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Equality of Opportunity: How to encompass Fifty Shades of Luck

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Abstract

Equality of opportunity is usually defined as a situation where the effect of circumstances on outcome is nullified (compensation principle) and the effort is acknowledged (reward principle). We propose a new version of the reward principle and we show that luck can be introduced in two ways in the definition of these principles, depending on whether the correlation between luck and circumstances should be nullified and whether the correlation between luck and effort should be rewarded. This leads to two distinct formulations (before-luck and after-luck) of the compensation and reward principles. Each combination of principles correspond to a particular view about how luck affects the opportunities of success. We also pay attention to the correlation between effort and circumstances which is dear to Roemer in a context of uncertainty.

Keywords: D63, J62, C14.

JEL Classification: Equality of Opportunity, luck, compensation principle, reward principle, moral hazard, first-order stochastic dominance, correlation, effort.

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1 Introduction

Perhaps there is no other philosophical debate more contentious than that over inequality and social justice. The seminal intuition of the theory of justice since John Rawls (1971) is that equality of outcomes is not necessarily fair. One needs to take into account the factors that determine observed outcomes to determine whether inequality of individual outcome is legitimate or not. Responsibility-sensitive egalitarianism, whose prominent advocates in philosophy are Dworkin (1981), Arneson (1989) and Cohen (1989)) and Roemer (1993,1998) and Fleurbaey (2008) in the economic literature, argues that the key criterion for assessing whether inequality should be seen as fair or unfair is individual responsibility. This amounts to distinguish between two sets of determinants of individual: circumstances include the determinants of outcome that lie beyond the realm of individual responsibility; effort gathers the determinants of which individuals are held responsible. Deciding what individuals should be held responsible for is of course a difficult issue that can be decided by each society on political grounds according to Roemer (1993). Once effort has been defined and observed, Equality of Opportunity (EOp) is usually considered to prevail if two conditions are met. First, individuals who exerted similar effort should face equal outcomes (compensation principle). Second, the impact of effort on the outcome should be respected somehow (reward principle).

Although equality of opportunity has been depicted as 'luck egalitarianism' by vocal critics (Anderson (1999), Hurley (2003)) with the idea that the "fundamental motivating aim of EOp is only to neutralize luck", the role of luck in the theory of equality of opportunity has largely been skimmed over in the literature while luck is pervasive in everyday life. In the commonsense view, luck is closely linked to randomness. In general, luck is understood as the chance happening of fortunate or adverse events. It would thus refer in this paper to the determinants of outcome that result from random processes. For instance Garcia-Gomez et al (2015) define luck as "everything which could not be predicted (ex-ante)". Randomness plays a role in individual outcomes: who you met in life, whether you were endowed with a particular gene,... are important determinants of individual success that at some point or another should be seen as random draws. The goal of the paper is to offer a framework that allows to encompass luck in formulating the basic requirements of equality of opportunity, the compensation and reward principles.

Of course, luck is already incorporated, as circumstances can be viewed as the luck of a birth lottery which allocates genes, families, and social environments to individuals. Birth lottery can be seen as an example of 'brute luck' which is randomly distributed across individuals all over the life, and is often unobservable to third parties (being in the right place at the right time). Dworkin (1981) is famous for having introduced the distinction between 'brute luck' and 'option luck' due to the outcome of chosen gambles. Dworkin's view was that no compensation is due to anyone who suffers a bad outcome due to option luck because person should be held responsible for her risk-taking. Fleurbaey (2008), however, contests this view. He splits gambles into two parts: the decision to

take the gamble, and the random draw. Let us view the risk-taking preference of the individual as a responsibility characteristic, and the outcome of the gamble as a circumstance – something over which the individual has no control. Fleurbaey then proposes to apply his two main equity solutions, Conditional-Equality and Egalitarian-Equivalence, to this particular context. Even if they disagree on the treatment of option luck, both Dworkin and Fleurbaey admit that luck can be absorbed in the dual world of effort and circumstances.

On the other hand, Lefranc, Pistolesi and Trannoy (2009b, LPT henceforth) believe that the project of separating influences into circumstances and effort is too binary. They call 'residual luck' a third determinant, and recommend something weaker than neutralizing residual luck. They only require that the correlation between such luck and circumstances be nullified. These authors are agnostic about what should be residual luck, although they point to a consensus that social background should be counted as a circumstance. These authors propose that a minimal requisite of EOp is that the residual luck should be equally distributed across types, at any given level of effort. This recommendation echoes Rawls's stance (1971, p.63) according to which "those who are at the same level of talent and ability, and have the same willingness to use them, should have the same prospects of success regardless of their initial place in the social system".

The present essay submits a more complete and deep discussion about how to incorporate luck into the definition of equality of opportunity. Our view is that the dual world may be a too straight jacket for encompassing many kinds of luck or many views about luck. We maintain that a more flexible approach with respect to luck may have sense and that in addition to the two boxes of effort and circumstances, it could be good to have a degree of freedom in defining a third box containing some residual luck factors according to the ethical preference of the social decision maker. Both principles remain in substance. They are just redefined with residual luck having a neutral status in the reward and compensation principles: residual luck appear in the clause if and has a "neutral status". The central message of the paper is that residual luck stands out as a intermediate factor between circumstances and effort. One important reason is that effort choice usually affects outcome in a non-deterministic way. In order to make this transparent, we borrow Savage's (1954) definition of an act to define effort in an incertain world.

This paper extends the analysis of luck of LPT in four important directions. First, while LPT only focused on the compensation principle, this paper considers both the principles of compensation and the principle of reward. As a matter of fact, we propose a new reward principle, which can be viewed as minimal and is compatible with the compensation principle. This minimal principle is then different from both the principle of natural and utilitarian reward. Second, in LPT we only consider luck before effort and here we also consider that good luck can go along with effort. In that case, luck is revealed to the agent after effort has been exerted, without being revealed to third parties and it leads to the so-called moral hazard problem in contract theory. We argue that this intricacy requires a more subtle stance for this kind of luck, since more effort can "cause"

a better luck. The third contribution is to discuss how the correlation of luck with effort and circumstances should be accounted for in the formulation of the compensation and reward principle. Should the correlation between luck and circumstances be viewed as a circumstance? Should the correlation between luck and effort be viewed as an effort? We show that various views can be taken leading to different formulations of the equality of opportunity requisite. Basically, two distinct formulations (before-luck and after-luck) of the compensation and reward principles can be defined. Each combination of principles correspond to a particular answer to the above questions about the status of the correlation of luck with effort or circumstances. The fourth contribution is to pay attention in an uncertain world to the correlation between effort and circumstances which is at the corner stone of the Roemer's theory of equality of opportunity and which distinguishes it from Fleurbaey's one. We examine the plausibility of a monotonic relation between effort and circumstances when the results are risky and the individual is rational and we exhibit the relation between the different requisites of EOp depending on whether the correlation between effort and circumstances is taken into account or not.

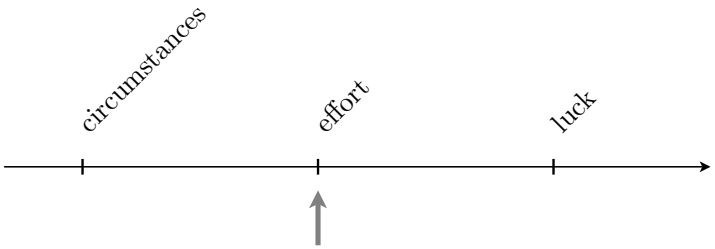
The rest of the paper is organized as follows. In section 2 we propose a framework to think about EOp in an uncertain world. Next, in section 3 we formulate a first version of the compensation and reward principle which are labeled the point-wise principles. They require that luck is revealed to the social observer. In the next section, we introduce a second version of the compensation and reward principle which are labeled the distribution-wise principles. They correspond to an ex-ante view before uncertainty is released. Section 4 illustrates how these principles may be useful in thinking about different kinds of luck in relation with how the correlation of luck with circumstances and effort should be taken into consideration. Section 5 concludes.

2 The Model

Before describing the model, a few general considerations are useful. First, responsibility-sensitive egalitarianism cannot be meaningful if individuals may lack the mental capacity to make their judgment and decision about their own life. They are mainly two categories that are concerned by this remark, teenagers before the age of consent and people with mental disorder. In particular all determinants of teenager outcomes are not fully under their control. We deduce that all factors that occur before the age of consent cannot be considered as responsibility characteristics. Consequently we focus on the situation of adults with full mental capacity and after the age of consent. Of course, the determinants which occur before the age of consent will continue to have some consequences on the process of individual outcome after the age of consent.

A second observation matters as well. When taking his decisions, the individual will likely know the circumstances, that is, all the events that happen before the age of consent are revealed to him. Think of family and social background and some biological traits, as beauty, height, some ability to learn and

Figure 1: Timeline



memory for instance.

The time line (see Figure 1) is basically as follows. On the one hand, circumstances occur first and are known when the individual is taking decisions. On the other hand, the impact of effort on outcome is mediated by luck in a way that the individual cannot fully predicted. Even if for the social observer innate talent and social background can be viewed as luck factors, the fact that they are past-luck for the individual when taking decisions vindicates that luck might be distinguished from circumstances. Indeed, the causal link between decisions and luck may be reversed. Past-luck can influence decisions whereas decisions can impact future luck.

Obviously, the individual is taking a lot of decisions and there will be an interplay between decision and luck in the long chain linking luck and decisions. Here, we will not attempt to describe this complex chain and the model is static.

Up to some point, the language of decision theory as proposed by Leonard Savage (1954) provides a useful starting point. Savage distinguished the set of states of nature from the set of consequences, and an act is simply an application from the former into the latter, that is, an act specifies a consequence for any state of the world. A decision is different from an act for Savage. A decision has been made when there was a choice between two or more acts. For instance it may happen that a worker has no choice about his working time in the actual firm he is employed. In that case, Savage will say that working time is not a decision. There are two differences with Savage. The first difference is we introduce another set, the set of circumstances, which are past-luck at the time the decision maker is observed making decisions. The second difference is that acts are costly which one authorizes them to label effort.

As was made obvious in the above development, there are four sets of variables in our model, named as outcomes, circumstances, states of nature and effort.

We assume that individual outcome can be measured by a positive scalar y. $\mathbb{R}_+$ is then the set of consequences in our model. We would like to clarify what we mean by individual outcome. First, it is a *gross* outcome in the sense that it does not include the cost of effort. We see the cost of effort as largely not

directly observable for public policy and we will come back to that point later on. Second, we focus on the *final* outcome enjoyed by individuals in the population, after any possible public policy. Our objective is limited in the sense that we are just willing to set up a model that characterizes a fair distribution of outcome y in the population, from the perspective of EOp. Contrary to Roemer (1998) and Fleurbaey (2008), we do not seek to define optimal policies from the point of view of EOp, given an unfair allocation. In fact, in several instances, the distinction between the distributions of outcome before and after public policy is difficult to draw and not particularly meaningful. For instance, it will be difficult to define the counterfactual corresponding to individual health without any public intervention. Even in the case of income, public education is likely the most powerful instrument to weaken the impact of circumstances, although it is hardly possible to estimate pre-education income. We should define a kind of production function the output of which being the outcome y, with inputs being the individual actions and public resources. Furthermore, defining and identifying an optimal EOp policy in fact call for a more complete framework, and would in particular require modeling how people react to circumstances and public policy in their choice of effort. The fact that we do not define explicitly the resources of the public sector as means for achieving EOp makes the usual definition of the reward principles (natural and utilitarian) meaningless in our framework. We have then to think of a principle of reward which makes sense in our model.

