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Has the world converged? A robust analysis of non-monetary bounded indicators*

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Abstract

Most non-monetary development indicators are bounded and many of them are presented in terms of either attainments or shortfalls. Whether an absolute approach or a relative approach should be undertaken to assess cross-country convergence of these indicators has been a subject of debate. Revisiting this debate, we provide three arguments explaining why a relative approach is misleading and, instead, an absolute approach is more appropriate. We assess the presence of absolute convergence across countries in several non-monetary development indicators by applying a number of absolute inequality indices. Although we find numerous instances of absolute convergence, these are rarely robust to alternative specifications of indices. We additionally contribute to the available methodological toolkit of convergence analysis by employing absolute-Lorenz curves to assess the robustness of absolute cross-country convergence, which is rarely conducted in the literature, and never to date with absolute-Lorenz curves. We also clarify the relationship between different relevant notions of egalitarian progress and elucidate how progress in these indicators relates to changes in their convergence using absolute Lorenz curves.

Keywords: Absolute convergence, non-monetary development indicators, absolute Lorenz curve, egalitarian progress, bounded indicators, consistent inequality indices.

JEL Classification: I31, O47, O57.

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1 Introduction

Global inequality has been at the forefront of the global development agenda. The concern has been reflected in the United Nation's Sustainable Development Goal 10: "Reduce inequality within and among countries".[1] The concern has also featured prominently in the recent World Bank's Global Monitoring Report (2015) on ending poverty and shared prosperity. Global inequality comprises assessments of inequality within countries as well as between countries.[2] The latter aspect has long been the subject of the "economic convergence" literature, seeking to ascertain whether the development outcomes (including both monetary and non-monetary indicators) of developing countries were catching up with those of more developed nations over time. Usually in this literature, the empirical assessments have been connected to theories predicting convergence or divergence across countries. However, how cross-country convergence is assessed, and the ensuing results, should have implications in shaping global policy directions.

During the last decades of the 20th century the bulk of convergence analyses focused on monetary indicators of wellbeing and development, chiefly GDP per capita. On the turn of the century most of this literature had concluded that the world's cross-country income distribution had become multi-modal and "convergence clubs" were emerging, with many developing countries lagging behind (for a good review of this early literature, see Islam, 2003). Recently, in the aftermath of the Chinese economic miracle and the decade-long commodity boom of the 21st century, Rodrik (2012, 2013) has revisited the empirical evidence on income and manufacturing productivity indicators. Among several assessments, he finds both evidence of absolute convergence and divergence in manufacturing productivity, depending on the manufacturing sector.

Meanwhile, the turn of the century also witnessed the proliferation of an empirical literature on cross-country convergence over non-monetary development indicators, such as life expectancy at birth, child mortality rates, adult literacy rates, and many others.[3] These studies tested for convergence (or lack thereof) in several non-monetary indicators over roughly the same period, between the 1960s and the 1990s. With variations in implementation, the main methods used were unconditional "beta-convergence" assessments, "sigma convergence" assessments, and inspections of density functions and transition matrices, usually constructed with kernel estimation methods. This literature produced mixed results. For instance, Neumayer (2003) and Kenny (2004) found convergence across most of the considered indicators; whereas Hobijn and

[1] See http://www.un.org/sustainabledevelopment/inequality/.

[2] Milanovic (2005) further introduces two different concepts of *international inequality* as well as a concept of *global inequality* in income. The first concept of international inequality is the unweighted inequality between country incomes; the second concept of international inequality is the population-weighted inequality between country incomes; and the third concept of global inequality is the inequality in incomes across individuals assuming the world as one country.

[3] See Micklewrite and Stewart (1999), Easterlin (2000), Hobijn and Franses (2001), Neumayer (2003), Dowrick et al. (2003), and Kenny (2004).

Franses (2001) found an increase in bimodality across most indicators and no more than 11% of countries in the same convergence club for any given indicator.

The assessment of convergence of the non-monetary development indicators used in the aforementioned studies may not be conceptually as straightforward as the assessment of convergence of monetary indicators. Monetary indicators are mostly unbounded from above and can increase without any constraint. By contrast, almost all non-monetary indicators have bounded domains, which means that they cannot improve over time in an unconstrained manner, in which case the application of tools such as the beta convergence assessment or relative inequality measures (e.g., the Gini coefficient, the coefficient of variation, etc.) to study convergence across countries may be questionable. Given that countries with higher levels of progress in any of these indicators cannot improve beyond a certain threshold, the likelihood of concluding convergence based on these assessment techniques is very high, *despite insufficient progress among the initially poor performing countries*. In turn, such a potentially misleading assessment may undermine the objective of 'not leaving anyone (any country in this case) behind'.[4]

Furthermore, many of the non-monetary development indicators can either be expressed in terms of *attainments* or *shortfalls* (Micklewrite and Stewart, 1999). For instance, in order to assess the overall health status of children in a country, we may either look at the child mortality rate or, equivalently, at its complement counterpart: the child survival rate. Being aware of this essentially arbitrary choice, Kenny (2004) provided justifications for preferring attainment definitions over their shortfall counterparts. However, Micklewrite and Stewart (1999) identified a more fundamental problem in applying relative inequality indices to these indicators: they cannot guarantee ranking bounded indicators *consistently*.[5] In other words, relative indices may conclude convergence across country attainments of an indicator (say, literacy rates), while simultaneously concluding divergence across country shortfalls (illiteracy rates) for the same indicator.[6] A consistent inequality assessment should ensure that the same convergence/divergence conclusion is reached whether the assessment is conducted across attainments of an indicator or its shortfalls counterpart.

In the face of these challenges, we revisit the assessment of cross-country convergence (or lack thereof) over fifteen bounded non-monetary development indicators, with recent data spanning various time periods, from an initial period in the 20th Century until 2010-13. We adopt an absolute approach (equivalently, *sigma convergence*) for the convergence assessment. An absolute approach not only imposes a more stringent condition on cross-country convergence,

[4] We borrow the terminology from the United Nations Association publication *Global Development Goals: Leaving no one behind*. The publication can be accessed at http://www.una.org.uk/content/global-development-goals-leaving-no-one-behind. For a recent assessment of progress on global health goals *within countries,* based on this concern of not leaving the poor behind, see Wagstaff et al. (2014).

[5] The *consistency* issue in inequality assessments with bounded indicators has also been well-known in the health inequality literature, where different consistent inequality indices have been proposed (see Erreygers, 2009; Lambert and Zheng, 2011; Aristondo and Lasso de la Vega, 2012; Chakravarty et al., 2013; Kjellson et al. 2015; Silber, 2015).

[6] Both Kjellson et al. (2015) and Silber (2015) provide examples with actual health data.

but also ranks bounded indicators consistently.[7] We employ four different absolute indices (i.e. not relying solely on one absolute index, as done in the literature with the commonly used standard deviation), which show absolute convergence for most of the selected bounded development indicators.

Even though the four absolute indices agree on convergence/divergence across various years for most indicators, there are some cases where these indices disagree with each other. In fact, even when these four indices agree, it cannot be guaranteed that further alternative legitimate choices of absolute indices would confirm the results. In other words, while generally robust to our specific choices of four inequality indices, *the trends we found were not guaranteed to be robust to every possible choice of absolute inequality index*. Thus, absolute convergence situations are not a priori robust to the choice of inequality indices, which leads us to our other significant contribution in the paper. In order to test whether our convergence/divergence analyses are *robust* to any choice of absolute inequality index, we resort to absolute Lorenz curves (Moyes, 1987). If two absolute Lorenz curves do not cross each other, then robust absolute convergence/divergence can be concluded; whereas if two absolute Lorenz curves intersect each other at least once, no robust absolute convergence/divergence conclusion can be made.[8] For each of the selected development indicators, we estimate the absolute Lorenz curves for all years and compare them with each other. Our robustness assessments show that for many development indicators most pair-wise year comparisons exhibit absolute Lorenz curve crossings and thus robust absolute convergence/divergence could not be concluded.

In an effort to interpret our empirical results, we also contribute methodologically to the assessment of convergence with bounded variables by providing valuable interpretations to absolute Lorenz curve comparisons, enabling us to identify the necessary and sufficient distributional changes that ensure egalitarian progress consistently. In addition to discussing how maximum inequality levels depend on mean attainment of bounded variables, we reach the fundamental conclusion whereby *robust inequality comparisons with bounded variables always favour global improvements (i.e. increases in the mean) that take place piece-meal, but spread evenly across several countries at the same time, over global improvements that fully move one country at a time from 0 to 1 in the indicator, while leaving other countries "behind"*.

The rest of the paper proceeds as follows. Section 2 provides three key reasons why we prefer an absolute approach, over a relative approach, for assessing convergence/divergence with bounded development indicators. Section 3 provides assessments of absolute or "sigma" convergence for various bounded development indicators using a battery of absolute inequality measures. Section

[7] Lambert and Zheng (2011) showed that only absolute inequality assessments can guarantee that inequality comparisons with bounded variables are consistent. Their work clearly suggests that, unless there are pressing reasons to choose attainments over their shortfall counterparts (or vice versa), only absolute convergence analyses can be deemed appropriate for bounded variables.

[8] In relation to the consistent robustness concern, Lambert and Zheng (2011) showed that, only in the case of inequality comparisons based on absolute indices, a distribution across attainments is unambiguously less unequal than another distribution of attainments if and only if the corresponding former distribution across shortfalls is unambiguously less unequal than the corresponding latter distribution across shortfalls.

4 provides a theoretical interpretation of our empirical results by way of a characterization of egalitarian progress in the context of bounded variables and absolute Lorenz curves. Section 5 then probes whether these convergence assessments are robust to alternative choices of plausible indices, using absolute Lorenz curves. Finally, section 6 provides concluding remarks.

2 What is the appropriate approach for measuring inequality for bounded variables?

Most non-monetary development indicators are bounded from above, meaning that it is not feasible for a country to reflect further improvements beyond strict maximum possible attainments in these indicators. Examples of these indicators abound, including *inter alia* mortality rates, literacy rates, enrolment rates at different educational levels, and percentages of people having access to various facilities. Once everyone in a country becomes literate or all children enrol in schools, then the country is considered to have achieved the strict maximum possible attainment in the relevant indicator.

There is also a second type of indicators which may not have strict upper bounds, but for a reasonably long time period these indicators may not be able to surpass a particular feasible upper bound. Examples include life expectancy at birth, fertility rate and mean years of schooling. The United Nations Development Program, for example, sets the upper bounds of country level life expectancy at birth at 85 years, and that of mean years of schooling at 15 years. These bounds are kept unchanged while assessing inter-temporal progress.

For simplicity, we assume that the attainment of a country i at any time period t for a hypothetical indicator x lies between 0 and 1 such that $x_{it} \in [0,1]$.[9] For many indicators, such as literacy rates or access rates to basic facilities, a score of 0 signifies the worst possible attainment and a score of 1 denotes the best possible attainment; whereas for other indicators, such as mortality rates, a score of 0 stands for the best possible attainment and a score of 1 reflects the worst possible attainment. The score between these two bounds is either monotonically increasing or decreasing with the level of attainment. We denote the distribution of attainments of a population of N countries at time period t by $X_t \coloneqq (x_{1t}, x_{2t}, \dots, x_{Nt})$ and the mean of the distribution at period t by: $\mu_X^t \equiv \frac{1}{N}\sum_{i=1}^{N} x_{it}$. The set of all the possible distributions *sharing the same mean value* μ_X^t is denoted by $\mathcal{X}_\mu$.

In order to assess whether the country-attainments have converged over time, we examine whether the inequality between them has decreased over time. We define an inequality index for bounded variables to be a mapping from the distribution of the bounded variable to the non-negative real line: $I(X): [0,1]^N \to \mathbb{R}_+$. The method that we employ for assessing inequality across the distribution of bounded variables is crucial in our study. There are two normative

[9] This is an assumption without loss of generality. The upper bound may be set at any positive value instead of 1. In this case, meaningful convergence analysis for any indicator can be conducted as long as the upper bound is kept unchanged over time for that indicator.

perspectives of reflecting inequality: the *relative perspective* and the *absolute perspective*.[10] The *relative perspective* requires the level of inequality to remain unaffected when all attainments are changed by the same proportion. On the other hand, the *absolute perspective* requires the level of inequality to remain unaltered when all attainments are changed by the same amount. There is another way of looking at these two perspectives. The relative perspective would conclude an unambiguous reduction in cross-country inequality as long as every poorer country reflects higher relative improvement (or growth) from its initial attainment; whereas the absolute perspective would conclude an unambiguous reduction in cross-country inequality as long as every poorer country reflects higher absolute improvement. We revisit these aspects in further detail in Section 4.[11] Looking at inequality from these two perspectives may provide very different snapshots as well as conflicting trends. In order to assess inequality involving bounded variables, we prefer the absolute perspective over its relative counterpart for three main reasons. Of these three reasons, in the first two we argue why it may not be ideal to use a relative perspective; whereas in the third reason we argue why we *should* use an absolute perspective.

The first reason for not using a relative perspective for assessing convergence with bounded variables stems from the normative argument that *no one in the development process should be left behind*. The relative perspective not only requires that inequality remains unchanged due to a proportional change in every country's attainment, but also ensures, as discussed earlier, that if initially poor-performing countries register slightly larger proportional improvements in their performance than the initially well-performing countries, inequality should fall. This argument goes in accord with the idea of (unconditional) beta-convergence frequently used in studying convergence in per capita GDP measures, whereby it is sufficient for the initially poor-performing countries to grow faster in order to ensure relative convergence immediately.[12] However, when an indicator is bounded, it is not feasible for the initially well-performing countries to continuously improve their performance at an equal or faster rate than the initially poor-performing countries, leading to a high likelihood of finding cross-country convergence.

