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Abstract

Economic insecurity is a term used to describe the uncertainty surrounding economic aspects of people's lives. Clearly, this is a multi-faceted issue and a comprehensive formal definition that subsumes all possible aspects of it is likely to remain difficult to be agreed upon for some time to come. We characterize a class of individual economic insecurity measures based on variations in economic resources. The measures involve three easily interpretable parameters and can be computed using currently available household longitudinal data. Our proposal provides a simple and intuitively appealing criterion to assist policy makers in assessing and ameliorating economic insecurity.

Keywords: Insecurity, Resource Variations, Geometric Discounting, Economic Index Numbers.

JEL Classification: D63.

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1 Introduction

Economic insecurity has received an increasing amount of public attention in the recent past and appears more and more frequently in policy debates. For example, the Great Recession, increasing job instability and job losses for many, a marked decline of the middle class, and numerous home foreclosures along with stagnant housing markets have affected the lives of a large number of individuals. As a consequence, household expectations regarding the general economic situation, the distinct danger of future unemployment, their financial situation and savings have dropped sharply, as recorded by the Consumer Economic Sentiment indicator of the European Commission. There is by now widespread agreement that economic instability is affecting people's lives considerably. Individuals feel economically insecure, a sentiment which is exacerbated by marked changes in the life cycle and unprecedented demographic changes, particularly those triggered by aging populations and mass migrations. These observations lead to the inevitable conclusion that we live in economically insecure times and that this situation is likely to continue for the foreseeable future.

The negative consequences of this development on people's wellbeing are many and diverse. Increased obesity (Smith, Stillman and Craig, 2013), rising suicide rates (Reeves, McKee and Stuckler, 2014), a deterioration of mental health (Rohde, Tang, Osberg and Rao, 2016) as well as long-lasting serious deficits in child and youth development (Kalil, 2013) have all been attributed, at least in part, to the devastating impact of increased economic insecurity.

Economic insecurity appears to be a term used to describe the uncertainty surrounding economic aspects of people's current lives. Clearly, economic insecurity is a multi-faceted issue and a comprehensive formal definition that subsumes all possible aspects of it is likely to remain difficult to be agreed upon for some time to come. A plausible definition of the term is that economic insecurity is related to the anxiety produced by the possible exposure to adverse economic events and by the anticipation of the difficulty to recover from them; the Business Dictionary (www.businessdictionary.com) defines economic security as "A situation of having a stable source of financial income that allows for the on-going maintenance of one's standard of living currently and in the near future." See Osberg (2015) for a thorough discussion.

There have been a few attempts to construct measures of economic insecurity. They include (i) the Economic Security Index by Hacker, Huber, Rehm, Schlesinger and Valletta (2010); (ii) a proposal by the International Labour Organization (2004) and by Osberg and Sharpe (2009); (iii) the index of Rohde, Tang and Rao (2014). The respective recommended measures can roughly be described as (i) the fraction of the population who experience a drop in disposable family income of at least 25% from the previous year and lack an adequate financial safety net; (ii) a weighted average of the 'scores' achieved by population groups in different attributes; (iii) the volatility that arises when incomes fall relative to the household's overall trend.

As opposed to some of the above, our focus is on individual measures of insecurity. Although we are well aware that the ultimate objective is to arrive at aggregate measures applicable to entire societies rather than merely to individuals, there are good reasons to restrict attention to the individual case first. Economic insecurity is a notion experienced at the individual level, just as is the sentiment of deprivation. As illustrated in Yitzhaki's (1979) seminal contribution and in most of the subsequent literature, it is plausible to start with the design of an individual value of deprivation, followed by an aggregation procedure to obtain a social deprivation index. The second step of aggregating individual levels of deprivation is typically achieved by employing a mean (the arithmetic mean, for instance) and the crucial problem is that of obtaining an individual measure in the first place. We follow this reasoning and proceed in an analogous manner here.

The availability of an individual measure has some attractive features. Given an individual insecurity index, it is possible to study its distribution over the entire population and analyze changes over time. The method allows for the intensity of insecurity in the tails of the distribution to be assessed (rather than just the prevalence) and, as such, the changing shape of the distribution provides useful information regarding the challenges that households face. In addition, this approach allows us to identify covariates of the index so that the individuals most exposed to economic insecurity can be identified, a fact that may be of great help in designing social safety nets and other aspects of economic policy. In a similar manner, the actions taken by insecure individuals and their choices can be further analyzed.

There are, of course, many aspects of life that may play an important role in assessing the economic insecurity faced by an agent. In an earlier contribution (Bossert and D'Ambrosio, 2013) we use a comprehensive notion of wealth as the relevant variable. By its nature, wealth confers economic security since it allows the individual to be prepared for emergencies and to consume out of it in case of severe illness or any other adverse situation caused by non-insurable risks. From an applied perspective, individual streams of wealth are rarely available with the consequence that wealth-based measures such as those proposed in Bossert and D'Ambrosio (2013) remain, at least for the time being, relatively difficult to implement. In longitudinal household data, it is standard practice to include information on household disposable income or household consumption. These are the best proxies of command over economic resources that researchers and policy makers can rely upon to evaluate the prospects of the individuals under analysis. In view of these considerations, we focus on the *resources* available to the individuals—whatever they may consist of—as the relevant variables and propose a measure of economic insecurity based on them.