We assume, in this section, that *all* the determinants of individual outcomes are observable. We define four generic sets of determinants of individual outcomes. The first set of determinants comprises individuals circumstances, denoted by $c \in \mathcal{C}^n$, where $\mathcal{C}^n$ is a space of dimension n which is not necessarily a metric or an ordered space. Using Roemer's terminology, we refer to a *type* as the set of individuals with similar circumstances. As noted before, it may be useful to describe circumstances as past-luck, past with respect to the time where the agent is taking his first decision.

The second set of determinants is residual luck, which is identical to the state of nature in decision theory. It is denoted by $\omega \in \Omega^q$, the set of state of nature which is not necessarily a metric or an ordered space. Ex-post, only one state of nature will be revealed to the individual. However, ex-ante, he does not know which state of nature will occur.

As been clear in the discussion, the set of circumstances and residual luck $\mathcal{C}^n \times \Omega^q$ describe factors that are beyond the control of the individual. Even if we have submitted a distinction between these two categories in terms of past and future, a positive feature, it must be emphasized that the model can also describe a distinction driven by normative reasoning.

As regard effort, we want to capture its main features in economic life which appears for instance in education (tertiary or vocational training), on the job market and undertaking a business, saving decisions on the life cycle, or life styles in health care. Effort is generally costly, multidimensional, all dimensions are not publicly revealed and the reward of effort is contingent to the realization of the state of the world. As did Savage, we can now just define effort as an

application from the set of circumstances and residual luck $\mathcal{C}^n \times \Omega^q$ into the set of consequences $\mathbb{R}_+$. That is, an effort will specify an outcome for any circumstance and any state of the world. And indeed, effort is an act as defined by Savage. However, it is more than that. John Roemer coined the term effort but he did not interpret this word literally. We here submit a literal interpretation of effort as an act inducing some cost for the agent. And for the very fact it introduces some cost, effort should be rewarded somehow. This is a competing idea with two ideas that have been proposed to support that responsibility characteristics should be rewarded. A first reason put forward by Gerald Cohen is that they are under control of the agent. A second rationale put forward by Ronald Dworkin and Marc Fleurbaey is that people should be held responsible for their preference. If we come back to the number of hours of work for a wage-earner particularly in a big firm within a quite regulated labor market like France, it is not fully convincing in all cases that the work duration is under control of the agent or that it results from a free optimization at least in the short run. In Savage's words, work duration will be an act but not a decision. The two popular views then vanish in this case whereas the cost argument can still be used to vindicate than someone who works in a firm in which the duration of work is 40 hours per week should be rewarded more than someone who works in a firm when it is not possible to work more than 35 hours per week. This cost interpretation of reward will be more convincing if there is a general agreement among the individuals about the cost of effort. And this is the route that we follow here.

We introduce our last set, the set of individual efforts, which is possibly multidimensional and is denoted by $e \in \mathcal{E}^p$, where $\mathcal{E}^p$ is not necessarily a metric space. However, there is a quasi-ordering $\succeq$ not necessarily complete that is defined on $\mathcal{E}^p$. $\succeq$ represents the agreement among individuals about how costly the effort is, as least in ordinal terms. We interpret the fact that $e \succ e'$ as meaning that all individuals agree that doing the multidimensional effort e is more costly than performing the multidimensional effort e' for all individuals in a given society. This quasi-ordering respects the natural order of vector in $\mathbb{R}^p$ (at least one component strictly greater and no component strictly lower) but it may go beyond this and capture a constant opinion across the population. For instance if we compare the multidimensional effort done by a surgeon and a teacher when lecturing assuming that both interventions last the same time, it is quite plausible that doing a surgical operation is universally viewed more painful than teaching. The assumption of the existence of a common partial ranking of effort levels can be stated using a formal decision model where the utility function is specified in the simple quasi-linear form (implying risk neutrality) as

$$u_i(y, v) = y - v_i(e) \tag{1}$$

where v denotes the disutility of effort (> 0). If we assume that $v_i(.)$ is increasing and convex, then the quasi-ordering will be given by the intersection of all orderings represented by the $v_i(.)$. The fact that the individuals interpret the effort levels in an unanimous way makes the support of the reward principle

stronger.

Individual realization of outcomes, circumstances, effort and residual luck, are thus denoted respectively by y, c, e and ω. We let Y denote the outcome function relating individual outcome to its determinants c, ω and e:

$$\begin{array}{rcl} \mathsf{Y} : \mathcal{C}^n \times \mathcal{E}^p \times \Omega^q \rightarrow & & \mathbb{R}_+ \\ (c, e, \omega) & \rightarrow & y = \mathsf{Y}(c, e, \omega) \end{array}$$

The outcome or reward function registers all effects of all the dimensions of public policy which is implemented in the society. The outcome function is then specific to a particular public policy and its knowledge is not sufficient to choose a particular policy to optimize some social welfare function. We should specify a possibility set of outcome functions.

It becomes more transparent why we said that y is the gross outcome and not the outcome net of the cost of effort. In the simplest case, the net outcome would be given by the expression (1) for all individuals. However, unless we suppose interpersonal ordinal comparison of the cost of effort (the $v_i(.)$ are the same up to the same increasing transformation) we cannot compare the outcome net of the cost of effort across the population. The focus on the gross outcome is therefore motivated by the fact that it is precisely the only variable across the population that can be steered by the public policy. It is almost clear that in all issues except some very specific ones, the social decision maker does not have all the information about the true cost of effort. It is indeed the basis of all principal agent theory and a second best optimization policy should be incentive compatible.

More generally, it is important to realize that our view of effort and luck is compatible with a structural model of the choice of effort. Although we do not need it for the following of our analysis and stating the compensation and reward principles, our model can be plugged into a rational choice model

3 The ex-post view: point-wise principles

We begin by stating a first version of the compensation and reward principles in the framework of the above model. The first version correspond to an ex-post view. All the information about the determinants of outcome have been disclosed to the social observer. We then explain by means of examples why luck may be isolated from a pure normative point of view. Finally we pay attention to the correlation between circumstances and effort.

3.1 point-wise principles

The above model can be both used for a positive analysis of social determinism and for a normative analysis. In the second one on which we focus here, there is some degree of freedom left to the social scientist or the social decision maker to substantiate the different categories of circumstances, luck and effort with

respect to the positive model. Defining the boundaries of these different "boxes" obviously overlap between the positive and normative views but they may not fully correspond. Let us develop this crucial point for the luck box.

A crucial distinction in the EOP approach is the distinction between circumstances and effort. The dominant view ascribes all the factors that individuals can be deemed responsible for to the category of effort and ascribes to circumstances all the factors beyond the scope of individual responsibility. Alternatively, Roemer (1998) suggests that the definition of the boundaries of circumstances and effort is a matter of political choice and should be made by society. The luck box is defined in a negative manner from the point view of equality of opportunity: it gathers all states of nature that are not considered as either circumstances or effort. Namely, it is not compulsory to nullify the impact of the luck factor on the outcome and this feature distinguishes them from circumstances. Simultaneously, it is not required that there these luck factors go along with a positive reward which make them distinct from effort. The reward associated to these luck factors remain unspecified from the very point of view of EOp and they remain in a kind of grey zone.

Some examples may need to be developed to convince the reader that there are some types of luck that might be too difficult to classify either as circumstances or effort. We postpone the full development of the argument after presenting the formal status of residual luck in the compensation and reward principles. Still, for the sake of illustration, consider the case of innate talent. We think that innate talent can be classified either as a circumstance, an effort or as luck. Those who will support the first view will correctly argue than innate talent is past-luck, it is luck that occurs before effort. On the opposite side, libertarian authors or authors that are listening to sirens' song from the libertarians such as Peter Vallentyne (1997) might argue inequality arising from genetic endowments are legitimate, in which case, they would like to include them as such in the definition of effort. In the middle, one can argue that there are talents that are revealed to the individual only when one becomes mature, like for instance the talents to become a politician. And still these talents were embodied in some sense since the very beginning of life. One can claim that to some extent high talented people do not know before choosing their job effort how much talented they are. If one follows this last intermediate stance about innate talent, then it belongs to the grey zone of luck factors. To make transparent that it is a normative statement that ultimately should be put forward to make a state of nature a residual luck factor, we will introduce the notation $l \in \mathcal{L}^q$ when we are referring about luck factors belonging to the grey zone. It can be viewed as a subset of the big set of states of nature Ω^q. In other words, when the matter is positive, we use ω, whereas l is used when the matter is normative.

It remains to present how this negative stance for residual luck factors appears in the statement of the two principles that characterized the requisite of equality of opportunity: the compensation principle and the reward princi-

ple.[1] Here both principles are defined in terms of properties that the function Y should satisfy. Its full knowledge is then required which means that these principles correspond to an ex-post view. All the information about the determinants of outcome have been disclosed to the social observer.

The compensation principle: circumstances should be irrelevant One of the key insights of equality of opportunity theories is that inequality arising from differential circumstances is morally offensive: fairness demands that individuals face similar outcomes, regardless of their circumstances. However, one has to be more specific because differences in effort are viewed as a source of legitimate inequality. Hence, the requirement of equality of opportunity should only apply among individuals with similar e. To summarize, circumstances define what should be compensated and effort defines the scope for relevant inter-individual comparisons. If all the determinants of outcomes belong to either c or e, the compensation principle can be formulated as follows:

The compensation principle is satisfied iff for any $e \in \mathcal{E}^p$, for any $c, c' \in C^m$,

$$\mathsf{Y}(c, e) = \mathsf{Y}(c', e)$$

This is what Fleurbaey (2008) refers to as equal outcome for equal responsibility when luck is absent.

Without luck, the conditional equality principle states that individuals should receive similar outcomes regardless of their circumstances, conditional on their effort. To extend the compensation principle in the presence of residual luck, one has decide whether the effect of luck should be fully preserved or whether differences in residual luck that are correlated to individual circumstances should also be nullified. This results in two distinct conditional equality principles of which only the first one is presented in this section. The point-wise view requires that the effect of luck on outcome be fully preserved. If so, the principle of conditional equality implies that individuals with similar effort and similar luck should enjoy similar outcomes. This leads to the point-wise compensation principle, which we state as follows:

Definition 1 *The point-wise compensation principle (PC) is satisfied iff : for any* $(c, c') \in \mathcal{C}^n \times \mathcal{C}^n$*, for any* $l \in \mathcal{L}^q$*, for any* $e \in \mathcal{E}^p$*, ,*

$$\mathsf{Y}(c, e, l) = \mathsf{Y}(c', e, l)$$

The issue of the correlation between luck and circumstances is escaped in the point-wise formulation of the principe of compensation. The positive or negative correlation between the random variable l and circumstances will remain unaffected. Note that luck goes along with effort as a conditioning variable. It seems as it is considered as an effort variable but the confusion will be cleared up with the statement of the point-wise reward principle.

