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Differences in needs and multidimensional deprivation measurement*

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Abstract

Individuals from different demographic population subgroups and households of different size and composition exhibit different needs. Multidimensional deprivation comparisons in presence of these differences in needs have yet to be analysed. This paper proposes a family of multidimensional deprivation indices that explicitly takes into account observed differences in needs across demographically heterogeneous units (i.e., either households of different size and composition or individuals of different population subgroups). The proposed counting family of multidimensional indices builds upon the Alkire and Foster methodology of poverty measurement (J. Public Econ. 95:476-487, 2011) and draws from the one-dimensional parametric equivalence scale literature. It aims to describe how much deprivation two demographically heterogeneous units with different needs must exhibit to be catalogued as equivalently deprived. Differences in needs are considered, in this context, as a legitimate source of differences in multidimensional deprivation incidence. Under this premise and through microsimulation techniques, applied over the 2013 Paraguayan household survey, I evaluate the measurement approaches contained in the proposed family of measures. The obtained results demonstrate that neglecting differences in needs yields biased multidimensional deprivation incidence profiles. Results also shed light on the ability of the proposed measures of this paper to effectively capture these differences in need.

Keywords: Multidimensional deprivation, poverty measurement, equivalence scales, heterogeneous households, individual heterogeneity.

JEL Classification: D63, I32.

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1 Introduction

There is increasing interest in measuring poverty by assessing deprivation in multiple dimensions of well-being rather than by exclusively evaluating the ability to consume market commodities. Within this growing literature, most of the applications of multidimensional deprivation measurement[1] use the Alkire and Foster (2011) method and either individuals or households as the unit of analysis.

However, differences in needs are present when measuring multidimensional deprivation across either individuals from different demographic sub-population groups or households of different sizes and compositions. While pregnant women, for instance, need access to antenatal health services, school-age children need access to basic education services. Deprivation of antenatal health services is thus relevant only to pregnant women, and access to basic education services is only relevant to school-age children. Similarly, households without children are not necessarily deprived in the absence of educational services and vaccinations, just as households without pregnant women are not necessarily deprived because of a lack of antenatal health services.

Differences in needs therefore pose comparability challenges when measuring multidimensional deprivation across demographically heterogeneous units, such as households of different sizes and compositions or individuals of different age ranges and genders.

A plethora of methods and techniques that account for differences in needs can be found in the one-dimensional welfare literature. Examples of this literature include Kapteyn and Van Praag (1978); Pollak and Wales (1979); Blundell and Lewbel (1991); Coulter et al. (1992b); Cowell and Mercader-Prats (1999); Duclos and Mercader-Prats (1999), and Ebert and Moyes (2003). They aim to provide societal profiles based on comparable household-based aggregates of income or expenditure obtained through the use of *equivalence scales.*

[1] Although the terms 'multidimensional deprivation' and 'multidimensional poverty' are used interchangeably in literature, throughout this article, the term 'multidimensional deprivation' is used to refer to indices that count the multiple deprivations jointly observed across a selected unit of analysis and, based on this counting procedure, identify the poor as the most deprived population. Examples of this long-standing literature are studies such as Townsend (1979), Atkinson and Bourguignon (1982), Mack et al. (1985), Callan et al. (1993), Feres and Mancero (2001), Atkinson (2002), Alkire and Foster (2011), and Aaberge and Brandolini (2014b).

In contrast to the one-dimensional welfare literature, comparisons of multidimensional deprivation between demographically dissimilar units have yet to be described. In fact, theoretically developed families of multidimensional indices such as those proposed by Tsui (2002); Bourguignon and Chakravarty (2003); Alkire and Foster (2011), and Seth (2010) have been developed exclusively using the individual as the unit of analysis, and do not discuss the arising comparability problems that heterogeneity in needs across units might pose.

This paper proposes a family of indices that measures multidimensional deprivation across demographically heterogeneous units while explicitly taking into account differences in needs across them. The proposed approach extends the Alkire and Foster (2011) counting family of multidimensional poverty indices, providing a wider set of indices that aims to adjust for observable differences in needs across demographically heterogeneous units. This is the methodological contribution of this paper to the multidimensional measurement literature.

The choice of the individual or the household as the unit of analysis is not arbitrary. It involves a normative decision to be made during the multidimensional measurement process. Household-based measures conceive households as cooperative units that jointly face the deprivation suffered by the household members, as, for instance, Angulo et al. (2015) discussed regarding the selection of the household as the unit of analysis for the Colombian Multidimensional Poverty Index. Individual-based measures, in contrast, allow the unmasking of differences in multidimensional deprivation across demographic sub-population groups, such as the case of gender differences analysed by Vijaya et al. (2014) for Karnataka, India or by Agbodji et al. (2013) for Burkina Faso and Togo.[2]

The family of indices that I propose in this article allows multidimensional deprivation to be measured using either individuals or households as the unit of analysis. The choice of individual or household is therefore open to be made according to the context of each application.

In the case of household-based multidimensional measures, the purpose of accounting for differences in needs is to enable pairs of households and thus different populations of households to be compared on a more equivalent basis. Similarly, in

[2]For a broad discussion of the different normative decisions embedded in multidimensional measurement, see Alkire et al. (2015a).

the individual-based case, the indices proposed in this paper aim to enable multidimensional deprivation comparisons of any two individuals and hence of different populations of individuals.

Furthermore, this family of measures describes, under equivalent normative considerations, the burden that multidimensional deprivation places on each unit of analysis. For instance, under an absolute normative perspective where each deprivation has an equal absolute value, multidimensional deprivation is described through count-based approaches to measurement. Conversely, under a relative normative perspective that conceives each unit of analysis as equivalently valuable, multidimensional deprivation is described in terms of share-based approaches to measurement. Intermediate normative perspectives, in contrast, lead to the expression of multidimensional deprivation as a mixture of count-based and share-based approaches to measurement.

I evaluate the effect of these different approaches to measurement on multidimensional deprivation incidence profiles. Specifically, using the ethical conditions set out by Fleurbaey and Schokkaert (2009) to catalogue differences in health outcomes as fair/legitimate and unfair/illegitimate, differences in needs are considered for the purpose of this paper as a source of fair and unavoidable differences in multidimensional deprivation incidence. Through microsimulation techniques and using the 2013 Paraguayan household survey, I seek to disentangle how much of the differences in multidimensional deprivation incidence profiles are observed because unaddressed differences in needs. The results of this evaluation demonstrate that unaddressed differences in needs yield multidimensional deprivation incidence profiles to reflect not only illegitimate differences in deprivation but also differences in needs that should be tackled by the measurement process. Failure to take differences in needs into account was found to cause biased multidimensional incidence profiles. Results also shed light on the ability of the proposed measures of this article to effectively capture these differences in need.

The starting point of this paper is the background literature that analyses welfare comparisons in the presence of heterogeneous needs. The next section presents an overview of this literature and the equivalence scale notion that seeds the family of indices proposed in this paper, along with a description of the relevant multidimensional measurement background literature.

2 Background

A plethora of methods and techniques from the one-dimensional literature attempts to assess welfare and inequality rankings while taking into account differences in needs between households. Examples of this in the literature are Kapteyn and Van Praag (1978); Pollak and Wales (1979); Blundell and Lewbel (1991); Coulter et al. (1992b); Cowell and Mercader-Prats (1999); Duclos and Mercader-Prats (1999), and Ebert and Moyes (2003). Within this literature, these technologies are known as *equivalence scales*. Their relevance, as pointed out by Cowell and Mercader-Prats (1999), is crucial for inequality and social welfare comparisons: "equivalence scales, by providing an interpersonally comparable measure of living standards, play a central role in the assessment of social welfare and income inequality. Failure to take account of the relationship between nominal and equivalized income can give a biased picture of both inequality and social welfare" (Cowell and Mercader-Prats, 1999, p. 409).

The proposed family of measures of this paper draws from this welfare equivalence scale literature to provide comparable measures of multidimensional deprivation for demographically heterogeneous units. We continue therefore describing, first this equivalence scale literature and then the relevant multidimensional measurement literature.

2.1 Welfare comparisons in the presence of heterogeneous needs

Equivalence scales have been used to allow the construction of societal measures of welfare and inequality based on comparable household measurements of income or expenditure (Fisher, 1987; Muellbauer, 1974). These scales intend to reflect the amount of income required for households of different sizes and compositions to have the same welfare level (Pollak and Wales, 1979; Nelson, 1993). An important emerging fact from reading this literature is that there is no universally correct equivalence scale. Different procedures are justified according to different circumstances.

From the empirical perspective, two main approaches to construct equivalence scales can be recognised: equivalence scales drawn from *econometric* techniques and equivalence scales that use *parametric* approaches. For a review of both branches of the empirical literature, see Cowell and Mercader-Prats (1999) and Flückiger (1999). Both econometric and parametric approaches are based on different normative values that determine the results. While econometric approaches vary across different functional forms used to model household preferences, parametric approaches are based on the selection of a set of parameters to typify the size and composition of the household. The following will briefly describe both approaches.

The most common econometric techniques implemented to derive equivalence scales consist of modelling demand functions using household budget data and then estimating the effect that non-income characteristics have over such demand (Coulter et al., 1992a). However, as Pollak and Wales (1979) pointed out, these type of scales are based on a household's demand preferences already constrained on the household demographic composition. Moreover, according to Blundell and Lewbel (1991, pp.50), scales revealed from demand data are based on conditional preferences, regardless of whether households choose demands and demographic attributes simultaneously, sequentially or independently. These types of equivalence scales are referred by Pollak and Wales (1979) as 'conditional' equivalence scales.

Conversely, 'unconditional' equivalence scales refer to the variation in income that households of different sizes and compositions require to achieve the same welfare level. However, this variation should be derived independently from the observed demographic profile of the household. According to Pollak and Wales (1979, pp.217), to derive unconditional scales, "welfare analysis must compare the well-being of a family in alternative situations which differ with respect to its demographic profile as well as its consumption pattern". In this vein, unconditional equivalence scales are not directly observable. For this particular type of scale, studies such as Blundell and Lewbel (1991), or, more recently, Hausman and Newey (2013), focus on estimating those unobserved parameters by using counterfactual techniques and applying sensible identifying assumptions.

The parametric approaches, on the other hand, have focused on providing a measurement approach that first takes into account the elasticity of the needs with respect to household size and then the different household compositions. Examples of these parametric technologies can be found in Atkinson and Bourguignon (1987); Buhmann et al. (1988); Coulter et al. (1992b), and Cowell and Mercader-Prats (1999). A general approach of this type of equivalence scale is analysed by

Buhmann et al. (1988) and Coulter et al. (1992b), in which they express household adjusted income (y_h) as a function of the observed household income (x_h), the size of the household (q_h) and a scale relativity parameter (θ):

$$y_h = \frac{x_h}{(q_h)^\theta}. \tag{1}$$

In this approach, needs are expressed in terms of the size of the household, and the scale relativity parameter varies from no adjustment of the household income by needs ($\theta = 0$) to a complete adjustment portrayed by the per capita household income ($\theta = 1$).

The family of measures proposed in this paper expressly draws from this parametric equivalence scale literature. Similar to the one-dimensional equivalence scale of Eq.(1), we use a parametric approach to measurement and emphasise needs under a scale relativity parameter θ. The proposed family of measures enhances multi-dimensional deprivation comparisons across either households of different sizes and compositions or individuals from different demographic sub-population groups. The approach aims to describe how much deprivation demographically heterogeneous units must exhibit to be catalogued as equivalently deprived. It allows societal multidimensional indices based on more comparable profiles than those available in multidimensional measurement literature.

2.2 Multidimensional deprivation measurement

Several conceptual approaches exist to measure well-being, and each chooses its specific conceptual focus: resources (income or others), basic needs, Sen's functionings or capabilities (Sen, 1993), rights, happiness and so on. In particular, the family of multidimensional measures proposed in this paper can be applied by different conceptual approaches.

However, the conceptual focus of any index and the selection of dimensions and indicators correspond to a normative selection to be taken for each specific context. For instance, the index currently in used by the Colombian government to track multidimensional poverty (the Colombian Multidimensional Poverty Index - CMPI) chose as focus a standard of living concept within which dimensions and indicators were selected (Angulo et al., 2015). The CMPI considered household deprivations as constitutive elements that describe the lack of a minimum standard of living. In particular, dimensions and indicators were selected by Angulo et al. (2015) using various criteria that range from literature-review-revealed relevant living standards for the Colombian context, to identified governmental priorities, and availability and reliability assessment of the data to be used. Other example is the Grenadian Living Conditions Index (GLCI) currently in use by the Grenadian government to target the most deprived population as eligible for social programs (Díaz et al., 2015). The GLCI also uses a living standard concept from which selected dimensions and indicators. But in contrast to the CMPI, the GLCI defined dimensions and indicators under a set of criteria correspondent to the targeting purpose of the measure. For example, the GLCI explicitly excluded from the set of indicators, variables that could be object of misreporting or that refer to a narrow time frame window to avoid capturing transient household living conditions. For a detailed discussion of the conceptual space, dimensions and indicators, to be chosen in the context of multidimensional deprivation measurement, see Alkire et al. (2015a).

Within the multidimensional literature, two alternative procedures identify the poor population and aggregating dimensions: the 'welfare approach' and the 'counting approach'. The first combines several dimensions into a single variable and sets a threshold to differentiate between poor and non-poor populations. The welfare approach has been studied by Bourguignon and Chakravarty (2003), Seth (2009), and Seth (2010), among others.

By contrast, the counting approach, as its name indicates, counts the number of dimensions in which persons suffers deprivation, and the identification of the poor person relies on defining how many dimensions must be deprived for someone to be categorized as multidimensionally deprived. Examples of these types of measures and analysis are proposed by Townsend (1979), Atkinson and Bourguignon (1982), Mack et al. (1985), Callan et al. (1993), Feres and Mancero (2001), Atkinson (2002), Aaberge and Brandolini (2014b), and Alkire and Foster (2011). Efforts have been made within the literature, such as Atkinson (2003), to analyse both approaches (welfare and counting) under a common framework. However, as pointed out by Aaberge and Brandolini (2014a), this discussion is still inconclusive.

The family of measures proposed in this article stands, specifically, within the counting multidimensional deprivation literature and builds upon Alkire and Foster (2011)'s methodology. For brevity, I henceforth refer to the multidimensional poverty measurement method proposed by Alkire and Foster (2011) using the abbreviation 'AF' or 'AF methodology'. I now continue describing this multidimensional deprivation measurement methodology, using a slightly modified notation.

2.3 The AF methodology

Consider a population consisting of $I \geq 1$ individuals evaluated across $J \geq 2$ indicators or dimensions. The AF method begins by defining an $I \times J$ matrix $\mathbf{A} = [a_{ij}]$, where each row corresponds to an individual and each column to the indicators quantifying the individuals' achievements such as education level, nutrition, health status, etc.[3] More precisely, the cell a_{ij} of the matrix $\mathbf{A}$ quantifies for the i-individual the j achievement. Each column is either a cardinal or an ordinal achievement indicator[4].

The AF methodology defines the i individual as deprived in the j dimension by placing a threshold z_j over a_{ij}. Then, whenever $a_{ij} < z_j$ the i individual is said to be j-deprived and the intensity of the suffered deprivation is described by:

$$g_{ij}^{\alpha} = \begin{cases} \left(\dfrac{z_j - a_{ij}}{z_j}\right)^{\alpha} & \text{if } a_{ij} < z_j \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

where $\alpha \geq 0$ is the poverty aversion parameter. The α parameter, first introduced in the poverty measurement literature by Foster et al. (1984) and used by Alkire and Foster (2011), assigns greater emphasis to the most deprived or lowest achieving individuals. The greater the value of α, the larger the accentuation of g_{ij}^{α} on the most deprived.

[3] In general, greater values of an achievement indicator refer to better-off conditions, and lower values of it refer to worse-off conditions.

[4] A cardinal indicator is such that any of its values measures the size of the achievement. This means that the comparison between any two given observed points of a cardinal indicator can be commensurate with the difference between their respective sizes. For instance, years of education is a cardinal achievement indicator because having two years of education can be considered double the number of one year of education. In contrast, an ordinal indicator does not allow measuring the size of the achievement, but rather only indicates a particular ordering between situations. An example of an ordinal achievement indicator is the self-assessment of health status, which takes the categories of "very poor", "poor", "good", and "very good". Note that in this case, we are unable to evaluate the 'size' of the situation. If we compare two observations, for instance, one person having very good health and another person having very poor health, we do not observe the size of the difference between the two situations. In this latter case, we only know that the first person has better off self-assessed health status than the second one, but we do not know the size of the difference in self-assessed health status between the two persons.

However, if the achievement variable is ordinal, the g_{ij}^{α} indicator is valid only for $\alpha = 0$, and g_{ij}^{0} takes the value of either 1 or 0, indicating the presence or absence of deprivation. Hence, as Alkire and Foster (2011) also discussed, any g_{ij}^{α} with $\alpha > 0$ can be defined only for cardinal indicators.

Given that most of the public policy indicators in current use are ordinal, our proposed methodology restricts g_{ij}^{α} strictly to the case of $\alpha = 0$. Henceforth, we denote it as simply g_{ij}.

The application of the z_j thresholds over the $\mathbf{A}$ matrix results in an $I \times J$ deprivation matrix $\mathbf{G} = [g_{ij}]$. Each row of the $\mathbf{G}$ matrix corresponds to an i individual and each column to a binary indicator of presence or absence of deprivation in each dimension.

The AF methodology continues by aggregating deprivations across dimensions for each i person with a c_i metric:

$$c_i = \sum_{j \in J} g_{ij}. \tag{3}$$

Then, a threshold k to identify the multidimensionally deprived is placed over the c_i metric. As a result, any i individual satisfying $c_i \geq k$ is identified as multidimensionally deprived.

Subsequently, the g_{ij} element from the $\mathbf{G}$ matrix is censored to zero in case the i individual is identified as not multidimensionally deprived, namely $g_{ij}(k) = 0$ for any i individual that satisfies $c_i < k$. Thus, $g_{ij}(k)$ denotes the i row and j column element of the $\mathbf{G}$ matrix after the identification of the multidimensionally deprived.

To obtain societal metrics, the simplest measure that AF proposes is the H-multidimensional deprivation incidence. This first metric corresponds to the proportion of people identified as multidimensionally deprived using the k threshold.

The second most important societal metric that AF proposes and that is currently in use by most of the applications of the method is the M_0-adjusted headcount ratio. AF defines the adjusted headcount ratio as $M_0 = \mu(g_{ij}(k))$, where $\mu(g_{ij}(k))$ corresponds to the average $g_{ij}(k)$ for $i = 1, 2, \ldots, I$ and $j = 1, 2, \ldots, J$.

3 The proposed family of multidimensional deprivation indices

This section motivates and presents the proposed family of multidimensional deprivation indices, as an extension of the AF methodology, which explicitly takes into account differences in needs among demographically heterogeneous units. The AF methodology and multidimensional methodologies available in the literature, such as Tsui (2002); Bourguignon and Chakravarty (2003); Seth (2009, 2010), and Rippin (2010) have all been developed using individuals as the unit of analysis and do not analyse the comparability problems that differences in needs might bring to multidimensional deprivation measurement.

In particular, when measuring deprivation, demographic heterogeneity plays a central role in the definition of what can be considered a lack of a minimum achievement. Children, for instance, can be considered deprived when they are not accessing basic education services, unlike adults, who can be considered deprived in the same education dimension when they do not know how to read and write. As another example, while adult populations that do not have access to job opportunities despite seeking them can be defined as deprived in employment, children cannot be defined as deprived in the absence of employment.

A long-standing tradition of policy indicators evaluates deprivation for each particular achievement over a specific sub-population of interest. For instance, one of the Millennium Development Goals launched by the United Nations Development Programme and adopted by several countries to be achieved by 2015 is universal primary education. Another MDG is universal access to reproductive health. Both access to primary education and access to reproductive health services are relevant for measurement only among their particular applicable populations, which are children 6 to 15 years of age and pregnant women, respectively.

