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A unified approach for measuring welfare protection under a decentralized framework

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# A unified approach for measuring welfare protection under a decentralized framework<sup>\*</sup>

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# Abstract

The existing literature on welfare decentralization has not produced a robust set of measures and properties, and no consensus has emerged on how differences in the protection provided by territorial programs should be aggregated into a composite index. The measurement of the level of protection provided by decentralized minimum income programs has often focused on one of two dimensions: adequacy or coverage. The rankings of regions in terms of the protection provided can be very different depending on the chosen outcome. In this paper, we introduce several properties that should be taken into account when measuring these differences. Based on social welfare functions, we propose a parameterized family of indices that satisfy those properties and combine both dimensions. Our empirical illustration using Spanish data confirms that focusing only on standard measures of adequacy may introduce an important bias, making it difficult to properly identify real differences in terms of social welfare among programs in different jurisdictions.

Keywords: welfare generosity, minimum income, social welfare, adequacy.

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#### **1. INTRODUCTION**

The potential effects of the decentralization of welfare benefits raise numerous interesting questions and have been a major focus of policy debates. In countries where these benefits are decentralized, inequalities in benefit adequacy and coverage have provoked great controversy and heated debates concerning their potential role in the fight against poverty. Problems of coordination and financing may produce a mosaic of highly varied programs—over and above the natural regional differences—with a striking disparity of regulations and results and, above all, large differences in the protection received by the poorest citizens. To the extent that equally poor people receive different treatment depending on where they live, horizontal inequity problems might arise.

The questions of how jurisdictions in a decentralized framework design their minimum income (welfare) programs and whether decentralization leads to large territorial differences in benefit levels and coverage have garnered a great deal of research attention over the years. Literature in this field has extensively examined the underprovision of welfare under decentralized designs using both partial equilibrium models of jurisdictions' behavior (Brown and Oates, 1987, Brueckner, 2000) and full general equilibrium models of the problem (Wheaton, 2000). Another large segment of the literature has analyzed the different ways in which jurisdictions respond to new policy environments (Soss et al., 2001), the impact of more decentralized rulemaking and authority on eligibility rules, level of benefits, sanctions and administrative procedures (Ziliak, 2007), and even partian differences (Leigh, 2008). Nonetheless, most of the extensive amount of research on welfare inequalities across jurisdictions has revolved around regions' strategic behavior (Shroder, 1995; Berry et al., 2003; Baicker, 2005a; Fiva and Rattsø, 2006; Dahlberg and Edmark, 2008) and the price and income effects of federal grants (Ribar and Wilhelm, 1999; Baicker, 2005b; Chernick, 1998, 2000; Marton and Wildasin, 2007; Toolsema and Allers, 2014).

Relatively little is known, however, about how differences across regions—in terms of the generosity or intensity of the social protection provided arising from welfare decentralization—should be measured. The existing literature has not produced a robust set of measures and properties, and no consensus has emerged on how the different outcomes of minimum income programs should be aggregated into a composite index. The measurement of the global effects of decentralized welfare benefits on inequality has usually focused on one of two dimensions (Figari *et al.*, 2013): adequacy or coverage. Recent evidence shows that welfare systems are quite insufficient to keep benefit levels in line with the general living standard (Van Mechelen and Marchal, 2013). The economic crisis also affected the abilities of governments to provide adequate levels of benefits (Marchal *et al.*, 2014). The growth of social needs has happened in parallel to a growing constraint on the allocation of resources. Depending upon a number of factors, these adequacy problems could vary greatly among different jurisdictions. How adequacy rates are measured is therefore very important. While most comparisons across regions or countries have been made in terms of the gaps between benefits and poverty lines (Ravallion, 2007), over the last years there has been a shift in focus towards measures that relate benefits to in-work or net disposable income (Immervoll, 2010; Vandenbroucke *et al.*, 2012).

On the other hand, the protection provided by welfare benefits can also be assessed in terms of the proportion of potential recipients who receive benefits. Eligibility and takeup rates stand as two of the main factors that drive this rate. As shown by Figari *et al.* (2013), in several countries, a large proportion of individuals are ineligible for welfare benefits even when they fall below a low poverty line. Eligibility rules limit coverage by design, either by introducing categorical conditions that exclude potential beneficiaries or by setting the income threshold for entitlement too low. High non take-up levels also involve that some eligible households do not receive benefits. Targeting errors (Duclos, 1996), stigma and transaction costs (Moffitt, 1983; Whelan, 2010), information asymmetries (Currie, 2006), expectations regarding long-term unemployment and/or levels of social assistance payments (Bargain *et al.*, 2012), or endogenous government policy (Ayala and Triguero, 2016) are some of the factors usually highlighted as key issues explaining why there is a gap between current and potential recipients.

Regional rankings and levels of inter-regional inequality can be very different depending upon the chosen outcome. Governments can choose to maintain high levels of generosity while promoting low access to benefits by different means. The political costs they face will differ depending on the chosen strategy. Consequently, rankings and social welfare assessments can be rather dissimilar depending on whether adequacy or coverage is chosen as the key indicator of the protection provided by welfare programs. To circumvent these problems, there is a need for research that provides a more complete assessment of these welfare schemes by considering both dimensions in a unique analytical framework. It is also important to analyze these differences in terms of social welfare. In this paper, we propose several properties that should be taken into account when assessing the protection provided by minimum income programs. In our approach, poorer recipients are considered more important than richer ones and, given a group of people with the same income level, small improvements in many individuals are preferred to a large improvement in one individual. Based on social welfare functions, we propose a parameterized family of indices that satisfy those properties and combine both the coverage and adequacy dimensions. The family takes into account both outcomes in such a way that the higher the coverage or adequacy rates, the higher the levels of the indices.

Some of the contributions of our approach are as follows: i) to the best of our knowledge, there are no measures that combine adequacy and coverage in welfare programs that can be interpreted in terms of social welfare; ii) we propose several properties that should be taken into account when assessing differences in the protection provided by minimum income programs; and iii) the methodology can be extrapolated to any decentralized welfare program.

To test the sensitivity and robustness of the proposed approach, we use data from Spanish regional welfare programs. In Spain, regional welfare schemes are the last safety net for low-income households and they are completely decentralized. The result is a mosaic of highly variable schemes, with a striking disparity of regulations and benefit levels across the different regions. We use data from the Income and Living Conditions Survey and the Labor Force Survey before and after the economic crisis.

