uttar Pradesh) had rates higher (more than 50 per 1000) than those in some of the poorest Sub-Saharan African countries such as Ethiopia, Malawi, Namibia, Rwanda.

Figure 2: Infant Mortality Rates across Indian States



Source: Census Bureau of India

Using data from two recent rounds of the Indian Census, namely 2001 and 2011, we calculate for each state *i* its share in the number of infants who survived their first year, on the basis of data on infant mortality rates (MR_i), birth rates (BR_i) and total population (TP_i).[16] More populous states such as Maharashtra, and Uttar Pradesh evidently had greater shares of the number of surviving infants, compared to less populous states such as Sikkim, and Andaman and Nicobar Islands.

[16] Data is available for all states (except Nagaland) and for 4 of the 7 union territories: http://censusindia.gov

In place of infant mortality rates, we compute infant survival rates (as did Grosse et. al., 2008).[17] State i's infant survival rates (SR_i) are derived by taking a linear transformation of the state's infant mortality rates$(SR_i = 1000 - MR_i)$. Thus state i's share in the number of infants who survived is calculated as follows.

$$share\ of\ survived\ infants_i = \frac{SR_i x BR_i x share\ of\ TP_i}{\sum SR_i x BR_i x share\ of\ TP_i} \tag{31}$$

Equation (31) shows that a state's share of survived infants is weighted by its share in total live births. Table 2 gives the estimates of some of the indices proposed in the Section 2 along with the bootstrap confidence intervals.[18] It is apparent from Equation (31) that a rise in a state's share in the number of surviving infants may be due to an increase in infant survival rates (SR_i) or in the state's birth rates (BR_i). [19]

In the non-anonymous case, we compare a state's share in 2001 with the same state's share in 2011. On the other hand in the anonymous case, we compare the share of a state which had rank i in 2011 with the share of the state which had rank i in 2001, these states being generally, but not necessarily, different. Overall we find that the estimated values of the various anonymous and non-anonymous indices are close but generally statistically different. This is because there was not much difference in the ranking of the states over time; only 8 out of 31 states changed

[17] An improvement in child mortality comes out as a lower value but this lower value is mathematically interpreted as deterioration. Since survival rates are positive entities the interpretation is easier and more intuitive.

[18] Bootstrap confidence intervals were derived as follows. Since infant survival rates are expressed in per thousand, we first made 1000 independent draws of numbers lying between 1 and 1000. For each state we compared each of these random numbers with the actual infant survival rate in this state. Whenever the random number was smaller than the actual infant survival rate of the state, we added 1 to the estimated infant survival rate for this state. Once the comparison was made with these 1000 random numbers we had a first estimate of infant survival rates in each state. We repeated this procedure 1000 times and then for each state we ranked the 1000 estimates of the infant survival rates by increasing values. The 5% and 95% confidence intervals was then obtained by writing down the $50^{th}$and $950^{th}$of these ranked infant survival rates.

[19] We undertook a so-called Shapley decomposition (Shorrocks, 2013) and found that most of the variation in states' share of surviving infants was due to variation in the states' birth shares. This is not surprising since changes over time in survival rates are by definition small when compared to changes in infant mortality rates. Results of the decomposition are available upon request to the authors.

their ranking by moving up/down by no more than 1 place; the Spearman's rank correlation for the two time periods was equal to 0.99.

Table 2: Estimates of Proposed Indices using Data on Infant Survival Rates in India

	Non Anonymous	**Anonymous**
Inequality of infant survival growth rates	I_N= 0.0405 *(0.0392 to 0.0418)*	I_A= 0.0378 *(0.0365 to 0.0389)*
Convergence of infant survival rates	C_N= 0.0098 *(0.0085 to 0.0111)*	C_A= 0.0102 *(0.0089 to 0.0115)*
Income related convergence of infant survival rates	C_N^* = -0.0282 *(-0.0301;-0.0265)*	C_A^* = -0.0245 *(-0.0260; -0.0221)*
Conditional (on income) convergence of infant survival rates	$C_N - C_N^*$ =0.0380 *(0.0351; 0.0411)*	$C_A - C_A^*$ =0.0347 *(0.0314; 0.0373)*

Source: Authors' calculations; 5% and 95% bootstrap confidence intervals are given in parentheses.

5.2 Inequality in the growth rates

As seen in Table 2, the indices measuring inequality in infant survival growth rates, in the non-anonymous $(I_N = 0.0405)$ as well as in the anonymous case$(I_A = 0.0378)$, are close to 0 though the confidence intervals show that they are statistically significantly different. Both estimates indicate that inequality in infant survival growth rates was low.