When it comes to measuring multidimensional deprivation, these differences in needs, reflected by the different populations where each indicator is applicable to be measured, bring comparability challenges to measuring how many dimensions in deprivation a particular individual or household might exhibit to be catalogued as multidimensionally deprived.

The applied multidimensional deprivation literature addresses these differences in needs by restricting individual-based measures of multidimensional deprivation to the analysis of demographically homogeneous individuals or, in the case of household-based measures, by either assuming the same set of needs across households or ignoring the fact that demographically dissimilar households have significantly different needs.

For instance, in terms of the individual-based applied multidimensional literature, a majority of these studies focus on measuring multidimensional deprivation among either children or adult populations. Examples of child multidimensional deprivation include studies such as Roelen et al. (2010), Roche (2013), Trani and Cannings (2013), Trani et al. (2013), and Qi and Wu (2014). Examples of studies that focus on multidimensional deprivation among an adult population include Oshio and Kan (2014) and Solaymani and Kari (2014).

In contrast, household-based applications of multidimensional deprivation measurement identify as most deprived those households that exhibit the largest number of dimensions in deprivation. Examples of such an empirical approach to track multidimensional deprivation among the population are the global MPI (Alkire et al., 2013) launched by the United Nations Development Program, the Mexican official methodology of poverty measurement (Coneval, 2010), the Colombian Multidimensional Poverty Index (Angulo et al., 2015), and the Chilean National Multidimensional Poverty Index (MDS, 2014). Along with these policy-oriented indices, there is an applied academic literature in which multidimensional deprivation is analysed using the household as the unit of analysis and consequently assuming the same set of needs across households. Examples include Alkire and Seth (2015), Alkire and Santos (2014), Ayuya et al. (2015), Bader et al. (2015), Cavapozzi et al. (2013), Mitra (2014), Alkire et al. (2015b), and Yu (2013). However, the larger and more demographically heterogeneous a household is, the greater its needs might be. Thus, small and demographically homogeneous households might register a systematically lower number of dimensions in deprivation, and conversely, larger and demographically heterogeneous households can exhibit a systematically larger number of dimensions in deprivation.

The family of indices proposed in this paper enables the measurement of multidimensional deprivation across heterogeneous units (i.e., households of different sizes and compositions or individuals from different demographic sub-population groups) while taking into account observable differences in need. We recognise the selection of the unit of analysis in multidimensional deprivation measurement as a normative exercise that the practitioner performs when designing the index of interest. As a result, we expressly set the household as the unit of analysis and present the household-based family of multidimensional indices. Then, as an extension of household-based measures, we present the proposed method to be applied for individual-based multidimensional deprivation measurement. The following sections describe these methods.

3.1 The household-based method

The proposed methodology of this paper begins by defining for each j achievement the sub-population group for which it is relevant to be measured. We call this the *applicable population* for achievement j, and we will measure the presence or absence of the j deprivation only within this set of sample units. This feature of our methodology captures individual differences in needs, corresponding to the traditional approach in the policy context to tracking indicators. With this feature, we bridge the gap between theoretically developed multidimensional indices and policy-oriented single indicators design.

This feature is formalized with an $I \times J$ matrix of applicable populations that we call $\mathbf{S}$. There are as many as J applicable sub-population groups, and any two applicable populations are not necessarily mutually exclusive. The cell s_{ij} of the matrix $\mathbf{S}$ is an indicator variable that takes a value of 1 if and only if the i-individual belongs to the applicable population of the j-achievement, and 0 if and only if the i-individual does not belong to the applicable population of the j-achievement. For instance, according to the Millennium Development Goals access to primary education is relevant to be measured among school-age children, then, cell s_{ij} takes a value of 1 whenever the i-individual is aged 6 to 15 years old and zero in case the individual is outside this age range.

We therefore define as unimportant for the measurement process any observed j achievement for the i person that does not belong to the applicable population of such achievement. Thus, the g_{ij} individual dimensional deprivation indicator evaluated on its applicable population is denoted by $g_{ij}(s_j)$ and takes the form of:

$$g_{ij}(s_j) = \begin{cases} 1 & \text{if } a_{ij} < z_j \text{ and } s_{ij} > 0 \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

where s_j denotes the applicable population of the j achievement.

Now, considering household as the unit of multidimensional deprivation analysis implies understanding the burden that deprivation places as shared among household members. As a result, we approach household-based multidimensional deprivation measurement trough combining the deprivation profiles of household members. First, household dimensional deprivation is measured and subsequently the burden of household multidimensional deprivation. The following describes these household metrics.

Each individual belongs to a particular h household, and each household contains q_h household members. Then, the d^{β}_{hj}-dimensional deprivation indicator for the h household and the j dimension is defined as:

$$d^{\beta}_{hj} = \begin{cases} \left(\sum_{i \in q_h} g_{ij}(s_j)\right)^{\beta} & \text{if } \sum_{i \in q_h} g_{ij}(s_j) > 0 \\ 0 & \text{otherwise,} \end{cases} \tag{5}$$

where $\beta \geq 0$ is the parameter of aversion to deprivation. Larger values of β assign increasing value to the most deprived dimensions (i.e, those with greatest number of j deprived household-members). Whenever $\beta = 0$, then household dimensional deprivation is expressed by a $\{0, 1\}$ indicator of absence or presence of at least one j deprived household-member. On the other hand, if $\beta = 1$, then dimensional deprivation is expressed by the count of deprived household members in the j dimension, and if $\beta = 2$, household dimensional deprivation is expressed as the quadratic expression of that count.

The β parameter of aversion to deprivation is analogous to the α parameter of poverty aversion introduced by Foster et al. (1984) and used by Alkire and Foster (2011) to assign increasing value to those dimensions with biggest *shortfall gap ratio* $((z_j - a_{ij})/z_j)$. Similar to the α parameter of the AF method, whenever $\beta = 0$, dimensional deprivation is expressed as an indicator of presence or absence of deprivation in the j dimension. However, while in the AF method $\alpha > 0$ can be used only in case the j dimension is captured by a cardinal achievement indicator, here $\beta > 0$ commensurates the household deprivation breadth in the j dimension, without necessarily enforcing the use of cardinal achievement indicators and in terms of the number of j deprived household-members.

As a result of the ordinal nature of most of policy indicators, current household-based applications of the AF method have been restricted to measure the burden that dimensional deprivation places on the household by indicating the presence or absence of at least one household member under deprivation in this dimension. This particular approach corresponds to using d_{hj}^{0} to express dimensional deprivation.

The use of $\beta = 0$, however, does not allow household metrics to be sensitive to increments in the number of deprived persons in an already deprived dimension. For instance, when evaluating access to primary education, a household with two school-aged children, one child attending school and the other not attending, registers $d_{hj}^{0} = 1$. Now, if this same household, as a result of a deprivation increment, increases its number of children who are not attending school to two, its d_{hj}^{0} indicator remains invariant.

In contrast, our proposed methodology enables expressing household dimensional deprivation with any $\beta > 0$, which produces a measure of dimensional deprivation that is sensitive to increments in the number of deprived persons in already deprived dimensions. For instance, in the example of the previous paragraph, if we set $\beta = 1$ and evaluate school attendance in the household with one deprived school-age child, then $d_{hj}^{1} = 1$. But if the household has two children deprived of school attendance, then $d_{hj}^{1} = 2$, a value that is twice as large that of the initial case. I further discuss and illustrate this advantage of the proposed method when discussing the properties of our societal metrics in Section 6 ahead on.

Still, not every household has the same set of dimensional needs. In fact, the number of j applicable household members generally varies across households. To account for this, we define n_{hj}^{β} to be the size of the h household needs on the j dimension:

$$n_{hj}^{\beta} = \begin{cases} \left(\sum_{i \in q_h} s_{ij}\right)^{\beta} & \text{if } \sum_{i \in q_h} s_{ij} > 0 \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

Two important cases are obtained by setting $\beta = 0$ and $\beta = 1$: n^0_{hj} indicates whether the household has need in the j dimension or not (i.e., has at least one household member that could suffer deprivation in such dimension); and n^1_{hj} informs the number of household members that exhibit need in the j dimension. For instance, in our same example of school attendance, since the h household has two school-age children, then we know that $n^0_{hj} = 1$ and $n^1_{hj} = 2$.

Using this n^{β}_{hj}-dimensional size of household needs from Eq. (6) we can express the size of household multidimensional needs as:

$$N^{\beta}_{h} = \sum_{j \in J} n^{\beta}_{hj}, \tag{7}$$

where N^0_h counts the number of dimensions that the h-household exhibit as need and N^1_h counts the number of achievements that the h-household exhibit as need.

The second stage of our proposed methodology consists of aggregating household deprivations across dimensions, discounted by needs, to obtain multidimensional profiles. In particular, we propose measuring the burden that multidimensional deprivation places on the household with a functional form that enables capturing either count-based, shared-based or a mixture of these two types of measures. In this vein and following Cowell and Mercader-Prats (1999) and Buhmann et al. (1988) one-dimensional equivalence scale presented in Eq. (1) from page 6, we express the burden of multidimensional deprivation as:

$$m^{\beta,\theta}_{h} = \begin{cases} \dfrac{\sum_{j \in J} d^{\beta}_{hj}}{\left(\sum_{j \in J} n^{\beta}_{hj}\right)^{\theta}} & \text{if } \displaystyle\sum_{j \in J} n^{\beta}_{hj} > 0 \\ 0 & \text{otherwise,} \end{cases} \tag{8}$$

where $\theta \geq 0$ is a deprivation response scale parameter that reflects the relativity of the response of the burden of deprivation to the scale of household needs. In the case that $\theta = 0$, we are in the presence of a count-based approach, and no discounting in needs is applied at all. Thus, the household is assumed as not receiving any advantage from the cooperative unit, and therefore, the burden that deprivation places on the household is not lightened to any degree from the scale of the needs. On the other hand, when $\theta = 1$, we are using a share-based approach. While the count-based structure places greater emphasis on larger households without accounting for any possible scale economy that might arise at this level, the share-based approach places greater emphasis on small households because they are more prone to registering the maximum possible burden of deprivation. Values of θ different than 0 or 1, aim to describe $m^{\beta,\theta}$ through an intermediate approach that lie in between of count-based and share-based perspectives. Henceforth, we use the notation $m^{\beta,\theta}$ to refer to the different values of the measure defined in Eq. (8) for the whole population of households as β or θ varies.

Current household-based policy-oriented indices that use the AF method measure the burden of multidimensional deprivation through the $m^{0,0}$ metric, which corresponds to counting the number of dimensions in deprivation. However, the $m^{0,0}$ metric does not discount by household needs at all. It does not differentiate the deprivation burden of non-deprived and non-applicable dimensions. This induces observing a systematically lower burden of multidimensional deprivation across small and demographically homogeneous households, as we empirically demonstrate in Section 5.

In contrast, our proposed $m^{\beta,\theta}$ family of measures with any $\theta > 0$ takes into account heterogeneous household needs within and across dimensions. Whenever $\theta > 0$, the burden of household multidimensional deprivation is discounted by the household needs and takes into account the scale advantages that the household receives to lighten the burden that deprivation places on it.

3.2 More on the $m_h^{\beta,\theta}$ proposed family of measures

At this point, the $m^{\beta,\theta}$ family of measures has been obtained upon first aggregating individuals' deprivation at the household level for each dimension and then aggregating deprivations across dimensions. This particular strategy is termed as a *first-individuals-then-dimensions* aggregating order. Nonetheless, a second possible course of action can consist of first aggregating dimensions in deprivation at the individual level to obtain individual multidimensional profiles and then aggregating across individuals to obtain household metrics. This second approach is referred as a *first-dimensions-then-individuals* aggregating order.

Each particular order leads to a different set of measures. My proposed household-based methodology is restricted to the use of a *first-individuals-then-dimensions* aggregating order. Table 1 describes the idea behind the four key most intuitive metrics that $m^{\beta,\theta}$ captures on the basis of this selected aggregating order. Only the members of the proposed family of household measures that use $\beta = 1$ are non-sensitive to the order in which they are constructed.

Though both aggregating orders enable household dimensional deprivation metrics to be cardinal rather than providing merely ordinal profiles, the first-individuals-then-dimensions selected order prevents invisibility of the multiple dimensions of deprivation. In other words, the opposite order would conduce the expression of $m_h^{0,\theta}$ in terms of the number of household members with at least one j deprived dimension, disregarding the number of dimensions of deprivation that the household may be exhibiting. Thus, it would not evidence the many different j dimensions of deprivation suffered by households at the same time. In addition, using a first-individuals-then-dimensions as the selected order enables my proposed family of measures to encompass the AF approach to measurement. Then, it allows comparability with regard to current household-based applications.

In the selected first-individuals-then-dimensions aggregating order, the use of $\beta = \{0, 1\}$ switches $m_h^{\beta,\theta}$ between being a count of household dimensions of deprivation (i.e., whether someone in the household is deprived, $\beta = 0$) and being a count of those members who are deprived ($\beta = 1$). Deprivation aversion captured by $\beta > 0$ assigns greater value to the most deprived dimensions.

In contrast, $\theta = \{0, 1\}$ switches $m_h^{\beta,\theta}$ between being a count-based measure of household deprivation (i.e., one in which the denominator is switched off, $\theta = 0$) and being a share-based measure ($\theta = 1$). Values of θ different from 0 and 1 aim to describe $m^{\beta,\theta}$ as an intermediate approach between share-based and count-based measures.

In general, count-based and share-based measures can be considered to capture two different conceptions of inequality. While count-based measures depict an 'absolute' conception of inequality, share-based measures a 'relative' conception of inequality. According to Kolm (1976a,b) and Shorrocks (1983), in the context of income inequality, a relative measure of inequality is one that remains invariant under a variation of income in the same proportion for all incomes in society. In addition, according to scholars, an absolute measure of inequality does not change under an equal absolute variation of income for all incomes in society. Absolute and relative measures of inequality have been analysed by the inequality literature under a

Table 1: Resulting measure of the $m_h^{\beta,\theta}$-burden of multidimensional deprivation across a selected combination of parameters and using a first-individuals-then-dimensions aggregating order

Combination of parameters	Resulting $m_h^{\beta,\theta}$ measure
$\beta = 0$, $\theta = 0$	Count of dimensions with at least one household member under deprivation. Measure comparable with the c metric of the AF method and termed the *dimensions-count-based* approach to measurement.
$\beta = 0$, $\theta = 1$	Share of possibly deprived dimensions. Measure termed the *dimensions-share-based* approach to measurement.
$\beta = 1$, $\theta = 0$	Count of household deprivations. Measure termed the *deprivations-count-based* approach to measurement.
$\beta = 1$, $\theta = 1$	Share of household possible deprivations. Measure termed the *deprivations-share-based* approach to measurement.

common framework as alternative approaches to measurement; examples include the studies of Kolm (1976a,b), and Shorrocks (1983). Intermediate indices of inequality have also been analysed by literature. Examples include Bossert and Pfingsten (1990) and Chakravarty and Tyagarupananda (2009), which express intermediate inequality indices as a mixture of relative and absolute measures of inequality.

In the specific case of the $m^{\beta,\theta}$ family of indices proposed in this paper, I follow the embodied intuition of the inequality literature and express the $m^{\beta,\theta}$-burden of multidimensional deprivation in terms of a θ parameter that allows us to capture different conceptions of inequality. The use of a count-based approach to measurement assigns an equal absolute value to each dimension or each deprivation. Whereas, under a share-based approach to measurement, the household burden of multidimensional deprivation is expressed in relation to the potential number of dimensions or deprivations that the household could possibly suffer.

Although the discussion about the pertinence of absolute, relative, or intermediate indices to analysis of the distribution of the population within a particular achievement might date from the 1970s, there remains little agreement about which approach is more pertinent for any society, mostly because they are based on value judgements about what can be considered just or unjust, so any decision must be context specific.

We now proceed to describe the method proposed in this paper to identify the most deprived households.

3.3 Identification of the multidimensionally deprived

For a given combination of β and θ, households exhibiting at least a k burden of multidimensional deprivation are identified as the multidimensionally deprived. Parameter k represents the multidimensional deprivation threshold above of which the most deprived household are observed. The k threshold takes values between zero and the maximum possible observable $m_h^{\beta,\theta}$. For instance, applications as the Colombian index of multidimensional poverty have set k, under a combination of statistical methods and empirical findings, as the 33% of the maximum weighted sum of dimensions on deprivation (Angulo et al., 2015). A similar 33% cut-off point over the weighted sum of deprivations have been used by Alkire and Santos (2010) for the global MPI and by Battiston et al. (2013) for a proposed index in the context of six Latin American countries. The plausible k is to be defined according to the context of each application.

Having set the k threshold, it naturally arises a binary indicator of presence or absence of multidimensional deprivation, p_h, as follows:

$$p_h = \begin{cases} 1 & \text{if } m_h^{\beta,\theta} \geq k \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

While applications of the AF method sort households under the basis of $m^{0,0}$ and households satisfying $m_h^{0,0} > k$ get identified as the multidimensionally deprived, the proposed methodology of this paper enables the identification of the most deprived to be done under the basis of any $m^{\beta,\theta}$. The implications that different $m^{\beta,\theta}$ measures have on identifying the multidimensionally deprived are investigated and discussed in Section 5 ahead on. We continue presenting the proposed methodology for aggregating household multidimensional deprivation at the society level.

3.4 The family of societal measures

Suppose that R is the total number of households. Then, as proposed by Alkire and Foster (2011), the simplest metric to represent the overall society multidimensional deprivation incidence is:

$$H = \mu(p_h), \tag{10}$$

where $\mu(p_h)$ corresponds to the average value of p_h for $h = 1, 2, \ldots, R$. In line with the AF methodology, H corresponds to the rate of societal multidimensional deprivation incidence. However, since the proposed methodology of this paper allows identifying the multidimensionally deprived population under the basis of any $m^{\beta,\theta}$, we denote the proportion of multidimensionally deprived population identified on the basis of a particular $m_h^{\beta,\theta}$ as $H(m^{\beta,\theta})$. This notation contains as special case $H(m^{0,0})$ or proportion of multidimensionally deprived population identified on the basis of the AF' $m^{0,0}$ sorting metric.

On the other hand, to construct societal metrics of the average burden that multidimensional deprivation places across households, and as the AF method proposes, we censor to zero any $m_h^{\beta,\theta}$ for non-multidimensionally deprived households, namely, $m_h^{\beta,\theta} = 0 \ \forall \ h \ s.t. \ p_h = 0$. We denote, therefore, the household burden of multidimensional deprivation after the identification of the multidimensionally deprived with the k threshold as $m^{\beta,\theta}(k)$. As a result, the societal mean burden of multidimensional deprivation is defined as:

$$MD^{\beta,\theta} = \mu(m^{\beta,\theta}(k)), \tag{11}$$

where $\mu(m^{\beta,\theta}(k))$ corresponds to the average value of $m^{\beta,\theta}(k)$ for $h = 1, 2, \ldots, R$. In this case, our $MD^{0,0}$ metric corresponds to the M_0 metric of the Alkire and Foster (2011) method. In comparison to the M_α family of measures of the AF method, our proposed $MD^{\beta,\theta}$ constitutes a broader set of measures that takes into account count-based, share-based and intermediate approaches to measure the burden that multidimensional deprivation places on the household.

In general, given the ordinal nature of policy indicators, most current applications on the Alkire and Foster (2011) method are able to describe societal multidimensional deprivation through $H(m^{0,0})$ and $MD^{0,0}$. Our proposed approach, in contrast, allows describing the multidimensional deprivation in terms of any $H(m^{\beta,\theta})$ and $MD^{\beta,\theta}$ with $\beta \geq 0$ and $\theta \geq 0$.