The structure of the paper is as follows. The next section proposes a general framework for defining an index of social welfare that considers adequacy and coverage in minimum income programs. In section three, we define this index based on social welfare functions. In section four, we illustrate the applicability of this index using a completely decentralized welfare system as reference. The paper ends with a brief list of conclusions.

3

# 2. ANALYTICAL FRAMEWORK

#### 2.1. General outline

Let  $\aleph = \{1, 2, ..., N\}$  be a population composed of N individuals with  $x = (x_i)_{i \in \aleph} = (x_1, x_2, ..., x_N)$  income distribution, which can be summarized by the cumulative distribution function F(x). Welfare programs aim to provide an adequate level of economic security (*B*) to individuals in the lower portion of the income distribution. Therefore, let us consider a minimum income program (MIP) and denote *B* as the level of guaranteed income. The income received from an MIP by an individual with income  $x_i$  is  $(B - x_i)$ . Let us denote by  $\wp = \{1, 2, ..., R\}$  the group of MIP recipients and by  $\mathcal{L} = \{1, 2, ..., L\}$  the target group or potential claimants on which we will focus, with  $\wp \subset \mathcal{L}$ .<sup>2</sup> We can consider different criteria to define the target group or potential claimants. As mentioned above, potential claimants can be defined by demographic categories or by an income threshold. In our approach, we will consider a general setting in which potential claimants are defined in terms of a level of income that is determined either exogenously or as a function of a measure of position in the income distribution.

**Definition 1**. An individual belongs to the target group  $\mathcal{L}$ :

$$i \in \mathcal{L}$$
 if  $x_i \leq k$ ,

where k is a certain level of income, with  $k \leq B$ .

Let us denote by  $y = (y_i)_{i \in \mathbb{N}} = (y_1, y_2, ..., y_{\mathbb{N}})$  the distribution of income after receiving benefits from an MIP and by  $y = (y_i)_{i \in \mathcal{L}} = (y_1, y_2, ..., y_L)$  the distribution of income of the target population after receiving an MIP. Then,  $y_i = B$  for all the recipients of MIP  $(i \in \mathcal{P})$ , and  $y_i = x_i$  for the individuals who do not receive MIP  $(i \in \overline{\mathcal{P}})$ . The level of protection provided by an antipoverty program has usually been measured according to two different dimensions: the percentage of potential claimants receiving this benefit and

 $<sup>^{2}</sup>$  There could be targeting errors if non-eligible households receive benefits. To simplify the analysis and focus on the distributional and welfare effects, we assume that there is a perfect allocation of resources.

the economic sufficiency that it provides. The first of these dimensions represents the coverage provided by the program (C), and its measurement is relatively straightforward. Measuring coverage would consist of dividing the number of recipients registered in the administrative files by the number of potential recipients.

**Definition 2.** The coverage *C* provided by an MIP is

$$C = R/L$$
[1]

where R is the number of recipients and L is the number of potential claimants.

There are different alternatives for estimating *L* using household income surveys. Among the different possibilities, we can in a very restrictive way consider *k* in Definition 1 as receiving no income (k = 0). We can also define the target group as those considered to be poor, which is reflected by k = z where z is the poverty line.

Summarizing the second dimension into a single measure is somewhat harder. Many of the recent proposals in the comparative analysis of adequacy place more emphasis on relative generosity than on economic sufficiency. A general approach consists of relating the level of benefits to a single measure that represents average living standards, such as median income.

**Definition 3.** The adequacy *A* of an MIP is

$$A = B/z$$
[2]

the ratio between the level of guaranteed income and the poverty line (z). We will consider that z is chosen such that z>B.

Sometimes these two dimensions—population coverage and adequacy—may produce conflicting results. In periods of economic downturn, for instance, governments might decide to modify some parameters (e.g., benefit levels) but use others (e.g., the proportion of claimants that enter the program) to prevent an increase in the number of welfare recipients and thus an increase in spending. It is useful to observe whether there are simultaneous improvements in both dimensions, opposite trends or overall gains despite possible declines in one of them. In this sense, it is important to have a measure that combines the results of the two dimensions to offer an overall picture of the protection provided by the program. This measure would make it possible to jointly consider adequacy and population coverage when measuring differences in the protection provided by welfare programs across jurisdictions.

#### 2.2. Desirable properties

The measure that quantifies the protection provided by an MIP is a function, P, defined over the range of income distributions,  $\mathfrak{I}$ , that allocates a real number to each possible income distribution:  $P: \mathfrak{I} \to \mathbb{R}$ . In the following, we show a list of desirable properties for any measure that quantifies the protection provided by an MIP. These properties are important because they will allow us to give shape to a concept that has not been previously defined in the relevant economic literature.

**Property 1**. Monotonicity with respect to coverage.

For any income distribution y with an associated coverage index C, and given k,

$$\frac{\partial P(y)}{\partial C} > 0$$

the index associated with the income distribution after receiving an MIP should increase with the proportion of recipients over the potential claimants *C*, *ceteris paribus*. That is, the greater the proportion of recipients, all other things being equal, the more protection the MIP provides, and therefore the greater the value of the index.

Property 2. Monotonicity with respect to adequacy.

For any income distribution y with an associated adequacy index A, and given z,

$$\frac{\partial P(y)}{\partial A} > 0$$

The higher the income provided by an MIP (B), keeping *z* and *C* fixed, the higher the index. That is, the higher the income provided by a MIP, all else being equal, the higher the protection provided.

Property 3. Replication invariance.

Let  $(y_i)_{i \in \aleph}$  be the income distribution after receiving MIP. Consider a new population in which the income distribution  $(y_i)_{i \in \aleph}$  is generated by the k-fold replication of an original income distribution  $(y_i)_{i \in \aleph}$ , then

$$P((y_i)_{i\in\aleph^k}) = P((y_i)_{i\in\aleph})$$

The index should not change if it is calculated on an income distribution that is generated by the k-fold replication of an original income distribution. This property allows comparisons of indexes for different populations in which the number of individuals is different.

### Property 4. Anonymity.

Let  $(y_i)_{i \in \aleph}$ ,  $(y'_i)_{i \in \aleph}$  be two distributions such that y' is obtained from y by a permutation of incomes, then

$$P(y') = P(y)$$

because the information provided by y is the same as that provided by y'. Therefore, the only thing that matters is income.