5.3 Convergence over time

Convergence in infant survival levels between 2001 and 2011 is measured by estimating the index C_N and C_A . The non-anonymous convergence index C_N is equal to 0.0098 and statistically significantly different from 0 suggesting mild β-divergence among states. Thus states with lower (higher) shares of survived infants in 2001 also had lower (higher) shares in 2011. The anonymous index C_A is equal to 0.0102 and statistically significantly different from 0 indicating that there was no evidence of σ - convergence. Both indices suggest survival levels did not converge much over time.

5.4 Income related convergence of infant survival growth rates

State income levels are measured as per capita net state domestic products at constant (2004-2005) prices. In the non-anonymous case, we rank all shares by increasing values of state average incomes in 2001 and estimate the index C_N^*. In the anonymous case, we rank shares in 2001 by increasing values of income in 2001 and shares in 2011 by increasing values of income in 2011 and estimate the index C_A^* . The indices measure the relationship between growth rates in survival levels and corresponding income levels. In both the cases, the estimated indices are negative and, though small in magnitude, they are significantly different from 0. Thus growth in survival levels was slightly higher, the lower the state income.

5.5 Conditional Convergence in growth rates

In the non-anonymous case, the indices C_N and C_A^* both use the same data on states shares. The difference between the two indices is that in the former, the shares are ranked by increasing values of infant survival shares in 2001, and in the latter, they are ranked by increasing values of income in 2001. If we take the difference between these two indices, a negative difference suggests conditional (on income) β -convergence. We find that the difference between the estimated indices is positive. Thus, growth rates in survival levels were generally higher for states having high values of other non-income determinants of these growth rates, implying conditional on income β -divergence. In the anonymous case, we take the difference between

C_A and C_A^*. Table 2 shows that the difference is positive and statistically significantly different than 0, suggesting again conditional (on income) σ -divergence in infant survival rates.

6. Concluding comments

This paper proposed a unified analytical framework to derive indices of inequality in growth rates, β -and $\sigma-$convergence. In the case of income it was shown that the computation of all these indices was based on the comparison of original (time 0) and final (time 1) income shares, the specificity of each measure depending first on whether a non-anonymous or an anonymous approach was taken, second on the ranking criterion selected to classify these income shares. The computation of inequality in growth rates required thus to rank the shares by the ratios of their values at times 1 and 0 while convergence estimates assumed that the shares were classified according to their values at time 0. In all cases we also defined what we called "equivalent growth rates", that is, a weighted average of the growth rates. In the case of inequality such an equivalent growth rate took into account the inequality of the growth rates and thus was smaller than the average growth rate. For convergence the equivalent growth rate turned out to be higher (smaller) than the average growth rate when growth was higher (smaller) for those who originally had a lower income share. The paper also showed that the same approach could be implemented to derive pro-poor growth rates, whether in the non-anonymous or in the anonymous case.

The analysis was then extended to derive generalized measures of inequality in growth rates or convergence, in the same way as inequality and mobility indices had previously been introduced in the literature, based on the notion of generalized Gini index. We also showed that a generalized convergence index can be decomposed to account for structural mobility and exchange mobility.

Finally the paper explained that the same kind of analysis could be applied to non-income indicators and could also allow one to analyze the link between these indicators and income. In other words on the basis of the analytical framework proposed in this paper it also possible to measure the convergence of non-income indicators, this convergence being estimated with respect to income. The methodology may be also applied to measure convergence, conditional on income.

An empirical illustration based on the Census data on Indian states was then presented to show the relevance of some of the concepts introduced in this paper. We found that there was not much inequality in the growth rates of infant survival levels among states. The evidence suggests that policies such as universal immunization of infants against measles were effective across states. In fact our estimates indicate that survival levels were slightly higher among states with lower per capita incomes. For example, Bihar which had the lowest per capita income in 2001, witnessed one of the fastest growth (38 percent) in the number of infants surviving their first year between 2001 and 2011. In this sense, growth in infant survival rates in India was pro-poor.

The empirical example illustrates that from a policy point of view, the analytical framework introduced in this paper will be useful in many ways. The unified approach makes it feasible to empirically estimate distributional indices of inequality, convergence, mobility and pro-poor growth even when data is limited and available in aggregate form such as population quintiles. Each of the proposed indices can be measured with cross-sectional data or panel data. Furthermore, the proposed indices facilitate analyzing changes in the distribution of non-income indicators and their eventual link to income growth. The rapid growth in the literature on multidimensional poverty in the recent years underscores the fact that growth in income is not sufficient to reduce poverty. The Millennium Development Goals emphasized universal primary education, promoting gender equality and improvement in maternal health, ensuring environmental sustainability. The paper offers a tool box to practitioners who are interested in measuring, for instance, pro-poorness of growth in non-income indicators.