Although it clearly is desirable to take into consideration additional variables of a different nature, we have not yet arrived at a satisfactory manner to aggregate these potentially very diverse attributes because many of them seem, to us at least, to be incommensurable. For instance, we find it difficult to formulate meaningful trade-offs between variations in wealth over time and sectoral unemployment rates or access to medical care, say. Our conclusion so far is that it may be considered too ambitions to come up with a single indicator in such circumstances and a more modest approach based on dominance criteria may be more suitable. We provide a more detailed discussion of this issue later in the paper.

What we are interested in is the subjective forecast of how well someone can handle a loss in the future. Past gains and losses determine the confidence an individual has today on how well she will manage if a similar situation arises. An individual who recovered from a negative shock (a first-down-then-up movement in resources) in the past is likely to be more confident when facing a loss: the encouraging experience reduces the person's insecurity. In contrast, someone who was never able to counterbalance a loss, or saw her resources increase and then decrease (a first-up-then-down movement), will consider a recovery to be less likely to occur in case of an adverse event. To capture this intuition, we focus on resource variations in the past up to the current period as the basic determinants of insecurity because these variations shape the extent of a person's self-confidence. Thus, the measures of individual insecurity we propose have as their domain resource streams of varying lengths. The length of these streams is not assumed to be fixed because individuals are of different (economic) ages in a given time period. Moreover, we allow resources to be negative, which is a realistic assumption. In our earlier contribution, we propose classes of linear measures of insecurity based on wealth streams and characterize them by means of sets of appealing axioms. These measures bear a formal resemblance to the generalized Gini indices familiar from the literature on the measurement of inequality. In particular, single-series Ginis and single-parameter Ginis (Donaldson and Weymark, 1980; Weymark, 1981; Bossert, 1990) have become standard tools in this area.

The application of one of the Gini-type measures alluded to above requires the choice of numerous parameter values. In particular, the main result reported in Bossert and D'Ambrosio (2013) involves two sequences of parameters—two parameter values (one for past gains, one for past losses) for each time period under consideration. Even if attention is restricted to a finite number of periods (which is a serious restriction because it is not clear by any means how many periods should be taken into consideration), this can amount to a rather formidable task. Thus, the flexibility afforded by this large class comes at a price: without further systematic restrictions, it may be difficult to make a sound choice of what may be considered 'suitable' parameter values. Moreover, the measures characterized in Bossert and D'Ambrosio (2013) fail to satisfy stationarity, a standard property in intertemporal economic models. Stationarity implies that no significance is attached to the way time periods are numbered and it is necessary to avoid behaviour where, with the mere passage of time, plans that seemed optimal yesterday need no longer be optimal today.

To address the above-described issues, we begin by stating a preliminary result that employs a system of axioms that differs from that of our earlier contribution. This leads to a characterization of a class of measures that are related but different from those characterized in Bossert and D'Ambrosio (2013). The first result of the present paper identifies the class of measures that can be expressed as a weighted sum of all period-to-period pairwise gains and losses, where the weights associated with the losses can differ form those assigned to the gains. Moreover, the weights are such that more recent gains (losses) are given higher weights than those that occur farther in the past. In contrast, the measures of Bossert and D'Ambrosio (2013) are such that the current level of the resource is added to the weighted sum of gains and losses. As a consequence, some members of our new class of measures are compatible with stationarity, whereas the measures in Bossert and D'Ambrosio (2013) are not.

To state our main result, we endow our model with more structure. In particular, we use a suitable formulation of the stationarity property that is familiar from the literature on intergenerational choice and a condition that we refer to as resource-variation monotonicity. The monotonicity property has an intuitive interpretation in the sense that it links the notion of economic insecurity to certain 'first-down-then-up' variations in economic resources, indicating the ability to recover from some losses, and to 'first-up-then-down' movements which are bound to induce a more pessimistic outlook. This feature will become clear once the property is defined formally later in the paper.

Our main result is the characterization of a class of individual insecurity measures the members of which are based on geometrically discounted resource differences. Only three parameters need to be chosen: a discount factor that is common to past gains and to past losses and two parameters that express the relative importance of aggregate losses and aggregate gains. A further result employs an additional axiom that gives priority to past losses over past gains, thereby providing a further restriction on the three parameters.

As is evident from our earlier remarks, we choose an axiomatic approach to the measurement of economic insecurity by formulating desirable properties of an individual index of individual insecurity and identifying the class of measures that satisfy all of these requirements. See Thomson (2001) for a thorough discussion of the usefulness of the axiomatic approach in numerous economic models.

The following section introduces the notation and our axioms. Section 3 contains the results and Section 4 provides a discussion of possible multi-criteria insecurity measures. Section 5 concludes. The proofs of our results are collected in the Appendix.

2 Preliminaries

We use $\mathbf{1}_m$ to denote the vector consisting of $m \in \mathbb{N} \setminus \{1\}$ ones. For any $T \in \mathbb{N}$, let $\mathbb{R}^{(T)}$ be the $(T+1)$-dimensional Euclidean space with components labeled $(T, \ldots, 0)$. Zero is interpreted as the current period and T is the number of past periods taken into consideration. We allow T to vary because people alive in the current period may have been born (or have become economic agents) in different periods. A measure of individual insecurity is a sequence of functions $I = \langle I^T \rangle_{T \in \mathbb{N}}$ where, for each $T \in \mathbb{N}$, $I^T\colon \mathbb{R}^{(T)} \to \mathbb{R}$. This index assigns a degree of insecurity to each individual resource stream $x = (x_T, \ldots, x_0) \in \mathbb{R}^{(T)}$. We allow resources to be negative. As can be seen from these definitions, we restrict attention to streams that involve at least one past period in addition to the current period; this is necessitated by the observation that our indexes are based on pairwise differences.