[1] See for instance Fleurbaey (2008), or Ramos and Van de Gaer (2012) for a recent survey.

The minimal reward principle: effort should pay Since effort is the only source of legitimate inequality in this model, exerting a greater effort may go along with a higher outcome. The EOp ethics goes beyond this mere chance of a better life. It transforms an option into an ethical requirement. Individuals exerting a greater effort must enjoy a higher level of outcome. The reward principle should be defined in an independent way of the compensation principle. We cannot then preclude that the compensation principle is not implemented in the definition of the reward principle. The inter-individual comparison should only apply to individuals within type, i.e. given the same level of circumstances c.

The reward principle is satisfied iff for any $c \in C^n$, for any $e, e' \in \mathcal{E}^p$, such that $e > e'$ (at least one component strictly larger),

$$\mathsf{Y}(c, e) > \mathsf{Y}(c, e')$$

This formulation of this principle appears to be minimal in only requiring that the outcome function be strictly increasing in effort. This formulation of the reward principle can be compared to other principles found in the literature.

Roemer does not include as an explicit requisite of EOp that effort should pay. He almost exclusively emphasizes the principle of equality of outcomes for a given effort, or if not possible, the second best solution that maximizes the outcome of the worst-off. Fleurbaey has however showed that Roemer implicitly appeals to a principle of reward. Fleurbaey coined the term of *utilitarian* reward principle to say that the redistribution of resources among individuals within a type arises from the maximization of a Benthamite social welfare function, something that Roemer recommends. Since utilitarianism corresponds to a zero-inequality aversion from the social planner, the pre-tax inequality among a type is not reduced except in specific cases, and if the effort pays before tax, it would go on after tax. And indeed, Roemer assumes that the outcome function before state intervention is monotone increasing in individual effort, although this is mostly seen as an identification condition, i.e. a way to infer individual effort, conditional on outcomes and circumstances. All in all, one can consider that Roemer solutions satisfy our minimal reward principle, even if it is not explicitly as such.

On the contrary, Fleurbaey (2008) discusses extensively the reward principle as being part of the equality of opportunity requisite. The framework of analysis is a discussion of public policy. In this context, the reward principle requires that compensation does not alter the *natural* reward of effort observed before policy intervention. In a nutshell it amounts to require that the increase in outcome associated with a rise in effort be unchanged after policy intervention. While we require that effort pays, we remain silent on the issue of how much effort should pay. In particular, we do not require that it pay more or less than in the "free" market situation. In particular, our principle is compatible with effort paying more than in the "free" market, as would be case for the working poor under a tax credit system such as the US EITC. Hence our formulation appears as a minimal reward requisite.

In the presence of residual luck, the reward principle can be stated in two distinct ways depending on to deal with the correlation between effort and luck. In this section, we focus on the first version of this principle where this correlation is not taking into account. It amounts to require that effort increases individual outcome, given individual circumstances and luck. We refer to this principle as the point-wise reward principle (PR).

Definition 2 *The point-wise reward principle (PR) is satisfied iff: for any* $c \in \mathcal{C}^n$*, for any* $l \in \mathcal{L}^q$*, for any* $e, e' \in \mathcal{E}^p$*, such that* $e > e'$ *(at least one component strictly larger),*

$$\mathsf{Y}(e, c, l) \geq \mathsf{Y}(e', c, l)$$

This is similar to the formulation of the reward principle above, where luck sides along with circumstances as a conditioning variable. It amounts to require that effort pays, given the revealed individual residual luck. Hence, it seems better suited to characterize situations where luck is antecedent and exogenous with respect to the choice of effort. The grey zone for residual luck means that it appears as a conditioning variable for both the compensation and reward principle.

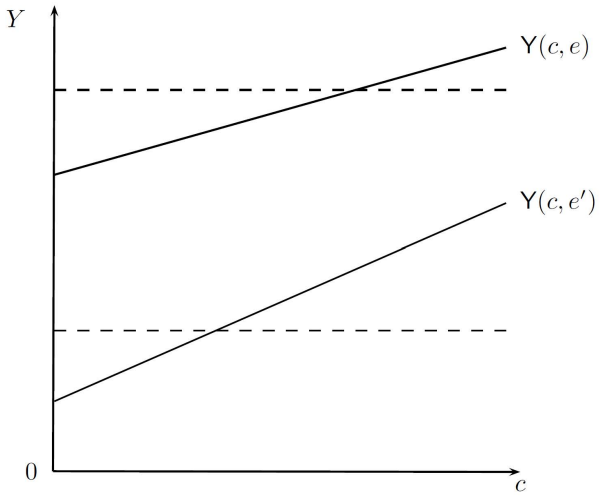
In the end, the combination of our two principles can be illustrated by the following figure borrowed from Fleurbaey and Schokkaert (2012). Individual outcome is represented on the vertical axis and circumstances are assumed to be captured by a positive scalar represented on the horizontal axis. Figure 2 is drawn for some value of the residual luck factor. The two increasing lines represent individual outcome as a function of circumstances c, for two levels of effort, e and e', with $e > e'$. Obviously, in this case, individual with greater values of c are advantaged. Conditional on c, outcome increase with e.

Our conditions require that conditional on effort, the gradient of outcome w.r.t the circumstances be nullified (compensation principle) and that the outcome lines be ranked by order of effort level. This situation is satisfied by the two dashed lines. Mathematically, it seems possible to find some outcome functions respecting both principles. However, it should be noted that this property should hold for any value of residual luck, which might in some practical cases be impossible to satisfy. Contrary to Fleurbaey, we do not require, as in the natural reward principle, that the differential reward associated with increased effort be equal to its observed value before policy intervention. In fact, in the case of figure 3.1 (and more generally whenever the cross-derivative of outcome w.r.t effort and circumstances is not zero), this cannot be uniquely defined and requires to single out a particular level of circumstances.

3.2 Why luck differs from circumstances

The fact that luck is different from effort does not need to be argued further. However, it may good to contend that luck should be treated in a different way from circumstances. We will basically develop two arguments. First, to some extent, gambling enhances welfare. Second genetic luck generally only deserves a partial and asymmetric compensation.

Figure 2: Outcomes, effort and circumstances



Gambling enhances welfare. A first argument for a differentiated treatment of option luck in the definition of equality of opportunity is that luck might be welfare improving. More precisely, the opportunity to engage into decisions that yield uncertain outcomes might, in some cases, enhance individual well-being, not only ex-ante but also ex-post. Although the result of a lottery is clearly out of control of the individual and cannot be claimed belonging to the responsibility sphere, organizing a compensation scheme of gambling may destroy welfare and hence be inefficient.

Bets and lotteries represent in developed societies a sizable share of individual resources. For instance, in France, aggregate (legal) bets amount, in 2012, to 32 Billions euros or 2% of national income. People's decision to enter typical lotteries might at first sight be difficult to explain without assuming risk-loving preferences. In general these lotteries are characterized by a relatively low-payout rate and (very) small odds of winning. However, some authors have noted that existing lotteries are often characterized by skewed-payoff distributions: losses are typically frequent and small and gains are usually very large and rare. If individuals exhibit preference for right-skewed lotteries, even risk-averse individuals might prefer to enter lotteries (see Golec and Tamarkin (1998) and Chiu (2010)).

People's decision to voluntary enter lotteries against the available alternative of not betting indicates, through a revealed preference argument, that gambling is welfare improving at least ex-ante. As a consequence, redistributing the gains of lotteries, i.e providing full insurance against this form of luck, would amount to a prohibition on gambling and would decrease welfare. As noted by Dworkin "if winners were made to share their winnings with losers, then no one would

gamble, as individuals, and the kind of life preferred by both those who in the end win and those who lose would be unavailable".[2]

There might of course be a conflict, when evaluating the impact of lotteries on individual welfare, between the ex-ante and ex-post perspectives. Most gamblers might be better off gambling ex-ante, in terms of expected welfare, and at the same time be worse off ex-post, once the dice has been rolled and they end up losing. In an egalitarian perspective, this might be an issue. To solve it, one needs to decide whether ex-ante or ex-post preferences should be taken into account when evaluating gambling decisions. Fleurbaey (2008) argues that one should give priority to ex-post preferences since individuals are better informed ex-post about their preferences. Fleurbaey adopts an ex-post view point for the application of the egalitarian- equivalent solution to option luck: it takes into account informed preferences, that is, preferences that the individual discover after the true state of nature. As a matter of example, Fleurbaey (2008 p.162) distinguishes between risk lovers and super risk lovers. If they lose, the former regret gambling ex-post whereas the latter do not. Following the distinction introduced by Kahneman, Wakker and Sarin (1997) between decision utility and experience utility, preferences that might be respected are 'experience' preferences, in the present case, the preferences of the super risk lover. Fleurbaey does not propose a compensation scheme for them and therefore Egalitarian-Equivalence leaves room for uninsured risky activities.

Empirically, it is possible to examine the social preferences for redistribution of the gains of gambling decisions by looking at the fiscal treatment of these gains. In France for instance, gains from the national lottery are exempted from taxation. This seems to indicate that gains from lotteries are fair, which is consistent with the ex-ante evaluation of gambling decisions or an ex-post view point if all losers are super risk lovers. In gambling, losers are loosing (relatively) small amounts of money. However, in other contexts, in decisions that looks like gambling such as off-piste skiing, the consequences of bad luck are far more serious up to fatal accident. In that case, calling for the mountain rescue service does not seem to be an issue in many if not all countries, meaning that the consequences of bad option luck are compensated as far as possible. Depending on the context, option luck can then classified as a circumstance or an effort variable.

Partial and asymmetric compensation of the genetic luck A second argument for considering a third generic class of variables, in the definition of equality of opportunity lies in actual preferences for redistribution. There are cases in which the type of redistribution favored by individuals seems to call for only a partial and asymmetric compensation of factors that nevertheless lie beyond the realm of individual responsibility.

Take for instance the case of genetic endowments. This represents a source of inequality that obviously lie beyond individual choice. Hence, a strict application of the criterion of individual responsibility would command that this

[2]Dworkin(2000 p. 75) quoted in Fleurbaey (2008)

source of inequality be fully neutralized. On the contrary, the standard libertarian argument would claim that genetic endowments are a fully legitimate source of inequality, since they constitutive of the individual. Hence, genetic endowments would count as effort and not be compensated, which can also be taken into account in the dual world. In many cases, however, individual conceptions of fairness might stand in a intermediate position between these two alternatives, and blend together the libertarian self-ownership arguments and the egalitarian responsibility perspectives (see Vallentyne 1997). For instance, people might want to compensate the effect of poor genetic endowments and at the same time leave individuals free to enjoy the benefit of a particularly favorable genetic luck. An example could be a situation in which, one supports the compensation of muscular dystrophia and at the same time let people enjoy the benefits of a particularly good health constitution. Similarly, some individuals might at the same time advocate for the compensation of unfavorable cognitive skills endowments, say through special education programs and support at the same time the possibility for individuals with special talents to develop them and enjoy the benefits thereof. In fact, such positions might echo the previous discussion on the preference for skewness in gambling contexts: the asymmetric compensation for luck would lead to a skewed distribution of the consequences of genetic luck that would be welfare enhancing, from the ex-ante perspective where genetic endowments have not been drawn.