#### Property 5. Focus.

Let  $(y_i)_{i \in \mathbb{N}}$ ,  $(y'_i)_{i \in \mathbb{N}}$  be two distributions such that  $y_i = y'_i \quad \forall i \in \mathcal{L}$ , then

$$P(y') = P(y)$$

The index should be independent of the incomes of individuals who are not in the target group (non-potential claimants).<sup>3</sup> This is analogous to the poverty focus axiom, which says that a poverty index is independent of non-poor incomes. This property allows us to compare the level of protection of an MIP in two different populations using a common criterion for defining potential claimants.

#### Property 6. Normalization.

If all the targeted individuals receive benefits from an MIP such that they reach an income level that represents a minimum living standard (e.g., the poverty line, B=z), then P(y) = 1, otherwise  $0 \le P(y) \le 1$ . This means that the maximum value of the index is obtained with an MIP that gives to all targeted individuals an income that corresponds to the minimum living standard.<sup>4</sup>

# Property 7. Scale invariance.

If  $\gamma$  is a positive scalar, then P( $\gamma y$ )=P(y). This implies that the function P(y) is homogeneous of degree 0 in the income distribution.

This property implies that the level of protection provided by a welfare program is essentially a relative concept and is independent of the absolute magnitude of the income notion.

<sup>&</sup>lt;sup>3</sup> There might be cases of some transfers going to non-potential claimants. One of the consequences of assuming the focus property is that this type of targeting error is considered here as an efficiency problem without distributional effects.

<sup>&</sup>lt;sup>4</sup> Remember that we are assuming that z is higher than B.

This property implies that the lower the initial level of income of the individual who receives the guaranteed income, the greater the index.

Let  $(x_i)_{i \in \mathbb{N}}$ ,  $(y'_i)_{i \in \mathbb{N}}$ ,  $(y''_i)_{i \in \mathbb{N}}$  be three distributions such that  $y''_l = B$  and  $y''_i = x_i, \forall i \neq l$  and  $i, l \in \mathcal{L}$ ; and y' is obtained from x in such a way that  $y'_j = B$  and  $y'_i = x_i, \forall i \neq j$  and  $i, j \in \mathcal{L}$ ; and  $x_j < x_l$ , then

$$P(y') > P(y'')$$

That is, transfers directed to poorer individuals are preferred and therefore associated with higher levels of the index.

Property 9. Preference for poorer recipients given a budget constraint

An immediate consequence of property 8 is that a higher value of the index entails higher spending. Taking this constraint into account, the total amount of transfers to be assigned (T) can be set as given. Given this fixed amount, the index is greater when the individual who receives the program is poorer.

Let  $(x_i)_{i \in \mathbb{N}}$ ,  $(y'_i)_{i \in \mathbb{N}}$ ,  $(y''_i)_{i \in \mathbb{N}}$  be three distributions such that y' is obtained from x in such a way that  $y''_l = B$  and  $y''_i = x_i, \forall i \neq l$  and  $i, l \in \mathcal{L}$ ; and  $y'_j = x_j + (B - x_l)$  and  $y'_i = x_i, \forall i \neq j$  and  $i, j \in \mathcal{L}$  and  $x_j < x_l$ , then

$$P(y') > P(y'')$$

This means that given two alternative programs that provide the same amount of transfers  $(B - x_l)$ , the one preferred is the one in which the recipients are poorer.<sup>5</sup>

<sup>&</sup>lt;sup>5</sup> We are aware that in this case individual j does not reach the guaranteed income after receiving the transfer. This is due to the budget constraint imposed.

Property 10. Preference for multiple small improvements.

Given a fixed amount of transfers *T* to be distributed, let  $(y'_i)_{i \in \aleph}$  and  $(y''_i)_{i \in \aleph}$  be two distributions derived from two alternative MIPs, such that in y', *T* is equally distributed among *m* recipients—all of whom have the same income before receiving an MIP and then income *B* after receiving it—while in y'', the whole amount of transfers *T* is assigned to *s* of the *m* recipients, being s < m. Then

That is, the level of protection of an MIP that provides income *B* to *m* individuals at cost *T* should be greater than it would be if a small proportion of those individuals receive *T* as MIP. It is preferable to achieve small improvements for many people than a larger improvement in a smaller set of individuals.<sup>6</sup>

# Property 11. Subgroup consistency

Given a society composed of G subgroups, exhaustive and mutually exclusive, it holds that

$$P(n, y) = \Phi(P(n^1, y^1), P(n^2, y^2), \dots, P(n^G, y^G))$$

where  $\Phi$  is a continuous and strictly increasing function in each of the first G arguments.

This property implies that that the index can be expressed as a function of the indices of different groups. It requires the index to be strictly increasing in the partial groups corresponding to different partitions instead of requiring additive decomposability. Note that this property does not indicate anything about the magnitude of the total increase in the global index as a result of an increase in the index for each of the groups and is therefore only a directional correspondence.

<sup>&</sup>lt;sup>6</sup> Again, in this case only one of the MIP alternatives guarantees income B to s individuals, while the other MIP guarantees a lower level of benefit to more individuals.

This property can be summarized as follows: if a given population subgroup's index increases, and everything else remains constant, then the index for the whole population should increase.

# **3. AN INDEX OF MINIMUM INCOME PROTECTION**

In this section, we build an index to measure the protection provided by an MIP. An appealing approach to constructing this index is to first specify a social welfare function to be used in comparing distributions and then derive an appropriate index to evaluate the protection provided by the MIP. Such an approach resembles the one proposed by Sen-Atkinson-Dalton for inequality indexes. The index is characterized by standard assumptions about social welfare functions, and it is later shown to satisfy the properties previously defined. In adopting the welfare-based approach to the index, the first step is to specify a social evaluation function. We consider a function defined over the potential claimants.<sup>7</sup> We assume that the social welfare function (SWF) is individualistic, strictly increasing, symmetric, and additive (Lambert, 1993; Cowell, 1995).

Individualistic means that our SWF depends on individuals' utilities and on nothing else. Given that our SWF is strictly increasing, social welfare increases when, ceteris paribus, any potential claimant's income rises. Our SWF is symmetric and, therefore, any permutation of individuals does not change social welfare (i.e., individuals play identical roles). Additivity implies that our SWF can be expressed as the summation of individuals' utilities, each individual having her own utility function that depends only on her income.