Our first axiom is a monotonicity property. First, it ensures that a gain in resources from the earliest period under consideration to the next is associated with a lower level of insecurity than a situation in which no change occurs between these two periods. Likewise, a loss in resources that occurs when moving from the earliest period to the following generates a higher level of insecurity compared to a move involving no change. Clearly, this property captures the notion of individual insecurity that is based on gains and losses.

Gain-loss monotonicity. For all $T \in \mathbb{N}$, for all $x \in \mathbb{R}^{(T-1)}$ and for all $a \in \mathbb{R}_{++}$,

$$I^T(x_{T-1} - a, x) < I^T(x_{T-1}, x) < I^T(x_{T-1} + a, x).$$

The following property ensures that a gain (loss) of a given magnitude reduces (increases) insecurity to a higher extent the closer to the present this gain (loss) occurs. Intuitively, this axiom requires that more recent experiences carry more weight than those that occur farther in the past. Thus, the requirement is very natural in our context.

Proximity monotonicity. For all $T \in \mathbb{N} \setminus \{1\}$, for all $t \in \{1, \ldots, T-1\}$ and for all $x \in \mathbb{R}^{(T)}$,

$$I^T(x_T, \ldots, x_{t+1}, x_{t+1}, x_{t-1}, \ldots, x_0) \geq I^T(x_T, \ldots, x_{t+1}, x_{t-1}, x_{t-1}, \ldots, x_0)$$
$$\Leftrightarrow \; x_{t+1} \geq x_{t-1}.$$

Homogeneity is a standard property in the theory of economic index numbers. It demands that if a resource stream is multiplied by a positive constant, insecurity is multiplied by the same constant.

Homogeneity. For all $T \in \mathbb{N}$, for all $x \in \mathbb{R}^{(T)}$ and for all $b \in \mathbb{R}_{++}$,

$$I^T(bx) = bI^T(x).$$

Our next property is that of translation invariance. It requires that if the same amount of the resource under consideration is added to the existing levels of the resource available in each period, the value of the insecurity measure is unchanged. As is the case for homogeneity, translation invariance is an axiom that is well-established in the literature.

Translation invariance. For all $T \in \mathbb{N}$, for all $x \in \mathbb{R}^{(T)}$ and for all $c \in \mathbb{R}$,

$$I^T(x + c\mathbf{1}_{T+1}) = I^T(x).$$

The final axiom used in our first result is a quasilinearity condition. It establishes a link between insecurity comparisons involving resource streams of different lengths. We use the term quasilinearity because of the structural similarity with quasilinear utility functions in consumer-demand theory; see, for example, Varian (1992, p. 154). In the insecurity context, quasilinearity states that the insecurity $I^T(x)$ associated with a resource stream

$x \in \mathbb{R}^{(T)}$ can be expressed as a quasilinear function involving the $T-1$ most recent resource levels $(x_{T-1}, \ldots, 0)$ and the resource difference $x_T - x_{T-1}$. The property is a variant of a well-known axiom phrased in the context of economic insecurity.

Quasilinearity. For all $T \in \mathbb{N} \setminus \{1\}$, there exists a function $F^T \colon \mathbb{R}^2 \to \mathbb{R}$ such that, for all $x \in \mathbb{R}^{(T)}$,

$$I^T(x) = I^{T-1}(x_{T-1}, \ldots, x_0) + F^T(x_T - x_{T-1}).$$

The main contribution of this paper lies in the use of two further axioms. The first is stationarity, the second is a monotonicity property with respect to specific variations in the resources available to an agent.

Stationarity. For all $r \in \mathbb{N}_0$, there exists an increasing function $G^r \colon \mathbb{R} \to \mathbb{R}$ such that, for all $t \in \mathbb{N}_0$ and for all $p, q, s \in \mathbb{R}$,

$$I^{t+r+2}(p, q, s\mathbf{1}_{t+r+1}) = G^r \left(I^{t+2}(p, q, s\mathbf{1}_{t+1}) \right).$$

Stationarity requires that the insecurity comparison associated with two specific streams is unchanged if two resource levels are shifted r periods into the past and the additional periods are assigned resource levels of s. For instance, if $t = 1$ and $s = 0$, the property requires that

$$I^{3+r}(p, q, 0, 0, \mathbf{0}_r)$$

can be expressed as an (r-dependent) increasing transformation G^r of

$$I^3(p, q, 0, 0).$$

Clearly, the axiom could be strengthened to include more complex streams but, for simplicity and ease of exposition, we state the above weak version that is sufficient for our purposes.

Our second additional axiom is the following monotonicity property.