These composite redistributive principles are hard to fit into the dual world. And this could motivate the introduction of a third category, luck, in a comprehensive model of compensation and reward. Still, one possibility of forcing asymmetric compensation into the dual model would be to split genetic luck into two components: the bottom of the distribution which would could as circumstances and be compensated and the top of the distribution which would be treated as effort. However, this does not allow to capture any redistributive preferences. There are for instance cases where partial compensation is implemented. This is for instance the case of health insurance in France , where good health is not taxed, bad health is compensated, and the compensation is furthermore gradual and increases with how serious the bad health condition is. The less lucky in terms of health are the reference type in the Fleurbaey terminology and are fully compensated. The second less lucky in terms of health are a little less compensated and so on.

3.3 The correlation between effort and circumstances

The principles laid out in this section can be applied for any possible partition of the determinants of outcomes between efforts and circumstances and residual luck. We now discuss various issues that pertains to correlation between effort and circumstances.

Assume for instance that the outcome of interest is disposable income and that the set of determinants of outcome consist in only two variables : parental social class and number of hours of work. The dominant criterion for partitioning determinants of outcome between effort and circumstances has been the

criterion of individual responsibility. According to this criterion, circumstances should gather those determinants that individuals are not held responsible for. Along these lines, our simple example would suggest to count parental social class as circumstances (people are seldom responsible for who their parents were) and hours of work as effort. However, hours of work and family background could well be related, for a variety of reasons, ranging from family connections to the intergenerational transmission of work ethics. We would like to develop the idea that they may be related through rational choice of effort, although to some respect uncertainty may make the correlation less plausible. We again consider the framework of our model, assuming that the individual chooses his effort by maximizing a utility function knowing the possible set figured out by the return function.

Let us assume that all variables are real variables and all functions are twice differentiable and that the decision of the individual obeys to Savage's axioms: there exists a subjective probability $\pi(\omega)$ and a concave utility function u such that the individual chooses his effort level (assuming for the to be unidimensional for the sake of simplicity) to maximize

$$\int_{\Omega^q} u(\mathsf{Y}(c,e,\omega))\pi(\omega)d\omega - v(e)$$

Here we do not assume the presence of moral hazard since $\pi(\omega)$ does not depend on effort. The FOC gives the usual condition that the expected marginal utility benefit should be equal to the deterministic marginal cost

$$E\left[u'(\mathsf{Y}(c,e,\omega))\mathsf{Y}'_e(c,e,\omega)\right] = v'(e)$$

and the SOC is supposed to be verified

$$A = E\left[((u''(\mathsf{Y}(c,e,\omega)\left(\mathsf{Y}'_e(c,e,\omega)\right)^2 + u'(\mathsf{Y}(c,e,\omega)\mathsf{Y}''_e(c,e,\omega)\right] - v''(e) < 0$$

which is true if the outcome function is concave in effort.

If we do a comparative static exercise looking at how the effort reacts to circumstances, we find that

$$\frac{de^*}{dc} = -\frac{E\left[u''(\mathsf{Y}(c,e,\omega)\mathsf{Y}'_c(c,e,\omega)\mathsf{Y}'_e(c,e,\omega)) + u'(\mathsf{Y}(c,e,\omega)\mathsf{Y}''_{ec}(c,e,\omega)\right]}{A} \quad (2)$$

It is tempting to assume that circumstances help to be successful ($\mathsf{Y}'_c(c,e,\omega) > 0$) and make the marginal return of effort higher $\mathsf{Y}''_{ec}(c,e,\omega) > 0$. And yet, we cannot sign the expression except in the case where the individual is risk neutral or maybe has a very low risk aversion. In case of risk neutrality, a rational choice of effort would go along with circumstances which introduces a positive correlation between effort and (favorable) circumstances. It is quite intuitive to see that risk aversion represents a driving force against providing an extra effort. In a risky environment, the benefit of an extra effort is lower for a risk

adverter. The intuition is also valid for exerting a higher effort to benefit from the advantage represented by circumstances.

From the inspection of formula (2), it results that if we assume that

$$\frac{\mathsf{Y}''_{ec}(c,e,\omega)}{\mathsf{Y}'_{e}(c,e,\omega)} > \mathsf{Y}'_{c}(c,e,\omega), \quad \forall\omega \tag{3}$$

that is, the cross partial derivative exceeds the product of the first order partial derivatives for any state of nature, and the agent has absolute risk aversion lower than 1, the result obtained with risk neutrality is still valid. If the inequality (3) is reversed and if the absolute risk aversion is greater than 1, we obtain the opposite result.

However, the condition (3) is too crude for it cannot be true in all the domain, and in particular, it cannot be true for any state of nature if we impose that the return function has constant return to scale (CRS). Indeed, requiring the opposite of condition (3) on the full domain $\mathcal{C}^n \times \mathcal{E}^p \times \Omega^q$ implies that

$$\mathsf{Y}'_{c}(\alpha c,\alpha e,\alpha\omega)\mathsf{Y}'_{e}(\alpha c,\alpha e,\alpha\omega) > \mathsf{Y}''_{ec}(\alpha c,\alpha e,\alpha\omega) \text{ for any } \alpha \in \mathbb{R}_{+}$$

If $\mathsf{Y}(c,e,\omega)$ is CRS, then $\mathsf{Y}'_{c}(c,e,\omega)$ and $\mathsf{Y}'_{e}(c,e,\omega)$ are homogeneous of degree 0 and $\mathsf{Y}''_{ec}(c,e,\omega)$ is homogeneous of degree -1. Then the above inequality implies that

$$\mathsf{Y}'_{c}(c,e,\omega)\mathsf{Y}'_{e}(c,e,\omega) > \frac{1}{\alpha}\mathsf{Y}''_{ec}(c,e,\omega), \forall\alpha \in \mathbb{R}_{+}$$

The LHS of this inequality which is equal to some positive real number must be higher than any positive real number, which is obviously impossible.

We deduce that less stringent assumption about marginal returns are in need. It turns out that we can impose weaker conditions to obtain a positive or negative relationship between effort and circumstances. For the sake of simplicity, we will assume that $q = 1$ and $\Omega = [0,1]$. Luck is good for success i.e., $\mathsf{Y}'_{\omega} > 0$ which really corresponds to what luck means in plain English.

- *Assumptions on the return function.*

1. $E\left[\mathsf{Y}''_{ec}(c,e,\omega)\right] > E\left[\mathsf{Y}'_{c}(c,e,\omega)\mathsf{Y}'_{e}(c,e,\omega)\right]$. Roughly speaking, this condition on expected returns means that the expected indirect effect of circumstances in terms of additional reward of effort is greater than the expected direct effect of circumstances. It should be true only on average which is considerably weaker than requiring that it should be true for any state of nature.
2. $\int_0^t \frac{\mathsf{Y}''_{ec}(c,e,\omega))}{E[\mathsf{Y}''_{ec}(c,e,\omega)]}\pi(\omega)d\omega \geq \int_0^t \frac{\mathsf{Y}'_{c}(c,e,\omega)\mathsf{Y}'_{e}(c,e,\omega)}{E[\mathsf{Y}'_{c}(c,e,\omega)\mathsf{Y}'_{e}(c,e,\omega)]}\pi(\omega)d\omega$, for $0 < t < 1$. In words, the random variable (r.v) $\mathsf{Y}''_{ec}(c,e,\omega))$ Lorenz- dominates the r.v $\mathsf{Y}'_{c}(c,e,\omega)\mathsf{Y}'_{e}(c,e,\omega)$. All together, the two conditions implies that the r.v. $\mathsf{Y}''_{ec}(c,e,\omega))$ Generalized-Lorenz dominates the r.v $\mathsf{Y}'_{c}(c,e,\omega)\mathsf{Y}'_{e}(c,e,\omega)$. However, this condition is not sufficient and we need dominance both in terms of expected values and Lorenz curves. The clue is always the

same, circumstances matter more for success through its indirect effect via effort than for its direct effect on the return function. However, all together, they are still weaker than the point-wise conditions (3).

These conditions makes the expected reward of circumstances sufficiently strong to counterweigh risk aversion.

Proposition 1 *Assume that the signs of the utility function alternate up to the fourth order. If the random variable* $\mathsf{Y}''_{ec}(c,e,\omega)$ *Lorenz-dominates and has a higher expectation than the random variable* $\mathsf{Y}'_{e}(c,e,\omega)\mathsf{Y}'_{c}(c,e,\omega)$ *and if the degree of absolute risk aversion is lower than 1 in a sufficiently large domain, then people who grew up with better circumstances will exert more effort* $\frac{de^*}{dc} > 0$.

Proof. See appendix ■

The class of utility functions satisfying both conditions on the signs of the partial derivatives and risk-aversion is far from being empty. For instance an appropriate sub class of CARA utility functions respects these conditions. The almost symmetric proposition holds as well showing this time that it may be possible to find a negative relationship between effort and circumstances. For the sake of completeness, we state the following corollary.

Proposition 2 *In addition to the usual signs on the first and second derivatives, let us require that* $u''' < 0$ *and* $u'''' > 0$. *If the random variable* $\mathsf{Y}'_{c}(c,e,\omega)\mathsf{Y}'_{e}(c,e,\omega)$ *Lorenz-dominates and has a higher expectation than the random variable* $\mathsf{Y}''_{ec}(c,e,\omega)$ *and if the degree of absolute risk aversion is larger than 1 on a sufficiently large domain, then people who grew up with better circumstances will exert more effort* $\frac{de^*}{dc} < 0$.

Proof. The logic of the proof is similar to the proof of the previous proposition. For the sake of completeness, it is provided in Appendix. ■

We are not claiming that all the sufficient conditions stated in the above propositions are plausible. Nevertheless even in an uncertain world a systematic positive or negative link between effort and circumstances might be found in the data which requires a further thought as suggested by Roemer. Of course, it might be difficult to detect this empirical evidence because effort is generally not fully observable. This is what a theoretical analysis was useful to think about the possible relation between effort and circumstances. It should be added that the model was not fully general since the cost of effort was not assumed to depend on circumstances and luck was not supposed to depend on effort (no moral hazard).

It is important to realize that in the framework of rational choice, the positive or negative relationship between effort and circumstances is a product of both the features of the preferences (the assumptions on the utility functions) and the features of the possibility set (the assumptions on the reward function). Let us agree that people should be responsible for their preferences as argued by Dworkin and Fleurbaey and that the return function is out of their control, a reasonable assumption, once the correlation between effort and luck (moral

hazard) has been put aside. The positive or negative relation is then a product of the intricacy between a responsibility and a non-responsibility characteristic. It is really a matter of the philosophical debate to consider this correlation as belonging to either set.