As a consequence of these properties, our SWF can be written as the summation of individuals' utilities using an increasing social utility function, which is shared by all of them and depends only on the individuals' own incomes (Cowell, 1995). To fully characterize our index, we need to impose two additional conditions on the social utility function. First, we assume that it is strictly concave. In other words, an increase in an individual's income, all else being equal, entails a larger change in the social utility function the lower the initial income of that individual. We also assume that the social

<sup>&</sup>lt;sup>7</sup> Because we are interested in evaluating MIPs, we do not consider the entire population and instead focus only on potential claimants. This allows the index to fulfill the focus property.

marginal utility has constant elasticity, given by the parameter  $\alpha$ -1, so that if an individual's income increases by 1 percent, then the marginal utility increases by  $\alpha$ -1 percent no matter her initial income level. Although an assumption of constant (relative) inequality aversion, often used in the literature on income inequality, is not necessary to define a reasonable index, we impose it to restrict the class of possible measures to a family parameterized by an inequality aversion parameter, which seems especially appealing given its intuitive interpretation.

This brings us to the following family of social utility functions with the common normalization parameters (Cowell, 1995):

$$U_{\alpha}\left(\frac{y_{i}}{z}\right) = \begin{cases} \frac{1}{\alpha} \left(\frac{y_{i}}{z}\right)^{\alpha} & \alpha \neq 0\\ ln\left(\frac{y_{i}}{z}\right) & \alpha = 0 \end{cases}$$
[3]

Then, our social welfare function takes the form<sup>8</sup>

$$W(y|z,\alpha) = \frac{1}{\alpha} \sum_{i=1}^{L} \frac{\left(\frac{y_i}{z}\right)^{\alpha}}{L}, \quad 0 \neq \alpha < 1$$
[4]

and

$$W(y|z,0) = \sum_{i=1}^{L} \frac{\ln\left(\frac{y_i}{z}\right)}{L}, \text{ when } \alpha = 0$$
[5]

The coefficient  $\alpha$  measures inequality aversion. We can calculate the equally distributed income level  $\xi$  such that  $W(y|z, \alpha) = W(\xi 1|z, \alpha)$  where 1 is a vector of ones. In these terms, a measure summarizing the protection provided by a minimum income program can be defined as:

<sup>&</sup>lt;sup>8</sup> We compare incomes and a certain living standard (e.g., the poverty line). As stressed by Ravallion (2007), we can assume that the government's aim for the program should be to provide cash transfers that are sufficient to bring everyone in each jurisdiction up to an income level that is sufficient to not be deemed "poor".

$$IMIP(y|Z,\alpha) = \frac{\xi}{Z}$$
[6]

The rationale for dividing  $\xi$  by z is that a situation in which an MIP is no longer needed is defined by all incomes being equal to z. Therefore,  $IMIP(y|z, \alpha)$  measures the ratio of the equally distributed income level in the current situation (after receiving MIP) and the ideal situation in which every potential claimant gets z. In other words, it is the proportionate welfare loss caused by having potential claimants with incomes below this threshold after receiving benefits.<sup>9</sup>

This comes down to

$$IMIP(y|z,\alpha) = \left\{ \frac{1}{L} \sum_{i=1}^{L} \left( \frac{y_i}{z} \right)^{\alpha} \right\}^{1/\alpha}, \quad 0 \neq \alpha < 1$$
[7]

that is the generalized mean of a function of normalized incomes of potential claimants. For  $\alpha = 1$  it is the arithmetic mean, for  $\alpha = -1$  it is the harmonic mean, and for  $\alpha = -\infty$  it is the minimum.

And this comes down to

$$IMIP(y|z,0) = \left\{ \prod_{i=1}^{L} \left(\frac{y_i}{z}\right) \right\}^{1/L}, \text{ when } \alpha = 0$$
[8]

In this case it is the geometric mean of a function of normalized incomes of potential claimants.

In a general setting in which an MIP provides an income level B, this expression can be decomposed into:

<sup>&</sup>lt;sup>9</sup> *IMIP*( $y|z, \alpha$ ) allows us to evaluate the distribution of incomes of potential claimants after receiving an MIP. If we want to assess the impact of an MIP, we should compare *IMIP*( $y|Z, \alpha$ ) and *IMIP*( $x|Z, \alpha$ ); that is *IMIP*(y) before and after receiving MIP.

$$IMIP(y|z,\alpha) = \left\{ \frac{1}{L} \sum_{i \in R} \left(\frac{B}{Z}\right)^{\alpha} + \frac{1}{L} \sum_{\substack{i \notin R \\ i \in \mathcal{L}}} \left(\frac{y_i}{Z}\right)^{\alpha} \right\}^{\frac{1}{\alpha}} = \left\{ \left(\frac{B}{Z}\right)^{\alpha} \frac{R}{L} + \frac{1}{L} \sum_{\substack{i \notin R \\ i \in \mathcal{L}}} \left(\frac{y_i}{Z}\right)^{\alpha} \right\}^{\frac{1}{\alpha}}, \quad [9]$$

and to

$$IMIP(y|z,0) = \left\{ \left(\frac{B}{Z}\right)^{R} \prod_{i \notin R, i \in \mathcal{L}} \left(\frac{y_{i}}{Z}\right) \right\}^{1/L}, when \ \alpha = 0$$
[10]

In addition to the possibilities of interpreting changes in any dimension (adequacy and coverage) in terms of social welfare and of assigning different weights to each dimension that represent alternative value judgments, the properties we have imposed on our SWF are consistent with the properties we want the index to satisfy for  $0 \le \alpha < 1$ , as defined in the previous section. (Proofs in the appendix).

Sometimes, these programs are targeted to households suffering from extreme poverty being a special case those who have no income. If we consider potential claimants as those with no income and that recipients receive B as minimum income protection, the index after receiving the benefit is:

$$IMIP(y|z,\alpha) = \left\{ \frac{1}{L} \sum_{i \in R} (B/z)^{\alpha} \right\}^{1/\alpha} = \frac{B}{Z} \left(\frac{R}{L}\right)^{1/\alpha}, 0 \neq \alpha \le 1$$
$$IMIP(y|z,0) = 0, \text{ when } \alpha = 0$$
[11]

In this case,  $IMIP(y|z, \alpha)$  is the adequacy rate weighted by a function of the coverage rate,  $(R/L)^{1/\alpha}$ . A remarkable advantage of this index is that depending on  $\alpha$ , different

weights can be given to the coverage rates to summarize alternative value judgments of the relevance of adequacy and coverage in the social welfare measurement of minimum income protection. Positive and lower values of  $\alpha$  represent larger weightings of coverage rates.