In order to illustrate our point, we show how inequality in internet access rates across countries has evolved across three time periods (1996, 2000, and 2005) using the relative perspective in Figure 1. The non-intersecting traditional Lorenz curves show that inequality in internet access rates has unambiguously improved between 1996 and 2000 and then again between 2000 and 2005. However the results in Figure 1 provide a misleading picture. In 1996, 72.1% of countries had less than one percent of population who used the internet in the last 12 months, whereas 7.9% of countries had more than five percent of population who used the internet in the last 12

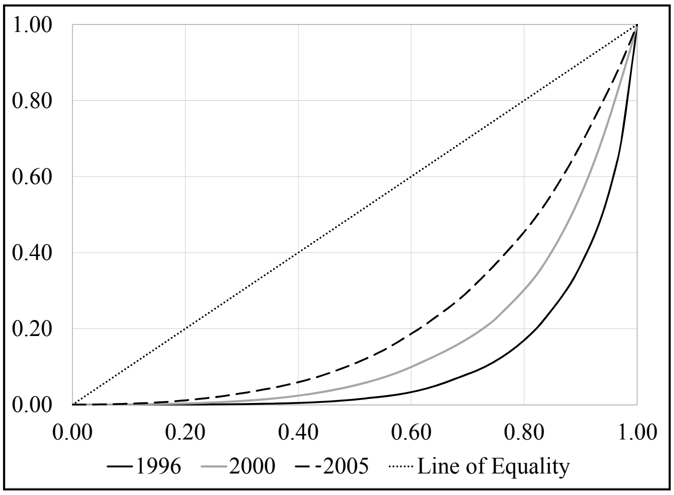
[10] An intermediate perspective has also been studied in the academic literature. See Zoli (2009) for a unified presentation of the three perspectives.

[11] The application of a relative perspective to evaluate convergence is similar to the concept of *beta convergence* assessment; whereas that of absolute perspective is essentially the so-called *sigma convergence* assessment (Barro and Sala-i-Martin, 2003).

[12] In a relatively short period, beta-convergence is necessary but insufficient to secure sigma-convergence. However, if beta-convergence continues uninterruptedly then it eventually leads to sigma-convergence. New disturbances may offset the process (Barro and Sala-i-Martin, 2003; p. 462). Nissanov and Silber (2009) elucidate how beta-convergence can be decomposed into a sigma-convergence component and two mobility components.

months. The average proportion of internet users in the 72.1% of countries that had less than 1% internet users in 1996 increased to 3% in 2000, and nearly another fourfold to 11.6% in 2005. However the average proportion of internet users in the 7.9% of countries that had more than five percent of internet users in 1996 increased to 39.6% in 2000, but to 71.1% (less than two-fold increase because it cannot increase more than 2.5 times due to the strict upper bound) in 2005. Despite a nearly four-fold increase between 2000 and 2005, the initially poor-performing countries in this indicator were left behind compared to the initially well-performing countries between 1996 and 2005; yet the analysis in Figure 1 concluded large satisfactory convergence across countries in 2000 and 2005 over 1996.[13]

Figure 1: Change in Inequality in Internet Access Rates across Countries



Our second reason for not using a relative perspective with bounded variables is that the relative perspective looks at inequality as a "pie-dividing" problem. In a pie-dividing problem, inequality is maximized when one person has ownership of the entire pie; whereas the rest is deprived of having any share of the pie. This scenario relies on three implicit assumptions: first, it is assumed that regressive transfers (from poorer to richer) are possible until one person owns everything; second, the pie is directly transferable between any two persons; third, the size of the pie does not matter, only its distribution does.

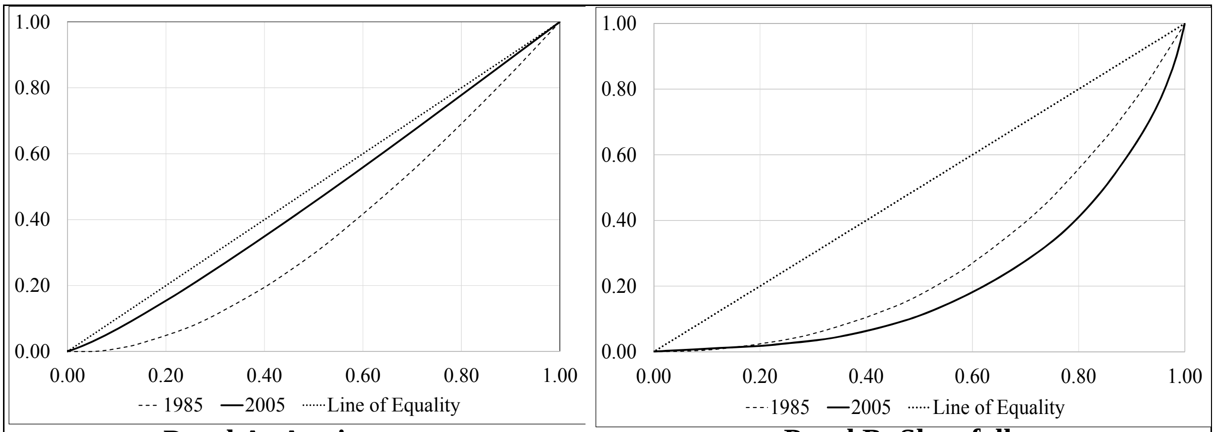
However, in the case of bounded variables all three assumptions can be deemed invalid. First, for a bounded variable (with a restriction that no person can have more than a certain amount of the pie), it is not possible for one person to have ownership of the entire pie. Second, the pie is not transferable, and in fact inequality cannot be presented as a pie-dividing problem at all. For example, if we treat a country as a person, then improvement in literacy rate of a country does not necessarily need to come at the cost of reducing literacy in another country. Third, focusing only on the distribution irrespective of the "size" creates some practical problems. For example,

[13] Grosse et al. (2008) provide similar reasons for preferring an absolute approach for non-income indicators due to their ordinal nature, but not necessarily referring to cardinal boundedness. In fact, measuring inequality across ordinal variables may require a completely different technique (see Allison and Foster, 2004).

in 1990, the average internet usage rate across all countries was 0.03%, where 87.9% of countries did not have any internet user at all, 7.3% of countries had less than 0.14% internet users and less than 5% of countries had between 0.3% and 0.8% internet users. The world was not so unequal in terms of the internet usage. In 2010, the average internet usage rate increased to 38.4% and every country had at least some internet users, yet a quarter of countries had on average less than 6% internet users while a quarter of countries had on average more than 75% internet users. A relative perspective would argue that the world had become much more equal in terms of internet usage in 2010 than the world was in 1990, which is hard to justify.

The first two reasons provide arguments for why we should not use a relative perspective while conducting convergence analysis for bounded indicators, but do not argue why we should necessarily use an absolute perspective. The third reason, however, argues why we should use an absolute perspective. The reason is that we want the assessment of inequality to be *consistent*. Many of the bounded development indicators can be presented in their complementary *shortfall* form. For example, instead of looking at what fraction of children has been immunized, an international policy advocate may want to emphasize the fraction of children not yet immunized in order to impose pressure on governments. In Figure 2, we present the traditional Lorenz curves reflecting the change between 1985 and 2005 in inequality in BCG immunization rates across countries from a relative perspective. In Panel A of the figure we present the change in inequality across attainments or immunization rates and in Panel B we present the change in inequality across shortfalls or non-immunization rates. Clearly, the dashed line in Panel A lies to the right of the solid line, implying that inequality in BCG immunization rates across countries has decreased between 1985 and 2005. The dashed line in Panel B however lies to the left of the solid line, meaning that inequality in BCG non-immunization rates across countries has increased between 1985 and 2005.[14]

Figure 2: Inconsistent Change in Inequality in BCG Immunization Rate across Countries



Panel A: Attainment **Panel B: Shortfall**

[14] We have also checked for the existence of unconditional beta-convergence analysis for 122 countries. The use of the BCG immunization rates (attainment) show statistically significant beta-convergence; whereas the use of the BCG non-immunization rates (short-falls) show statistically significant beta-divergence.

While there can be *ad hoc* reasons to choose an attainment definition of the variable over a shortfall (e.g. see Kenny, 2004), the choice is otherwise intrinsically arbitrary. However this "dual nature" of the bounded variable poses a challenge for the most basic form of convergence analysis, namely tracking an inequality index across countries over time. Should the inequality comparison reverse only because inequality is assessed across short-falls rather than attainments? It is hard to find a justification in support of this phenomenon. In order to avoid such inconsistencies, a consistent measure of inequality should be used.[15] A consistent inequality measure ensures that the inequality comparison is not reversed just because the country performance is gauged in terms of shortfalls rather than attainments.

Let us define the complementary *shortfall* counterpart of a development indicator as: $y_{it} \equiv 1 - xit$ and denote the distribution of shortfalls of the population of N countries at time period t by $Y_t \coloneqq (y_{1t}, y_{2t}, \dots, y_{Nt})$. If an inequality measure is consistent, then $I(X_{t_1}) < I(X_{t_2})$ if and only if $I(Y_{t_1}) < I(Y_{t_2})$. Micklewrite and Stewart (1999) were the first to note that only an absolute inequality index could be consistent according to this definition. Then Lambert and Zheng (2011) formally identified the precise classes of absolute inequality measures guaranteeing the consistency of inequality comparisons based on bounded variables. Thus, for instance, the popular coefficient of variation (e.g. used by Neumayer, 2003; and Kenny, 2004) does not guarantee consistent inequality comparisons, since it is a relative inequality measure.

3 Have the bounded development indicators converged? An absolute perspective

In this section we explore whether several bounded non-monetary development indicators have converged or diverged over time from an absolute perspective. We classify these indicators into three categories: health indicators, education indicators and access rate indicators. Among health indicators, we look at various child immunization rates, child mortality rates, life expectancy at birth and fertility rate; among education indicators, we look at child school enrolment rates, youth literacy rates, mean years of schooling and expected years of schooling; and among access indicators, we look at access rate to improved sanitation facilities and improved drinking water sources, and the internet usage rate. Detailed definitions of these indicators, together with the number of countries and time spells covered can be found in Appendix I. For each indicator, we only take into account those countries for which we were able to secure data for all considered years. In this paper, we consider each country as a separate entity and are interested in their convergence. Therefore, we choose not to weight countries by their population sizes.

We assess convergence using certain absolute inequality measures. A plethora of absolute measures have been proposed in the inequality measurement literature. The typical absolute

[15] In addition to Micklewrite and Stewart (1999), a number of studies in the health economics literature have observed this inconsistency and proposed using a consistent measure of inequality. See, for example, Erreygers (2009), Lambert and Zheng (2011), Aristondo and Lasso de la Vega (2012), Chakravarty et al. (2013), Kjellson et al. (2015), Silber (2015). For an example applying an absolute measure to assess inequality in the poverty counting framework, see Seth and Alkire (2014).

inequality measure used for assessing sigma-convergence is the *standard deviation* (Barro and Sala-i-Martin, 2003). Given that inequality measures aim to provide complete rankings and different inequality measures may disagree with each other, we use four different absolute inequality measures in order to assess sigma-convergence. The first one is the variance (I_V), which is the decomposable version of the standard deviation, and the second is the absolute Gini coefficient (I_{AG}).[16] The other two measures (I_{CCD1} and I_{CCD2}) are from the family of absolute inequality measures proposed by Chakravarty *et al*. (2013).[17]

$$I_V(X_t) = \frac{1}{N}\sum_{i=1}^{N}(x_{it} - \mu_X^t)^2, \tag{1}$$

$$I_{AG}(X_t) = \frac{1}{2N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left|x_{ij} - x_{jt}\right|, \tag{2}$$

$$I_{CCD1}(X_t) = \ln\left[\frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\exp\left(\left|x_{ij} - x_{jt}\right|\right)\right], \tag{3}$$

$$I_{CCD2}(X_t) = 100 \times \ln\left[\frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\exp\left(0.01 \times \left|x_{ij} - x_{jt}\right|\right)\right]. \tag{4}$$

Absolute inequality indices satisfy certain crucial properties; including being *translation invariant* but not scale invariant. Formally, by translation invariant we mean that: $I(X_t) = I(Y_t)$, where $Y_t = X_t + \delta_N$ and δ_N is an N-dimensional vector of (positive or negative) constants: $\delta_N = (\delta, \delta, \dots, \delta)$, such that $y_{it}, x_{it} \in [0,1]$ for all i respecting the bounds of the variable. By scale invariant we mean that: $I(X_t) = I(Y_t)$, where $Y_t = \lambda X_t$ and $\lambda > 0$ such that $y_{it}, x_{it} \in [0,1]$ for all i respecting the bounds of the variable. In addition, absolute inequality indices fulfil *symmetry*, whereby a permutation of x_{it} across countries should not change the value of the inequality index; and *population principle*, whereby a replication of each country by the same factor should not affect the inequality index. Finally, if we define a *regressive transfer* as any transfer of a positive amount γ from a worse-off country i to a better-off country j, then inequality indices should fulfil regressive-transfer sensitivity whereby $I\left(X_{t_2}\right) > I\left(X_{t_1}\right)$ if X_{t_2} is obtained from X_{t_1} through a regressive transfer (or a sequence thereof).[18]