The properties that make the proposed family of societal measures satisfactory for the purposes of multidimensional deprivation measurement are investigated and discussed in Section 6 ahead on.

3.5 Weights

For completeness purposes and to guide applications where dimensions have different relative importance across each-other, in this section we introduce and describe a weighting system to differentiate these relative importances. We therefore, introduce the $\mathbf{w} = (w_1, w_2, \ldots, w_J)$ vector of non-negative importance weights, where $w_j \geq 0$ denotes the relative importance weight for the j achievement in the overall deprivation evaluation, and satisfies $\sum_{j=1}^{J} w_j = 1$. This weighting system can be used to aggregate deprivations across the J dimensions and obtain the burden of multidimensional deprivation as:

$$\mathbb{m}_{\mathbb{h}}^{\beta,\theta} = \begin{cases} \dfrac{\sum_{j \in J} w_j d_{hj}^{\beta}}{\left(\sum_{j \in J} w_j n_{hj}^{\beta}\right)^{\theta}} & \text{if } \displaystyle\sum_{j \in J} w_j n_{hj}^{\beta} > 0 \\ 0 & \text{otherwise.} \end{cases} \tag{12}$$

This $\mathbb{m}_{\mathbb{h}}^{\beta,\theta}$-burden of multidimensional deprivation represents the $\mathbf{w}$ scaled variant of Eq.(8). The application of the $\mathbf{w}$ dimensional weights produces, subsequently, societal measures H and $MD^{\beta,\theta}$ to be updated using this $\mathbf{w}$ scaled variant of $m_h^{\beta,\theta}$.

The selection of these dimensional weights can be devised according to the purpose of the measure and by different alternative procedures such as normative selection or data-driven techniques. For a discussion of alternatives to setting weights in a multidimensional index, see Decancq and Lugo (2013).

3.6 The individual-based scenario

Whenever individuals, rather than households, are selected as the unit of multidimensional deprivation analysis, differences in needs are observed across different demographic sub-population groups, as for instance across population from different ranges of age or gender. While pregnant women, for instance, need to access to antenatal health services, school aged children need to access to basic educative services. Deprivation in antenatal health services is, therefore, relevant to be measured exclusively across pregnant women, as it is access to basic educative services across school aged children.

Given that individuals from different demographic sub-population groups exhibit differences in needs, current applications of the Alkire and Foster (2011) multidimensional deprivation method, that use the individual as the unit of analysis, tackle these differences in needs by restricting the analysis to arguably homogeneous demographic sub-population groups and a set of comparable indicators. For instance, Oshio and Kan (2014) studied the association between multidimensional poverty and health among individuals aged 20 to 59 years old, using for the multidimensional poverty index indicators such as: low education attainment, non-coverage to public pension and household income. Batana (2013), on the other hand, studied multidimensional poverty in fourteen Sub-Saharan African countries by restricting the analysis to women between 15 and 49 years old and using indicators of assets, access to health services, schooling and empowerment. Also, examples of multidimensional poverty focused on children are Trani and Cannings (2013), Qi and Wu (2014), Roelen et al. (2010), and Trani et al. (2013) for Western Darfur (Sudan), China, Vietnam and Afghanistan, respectively. To date, no application of individual-based multidimensional deprivation taking into account the whole age range of the population can be found.

The methodology introduced here can be used to enable individual-based multidimensional deprivation measurement in presence of different needs across demographically heterogeneous sub-population groups. In the context of this paper, this approach is named as the individual-based scenario, as it is derived as a special case of the previously described household-based measures.

Particularly, in this proposed individual-based scenario each household in the society is assumed as consisting of one member, which simply implies each person is in its own household. The afore-presented household-based measures are consequently derived. Hence, the dimensional deprivation indicator and the burden of multidimensional deprivation, both are obtained without aggregating at the household level.

Specifically, given that the d_{hj}^{β}-dimensional deprivation indicator for the h household in the j dimension, was developed as an aggregation of the household members' $g_{ij}(s_j)$ individual deprivation indicators to the power of β (Eq.(5)); then, this aggregation and the β parameter have no relevance in an individual-based scenario because in this case the resulting measure is always a binary variable of presence or absence of deprivation, which is simply $g_{ij}(s_j)$. Consequently, the $m_h^{\beta,\theta}$-burden of multidimensional deprivation for the h household (Eq.(8)), becomes also non-sensitive to different values of β and expressed independently for each i individual. We denote this variant of Eq.(8) as m_i^{θ}.

Still, the use of the θ parameter in the individual-based scenario expresses the responsiveness of deprivation to the size of the individual's needs. Similar to the household-based case, in the individual-based scenario, the use of the θ parameter allows expression of the multidimensional deprivation burden that the i individual suffers, either as a count of dimensions on deprivation, a proportion of dimensions of deprivations or any mixture of these two types of measures.

The use of this individual-based scenario naturally produces an identification of the most deprived to be done sorting individuals with any m_i^{θ} measure and defining as multidimensionally deprived those satisfying $m_i^{\theta} > k$. Societal measures H and MD, are therefore, developed using the individual-based variants of the measures.

The individual-based proposed approach with $\theta = 0$, worth noting, corresponds to the individual-based AF methodology. In this case the proportion of multidimensionally deprived individuals is expressed by $H(m_i^0)$ and the MD^0 metric results equivalent to the AF metric M_0.

Another approach to measure individual-based multidimensional deprivation might be, for instance, setting a weighting system to account for the observed heterogeneous needs. This means using a dimensional weighting system ($\mathbf{w} = (w_1, w_2, \ldots, w_J)$) differentiated by sub-population groups. With such an approach, it is possible to ensure that each sub-population group exclusively weights their relevant indicators, such that the sum across w_j adds to 1, for each sub-population group. It is worth noting, however, that in this case each dimension results in not having the same normative value across individuals in society and m^{θ} is always restricted to the share-based approach.

We now proceed to evaluate throughout different methods the implications of using different possible measures to identify the multidimensionally deprived population. The analysis is carried out making use of the data that is presented in the next section.

4 Data

For the empirical analysis of this paper, a household-based multidimensional deprivation index is built using the 2013 Paraguayan Household Survey (PHS). The PHS is a cross-sectional living conditions survey that has been collected yearly since 1984 by the Paraguayan National Statistical Department. Referred to as the *Encuesta Permanente de Hogares*, it captures a broad range of living condition indicators. The survey aims to provide national estimates for income poverty, inequality, and some key quality of life descriptors. The questionnaire of the PHS 2013 includes information regarding education, health, the labour market, individual income, dwelling conditions, and international migration and a special module for agriculture and forestry activities.

The PHS 2013 used a two-stage, clustered probabilistic sample design that was stratified in the first stage by 31 geographical domains. The strata corresponded to rural and urban areas of 15 out of the total 17 Paraguayan counties (*departamentos*) and the national capital of Asunción. The sample allows for total national, urban, and rural area estimates, as well as for disaggregation throughout seven geographic domains. The first geographic domain corresponds to Asunción, the Paraguayan capital city. The next five domains correspond to the national counties of San Pedro, Caaguazú, Itapúa, Alto de Paraná, and Central. The seventh and last domain corresponds to the aggregation of the 12 remaining Paraguayan countries. In 2013, the PHS was collected from a sample of 21,207 persons across 5,424 households.

Table 2 describes the items included within the multidimensional deprivation index constructed for the analysis purposes of this article. In particular, this index example captures information on health, education, and dwelling conditions across five deprivation indicators: health insurance non-coverage, non-access to health services, non-school attendance, low educational achievement, and sub-standard housing.

Table 2: Example of multidimensional indicator: Dimensions, indicators, weights, applicable population and deprivation criteria

Well-being dimension	Deprivation indicator	Applicable population where the indicator is relevant to be measured	A person from the applicable population is deprived if:
Health	Health insurance non-coverage	Any person	Does not have access to health insurance coverage.
	Non-access to health services	Any person that was sick or had an accident during the 90 days previous to the interview	Did not receive institutional care*.
Education	Non-school attendance	5 - 17 years old population	Is not attending school.
	Low educational achievement	Population 18 years old and over	Has less than 9 years of completed education.
Dwelling conditions	Sub-standard housing	Any person	Lacks at least 2 of the following 3 dwelling conditions: flooring different from earth or sand; adequate material of ceilings**; adequate material of walls***.

Notes: *Institutional care corresponds to attention received by a professional health worker (physicist, nurse, dentist or professional midwife) in private or public health institution (It is not a health care institution: pharmacy, empirical medicine man store, own house, other's house). **Inadequate ceiling material refers to the following: Straw, eternit, clapboard, palm trunk, cardboard, rubber, packaging timber, other. ***Inadequate wall materials refer to the following: wattle, mud, wood, palm trunk, cardboard, rubber, wood, another material, or no wall at all.

Of the 21,207 interviewed individuals for PHS 2013, we excluded from the analysis 264 observations that do not belong to the household unit (i.e., domestic personnel), and 34 observations were also excluded because of non-response to at least one of the five considered indicators. Thus, our effective sample comprises 20,909 interviewed persons across 5,423 households.

5 Evaluating measures

Examples of applications of the Alkire and Foster (2011) method that select household as the unit of analysis are Alkire et al. (2013), Alkire et al. (2015b), Angulo et al. (2015), Alkire and Seth (2015), Alkire and Santos (2014), Ayuya et al. (2015), Bader et al. (2015), Cavapozzi et al. (2013), Mitra (2014), Alkire et al. (2015b), and Yu (2013). This literature measures the burden of multidimensional deprivation through the household count of deprived dimensions, strategy termed in Table 1 the *dimensions-count-based* approach to measurement. In this section, I evaluate the effects on multidimensional deprivation profiles of using such an approach and compare it to those obtained using other members of the family of measures proposed in this paper.

5.1 Observed multidimensional deprivation incidence profiles

As described on Section 3.1, a dimensions-count-based approach implies measuring household dimensional deprivation in terms of whether or not there is at least one household member facing deprivation, and subsequently, households are compared in terms of the number of deprived dimensions. Multidimensionally deprived households are those exhibiting a majority of these deprived dimensions.

Table 7 within the Appendix presents the proportion of households with at least one deprived household member from each of the five dimensions considered in this application. This corresponds to the mean d^0_{hj}-dimensional deprivation indicator across the 5,423 observed Paraguayan households by household size. It can be seen that larger households exhibit a larger proportion of dimensional deprivation than smaller households. The positive relation between household size and dimensional deprivation is observed because the number of persons in the applicable population increases as the household size increases. The dimensions prone to this effect are health insurance non-coverage, non-access to health services, non-school attendance, and low educational achievement. Take, for instance, the non-school attendance indicator in Table 7, which is applicable for children 5 to 17 years of age. One-person households are rarely composed by this population subgroup because school-age children cannot form a household. Therefore, the proportion of households consisting of one person that are dimensionally deprived in school attendance is 0%. Conversely, 21.4% of households consisting of seven or more persons are deprived of school attendance because they contain in average 4 children.

If, subsequently, household dimensions of deprivation are counted and the multidimensionally deprived households are those with the largest count of these dimensions on deprivation, larger and more heterogeneous households tend to be identified as the most deprived. The following elaborates further on this:

With the purpose of comparing the multidimensionally deprived population of households identified using different $m^{\beta,\theta}$ measures, households are sorted on the basis of each $m^{\beta,\theta}$ score and the first 40% most deprived (2,168 households) are identified as multidimensionally deprived. The population of households identified as the most deprived using the dimensions-count-based approach ($m^{0,0}$) is compared with regard to those obtained using the other three $m^{\beta,\theta}$ measures described in Table 1: the dimension-share-based approach ($m^{0,1}$), the deprivations-count-based approach ($m^{1,0}$), and the deprivations-share-based approach ($m^{1,1}$).

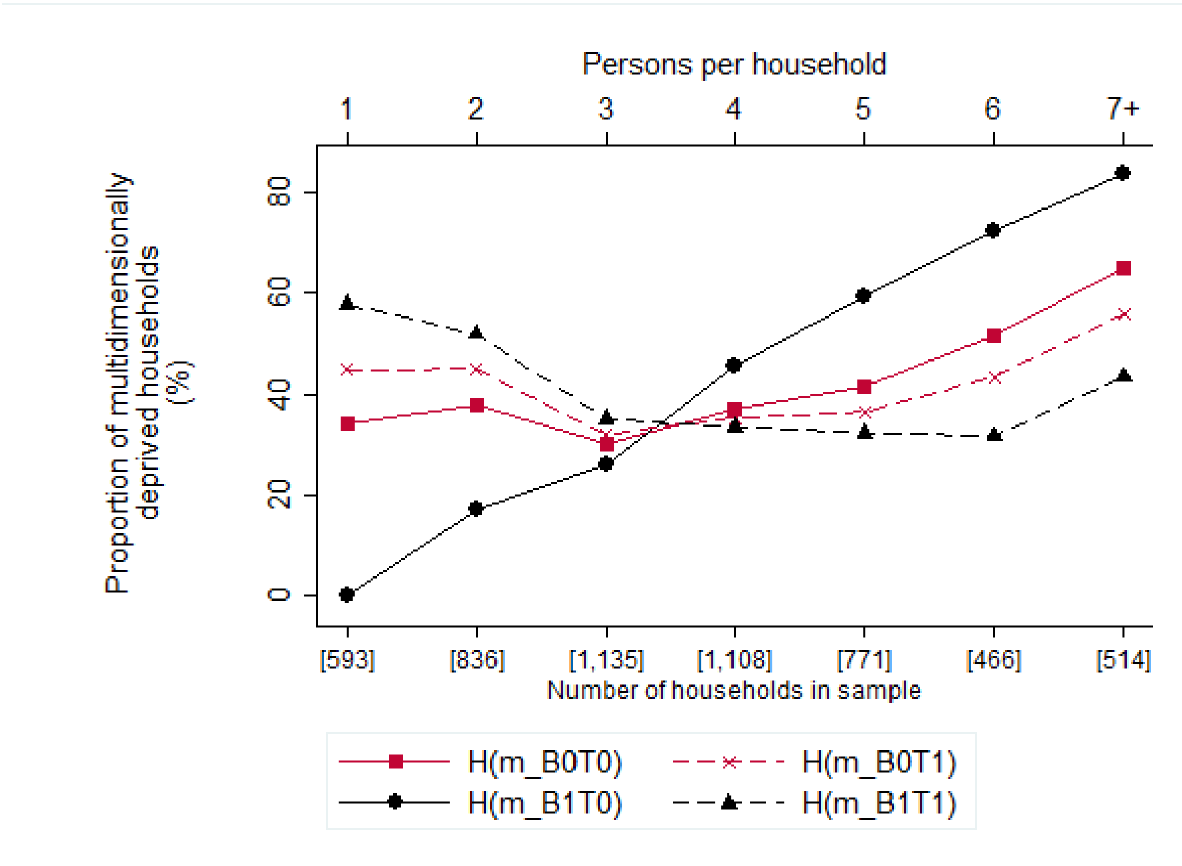
Note that identifying a fixed share of the population (40% in this case) as the most deprived is different from placing a particular k multidimensional threshold over the $m^{\beta,\theta}$ score. Given that the range of variability of $m^{\beta,\theta}$ varies along the β and θ parameters, the use of a fixed share of households enables us to compare the different deprived populations on an equal basis. The particular 40% share of households arose as a plausible natural breaking point in the distribution of deprivations observed by the multidimensional index in the analysis. Nonetheless, in Section 5.5.2 I test the robustness of the obtained results under other different possible shares of the population.

Figure 1 plots the obtained H-multidimensional deprivation incidence by household size for the four $m^{\beta,\theta}$. No adjustment by differences in needs corresponds to measures that use $\theta = 0$: the dimensions-count-based and the deprivations-count-based approaches. In the figure, the results obtained upon sorting households under a dimensions-count-based approach ($m^{0,0}$) are plotted by square markers. The profile obtained on the basis of a deprivations-count-based approach ($m^{1,0}$) is plotted by circle markers in the figure. The vertical axis corresponds to the proportion of households of each size identified as multidimensionally deprived. For instance, out of the total observed 514 households consisting of seven or more persons, in about 80% of them are identified as multidimensionally deprived when a deprivation-count-based approach is used.

As expected, the results indicate that the H-multidimensional deprivation incidence varies across household size and measures. The profiles obtained upon $m^{\beta,\theta}$ measures that do not account for needs ($m^{0,0}$ and $m^{1,0}$) show the greatest proportion of multidimensionally deprived among large households, as well as, the lowest proportion among small households. In particular, when using the AF-proposed $m^{0,0}$, households consisting of seven or more persons register 29.2 percentage points more multidimensional deprivation incidence than households consisting of one person.

Any $\theta > 0$ enables the burden of household multidimensional deprivation to be adjusted by household needs, increasing the amount of the adjustment as θ increases. Then, contrary to count-based approaches, a deprivations-share-based approach (triangle markers in the figure) produces 57.8% of households consisting of one person being catalogued as multidimensionally deprived and 43.6% of households consisting of seven or more persons being catalogued as multidimensionally deprived. Thus, in this case, a 14.2 p.p. higher incidence of multidimensional deprivation is observed among smaller households than across larger households.

Figure 1: Proportion of multidimensionally deprived households, $H(m^{\beta,\theta})$, across household size



Source: PHS 2013.

These descriptive statistics suggest that identifying the most deprived on the basis of a household burden of multidimensional deprivation not adjusted by household needs results in greater H-deprivation incidence among larger households. Multidimensional deprivation incidence among larger households reduces as the adjustment by the size of the needs increases. The use of different $m^{\beta,\theta}$ measures to sort households produces different profiles of multidimensional deprivation incidence, and these results are driven by the size of the household needs.

What should we make of these differences? On one hand, as particular studies from the one-dimensional equivalence scale literature suggest, one can argue that there is no correct or incorrect equivalence scale and that different measures are justified according to different circumstances (Cowell and Mercader-Prats, 1999, pg.409). In this vein, the selection of the measure to describe household multidimensional deprivation constitutes a context-specific normative definition. While count-based approaches ($\theta = 0$) give either to each dimension (using $\beta = 0$) or to each deprivation ($\beta = 1$) an equal absolute value in the measurement of the burden of multidimensional deprivation, deprivation share-based approaches ($\theta = 1$) give an equal absolute value to each household, disregarding its demographic composition and taking into account the scale economies that arise at this level. An intermediate normative perspective corresponds to set the θ parameter between these two solutions. The value of θ reflects the responsiveness of the burden of deprivation to the scale of needs; values of θ close to zero convey a lower response of the burden of multidimensional deprivation to the size of the needs. Conversely, values of θ close to one convey a greater response of the burden of deprivation to the size of the needs.

On the other hand, researchers can consider, as we do in this paper, differences in need as a 'legitimate' source of variation in the observed deprivation profiles. In such circumstances, we are interested in determining how much of the observed profile results from these legitimate differences in needs and how much of it can be attributed to actual unfair or 'illegitimate' differences in deprivation. As a result, I will set out in Section 5.2 a framework based on Fleurbaey and Schokkaert (2009) to show how effectively each $m^{\beta,\theta}$ captures strictly illegitimate differences in deprivation. The methodology for approaching such a framework and the results of the evaluation are presented in Sections 5.3 and 5.4, respectively.

5.2 Evaluation framework: fundamentals

Here, we begin defining legitimate and illegitimate differences in multidimensional deprivation. We follow the framework set up by Fleurbaey (2007) in social choice on equity, responsibility and fairness, and in particular the proposed approach of Fleurbaey and Schokkaert (2009) to analyse fair and unfair health and healthcare inequalities. In this framework, differences in achievement levels (such as health or educational attainment) are seen as caused by myriad factors, some of which can be catalogued as producing fair differences and others as producing unfair differences.