We recall here that two answers have been brought to this question in the philosophical debate. A first possibility is to consider that, despite their correlation, the various variables that compose circumstances and effort are meaningfully defined and should be used as such when applying the compensation and reward principal. Brian Barry was a prominent advocate of this stance to what LPT refer to as absolute effort. In our example, this would amount to require that the number of hours worked should be respected as such when applying both principles. We can add that according to the cost-view of responsibility an extra absolute effort means that the individual has supported an extra cost and this extra cost should be rewarded. And indeed, it is interesting to note that this is exactly this cost argument that was emphasized by Brian Barry about Asian students versus other students: "...i*f reward is due to effort, then the Asian students should receive more reward than the other children, for they really try harder*'[3].

The second possibility is suggested by Roemer (1993). Roemer argues, against the previous view, that the existence of a correlation between effort and circumstances is not possible, if circumstances and effort are defined in a way that is consistent with the principle of responsibility. For him, if individuals are not responsible for their circumstances but are responsible for effort, the definition of effort needs to be purged of any residual influence of circumstances on individual effort. In our example, the fact that lower social class origin would lead to lower hours of work calls for compensation. The correlation between effort and the type is a type characteristics. This leads to redefine effort in such a way that it would be, by construction, independent of circumstances. LPT refer to this as relative effort.[4]

One way to build relative effort from absolute effort to take the rank in the distribution of absolute effort conditional on circumstances. Define $F(e|c)$ the cumulative distribution of effort in the population conditional on circumstances c. The relative effort in a type c is defined by $e_c = F(e|c)$, belongs to $[0, 1]$ and is identically distributed across all types. This conditional distribution can only be observed ex-post, that is after the effort has been exerted. In the frame of our model, if effort is chosen from a rational model, one should observer only one effort level per type if anyone has the same risk preferences and disutility for effort in a type. The choice of effort within a type should then reflect a difference of preference somehow and should be respected according to the mantra of EOp. The relative view of effort endorsed by Roemer suggests to measure effort in a way that depends on the population distribution of the relevant variables.

[3] Roemer (1998) p.21.

[4] "*The choice of a degree of effort (as measured by the percentile of effort levels within a type) as the relevant metric for how hard a person tried, is justified by a view that, if we could somehow disembody individuals from their circumstances, then the distribution of the propensity to exert effort would be the same in every type*" Roemer (1998).

Henceforth, effort is not solely defined on the basis of individual characteristics taken in isolation.

One can ask whether the way effort is defined, relative vs absolute, matters for the requirements of equality of opportunity. It is key here to observe that the reward principle is a within-type principle whereas that the compensation principe is a between-type principle. The point-wise principle requires that within a type, the outcome will be non decreasing with effort. Since relative effort is an increasing function of absolute effort, it is immediate that if the point-wise principe holds for either definition of effort, it will hold for the other. We conclude that it does not matter whether the reward principle is defined or checked for the relative or absolute version of effort.

It turns out that it is not the case for the principle of compensation and in fact for the very reason that the principle of reward holds. The principe of compensation with relative effort means that for any $(c, c') \in \mathcal{C}^n \times \mathcal{C}^n$, for any $l \in \mathcal{L}^q$, for any $e_c, e_{c'} \in [0,1]^2$ such that $e_c = e_{c'}$,

$$\mathsf{Y}(c, e_c, l) = \mathsf{Y}(c', e_{c'}, l)$$

Let first establish that if the principle of compensation holds for the absolute effort, it is violated for the relative one. For any $t \in [0,1]$, consider the relative effort levels e_c and $e_{c'}$ such that $t = e_c = e_{c'}$. Then, there exists absolute effort levels, in each type, e and e' such that:

$$e = F^{-1}(t|c) \text{ and } e' = F^{-1}(t|c')$$

and assume that $e \neq e'$ and wlog $e > e'$.

The principle of compensation requires that the return function does not depend on circumstances for any level of absolute effort and any state of the world. Then in particular for e and e', we have

$$\begin{aligned} \mathsf{Y}(c, e, l) &= \mathsf{Y}(c', e, l) \\ \mathsf{Y}(c, e', l) &= \mathsf{Y}(c', e', l) \end{aligned}$$

But since the principle of reward holds

$$\mathsf{Y}(c, e, l) > \mathsf{Y}(c, e', l)$$

and the above inequalities imply that

$$\mathsf{Y}(c, e, l) > \mathsf{Y}(c', e', l)$$

which means that

$$\mathsf{Y}(c, t = e_c, l) > \mathsf{Y}(c', t = e_{c'}, l)$$

which violates the point-wise reward principle applied to relative effort.

Conversely, it seems clear that if the compensation principle applied to relative effort holds, the principle of compensation applied to absolute effort is violated, provided that the principle of reward is verified. The following proposition makes stock of the previous findings.

Proposition 3 *Suppose that the principle of reward holds. It holds for the absolute version of effort iff it holds for the relative version of effort. Then if there exists two types for which the conditional distribution of efforts differ, then the principle of compensation cannot hold both for absolute effort and relative effort.*

4 The ex-ante view: the distribution-wise principles

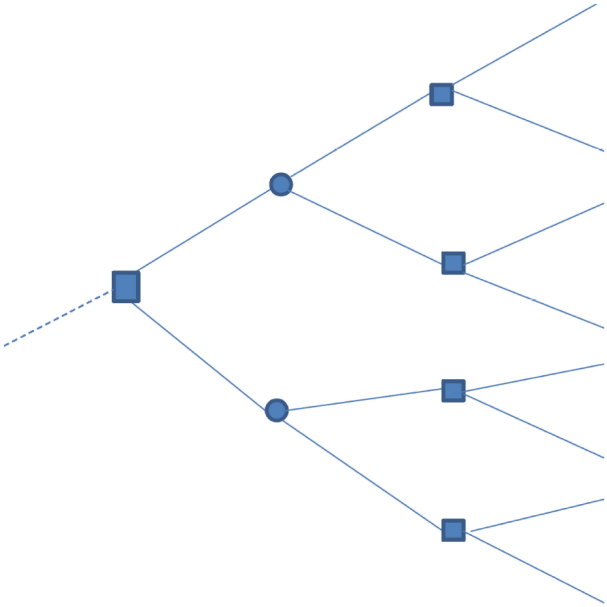
In the previous section we adopted an ex-post perspective, in the sense that equality of opportunity was assessed once all uncertainty had been resolved. Alternatively, one can endorse an ex-ante point of view, and assess equality of opportunity before the individual knows the state of nature that will prevail (for a discussion about ex-post and ex-ante see Fleurbaey (2010)). We first argue that the ex-ante view is particularly needed to cope with moral hazard. We then state an ex-ante view of the compensation and reward principles. Finally we clarify that the choices between the ex-ante and ex-post views depend on how we want to deal with the correlation of luck and effort and the correlation of luck and circumstances. We finally give examples that illustrate how we can embrace all types of luck by mixing the ex-ante and ex-post perspectives.

4.1 The distinctive feature of moral hazard

We now suppose that we can describe the process by which the outcome y has been obtained by a decision tree under uncertainty where the individual plays against nature. A decision tree comprises decision nodes indicated by a square, chances nodes represented by a circle and end nodes. Figure 3 features such an example for an individual who decides twice and for whom the first decision is followed by the intervention of nature. The end nodes are elements of $\mathbb{R}_+$. The starting gate at the age of consent (the first decision node) depends itself on her background (genes, family and social context) that can be seen as a result of a giant lottery, the birth lottery and is featured by a dashed arrow. It is uncontroversial at least from an EOp perspective that starting gate positions should be equalized across individuals. So the following discussion mainly focuses on the interplay between decisions and nature, or in the language of EOp between effort and luck, moral hazard in a nutshell.

Moral Hazard: Heaven helps those who help themselves Let moral hazard designate the random factors drawn after effort choice has been exercised and that are correlated with effort. For at least moral hazard, we argue that the dual model is a straight jacket and that this form of luck stands in an intermediate position between effort and circumstances, as far as the reward and compensation principles are concerned. It does not belong to effort, as it incorporates exogenous factors that fall outside the realm of individual responsibility. Furthermore, there are no ethical reasons to require that they be rewarded. Yet, unlike circumstances, it should not necessarily be compensated.

Figure 3: Decision tree



Even if a good deal of the discussion in the literature about luck has been focused on gambling, gambling per se is far less ubiquitous than moral hazard. The action precedes the lottery in moral hazard as in option luck but in addition, the action is costly in terms of true effort or resources (preventing risks). Let us come back to Figure 3 in which individuals choose their effort level. In this ex-ante perspective, some individual circumstances are known. Yet other factors are not fully determined and are ex-ante uncertain. They can be considered as random and their occurrence lies beyond the realm of individual responsibility. However, to some extent, the random distribution of these unknown factors is not independent from the choice of effort. In particular, choosing a higher level of effort may bring better draws of these random factors, in various ways. For instance, the distribution of random factors attached to higher effort might be better, in the sense of first-order stochastic dominance: as an example, the distribution of exam grades for a student working harder might dominate the distribution of grades he would have if he studies less. In other cases, the benefits of effort might take the form of a safer outcome distribution, in the sense of second-order stochastic dominance, as in the case of voluntary and costly insurance. These forms of correlation between effort and random factors are probably widespread. Educational or job search decisions, to name just a few, are key responsibility variables of which the consequences are not deterministic. Hence, it seems important to account, in the definition of equality of opportunity, for the correlation between effort and subsequent

random factors.

This correlation raises interesting issues for the definition of equality of opportunity. If effort is a legitimate source of inequality, it seems natural that respecting the effect of effort on outcomes also requires respecting the consequences of effort in terms of more favorable luck distribution. This should be reflected in the principle of reward. Let us observe that the point-wise reward principle is useless to this respect since it gets rid of the correlation between effort and luck. At the same time, luck remains distinct from effort, as luck is not per se a source of individual merit and the principe of reward should not apply to luck as it applies to effort. It seems clear that the principles of reward should be redefined to account for moral hazard. It is the aim of the following subsection.

4.2 Distribution-wise principles

Individual realization of outcomes, circumstances, effort and residual luck, are still denoted respectively by y, c, e and l. Each of these variables can be seen as randomly distributed in the population. We use upper case letters Y, C, E and L to denote the corresponding random variables in the population. f_X denotes the distribution function of variable X in the population and F_X denotes the associated cumulative distribution function. Lastly, we denote by $f_{X|Z}$ the distribution in the population of variable X conditional on variable Z. The ex-ante perspective defines the requisite of equality of opportunity in terms of these distributions, which vindicates that we can name them the distribution-wise principles.

Two remarks are in order to better understand the meaning of the ex-ante approach. First, since it is quite implausible that the distribution of luck is known ex-ante in almost cases, the major source of information about the distribution of luck is obtained, once all the uncertainty is resolved. The frequency approach of probability will be useful to build an empirical CDF of luck. The ex-ante perspective is then based on ex-post data. Second it is important to realize that the only factor that is not fixed when computing the CDF of outcome conditional on effort and circumstances is luck. Namely the distribution of luck for a given effort and circumstance is what makes the distribution of outcome conditional on the same variables non-degenerate, i.e. distinct from a single mass-point distribution. Mathematically, define $\mathcal{L}(y, c, e) = \{l \in \mathcal{L}^q \mid \mathsf{Y}(c, e, l) \leq y\}$. Then by definition,

$$F_{Y|C,E}(y \mid c, e) = \int_{l \in \mathcal{L}(y,c,e)} f_{L|C,E}(l \mid c, e) dl.$$

We first examine the minimal reward principle and next turn to the compensation principle.