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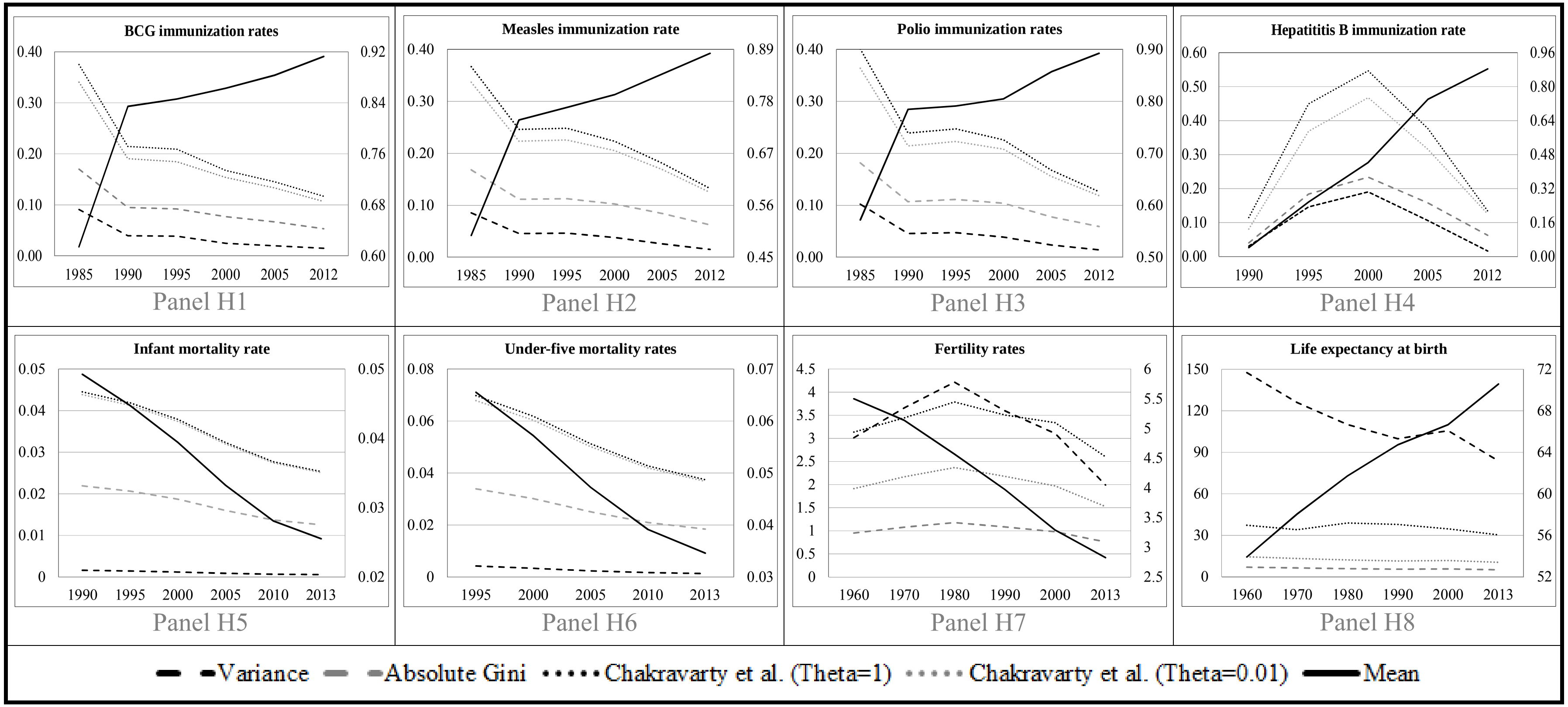
[16] The variance and the absolute Gini coefficient fall in the two families of consistent absolute inequality indices proposed by Lambert and Zheng (2011, p. 216). The family of indices that include the variance is $I_{RI}(X_t) = \frac{1}{N}\sum_{i=1}^{N} u(x_{it} - \mu_X^t)$ where u is strictly convex, twice differentiable and $u(z) = u(-z)\ \forall z \neq 0$; whereas the family of indices that include the absolute Gini coefficient is $I_{RD}(X_t) = \frac{1}{N}\sum_{i=1}^{N} \omega(p_{it})(x_{it} - \mu_X^t)$, where $p_{it} = (2i-1)/2N$ is the rank of x_{it} and $\omega(p_{it})$ a strictly increasing function such that $\omega(1 - p_{it}) = -\omega(p_{it})$.

[17] The general class of indices proposed by Chakravarty *et al*. (2013) is $I(X_t; \theta) = \frac{1}{\theta}\ln\left[\frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\exp\left(\theta\left|x_{it} - x_{jt}\right|\right)\right]; \theta > 0$.

[18] As discussed in the previous section, the bounded indicators that we analyze in this paper are not directly transferable and therefore the applicability of the regressive transfer should be seen as if there were two observed periodic distributions X_{t_1} and X_{t_2}, such that, in principle, X_{t_2} could be obtained from X_{t_1} through a sequence of regressive transfers, and then X_{t_2} would be deemed less unequal than X_{t_1}.

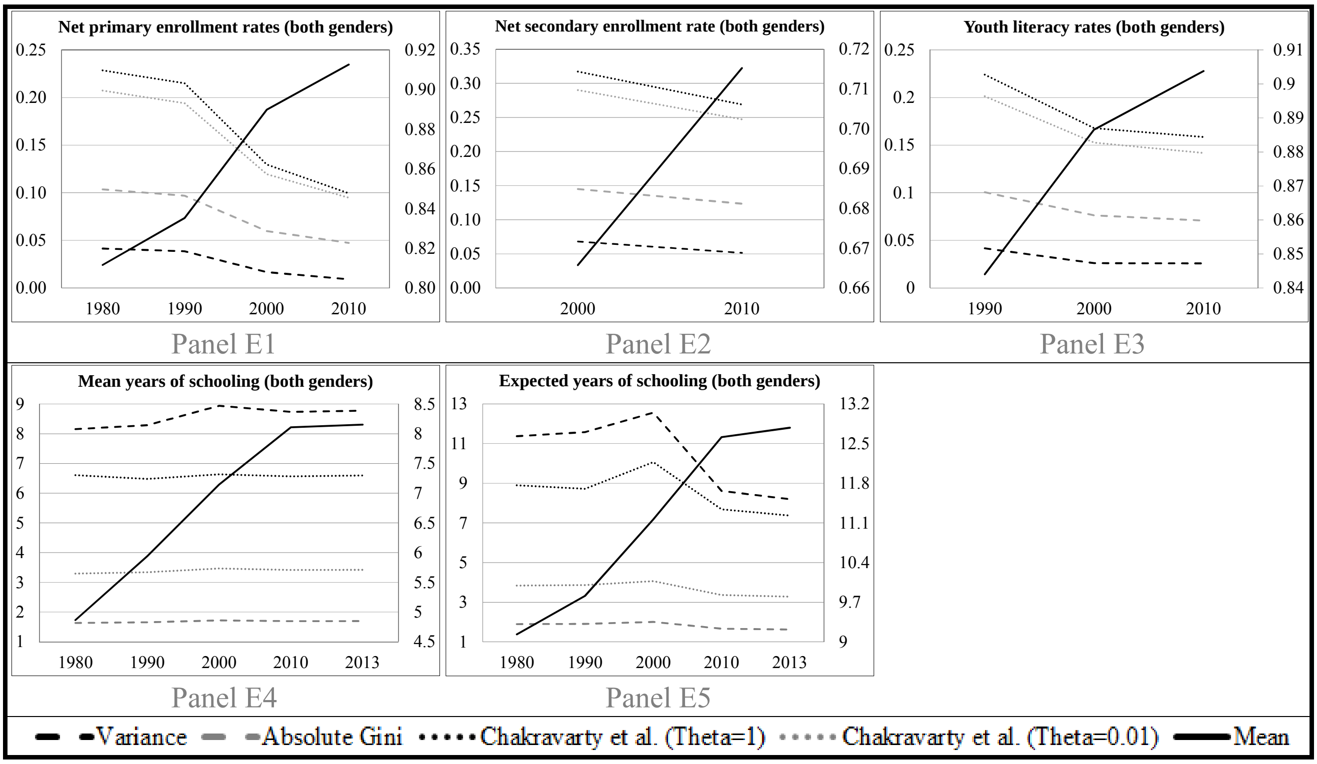
Figure 3: Convergence or Divergence across Health Indicators by Different Absolute Inequality Measures



In each diagram, the left vertical axis measures absolute inequality across countries and the right vertical axis measures mean attainment across countries.

In Figure 3-Figure 5, we show how the indicators under consideration have converged or diverged over time, presenting their changes in means (right vertical axes) and the levels of absolute inequality (left vertical axes) using four absolute inequality indices in Equations (1)-(4). We observe absolute convergence throughout for most of the indicators, with a few exceptions. Let us first look at the health indicators presented in Figure 3. In the first row (Panels H1-H4), we present the changes in mean and cross-country convergence in four immunization rates: BCG, Measles, Polio and Hepatitis-B. The first three immunization indicators show similar mean improvement patterns between 1985 and 2012 as well as similar type of cross-country convergence. There were drastic improvements in means and fast convergence between 1985 and 1990. Between 1990 and 2012, the improvements in means and convergence have been slower but steady. For BCG immunization rate though we observe the increase in mean performance was accompanied by absolute convergence but it was not monotonic across all inequality measures. Between 1990 and 1995, the variance reported a mild temporary increase.

Figure 4: Convergence or Divergence across Education Indicators by Different Absolute Inequality Measures



In each diagram, the left vertical axis measures absolute inequality across countries and the right vertical axis measures mean attainment across countries.

The convergence pattern is different for Hepatitis-B immunization rates. The mean attainment was very low (4%) to begin with, in 1990, and then increased during the accounting period reaching 81% in 2012. Countries initially diverged due to non-uniform progress until 2000, but then after the mean attainment reached a level of 45%, countries started converging. This should be expected because countries that made big strides in the 1990s could not improve much faster due to the strict upper bound. For the Hepatitis-B indicator, we observe an improvement pattern resembling a 'Kuznets curve'. This is an issue we revisit in the next section.

The next row in Figure 3 presents the changes in means and cross-country convergence in infant mortality rate, under-5 mortality rate, fertility rate and life expectancy rate.[19] Like the first three immunization indicators, both types of child mortality rate showed steady improvements between 1990 and 2013 as well as steady absolute convergence. Fertility rate however imitate the convergence pattern of Hepatitis-B immunization rate. The mean fertility rate fell steadily from nearly 5.5 births per woman in 1960 to less than three births per woman in 2013. This steady reduction between 1960 and 1980 was accompanied by cross-country divergence, but since 1980, the reduction in mean was accompanied by reduction in absolute equality and thus absolute convergence. The final health indicator that we analyze is life expectancy at birth. The mean attainment in this indicator had gradually improved from nearly 54 years in 1960 to around 70 years in 2013, but the cross-country convergence was slightly wobbling between 1970 and 2000. Also the absolute indices clearly disagreed with each other on convergence. According to CCD ($\theta = 1$) index, divergence occurred between 1970 and 1980; whereas there was a slight divergence between 1990 and 2000 according to the variance. In each period, these two indices disagreed with other indices.

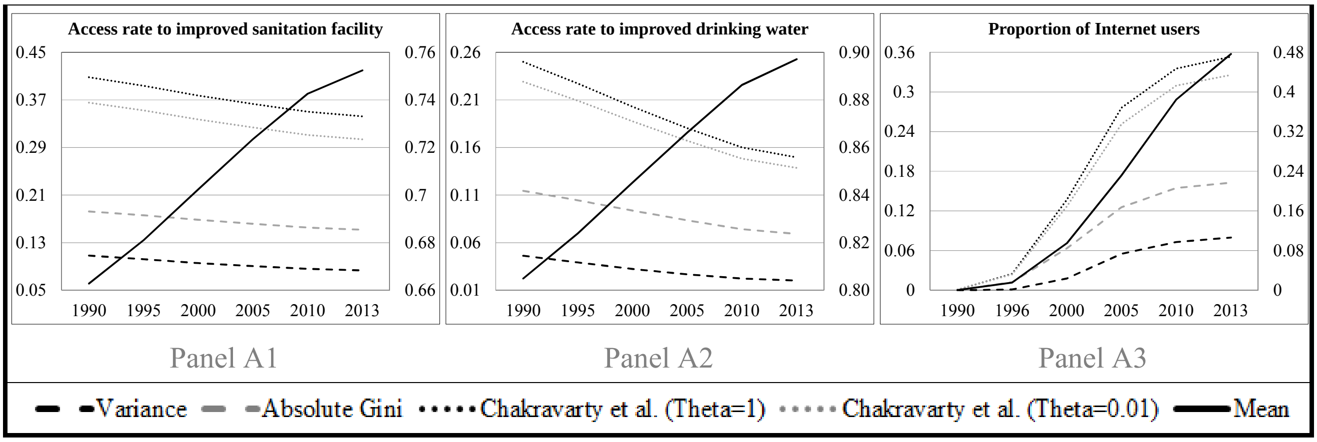
In Figure 4, we analyse five education indicators: net primary enrolment rate, net secondary enrolment rate, youth literacy rate, mean years of schooling, and expected years of schooling. The first three of these five indicators have a strictly upper bound. We present the change in mean attainment and convergence/divergence of these indicators separately for males and females in Appendix II. All five indicators steadily improved on average during the corresponding study periods but cross-country convergence was not steady for all of them. The first three indicators with strict upper bound converged throughout the study periods, but the last two indicators did not. In fact, both mean years of schooling and expected years of schooling diverged between 1980 and 2000 despite drastic improvement in mean attainments (indices disagreed though about convergence/divergence between 1980 and 1990). Since 2000, expected years of schooling reflected cross-country convergence, while mean years of schooling did not. Looking at the convergence pattern by gender in Appendix II, we find that the cross-country convergences were at least as fast (i.e. equal or steeper reduction in absolute inequality) among females as among males in both net primary and secondary enrolment rates as well as youth literacy rate. This positive finding was slightly marred by the cross-country divergence in female mean years of schooling.