In particular, for the case of health and healthcare inequalities, Fleurbaey and Schokkaert (2009) defined as legitimate or fair those differences attributed to causes that fall under individuals' responsibility. Legitimate differences in this context correspond, therefore, to those derived from preferences.

In this vein, for the purposes of this paper, I define as illegitimate or unfair any difference in deprivation related to situations out of the control of the individual and so not her/his responsibility. From this ethical perspective, we are interested in multidimensional deprivation incidence profiles arising exclusively from illegitimate causes.

Now, we consider differences in need. Implicit in my approach is that an individual can be regarded as deprived by a particular indicator only if it measures an achievement that can be viewed as something that this individual legitimately needs. Needs differ across dimensions of multidimensional deprivation by population subgroup.

For instance, while adults who do not have work opportunities despite looking for them can be catalogued as employment deprived, children cannot be catalogued as deprived in the absence of employment. Conversely, children under 11 years old who are forced to work would be catalogued as deprived. Children are accountable on other deprivations that are relevant to them, such as access to education services. As such, adults and children have different sets of needs. While adults need access to job opportunities and are considered employment deprived whenever they do not have access to them, children need access to basic school services and are considered educationally deprived if they lack such access.

In my framework, differences in deprivation caused by differences in need between population subgroups are seen as fair (and, in some cases, even desirable). As a result, needs are incorporated into my multidimensional deprivation family of indices by excluding from the calculations all dimensions that do not correspond to needs for a particular individual. In this context, an analyst who viewed these differences as fair could seek to equivalise the burden of deprivation for two households with different sets of needs by using a share-based approach; to the extent that the many fair causes of deprivation are correlated with the demographic characteristics of the population subgroups, this equivalised comparison would be solely in terms of deprivation resulting from unfair causes.

Having defined differences in needs as a source of legitimate differentials in multidimensional deprivation incidence, the evaluation presented in this section seeks to disentangle how much of the difference in multidimensional deprivation incidence that a particular $m^{\beta,\theta}$ measure produces are attributable to unavoidable differences in needs and therefore can be catalogued as legitimate. To undertake such an evaluation, I define a desirability condition which should be satisfied by any multidimensional deprivation incidence profile. Based on this condition, the performance of the profiles is evaluated. The next paragraph describes this desirability condition.

Desirability condition. An *unbiased* multidimensional deprivation incidence profile is such that it is unable to distinguish between two population groups that have no illegitimate differences in deprivation between each other but only different sets of needs. As such, any two households in a household-based scenario or any two individuals in an individual-based scenario with no illegitimate difference in deprivation between the two of them must have the same multidimensional deprivation incidence.

The intuition behind this desirability condition is that a fair state is such that no differences in deprivation exist as a result of illegitimate sources. This condition aims to test the ability of a multidimensional deprivation incidence profile to reflect the presence of such a fair state.

If we can confirm that a multidimensional deprivation incidence profile based on a particular $m^{\beta,\theta}$ measure equivalently classifies (as either multidimensionally deprived or non-deprived) any two households with differences in $m^{\beta,\theta}$ caused only by differences in needs and not by differences resulting from illegitimate sources, we also know that the differences evidenced by this particular profile are entirely a result of illegitimate sources. Hence, multidimensional deprivation incidence profiles that are unable to portray the same incidence between two households with differences in $m^{\beta,\theta}$ caused only by differences in needs are said to provide a *biased* picture of societal multidimensional deprivation incidence.

Nonetheless, following Fleurbaey and Schokkaert (2009), another possible course of action can involve setting an alternative condition. In this alternative condition, an unbiased multidimensional deprivation incidence profile is understood such that no influence of legitimate factors exists. However, this alternative condition is neither formalised nor tested in this paper, and we focus entirely on evaluating whether or not the incidence profiles that we obtain on the basis of particular $m^{\beta,\theta}$ measures satisfy our basic premise or desirability condition.[5] The next section describes the specific methodological setting proposed for the evaluation of multidimensional deprivation incidence profiles under the selected basic premise or desirability condition.

5.3 Method

Section 5.1 described the behaviour of multidimensional deprivation incidence profiles based on different $m^{\beta,\theta}$ measures. In the factual observed case of the 2013 Paraguayan example, as in any other observed factual case, differences in deprivation originating from legitimate and illegitimate sources are not straightforwardly differentiated. Based on the discussion in Section 5.2, I am interested in evaluating the behaviour of the measures proposed in this paper in situations where there are no unfair differences between people and households but only fair differences. To do this, I use microsimulation methods to generate counterfactual deprivation profiles for the 2013 Paraguayan example.

[5] While our selected desirability condition seeks to rule out illegitimate differences in deprivation and therefore observe any remaining "fairness gap" (a term used by Fleurbaey and Schokkaert (2009) to describe the condition that we select as desirable to be satisfied by multidimensional deprivation incidence profiles), the second alternative course of action, in contrast, aims to rule out legitimate differences. As the scholars discussed, these two conditions cannot be attained simultaneously. In particular, if the alternative condition is satisfied, it is possible that no difference in multidimensional deprivation incidence between two households/individuals will be observed because they have identical legitimate sources of deprivation, so no legitimate difference in deprivation remains. However, these two households still have illegitimate differences in deprivation, and the measure would depict them as equivalently deprived. For the purposes of this paper, this situation is considered ethically undesirable, so we deliberately focus on evaluating our measures only in terms of the selected desirability condition.

Microsimulation methods allow the generation of counterfactual states by applying deterministic or stochastic rules to simulate changes in the state or behaviour of the population and then enable analysis of the outcomes of those rules. This technique operates at the micro level (for instance, individuals or households) and allows analysis at any relevant level of aggregation (Figari et al., 2014, p.4). In particular, static microsimulation techniques fix the characteristics of the micro units and have been utilised in the economic literature to, for instance, assess the impact of policy reforms and describe the optimal design policy (Blundell, 2012).

For the purposes of this paper, a static microsimulation method is used. The evaluation of multidimensional deprivation incidence profiles is approached as a "controlled experiment" (a term used by Figari et al. (2014) to describe microsimulation techniques) with the data to determine the ability of particular $m^{\beta,\theta}$ measures to enable the observation of unbiased incidence profiles, as defined by our desirability condition.

As such, the demographic structure of the population and the characteristics of the household that describe differences in need are set as invariant, a counterfactual scenario with no illegitimate difference in deprivation is built, and subsequently, the performance of our $m^{\beta,\theta}$ measures is evaluated under these circumstances.

The counterfactual scenario of no illegitimate difference in deprivation is created by distributing deprivation completely at random across individuals and households. In other words, we fix the characteristics of the sample members (including whether or not they are members of applicable population subgroups) and then, for each j-dimension, randomly allocate whether or not they are in deprivation. The random allocation is done by sampling without replacement from the observed deprivation so that the total number of deprived people is the same in the counterfactual and factual samples.

The random distribution of deprivation emulates no unfair difference because it is not related to any individual or household characteristics and thus not a result of any underlying behaviour or characteristic. By building a (counterfactual) population in which there is no difference in deprivation resulting from unfair causes, I can determine whether a multidimensional incidence profile based on a particular $m^{\beta,\theta}$ measure is able to make an unbiased comparison.

Any multidimensional deprivation incidence profile satisfying the desirability condition must exhibit no relation between multidimensional deprivation incidence and the size of household needs in this counterfactual scenario.

Different approaches can nonetheless be used to measure the size of household needs. The proposed methodology of this paper measures, the size of household multidimensional needs by the N_h^{β} indicator, as per defined in Eq.(7) on page 15. Then, the first alternative is to use the N_h^0-count number of dimensions that the h-household needs as measure of the size of household needs. A second alternative is to use the N_h^1-count number of achievements that the h-household needs. And, a third alternative measure of the size of household needs, could be simply household size. The evaluation results presented in the next section are developed using N_h^0 as measure of the size of household needs.[6] However, as a robustness analysis, Section 5.5.1 presents the results obtained using either N_h^1 or household size, as alternative measures.

Then, I approach the evaluation of each profile in the counterfactual state of no unfair difference, via a comparison of multidimensional deprivation incidence and the size of households needs. For this purpose I use the linear regression $p_h = \rho + \delta N_h^0$, where p_h is the binary indicator of the presence or absence of multidimensional deprivation in the h-household, ρ is the intercept term, and δ is the regression coefficient of interest. This δ regression coefficient captures the difference in p_h-multidimensional deprivation incidence that can be attributable to the size of household needs.

A profile that satisfies the desirability condition must reflect no difference in multidimensional deprivation incidence given by households' different needs. Differences across the size of household needs are said to be fair because they depict unavoidable and legitimate differences in need.

Finally, two additional important caveats. First, because of the randomness of the allocation of deprivation, one could argue that a particular population subgroup might have a larger incidence of deprivation than another, simply as a result of

[6]We remark that this measure takes into account, the number of persons in the household and its composition with respect to the dimensions captured by the multidimensional index. For instance, take household A and B, both consisting of two persons each. Household A, consisting of one adult person and one toddler. In the index example, this household may be scored as deprived in four out of the five considered dimensions. In contrast, household B, consisting of one adult and a 10-year-old child, may be scored as deprived in all five considered dimensions. In this case, household size does not capture the difference in possible deprivations that these two households of the same size have.

this randomness. To overcome these possible random differences among population subgroups, the counterfactual scenario with no illegitimate difference in deprivation among households was simulated 1,000 independent times, and the results that I describe below correspond to the distribution of these 1,000 independent simulations.

Second, following Coulter et al. (1992a), I evaluate multidimensional deprivation profiles based on the $m^{\beta,\theta}$ measures, where $\beta = \{0, 1\}$ and θ takes 100 distinct values from the interval $[0, 1]$. This means that a counterfactual scenario with no illegitimate difference in deprivation is simulated 1,000 independent times, and in each scenario, I evaluate the results of multidimensional deprivation profiles obtained upon the $m^{\beta,\theta}$ proposed measures of this paper using the key parametric values of β and θ across their full range. The resulting collection of estimates approximates the distribution of the index over the counterfactual scenario's outcomes.

5.4 Results

In this section, I present the evaluation results of multidimensional profiles developed under $m^{\beta,\theta}$ measures that do not adjust the burden of multidimensional deprivation by differences in needs (using $\theta = 0$). I then compare these results with those obtained from measures that adjust by differences in need (measures with $\theta > 0$). At this stage, it should be recalled that θ reflects the response of the burden of deprivation to the scale of household needs. Values of θ close to zero reflect a low response of the burden of deprivation to the scale of household needs and values of θ close to one reflect a greater adjustment of the burden of deprivation by the size of household needs. This θ parameter was included to account for differences in needs when comparing household's multiple deprivations, as the applied literature on income and expenditure household-based measures does to compare household's welfare.[7]

Figure 2 on page 38 plots the results of this evaluation. The horizontal axis in the figure corresponds to the range of θ parameters used to calculate the $m^{\beta,\theta}$ measure. The first value of this range corresponds to $\theta = 0$ (no adjustment for the size of household needs), the adjustment by the size of household needs increases as θ increases. The last value on the right-hand side of the horizontal axis corresponds to $\theta = 1$. The vertical axis in the figure represents the magnitude in percentage points

[7]Two examples of this literature are Buhmann et al. (1988) and Coulter et al. (1992b)

of the estimated δ regression coefficient of the effect that the N_h^0-size of household needs has on p_h.

One estimated δ regression coefficient is obtained in each of the 1,000 simulations, thus, each δ coefficient measures the strength of the relationship between p_h-multidimensional deprivation incidence and the size of household needs in the counterfactual scenario of no ilegitimate difference in deprivation. The 1,000 obtained δ coefficients describe the distribution of this relation in the (counterfactual) population in which there is no difference in deprivation resulting from unfair causes. The mean of this obtained regression coefficient across the 1,000 simulations is used as measure of central tendency of the behaviour of δ.

In Figure 2, each marker represent this central tendency measure of the δ regression coefficient obtained from using a particular $m^{\beta,\theta}$ measure. The shaded zone around the markers represents the range of variability of 95% of these 1,000 obtained estimates of δ. Any measure that properly accounts for legitimate differences in needs is, ideally, expected to have a distribution with a mean of zero and a narrow spread (such as 95% of the values within that narrow interval).

As observed, the mean of the obtained δ regression coefficient across the 1000 simulations, when using $m^{0,0}$ to sort and identify households is 17.8 percentage points (p.p.), with a range of variability of 95% of its values between 16.3 and 19.4 p.p. This result indicates, that comparing households on the basis of the widely used AF dimensions-count-based approach ($m^{0,0}$) does not permit an unbiased incidence profile. The simulation results of using this metric show a distribution of estimates far above the desirable zero mean, and their values are concentrated around this positive mean.

Similarly, the mean across the 1,000 simulations of the δ regression coefficient between p_h and the size of the needs, obtained when measuring the burden of multidimensional deprivation on the basis of the deprivations-count-based approach to measurement ($m^{1,0}$), results to be 21.9 p.p, with 95% of its values between 20.7 and 23.2 p.p.

A positive δ regression coefficient observed across all the 1,000 counterfactual scenarios when measuring the burden of multidimensional deprivation by any of these two metrics (the dimensions-count-based approach and the deprivations-count-based approach) indicates that these both metrics produce multidimensional deprivation incidence p_h to be correlated with the size of the household needs. This occurs even when households do not have any illegitimate difference in deprivation among them.

When the $m^{0,0}$ metric is used to sort and identify multidimensionally deprived households, an additional dimension that households exhibit as need increases by an average of 17.8 p.p. the ability of the household to be classified as multidimensionally deprived. Similarly, when $m^{1,0}$ is used to sort households, an additional possible household scoring dimension increases multidimensional deprivation incidence by an average of 21.9 p.p.

These results demonstrate that count-based measures cause any two households with different sizes of household needs to show different multidimensional deprivation incidence even if there is no illegitimate difference in deprivation between the two of them. Thus, these two metrics proved unable to properly capture a state in which there are no unfair differences in deprivation between households.

On the other hand, sorting households using a share-based approach to measurement, either an $m^{0,1}$ or an $m^{1,1}$ metric, does not permit unbiased multidimensional deprivation incidence profiles. The distribution of the obtained δ regression coefficient in these two cases is concentrated far below zero, and the interval of 95% of their values is narrow around the negative mean of the 1,000 obtained δ regression coefficients. An negative mean across the simulations of the δ regression coefficient, indicates that the metric used to sort and identify households does not effectively addressed differences in need. It produces multidimensional deprivation incidence to decrease systematically as the size of household needs increases.

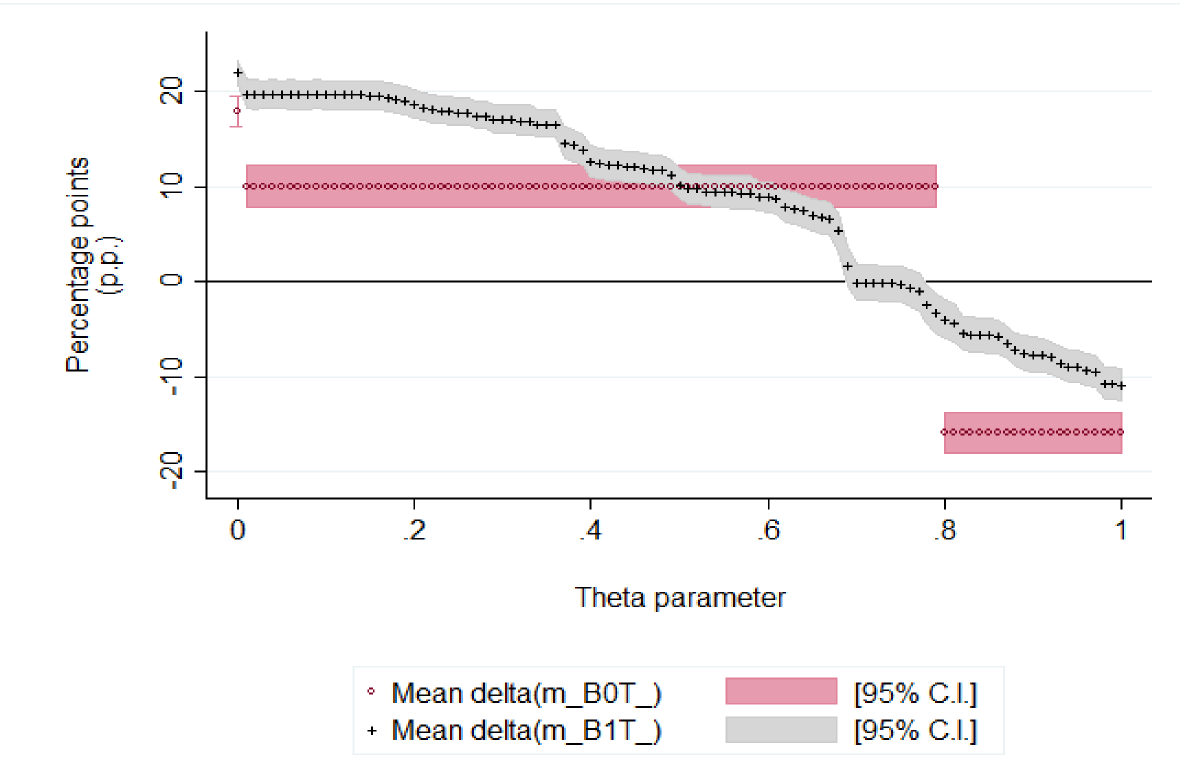
For instance, the use of a deprivations-share-based approach to measurement ($m^{1,1}$) to sort households produces a distribution of the 1,000 obtained δ regression coefficient concentrated around -10.9 p.p., and the distribution of 95% of the estimates varies between -12.6 and -9.1 p.p. This means that, even when there is no difference in illegitimate deprivation between households, the use of an $m^{1,1}$ measure to sort and identify multidimensionally deprived households produces an additional dimension that the household exhibits as a need to reduce the ability of this household to be classified as multidimensionally deprived at 10.9 p.p.

Whereas count-based approaches cause a biased picture of household-based multidimensional deprivation profiles, larger and more heterogeneous households are more likely to be identified as the most deprived. Share-based approaches invert these results, producing also a biased picture of household-based multidimensional deprivation profiles. In the latter case, in contrast to count-based approaches, small and homogeneous households tend to be more likely to be identified as the most deprived, but only about half as often as in count-based approaches.

Nonetheless, sorting households in these counterfactual states based on any $m^{\beta,\theta}$ measures that use $\beta = 1$ and a value θ between 0.69 and 0.77 satisfies the desirability condition for the particular case of 2013 Paraguayan index example. Any of these metrics produces a distribution of the obtained 1,000 δ regression coefficients between $p_h(m_h^{\beta,\theta})$ and N_h^0 with values very close to zero and a narrow spread of the distribution around this value. These results suggest that, in the case of the 2013 Paraguayan example, those metrics enable us to depict as equivalently deprived households with no illegitimate difference in deprivation but only differences in needs among them.

Measuring the household multidimensional deprivation based on a burden with

Figure 2: Simulation results: distribution of the obtained δ regression coefficient in percentage points (p.p.) when using $m^{\beta,\theta}$ to sort and identify the most deprived households



Source: PHS 2013. Notes: Estimated population means produced based on a sample of 5,423 households. Results obtained by simulating 1,000 independent times a random allocation of deprivation across the observed households, keeping constant the demographic configuration of the households and the societal amount of deprivation in each indicator. Shaded areas denote 95% of the obtained δ estimates. The lower limit corresponds to the δ value at the 0.025 percentile and the upper limit to the δ value at the 0.975 percentile.

a larger aversion to deprivation parameter, such as $\beta = 1$, in comparison to measuring it with a smaller aversion to deprivation parameter, such as $\beta = 0$, shows the distribution of the estimated δ coefficient increasing the adjustment by the size of the needs as long as we increase the θ deprivation response scale parameter.