The distribution-wise reward principal As previously discussed, the minimal reward principle requires that effort should pay. In the previous section, we

required that the outcome function be increasing in effort, given circumstances. This is consistent with the view that circumstances are exogenously given and that effort choice is made given circumstances.

However, as previously discussed, there might be cases where part of the payoff to increased effort is to bring more favorable luck. In this case, conditioning on luck will absorb part of the benefit of effort. To allow for the fact that the benefit of higher effort might take the form of better luck, one might condition only on circumstances when defining the principal of minimal reward. Hence, we may want to impose that effort pays in terms of the overall distribution from which individuals draw their outcome but not necessarily for each degree of luck. This leads to a second formulation of the reward principle that we define as the distribution-wise reward principal (DR).

Definition 3 *The distribution-wise reward principle (DR) is satisfied iff: for any $c \in \mathcal{C}^n$, for any $e, e' \in \mathcal{E}^p$, such that $e > e'$ (at least one component strictly larger), we have :*

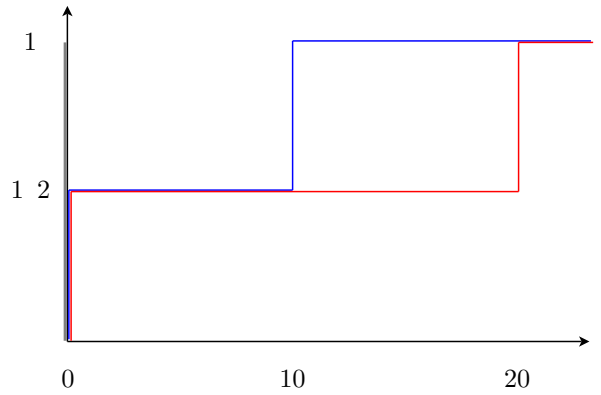
$$F_{Y|C,E}(\ |\ c, e) \succeq_{FSD} F_{Y|C,E}(\ |\ c, e')$$

where $\succeq_{FSD}$ denotes first-order stochastic dominance

The perspective behind the DR principle can be seen as an ex-ante point of view where effort is chosen before luck is determined and potentially influences the luck draw. This justifies why, in this perspective, individual outcomes need to be evaluated on the basis of the distribution individuals face. Since uncertainty is an integral part of this distribution and individuals cannot be assumed to be risk neutral, the outcome prospects cannot be rightfully summarized by the expected (mean) outcome and the full distribution must be considered. Of course, this makes the formulation of the reward principle more complicated. To translate the idea that effort should pay, one may require that the distribution of individual outcome be strictly better for greater levels of effort, in the sense of first-order stochastic dominance. This may appear to be too restrictive and would not allow to fully account for the relationship between effort and luck. For instance, as discussed above, effort might take the form of greater prudence exerted to access a safer distribution. In this case, it might be thought that requiring first-order dominance might be excessive. However, we claim that because y is the gross outcome and not the outcome net of effort, the distribution-wise reward principle allows to cover all risk situations. We will proceed by means of an example. Let us consider the case of betting.

So consider flipping a coin with three possible stakes 0, 5 and 10. We consider a fair bet and then if lucky you gain 10 if you bet 5 and 20 if you bet 10. The ranking of the efforts is non ambiguous, $0 < 5 < 10$. The CDF of the prospects are drawn in the below figure and it appears that the CDF for the higher effort (betting 10) first-order stochastic dominates the CDF for the intermediate effort (betting 5) and the CDF for the null effort (the spike at 0). The CDF which are drawn in Figure 4 are the CDF corresponding to the gross outcome.

Figure 4: First-order stochastic dominance for betting



Obviously, it is well-known that in this example, the ranking of the distribution of net outcomes will be exactly the opposite in terms of second-order stochastic dominance (SSD), that is, the distribution of net prospects if not betting will SSD dominate the distribution of net prospects if betting 5 and so on.

We claim that, far from being a restrictive case, this example conveys a widespread intuition about risky situations. If we deliberately get rid of the undertaking cost, it must be that gross-income prospects corresponding to greater effort should first-order stochastic dominates gross-income prospects for lower effort. It is a necessary condition for investing in prevention be rewarding. Namely, the net-income prospect corresponding to greater effort should at least SSD dominate the net-income prospect corresponding to less effort. For this intuition to be valid, we must be prepared to be flexible enough to cope with a positive or negative view of undertaking risks. In the instance of binge drinking, we may endorse a negative view leading to consider that not drinking represents a greater effort than drinking. The effort could be measured by the number of glasses that you resist to drink. The more you abstain, the greater the effort.

To summarize, the two reward principles impose distinct restrictions on the effect of effort on individual outcomes. These restrictions differ in the way the correlation between effort and luck is taken into consideration when judging the effect of effort on outcomes. The point-wise reward principle only focuses on the direct effect of effort on outcome. On the contrary, the distribution-wise principle requires that a rise in effort improves the prospect for outcome, through two possible effects : the direct effect of effort on outcome and the indirect effect of effort on the distribution of luck.

Without further assumptions, the two principles are independent. However, if there is no correlation between the distribution of effort and luck, basically, both principles mixed up, as stated by the two following propositions.

Proposition 4 *If the distribution of l is independent of e, PR implies DR.*

Proof. By definition, $F_{Y|C,E}(u \mid c,e) = \int_{l \in \mathcal{L}(y,c,e}} f_{L|C,E}(l \mid c,e)dl$. Under the assumption of independence, the integrand in the equation for $F_{Y|C,E}(u \mid c,e)$ is independent of e. Assumption PR further implies that $\mathcal{L}(y,c,e) \subset \mathcal{L}(y,c,e)$ for $e > e'$ since $\mathsf{Y}(c,e,l) \geq \mathsf{Y}(c,e',l)$. Hence the result. ■

The reciprocal is not true, as demonstrated by the following case of pure option luck : $l \sim \mathsf{U}_{[0,1]}$, $e \in [0,1]$, $\mathsf{Y}(e,l) = (l-.5)(1-e)$. The converse proposition is hardly more difficult to prove.

Proposition 5 *If the distribution of l is independent of e, if luck is uni-dimensional and if Y is monotonically increasing in l, DR implies PR.*

Proof. If l is uni-dimensional and if Y is monotonically increasing in l, the income rank in the distribution of $F_{Y|C,E}(| \ c,e)$ is identical to the luck rank in the distribution $F_{L|C,E}(| \ c,e)$. Furthermore, if the distribution of l is independent of e, we have : $\forall e, \ f_{L|C,E}(l \mid c,e) = f_{L|C}(l \mid c)$. Hence, under the above assumptions, two individuals with similar circumstances and different effort who sit at the same rank of their income distributions, conditional on effort and circumstances have exerted similar effort.
Define $F^{-1}_{Y|C,E}(q \mid c,e)$ the quantile function, conditional on c and e. DR1 implies that for any $e > e'$ and any $q \in [0,1]$, $F^{-1}_{Y|C,E}(q \mid c,e) \geq F^{-1}_{Y|C,E}(q \mid c,e')$. Noting that $F^{-1}_{Y|C,E}(q \mid c,e) = \mathsf{Y}(c,e,l)$ for $l = F^{-1}_{L|C}(q \mid c)$, we get : $\forall c, \forall e > e', \forall l \ \mathsf{Y}(c,e,l) \geq \mathsf{Y}(c,e',l)$ ■

Finally, it can be underscored that since the distribution-wise reward principle is a within-type principle, it does not matter whether we retain the absolute or relative version of effort.

The distribution-wise compensation principle LPT already propose the ex-ante version of the compensation principle. It rests on the view that differences in luck across individuals should not necessarily be preserved, contrary to differences in effort. They suggest that luck factors should be subject to a limited form of compensation : equality of opportunity does not require that the effect of luck be nullified (which may be impossible to implement without a perfect knowledge of the decision tree of all individuals in society) but that luck factors are even-handed distributed, given circumstances and effort. Hence, the conditional equality principle should hold across all individuals who exert similar effort, regardless of their luck and for all possible circumstances. This view defines the distribution-wise conditional equality principle (DC), which we define as follows :

Definition 4 *The distribution-wise compensation principle (DC) is satisfied iff : for any $(c,c') \in \mathcal{C}^n \times \mathcal{C}^n$, for any $e \in \mathcal{E}^p$, ,*

$$F_{Y|C,E}(\ | \ c,e) = F_{Y|C,E}(\ | \ c',e)$$

The difference in the two versions of the compensation principle stems from the possible influence of individual circumstances on luck. Given effort, the point-wise principle allows for differences in outcomes across individuals with distinct circumstances, but only to the extent that these differences are mediated by differences in luck. On the contrary, the distribution-wise principle rules out all differences in outcome that can be linked to differential circumstances. It is also important to note that in the two principles, outcome comparisons are always restricted to individuals with similar effort. Lastly, without further assumptions, the two principles are independent. Of course, absent the correlation between circumstances and luck, the two compensation principles boil down.

Proposition 6 *If the distribution of l is independent of c PC implies DC.*

Proof. Define $\mathcal{L}(y,c,e) = \{l \in \mathcal{L}^q \mid \mathsf{Y}(c,e,l) \leq y\}$.
By definition, $F_{Y|C,E}(u \mid c,e) = \int_{l \in \mathcal{L}(y,c,e}} f_{L|C,E}(l \mid c,e)dl$. The independence assumption implies that the integrand in the previous expression is independent of c : $\forall c,\ f_{L|C,E}(l \mid c,e) = f_{L|E}(l \mid e)$. Assuming PR is satisfied further implies that the domain of integration is also independent of c. Hence, the result. ■

The converse proposition is not always true if luck is multi-dimensional.

Proposition 7 *If the distribution of l is independent of c, if luck is uni-dimensional and if Y is monotonically increasing in l, DC implies PC.*

Proof. proof by contradiction. Assume that PC is not satisfied in the neighborhood of some $l^\star$ for some effort e. More precisely, assume that for e $\mathsf{Y}(c,e,l) = \mathsf{Y}(c',e,l)$ for $l \leq l^\star$ and that $\mathsf{Y}(c,e,l) \geq \mathsf{Y}(c',e,l)$ for $l \in [l^\star, l^\star + u]$ with u arbitrarily small and the inequality is strict for at least one l. We can show by integration that : $F_{Y|C,E}(y \mid c,e) = F_{Y|C,E}(y \mid c',e)$ for $y \leq \mathsf{Y}(c,e,l^\star)$ and $F_{Y|C,E}(y \mid c,e) < F_{Y|C,E}(y \mid c',e)$ for $y = \mathsf{Y}(c,e,l^\star + u)$. ■

This result does not generalizes to multi-dimensional cases as demonstrated by the following example : two dimensions of luck, l_1 and l_2, iid; two circumstances c and c'; $\mathsf{Y}(c,e,l_1,l_2) = el_1$; $\mathsf{Y}(c',e,l_1,l_2) = el_2$.