#### 4. AN EMPIRICAL ILLUSTRATION

# 4.1. Data

We use data from Spanish welfare programs to test the sensitivity and robustness of the proposed approach. The primary reason for this choice is that minimum income protection in Spain is completely decentralized. Since their beginning, regional initiatives have been handicapped by serious problems in co-ordination and financing without any type of transfers to the regions from the central government. This shortcoming, over and above the natural regional differences, has produced a mosaic of highly varied schemes with a striking disparity of regulations and results and, above all, a certain widening of the differences that the poorest citizens experience regarding rights and resources.

The number of people involved represents the growing scope of the programs and their contribution to regional social policy. Whatever the case, in terms of the possible lessons this extreme model of fiscal federalism may offer, the key question is the extent of inequality in the protection offered by each region. As previously mentioned, there are two possible ways to look at the different inequality sources embedded in this design: adequacy and coverage of potential claimants.

Adequacy rates can be estimated by comparing the level of benefits in each region with the national poverty line<sup>10</sup>. Every year, the Department of Social Services and Equality of the central government publishes the number of benefit recipients and the amounts set for these benefits in each region. Table 1 provides a thumbnail sketch of the existing differences in the level of benefits for different types of households. While benefits are

<sup>&</sup>lt;sup>10</sup> A similar analysis could be conducted by considering regional instead of national thresholds. National lines allow us to establish a general scheme of how regions compare with national standards. In contrast, regional poverty lines allow us to gauge intraregional poverty according to cost-of-living differences.

considerably above the average in some regions, they are clearly below in others. These differences are particularly striking in the case of households with more members.

# [TABLE 1]

Given the level of benefits in each region for different types of households, adequacy rates can be estimated by comparing these amounts with the corresponding poverty lines. We use the Spanish sample from the 2008 and 2014 EU Survey on Income and Living Conditions (EU-SILC). This sample consists of information about 13,000 households comprising approximately 37,000 individuals. The income variable we use to set the poverty line is annual disposable income. It includes household monetary income after direct taxes and social security contributions, including earnings, cash property income, regular social transfers, private transfers and other cash income. It does not include in-kind earnings or imputed rents. This variable is adjusted for each household by the so-called modified OECD equivalence scale.<sup>11</sup>

The coverage rates can be calculated by comparing the number of current recipients with the corresponding number of potential claimants in each region. Annual data on the distribution of recipients across regions are also provided by the Department of Social Services and Equality of the central government. Figure 1 illustrates how the recipients are unevenly distributed by regions. This marked concentration is rooted not in the changes that took place in the crisis or in how each region has been able to meet the dramatic increase in the demand for these services but in the initial design of these schemes. They are dependent on the initiative of each regional government and thus largely on the available resources and the regions' unequal budgetary abilities.

#### [FIGURE 1]

In any case, the key issue here is not how many recipients there are in each region, a number that may depend on the size of the population or the shape of the income distribution, but how each regional program provides coverage to potential recipients.

<sup>&</sup>lt;sup>11</sup> The modified OECD equivalence scale assigns a value of 1 to the first adult in the household, 0.5 to every other adult, and 0.3 to each child.

One way to estimate the number of potential recipients is to identify eligible recipients in the abovementioned survey. The problem with this option is that the recipients of these benefits are poorly coded in the survey and, in addition, data about household incomes suffers from major problems of underreporting. One possible alternative is considering as potential claimants all households in each region that do not earn any income from labor and do not receive any benefits from Social Security transfers (i.e., pensions or other benefits) or from unemployment insurance or assistance payments. In theory, these poor households should be receiving benefits from the last safety net or, in other words, the minimum income protection provided by each region.<sup>12</sup> The data we use to estimate potential claimants in this way come from the Spanish Labor Force Survey. This survey is conducted quarterly by the National Institute of Statistics (INE). The sample size of the survey is 60,000 households comprising approximately 190,000 individuals.

### 4.2. Results

Table 2 presents the adequacy and coverage rates for all regions in 2007 and 2013. The table gives general support to the notion that there are very marked differences across the regions for both dimensions. Adequacy rates are twice higher in regions such as Basque Country and Navarra than in others such as Valencia and la Rioja. Differences in coverage rates are even larger. In two regions, the number of recipients is higher than the number of no income households<sup>13</sup>, but in others, the programs cover less than 10 percent of these households. It must be noted that some regional governments pay benefits that act as tax credits to households whose earnings are clearly insufficient to meet their family needs while others restrict these benefits to households with no labor income.

# [TABLE 2]

<sup>&</sup>lt;sup>12</sup> The actual questions in the Labor Force Survey from which this information comes do not completely rule out the possibility that some individuals in the sample could receive some income from independent means (i.e., renters). In any case, classification within the no income households implies that a person cannot cohabit with any others who are receiving labor income, unemployment benefits or any type of pension. Using data from the Spanish sample of EU-SILC, we have determined that only 0.6 percent of households without any other income would have property and investment income over the annual poverty threshold.

<sup>&</sup>lt;sup>13</sup> Coverage rates are top coded at 1. We impose that regions with a number of recipients higher than the one corresponding to no income households have coverage rates equal to 1.

While adequacy rates increased in most regions during the economic crisis, in some cases the programs were overwhelmed by the growing demand for benefits. The result was a significant reduction in the percentage of potential recipients covered by the programs in many regions. In other cases, governments made a great effort to offset the increase in poverty. In any case, changes in adequacy must be interpreted by taking into account the relative approach in the measurement of poverty. Because poverty thresholds fell drastically with the reduction in median income during the crisis, the simple act of maintaining the level of benefits—or even slightly reducing it—almost automatically improved adequacy rates. It is rather remarkable, then, that the relationship between the amounts and thresholds fell in some regions in the time period under study.

Above all, the most noteworthy result in the analysis of adequacy and coverage rates is the change in regional rankings that occurs when we move from one indicator to another. While there are very few changes in the positions of the regions that have better indicators in the two dimensions, movement in the rankings is much greater for others, especially for regions at the bottom of the respective distributions.