In summary, the simulation results presented in this section indicate that neglecting differences in needs yields a biased picture of household-based multidimensional deprivation incidence profiles. While count-based approaches produce larger multidimensional deprivation incidence among households with larger sizes of needs, share-based approaches produce larger incidence among households with smaller sizes of needs. The degree to which we must account for these differences in need, therefore, stands out as relevant. Particular members of the proposed family of measures of the burden of multidimensional deprivation proved able to depict as equivalently deprived households with different sizes of needs, permitting unbiased multidimensional deprivation incidence profiles. We now analyse the robustness of these obtained results under alternative considerations.

5.5 Alternative specifications

To investigate whether or not the obtained results are robust to alternative considerations, three different sources of possible variation in the aforementioned methodological approach are implemented. First, measures are evaluated using alternative functional forms. Second, we analyse whether or not the obtained results are robust under alternative combinations of relative importance for the five considered indicators in the 2013 PHS index. Third, the robustness of the results is studied under different shares of the population to identify the most deprived population of households. This section presents the results of these three complementary analyses.

5.5.1 On the functional form

Section 5.4's results were derived assuming that the relationship between the p_h indicator of the presence or absence of multidimensional deprivation and the size of household needs, in the counterfactual scenario with no difference in illegitimate deprivation among households, was linear (if such relation exists). The size of household needs was measured as the number of dimensions that the household exhibit as need in the multidimensional index. This section presents the results of an evaluation of incidence profiles using two other specifications of the size of household needs, and assuming a not necessarily linear relation between p_h and the size of household needs.

Figure 3 summarizes the distribution of the δ regression coefficients obtained across the 1,000 simulated counterfactual scenarios of no illegitimate difference in deprivation among households, when using two different specifications of the size of household needs. While Figure 3.a presents the results of the δ regression coefficient obtained from using the linear regression $p_h = \rho + \delta N_h^1$, where N_h^1 measures the size of household needs as the number of achievements that the household exhibit as needs. Figure 3.b presents the results of this δ regression coefficient in the case we measure the size of household needs as simply household size.

As expected, the obtained δ regression coefficient estimates vary as the specification of the size of household needs vary. However, the distribution of estimates show consistent results across the different implemented specifications of the size of household needs. In particular, sorting households under the basis of a measure that does not account for differences in needs (the $m^{0,0}$ and the $m^{1,0}$ metric) resemble across the 1,000 simulations and three different specifications of the size of household needs, a δ regression coefficient far above zero and with a narrow interval of 95% of its values concentrated around the positive mean of estimates. This result confirms that profiles using these two metrics do not satisfy the desirability condition because they do not permit unbiased multidimensional deprivation incidence profiles.

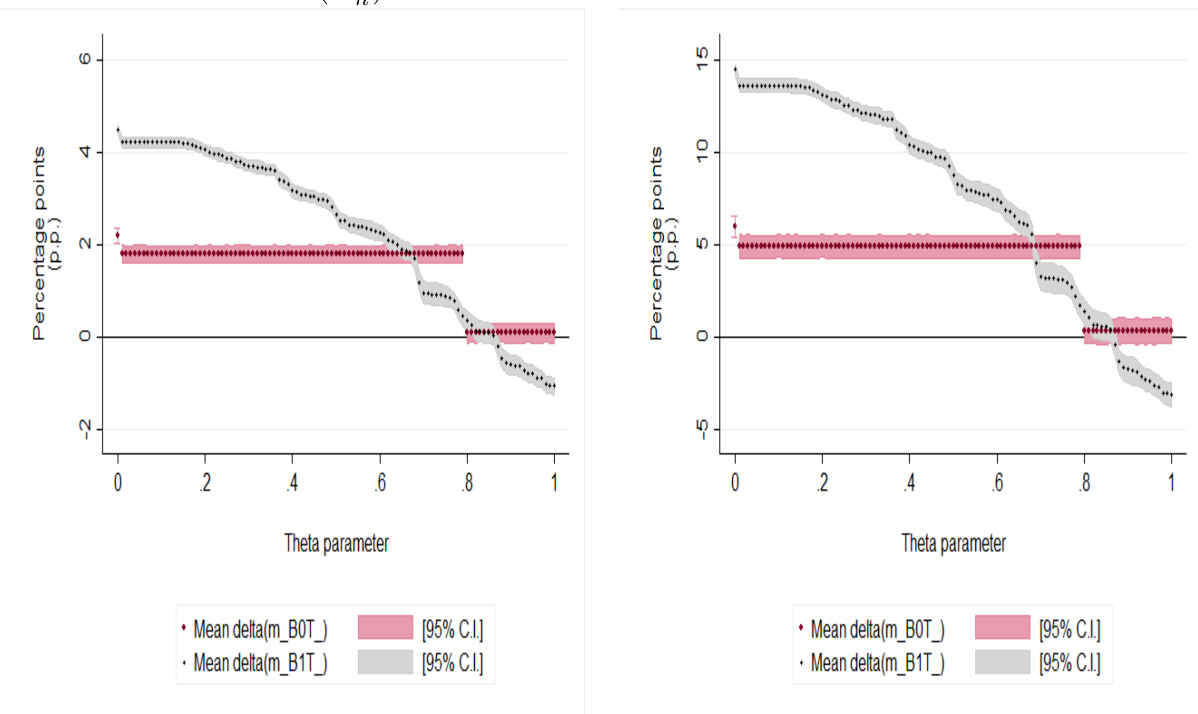
For instance, when using the dimensions-count-based approach to measurement (the $m^{0,0}$ metric) to sort and identify households, an increase of one household achievement in need leads to an increase of 2.2 p.p. in the multidimensional deprivation incidence. This $m^{0,0}$ approach to measurement also produces an average 6.0 p.p. larger incidence of multidimensional deprivation as the number of persons in the household increases by one. Thus, even when households have no illegitimate difference in deprivation among them, the use of $m^{0,0}$ yields biased multidimensional incidence profiles. The multidimensional deprivation incidence increases as household needs increase.

In the case of a deprivations-share-based approach ($m^{1,1}$), the results of the alternative specifications of the size of household needs confirm that using the $m^{1,1}$

Figure 3: Simulation results: distribution of the obtained δ regression coefficient in percentage points (p.p.), using two different specifications of the size of household needs

(a) Size of household needs: number of achievements that the household exhibit as need (N_h^1)

(b) Size of household needs: number of persons per household



Source: PHS 2013. Note: Notes: Estimated population means produced based on a sample of 5,423 households. Results obtained by simulating 1,000 independent times a random allocation of deprivation across the observed households, keeping constant the demographic configuration of the households and the societal amount of deprivation in each indicator. Shaded areas denote 95% of the obtained δ estimates. The lower limit corresponds to the δ value at the 0.025 percentile and the upper limit to the δ value at the 0.975 percentile.

metric also yields a biased picture of the multidimensional deprivation incidence. The obtained distribution of the δ regression coefficient across the three alternative specifications of the size of household needs ranges around negative values far from the desirable zero mean.

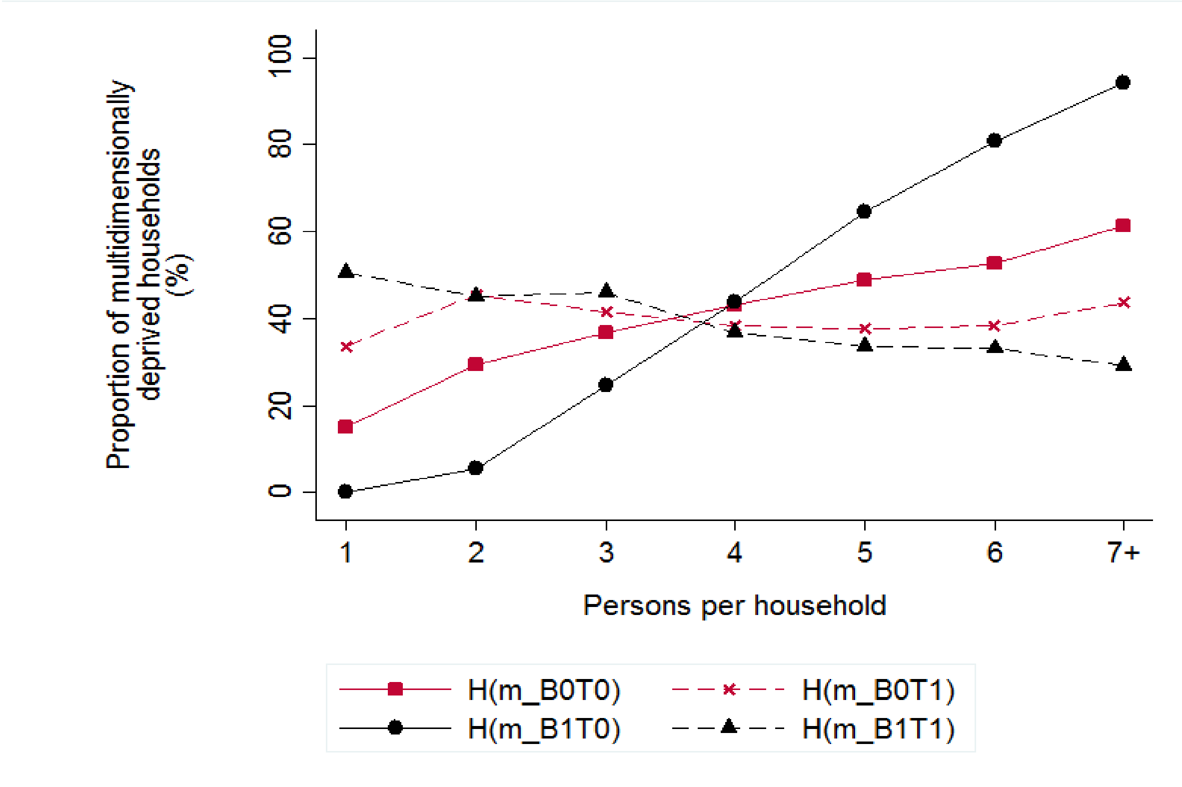
On the other hand, analysts might find context-specific applications where the factual relation between the multidimensional deprivation incidence and the size of household needs is not linear, and we observe other shapes, such as U shapes or tick shapes. In this case, again, as either of these shapes corresponds to a factual observed case, we do not know how much of the difference in multidimensional deprivation incidence observed between households of different sizes of needs corresponds to legitimate differences in deprivation and how much of it can be attributed to illegitimate differences in deprivation. Thus, the important relationship to analyse is between the multidimensional deprivation incidence and the size of household needs in a counterfactual scenario with no illegitimate difference in deprivation among households. In such scenario is where we can determine whether or not the index is providing an unbiased multidimensional deprivation profile.

Let us recall that the proposed desirability condition seeks to identify as equivalently deprived households of different needs when no illegitimate difference in deprivation exists because differences in needs are understood as legitimate sources of differences. Then, if we observe a relation between household size (for instance) and multidimensional deprivation incidence in a counterfactual scenario with no difference in illegitimate deprivation, regardless of the shape of the relationship, the profile does not satisfy the desirability condition and is said to portray a biased picture of multidimensional deprivation.

As such, one could argue that, in these cases, the proposed linear regression approach to compare p_h and the size of household needs in the counterfactual scenario might be mistaken because such an approach could reveal no relation between p_h and the size of household needs, but still there could be an underling relationship between them. To analyse these possible alternative situations, the most straightforward recommended evaluation is to build independent simulations of the counterfactual state with no illegitimate difference in deprivation among households and, rather than using a linear regression to measure the relationship between p_h and the size of household needs, using a graphic representation of the relation of p_h and household size, as Figure 4 shows.

A measure satisfying the desirability condition should indicate no relation between household size and the incidence of multidimensional deprivation in the counterfactual scenario. For instance, we observe in Figure 4 that all four evaluated measures register a specific relation between H and household size. As an example, if we use the AF proposed $m^{0,0}$ to identify multidimensionally deprived households and compare the H incidence of multidimensional deprivation between households consisting of one person and households consisting of two persons, multidimensional deprivation in the counterfactual scenario between the two groups of households differs by 11.6 percentage points. If we compare the obtained mean between each consecutive pair of household groups, for any compared pair of household groups the obtained mean of H increases by at least 4 p.p. as household size increases. When comparing households consisting of seven or more persons and households consisting

Figure 4: Simulation results: $H(m^{\beta,\theta})$-proportion of multidimensionally deprived households, across household size



Source: PHS 2013. Notes: Estimated population means produced based on a sample of 5,423 households. Results obtained by simulating 1,000 independent times a random allocation of deprivation across the observed households, keeping constant the demographic configuration of the households and the societal amount of deprivation in each indicator.

of one person, we observe a difference in multidimensional deprivation between the two groups of households of 46.2 percentage points. If the measure would portray an unbiased profile, there should be no difference or a difference very close to zero between the incidence of these groups of households because we are evaluating the results in the counterfactual state with no illegitimate difference in deprivation among households. These results indicate a positive linear relation between household size and H, a result consistent with the δ regression coefficients obtained in Section 5.4.

Graphic representations can nonetheless sometimes be difficult to interpret. Thus, another possible methodological approach to understand the underlying relation between household size and the p_h multidimensional deprivation incidence in the counterfactual scenario with no illegitimate source of deprivation is by measuring the size of household needs with dummy variables of different household sizes and regressing them against the p_h indicator obtained for the counterfactual scenario. The joint significance of the estimated relationship between the dummies and p_h

is tested as equal between each other and to zero. This is repeated for each independently simulated counterfactual scenario. For instance, for the particular case of the $m^{0,0}$ metric in the 2013 PHS application, I built seven dummies of different household sizes and regress p_h against six of them. This, in each of the 1,000 independent randomly simulated scenarios of no illegitimate difference in deprivation among households. In all of the 1,000 regressions the null hypothesis of equal relationship between the dummies and p_h was rejected. These 1,000 consistent results confirm that measuring the burden of multidimensional deprivation with a $m^{0,0}$ metric does not permit households of different size with no illegitimate difference in deprivation to be classified as equivalently deprived. If, for instance, the same approach is used to evaluate a profile based on an $m^{1,0.87}$ metric, in 80.8% of the 1,000 counterfactual scenarios, we cannot reject the null hypothesis of an equal estimated relationship among the six dummies for household size and p_h. These results indicate that the $m^{0,0}$ metric does not permit an unbiased multidimensional deprivation profile in our 2013 Paraguayan application.

We now go on to present the results obtained from evaluating multidimensional deprivation incidence profiles under two other possible sources of methodological variation.

5.5.2 Alternative population shares

The empirical results presented in Section 5.4 correspond to the Paraguayan index example developed on the basis of identifying the 40% most deprived households, according to each $m^{\beta,\theta}$ measure, as the multidimensionally deprived population. In this section, we evaluate the behaviour of the counterfactual scenario results, but rather than selecting the 40% most deprived households, the results are obtained using three different shares of the population: 20%, 30%, and 50% of the population of households. The first column of Figure 5 on page 48 shows the results of using these three alternative shares of the multidimensionally deprived population.

The results obtained from using these three alternative population shares are consistent with the results obtained for the identification of 40% of the population of households as multidimensionally deprived (Section 5.4' results). When sorting households on the basis of an $m^{\beta,\theta}$ measure that is not adjusted for differences in need using the widely used AF $m^{0,0}$ measure or the $m^{1,0}$ metric, both produce a δ

relation between the incidence of multidimensional deprivation and the size of the household needs in all 1,000 simulated scenarios that is always larger than 10 p.p. In these three alternative methodological specifications, neither the AF dimension count-based approach nor the deprivations-count-based-approach satisfies the desirability condition.

In the case of a deprivations-share-based approach ($m^{1,1}$), the results of the three alternative population shares are also consistent with the results obtained on the basis of identifying 40% of the population of households. The distribution of the estimates of the relation between the incidence of multidimensional deprivation and size of household needs ranges around values still not around the desirable zero mean, but in about half the size of mean δ obtained by the $m^{0,0}$ metric. The use of the $m^{1,1}$ metric also yields a biased picture of the multidimensional deprivation incidence.

The distribution of δ estimates around the desirable zero mean and with a narrow interval of 95% of its values concentrated around it varies as the population share of households identified as multidimensionally deprived varies. Identifying the 40% most deprived households as multidimensionally deprived and using $m^{\beta,\theta}$, where $\beta = 1$ and $\theta = [0.69, 0.77]$, results in this desirable distribution; the identification of different population shares as multidimensionally deprived produces different sets of θ parameters exhibiting this desirable distribution. The selection of parametric values of β and θ to describe the burden of household multidimensional deprivation under the proposed methodology of this article is advised in light of robustness checks using different multidimensional deprivation thresholds and specifications.

5.5.3 Alternative weighting systems

The empirical results presented in Section 5.4 correspond to our Paraguayan index example without applying any **w** dimensional weighting system. This implies that, in practice, the relative importance of each of the five considered indicators represents one-fifth of the whole index. This section evaluates the performance of the multidimensional deprivation profiles in the counterfactual scenario with no difference in deprivation resulting from illegitimate causes under alternative **w** dimensional weighting specifications. Each alternative specification involves applying a specific

w dimensional weighting system. Table 3 includes the different specifications of analysed **w** dimensional weighting systems.

The first set of weights (weighting system 1 in Table 3) represents the mainstream approach to set weights in the current multidimensional deprivation literature, a nested weighting structure. This commonly used structure of weights applies equal relative importance to each dimension and each indicator within each dimension. It corresponds to the particular approach used by Alkire et al. (2013), Alkire et al. (2015b), and Angulo et al. (2015), among others.

The second alternative weighting system aims to specify the index through a combination of indicators that balance the applicable sub-population groups within each dimension. This type of balancing procedure was proposed by Alkire (2015), as an alternative methodological approach to account for differences in need. It implies each well-being dimension to account all population subgroups with one applicable deprivation indicator. Note that, when introducing the five considered indicators for the Paraguayan illustration (Table 2 above), the dwelling conditions and health dimensions both include a set of indicators that are applicable to any person; the education dimension, in contrast, includes two indicators that together do not cover all population subgroups. In particular, the education dimension does not include any indicator for children under five years of age. Thus, in the Paraguayan example, a balancing procedure can be done by either including an additional indicator in the education dimension, which must be applicable exclusively to children under five years of age, or excluding the education dimension from the index. We adopt a balancing procedure here by excluding the education dimension from the index, as shown in weighting system 2 of Table 3.

In the vein of this type of balancing procedure, another possible course of action to account for differences in need across population subgroups could be the application of a set of weights that vary across groups that exhibit different sets of needs. This procedure would consist of defining a set of weights such that the sum of them across indicators is one per each population group that is accounted in the index through a different set of indicators. This corresponds to the third analysed alternative weighting system and is shown by the last three columns of Table 3.