Resource-variation monotonicity. For all $t \in \mathbb{N}$, for all $p \in \mathbb{R}$ and for all $q \in \mathbb{R}_{++}$,

$$\begin{aligned} I^{t+2}(p, p, p+q, p\mathbf{1}_t) \; > \; I^{t+2}(p, p+q, p\mathbf{1}_{t+1}) \; > \; I^{t+2}(p\mathbf{1}_{t+3}) \; > \\ I^{t+2}(p, p-q, p\mathbf{1}_{t+1}) \; > \; I^{t+2}(p, p, p-q, \mathbf{1}_t). \end{aligned} \quad (1)$$

Loosely speaking, resource-variation monotonicity is motivated by the following consideration. First, it is assumed that a first-up-then-down move of a given magnitude in two consecutive periods involves an increase in insecurity, whereas the reverse is true for an analogous first-down-then-up move. The motivation of this hypothesis is straightforward: a ceteris-paribus increase in resources followed by a decrease leaves the agent with a net increase of insecurity because the more recent move is associated with a downwards trajectory, whereas a down-and-up move leads the agent to be more confident because of the recovery aspect of such a change. Moreover, if an increase in resources in a given period is followed by a decrease of the same magnitude in the next period, the insecurity attached with this stream exceeds that of a stream that shifts this up-and-down move one period into the past. Analogously, if a decrease in resources is followed by an increase of the same magnitude in the next period, shifting this down-and-up move one period into the past increases insecurity. These hypotheses reflect the observation that more recent experiences are more influential than those that occur farther in the past. As is the case for stationarity, we limit the scope of the axiom to a relatively small class of cases (which is sufficient for our purposes) for simplicity and ease of exposition. See Figures 1 to 5 for an illustration of the axiom with $p = 0$ and $q = 1$, where the resource streams are listed in decreasing order of insecurity. That is, the stream in Figure 1 has a higher insecurity than that in Figure 2 which, in turn, has a higher insecurity than that in Figure 3, and so on.

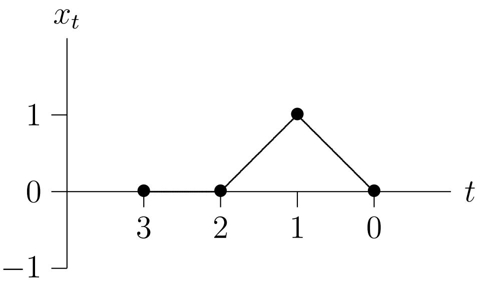

Figure 1: The resource stream $x^1 = (0, 0, 1, 0)$.

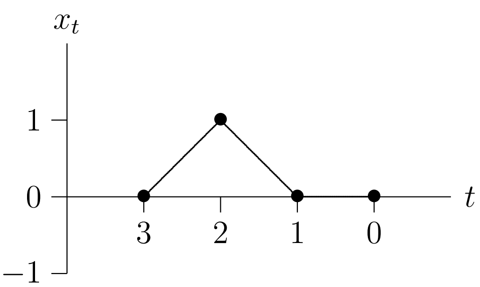

Figure 2: The resource stream $x^2 = (0, 1, 0, 0)$.

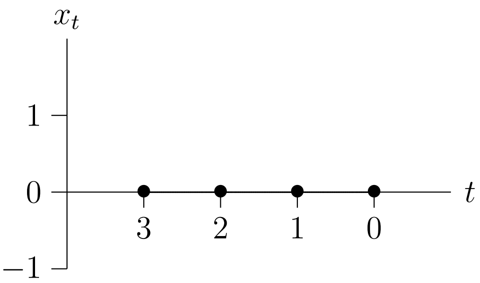

Figure 3: The resource stream $x^3 = (0, 0, 0, 0)$.

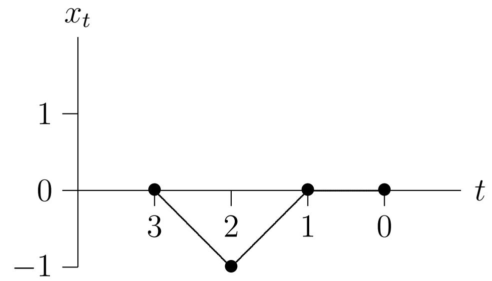

Figure 4: The resource stream $x^4 = (0, -1, 0, 0)$.

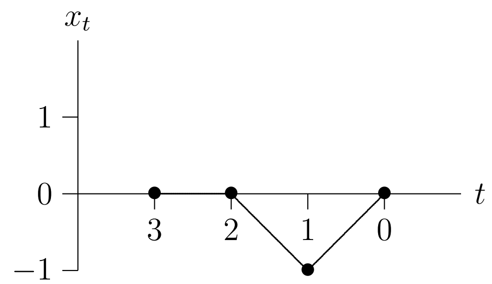

Figure 5: The resource stream $x^5 = (0, 0, -1, 0)$.

3 Results

As a preliminary step, we identify the insecurity measures that satisfy our first five axioms of gain-loss monotonicity, proximity monotonicity, homogeneity, translation invariance and quasilinearity. Although somewhat related to the first characterization result in Bossert and D'Ambrosio (2013), the following theorem characterizes a different class and its proof differs from that in our earlier contribution.