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Appendix A: Proofs of the Properties

The non-anonymous case

Measuring inequality in growth rates

i) When the growth rates η_i are not all identical, $\eta_E(I_N)$ will always be smaller than $\overline{\eta}$ so that $0 < I_N < 1$

Proof: Since in this instance the coefficient α_i is higher, the higher η_i, the function $f = \alpha_i \eta_i$ not only rises with η_i but it increases at an increasing rate. Note also that $\overline{\eta}$ may be expressed as

$$\overline{\eta} = \left(\frac{\Delta\bar{x}}{\bar{x}}\right) = \left(\frac{(1/n)\sum_i \Delta x_i}{(1/n)\sum_i x_i}\right) = \sum_i \left(\frac{\Delta x_i}{x_i}\right)\left(\frac{x_i}{\sum_i x_i}\right) = \sum_i s_i \left(\frac{\Delta x_i}{x_i}\right) = \sum_i s_i \eta_i \qquad \text{(A-1)}$$

Combining (10) and (A-1) we may also write that

$$\overline{\eta} = \sum_i \alpha_i \overline{\eta} = \sum_i \alpha_i \left(\sum_i s_i \eta_i\right) \qquad \text{(A-2)}$$

Comparing then (12) and (A-1) and given the shape of the function $f = \alpha_i \eta_i$ we easily conclude, using properties of convex functions, that $\overline{\eta} \geq \eta_E(I_N)$.

Analyzing $\beta-$ convergence of income shares over time

i) The equivalent growth rate $\eta_E(C_N)$ may be greater or smaller than $\overline{\eta}$

Proof: Combining (8) and (12) we may write that

$$\eta_E = \sum_{i=1}^{n} \alpha_i \eta_i = \sum_{i=1}^{n} \left\{ s_i \left[s_i + 2\sum_{j<i} s_j \right] \right\} \eta_i = \sum_{i=1}^{n} s_i a_i \eta_i \qquad \text{(A-3)}$$

where $a_i = \left[s_i + 2\sum_{j<i} s_j \right]$.

Although a_i will be higher, the poorer the individual at time 0, we observe, given that $\overline{\eta} = \sum_i s_i \eta_i$, that $(s_i a_i)$ may be higher or smaller than s_i so that there will be no clear link between the value of the weights $\alpha_i = s_i\left[s_i + 2\sum_{j<i} s_j \right]$ and the original shares s_i at time 0. We therefore cannot know whether $\eta_E(C_N)$ will be higher or smaller than $\overline{\eta}$.

iii) The index C_N varies between -1 and +1.

Proof: Using (16) it is first easy to see that when $s_i = w_i \ \forall i, C_N = 0$.

Assume now that the income shares at time 0 are all equal to $(1/n)$ except for two individuals, the poorest one whose share is $((1/n) - \varepsilon)$ and the richest one whose share is $((1/n) + \varepsilon)$ where ε is infinitesimal. If at time 1 the shares w_i are such that $w_1 = \cdots = w_i = \cdots w_{n-1} = 0$

while $w_n = 1$, it is easy to prove, using (16) and ranking both set of shares (the shares s_i and w_i) by increasing values of the shares s_i, that $C_N \rightarrow 1$ as $n \rightarrow \infty$.

Assume now that the income shares at time 0 are all equal to $(1/n)$ except for two individuals, the poorest one whose share is $\left((1/n) - \varepsilon\right)$ and the richest one whose share is $\left((1/n) + \varepsilon\right)$ where ε is infinitesimal. If at time 1 the shares w_i are such that $w_2 = \cdots = w_i = \cdots w_{n-1} = 0$ while $w_1 = 1$, it is again easy to prove, using (16) and ranking both set of shares (the shares s_i and w_i) by increasing values of the shares s_i, that $C_N \rightarrow -1$ as $n \rightarrow \infty$.

v) When two individuals swap their income, the impact of such a change on C_N will be negative and greater in absolute value, the greater the income gap between the individuals who swapped their incomes.

Proof: Assume that individuals h and k swapped their incomes and that this is the only change that occurred between times 0 and 1. The original shares are ranked by increasing values so that $s_h < s_k$. Define Δ as $\Delta = s_k - s_h$. Expression (15) will then be written as

$$C_N = s^{'} Hw = s^{'} Hw - s^{'} Hs = s' H(w - s) \tag{A-4}$$

since $s' Hs = 0$.