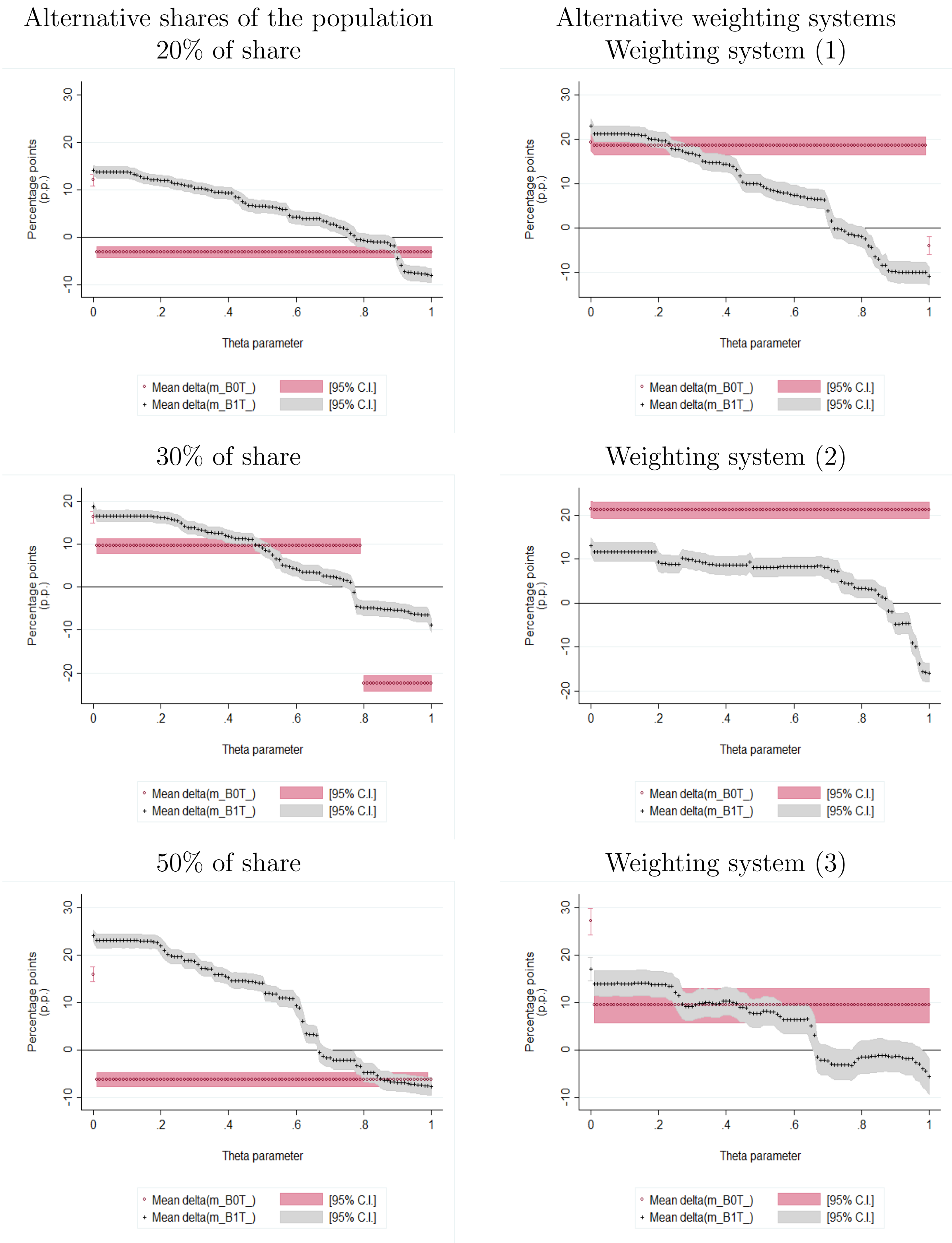
The evaluation results obtained from using these three alternative weighting systems are presented in the second column of Figure 5. Consistent with the results shown in Section 5.4, the three alternative specifications of weights show that using the AF dimensions-count-based approach to measurement ($m^{0,0}$) results in multidimensional deprivation increasing as the size of household needs increases. This, even households have no illegitimate difference in deprivation among each other. Again, the distribution of the estimates of this measure across the three different weighting systems and the 1,000 independently simulated scenarios shows always a δ regression coefficient between the multidimensional deprivation incidence and the size of household needs larger than zero of more than 16 p.p. These alternative specifications results confirm that, despite the use of balancing procedures or alternative weighting systems to address households' differences in needs, using the $m^{0,0}$ metric to compare households and identify the most deprived yields a biased picture of multidimensional deprivation profiles.

These alternative specification results also confirmed that using a dimensions-share-based approach ($m^{1,1}$) to sort households and identify the most deprived generates a biased picture of the multidimensional deprivation incidence. A multidimensional deprivation incidence profile that uses $m^{1,1}$, as well as when using $m^{0,0}$, does not satisfy the basic premise or desirability condition. In the case of $m^{1,1}$, the distribution of δ is concentrated around negative values smaller than -2 p.p. Thus, the multidimensional deprivation incidence systematically decreases as the size of the household needs increases when using a deprivations-share-based approach to measurement.

Table 3: Alternative implemented weighting systems

Well-being dimension	Deprivation indicator	Weighting system				
		(1)	(2)	(3)		
		Any person	Any person	0-4 years old	5-17 years old	18+ years old
Health	Health insurance non-coverage	1/6	1/4	1/4	1/6	1/6
	No access to health services	1/6	1/4	1/4	1/6	1/6
Education	Non-school attendance	1/6	0	0	1/3	0
	Low educational achievement	1/6	0	0	0	1/3
Dwelling conditions	Sub-standard housing	1/3	1/2	1/2	1/3	1/3

Figure 5: Simulation results: distribution of the obtained δ regression coefficient in percentage points (p.p.) when using $m^{\beta,\theta}$ to sort and identify the most deprived households



Source: PHS 2013. Notes: Estimated population means produced based on a sample of 5,423 households. Results obtained by simulating 1,000 independent times a random allocation of deprivation across the observed households, keeping constant the demographic configuration of the households and the societal amount of deprivation in each indicator. Shaded areas denote 95% of the obtained δ estimates. The lower limit corresponds to the δ value at the 0.025 percentile and the upper limit to the δ value at the 0.975 percentile.

As a result, this empirical evaluation has shown under different alternative considerations that measuring the burden of multidimensional deprivation without accounting for differences in need produces a biased multidimensional deprivation incidence profile because it captures not only illegitimate differences in deprivation but also unaddressed legitimate differences in needs. The case of a deprivations-share-based approach to measurement is similar. Though a deprivations-share-based approach address differences in needs, this approach to measurement overshoots the results. Like count-based approaches, share-based approaches lead to biased multidimensional deprivation incidence profiles. Neither approach satisfies the desirability condition outlined in this paper to be attained by the multidimensional deprivation incidence.

Nonetheless, other different combinations of β and θ to describe the burden of household multidimensional deprivation in the context of the 2013 Paraguyan application have proved to reveal unbiased multidimensional deprivation incidence profiles. The selection of parametric values of β and θ to describe this burden is advised in light of robustness checks using different multidimensional deprivation thresholds and specifications.

Before moving on to investigate the properties exhibited by the family of measures proposed in this paper, I briefly discuss in the next section the case when multidimensional deprivation is evaluated rather than at the household at the individual level; the methodology presented in Section 3.6 is named 'the individual-based scenario'.

5.6 The individual-based scenario

This section illustrates the empirical behaviour of the multidimensional deprivation measurement methodology proposed in this article in an individual-based scenario and evaluates its proposed measures. For this purpose, the same 2013 PHS indicators used for analysis in previous sections are used here. However, as described when outlining the methodology for individual-based multidimensional deprivation measurement in the presence of differences in needs (Section 3.6), household-based aggregates are not pursued here. In contrast, in the individual-based scenario, each individual is considered its own household and the burden of multidimensional deprivation is measured by an m^{θ} metric. Measuring the burden of multidimensional

Table 4: Observed individual dimensional deprivation

Deprivation indicator	Applicable population where the indicator is relevant	Number of observed persons in the applicable population	Proportion (%) of deprived persons in the applicable population
Health insurance non-coverage	Any person	20,909	71.9
No access to health services	Any person that was sick or had an accident during the 90 days previous to the interview	7,199	23.8
Non-school attendance	5 - 17 years old population	5,706	6.7
Low educational achievement	18 years old population and over	13,406	47.9
Sub-standard housing	Any person	20,909	24.6

Source: PHS 2013.

deprivation without accounting for differences in need imply setting $\theta = 0$, which is simply the number of dimensions of deprivation that each individual exhibits. This corresponds to the AF method.

Table 4 presents for each 2013 PHS considered deprivation indicator the number of observed persons in its applicable population and the proportion of deprived persons within it. These five deprivation indicators are subsequently combined to depict the burden that multidimensional deprivation places on each individual. As a result, we obtain the m^{θ} index that takes values according to the used θ parameter of responsiveness of deprivation with regard to the level of needs.

A burden of multidimensional deprivation that does not account for the size of individual needs counts the number of deprivations that each individual exhibits. However, given that the accounted needs vary across three population subgroups (children under five years of age, 5- to 17-year-old children, and those 18 years of age and over), the use of m^0 leads to a smaller mean burden of multidimensional deprivation among the population groups with a smaller number of accounted dimensions. This is the case for children under five years of age. This population subgroup is recorded as having need with respect to three out of the five considered indicators, whereas children from 5 to 17 years of age and the population 18 years of age and older may be recorded as exhibiting a in four out of the five considered indicators.

The proposed methodology of this paper seeks to enable multidimensional deprivation measurement in the presence of differences in need. The same methodological approach used for the household-based scenario is followed here. Observed differences in the unadjusted burden of multidimensional deprivation across demographic heterogeneous groups capture, in addition to illegitimate differences in deprivation, differences in deprivation caused by legitimate and unavoidable differences in needs. Thus, to evaluate measures, we use the desirability condition outlined on page 31, which states that any two individuals with no illegitimate difference in deprivation between them must have the same multidimensional deprivation incidence.

In a counterfactual scenario with no illegitimate difference in deprivation among individuals but only differences in need across them, we analyse the relationship between the multidimensional deprivation incidence (p_i) and the size of individual needs. Similar to the household-based analysis, here I approach this analysis via the linear regression $p_i = \rho + \delta N_i$, where N_i represents the size of individuals multidimensional needs and is measured by the number of achievements, i.e., indicators, the individual exhibits as needs.

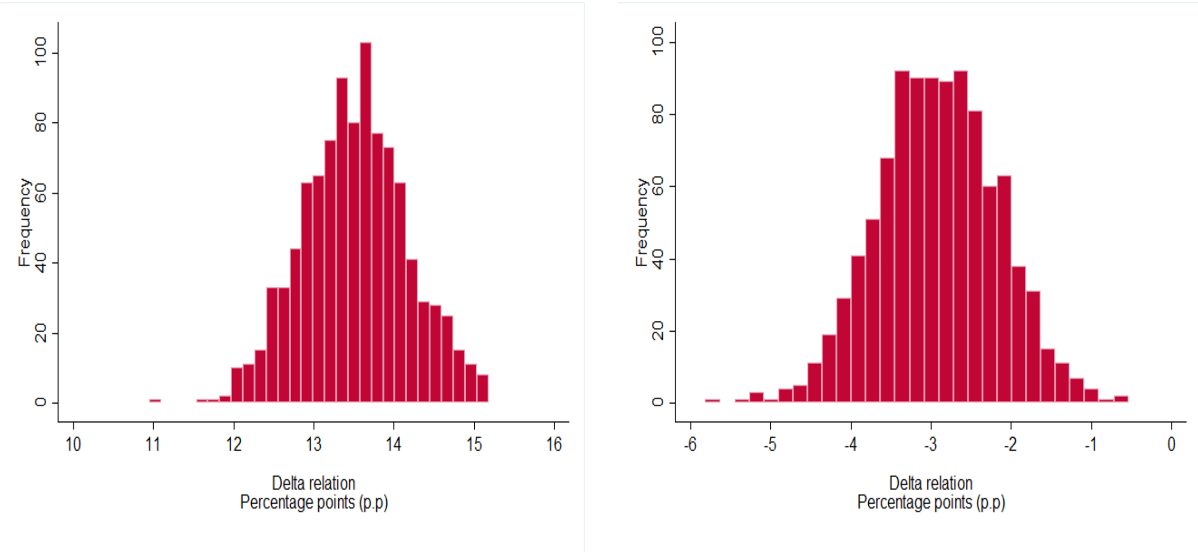
Figure 6 plots the evaluation results of m^0 and m^1. The distribution of obtained δ estimate coefficient, when using m^0 to rank individuals, register positive values across all the 1,000 simulated scenarios and ranges between 11.0 and 15.2 p.p. In these counterfactual scenarios of randomly allocated deprivation, one additional dimension increases the multidimensional deprivation incidence by an average of 13.5 p.p. These results indicate that the m^0 metric leads to a biased multidimensional deprivation incidence profile.

A multidimensional profile based on a share-based measure does not satisfy the desirability condition. As observed from Figure 6.b, the estimates of the δ regression coefficient across the 1,000 simulations range between -5.8 and -0.5 p.p., and the mean of these estimates is concentrated at 2.9 p.p. below zero.

The evaluation results of these two metrics (m^0 and m^1) were obtained upon identifying as multidimensionally deprived the 40% most deprived population. Other

Figure 6: Simulation results: distribution of the obtained δ regression coefficient in percentage points (p.p.) using two different m^θ measures

(a) m^0 (b) m^1



Source: PHS 2013. Notes: Estimated population means produced based on a sample of 20,909 individuals. Results obtained by simulating 1,000 independent times a random allocation of deprivation across the observed individuals, keeping constant the demographic configuration of the population and the societal amount of deprivation in each indicator.

alternative population shares were also used to identify the multidimensionally deprived population (30% and 20%), and the results proved robust under these other two population shares.

We now move on to describe our proposed family of measures in terms of the characteristics that make it subject to evaluation as suitable for the purpose of multidimensional deprivation measurement.

6 On the properties

Following the classification of properties for income or expenditure-based poverty measures proposed by Foster (2006) and generalising this classification for the multidimensional poverty measures, this section first investigates the properties that make the societal measures proposed in this paper non-sensitive to some aspects of the distribution, namely *Scale invariance*, *anonymity*, *replication invariance*, and *focus*. Subsequently, it analyses the features that reflect a proper orientation of the proposed societal measures, namely the *dominance properties*. This section completes the discussion, examining how *transfers*, *decomposability*, and *continuity* behave in the context of the family of measures proposed in this paper.

Scale invariance

The income or expenditure poverty measurement literature has traditionally used a *scale invariance* or *normalization* property to ensure societal measures are expressed in relation to the poverty line. In particular, this is the approach used by Foster et al. (1984) and described in detail by Foster (2006). Along with the *anonymity* and *replication invariance* property, which I discuss ahead on this section, this scale invariance property has been used in literature to enable comparisons of the incidence, depth and severity of poverty across societies of different sizes.

For the particular case of the H and $MD^{\beta,\theta}$ measures of multidimensional deprivation proposed in this paper, the range of variability vary along β and θ vary. The following focuses on describing the range of variability of these societal measures.

A relative approach to measurement is used excursively by H and $MD^{\beta,1}$ measures. As such, they take values from the interval $[0, 1]$. While H expresses the incidence of multidimensional deprivation in relation to the size of the population of households, $MD^{\beta,1}$ expresses multidimensional deprivation in relation to the size of household needs.

In particular, in an individual-based scenario (i.e., any i individual is its own household), if all individuals in society are identified as multidimensionally non-deprived, then $H = 0$. In addition, if all individuals in the society are identified as multidimensionally deprived, then $H = 1$.

Similarly, in the household-based scenario, if all households in the society are non-multidimensionally deprived, then $H = 0$, and if all households in the society are non-multidimensionally deprived, then $H = 1$. However, in such household-based scenario, multidimensional deprivation is not evaluated at the individual level. Therefore, in such a case, $H = 1$ does not imply that all individuals in society are multidimensionally deprived. It implies, in contrast, that all individuals in the society belong to a multidimensionally deprived household. Similarly, $H = 0$ does not mean that all individuals in the society are non-multidimensionally deprived; it reflects that all individuals in society belong to a non-multidimensionally deprived household.

In terms of the $MD^{\beta,1}$ metrics, while any $MD^{\beta,1} = 0$ indicates all i household members are non-deprived in all their j applicable dimensions, the interpretation of

$MD^{\beta,1} = 1$ vary as β varies. Only $MD^{1,1} = 1$ indicates all i household members are deprived in all their j applicable dimensions. For instance, $MD^{0,1} = 1$ rather indicates all households have at least one j deprived household member in all their applicable dimensions.

On the other hand, any $MD^{\beta,\theta}$, where $\theta \in [0, 1)$ uses either an absolute or an intermediate approach to measurement. These societal measures, therefore, take values not necessarily from the interval $[0, 1]$, but rather from the interval $[0, \mu(N_h^\beta)]$, where $\mu(N_h^\beta)$ corresponds to the average value of N_h^β for $h = 1, 2, \ldots R$, and N_h^β corresponds to the size of household needs for the h household – as is discussed on page 15. Worth noting then, that the use of these metrics does not lead societies with different sizes of needs to reach the same $MD^{\beta,\theta}$ value whenever all i household members are j deprived. This result is consistent with the absolute or intermediate approach used for measurement.

To illustrate the range of scale of the most important societal measures of the proposed methodology of this paper, the Paraguayan index example is used. In particular, the first row within Table 5 below in page 67 shows the observed H and $MD^{\beta,\theta}$, where $\beta = \{0, 1\}$ and $\theta = \{0, 1\}$ in the 2013 PHS. To develop this observed case, any h household satisfying $m_h^{1,0.87} > 0.65$ is identified as multidimensionally deprived. Then, 40% of the 2013 Paraguayan households in the sample is identified as multidimensionally deprived.[8] The results on only these five metrics are analysed as they are the most important societal measures of my methodology. Henceforth in this section I focus on the analysis of these five societal measures.

Subsequently, two scenarios worth analysing are simulated: first, the scenario where all household members are assumed as non-deprived in all their relevant indicators. Second, the scenario where all household-members are assumed as deprived in all their relevant dimensions. In each of the simulations, any h household satisfying $m_h^{1,0.87} > 0.65$ is identified as multidimensionally deprived. The second and third rows in Table 5 include the results of the fully non-deprived scenario and fully deprived scenario, respectively.

[8] Although this particular measure of the household burden of multidimensional deprivation ($m_h^{1,0.87}$) was selected in light of the findings of Section 5 above, here in this Section is used only for illustrative purposes. As such, the examples presented in this section can be equivalently derived from different $m^{\beta,\theta}$ measures and different k-thresholds.

It can be seen from the table that the fully non-deprived scenario results in all measures having a value of zero. Now, if it is assumed that all individuals in society have all their a_{ij} applicable achievements in deprivation and the $m_h^{1,0.87}$-burden of multidimensional deprivation is evaluated again, as is seen from Row (3) in the table, 100% of households result in being identified as multidimensionally deprived ($H = 1$). Also, in this fully deprived scenario, it is observed that the $MD^{\beta,\theta}$ measures that use a shared based approach to measurement, namely the $MD^{0,1}$ and $MD^{1,1}$ measures, exhibit a value of 1.0. One the one hand, the value $MD^{0,1} = 1$ indicates that households in society have an average of 100% of their applicable dimensions in deprivation. Similarly, $MD^{1,1} = 1$ indicates that households in society have on average 100% of their applicable achievements in deprivation.

Furthermore, both measures $MD^{0,0}$ and $MD^{1,0}$ in the fully deprived scenario take the value of the societal mean of household needs. For instance, in the 2013 Paraguayan application, this fully deprived scenario shows $MD^{0,0} = 4.2$ and $MD^{1,0} = 12.6$, meaning that, households in society have in average 4.2 dimensions in deprivation and 12.6 deprived achievements; values that in turn represent the average societal size of household needs.

Anonymity

The poverty measurement literature and, in particular, the multidimensional literature characterise some families of societal measures under a *symmetry* or *anonymity* property. For instance, according to Alkire and Foster (2011), the symmetry property that their family of measures uses ensures that societal metrics are not being constructed under the basis of greater emphasis on some population subgroups over others. This property is also used by multidimensional measures, such as the ones proposed by Tsui (2002), Bourguignon and Chakravarty (2003), and Seth (2010), among others. Bourguignon and Chakravarty (2003) defined their measures as symmetric since any person's characteristics, other than the multiple well-being dimensions considered for the measure, are set as not relevant in the measurement process of their measures. Similarly, Seth (2010) suggested that the identities of the individuals are not ethically significant in the measurement process. As such, individuals within society are considered anonymous. I henceforth refer to this measurement property as *anonymity*.

In practice, however, assuming anonymity of individuals that exhibit different needs, without accounting for these differences in need, results in biased multidimensional incidence profiles. The text that follows elaborates further on this.