In Figure 5, we analyse three access-rate indicators: access to improved sanitation facility, access to improved drinking water, and internet usage rate. Although all three indicators improved steadily on average, only the first two also converged over time. Our finding on absolute

[19] Following convention, mortality rates are presented as deaths per thousands, thereby ranging from 0 to 1.

divergence in internet usage rates clearly contrasts with the finding from the relative perspective presented in Figure 1 earlier, which concluded convergence in cross-country internet usage rates between 1990 and 2013. Note that the divergence pattern that we observe for internet usage rate resembles the divergence pattern of the Hepatitis-B indicator between 1990 and 2000 until the average rose to around 0.45 (Panel H4 of Figure 3). The Hepatitis-B indicator showed steady convergence after 2000. Given that the mean internet usage rate has reached almost 50% by the second decade of the XXI Century, absolute convergence may result in subsequent years.

Figure 5: Convergence or Divergence across Access Rates Indicators by Different Absolute Inequality Measures



In each diagram, the left vertical axis measures absolute inequality across countries and the right vertical axis measures mean attainment across countries.

Using four different absolute inequality indices, we found absolute convergence experienced by most of the considered indicators. We also found that the four inequality indices were in accord for most year-to-year comparisons, but not all. In other words, the changes were not robust across all indices. For instance, between 1990 and 1995, the variance indicated a mild divergence across countries in BCG immunization rate while other absolute inequality measures reported convergence. Similarly, for Hepatitis B immunization rate, the level of absolute inequality was lower in 2005 than in 1995 according to the variance, but was higher for the rest of the indices.

Even though these four indices agreed in most of the comparisons, we cannot be sure that other equally appropriate absolute indices would also agree without further testing. With relative inequality indices it is usually possible to predict with little error whether a comparison is robust to any index choice by deploying three or four indices that emphasize inequalities in different parts of the variable's distribution (Shorrocks and Slottje, 2002). Unfortunately, Lambert and Zheng (2011; theorem 6, p. 217) showed that consistent absolute inequality indices are insensitive to any differential effect of progressive or regressive transfers over different parts of the distribution. Therefore there is no smart choice of absolute inequality indices enabling us to predict a robust comparison. But trying out every conceivable suitable inequality index is also impractical. Hence, there is the need for submitting these inequality comparisons (used to assess cross-country convergence) to alternative robustness tests. In the next section, we explore how we can probe the robustness of convergence assessments to different choices of absolute inequality indices.

4 Robustness of absolute convergence and egalitarian progress

In order to test whether the absolute convergences are robust to different choices of absolute inequality indices, we use the absolute Lorenz curves introduced by Moyes (1987). We outline the concept using some additional notation. Let us denote the *ordered distribution* corresponding to X_t by X_t^*, whose i^{th} element is denoted by x_{it}^* such that $x_{it}^* > x_{i't}^*$ whenever $i > i'$. That is, in X_t^*, the elements of X_t have been reordered in ascending order. Let us denote the percentage of people with achievements no larger than x_{it}^* by p_{it}. Thus, by construction, $p_{it} > p_{i't}$ whenever $i > i'$ and $p_{Nt} = 1$. Using sums, the absolute Lorenz curve may be defined as:

$$L(X_t; p_{kt}) \equiv \frac{1}{N}\sum_{i=1}^{k}(x_{it}^* - \mu_X^t)\,; k = 1,2, \dots, N. \tag{5}$$

For the convenience of discussion we present and use the continuous version of the absolute Lorenz curve definition. Let us define the percentile of population by p and the quantile function of X_t by $Q_X^t(p)$. Then the absolute Lorenz curve for a given percentile: $L(X_t; p)$ is a mapping from X_t onto the non-positive segment of the real line: $L(X_t; p): X_t \to \mathbb{R}_-$ and can be defined by:

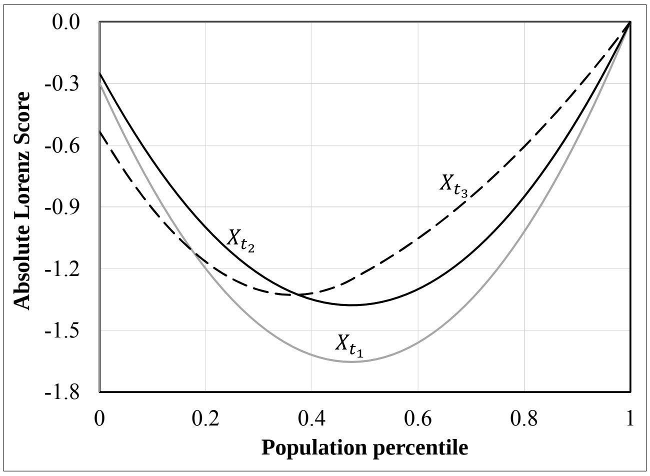
$$L(X_t; p) \equiv \int_0^p (Q_X^t(\wp) - \mu_X^t)d\wp\,; p \in [0,1]. \tag{6}$$

How is the absolute Lorenz curve useful for our purpose? Moyes (1987, proposition 3.1) showed that whenever the absolute Lorenz curve of a distribution X_{t_1} is never above that of another distribution X_{t_2}, and at least once strictly below, then all absolute inequality indices satisfying the four properties defined in the previous section (symmetry, population principle, regressive-transfer sensitivity and translation invariance) will deem X_{t_1} more unequal than X_{t_2}. Technically, for any absolute inequality measure I satisfying the four above properties, $I(X_{t_1}) > I(X_{t_2})$ if and only if $L(X_{t_1}, p) \leq L(X_{t_2}, p)$ for all $p \in [0,1]$ and $L(X_{t_1}, p) < L(X_{t_2}, p)$ for some p. This is a strong result because if the absolute Lorenz curve of a distribution of achievements for an indicator at a particular time period dominates the absolute Lorenz curve of the distribution of achievements in the following period, then all absolute inequality measures satisfying the four properties would conclude convergence in that indicator.

This result is also quite powerful in terms of the consistency requirement that we discussed in section 2: Lambert and Zheng (2011) showed that whenever absolute Lorenz dominance of X_{t_2} over X_{t_1} occurs, there is also absolute Lorenz dominance of Y_{t_2} over Y_{t_1}. This means that *in* *order to test for robustness we can choose either attainment representations or shortfall representations of the bounded variables.*

It is worth mentioning some details pertaining to the shape of the absolute Lorenz curve. First, the value of the absolute Lorenz at $p = 1$ is zero, or $L(X_t, 1) = 0$, because the mean-centred sum of all attainments is equal to zero. Second, absolute Lorenz curves are convex which is easy to note as X_t has been reordered in ascending order. Third, let us define a percentile $p' \in [0,1]$ such that $Q_X^t(p') = \mu_X^t$ for a continuous distribution.[20] Then an absolute Lorenz curve is downward sloping for all $p \in [0, p')$, reaches its minimum value (i.e. its maximum absolute value) at p' and is upward sloping for all $p \in (p', 1]$.

Figure 6: Absolute Lorenz Curves and the Ambiguity of Absolute Convergence



In Figure 6, we present the absolute Lorenz curves for three hypothetical distributions of achievements at three different time periods: X_{t_1} (solid gray line), X_{t_2} (solid black line) and X_{t_3} (dashed line). Clearly, the absolute Lorenz curve for period t_2 lies everywhere above the absolute Lorenz curve for period t_1. Absolute inequality has unambiguously decreased from period t_1 to t_2, i.e. absolute convergence has occurred. However, in period t_3 the absolute Lorenz curve intersects both the absolute Lorenz curves in periods t_1 and t_2. Hence we cannot claim that convergence has unambiguously occurred between t_1 and t_3, or between t_2 and t_3. In other words, while some absolute inequality indices would conclude convergence, we cannot rule out the possibility that some other indices would disagree. Now we discuss some policy- and welfare-relevant interpretations of the shapes of the absolute Lorenz curves.

Interpreting Egalitarian Progress with Absolute Lorenz curves

The shapes of the absolute Lorenz curves presented in Figure 6 convey more useful information than just a test for the robustness of inequality comparisons. We have already observed that when

[20] For a discrete distribution, the percentile $p' \in [0,1]$ may be so defined that $Q_X^t(p') < \mu_X^t$ and $Q_X^t(1 - p') \geq \mu_X^t$.

an absolute Lorenz curve lies further away from the horizontal axis than another absolute Lorenz curve for every percentile, then the level of absolute inequality is higher for the latter Lorenz curve. However, what shape does a Lorenz curve take when the level of inequality is largest for a given level of mean achievement? Furthermore, given that our paper is about exploring the convergence of cross-country progress, can we infer what type of changes within the cross-country distribution ensures that the progress is egalitarian from an absolute perspective?

Let us first characterize the shape of the absolute Lorenz curve with bounded variables when the level of maximum inequality is reached for a given level of mean attainment. We denote the set of all possible distribution of attainments bounded between 0 and 1 with the same level of mean attainments μ_X^t by $\mathcal{X}_\mu$ and the partial derivative of $L(X_t, p)$ at percentile p by $L_p(X_t, p)$. The situation of maximum inequality *for a given value of the mean* is then characterized by the following theorem:

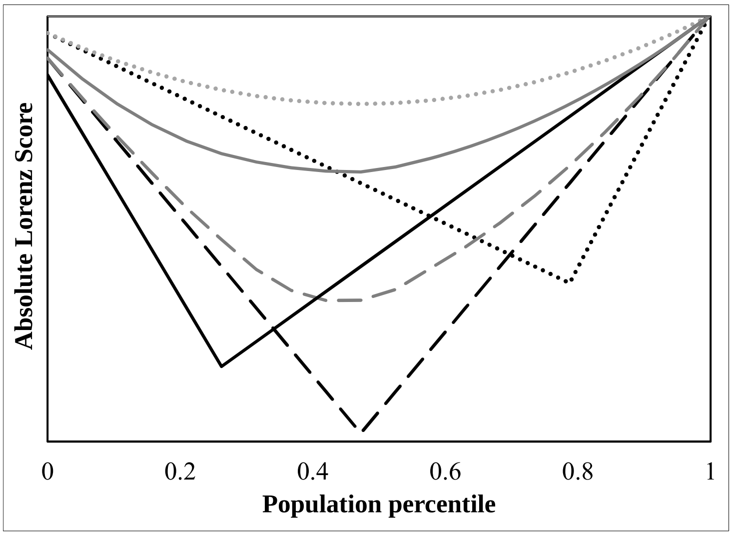
Theorem 1: The following statements are equivalent.

(a) $X_t \in \mathcal{X}_\mu$ comprises a proportion q_t of countries for which $x_{it} = 1$, and a proportion $(1 - q_t)$ of countries for which $x_{it} = 0$.
(b) $L(X_t, p) \leq L(X_{t'}, p)$ for all $p \in [0,1]$ and for all $X_{t'} \in \mathcal{X}_\mu$.
(c) $L_p(X_t, p) = -\mu_X^t$ for all $p \in [0,1 - q_t)$ and $L_p(X_t, p) = 1 - \mu_X^t$ for all $p \in (1 - q_t, 1]$.
(d) $L(X_t, 1 - q_t) = -\mu_X^t(1 - \mu_X^t)$

Proof: See Appendix III. ■

Theorem 1 characterizes X_t as the distribution exhibiting the highest level of absolute inequality possible across all distributions in $\mathcal{X}_\mu$. Distribution X_t comprises a proportion q_t of countries with $x_{it} = 1$ and the remainder with $x_{it} = 0$. Hence the shape of the $L(X_t, p)$ is an inverted triangle with lower vertex having coordinates $(1 - q_t, -\mu_X^t(1 - \mu_X^t))$. Any other absolute Lorenz curve based on a distribution *with the same mean* will be "contained" between $L(X_t, p)$ and the horizontal axis (shown by part (b) of Theorem 1). Let us look at the illustrations in Figure 7, where the solid (both black and gray) absolute Lorenz curves correspond to distributions with a mean of 0.2, the dashed (both black and gray) absolute Lorenz curves correspond to distributions with a mean of 0.5, and the dotted (both black and gray) absolute Lorenz curves belong to distributions with a mean of 0.7. The inverted triangular absolute Lorenz curves (black) represent the distributions featuring maximum inequality for each of the three mean values.

Figure 7: Absolute Lorenz curves of three hypothetical distributions of bounded variables



A key feature of absolute Lorenz curves is precisely the dependence of the lowest possible value of the absolute Lorenz curve on μ_X^t. When we allow the mean to change, then the lowest possible value is minimized (i.e. maximized in absolute value) when $\mu_X^t = 0.5$, as depicted by the black dashed Lorenz curve in Figure 7, which has potentially important empirical implications. If the change in inequality is robust over time as presented in the hypothetical example in Figure 7 (gray absolute Lorenz curves), then we would witness robust empirical "Kuznets curves" with bounded variables, whereby inequality roughly increases when the mean rises from $\mu_X^t = 0$ to $\mu_X^t = 0.5$, and then decreases as μ_X^t moves toward 1.[21] However, the movement of the mean from 0 to below 0.5 does not guarantee a continuous increase in inequality throughout the accounting period; while the movement of the mean from above 0.5 toward 1 does not guarantee a constant decrease in inequality throughout the accounting period either.