We then observe that

$$C_N = s' H(w - s) = [s_1, \dots, s_{h-1}, s_h, s_{h+1}, \dots, s_{k-1}, s_k, s_{k+1}, \dots, s_n]' Hv \tag{A-5}$$

where v', the row vector corresponding to the column vector v is written as

$$v = [0, \dots, 0, \Delta, 0, \dots, 0, (-\Delta), 0, \dots, 0] \tag{A-6}$$

Combining (A-5) and (A-6) it is easy to derive that

$$C_N = s^{'} H(w - s) = s_h(-\Delta) + \sum_{j=h+1}^{k-1} s_j(-\Delta) + s_k(-\Delta) < 0 \tag{A-7}$$

Appendix B: Simple Numerical Illustrations of the Proposed Indices

Table B1: Non-Anonymous I

	Income		Convergence Index			Inequality Index		
Obs.	Time 0	Time 1	Obs.	(s_i)	(w_i)	Obs.	(s_i)	(w_i)
A	100	60	E	0.02	0.33	C	0.20	0.10
B	80	100	D	0.06	0.03	A	0.40	0.20
C	50	30	C	0.20	0.10	D	0.06	0.03
D	15	10	B	0.32	0.33	B	0.32	0.33
E	5	100	A	0.40	0.20	E	0.02	0.33
$\overline{\eta}$=20%	250	300	C_N =-0.34	$\eta_E(C_N) = 60.9\%$		$I_N = 0.43$	$\eta_E(I_N) = -31.4\%$	

Source: Authors' calculations

Note: Using (11), we know, since $0 \leq J \leq 1$, that $0 \leq \left\{\frac{[1+\eta_E(I_N)]}{[1+\overline{\eta}]} = (1-J)\right\} \leq 1$. As a consequence $0 \leq \left(1+\eta_E\right) \leq (1+\overline{\eta})$. It follows that $-1 \leq \eta_E \leq \overline{\eta}$ so that η_E may be negative.

Table B2: Anonymous Case

Convergence				Inequality			
Obs.	(s_i)	Obs.	(w_i)	Obs.	(s_i)	Obs.	(w_i)
E	0.02	D	0.03	A	0.40	B	0.33
D	0.06	C	0.10	C	0.20	A	0.20
C	0.20	A	0.20	B	0.32	E	0.33
B	0.32	E	0.33	E	0.02	D	0.03
A	0.40	B	0.33	D	0.06	C	0.10
$\overline{\eta} = 20\%$	C_A =-0.091	$\eta_E(C_A) = 30.9\%$			$I_N = 0.096$	$\eta_E(I_A)$= 8.5%	

Source: Authors' calculations

Equivalent growth rate among the poor:

Refer to the data of Table B1 and assume that the poverty line z is equal to half the median income that is 25. In the non-anonymous case, there are two poor individuals at time 0, with incomes of 5 and 15. Their respective incomes at time 1 are 100 and 10. The average rate of income growth in the population is $\overline{\eta}$=20%. It is easy to check, using (18), that in such a case, $\eta_E(P_N) = 465.1\%$. In the anonymous case, the incomes of the two lowest quintiles at time 0 are 5 and 15. At time 1 the incomes of the two lowest quintiles are 10 and 30. The average rate of income growth is, as before, 20%. But using (18), we derive that $\eta_E(P_A) = 100\%$.

Table B3: Generalized Inequality Indices and Equivalent Growth Rates

Values of γ	$\widetilde{I_N}$	$\widetilde{\eta}_E(\widetilde{I_N})$	$\widetilde{I_A}$	$\widetilde{\eta}_E(\widetilde{I_A})$
2	0.429	-31.4%	0.096	8.5%
3	0.477	-37.3%	0.128	4.7%
5	0.497	-39.7%	0.153	1.6%
10	0.499	-40%	0.166	0.1%
100	0.5	-40%	0.167	0%

Source: Authors' calculations; Refer to the data of Table B1

Table B4: Generalized Convergence indices and Equivalent Growth Rates

Values of γ	$\widetilde{C_N}$	$\widetilde{\eta}_E(\widetilde{C_N})$	$\widetilde{C_A}$	$\widetilde{\eta}_E(\widetilde{C_A})$
2	-0.341	60.9%	-0.091	30.9%
3	-0.627	95.3%	-0.149	37.9%
5	-1.166	159.9%	-0.233	47.9%
10	-2.499	319.9%	-0.379	65.4%
100	-13.53	1643.6%	-0.666	99.9%

Source: Authors' calculations; Refer to the data of Table B1.

Note: The very high equivalent growth rates when γ is high is a consequence of the numerical example of Table B1 where the poorest individual's income increases from 5 to 100.

Table B5: Decomposition of Generalized Convergence Index (non-anonymous case)

Values of γ	Convergence $\widetilde{C_N}$	Structural Mobility $\widetilde{I_N}$	Exchange Mobility $\left[\sum_{i=1}^{n}(c_i - d_i)\left(\frac{w_i}{s_i}\right)\right]$
2	-0.341	0.429	-0.77
3	-0.627	0.477	-1.105
5	-1.166	0.497	-1.663
10	-2.499	0.499	-2.998
100	-13.53	0.5	-14.03

Source: Authors' calculations; Refer to the data of Table B1