As discussed in previous sections, needs differ across dimensions of multidimensional deprivation by population subgroup. While a particular population subgroup can be catalogued as deprived in a certain j dimension because it lacks an achievement level that is considered as needed, this does not necessarily mean that all demographic sub-population groups that lack such an achievement level can be catalogued as deprived. Therefore, an individual can only be regarded as deprived by a particular indicator if it measures an achievement which can be viewed as something this individual legitimately needs.

In the methodological approach of this paper, differences in needs are considered as fair sources of differences in deprivation; therefore they are tackled throughout the measurement process. Neglecting heterogeneity in needs, as the empirical findings of Section 5.4 above have demonstrated, does not enable unbiased multidimensional deprivation incidence profiles. Then, assuming anonymity across individuals without taking into account their heterogeneity in needs led to a biased picture of the incidence of societal multidimensional deprivation.

In the one-dimensional welfare measurement literature, as pointed out by Coulter et al. (1992a), heterogeneity in needs has been tackled by either measuring each persons well-being by using a common metric that incorporates the information on heterogeneity and then aggregating across persons using the anonymity property or, alternatively, by dropping the anonymity property and accounting for the heterogeneity with, for instance, a weighting system that reflects those heterogeneous needs. The approach that I present in this paper follows the first methodological strategy. Heterogeneity in needs across units is tackled by the measurement process and then the anonymity property is used.

Therefore, societal measures H and $MD^{\beta,\theta}$, for any combination of β and θ parameters, are meant to be non-sensitive to permutations of the units where the identification of the multidimensionally deprived population occurs. In the individual-based scenario, this means that societal measures are non-sensitive to rearrangements of individuals across the population. Similarly, in the household-based case, societal measures are meant to be non-sensitive to permutations of households within society and implicitly are also non-sensitive to permutations of individuals within households. These two characteristics of societal measures are termed, for the purposes of this paper, as *Household anonymity* and *Within household anonymity*, respectively.

To illustrate these two characteristics of H and $MD^{\beta,\theta}$, I simulate in the context of the 2013 Paraguayan household-based index example these two types of permutations of the population. First, 1,000 independent and random permutations of the population of households are simulated; second, 1,000 independent and random permutations of individuals within each household are simulated. The observed deprivation in each household and its demographic configuration is kept as constant. Then, H and $MD^{\beta,\theta}$ are evaluated in the observed case and after each simulation.

The observed mean of these five societal metrics before any permutation correspond to the observed case (Row (1) from Table 5). Row (4) from Table 5 shows the mean of each of the five analysed metrics across the 1,000 simulated permutations of households. Any measure non-sensitive to permutations of households would show a non-zero difference between Row (4) and Row (1). This obtained difference is shown in Row (5) of the table. It is seen in the table that, as expected, any of the five analysed measures is sensitive to permutations of households across society as they show a zero difference between the simulated and the observed case. This result illustrates Household anonymity for the five analysed measures.

On the other hand, Row (6) from Table 5 shows, for each of the five analysed societal metrics, the obtained mean across 1,000 simulations of random rearrangements of individuals within each household. Any measure sensitive to permutations of individuals within the household would show a non-zero difference between Row (6) and Row (1). It is observed, from Row (7) in the table, that any of the five analysed measures is sensitive to permutations of individuals within households. This result illustrates Within household anonymity for the five analysed measures.

It should be emphasised that permutations of individuals across households, in the household-based scenario, resemble either demographic changes or transfers across units, to which the proposed household-based measures of this article are sensitive. I further elaborate on these sensitivities on page 61 and page 66, when discussing the proposed dominance properties and how transfers behave in this context.

Population replication invariance

This measurement characteristic makes the H and $MD^{\beta,\theta}$ societal measures comparable across differently sized populations. This implies that, for a particular society made of R households, if we replicate $t \geq 2$ times these R households, the society level multidimensional measures, H and $MD^{\beta,\theta}$, will remain unaltered for any combination of β and θ parameters.

To illustrate this proposed characteristic for the H and $MD^{\beta,\theta}$ societal measures, in the context of the Paraguayan index example, I replicate t times the 5,423 PHS 2013 observed households. In this case t was defined as a random integer number $t \in [2, 1000]$. After such replication of the Paraguayan households in sample, I evaluate H and $MD^{\beta,\theta}$. This replication was repeated 1,000 independent times. The mean H and $MD^{\beta,\theta}$ obtained across these 1,000 independent replications is shown in Row (8) of Table 5. Any measure sensitive to replications of the population would show a non-zero difference between Row (8) and Row (1) of the table. As expected, any of the five analysed measures result in being sensitive to replications of the population of households. This result illustrates Population replication invariance in the five analysed measures.

Focus

An individual-based family of measures, such as one proposed by Alkire and Foster (2011), considers as non-relevant the sensitivity of societal measures to two types of increments in achievement levels: first, increments of achievement levels in the non-multidimensionally deprived population, and second, increments of achievement levels in non-deprived dimensions. The authors termed the ability of their measures to be non-sensitive to these two types of increments as *poverty focus* and *deprivation focus*, respectively.

In light of these two properties, the $MD^{\beta,\theta}$ proposed family of measures of this article consider non-relevant the sensitivity of societal measures to the following types of increments in achievement levels: i) increments in the j achievement level among individuals that belong to a non-multidimensionally deprived household; ii) increments in the j achievement level among j non-deprived individuals; and iii) increments in the j achievement level among individuals that do not belong to the j applicable population subgroup.

Therefore, societal measures H and $MD^{\beta,\theta}$, for any combination of β and θ parameters, are meant to be non-sensitive to these type of increments in achievement levels. One the one hand, the non-sensitivity of the measures to increments in achievement levels among individuals that belong to non-multidimensionally deprived households, in the context of this article, is termed *Multidimensional deprivation focus* (MDF). In comparison to the AF method, this characteristic is analogous to the poverty focus property proposed by the AF method.

It is worth noting, however, that in a household-based scenario, this multidimensional deprivation focus property enforces societal measures to be non-sensitive to increments in achievements of both deprived and non-deprived individuals that belong to multidimensionally non-deprived households. Identifying the multidimensionally deprived population at the household level is based on considering the household as a single unit. As such, it prevents observing multidimensionally deprived and multidimensionally non-deprived individuals within the same household. It produces measures to be non-sensitive to improvements or declines in achievement levels of deprived individuals that might have a large number of dimensions in deprivation but that do not belong to multidimensionally deprived households. This is in fact the case of any societal measures based on household-based metrics, either $H(m^{0,0})$ and M_0 from the AF method or the proposed $H(m^{\beta,\theta})$ and $MD^{\beta,\theta}$ measures of this article.

On the other hand, as discussed when introducing the proposed methodology of this article, every achievement is not necessarily relevant to be measured across any i person. Then, the $MD^{\beta,\theta}$ proposed measures uncover this consideration. This means that the $MD^{\beta,\theta}$ family of measures considers non-relevant achievement increments, not only among j non-deprived individuals but also among individuals that do not belong to the j applicable population. In the context of this article, this property is termed *applicable deprivation focus*.

To illustrate both focus proposed properties: multidimensional deprivation focus and applicable deprivation focus, particular increments in achievement levels are simulated in the context of the 2013 PHS index example. Each simulation is repeated 1,000 independent times. To show the sensitivity of societal measures to these increments, H and $MD^{\beta,\theta}$ are evaluated before and after each simulated increment. The following describes these simulations and the obtained results.

Multidimensional deprivation focus. An increment in the educational achievement indicator is simulated among individuals that belong to a non-multidimensionally deprived household. As such, 50% of the 18 years old and over individuals that belong to non-multidimensionally deprived households are sampled without replacement. Among them, one additional year of education is simulated. This population corresponds to 4,050 individuals out of the 20,909 individuals in the sample of the PHS 2013. Societal measures are evaluated after each of the 1,000 independent simulations. Row (10) in Table 5 shows the mean of the obtained measures after the simulations.[9]

Any measure not satisfying the Multidimensional deprivation focus property would show a non-zero difference between Row (10) and Row (1) of the table. It is observed in Row (11) of the table that the five analysed measures result in being non-sensitive to increments in achievement levels of individuals belonging to multidimensionally non-deprived households. This result illustrates Multidimensional deprivation focus for the five analysed measures.

Applicable deprivation focus. In this case, we illustrate the sensitivity of societal H and $MD^{\beta,\theta}$ to increments in the j achievement level among individuals that do not belong to the j applicable population subgroup. In particular, an increment in one year of education among 50% of the under 18 years old individuals belonging to a multidimensionally deprived household, is simulated 1,000 independent times. This population corresponds to 1,695 individuals in the PHS 2013 sample. The mean H and $MD^{\beta,\theta}$ obtained across the simulations is shown in Row (12) from Table 5.

Any measure not satisfying the Applicable deprivation focus property would show a non-zero difference between Row (12) and Row (1) in the table. It is observed in Row (13) of the table that the five analysed societal measures result being non-sensitive to increments in achievement levels among individuals that do not belong to the applicable population subgroup. This result illustrates Applicable deprivation focus for the five analysed measures.

[9]For the purposes of this illustration, the 50% share of this population subgroup was selected to ensure observing a big enough change in the measure displayed with two decimals of precision in Table 5. However, the example presented in this section can be analogously derived using different shares of the population or indicators.

We now discuss the set of properties that depict the orientation and desirable sensitivities of our measures, which are termed by Foster (2006) as dominance properties.

Dominance properties

According to Foster (2006), dominance properties are the characteristics of a poverty measurement that describe the ability of the metric to reflect improvements or declines among the poor population. They aim to resemble the proper orientation of societal metrics. In this regard, the characterization of the family of measures of this article, which is described below, draws and extends the dominance properties proposed by Alkire and Foster (2011) for multidimensional measures of poverty.

Here, I define three relevant types of improvements in achievement levels: i) *applicable achievement increment*, ii) *deprivation reduction among the multidimensionally deprived*, and iii) *dimensional deprivation reduction among the multidimensionally deprived*.

First, an *applicable achievement increment* occurs whenever the i individual that belongs to the s_j applicable population subgroup increases its a_{ij} achievement level by a constant $\gamma > 0$. This means that the a'_{ij} achievement for the i household-member and the j dimension is obtained by an increment of a constant $\gamma > 0$, such that $a'_{ij} = a_{ij} + \gamma$ for any person i satisfying $i \in s_j$.

Now, let assume that this i individual is deprived in the j dimension and belongs to a multidimensionally deprived household. Then, a *deprivation reduction among the multidimensionally deprived* occurs whenever this i individual increases his/her welfare in the j achievement, and this improvement changes his/her status from deprived to non-deprived.

Hence, in addition to be an applicable achievement increment, a deprivation reduction among the multidimensionally deprived makes this individual, no longer j-deprived due to this welfare improvement. This means that the a'_{ij} achievement for the i individual in the j dimension, obtained by an increment of a constant $\gamma > 0$ such that $a'_{ij} = a_{ij} + \gamma$, for any person i satisfying $i \in s_j$, $a_{ij} < z_j$, $i \in h$ $s.t.$ $p_h = 1$, is a deprivation reduction among the multidimensionally deprived whenever $a'_{ij} \geq z_j > a_{ij}$.

Nonetheless, the household to which this i individual belongs might be still having any other household member j-deprived, therefore continuing to be deprived in such dimension. Then, a *dimensional deprivation reduction among the multidimensionally deprived* is an improvement such that it involves an applicable achievement increment that produces a deprivation reduction among the multidimensionally deprived and also a change in the household status from deprived to non-deprived in such a dimension.

This means that the a'_{ij} achievement for the i individual and the j dimension, obtained by an increment of a constant $\gamma > 0$ such that $a'_{ij} = a_{ij} + \gamma$ for any person i satisfying $i \in s_j$, $a_{ij} < z_j$, $i \in h \ s.t. \ p_h = 1$, produces $a'_{ij} \geq z_j > a_{ij}$ and $d^{\beta'}_{hj} = 0$, where the d^{β}_{hj}-dimensional deprivation indicator for the h household and the j dimension before this achievement increment is $d^{\beta}_{hj} > 0$ and after the achievement increment corresponds to $d^{\beta'}_{hj} = 0$.

Having defined these three different types of increases in welfare as relevant, the following three properties to characterise the $MD^{\beta,\theta}$ family of measures arise:

- Weak Achievement Monotonicity (WAM). Multidimensional deprivation $MD^{\beta,\theta}$ satisfies weak achievement monotonicity if $MD^{\beta,\theta}$ does not increase due to an applicable achievement increment.

- Deprivation Monotonicity (DM). Multidimensional deprivation $MD^{\beta,\theta}$ satisfies deprivation monotonicity if $MD^{\beta,\theta}$ decreases due to a deprivation reduction among the multidimensionally deprived.

- Dimensional Deprivation Monotonicity (DDM). Multidimensional deprivation $MD^{\beta,\theta}$ satisfies dimensional deprivation monotonicity if $MD^{\beta,\theta}$ decreases due to a dimensional deprivation reduction among the multidimensionally deprived.

Although not every measure satisfies the three proposed dominance properties, each combination of β and θ parameters enforces different properties. In particular, any $MD^{\beta,\theta}$ is proposed to satisfy WAM and DDM, and $MD^{\beta,\theta}$, where $\beta > 0$ to satisfy DM.

The behaviour of these three properties in the proposed family of measures is analysed using microsimulation techniques. Specifically, each of the three different types of improvements in welfare in the 2013 Paraguayan example (an applicable achievement increment, a deprivation reduction among the multidimensionally deprived and a dimensional deprivation reduction among the multidimensionally deprived) are simulated, and societal measures before and after the simulations are evaluated. These simulations aim to illustrate the orientation of the proposed family of measures.

Each of the three different types of improvements in welfare are simulated 1,000 independent times. After each simulation, the resulting societal H and $MD^{\beta,\theta}$ metrics are evaluated and Table 5 presents the obtained average across 1,000 independent simulations performed. The next paragraphs describe these simulations and the obtained results.

According to the definition used in the 2013 PHS multidimensional index example, individuals 18 years of age or older that have less than 9 years of completed education are considered as deprived in educational achievement i.e., having low educational achievement. The 2013 PHS has a sample of 13,389 interviewees that are 18 years of age or older. Before any achievement increment was simulated, this population exhibited an average of 8.9 years of education.

Then, an achievement increment in the educational achievement indicator is simulated by sampling without replacement, out of the total 13,389 observed individuals, 50% of those having less than 8 completed years of education and belonging to a multidimensionally deprived household. This sample corresponds to 1,972 individuals. The number of years of education for each of the sampled individuals is incremented by one. Although this sample experienced this one-year increment, the additional year does not change their deprivation status. After the achievement increment, the 18 years of age and older population had an average of 9.1 years of completed education.

The mean results of societal measures obtained after these 1,000 independent simulations are presented in Row (14) from Table 5. Any measure satisfying the WAM property would show a zero or negative difference between the simulated scenario and the observed case. Row (15) from the table, shows this obtained difference. It is observed that any of the five different societal analysed measures increased as a result of an achievement increment. These results indicate that these measures satisfy the proposed WAM property.

After simulating an achievement increment that, although constitutes an improvement in welfare for some individuals but does not alter their deprivation status because is not large enough to remove individual deprivation, I simulate an achievement increment such that deprivation no longer is observed, namely a deprivation reduction among the multidimensionally deprived.

In particular, out of the 13,389 PHS 2013 individuals 18 years of age or older, 4,150 are deprived in educational achievement and belong to a multidimensionally deprived household. Out of those 4,150 individuals, 3,533 belong to households that along to be multidimensionally deprived also exhibit more than one deprived person in educational achievement.

To simulate a deprivation reduction among the multidimensionally deprived, a random sample without replacement of 50% of these 3,533 individuals is drawn. Only one person per household belongs to this randomly selected sample. In total, the sample is made up of 719 individuals. Each of the sampled individual change his/her deprivation status from deprived to non-deprived. It is worth noting that although these sampled individuals experience an improvement in welfare that removes their deprivation status in educational achievement, this improvement does not change the household deprivation status because they were not the only household members facing low educational achievement.

Before the simulated deprivation reduction, 47.9% of the 18 years old and older population had low educational achievement; after the simulated deprivation reduction, this rate became 42.5%. The mean result of the 1,000 simulations is displayed in Row (16) of Table 5. The results suggest that, keeping constant the households identified as multidimensionally deprived, societal values of $MD^{\beta,\theta}$ with $\beta > 0$ decrease after a deprivation reduction among the multidimensionally deprived. This result illustrates the DM behaviour in my proposed family of indices.

The third type of simulated welfare increment is a dimensional deprivation reduction among the multidimensionally deprived. In the first two types of simulated welfare increments, although the i individual increases her/his welfare due to no longer being deprived in educational achievement, other household members might be still deprived in the same dimension. Therefore, an achievement increment or a deprivation reduction among the multidimensionally deprived does not necessarily change the household status from deprived to non-deprived.

In fact, when analysing the PHS 2013, from the 40.2% of households identified as multidimensionally deprived, 4,150 adults are deprived in educational achievement but only 617 of them belong to a household where they are the only person having low educational achievement. Accordingly, household dimensional deprivation is only removed in virtue of an improvement in welfare when any of those 617 individuals suffer a deprivation reduction among the multidimensionally deprived. To simulate the welfare increment being a dimensional deprivation reduction among the multidimensionally deprived, a random sample of 50% of those 617 individuals is drawn. The sample is made up of 380 individuals.

Before the simulation of dimensional deprivation reduction among the multidimensionally deprived, 67.0% of interviewed households were deprived in educational achievement; after the simulated deprivation reduction, the rate of low educational achievement among household decreased to 60.0%. Row (18) from Table 5 shows the mean result of the 1,000 simulations. The results suggest that societal $MD^{\beta,\theta}$ falls due to a dimensional deprivation reduction among the multidimensionally deprived, a result that is consistent with the proposed DDM property.

The results of these simulations, in summary, confirm the proper orientation of the proposed $MD^{\beta,\theta}$ family of indices. While WAM enforces the $MD^{\beta,\theta}$ multidimensional deprivation measures to not increase as a result of any increment in welfare, DDM makes $MD^{\beta,\theta}$ decrease if any multidimensionally deprived household reduces its number of deprived dimensions. DM ensures that $MD^{\beta,\theta}$ decreases if any j-deprived i individual belonging to a multidimensionally deprived household reduces his/her number of suffered deprivations.

One the one hand, in terms of an individual-based scenario and in comparison to the AF set of properties, the proposed WAM and DDM properties of this article result equivalently to the AF's proposed *weak monotonicity* and *dimensional monotonicity* properties, respectively.

In terms of a household-based scenario and given that the $MD^{0,0}$ metric of the proposed family of measures represents the implemented M_0 measure that applications of the AF method use[10], it is observed from Table 5 that $MD^{0,0}$ is non-sensitive to changes in the number of household deprivations, unless these changes imply a

[10]Examples of these applications are Alkire et al. (2013), Alkire et al. (2015b), Angulo et al. (2015), Alkire and Seth (2015), Alkire and Santos (2014), Ayuya et al. (2015), Bader et al. (2015), Cavapozzi et al. (2013), Mitra (2014), Alkire et al. (2015b), and Yu (2013)

change of the household dimensional status from deprived to non-deprived. Then, this particular metric does not unambiguously fall due to reductions in the number of household deprivations. This, in contrast to the $MD^{\beta,\theta}$ measures, with $\beta > 0$, which satisfy DM.