Let us now discuss what type of distributional changes can be robustly ordered with an absolute Lorenz egalitarian criterion, and what type of distributional changes cannot be robustly ordered. We present the second situation first. To begin with consider a situation where a distribution X_1 is made of a $(1 - q_1)$ proportion of countries with $x_{i1} = 0$ and the remainder $q_1 < 1$ proportion of countries with $x_{i1} = 1$. This is the most unequal situation among all distributions with the same mean as X_1. Suppose a distribution X_2 is obtained from X_1 by improving the situation of country i' such that $x_{i'1} = 0$ but now $x_{i'2} = 1$, all else equal. Thus, in distribution X_2, there are $(1 - q_2)$ proportion of countries with $x_{i2} = 0$ and the remainder $q_2 > 0$ proportion of countries with $x_{i2} = 1$, such that $q_1 < q_2 < 1$. Intuitively, the attainment of country i' has been improved from 0 to 1 which has also increased the mean attainment, albeit leaving the rest of the countries with 0 attainment behind. We consider this situation as non-rankable because even though inequality between country i' and the q_1 countries has disappeared, it has come at the cost of

[21] Clearly, in our illustration as the mean increases from 0.2 to 0.5 the potential "scope" for more unequal distributions increases, and then when the mean increases further from 0.5 to 0.7, that "scope" decreases in turn.

leaving the rest behind, thereby widening inequality between country i' and the $1-q_2$ countries. Denoting the set of all possible distribution of attainments bounded between 0 and 1 by $\mathcal{X}$, the following theorem summarizes this result:

Theorem 2: For any $X_{t_1}, X_{t_2} \in \mathcal{X}$, if $0 < \mu_X^{t_1} < \mu_X^{t_2} < 1$, and $x_{it} = 0 \vee 1$ for all i and for $t = t_1$, t_2, then there exists some p, $p' \in [0,1]$ with $p \neq p'$ such that $L(X_{t_1}, p) < L(X_{t_2}, p)$ and $L(X_{t_1}, p') > L(X_{t_2}, p')$.

Proof: see Appendix III. ■

Theorem 2 thus states that the absolute Lorenz curves of distributions characterized by maximum inequality and different means always cross.[22]

We move now to characterizations of egalitarian progress with absolute Lorenz curves. Specifically we ask: under what conditions can progress in mean attainment be deemed robustly more egalitarian? Suppose the initial distribution is X_{t_1}, which changes to X_{t_2} over time. Since progress occurs between periods t_1 and t_2, we assume $\mu_{t_2} > \mu_{t_1}$. Let us denote the absolute change in every percentile by $s(p) = Q_X^{t_2}(p) - Q_X^{t_1}(p)$. Then $s \equiv \int_0^1 s(\wp)d\wp = \int_0^1 [Q_X^{t_2}(p) - Q_X^{t_1}(p)]d\wp = \mu_X^{t_2} - \mu_X^{t_1}$. That is, the average absolute change across percentiles is essentially the difference in means between two periods.[23] Using the formulation in Equation (6), the difference between the absolute Lorenz curves for two periods can be presented as:

$$L(X_{t_2}; p) - L(X_{t_1}; p) \equiv p\left[\frac{1}{p}\int_0^p s(\wp)d\wp - s\right] \equiv pD(p);\ \ p \in [0,1]. \quad (7)$$

What is $D(p)$ intuitively? Note that $\mu_X^{L,t}(p) = \left[\int_0^p Q_X^t(p)d\wp\right]/p$ is the *lower partial mean* for percentile p (using the terminology of Foster et al., 2013) or the mean attainment of the bottom p percent of countries, and so $s_L(p) = \left[\int_0^p s(\wp)d\wp\right]/p$ is the change in the lower partial means for the bottom p percent of countries.[24] Thus, $D(p)$ is the difference between the change in lower partial means among the bottom p percent of countries and the overall change in means across

[22] The consistency requirement itself also implies that, in some pair-wise comparisons of distributions with maximum inequality, we cannot claim that one distribution is more unequal. Imagine we have X_1 and X_2, both featuring maximum inequality, with $\mu_X^1 = 0.3$ and $\mu_X^2 = 0.7$. The consistency requirement demands that if we deem X_1 more unequal than X_2, then we should also declare Y_1 with $\mu_Y^1 = 0.7$ more unequal than Y_2 with $\mu_Y^2 = 0.3$. But this would not make any sense, since Y_1 is identical to X_2, and Y_2 is the same as X_1.

[23] Note that if we have a panel dataset, computing every $s(p)$ requires pairing the countries for two different periods who happen to be in the same percentile rank. Only in the absence of re-rankings we should expect these two countries to be one and the same.

[24] This statistic is linked to the Generalized Lorenz (GL) curve. At any percentile p, the height of the GL curve is the lower partial mean times the corresponding percentile p.

two periods. When can we say that the progress between two periods t_1 and t_2 has been egalitarian by an absolute approach? The following theorem provides the answer:

Theorem 3: For any $X_{t_1}, X_{t_2} \in \mathcal{X}$ and $\mu_X^{t_2} > \mu_X^{t_1}$, $L(X_{t_2}, p') > L(X_{t_1}, p') \forall p \in [0,1]$ with at least one inequality holding strictly if and only if $D(p) \geq 0\ \forall p \in [0,1]$ with at least one inequality holding strictly.

Proof: Straightforward by inspection of equation (7). ■

The intuition behind Theorem 3 is quite interesting. It states that in order to obtain an egalitarian progress in period t_2 over period t_1, we require the difference in equation (7) to be non-negative for all p and strictly positive for some p. Intuitively, the theorem states that in order to have absolute Lorenz dominance, the absolute change in the lower partial mean attainment of every bottom p percent of countries, $s_L(p)$, has to be at least as large as the overall absolute change in means s and strictly larger for at least one percentile.[25,26]

Theorems 2 and 3 together state that *robust inequality comparisons with bounded variables always favour global improvements (i.e. increases in the mean) that take place piece-meal, but spread evenly across several countries at the same time, over global improvements that fully move one country at a time from 0 to 1 in the indicator, while leaving other countries "behind".*

For example, imagine X_{t_1} featuring ten countries, of which five have 100% immunization rate and the rest have 0% immunization rate. Then in the next period, suppose there are two scenarios: (a) $X_{t_{2a}}$, where now six countries have 100% and the rest have 0%; and (b) $X_{t_{2b}}$, where five countries have 100% and the rest have 20% each. A robust and consistent comparison will not be able to rank X_{t_1} and $X_{t_{2a}}$ unanimously, due to Theorem 2. By contrast, $X_{t_{2b}}$ is robustly, and consistently, less unequal than X_{t_1} according to Theorem 3.

Strictly progressive absolute improvement

Note however that the restriction $D(p) \geq 0\ \forall p \in [0,1]$ does not necessarily imply that countries in lower percentiles had necessarily larger absolute progress than their counterparts at higher percentiles. We define a concept called *strictly progressive absolute improvement* which goes strongly with the idea of "not leaving anyone behind". We refer to an overall improvement or an

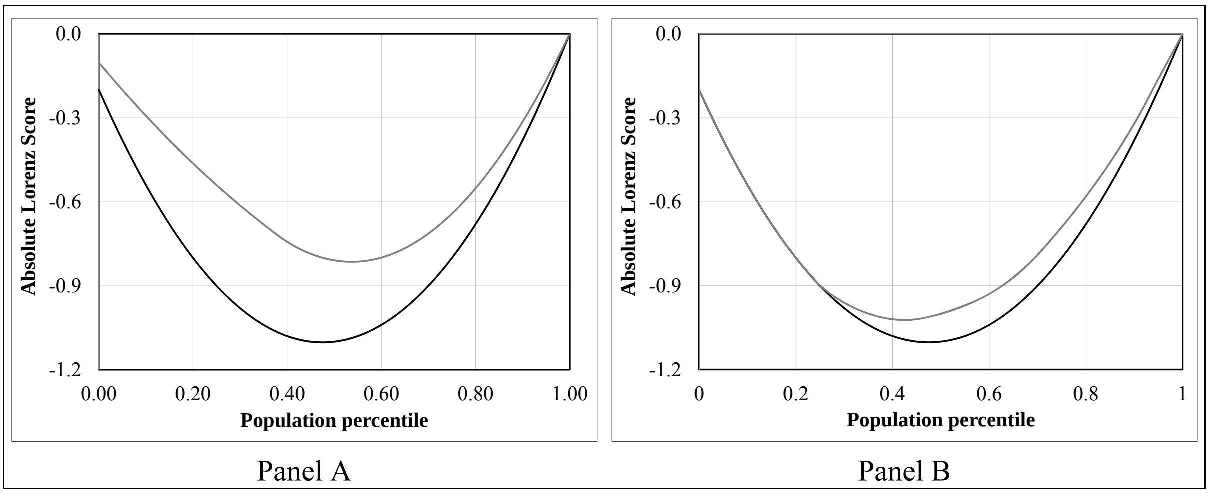
[25] Theorem 3 connects nicely with the concept of strong absolute pro-poor growth (Grosse et al., 2008) whose existence, by definition, requires the absolute change in the lower partial means for the bottom p percent of countries to be higher than the absolute change in means. Essentially, Theorem 3 states that absolute Lorenz dominance of the second period over the first is tantamount to strong absolute pro-poor growth *for every percentile partition*. For a review and systematic treatment of the connection between pro-poor growth and relative convergence see Dhongde and Silber (2016).

[26] Interestingly, if we have a situation where $\mu_X^{t_2} > \mu_X^{t_1}$, yet $L(X_{t_2}; p) = L(X_{t_1}; p)$ for all $p \in [0,1]$, then this is a situation of translation invariance whereby all countries have experienced an equal absolute amount of improvement.

increase in mean attainment as 'strictly progressive absolute improvement' if $s(p) \geq s(p')$ for any $p < p'$ with at least one strict inequality; or, if s is differentiable, then $s_p(p) \equiv ds(p)/dp \leq 0$ for all $p \in [0,1]$ and $s_p(p) < 0$ for some p. In these circumstances, if we plotted $s(p)$ against p we would obtain a downward-sloping absolute growth incidence curve (Klasen, 2008).

Let us illustrate the concept with the two cases of robust absolute convergence in Panels A and B of Figure 8, supposing that progress has occurred overall in both cases. In both panels, the solid black curve represents the Lorenz curve of the initial distribution X_{t_1}; while the solid gray curve represents the Lorenz curve of the final distribution X_{t_2}. Although robust absolute convergence has taken place in both cases, in Panel A convergence has been accompanied by strictly progressive absolute improvement, whereas in Panel B, robust absolute convergence has taken place by leaving the poorest countries behind (albeit anonymously).

Figure 8: Robust absolute convergence and Strictly progressive absolute improvement



It turns out that a strictly progressive absolute improvement leads to a robust reduction in absolute inequality, which is summarized in the following corollary:

Corollary 1: For any X_{t_1}, $X_{t_2} \in \mathcal{X}$, if X_{t_2} is obtained from X_{t_1} by a strict progressive absolute improvement, then $L(X_{t_2}, p) > L(X_{t_1}, p) \forall p \in [0,1]$ with at least one strict inequality.

Proof: Note that $s_p(p) \leq 0$ for all p and $s_p(p) < 0$ for some p effectively transforms $pD(p)$ into a reverse absolute Lorenz curve, which by definition has to be non-negative. Then by Theorem 3 we get $L(X_{t_2}, p) \geq L(X_{t_1}, p)$ $\forall p \in [0,1]$. ■

Strictly progressive absolute improvement across the whole percentile domain is a stronger egalitarian requirement in order to guarantee that no one is left behind. According to Corollary 1, strictly progressive absolute improvement is *sufficient* (but not necessary) to ensure robust absolute convergence.

Strictly progressive relative improvement

Finally, we consider how country-wise growth rates, subject to their initial levels of mean attainment in indicators relate to robust absolute convergence. Let us define the relative change in the lower partial mean attainments for percentile p as: $g_L(p) \equiv s_L(p)/\mu_X^{L,t_1}(p)$; and the relative change in the overall mean attainments as: $g \equiv s/\mu_X^{t_1}$. Note that $g_L(p)$ is nothing but the relative change in the Generalized Lorenz curve at percentile p.[27] Let us define *strictly progressive relative improvement* as $g_L(p) \geq g$ for all $p \in [0,1]$ with at least one strict inequality. This requires that the relative growth in the lower partial mean for every percentile p is at least as large as the overall growth of mean attainment and strictly larger for at least one percentile. The following corollary summarizes the relation between robust absolute convergence and strict progressive relative improvement:

Corollary 2: For any $X_{t_1}, X_{t_2} \in \mathcal{X}$, if $L(X_{t_2}, p) \geq L(X_{t_1}, p)\ \forall p \in [0,1]$ (with at least one strict inequality) then it must be the case that X_{t_2} is obtained from X_{t_1} by strict progressive relative improvement.

Proof: From Theorem 3 we know that if $L(X_{t_2}, p) \geq L(X_{t_1}, p)$ for all $p \in [0,1]$ (with at least one strict inequality) then it must be the case that $s_L(p) \geq s$ for all $p \in [0,1]$ (with at least one strict inequality). Now if we rewrite both sides of this inequality in terms of the growth rates, we get: $s_L(p) = g_L(p)\mu_X^{L,t_1}(p) \geq g\mu_X^{t_1}$. Since we know by definition of quantiles that $\mu_X^{L,t_1}(p) \leq \mu_X^{t_1}$, then it must be the case (as a necessary but insufficient condition) that: $g_L(p) \geq g$ for all $p \in [0,1]$. ■

5 Has the cross-country absolute convergence/divergence been robust?

In section 3, we saw that most of the development indicators reflected improvements as well as convergence over time using four absolute inequality indices. How robust were these convergence experiences? We now assess the robustness of these cross-country convergence experiences using the absolute Lorenz curve framework presented in the previous section.