# [TABLE 3]

These differences complicate the overall assessment of the protection in terms of social welfare provided by minimum income programs in each region. One way to address this problem in a way that jointly considers the two dimensions is by using the measure proposed in the previous section. Table 3 presents estimates of the index for the different regions in 2007 and 2013 using a couple with two children as a unit of reference. Several points are worth mentioning. First, by giving positive and low values to  $\alpha$ , the index becomes much lower in the regions with the lowest coverage rates and much higher in the regions with the highest rates.<sup>14</sup> Second, in two-thirds of the regions, the programs improved both the protection provided and the social welfare levels expressed in the terms

<sup>&</sup>lt;sup>14</sup> Because weightings,  $\left(\frac{R}{L}\right)^{1/\alpha}$ , are an increasing and concave function of  $\alpha$ , positive but close to zero values of this parameter can yield almost zero values for the proposed measure (IMIP) when R < L. If  $\alpha$  approaches 0, the weighting scheme assigns a value to the coverage rate such that the index shows very low values. It is only in the case of values of R close to L that adequacy has a determining role. In that case, the value given to  $\alpha$  is less important because the weighting corresponding to the coverage rates is going to be very close to 1.

set out above. In contrast, in other regions, the overall levels of social welfare achieved by these programs decreased during the economic crisis.

Third, while the index drastically changes as different values of  $\alpha$  are considered, there are not, however, significant variations in the regional rankings when  $\alpha$  is reduced from 1 by less than 0.5. In that scenario, nine regions change their order in the ranking but never by more than two positions. When  $\alpha$  changes from 1 to 0.25, more re-rankings occur and affect thirteen of the seventeen regions in a range that goes from one to six positions.

Therefore, taking into account how these programs provide coverage to no income households qualifies decisively the overall assessment of minimum income schemes in terms of social welfare. Assessing welfare gains by focusing only on standard measures of adequacy may introduce an important bias, making difficult the proper identification of the real differences between programs of different jurisdictions. Our measure allows the calibration of the relevance of both dimensions depending on the value judgments of the social evaluator.

#### **5. CONCLUSION**

Relatively little is known about how to comprehensively assess the protection provided by welfare programs in a decentralized framework. In this paper, we propose a measure that uses a social welfare approach to evaluate the protection provided by minimum income programs. We have built a new index that combines adequacy (benefits over poverty thresholds) and coverage (proportion of recipients over potential claimants). Unlike most approaches that measure the differences between the protection provided by these programs across different jurisdictions, we analyze both dimensions using a unified framework.

The resulting family of indices can be interpreted in terms of social welfare. A contribution of this paper is the definition of a set of properties that should be taken into account when measuring the protection provided by the different territorial schemes. Some of the main properties of social welfare and inequality analysis have been extended to study possible differences in the outcomes of these programs. As a result, the indices

19

are characterized in terms of assumptions about social welfare functions, and we have shown how the defined properties are satisfied.

In our family of indices, the poorest recipients are considered more important than the other recipients and, given a group of people with the same income level, small improvements for many individuals are preferred to a large improvement for one individual. Additionally, the indices take into account coverage and adequacy rates in such a way that the higher the rate is for one of these, the higher the level of the indices. A remarkable advantage of the approach is that different weights can be given to the coverage rates to summarize alternative value judgements about the relevance of each dimension in the social welfare measurement of minimum income protection. We believe this is an important contribution from the perspective of the decision-making process.

As an illustration, we have analyzed the Spanish regional levels of coverage, adequacy and the proposed measure (IMIP) to test the possibilities of our approach. We observe that there are changes in regional rankings when we move from coverage to adequacy rates. These differences complicate the overall assessment of the social protection provided by minimum income programs in terms of social welfare in each region. When we assign a higher weight to the coverage rates instead of considering only adequacy outcomes, re-rankings affect two thirds of the regions in a range that goes from one to six positions. Therefore, assessing welfare gains by focusing only on standard measures of adequacy makes difficult the proper identification of the real differences between programs of different jurisdictions. Our measure allows the calibration of the relevance of adequacy and coverage by considering different levels of sensitivity to changes in the lower part of the income distribution.

In short, from a policy point of view, it is important to analyze not only the evolution of the basic outcomes of regional welfare systems but also aspects of the distributional change that may take place when the different dimensions are jointly considered. Our proposal may help in achieving a better understanding of the differences in performance of regional welfare programs by considering adequacy, coverage and measures that combine both dimensions under a unified framework.

20

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# Appendix: Proving that IMIP satisfies the basic properties

It is easy to deduce the fulfillment of properties 2-6 from the definition of the index. Property 1 (monotonicity with respect to coverage) is satisfied for  $\alpha > 0$ . Therefore, we restrict to  $0 < \alpha < 1$ .

# Property 7. Scale invariance.

The function P(y) is homogeneous of degree 0 if z is defined as a function of y of degree 1. This is, for instance, the case when the poverty line is defined as a proportion of median income or as a percentage of the mean of the distribution. These are the most common references for the standard of living in developed countries.

Property 8. General preference for poorer recipients.

We have:

$$IMIP(y'|z,\alpha) = \left\{ \frac{1}{L} \left(\frac{B}{z}\right)^{\alpha} + \frac{1}{L} \sum_{\substack{i \in \mathcal{L} \\ i \neq j}} \left(\frac{x_i}{z}\right)^{\alpha} \right\}^{1/\alpha}$$

and

$$IMIP(y''|z,\alpha) = \left\{ \frac{1}{L} \left(\frac{B}{z}\right)^{\alpha} + \frac{1}{L} \sum_{\substack{i \in \mathcal{L} \\ i \neq l}} \left(\frac{x_i}{z}\right)^{\alpha} \right\}^{1/\alpha}$$

We know that  $\frac{1}{L} \sum_{\substack{i \in \mathcal{L} \\ i \neq j}} \left(\frac{x_i}{z}\right)^{\alpha} - \frac{1}{L} \sum_{\substack{i \in \mathcal{L} \\ i \neq l}} \left(\frac{x_i}{z}\right)^{\alpha} = \frac{1}{L} \frac{x_l - x_j}{z} > 0$ . Therefore,  $IMIP(y'|z, \alpha) > IMIP(y''|z, \alpha)$ .