Theorem 1. *A measure of individual economic insecurity I satisfies gain-loss monotonicity, proximity monotonicity, homogeneity, translation invariance and quasilinearity if and* *only if there exist decreasing functions* $\ell \colon \mathbb{N} \to \mathbb{R}_{++}$ *and* $g \colon \mathbb{N} \to \mathbb{R}_{++}$ *such that, for all* $T \in \mathbb{N}$ *and for all* $x \in \mathbb{R}^{(T)}$,

$$I^T(x) = \sum_{\substack{t \in \{1,\dots,T\}: \\ x_t > x_{t-1}}} \ell(t)\,(x_t - x_{t-1}) + \sum_{\substack{t \in \{1,\dots,T\}: \\ x_t < x_{t-1}}} g(t)\,(x_t - x_{t-1})\,. \tag{2}$$

Independent of the choice of the functions ℓ and g, any constant resource stream $s\mathbf{1}_{T+1}$ with $s \in \mathbb{R}$ and $T \in \mathbb{N}$ is assigned an insecurity value of zero according to the indices described in Theorem 1. This observation follows immediately because all period-to-period differences are equal to zero in this case. As already pointed out, the above measures differ from those characterized in Bossert and D'Ambrosio (2013). In particular, according to the indices proposed in our earlier paper, a constant stream $s\mathbf{1}_{T+1}$ has a level of insecurity of minus s—that is, (ceteris-paribus) higher current resources are associated with lower insecurity values. That the classes proposed here do not possess this property may very well be viewed as a shortcoming. However, this is not necessarily the case, depending on the interpretation of the resource variable. If x represents a wealth stream, the current level of wealth can be seen as a buffer stock that can be used to absorb adverse events. This is no longer obvious if the resource under consideration is income which is typically considered as representing a flow rather than a stock and, thus, the argument would seem to have much less force. Moreover, we reiterate that the indices of our earlier contribution fail to be stationary. Thus, there is a trade-off to be taken into account when assessing their merits relative to the present proposal.

The measures identified in the above theorem have a simple structure. They are based on the sum of weighted period-to-period gains and losses, where the weights assigned to losses and those assigned to gains (given by the functions ℓ and g) can be different. Moreover, the only additional restriction imposed on these weight functions is that more recent periods are assigned higher weights than those farther in the past. Clearly, this allows for a rather large class of measures and the selection of suitable weights can present a formidable task. Perhaps more importantly, some of these weight functions are associated with rather counter-intuitive properties such as time-inconsistent choices. It is with these considerations in mind that we narrow down this class by imposing stationarity and resource-variation monotonicity.

The following result characterizes insecurity measures that employ geometric discounting. Not surprisingly, geometric discounting follows from stationarity. The relative weights of aggregate losses and gains are expressed by means of the positive parameters

ℓ_0 and g_0. It it worth emphasizing that the discount factor δ that applies to losses must be the same as that attached to gains. Furthermore, the possible values of δ must be below the smaller of the two ratios ℓ_0/g_0 and g_0/ℓ_0, the other two parameters of the class characterized below. These parameter restrictions are consequences of resource-variation monotonicity. Clearly, higher values of δ correspond to higher importance attached to previous experiences.

Theorem 2. *A measure of individual economic insecurity I satisfies gain-loss monotonicity, proximity monotonicity, homogeneity, translation invariance, quasilinearity, stationarity and resource-variation monotonicity if and only if there exist $\ell_0, g_0 \in \mathbb{R}_{++}$ and $\delta \in (0, \min\{\ell_0/g_0, g_0/\ell_0\})$ such that, for all $T \in \mathbb{N}$ and for all $x \in \mathbb{R}^{(T)}$,*

$$I^T(x) = \ell_0 \sum_{\substack{t \in \{1,\ldots,T\}: \\ x_t > x_{t-1}}} \delta^{t-1} (x_t - x_{t-1}) + g_0 \sum_{\substack{t \in \{1,\ldots,T\}: \\ x_t < x_{t-1}}} \delta^{t-1} (x_t - x_{t-1}). \tag{3}$$

As we do in Bossert and D'Ambrosio (2013), the class of measures identified in the above theorem can be narrowed down further by imposing a requirement that we refer to as loss priority. It demands that a ceteris-paribus loss in a given period has a stronger impact on individual insecurity than a ceteris-paribus gain of the same magnitude in the same period. This axiom imposes further restrictions on the relationships between the parameters ℓ_0, g_0 and δ.

Loss priority. For all $T \in \mathbb{N}$, for all $x \in \mathbb{R}^{(T-1)}$ and for all $f \in \mathbb{R}_{++}$,

$$I^T(x_{T-1} + f, x) - I^T(x_{T-1}, x) > I^T(x_{T-1}, x) - I^T(x_{T-1} - f, x).$$

If loss priority is added to the axioms of Theorem 2, it follows immediately that ℓ_0 (the weight assigned to aggregate discounted losses) must exceed g_0 (the weight associated with aggregate discounted gains). This implies that

$$\frac{g_0}{\ell_0} < 1 < \frac{\ell_0}{g_0}$$

and the minimum of the two ratios is obtained at g_0/ℓ_0. Thus, we obtain the following result.

Theorem 3. *A measure of individual economic insecurity I satisfies gain-loss monotonicity, proximity monotonicity, homogeneity, translation invariance, quasilinearity, stationarity, resource-variation monotonicity and loss priority if and only if there exist $\ell_0, g_0 \in \mathbb{R}_{++}$ and $\delta \in (0, g_0/\ell_0)$ such that $\ell_0 > g_0$ and, for all $T \in \mathbb{N}$ and for all $x \in \mathbb{R}^{(T)}$,*

$$I^T(x) = \ell_0 \sum_{\substack{t \in \{1,\ldots,T\}: \\ x_t > x_{t-1}}} \delta^{t-1} (x_t - x_{t-1}) + g_0 \sum_{\substack{t \in \{1,\ldots,T\}: \\ x_t < x_{t-1}}} \delta^{t-1} (x_t - x_{t-1}).$$

We propose to use a member of the class identified in Theorem 3 to measure economic insecurity. The loss-priority property is akin to the loss-aversion assumption in decision theory and it would seem that it adequately captures the attitude of households that are concerned with their ability to absorb adverse shocks.