[27] While discussing pro-poor growth, Foster et al., (2013) suggests looking at growth of the Generalized Lorenz curve at every percentile creating a Generalized Lorenz growth curve. A growth experience is considered pro-poor if the Generalized Lorenz growth curve lies above the overall growth in mean attainment.

Table 1 presents year-to-year absolute Lorenz dominance results for each indicator, based on the analysis in Figure 6. In the first column, we mention the health, education and access-rate indicators. The second column shows the corresponding years of study for each indicator. Finally, in the third column we report the Hasse diagram describing which of these years absolute Lorenz dominates the other years. An arrow from one year (t_2) to another year (t_1), or $t_2 \longrightarrow t_1$, means that the level of absolute inequality is robustly lower in year t_2 compared to year t_1. In other words, the cross-country absolute Lorenz curve for year t_2 lies everywhere above the cross-country absolute Lorenz curve for year t_1 as in Figure 6. We present the year-wise cross-country absolute Lorenz curves for the selected development indicators in Figure 9.

We find that many seemingly converging situations are actually not robust, which means that there would be at least one absolute inequality measure disagreeing with the analysis based on the four indicators in section 3.

Let us first look at the immunization rate indicators. Even though the means of the first three immunization rate indicators improved steadily between 1985 and 2012 and the countries showed convergence by four inequality measures (except variance between 1990 and 1995 for BCG), we do not find many pair-wise year comparisons to be fully robust. It is evident from Panel H1 of Figure 9 that several absolute Lorenz curves for BCG crossed each other. Only each one of years 2000, 2005 and 2012 is robustly more equal than year 1995. We may thus say that the cross-country convergence for BCG immunization rates was robust in 2000, 2005 and 2012 over 1995. Like BCG, for the measles immunization rate and the polio immunization rate, some year-wise comparisons were robust. For measles the cross-country inequality in 2012 was unambiguously lower than the rest of the years, meaning robust convergence occurred in 2012. For the polio immunization rate, only the comparison between 2005 and 2012 was robust; hence we cannot conclude that robust convergence occurred between 2012 and other preceding years.

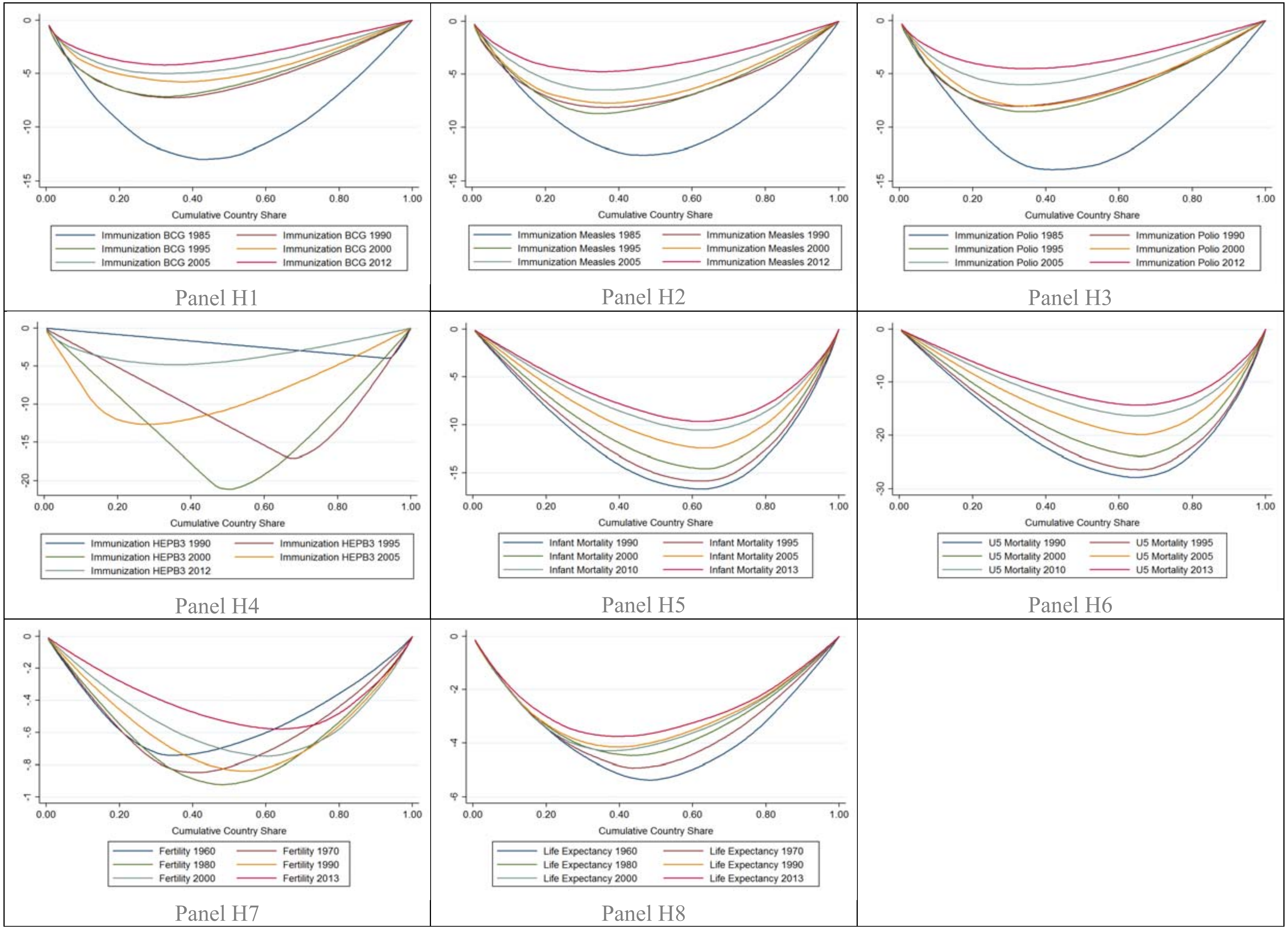
The case of Hepatitis-B immunization rate is quite interesting. Its mean in 1990 was very low and then it increased drastically until 2012; whereas inequality was very low to begin with, increased until 2000 and then went down. Although the level of inequality increased and then decreased, robust convergence was obtained only in 2012 over 2005 as is evident from Table 1. The rest of the absolute Lorenz curves intersected each other.

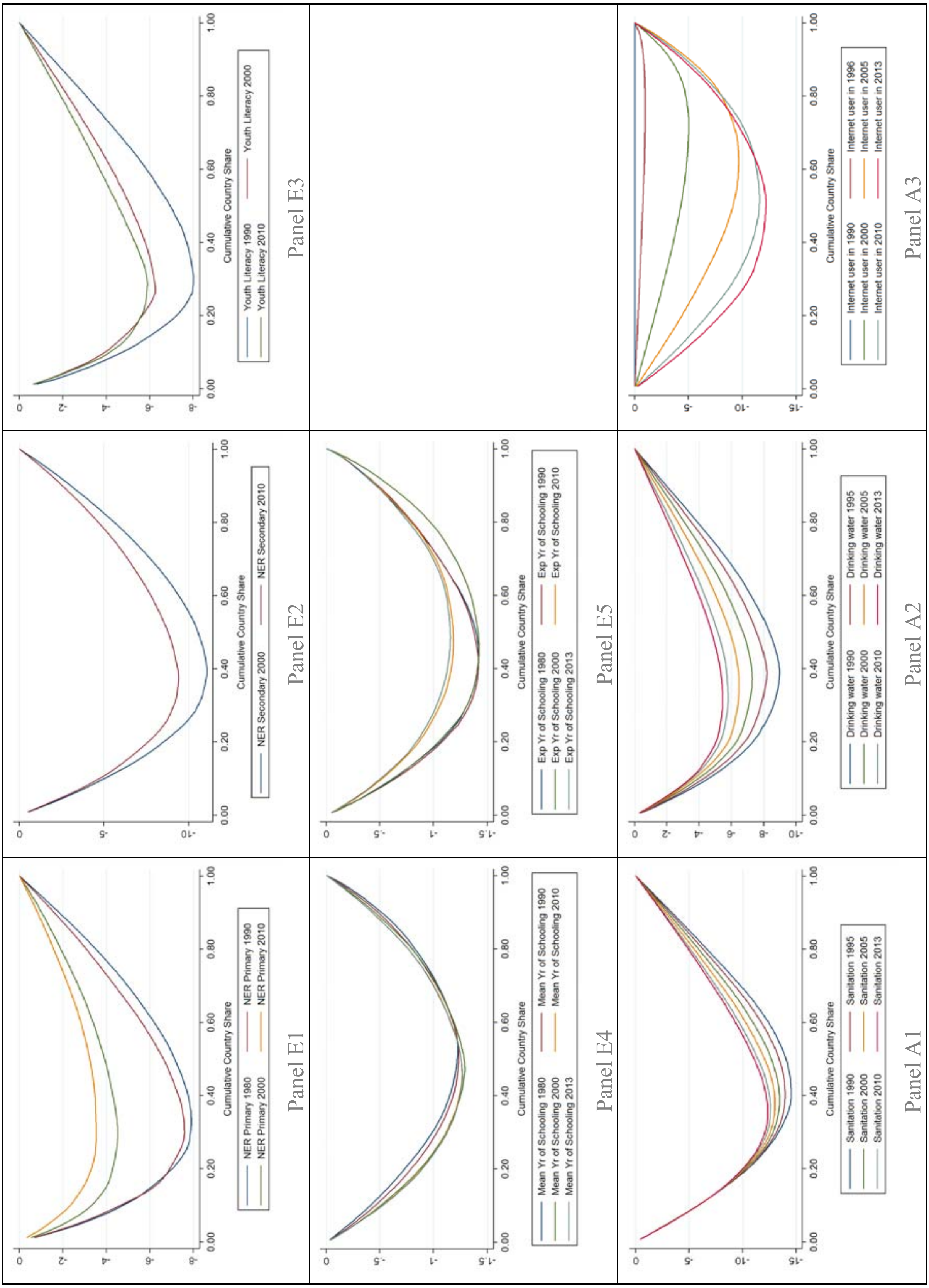
Unlike the immunization rates, more pair-wise comparisons were robust for both infant-mortality rates and under-5 mortality rates. For infant mortality rate, we find that 2013 is robustly less unequal than both 2010 and 2005, and in turn, both years are robustly less unequal than 2000. Otherwise we do not have any more robust pair-wise year comparisons. For under-five mortality rate, we find that 1995 is robustly more unequal than all the subsequent years. In turn, 2010 is robustly less unequal than all the other considered years, except for 2013 and 1990 (the curves cross; see absolute Lorenz curves in Panel H6 of Figure 9). For fertility rate, we did not find any pair-wise comparisons to be robust as all curves intersect each other, but for life expectancy, we find 2013 to be robustly less unequal than any of the previous five decades.

Table 1: Absolute Lorenz dominance for each indicator across years using Hasse diagrams

Indicator	Years of analysis	Hasse Dominance Diagram
BCG immunization rate	1985, 1990, 1995, 2000, 2005, 2012	2012 → 1995 2005 → 1995 2000 → 1995
Measles immunization rate	1985, 1990, 1995, 2000, 2005, 2012	2012 → 2005, 2000, 1995, 1990, 1985
Polio immunization rate	1985, 1990, 1995, 2000, 2005, 2012	2012 → 2005
Hepatitis-B immunization rate	1990, 1995, 2000, 2005, 2012	2012 → 2005
Infant mortality rate	1990, 1995, 2000, 2005, 2010, 2013	2013 → 2010, 2005 2010 → 2000 2005 → 2000
Under-five mortality rate	1990, 1995, 2000, 2005, 2010, 2013	2013 → 2005 2010 → 2005, 2000 2005 → 1995 2000 → 1995
Fertility rate	1990, 1995, 2000, 2005, 2012	No robust pair-wise dominance
Life expectancy at birth	1960, 1970, 1980, 1990, 2000, 2013	2013 → 2000, 1990, 1980, 1970 1970 → 1960
Net primary enrolment rate	1975-1984, 1985-1994, 1995-2004, 2005-2015	2005-15 → 1985-94, 1975-84 1995-04 → 1985-94, 1975-84
Net secondary enrolment rate	1995-2004, 2005-2014	No robust pair-wise dominance
Youth literacy rate	1985-1994, 1995-2004, 2005-2014	1995-04 → 1985-94
Mean years of schooling	1980, 1990, 2000, 2010, 2013	No robust pair-wise dominance
Expected years of schooling	1980, 1990, 2000, 2010, 2013	2013 → 2000 2010 → 2000
Access to improved sanitation facilities	1990, 1995, 2000, 2005, 2010, 2013	No robust pair-wise dominance
Access to improved drinking water source	1990, 1995, 2000, 2005, 2010, 2013	2013 → 2000 2010 → 2000, 1995 2005 → 1995 2000 → 1990 1995 → 1990
Internet usage rate	1990, 1995, 2000, 2005, 2010, 2013	1990 → 2000 1996 → 2000 2000 → 2005, 2010, 2013