The case where effort is defined in a relative way has interesting implications for checking the distribution-wise principle of compensation. In this case, effort is distributed identically across all types. It is possible to reformulate the compensation principle as a constraint on the distribution of outcome within types. Define $F(y|c)$ the cumulative distribution of outcome in the population conditional on circumstances c. We can establish the following proposition.

Proposition 8 *If effort is relative, the distribution-wise compensation principle requires : $\forall c, c' \in C^m, \forall y \in^+,\ F(y|c) = F(y|c')$*

Proof. See LPT ■

4.3 The correlation between luck, circumstances and effort

To summarize, the extension of the reward and conditional equality principles in the presence of luck leads to two sets of equality of opportunity principles. In the point-wise principles, lucks stands in an intermediate position between effort and circumstances. In the compensation principle, luck is treated as effort whereas luck counts as circumstances in the reward principle. In the distribution-wise principles, luck matters through its overall effect on the distribution of outcome faced by individuals with given effort and circumstances. The distribution-wise compensation principle requires that, given effort, the distribution of outcomes that results from the influence of luck should be independent of their circumstances. The distribution-wise reward principle requires that, given circumstances, a higher level of effort translates into a more favorable outcome distribution. We now discuss the relevance of the two perspectives for dealing with different kinds of luck.

The main divide between the point-wise and the distribution-wise principles lies in the way they account for the correlation between luck on the one hand, and circumstances and effort on the other hand. However let us remark that in the compensation principle, the correlation between luck and effort is irrelevant. Similarly, in the reward principle, the correlation between luck and circumstances is irrelevant.

Under the point-wise version of each principle, the possible correlation between luck and circumstances or effort is treated as residual luck in both principles. In particular, the correlation between circumstances and luck does not call for compensation and the correlation between effort and luck does not call for any reward.

On the contrary, under the distribution-wise principles, the correlation between residual luck and circumstances or effort matters. Hence, according to the distribution-wise reward principle, the part of luck that is correlated with circumstances should be nullified. It is considered as a circumstance. And according to the distribution-wise reward principle, the part of luck that is correlated with effort is included in the reward of effort.

The views endorsed in the definition of each principle can be taken independently. The above discussion suggests four distinct ways of defining equality of opportunity, depending on judgements about how the correlation between luck on the one hand, and circumstances and effort on the other hand, should be treated. These fours distinct cases are presented in table below.

Can we find a type of luck that corresponds to each cell of the table? Almost. Let us consider the following time line where genetic luck precedes circumstances which are strictly defined as background variables. Next comes what we name accidental luck, as luck in the marriage market which intervenes before effort but after circumstances. Finally, moral hazard is coming after effort with a draw specific for each level of effort.

Table 1: Four combinations of EOP principles

	$E \cap L$ counts as effort	$E \cap L$ counts as luck
$C \cap L$ counts as circumstances	DC, DR	DC, PR
$C \cap L$ counts as luck	PC, DR	PC, PR

Table 2: Sorting of luck types and EOp principles

	$E \cap L$ counts as effort	$E \cap L$ counts as luck
$C \cap L$ counts as circumstances	DC, DR *Moral Hazard*	DC, PR *Accidental luck*
$C \cap L$ counts as luck	PC, DR (?)	PC, PR *Genetic talent*

The proposed assignment of the different considered types of luck, genetic luck, accidental luck and moral hazard is summed up by the following table.

We have already argued that for moral hazard (entrepreneurial luck providing an example) we should apply the distribution-wise reward principle and treat the correlation between luck and effort as a component of effort. Let us now turn to the correlation between luck and circumstances. In the case of entrepreneurial luck, it might also be the case that the return on entrepreneurial activities depend on circumstances. In this case, compensation for circumstances should probably be extended to the benefits or disadvantages of circumstances that occur through the fact that individuals draw luck from a more or less favorable distribution of returns. In other words, we might want to leave individuals bear the consequences of their risky choice but net of the effect of circumstances. In this case, we would consider that the correlation between luck and circumstances should be considered as a circumstance and apply the distribution-wise compensation principle.

Now consider the case of accidental luck where luck is determined before effort is chosen. It might be that better luck induces individuals to expand more effort. For instance, outcomes might depend positively on individual's luck on the marriage market and society might decide that this form of luck does not in itself call for compensation. Still, at the same time, the correlation of this accidental luck with circumstances should certainly be considered as a circumstance and calls for a compensation. Now consider two individuals with the same intrinsic disposition for effort but with different accidental luck. Although as a result of luck, one will end up expanding more effort. The reward for additional effort will be guaranteed anyway by any reward principle. Now the issue is to know whether he is more deserving because more lucky. The answer is no because luck happens before effort. Hence in this case, we should condition on luck in the formulation of the reward principle and apply the point-

wise reward principle, i.e. treat the correlation between luck and effort as luck.

Yet in other case, on the contrary, some might want to give priority to luck over circumstances. For instance, some people might agree that individuals should be allowed to enjoy the benefits of their innate ability, say a particular gift for music, or genetic endowment, say beauty, irrespective of the family background they were born to. Suppose that all alleles linked to the physical appearance are observable. Apparently, a premium is attached to beauty of a person: According to Hamermesh and Biddle (1994) this premium is about 6% of earnings for good looking for men and 12% for women. Do we want to fully neutralize the likely correlation between social background and beauty or do we consider that it is a "natural" inequality? In this case, the possible correlation between ability and circumstances should be treated as luck and one should apply the point-wise compensation principle. What about the correlation between beauty and effort? For the very reason that innate talent likely precedes effort, it will be difficult to assimilate the correlation between effort and genetic luck as effort. Hence, the application of the point-wise reward principle seems enough.

As a matter of rule, we suggest that if luck precedes effort, the correlation between effort and luck will be counted for luck. On the opposite, if luck is posterior to effort, the correlation between effort and luck will be counted for effort. As regards the correlation between luck and circumstances, if luck precedes circumstances, then it appears as luck whereas if it is posterior to circumstances it will be considered as circumstances. A question mark appears in the above table in the bottom left cell. Indeed, this reasoning prevents to find a luck factor that is considered as a circumstance (it should be anterior to circumstance) and as an effort (it should be posterior to effort), since circumstances precedes luck,.

5 Conclusion

In this paper, we have argued that the strict dichotomy between effort and circumstances provides a restrictive framework for defining equality of opportunity, defined as a combination of a compensation principle and a reward principle. We submit that a comprehensive definition of equality of opportunity needs to take into account a third generic category, which we call *residual luck* and that should be distinguished from effort and circumstances. *Residual luck* gathers the factors that (i) do not call for compensation, as long as they affect outcomes in a neutral way, given circumstances and (ii) do not imply any specific reward requirement. The main reason for endorsing such a proposition is that effort might stochastically influence the realization of some individual outcomes, as in moral hazard. As such, these determinants cannot be treated as circumstances or effort strictly speaking. Individual are only "partly responsible" for them. They call for compensation but only to the extent that they are related to individual circumstances. They should be respected, to the extent that they are shaped by effort choice. Lastly, they should not be seen in and of themselves, as a source of merit in the formulation of reward principles.

When introducing residual luck in the definition of the compensation and reward principles, residual luck stands as an intermediate category between circumstances and effort. For both principles, residual luck appears as a variable that restricts the scope of the compensation and the reward requisites. The extent of this restriction depends on whether the correlation of luck with effort is treated as effort or luck and the correlation of effort and circumstances is considered as circumstances or luck.

Of course, these definitions are provided under the assumption that the determinants of individual outcomes are fully observable. In practice, this will rarely be the case. Hence it would be necessary to discuss how the various notions of equality of opportunity examined in this paper could be identified empirically under observational constraints. We leave this discussion to future developments.

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6 Appendix: Proof of propositions.

6.1 Proof of Proposition 3

Working with the numerator of (2)

$$N = E(u^{''}(\mathsf{Y}(c,e,\omega)\mathsf{Y}'_c(c,e,\omega)\mathsf{Y}'_e(c,e,\omega)) + E(u'(\mathsf{Y}(c,e,\omega)\mathsf{Y}''_{ec}(c,e,\omega)))$$

we want to prove that this expression is positive under assumptions.Using the fact that u' is convex, then we can bind from below the second term which is positive and similarly using the fact that $u^{''}$is concave, we can bind from above the first term which is negative. We use the integral version of the Jensen inequality. Suppose Ω is a measurable subset of the real line with μ some measure, g any

real-valued measurable function, f any positive real-valued measurable function and ϕ any convex function over the range of g, then

$$\varphi\left(\frac{\int_\Omega gfd\mu}{\int_\Omega f(x)d\mu}\right) \leq \frac{\int_\Omega \varphi(g)fd\mu}{\int_\Omega f(x)d\mu}$$

Applying the Jensen inequality with u'convex to the second term of N, we get

$$u'(\frac{1}{\mathsf{Y}''_{ec}(c,e,\omega))}\int_\Omega(\mathsf{Y}(c,e,\omega)\mathsf{Y}''_{ec}(c,e,\omega))\pi(\omega)d\omega) \leq \frac{1}{E(\mathsf{Y}''_{ec}(c,e,\omega))}\int_\Omega u'(\mathsf{Y}(c,e,\omega)\mathsf{Y}''_{ec}(c,e,\omega))\pi(\omega)d\omega$$

Or defining $\mathsf{y}''_{ec}(c,e,\omega)) = \frac{\mathsf{Y}''_{ec}(c,e,\omega))}{E(\mathsf{Y}''_{ec}(c,e,\omega))}$

$$E(\mathsf{Y}''_{ec}(c,e,\omega))u'(E(\mathsf{Y}(c,e,\omega)\mathsf{y}''_{ec}(c,e,\omega)) \leq E(u'(\mathsf{Y}(c,e,\omega)(\mathsf{Y}''_{ec}(c,e,\omega))$$

Applying once again the above Jensen inequality with $u^{''}$concave, one gets

$u^{''}(\frac{1}{E(\mathsf{Y}'_c(c,e,\omega)\mathsf{Y}'_e(c,e,\omega))}\int_\Omega(\mathsf{Y}(c,e,\omega)\mathsf{Y}'_c(c,e,\omega)\mathsf{Y}'_e(c,e,\omega)\pi(\omega)d\omega) \geq$
$\frac{1}{E(\mathsf{Y}'_c(c,e,\omega)\mathsf{Y}'_e(c,e,\omega))}(\int_\Omega u^{''}(\mathsf{Y}(c,e,\omega))\mathsf{Y}'_c(c,e,\omega)\mathsf{Y}'_e(c,e,\omega)\pi(\omega)d\omega)$
Defining $\mathsf{y}'_c(c,e,\omega)\mathsf{y}'_e(c,e,\omega) = \frac{\mathsf{Y}'_c(c,e,\omega)\mathsf{Y}'_e(c,e,\omega)}{E(\mathsf{Y}'_c(c,e,\omega)\mathsf{Y}'_e(c,e,\omega))}$ one finally obtains

$$E(\mathsf{Y}'_c(c,e,\omega)\mathsf{Y}'_e(c,e,\omega))u^{''}(E(\mathsf{Y}(c,e,\omega)\mathsf{y}'_c(c,e,\omega)\mathsf{y}'_e(c,e,\omega)) \geq E(u^{''}(\mathsf{Y}(c,e,\omega))\mathsf{Y}'_c(c,e,\omega)\mathsf{Y}'_e(c,e,\omega))$$

Then

$$N > E(\mathsf{Y}'_c(c,e,\omega)\mathsf{Y}'_e(c,e,\omega))u^{''}(E(\mathsf{Y}(c,e,\omega)\mathsf{y}'_c(c,e,\omega)\mathsf{y}'_e(c,e,\omega))+E(\mathsf{Y}''_{ec}(c,e,\omega))u'(E(\mathsf{Y}(c,e,\omega)\mathsf{y}''_{ec}(c,e,\omega))$$

We want to show that the RHS of the above expression is positive
That is:

$$1 > \frac{E(\mathsf{Y}'_c(c,e,\omega)\mathsf{Y}'_e(c,e,\omega))}{E(\mathsf{Y}''_{ec}(c,e,\omega))}\frac{-u^{''}(E(\mathsf{Y}(c,e,\omega)\mathsf{y}'_c(c,e,\omega)\mathsf{y}'_e(c,e,\omega))}{u'(E(\mathsf{Y}(c,e,\omega)\mathsf{y}''_{ec}(c,e,\omega))}$$

We already know that the first ratio is lower than 1 by condition 1. To conclude, we need to establish that the second ratio is also lower than 1. In fact we will find a bind from above for the second ratio and we will prove that it is lower than 1.