The following example illustrates the class of measures characterized in Theorem 3.

Example 1. *Throughout the example, suppose that $T = 3$ and the weights assigned to aggregate losses and gains are $\ell_0 = 1$ and $g_0 = 15/16$.*

(a) Consider the stream $x^1 = (4, 12, 12, 16)$. We obtain

$$I^3(x^1) = g_0 \left(\delta^2(4-12) + \delta^1(12-12) + \delta^0(12-16)\right) = -\frac{15}{2}\delta^2 - \frac{15}{4} < 0.$$

Because the available resources never decrease from one period to the next, the agent never experiences any losses and, as a consequence, the resulting insecurity value is negative for any choice of the discount factor $\delta \in (0, 15/16)$. In general, any stream without losses and at least one gain has a negative insecurity value and, thus, represents less insecurity than any constant stream.

(b) Now consider the reverse stream $x^2 = (16, 12, 12, 4)$. It follows that

$$I^3(x^2) = \ell_0 \left(\delta^2(16-12) + \delta^1(12-12) + \delta^0(12-4)\right) = 4\delta^2 + 8 > 0.$$

The agent never experiences any gains and, thus, the resulting insecurity value is always positive. Clearly, any stream without gains and at least one loss is associated with a positive insecurity value and therefore is more insecure than any constant stream.

(c) Let $x^3 = (16, 4, 4, 12) \in \mathbb{R}^{(3)}$. In this stream, the individual experiences a loss of 12 when moving from three periods ago to two periods ago, no change in the period that follows and, finally, a gain of 8 in the move from the previous period to today. For any discount factor $\delta \in (0, 15/16)$, the corresponding value of the insecurity index is

$$I^3(x^3) = \ell_0 \delta^2 (16-4) + g_0 \delta^0 (4-12) = 12\delta^2 - 15/2.$$

For any value of δ above $(1/2)\sqrt{5/2}$, the index value is positive (and, thus, x^3 is associated with more insecurity than the insecurity of a constant stream); if δ is less than $(1/2)\sqrt{5/2}$, insecurity is lower than that resulting from a constant stream.

(d) Finally, consider the stream $x^4 = (4, 16, 16, 8)$. It follows that

$$I^3(x^4) = g_0\delta^2(4-16) + \ell_0\delta^0(16-8) = -\frac{45}{4}\delta^2 + 8.$$

For any value of δ below $(4/3)\sqrt{2/5}$, the index value is positive and x^4 is associated with more insecurity than the insecurity of a constant stream.

4 Beyond resources: a conceptual difficulty

A possibility for enriching the setting analyzed in this paper consists of adding variables that might be relevant for individual insecurity, provided that data on these variables are available. For example, streams of employment rates (which may be sector-specific or may apply to the entire economy) are obvious candidates. In general, suppose that there are $K \geq 2$ variables $y^1, \ldots, y^K$ that are considered relevant for measuring economic insecurity. Thus, instead of a single stream of economic resources $x = (x_T, \ldots, x_0)$, we now have a K-tuple $y^1 = (y^1, \ldots, y^K)$ where, for each $k \in \{1, \ldots, K\}$, $y^k = (y^k_T, \ldots, y^k_0)$ is the stream associated with the k^{th} variable. We can summarize the relevant information in a $K \times (T+1)$ insecurity matrix

$$Y^T = \begin{pmatrix} y^1_T & \cdots & y^1_0 \\ \vdots & & \vdots \\ y^K_T & \cdots & y^K_0 \end{pmatrix}$$

where each row $k \in \{1, \ldots, K\}$ corresponds to a variable stream y^k of length $T+1$. Note that the length of a stream is determined by the number of periods for which the agent is included in the data and, thus, is not variable-specific. Of course, streams with different lengths may be associated with different matrices, as is the case for the model examined earlier—recall that it is essential to be able to compare matrices Y^T and $Z^{T'}$ when T and T' differ.

It may seem tempting to define an overall index of individual insecurity by considering a function that assigns an insecurity value to each possible insecurity matrix. However, we have some concerns regarding the suitability of such an approach. Variables such as wealth and employment rates, for instance, differ substantially in nature and may very well be considered incommensurable. Thus, it is by no means obvious how some determinants of insecurity are to be traded off against others. We therefore advocate a more modest proposal that consists of applying the relevant desiderata by means of a dominance criterion, thus allowing for some non-comparability that is intended to reflect the underlying incommensurability of the variables employed. More precisely, suppose that, for each variable y^k, we determine a variable-specific indicator of economic insecurity I^k—that is, one index for each determinant of insecurity. The dominance criterion we envision can now be expressed by declaring an individual insecurity matrix Y^T to be associated with at least as much economic insecurity as a matrix $Z^{T'}$ if and only if

$$I^k(y^k_T, \dots, y^k_0) \geq I^k(z^k_{T'}, \dots, z^k_0) \text{ for all } k \in \{1, \dots, K\},$$

that is, if and only if all variable-specific indicators I^k assign levels of insecurity to the k^{th} row of Y^T that are at least as high as those assigned to the k^{th} row of $Z^{T'}$. Clearly, this results in a quasi-ordering (a reflexive and transitive but not necessarily complete relation) defined on possible K-tuples of streams of relevant variables.