Figure 9: Year-wise absolute Lorenz curves for different Development Indicators




Panel E1
NER Primary 1980
NER Primary 1990
NER Primary 2000
NER Primary 2010
Cumulative Country Share
Panel E2
NER Secondary 2000
NER Secondary 2010
Cumulative Country Share
Panel E3
Youth Literacy 1990
Youth Literacy 2000
Youth Literacy 2010
Cumulative Country Share
Panel E4
Mean Yr of Schooling 1980
Mean Yr of Schooling 1990
Mean Yr of Schooling 2000
Mean Yr of Schooling 2010
Mean Yr of Schooling 2013
Cumulative Country Share
Panel E5
Exp Yr of Schooling 1980
Exp Yr of Schooling 1990
Exp Yr of Schooling 2000
Exp Yr of Schooling 2010
Exp Yr of Schooling 2013
Cumulative Country Share
Panel A1
Sanitation 1990
Sanitation 1995
Sanitation 2000
Sanitation 2005
Sanitation 2010
Sanitation 2013
Cumulative Country Share
Panel A2
Drinking water 1990
Drinking water 1995
Drinking water 2000
Drinking water 2005
Drinking water 2010
Drinking water 2013
Cumulative Country Share
Panel A3
Internet user in 1990
Internet user in 1996
Internet user in 2000
Internet user in 2005
Internet user in 2010
Internet user in 2013
Cumulative Country Share

We next look at the five education indicators. For net primary enrolment rate we find that the Lorenz curves of years 2000 (1995-2004) and 2010 (2005-2014), each separately, dominate both the Lorenz curves of years 1980 (1975-84) and 1990 (1985-1994), which can be observed from Panel E1. No dominance in convergence is found between years 2000 (1995-2004) and 2010 (2005-2014) and between years 1980 (1975-84) and 1990 (1985-1994). Unlike net primary enrolment ratio, the convergence was not robust for net secondary enrolment ratio. Youth literacy rate reflects increase in the mean, from a period baseline above 0.5 (84%) and convergence by all four inequality indices, but we find that only the comparison between 1990 (1985-1994) and 2000 (1995-00) to be fully robust (with the latter showing less inequality; the other absolute Lorenz curves cross, see Panel E3 of Figure 9). Mean years of schooling did not show any sign of convergence. Although countries diverged initially in this indicator, the divergence was not robust. For expected years of schooling, inequality fell drastically after 2000 according to all four absolute indices; and both 2010 and 2013 were robustly less unequal than 2000.

We finally look at the access indicators: improved sanitation facilities, improved drinking water, and internet usage rate. The increase in the mean rate of access to sanitation facilities, from a period baseline above 0.5 (66% in 1990), is accompanied by absolute inequality reductions, all of which are monotonic, but as shown in Panel A1, we do not find any pair-wise year comparisons which are fully robust to any choice of inequality index. Unlike the access to sanitation facilities, we find more robust convergence for access to improved drinking water. Its mean increases from a period baseline well above 0.5 (80% in 1990) and we find that the initial year 1990 is robustly more unequal than all the subsequent years. In turn, 2005, 2010 and 2013, each separately dominate 1990, 1995, and 2000; i.e. the three later years are robustly less unequal, but their respective absolute Lorenz curves cross between each other (see Panel A2).

The internet usage rate shows that, overall, the increase in the mean, from a period baseline well below 0.5 (just above 0% in 1990) until a final period value just below 0.5 (48% in 2013), is accompanied by absolute inequality increases, all of which are monotonic. Meanwhile, we find that the last three years, 2005, 2010 and 2013, are all each separately dominated by 1990, 1996, and 2000; i.e. the three later years are robustly more unequal, although their respective absolute Lorenz curves cross between each other. Likewise 1990 and 1996 are robustly the least unequal years (see absolute Lorenz curves in Panel A3).

We thus find that many seemingly converging situations based on our analysis in Section 3 were actually not robust. There are only two indicators among the set considered in this paper where we find that robust convergence has occurred in the final year compared to all the previously considered years. These are measles immunization rate and life expectancy at birth. The infant mortality rate, net primary enrolment rate and access to improved drinking water indicators have shown some sign of absolute convergence. For example, infant mortality has shown robust convergence in 2013 compared to 2000, 2005 and 2010 but not compared to 1990 and 1995; whereas net primary enrolment rate has converged in 2005-15 compared to 1975-84 and 1985-

94. The access to improved drinking water has also shown some sign of robust absolute convergence. Except for this handful of indicators, no sign of robust absolute convergence has been observed; internet usage rates have diverged in 2013 compared to 1990, 1996 and 2000.

Finally, we discuss some interesting cases regarding how the shapes of the Lorenz curves relate to the forms of egalitarian progress discussed in the previous section. The first case is attributed to Hepatitis-B from Panel H4 of Figure 3. Although the mean increased from $\mu_{HB}^{1990} = 0.04$ to $\mu_{HB}^{2000} = 0.41$, and then to $\mu_{HB}^{2012} = 0.81$, when we compute the absolute Lorenz curves presented in Panel H4 of Figure 9, we note that in 1990 the absolute Lorenz curve lies very close to the horizontal axis meaning the countries were in a near-egalitarian situation of low attainment. A similar situation occurs in 2012, but now the countries are again in a near-egalitarian situation of high attainment. By contrast, in 2000, we witness higher inequality assessed by all four inequality measures presented in Panel H4 of Figure 3. This case is analogous to the example presented in Figure 7, but the difference is that in the case of Hepatitis-B all three absolute Lorenz curves crossed each other and a robust conclusion could not be reached.

The second case is attributed to several indicators: access to sanitation facilities, access to drinking water, life expectancy, and some education indicators. The shapes of the absolute Lorenz curves are analogous to the case presented in Panel B of Figure 8. The overall improvements in these indicators have not been inclusive for the poorest countries, despite some pair-wise year comparisons being robust. In order to formally check whether any pair-wise year comparison satisfied the requirement of strictly progressive absolute improvement, we applied the test in Corollary 1 to each pair-wise absolute Lorenz comparison. We found, quite interestingly, that none of the pair-wise comparisons actually satisfied this requirement. Thus, although there were signs of convergence for some indicators across some periods, none of these convergences was strictly egalitarian from an absolute perspective.

6 Concluding Remarks

The convergence literature has made a remarkable effort in implementing several convergence measurement tools in order to capture different dimensions and notions of the phenomenon (e.g. absolute versus relative convergence, formation of "convergence clubs" versus twin peaks, etc.). This paper contributes to the existing convergence literature in three different ways. First, we discuss why it is more appropriate to pursue an absolute approach rather than a relative approach for assessing convergence when the underlying feasible progress of non-monetary indicators is bounded, and especially when many of these bounded indicators can be measured either with attainments or with shortfalls; a choice which is essentially arbitrary. Second, although the literature has borne witness to the absolute approach to assess convergence, the literature has rarely paused to question whether any of these assessments and tools is fully robust to alternative implementations. Our second contribution thus seeks to highlight the importance of checking whether the absolute convergence/divergence trends are robust to alternative legitimate choices of absolute inequality indices. We perform this check not only by deploying a myriad of appropriate inequality indices, but chiefly by implementing absolute Lorenz dominance tests. Third, we discuss in details the implications pertaining to the changes in shapes of the absolute Lorenz curves as well as the ways in which absolute robust convergence is related to different types of egalitarian progress.

Using four different absolute inequality measures to assess convergence over time for over fifteen bounded non-monetary development indicators, we obtained different results, depending on the indicator under inspection. In some cases, we found that the four inequality indices would immediately disagree in their reported trends; whereas, in many cases we found that even though the four inequality indices were in agreement, the absolute Lorenz curves crossed. This meant that there were other unused indices which would produce disagreeing trends. However, we also found situations in which the inequality trends, or at least segments of the trends, were fully robust (e.g., year 1990 was unequivocally the most unequal year in the case of access to water). These multiple situations strongly highlight the importance of conducting robustness assessments even in such seemingly narrow analysis scenarios as that of absolute convergence with bounded variables.

We would also like to emphasize that the concern for robustness of convergence assessments to the choice of inequality indices is not only relevant for bounded indicators. Absolute Lorenz comparisons can (and should) be applied to any assessment of absolute convergence. By contrast, a pending question is whether, for the sake of consistency, we should also restrict convergence analyses to their absolute version in the cases of variables with “soft bounds” or indicators characterized by crisp lower bounds but fuzzy upper bounds (e.g., indicators such as expected years of schooling and life expectancy in years). In this paper, we have applied the assessment tools of the absolute approach to these indicators. Even though there is no sharp upper bound for any of these variables, we are confident that their domain is not open-ended. Should practitioners decide to treat these indicators as “hard-bound”, the methods proposed in this paper would be suitable.

Finally, we formally studied the relationship between progress in mean attainment of bounded variables and absolute convergence. These relationships are very helpful in interpreting the empirical results. In particular we elucidated three key features: (a) how the scope of maximum possible inequality is a parabolic function of mean attainment (which allows for the prospects of a Kuznets curve); (b) the necessary and sufficient conditions for egalitarian progress, along with conditions which are only sufficient, and others which are only necessary; (c) the impossibility to rank situations of maximum inequality with different means in a robust manner. In turn, these formal results provided the basis for our main interpretative message in this paper: r*obust inequality comparisons with bounded variables always favour global improvements (i.e. increases in the mean) that take place piece-meal, but spread evenly across several countries at the same time, over global improvements that fully move one country at a time from 0 to 1 in the indicator, while leaving other countries “behind”*. This assessment relied on an anonymous perspective in which only relative positions in the distribution, but not countries' identities, matter. Future empirical and methodological work should also focus on the interpretation and elucidation of egalitarian progress with bounded variables, but in two more complex scenarios: (1) a non-anonymous context more akin to a mobility analysis; and (2) population-weighted indicators, which entail a convergence trend also potentially affected by changes in the world population distribution.

References

Allison, R. Andrew, and James E. Foster (2004) "Health inequality using qualitative data", *Journal of Health Economics*, 23: 505-524.

Barro, Robert, and Xavier Sala-i-Martin (2003) *Economic Growth (2nd Edition)*, The MIT Press.

Chakravarty, Satya, Nachiketa Chattopadhyay, and Conchita D'Ambrosio (2013) "On a family of achievement and shortfall inequality indices", ECINEQ Working Paper 2013-300.

Dhongde, Shatakshee, and Jacques Silber (2016) "On distributional change, pro-poor growth and convergence", *Journal of Economic Inequality*, forthcoming.

Dowrick, Steve, Yvonne Dunlop, and John Quiggin (2003) "Social indicators and comparisons of living standards", *Journal of Development Economics*, 70: 501-29.

Easterlin, Richard (2000) "The worldwide standard of living since 1800", *Journal of Economic Perspectives*, 14(1): 7-26.

Erreygers, Guido (2009) "Can a single indicator measure both attainment and shortfall inequality?", *Journal of Health Economics*, 28: 885-93.

Foster, James, Suman Seth, Michael Lokshin, and Zurab Sajaia (2013) *A Unified Approach to Measuring Poverty and Inequality--Theory and Practice: Streamlined Analysis with ADePT Software*. Washington, DC: World Bank.

Grosse, Melanie, Kenneth Harttgenm and Stephan Klasen (2008) "Measuring pro-poor growth in non-income dimensions", *World Development* 36(6): 1021-47.

Hobijn, Bart, and Philip Franses (2001) "Are living standards converging?", *Structural change and economic dynamics*, 12: 171-200.

Islam, Nazrul (2003) "What have we learnt from the convergence debate?", *Journal of Economic Surveys*, 17(3): 309-62.

Kenny, Charles (2004) "Why are we worried about income? Nearly everything that matters is converging", *World Development*, 33(1): 1-19.

Klasen, Stephan (2008) "Economic growth and poverty reduction: measurement issues using income and non-income indicators", *World Development*, 36(3): 420-45.

Kjellsson, Gustava, Ulf Gerdtham, and Dennis Petrie (2015) "Lies, damned lies, and health inequality measurements. Understanding the value judgments", *Epidemiology*, 26(5): 673–80.

Lasso de la Vega, Casilda, and Oihana Aristondo (2012) "Proposing indicators to measure achievement and shortfall inequality consistently", *Journal of Health Economics*, 31: 578-83.

Lambert, Peter, and Buhong Zheng (2011) "On the consistent measurement of attainment and shortfall inequality", *Journal of Health Economics*, 30: 214-9.

Micklewright, John, and Kitty Stewart (1999) "Is the Well-being of Children Converging in the European Union?" *The Economic Journal*, 109(459): 692-714.

Milanovic, Branko (2005), Worlds Apart: Global and International Inequality 1950-2000, Princeton University Press.

Moyes, Patrick (1987) "A new concept of Lorenz domination", *Economic Letters*, 23(2): 203-7.

Neumayer, Eric (2003) "Beyond income: convergence in living standards, big time", *Structural change and economic dynamics*, 14: 275-96.

Nissanov, Zoya, and Jacques Silber (2009) "On pro-poor growth and the measurement of convergence", Economic Letters, 105: 270-2.

Rodrik, Dani (2012) "The future of economic convergence", in Federal Reserve Bank of Kansas City, *Achieving maximum long-run growth*, Federal Reserve Bank of Kansas City.

Rodrik, Dani (2013) "Unconditional convergence in manufacturing", *Quarterly Journal of Economics*, 165-204.

Seth, Suman, and Sabina Alkire (2014) "Did poverty reduction reach the poorest of the poor? Assessment methods in the counting approach", OPHI Working Paper 77.