Finally, a desirable dominance characteristic of a multidimensional deprivation measure is the ability of the measure to fall unambiguously under any applicable achievement increment, even if such achievement increment does not remove deprivation. The AF method termed this property as *monotonicity.* Any measure satisfying monotonicity would produce a decrease in the societal value of $MD^{\beta,\theta}$ because an applicable achievement increment. In practical terms, if for instance $MD^{\beta,\theta}$ would satisfy monotonicity, we would observed in the simulation results presented in Row (14) from Table 5 a non-zero difference with regards to the observed scenario (Row (1)). However, since my measures are built on the basis of counting deprivations, they are not able to document this type of welfare improvement. In the case of the AF method, any M_α with $\alpha > 0$ is meant to satisfy monotonicity. However, M_α with $\alpha > 0$ are metrics not commonly used in the applied literature because they require all considered achievement indicators to be cardinal. Given the ordinal nature of the majority of policy indicators, the AF's monotonicity property is therefore hardly exhibited.

Transfers

Another dominance property widely analysed by the income-based poverty measurement literature is the sensitivity of the measures to progressive transfers, which are transfers of income from a poor person to any other person that is poorer. In such a case, poverty measures are desired to decrease as a result of this type of change in the income distribution. This measurement sensitivity has been analysed by Sen (1976) and Kakwani (1980), among others.

For multidimensional measures, on the other hand, Bourguignon and Chakravarty (2002), Foster et al. (2005), Alkire and Foster (2011), and Chakravarty and Silber (2008) have proposed their societal measures to be sensitive to progressive transfers. Nonetheless, in the case of multidimensional deprivation indices, this type of transfer principle is not necessarily compelling for all the dimensions included within a particular index. For instance, it is not a compelling argument to desire sensitivity of the measures to transfers of good health from one individual to another or to desire

Table 5: Simulation results: Mean H incidence of multidimensional deprivation and mean $MD^{\beta,\theta}$ burden of multidimensional deprivation

		$MD^{\beta,\theta}$			
	H	$\beta = 0$		$\beta = 1$	
		$\theta = 0$	$\theta = 1$	$\theta = 0$	$\theta = 1$
(1) Observed	0.40	1.18	0.28	3.53	0.28
Scale invariance					
(2) Fully non-deprived scenario	0.0	0.0	0.0	0.0	0.0
(3) Fully deprived scenario	1.0	4.2	1.0	12.6	1.0
Household anonymity					
(4) Simulated	0.40	1.18	0.28	3.53	0.28
(5) Difference (4)-(1)	0.00	0.00	0.00	0.00	0.00
Within household anonymity					
(6) Simulated	0.40	1.18	0.28	3.53	0.28
(7) Difference (6)-(1)	0.00	0.00	0.00	0.00	0.00
Population replication invariance					
(8) Simulated	0.40	1.18	0.28	3.53	0.28
(9) Difference (8)-(1)	0.00	0.00	0.00	0.00	0.00
Multidimensional deprivation focus					
(10) Simulated	0.40	1.18	0.28	3.53	0.28
(11) Difference (10)-(1)	0.00	0.00	0.00	0.00	0.00
Applicable deprivation focus					
(12) Simulated	0.40	1.18	0.28	3.53	0.28
(13) Difference (12)-(1)	0.00	0.00	0.00	0.00	0.00
Weak achievement monotonicity					
(14) Simulated	0.40	1.18	0.28	3.53	0.28
(15) Difference (14)-(1)	0.00	0.00	0.00	0.00	0.00
Deprivation monotonicity					
(16) Simulated	0.40	1.18	0.28	3.39	0.27
(17) Difference (16)-(1)	0.00	0.00	0.00	-0.13	-0.01
Dimensional deprivation monotonicity					
(18) Simulated	0.40	1.11	0.27	3.46	0.26
(19) Difference (18)-(1)	0.00	-0.07	-0.01	-0.07	-0.02

Source: PHS 2013. Note: population means developed under the basis of a sample of 5,423 Households. Households satisfying $m^{1,0.87} > 0.65$ are identified as the multidimensionally deprived.

sensitivity of the measures to transfers of educational achievement from one person to another.

Sensitivity to transfers is relevant when describing multidimensional deprivation through indicators circumscribed to resources, such as monetary, health, or educational resources. In these cases, sensitivity of the societal measures to transfers from one individual with a larger amount of those resources to an individual with smaller amount of them are seen to be desirable for the purposes of distributive analysis, as pointed out by the poverty measurement literature. However, when the deprivation indicators describe a lack of outcomes, such as the absence of good health or nutritional status for example, this desired sensitivity might be losing its purpose.

As Aaberge and Brandolini (2014a) pointed out, the analysis of the sensitivity of multidimensional measures to changes in the distribution of deprivations is an area that requires further research. In particular, for the proposed $MD^{\beta,\theta}$ family of indices of this article, two types of sensitivities arise as being relevant to analyse with regards to transfers: the sensitivity of societal measures to demographic re-arrangements resembled by permutations of individuals across households and transfers of resources from better off to worse off individuals (within the multidimensionally deprived population). I would like to ensure that my measures are based on a progressive transfer of resources or individuals across households. However, the complexity involved in the possible compensation dynamics between attributes and the analysis of the mechanisms throughout demographic reconfigurations require more detailed research that is out of the scope of this article. For the time being, the dominance properties outlined in the previous section (WAM, DM and DDM) assure that the proposed $MD^{\beta,\theta}$ measures have the proper orientation if any of these transfers result in either an achievement increment, a deprivation reduction, or a dimensional deprivation reduction.

Decomposability

The poverty measurement literature defines as decomposable any metric that can be expressed as a weighted average of subgroup estimates, where weights are population subgroup shares. The references Foster et al. (1984), Tsui (1999), and Alkire and Foster (2011) refer to this property as *decomposability*, and Bourguignon and Chakravarty (2003) refers to it as *subgroup decomposability*.

Subgroup decomposability allows consistent decompositions of the societal measure into population subgroups. In particular, my H and $MD^{\beta,\theta}$ societal measures are able to be expressed as a weighted average of the multidimensional deprivation level observed across subgroups of households, where the weight of each is the share of households that each subgroup represents.

As an example, if I sort Paraguayan households under the basis of $m^{1,0.87}$ and identify as multidimensionally deprived those satisfying $m^{1,0.87} > 0.65$, this leads to 40.3% of the total 5,423 households being identified as multidimensionally deprived. This result can be decomposed further by sub-population groups such as household sizes or counties. For illustrative purposes, we show the decompositions by county in Table 6 below. The last row in the table corresponds to the overall societal estimate, and all seven previous rows correspond to each subgroup of households by county. If I obtain the share of households by county based on the figures included in Column 1 and use those shares as weights to calculate the weighted sum of each of the measures across household sizes, the results correspond to the overall societal figures. As Alkire and Foster (2011) pointed out for their family of measures, this measurement feature becomes an important technology for policy purposes. It allows the design and evaluation policy interventions for specific population subgroups.

However, two caveats are worth noting. First, an identification of the multidimensionally deprived at the household level produces societal measures to not be decomposable by individual population subgroups (ranges of age, gender, or ethnicity) disregarding the household where they belong. Including current household-based applications of multidimensional deprivation measurement, H and $MD^{\beta,\theta}$ are not the exception in this case.

Second, following the Alkire and Foster (2011) proposed notion of dimensional decomposability for any of their M_α metrics, the $MD^{\beta,\theta}$ measures with $\theta = 0$ can be decomposed to estimate the contribution of each j dimension in the overall societal measure. Then, $MD^{\beta,0}$ can be, alternatively to Eq. (11), expressed as the following:

$$MD^{\beta,0} = \sum_{j \in J} \mu(d_{hj}^{\beta}(k)) \; / \; J, \tag{13}$$

where $d_{hj}^{\beta}(k)$ is the household dimensional deprivation indicator censored to zero for any non-multidimensionally deprived household, and $\mu(d_{hj}^{\beta}(k))$ corresponds to

Table 6: County specific Paraguayan results

	Number of households	H	$MD^{\beta,\theta}$			
			$\beta=0$		$\beta=1$	
			$\theta=0$	$\theta=1$	$\theta=0$	$\theta=1$
	(1)	(2)	(3)	(4)	(5)	(6)
Asunción	691	0.1	0.3	0.1	0.8	0.1
San Pedro	469	0.6	1.8	0.4	5.8	0.4
Caaguazú	758	0.6	1.8	0.4	5.5	0.4
Itapúa	456	0.5	1.5	0.3	4.3	0.3
Alto de Paraná	826	0.5	1.5	0.4	4.7	0.4
Central	1,141	0.2	0.5	0.1	1.4	0.1
12 remaining counties	1,082	0.5	1.4	0.3	3.9	0.3
Total Paraguay	5,423	0.4	1.2	0.3	3.5	0.3

Source: PHS 2013. Note: Multidimensionally deprived households are identified as those satisfying $m^{1,0.87} > 0.65$.

the average value of $d^{\beta}_{hj}(k)$ for $h = 1, 2, \ldots, R$. Hence, the contribution of the j dimension in the $MD^{\beta,0}$ societal measure corresponds to $\big(\mu(d^{\beta}_{hj}(k)) \;/\; J\ \big)/MD^{\beta,0}$.

Continuity

Continuity in the multidimensional measurement literature has been used, for instance, by Bourguignon and Chakravarty (2003) to characterise welfare multidimensional measures. According to the scholars, it ensures a well-behaved functional form that would produce no abrupt jumps given changes in achievements. My proposed approach may have more than one possible source of discontinuity. First, I propose counting dimensions on deprivation and household deprivations, thus, household metrics are counting indicators that belong to the set of natural numbers. Second, given that two identification procedures are used (the first one identifies individuals in deprivation and the second identifies multidimensionally deprived households). Therefore, continuity is not expected to be achieved.

Nonetheless, despite the lack of continuity of the metrics, they are still cardinal indicators. The proposed methodology of this article exploits the ordinal nature of most of the policy indicators currently in use. Using either count-based, share-based, or a mixture of both approaches, leads to cardinal metrics of the household burden of multidimensional deprivation being developed. The cardinal nature of my proposed $m^{\beta,\theta}$ metrics allow the comparison of the size of this observed burden of multidimensional deprivation across any two given households, which provides policy makers a technology that allows ranking households from most deprived to least deprived; this reveals an important technique for targeting the most deprived households in developing countries.

7 Discussion and concluding remarks

This paper proposes a family of multidimensional deprivation indices that takes into account differences in need that demographically heterogeneous units (i.e., either households of different size and composition or individuals of different population subgroups) exhibit.

In particular, when measuring deprivation, demographic heterogeneity plays a central role in the definition of who can be considered as lacking a minimum achievement. Children, for instance, can be considered deprived when they are not accessing basic education services, unlike adults, who can be considered deprived in the same education dimension when they do not know how to read and write. As another example, while adult populations that do not have access to job opportunities despite seeking them can be defined as deprived in employment, children cannot be defined as deprived in the absence of employment. Different sub-population groups exhibit different sets of needs.

These differences in needs are captured in this paper by defining who is accountable to be defined as deprived in each indicator. As such, not every individual is understood as legitimate needing each achievement. Whereas deprivation within the applicable population subgroup of each achievement is set as unfair and avoidable, differences in achievement levels within the non-applicable population subgroup are catalogued as fair.

The definition of these applicable populations results in being key normative definition in the proposed methodology of this article. They are advised to be made using the available international indicator definitions. For instance, the International Labour Organization describes in its regulations the age ranges defined as suitable to measure labour market indicators such as employment, unemployment, child labour and so forth. Another example is regarding education indicators, in which the ages more suitable to measure enrolment and school lag vary according to each context. As such, the definition of these age ranges should generally follow the context-specific norms.

When it comes to measuring multidimensional deprivation, these differences in needs bring comparability challenges to measuring how many dimensions in deprivation a particular individual or household might exhibit to be catalogued as multidimensionally deprived. This paper addresses these comparability challenges by expressly drawing from the parametric one-dimensional equivalence scale literature and presenting a methodology of multidimensional deprivation measurement that describes how much deprivation demographically heterogeneous units with different needs must exhibit to be catalogued as equivalently deprived. The proposed methodology allows societal multidimensional indices based on comparable individual / household estimates.

Selecting either of these two units of analysis involves normative criteria to be consider by the analyst, according to the purposes of each application. While individual-based measures allow the unmasking of differences in multidimensional deprivation across demographic sub-population groups, household-based measures conceive households as cooperative units that jointly face the deprivation suffered by the household members.

The proposed technology of this paper is meant to be 1) applicable for the purposes of policy and 2) suitable for contexts either where multidimensional deprivation is aimed to be measured at the individual level across a wide range of indicators with different applicable population groups, or where public policies are designed to be targeted at the household level, or where risk or resources are arguably pooled across household members.[11]

[11]In particular, household-based measures proposed in this article are developed under the assumption that household members jointly face deprivation, whenever it occurs to a particular member. As a matter of fact, the household burden of multidimensional deprivation is expressed

To measure the burden of multidimensional deprivation, different approaches that range from count-based (absolute) to share-based (relative) and intermediate approaches to measurement were proposed. The empirical obtained results indicate that identifying the multidimensional deprived population on the basis of these different approaches to measurement produce significantly different multidimensional deprivation profiles. Then, the selection of the most appropriate approach to measure the burden of multidimensional deprivation and then sorting and identifying the multidimensionally deprived population under these bases constitutes an important normative definition.

One the one hand, this most appropriate approach to measurement can be selected under normative considerations. As such, one could argue that there is no correct or incorrect approach to measurement and the most appropriate measure should be justifiable according to each context. While count-based approaches either give to each dimension or to each deprivation (depending on the selected combination of parameters) an equal absolute value in the measurement of the burden of multidimensional deprivation, share-based approaches give an equal absolute value to each unit of analysis, disregarding the size of its needs. An intermediate normative perspective approach corresponds to a combination of parameters in between these two solutions.

On the other hand, the second possible course of action corresponds to determine the combination of parameters that enables unbiased societal multidimensional deprivation incidence profiles. This combination of parameters can be obtained, as analysed in Section 5, by defining as unbiased any multidimensional deprivation

as an additive function of the individual members deprivation status and seeks to take into account the different set of needs that demographically dissimilar households experience. However, despite collective-based decision making being studied by the economic literature, such as by Chiappori (1992) or Corfman and Lehmann (1987), who model and analyse this type of decision making process, there is no consensual evidence of whether or not households behave as a collective unit. A classic example in the literature of risk pooling evidence among household members to protect the collective unit from adverse shocks is the Townsend (1994) study of three poor high-risk Indian villages. In that particular setting, Townsend (1994) found that contemporaneous household consumption is not dramatically influenced by transitory shocks, such as unemployment or sickness. Nonetheless, there is also evidence that individual risk is only partially pooled among household members because competitive objectives among them might arise. Examples of this evidence are studies such as Hayashi et al. (1996), Doss (2001), and Dercon and Krishnan (2000). All of them suggest the absence of full risk pooling among members but partial and heterogeneous risk pooling depending on characteristics such as age, gender, and cultural traditions, in each of those analysed contexts.

incidence profile able to classify as equivalently deprived any two households / individuals with no unfair difference in deprivation but only differences in needs, which are considered for the purposed of this paper as fair and legitimate. As such, a counterfactual scenario of no illegitimate difference in deprivation is advised to be built, measures in such a scenario should be evaluated.

In particular, multidimensional deprivation incidence profiles based on my proposed measures were evaluated under these circumstances and in use of an 2013 Paraguayan index example. The obtained simulation results indicate that neglecting differences in needs yields a biased picture of household-based multidimensional deprivation incidence profiles. Count-based approaches produced larger multidimensional deprivation incidence among households with larger sizes of needs, despite having no illegitimate difference in deprivation. Share-based approaches, on the contrary, produced larger multidimensional deprivation incidence among households with smaller sizes of needs. The degree to which we must account for these differences in need, therefore, stands out as relevant.

These obtained results proved to be robust under alternative considerations. In particular, the behaviour of the measures was analysed using different specifications of the size of household needs, alternative shares of the population to be identified as multidimensionally deprived and alternative weighting systems to address differences in needs. Also, balancing procedures as the proposed by Alkire (2015), which imply in each well-being dimension to account all population subgroups with one applicable deprivation indicator, were as well discussed and evaluated as alternative methodological approaches. Across all these robustness checks, measuring the burden of deprivation by the widely used household count of dimensions in deprivation and identify the most deprived on these basis confirms to yield biased multidimensional deprivation incidence profiles.

Nonetheless, particular members of my proposed family of measures of the burden of multidimensional deprivation proved able to depict as equivalently deprived households with no illegitimate difference in deprivation but only different sizes of needs. They, therefore, confirm permitting unbiased multidimensional deprivation incidence profiles in the context of the Paraguayan index example. The selection of the context-specific measure to describe the burden of multidimensional deprivation is advised in light of simulating counterfactual scenarios of no illegitimate difference in deprivation and robustness checks that use different multidimensional deprivation thresholds and specifications.

Yet, within this framework the limitations that a parametric equivalence scale of the type that is proposed in this article can be recognized. First, it does not differentiate between needs and preferences. Being that the ultimate purpose of a multidimensional measure of deprivation is to capture unfair disadvantage, differences in deprivation due to other fair sources should be also taken into account, an example being differences in preferences. For the sake of simplicity and as a first effort in literature to provide an equivalence scale tool to enhance household or individual comparability for multidimensional deprivation measurement in the presence of differences in need, the analysis of this paper so far focuses on accounting for these differences in need and leaves for further research the effect that other sources of fair differences, such as preferences, might have over multidimensional deprivation incidence profiles. Analysis of the relation between multidimensional poverty and preferences can be found in Decancq et al. (2014).

Second, differences in needs are accounted for based on a still limited number of observable attributes (i.e., household size, composition, or age and gender). Third, the proposed methodology does not address the complexities when complementarity and substitutability among dimensions can be observed.

Moreover, as pointed out by Pollak and Wales (1979) , Fisher (1987), and Blundell and Lewbel (1991) and discussed in Section 2.1, household current demographic composition that leads to differences in need might be driven by previous deprivation status as well. For instance, a particular household consisting of two adults and five children might be this size not only because both adults have a preference for many children, but also because they did not have access to pregnancy prevention education or could not afford using some form of birth control. Then, household composition not only reflects needs or preferences, catalogued in this paper as producing fair differences in deprivation among households, but also current household compositions might be a reflection of avoidable and unfair previous states of deprivation. For the sake of simplicity and as a first effort in literature to provide an equivalence scale tool to enhance household or individual comparability for multidimensional deprivation measurement, the analysis of this paper so far has taken household demographic composition to be completely within the space of individuals' responsibility. This is a complex issue that is left for further research.

Nonetheless, my proposed approach allows multidimensional measures to be constructed upon unit comparisons (either households or individuals) that account for differences in need that previous measures have failed to take into account. According to Elster and Roemer (1991, pp.1), any notion of well-being not only should be based on appropriately operationalised interpersonal comparisons, but also should be adequate for the purposes of distributive justice. Then, the family of measures presented in this article is proposed to be applicable to enhance comparability across demographically heterogeneous units that exhibit different needs. It is also proposed to be adequate for the purposes of multidimensional deprivation measurement, as analysed in Section 6 when discussing the set of properties that make the proposed family of measures satisfactory for the purposes of multidimensional deprivation measurement.

Appendix

Table 7: Proportion of households with at least one deprived person from the applicable population (%)

	Persons per household							Total
	1	2	3	4	5	6	7 or more	
(1) Health insurance non-coverage	70.3	75.2	79.9	81.0	84.3	91.4	93.8	81.3
(2) No access to health services	12.5	17.7	15.1	19.7	20.1	23.6	29.6	19.0
(3) Non-school attendance	0.0	1.9	2.9	4.4	5.8	9.7	21.4	5.5
(4) Low educational achievement	61.4	64.8	57.8	65.4	68.6	78.3	88.3	67.0
(5) Sub-standard housing	25.5	25.0	18.5	19.9	23.5	24.9	34.2	23.3
	Sample number							
Number of households	593	836	1,135	1,108	771	466	514	5,423
% of individuals	2.8	8.0	16.3	21.2	18.4	13.4	19.9	100

Source: PHS 2013.

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