5 Concluding remarks

As far as possible applications of our results that go beyond the measurement of this socially important phenomenon are concerned, there appear to be several promising avenues to explore. For instance, it may be useful to study the effects of insecurity on voting behavior. There clearly are examples (some in the not-too-distant past) in which the hypothesis that the outcomes of an election were, to some extent, influenced by sentiments of insecurity would appear to be quite plausible. Moreover, as Hacker (2008) puts it, the 'decline of the American dream' may very well turn out to be closely related to people's sense of insecurity and their awareness of being in a precarious situation.

Economic insecurity clearly is not restricted to a small number of societies and does not respect national boundaries. We expect that its analysis will only become more relevant in the times that lie ahead. Issues involving social cohesion, the alarming increase in inequality that can be observed across the globe, and the perceived and actual effects of public-policy choices all seem to be profoundly affected by the extent to which individuals consider themselves to be secure with respect to their economic circumstances. We hope that the measures presented here (and possible refinements that may be developed in future work) can assist in addressing some of these challenging problems.

Appendix: Proofs

Proof of Theorem 1. 'If.' That gain-loss monotonicity is satisfied follows because the functions ℓ and g are positive-valued. Proximity monotonicity is satisfied because ℓ and g are decreasing. Furthermore, it is immediate that homogeneity and translation invariance are satisfied. Finally, to see that quasilinearity is satisfied, define the function $F^T : \mathbb{R} \to \mathbb{R}$ by letting

$$F^T(y) = \begin{cases} \ell(T)y & \text{if } y \geq 0 \\ g(T)y & \text{if } y < 0 \end{cases} \tag{4}$$

for all $y \in \mathbb{R}$.

'Only if.' Suppose that I satisfies the axioms of the theorem statement. We prove the requisite implication by inductively constructing the functions ℓ and g.

Step 1. Let $T = 1$ and $x = (x_1, x_0) \in \mathbb{R}^{(T)}$.

Case (1.i). If $x_1 = x_0$, the application of translation invariance with $c = -x_0$ yields

$$I^1(x_1, x_0) = I^1(x_0, x_0) = I^1(x_0 - x_0, x_0 - x_0) = I^1(0, 0).$$

Homogeneity implies that $I^1(b \cdot 0, b \cdot 0) = bI^1(0, 0)$ for all $b \in \mathbb{R}_{++}$ and, thus, it follows that $I^1(x_1, x_0) = 0$ whenever $x_1 = x_0$.

Case (1.ii). If $x_1 > x_0$, translation invariance with $c = -x_0$ implies

$$I^1(x_1, x_0) = I^1(x_1 - x_0, 0).$$

Using homogeneity with $b = x_1 - x_0 > 0$, it follows that

$$\begin{aligned} I^1(x_1, x_0) &= I^1(x_1 - x_0, 0) = I^1((x_1 - x_0) \cdot 1, (x_1 - x_0) \cdot 0) \\ &= (x_1 - x_0)I^1(1, 0) = \ell(1)(x_1 - x_0), \end{aligned}$$

where $\ell(1) = I^1(1, 0)$. It follows from gain-loss monotonicity that $\ell(1) > 0$.

Case (1.iii). If $x_1 < x_0$, translation invariance with $c = -x_0$ implies

$$I^1(x_1, x_0) = I^1(x_1 - x_0, 0)$$

and, using homogeneity with $b = -(x_1 - x_0) > 0$, we obtain

$$\begin{aligned} I^1(x_1, x_0) &= I^1(x_1 - x_0, 0) = I^1(-(x_1 - x_0) \cdot (-1), -(x_1 - x_0) \cdot 0) \\ &= -(x_1 - x_0)I^1(-1, 0) = -g(1)(x_1 - x_0), \end{aligned}$$

where $g(1) = I^1(-1, 0)$. That $g(1) > 0$ follows again from gain-loss monotonicity.

Combining theses three cases, it is immediate that (2) applies for $T = 1$. Note that only gain-loss monotonicity, homogeneity and translation invariance are required in this step of the proof.

Step 2. Now suppose that $T \geq 2$ and, for all $x \in \mathbb{R}^{(T-1)}$,

$$I^{T-1}(x) = \sum_{\substack{t\in\{1,\ldots,T-1\}:\\ x_t > x_{t-1}}} \ell(t)\,(x_t - x_{t-1}) + \sum_{\substack{t\in\{1,\ldots,T-1\}:\\ x_t < x_{t-1}}} g(t)\,(x_t - x_{t-1}) \tag{5}$$

where $\ell(1) > \ldots > \ell(T-1) > 0$ and $g(1) > \ldots > g(T-1) > 0$. The conjunction of quasilinearity and (5) implies that there exists a function $F^T \colon \mathbb{R} \to \mathbb{R}$ such that, for all $x \in \mathbb{R}^{(T)}$,

$$\begin{aligned} I^T(x) &= I^{T-1}(x_{T-1}, \ldots, x_0) + F^T(x_T - x_{T-1}) \\ &= \sum_{\substack{t\in\{1,\ldots,T-1\}:\\ x_t > x_{t-1}}} \ell(t)\,(x_t - x_{t-1}) + \sum_{\substack{t\in\{1,\ldots,T-1\}:\\ x_t < x_{t-1}}} g(t)\,(x_t - x_{t-1}) + F^T(x_T - x_{T-1}). \end{aligned} \tag{6}$$

Homogeneity implies that $F^T(b(x_T - x_{T-1})) = bF^T(x_T - x_{T-1})$ for all $b \in \mathbb{R}_{++}$. As in Step 1, we distinguish three cases.