Shorrocks, Anthony, and Daniel Slottje (2002) "Approximating Unanimity Orderings: An Application to Lorenz Dominance", *Journal of Economics*, supplement 9: 91-117.

Silber, Jacques (2015) "On inequality in health and pro-poor development: the case of Southeast Asia", *Journal of Economic Studies*, 42(1): 34-53.

United Nations (2015) *Goal 10: Reduce inequality within and among countries*, http://www.un.org/sustainabledevelopment/inequality/

Wagstaff, Adam, Caryn Bredenkamp, and Leander Buisman (2014) "Progress in global health goals: are the poor being left behind?", *World Bank Research Observer*, 29: 137-62.

World Bank (2015) *Global Monitoring Report 2014/2015: Ending poverty and sharing prosperity*, The World Bank.

Zoli, Claudio (2009) A generalized version of the inequality equivalence criterion: a surplus sharing characterization, complete and partial orderings. In: de Swart, H. (Ed.), *Logic, Game Theory and Social Choice*. University Press, Tilburg, pp. 427–441.

Appendix I

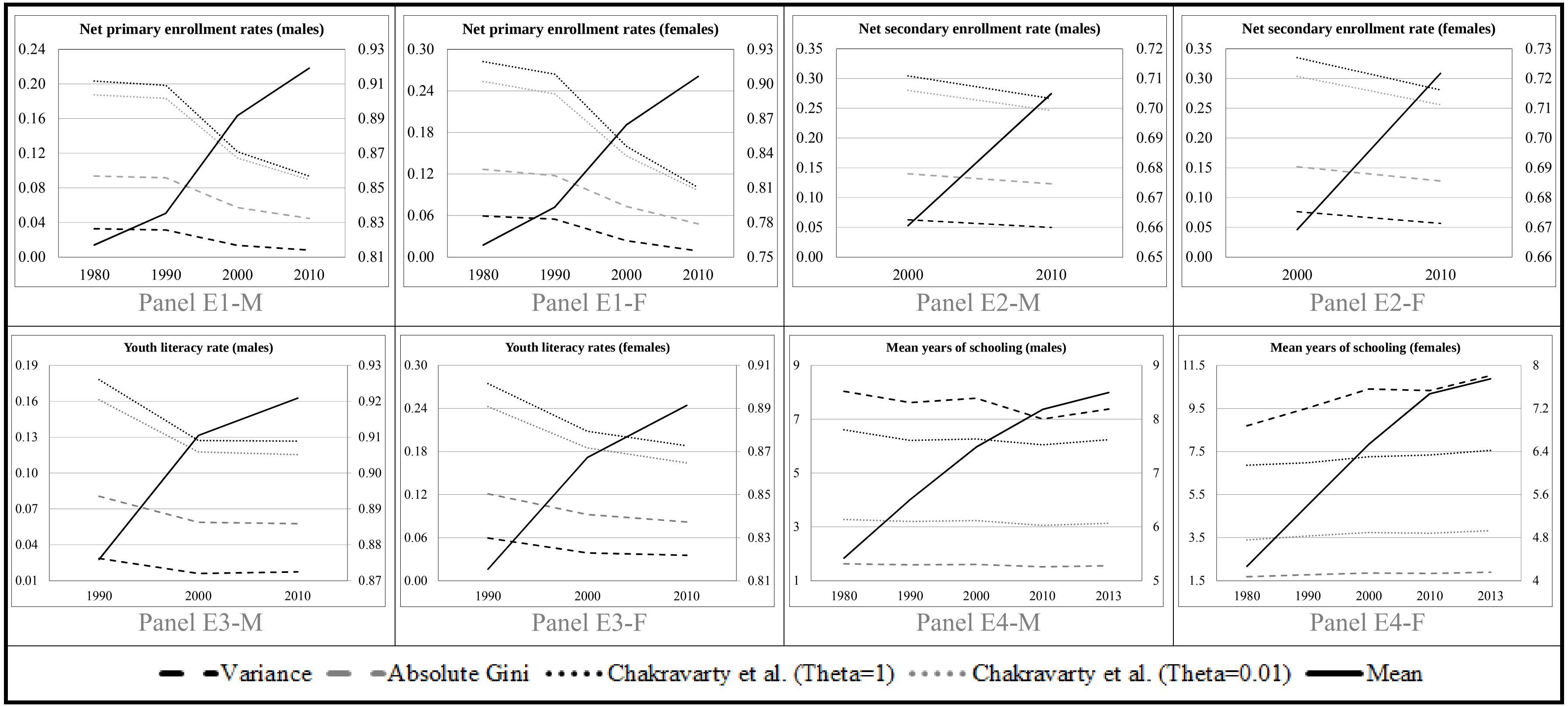
This appendix lists the development indicators, along with the number of countries for which data are available given the selection of years to study, the definition of the indicators, and the selection of years. We assembled the datasets from: http://data.un.org/Explorer.aspx (United Nations Data Website).

Indicator	Number of Countries	Definition	Years of analysis
Health Indicators			
Measles immunization rate (0-100%)	164	Percentage of children under one year of age who have received at least one dose of a measles vaccine.	1985, 1990, 1995, 2000, 2005, 2012
Polio immunization rate (0-100%)	164	Percentage of children under one year of age who have received at least one dose of a polio vaccine (Pol3).	1985, 1990, 1995, 2000, 2005, 2012
BCG immunization rate (0-100%)	130	Percentage of children under one year of age who have received at least one dose of a BCG vaccine.	1985, 1990, 1995, 2000, 2005, 2012
Hepatitis-B immunization rate (0-100%)	152	Percentage of children under one year of age who have received at least one dose of a Hepatitis-B vaccine.	1990, 1995, 2000, 2005, 2012
Infant mortality rate (0-1000)	195	Number of infants dying before reaching one year of age, per 1,000 live births in a given year	1990, 1995, 2000, 2005, 2010, 2013
Under-five mortality rate (0-1000)	195	Probability per 1,000 that a new-born baby will die before reaching age five, if subject to age-specific mortality rates of the specified year	1990, 1995, 2000, 2005, 2010, 2013
Fertility rate	190	The number of children that would be born to a woman if she were to live to the end of her childbearing ages and bear children in accordance with current age-specific fertility rates	1990, 1995, 2000, 2005, 2012
Life expectancy at birth	190	The number of years a newborn infant could expect to live if prevailing patterns of age-specific mortality rates at the time of birth stay the same throughout the infant's life.	1960, 1970, 1980, 1990, 2000, 2013
Education Indicators			
Net primary enrolment rate [boys, girls, and both] (0-100%)	87	Ratio of children of official school age who are enrolled in school in primary education to the population of the corresponding official school age according to the International Standard Classification of Education of UNESCO	1975-1984, 1985-1994, 1995-2004, 2005-2015
Net secondary enrolment rate [boys, girls, and both] (0-100%)	116	Ratio of children of official school age who are enrolled in school in secondary education to the population of the corresponding official school age according to the International Standard Classification of Education of UNESCO	1995-2004, 2005-2014

Indicator	Number of Countries	Definition	Years of analysis
Youth literacy rate [boys, girls, and both] (0-100%)	84	Proportion of people aged 15-24 who can both read and write, with understanding, a short simple statement on their everyday life	1985-1994, 1995-2004, 2005-2014
Expected years of schooling (children)	158	Number of years of schooling that a child of school entrance age can expect to receive if prevailing patterns of age-specific enrolment rates persist throughout the child's life	1980, 1990, 2000, 2010, 2013
Mean years of schooling (adults)	143	Average number of years of education received by people ages 25 and older, converted from education attainment levels using official durations of each level	1980, 1990, 2000, 2010, 2013
Access Indicators			
Internet usage rate (0-100%)	165	Percentage of population who have used the internet (from any location) in the last 12 months. Internet can be used via a computer, mobile phone, personal digital assistant, games machine, digital TV etc.	1990, 1995, 2000, 2005, 2010, 2013
Rate of access to improved sanitation facilities (0-100%)	163	Percentage of the population with access to improved sanitation facility. Improved sanitation facilities are likely to ensure hygienic separation of human excreta from human contact. They include flush/pour flush (to piped sewer system, septic tank, pit-latrine), ventilated improved pit (VIP) latrine, pit latrine with slab, and composting toilet.	1990, 1995, 2000, 2005, 2010, 2013
Rate of access to improved drinking water source (0-100%)	172	Share of the population with access to improved drinking water source. Improved drinking water sources include piped water on premises (piped household water connection located inside the user's dwelling, plot or yard), and other improved drinking water sources (public taps or standpipes, tube wells or boreholes, protected dug wells, protected springs, and rainwater collection).	1990, 1995, 2000, 2005, 2010, 2013

Appendix II

Convergence or divergence across gender-wise education indicators by different absolute inequality measures



In each diagram, the left vertical axis measures absolute inequality across countries and the right vertical axis measures mean attainment across countries.

Appendix III

Proof of Theorem 1:

First we prove that (a) ↔ (c). Clearly if X_t is defined by (a) then $L(X_t, p) = \int_0^p (0 - \mu_X^t) d\wp = -\mu_X^t p$ for all $p \in [0,1 - q_t]$ and $L(X_t, p) = \int_0^p (1 - \mu_X^t) d\wp = (1 - \mu_X^t)p$ for all $p \in (1 - q_t, 1]$. Hence the slopes prescribed by (c) ensue. Now (a) is also necessary for (c) to occur, which we show by contradiction. Suppose there is an x_{it} such that $0 < x_{it} < 1$. Clearly, (c) cannot be true. Therefore, (a) and (c) imply each other.

Next we prove that (a) ↔ (d). Given the definition of X_t it must be the case that: $L(X_t, 1 - q_t) = \int_0^{1-q_t} (0 - \mu_X^t) d\wp = -\mu_X^t (1 - q_t)$. But note that the mean of X_t is: $\mu_X^t = 1 \times q_t + 0 \times (1 - q_t) = q_t$. Hence $L(X_t, 1 - q_t) = -\mu_X^t (1 - \mu_X^t)$. This proves that (a) implies (d). But (a) is also necessary for (d). Imagine a distribution $X_{t'}$ with the same μ_X^t as X_t and a proportion q_t of elements above the mean, but one of the $1 - q_t$ elements below the mean is strictly positive (compensating with an element above the mean which is strictly below 1). Then in that case it is clear that $L(X_\tau, 1 - q_t) > \int_0^{1-q_t} (0 - \mu_X^t) dp = -\mu_X^t (1 - q_t)$.

Finally, we prove that (a) ↔ (b). Let's focus on the case where $X_{t'}$ has at least two elements different from X_t (if only one element is different then the means cannot be identical and the two distributions would not belong together in $\mathcal{X}_\mu$; and when the two distributions are equal we get, trivially: $L(X_t, p) = L(X_{t'}, p)$ for all p). We start comparing the Lorenz curves in the interval $p \in [0,1 - q_t]$. In that interval: $L(X_{t'}, p) \geq \int_0^p (0 - \mu_X^t) d\wp = -\mu_X^t p = L(X_t, p)$, with strict inequality if at least one of the different elements is in the same interval. Then we compare the curves in the interval $p \in [1 - q_t, 1]$. If we find the value of the Lorenz curves, by integrating leftward from $p = 1$, then we will find again that: $L(X_{t'}, p) \geq \int_p^1 (1 - \mu_X^t) d\wp = -(1 - \mu_X^t)(1 - p) = L(X_t, p)$, with strict inequality if at least one of the different elements is in the same interval. This is the proof that (a) implies (b). Necessity of (a) can be proved in different ways. One straightforward manner is to choose any X_t different from the definition in (a) and then find $X_{t'}$, with the same mean, such that the Lorenz curves of the two distributions either cross, or that of $X_{t'}$ is always below. ■

Proof of Theorem 2:

Let $q_{t_1} > 0$ be the proportion of countries in X_{t_1} with $x_{it_1} = 1$, and $(1 - q_{t_1})$ be the corresponding proportion of countries with $x_{it_1} = 0$. For X_{t_2}, the respective proportions are given by $q_{t_2} > 0$, and without loss of generality, suppose: $q_{t_1} < q_{t_2} < 0.5$. If we compute both Lorenz curves at $p = 1 - q_{t_2}$, we get: $L\left(X_{t_1}, 1 - q_{t_2}\right) = -q_{t_1}(1 - q_{t_2})$ (since $\mu_X^{t_1} = q_{t_1}$), and $L\left(X_{t_2}, 1 - q_{t_2}\right) = -q_{t_2}(1 - q_{t_2})$. Clearly: $L\left(X_{t_1}, 1 - q_{t_2}\right) > L\left(X_{t_2}, 1 - q_{t_2}\right)$. Now we compute both Lorenz curves at $p = 1 - q_{t_1}$. On this occasion we get: $L(X_{t_1}, 1 - q_{t_1}) = -q_{t_1}(1 - q_{t_1})$ and $L(X_{t_2}, 1 - q_{t_1}) = -q_{t_2}(1 - q_{t_2}) + [1 - q_{t_2}][q_{t_2} - q_{t_1}] = -q_{t_1}(1 - q_{t_2})$. Hence now: $L(X_{t_1}, 1 - q_{t_1}) < L(X_{t_2}, 1 - q_{t_1})$.

The same reasoning can be applied to a general example in which $1 > q_{t_1} > q_{t_2} > 0.5$, and one in which $1 > q_{t_1} > 0.5 > q_{t_2}$. In both cases curve crossings will be obtained, thereby ruling out robustness. ■

Appendix IV

Absolute Lorenz curves for primary and secondary net enrolment rates (NER) and and youth literacy rates for girls and boys