# Property 9. Preference for poorer recipients given a budget restriction.

To prove it, we rewrite  $B=x_l + T$ , then  $y'_j = x_j + T$ , therefore

$$IMIP(y'|z,\alpha) = \left\{ \frac{1}{L} \left( \frac{x_j + T}{z} \right)^{\alpha} + \frac{1}{L} \sum_{\substack{i \in \mathcal{L} \\ i \neq j}} \left( \frac{x_i}{z} \right)^{\alpha} \right\}^{1/\alpha}$$

and

$$IMIP(y''|z,\alpha) = \left\{ \frac{1}{L} \left( \frac{x_l + T}{z} \right)^{\alpha} + \frac{1}{L} \sum_{\substack{i \in \mathcal{L} \\ i \neq l}} \left( \frac{x_i}{z} \right)^{\alpha} \right\}^{1/\alpha}$$

We know that  $\frac{1}{L} \sum_{\substack{i \in L \\ i \neq j}} \left( \frac{x_i}{z} \right)^{\alpha} - \frac{1}{L} \sum_{\substack{i \in L \\ i \neq l}} \left( \frac{x_i}{z} \right)^{\alpha} = \frac{1}{L} \frac{(x_l - x_j)}{z}$  and given the concavity imposed on the utility function  $\frac{1}{L} \left[ \left( \frac{x_j + T}{z} \right)^{\alpha} - \left( \frac{x_j}{z} \right)^{\alpha} \right] > \frac{1}{L} \left[ \left( \frac{x_l + T}{z} \right)^{\alpha} - \left( \frac{x_l}{z} \right)^{\alpha} \right]$  for  $0 < \alpha < 1$ . Therefore,  $IMIP(y'|z, \alpha) > IMIP(y''|z, \alpha)$ .

For = 0:

$$IMIP(y'|z,\alpha) = \left\{ \left(\frac{x_j + T}{z}\right) \left(\frac{x_l}{z}\right) \prod_{i \notin R, i \in \mathcal{L}, i \neq j, l} \left(\frac{x_i}{z}\right) \right\}^{1/L}$$

$$IMIP(y''|z,\alpha) = \left\{ \left(\frac{x_l + T}{z}\right) \left(\frac{x_j}{z}\right) \prod_{i \notin R, i \in \mathcal{L}, i \neq j, l} \left(\frac{x_i}{z}\right) \right\}^{1/L}$$

We know that  $\left(\frac{x_j+T}{z}\right)\left(\frac{x_l}{z}\right) > \left(\frac{x_l+T}{z}\right)\left(\frac{x_j}{z}\right)$ . Therefore, IMIP(y'|z, 0) > IMIP(y''|z, 0).

Property 10. Preference for multiple small improvements.

To prove it, we consider, without loss of generality, that there are *m* potential claimants with income  $x_i$ . Then, multiple small improvements of amount *T* results in *y*', with

$$IMIP(y'|z,\alpha) = \left\{ \frac{m}{L} \left( \frac{x_j + T}{z} \right)^{\alpha} + \frac{1}{L} \sum_{\substack{i \notin R \\ i \in \mathcal{L} \\ i \neq j}} \left( \frac{x_i}{Z} \right)^{\alpha} \right\}^{1/\alpha}$$

And s improvements of the same total amount (mT) results in y", with

$$IMIP(y''|z,\alpha) = \left\{ \frac{s}{L} \left( \frac{x_j + \frac{mT}{s}}{z} \right)^{\alpha} + \frac{(m-s)}{L} \left( \frac{x_j}{z} \right)^{\alpha} + \frac{1}{L} \sum_{\substack{i \notin R \\ i \notin J \\ i \neq j}} \left( \frac{x_i}{z} \right)^{\alpha} \right\}^{1/\alpha}$$

The last addend in each expression cancels out. Given that the utility function is concave and that  $\frac{x_j+T}{z}$  can be rewritten as  $\frac{(m-s)x_j}{mz} + s\frac{x_j+\frac{mT}{s}}{mz}$ , it follows that  $\left(\frac{x_j+T}{z}\right)^{\alpha} > \frac{m-s}{m}\left(\frac{x_j}{z}\right)^{\alpha} + \frac{s}{m}\left(\frac{x_j+\frac{mT}{s}}{z}\right)^{\alpha}$ . Then,  $\left\{\frac{m}{L}\left(\frac{x_j+T}{z}\right)^{\alpha}\right\}^{1/\alpha} > \left\{\frac{s}{L}\left(\frac{x_j+\frac{mT}{s}}{z}\right)^{\alpha} + \frac{(m-s)}{L}\left(\frac{x_j}{z}\right)^{\alpha}\right\}^{1/\alpha}$  and  $IMIP(y'|z,\alpha) > IMIP(y''|z,\alpha)$ 

The same applies for  $\alpha = 0$ 

Multiple small improvements of amount T result in y', with

$$IMIP(y'|z,\alpha) = \left\{ \left(\frac{x_j + T}{z}\right)^m \prod_{i \notin R, i \in \mathcal{L}, i \neq j} \left(\frac{x_i}{Z}\right) \right\}^{1/L}$$

And s improvements of the same total amount (mT) results in y", with

$$IMIP(y''|z,\alpha) = \left\{ \left( \frac{x_j + \frac{mT}{s}}{z} \right)^s \left( \frac{x_j}{z} \right)^{m-s} \prod_{i \notin R, i \in \mathcal{L}, i \neq j} \left( \frac{x_i}{z} \right) \right\}^{1/L}$$

As the last component in each expression cancels out, and given that

$$\left(\frac{x_j+T}{z}\right)^m = \sum_{k=0}^s \binom{m}{k} \left(\frac{x_j}{z}\right)^{m-k} \left(\frac{T}{z}\right)^k + \delta \quad \text{with } \delta > 0$$

and

$$\left(\frac{x_j + \frac{mT}{s}}{z}\right)^{ms} = \sum_{k=0}^{s} {\binom{s}{k} \left(\frac{x_j}{z}\right)^{m-k} \left(\frac{mT}{s}}{z}\right)^k}$$

to prove the property it is sufficient if the following expression is satisfied

$$\binom{m}{k} > \binom{s}{k} \left(\frac{m}{s}\right)^k, \ 0 \le k \le s$$

As  $\binom{m}{k} = \frac{m (m-1)...(m-k+1)}{k!}$  and  $\binom{s}{k} \left(\frac{m}{s}\right)^k = \frac{s (s-1)...(s-k+1)}{k!} \left(\frac{m}{s}\right)^k$  after some manipulation, we find that

$$\frac{m\left(m-1\right)\ldots\left(m-k+1\right)}{m^{k}} > \frac{s\left(s-1\right)\ldots\left(s-k+1\right)}{s^{k}}$$

that is satisfied for s<m, and consequently

$$\left(\frac{x_j+T}{z}\right)^m > \left(\frac{x_j+\frac{mT}{s}}{z}\right)^{ms} \forall s < m,$$

which completes the proof.