For that, we need to know whether $\int_\Omega(\mathsf{Y}(c,e,\omega)\mathsf{y}'_c(c,e,\omega)\mathsf{y}'_e(c,e,\omega)\pi(\omega)d\omega) \gtrless \int_\Omega(\mathsf{Y}(c,e,\omega)\mathsf{y}''_{ec}(c,e,\omega))\pi(\omega)d\omega$

or equivalently $\int_\Omega(\mathsf{y}'_c(c,e,\omega)(\mathsf{y}'_e(c,e,\omega) - \mathsf{y}''_{ec}(c,e,\omega))(\mathsf{Y}(c,e,\omega)\pi(\omega)d\omega \gtrless 0$

Integrating by parts the above expression, we obtain

$$\left[\int_0^1 (\mathsf{y}'_c(c,e,\omega)(\mathsf{y}'_e(c,e,\omega) - \mathsf{y}''_{ec}(c,e,\omega))\pi(\omega)d\omega\right](\mathsf{Y}(c,e,1)$$
$$-\int_0^1 (\mathsf{Y}'_\omega(c,e,t)\left(\int_0^t (\mathsf{y}'_c(c,e,\omega)(\mathsf{y}'_e(c,e,\omega) - \mathsf{y}''_{ec}(c,e,\omega))\pi(\omega)d\omega\right)\pi(t)dt$$

Since by definition $\int_\Omega \mathsf{y}''_{ec}(c,e,\omega))\pi(\omega)d\omega = \int_\Omega \mathsf{y}'_c(c,e,\omega)\mathsf{y}'_e(c,e,\omega)\pi(\omega)d\omega = 1$
the first term cancel out and by condition 2 the third term is positive.

Then
$E(\mathsf{Y}(c,e,\omega)\mathsf{y}'_c(c,e,\omega)\mathsf{y}'_e(c,e,\omega)) > E(\mathsf{Y}(c,e,\omega)\mathsf{y}''_{ec}(c,e,\omega))$.
Because $u'$is decreasing, the above inequality implies
$u'\left(E(\mathsf{Y}(c,e,\omega)\mathsf{y}'_c(c,e,\omega)\mathsf{y}'_e(c,e,\omega)\right) < u'\left(E(\mathsf{Y}(c,e,\omega)\mathsf{y}''_{ec}(c,e,\omega))\pi(\omega)d\omega\right)$

Implying that

$$\frac{-u''(E(\mathsf{Y}(c,e,\omega)\mathsf{y}'_c(c,e,\omega)\mathsf{y}'_e(c,e,\omega))}{u'\left(E(\mathsf{Y}(c,e,\omega)\mathsf{y}'_c(c,e,\omega)\mathsf{y}'_e(c,e,\omega)\right)} > \frac{-u''(E(\mathsf{Y}(c,e,\omega)\mathsf{y}'_c(c,e,\omega)\mathsf{y}'_e(c,e,\omega))}{u'(E(\mathsf{Y}(c,e,\omega)\mathsf{y}''_{ec}(c,e,\omega))}$$

Since we have assumed that $1 \geq -\frac{u''(.)}{u'(.)}$ the proof is complete. QED

6.2 Proof of Proposition 4

Applying the Jensen inequality with u'concave to the second term of N, we get

$$u'(\frac{1}{\mathsf{Y}''_{ec}(c,e,\omega))}\int_\Omega (\mathsf{Y}(c,e,\omega)\mathsf{Y}''_{ec}(c,e,\omega))\pi(\omega)d\omega) \geq \frac{1}{E(\mathsf{Y}''_{ec}(c,e,\omega))}\int_\Omega u'(\mathsf{Y}(c,e,\omega)\mathsf{Y}''_{ec}(c,e,\omega))\pi(\omega)d\omega$$

or

$$E(\mathsf{Y}''_{ec}(c,e,\omega))u'(E(\mathsf{Y}(c,e,\omega)\mathsf{y}''_{ec}(c,e,\omega)) \leq E(u'(\mathsf{Y}(c,e,\omega)(\mathsf{Y}''_{ec}(c,e,\omega))$$

Applying once again the above Jensen inequality with u''convex, one gets

$u''(\frac{1}{E(\mathsf{Y}'_c(c,e,\omega)\mathsf{Y}'_e(c,e,\omega))}\int_\Omega (\mathsf{Y}(c,e,\omega)\mathsf{Y}'_c(c,e,\omega)\mathsf{Y}'_e(c,e,\omega)\pi(\omega)d\omega) \leq$
$\frac{1}{E(\mathsf{Y}'_c(c,e,\omega)\mathsf{Y}'_e(c,e,\omega))}(\int_\Omega u''(\mathsf{Y}(c,e,\omega))\mathsf{Y}'_c(c,e,\omega)\mathsf{Y}'_e(c,e,\omega)\pi(\omega)d\omega)$
one finally obtains

$$E(\mathsf{Y}'_c(c,e,\omega)\mathsf{Y}'_e(c,e,\omega))u''(E(\mathsf{Y}(c,e,\omega)\mathsf{y}'_c(c,e,\omega)\mathsf{y}'_e(c,e,\omega)) \leq E(u''(\mathsf{Y}(c,e,\omega))\mathsf{Y}'_c(c,e,\omega)\mathsf{Y}'_e(c,e,\omega))$$

Then

$$N < E(\mathsf{Y}'_c(c,e,\omega)\mathsf{Y}'_e(c,e,\omega))u''(E(\mathsf{Y}(c,e,\omega)\mathsf{y}'_c(c,e,\omega)\mathsf{y}'_e(c,e,\omega))+E(\mathsf{Y}''_{ec}(c,e,\omega))u'(E(\mathsf{Y}(c,e,\omega)\mathsf{y}''_{ec}(c,e,\omega))$$

We want to show that the RHS of the above expression is negative

That is:

$$1 \leq \frac{E(\mathsf{Y}'_c(c,e,\omega)\mathsf{Y}'_e(c,e,\omega))}{E(\mathsf{Y}''_{ec}(c,e,\omega))} \frac{-u''(E(\mathsf{Y}(c,e,\omega)\mathsf{y}'_c(c,e,\omega)\mathsf{y}'_e(c,e,\omega))}{u'(E(\mathsf{Y}(c,e,\omega)\mathsf{y}''_{ec}(c,e,\omega))}$$

We already know that the first ratio is greater than 1 by condition 1. To conclude, we need to establish that the second ratio is also larger than 1. In fact we will find a bind from below for the second ratio and we will prove that this bind is lower than 1.

For that, we need to know whether $\int_\Omega (\mathsf{Y}(c,e,\omega)\mathsf{y}'_c(c,e,\omega)\mathsf{y}'_e(c,e,\omega)\pi(\omega)d\omega) \gtrless \int_\Omega (\mathsf{Y}(c,e,\omega)\mathsf{y}''_{ec}(c,e,\omega))\pi(\omega)d\omega$

or equivalently $\int_\Omega (\mathsf{y}'_c(c,e,\omega)(\mathsf{y}'_e(c,e,\omega) - \mathsf{y}''_{ec}(c,e,\omega))(\mathsf{Y}(c,e,\omega)\pi(\omega)d\omega \gtrless 0$

Integrating by parts the above expression, we obtain

$$\left[\int_0^1 (\mathsf{y}'_c(c,e,\omega)(\mathsf{y}'_e(c,e,\omega) - \mathsf{y}''_{ec}(c,e,\omega))\pi(\omega)d\omega\right](\mathsf{Y}(c,e,1)$$
$$-\int_0^1 (\mathsf{Y}'_\omega(c,e,t)\left(\int_0^t (\mathsf{y}'_c(c,e,\omega)(\mathsf{y}'_e(c,e,\omega) - \mathsf{y}''_{ec}(c,e,\omega))\pi(\omega)d\omega\right)\pi(t)dt$$

Since by definition $\int_\Omega \mathsf{y}''_{ec}(c,e,\omega))\pi(\omega)d\omega = \int_\Omega \mathsf{y}'_c(c,e,\omega)\mathsf{y}'_e(c,e,\omega)\pi(\omega)d\omega = 1$ the first term cancel out and by condition 2 the third term is negative.

Then

$E(\mathsf{Y}(c,e,\omega)\mathsf{y}'_c(c,e,\omega)\mathsf{y}'_e(c,e,\omega)) < E(\mathsf{Y}(c,e,\omega)\mathsf{y}''_{ec}(c,e,\omega))$.

Because $u'$is decreasing, the above inequality implies

$u'\left(E(\mathsf{Y}(c,e,\omega)\mathsf{y}'_c(c,e,\omega)\mathsf{y}'_e(c,e,\omega)\right) > u'\left(E(\mathsf{Y}(c,e,\omega)\mathsf{y}''_{ec}(c,e,\omega))\pi(\omega)d\omega\right)$

Implying that

$$\frac{-u''(E(\mathsf{Y}(c,e,\omega)\mathsf{y}'_c(c,e,\omega)\mathsf{y}'_e(c,e,\omega))}{u'\left(E(\mathsf{Y}(c,e,\omega)\mathsf{y}'_c(c,e,\omega)\mathsf{y}'_e(c,e,\omega)\right)} < \frac{-u''(E(\mathsf{Y}(c,e,\omega)\mathsf{y}'_c(c,e,\omega)\mathsf{y}'_e(c,e,\omega))}{u'(E(\mathsf{Y}(c,e,\omega)\mathsf{y}''_{ec}(c,e,\omega))}$$

Since we have assumed that $1 \leq -\frac{u''(.)}{u'(.)}$ the proof is complete.QED