**Property 11**. Subgroup consistency is satisfied because IMIP is an increasing transformation of canonical indices (see Foster and Shorrocks, 1991).

		2007		2013				
Region	Single person	Couple, 2 children	Single-		Couple	Single-		
			parent,	Single person	2 children	parent,		
		2 chinarch	2 children		2 children	2 children		
Andalusia	353.8	490.7	445.1	400.1	555.0	503.3		
Aragón	336.0	629.0	524.2	441.0	749.7	661.5		
Asturias	396.7	610.9	547.4	443.0	682.1	611.3		
Balearic Islands	364.5	583.2	546.7	425.7	681.1	638.6		
Canary Islands	342.8	410.5	376.6	472.2	584.0	534.3		
Cantabria	286.8	418.2	383.6	426.0	585.8	532.5		
Castile-La Mancha	349.4	464.8	426.3	372.8	454.8	413.8		
Castile and León	374.4	499.2	464.3	426.0	639.0	596.4		
Catalonia	385.0	514.2	473.8	423.7	589.6	534.3		
Extremadura	374.4	494.2	454.3	399.4	585.8	532.5		
Galicia	374.4	524.2	484.2	399.4	516.5	463.3		
Madrid	340.0	578.0	510.0	375.6	532.5	532.5		
Murcia	300.0	498.0	422.0	300.0	498.0	442.0		
Navarre	456.5	656.2	599.1	548.5	898.0	832.8		
Basque Country	585.6	831.9	818.6	662.5	941.1	941.1		
Rioja	335.4	518.0	464.7	372.8	372.8	372.8		
Valencia	364.5	414.5	400.5	385.2	434.9	416.2		
Mean	371.8	537.4	490.7	427.9	605.9	562.3		

Table 1.	Minimum 1	Income B	senefits in	Spanish [	<b>Regions</b> ,	2007 a	and 2013 (	euros)
				1				. /

**Source:** *El Sistema público de Servicios Sociales. Informe de Rentas Mínimas de Inserción.* Ministerio de Sanidad, Servicios Sociales e Igualdad.

Rank	nk Coverage rate, 2007		Adequacy rate, 2007		Coverage rate, 2013		Adequacy rate, 2013	
1	Basque Country	1.00	Basque Country	0.66	Basque Country	1.00	Basque Country	0.77
2	Asturias	1.00	Navarre	0.52	Navarre	1.00	Navarre	0.73
3	Cantabria	0.73	Aragón	0.50	Asturias	0.68	Aragón	0.61
4	Navarre	0.51	Asturias	0.48	Cantabria	0.61	Asturias	0.55
5	Galicia	0.38	Balearic Islands	0.46	Rioja	0.58	Balearic Islands	0.55
6	Murcia	0.36	Madrid	0.46	Aragón	0.51	Castile and León	0.52
7	Andalusia	0.29	Galicia	0.42	Castile and León	0.34	Catalonia	0.48
8	Madrid	0.26	Rioja	0.41	Galicia	0.33	Cantabria	0.48
9	Catalonia	0.24	Catalonia	0.41	Andalusia	0.32	Extremadura	0.48
10	Aragón	0.14	Castile and León	0.40	Catalonia	0.26	Canary Islands	0.47
11	Castile and León	0.14	Murcia	0.40	Madrid	0.23	Andalusia	0.45
12	Balearic Islands	0.11	Extremadura	0.39	Valencia	0.14	Madrid	0.43
13	Canary Islands	0.11	Andalusia	0.39	Canary Islands	0.12	Galicia	0.42
14	Valencia	0.09	Castile-La Mancha	0.37	Murcia	0.12	Murcia	0.40
15	Rioja	0.09	Cantabria	0.33	Balearic Islands	0.09	Castile-La Mancha	0.37
16	Extremadura	0.06	Valencia	0.33	Extremadura	0.08	Valencia	0.35
17	Castile-La Mancha	0.04	Canary Islands	0.33	Castile-La Mancha	0.05	Rioja	0.30

 Table 2. Adequacy and coverage rates (couple, 2 children)

		2007			2013	
Region	$(\alpha = 0.5)$	$(\alpha = 0.75)$	$(\alpha = 1)$	$(\alpha = 0.5)$	$(\alpha = 0.75)$	$(\alpha = 1)$
Andalusia	0.00	0.03	0.07	0.00	0.05	0.10
Aragón	0.00	0.01	0.04	0.04	0.16	0.25
Asturias	0.49	0.49	0.49	0.12	0.26	0.33
Balearic Islands	0.00	0.01	0.03	0.00	0.00	0.02
Canary Islands	0.00	0.00	0.02	0.00	0.01	0.03
Cantabria	0.10	0.18	0.22	0.07	0.18	0.25
Castile-La Mancha	0.00	0.01	0.03	0.01	0.06	0.12
Castile and León	0.00	0.00	0.00	0.00	0.00	0.01
Catalonia	0.00	0.02	0.06	0.00	0.03	0.08
Extremadura	0.00	0.00	0.01	0.00	0.00	0.02
Galicia	0.01	0.06	0.11	0.00	0.05	0.10
Madrid	0.00	0.03	0.07	0.00	0.02	0.06
Murcia	0.01	0.05	0.10	0.00	0.01	0.02
Navarre	0.04	0.14	0.21	0.73	0.73	0.73
Basque Country	0.66	0.66	0.66	0.77	0.77	0.77
Rioja	0.00	0.00	0.02	0.03	0.10	0.15
Valencia	0.00	0.00	0.01	0.00	0.01	0.02

# Table 3. Results of the index (couple, 2 children)



# Figure 1. Number of recipients in each region as a percentage of total population,

**Source:** *El Sistema público de Servicios Sociales. Informe de Rentas Mínimas de Inserción.* Ministerio de Sanidad, Servicios Sociales e Igualdad.