Case (2.i). If $x_T = x_{T-1}$, homogeneity implies $F^T(b \cdot 0) = bF^T(0)$ for all $b \in \mathbb{R}_{++}$ and hence $F^T(0) = 0$.

Case (2.ii). If $x_T > x_{T-1}$, homogeneity with $b = x_T - x_{T-1} > 0$ implies

$$F^T(x_T - x_{T-1}) = F^T((x_T - x_{T-1}) \cdot 1) = (x_T - x_{T-1})F^T(1) = \ell(T)(x_T - x_{T-1})$$

where $\ell(T) = F^T(1)$. Gain-loss monotonicity and proximity monotonicity together imply that $0 < \ell(T) < \ell(T-1)$ so that the induced function ℓ is positive-valued and decreasing.

Case (2.iii). If $x_T < x_{T-1}$, homogeneity with $b = -(x_T - x_{T-1}) > 0$ implies

$$F^T(x_T - x_{T-1}) = F^T(-(x_T - x_{T-1}) \cdot (-1)) = -(x_T - x_{T-1})F^T(-1) = g(T)(x_T - x_{T-1})$$

where $g(T) = -F^T(-1)$. Gain-loss monotonicity and proximity monotonicity together imply that $0 < g(T) < g(T-1)$ and, thus, g is positive-valued and decreasing.

Thus, combining the observations of cases (2.i) to (2.iii), it follows that (4) is satisfied.

Substituting back into (5), we obtain

$$\begin{aligned}
I^{T-1}(x) &= \sum_{\substack{t\in\{1,\ldots,T-1\}:\\ x_t>x_{t-1}}} \ell(t)\left(x_t-x_{t-1}\right)+\sum_{\substack{t\in\{1,\ldots,T-1\}:\\ x_t<x_{t-1}}} g(t)\left(x_t-x_{t-1}\right)\\
&+\begin{cases}\ell(T)\left(x_T-x_{T-1}\right) & \text{if } x_T-x_{T-1}\geq 0\\ g(T)\left(x_T-x_{T-1}\right) & \text{if } x_T-x_{T-1}<0\end{cases}\\
&= \sum_{\substack{t\in\{1,\ldots,T\}:\\ x_t>x_{t-1}}} \ell(t)\left(x_t-x_{t-1}\right)+\sum_{\substack{t\in\{1,\ldots,T\}:\\ x_t<x_{t-1}}} g(t)\left(x_t-x_{t-1}\right)
\end{aligned}$$

which completes the proof. ■

Proof of Theorem 2. 'If.' That gain-loss monotonicity, proximity monotonicity, homogeneity, translation invariance and quasilinearity are satisfied follows from Theorem 1.

To see that stationarity is satisfied, let $G^r(y)=\delta^r y$ for all $r\in\mathbb{N}_0$ and for all $y\in\mathbb{R}$ in the definition of the axiom.

Finally, we prove that resource-variation monotonicity is satisfied. Let $p\in\mathbb{R}$ and $q\in\mathbb{R}_{++}$. Substituting (3) in (1) yields

$$q\left[\ell_0\delta^{t-1}-g_0\delta^t\right] \;>\; q\left[\ell_0\delta^{t}-g_0\delta^{t+1}\right] \;>\; 0 > \; q\left[\ell_0\delta^{t+1}-g_0\delta^t\right] \;>\; q\left[\ell_0\delta^{t}-g_0\delta^{t-1}\right]$$

which is equivalent to

$$\delta^{t-1}\left[\ell_0-g_0\delta\right] \;>\; \delta^{t}\left[\ell_0-g_0\delta\right] \;>\; 0 \;>\; \delta^{t}\left[\ell_0\delta-g_0\right] \;>\; \delta^{t-1}\left[\ell_0\delta-g_0\right]. \tag{7}$$

These inequalities are satisfied because $\delta\in(0,\min\{\ell_0/g_0,g_0/\ell_0\})$.

'Only if.' Suppose that I satisfies the required axioms. By Theorem 1, there exist decreasing functions $\ell\colon\mathbb{N}\to\mathbb{R}_{++}$ and $g\colon\mathbb{N}\to\mathbb{R}_{++}$ such that (2) is satisfied. It remains to be shown that there exist $\ell_0,g_0\in\mathbb{R}_{++}$ and $\delta\in(0,\min\{\ell_0/g_0,g_0/\ell_0\})$ such that $\ell(t)=\ell_0\delta^{t-1}$ and $g(t)=g_0\delta^{t-1}$ for all $t\in\mathbb{N}$.

First, we identify the class of parameter functions ℓ that apply to the losses experienced in each period. This part of the proof parallels that of Theorem 4 in Blackorby, Bossert and Donaldson (1997). Let $p,q\in\mathbb{R}_+$ be such that $p\geq q$ and let $s=0$. Substituting (2) in the definition of stationarity, we obtain

$$\ell(t+r+1)q+\ell(t+r+2)(p-q)=G^r\left(\ell(t+1)q+\ell(t+2)(p-q)\right)$$

for all $t,r\in\mathbb{N}_0$ or, setting $u^0=q$ and $u^1=p-q$,

$$\ell(t+r+1)u^0+\ell(t+r+2)u^1=G^r\left(\ell(t+1)u^0+\ell(t+2)u^1\right) \tag{8}$$

for all $t, r \in \mathbb{N}_0$ and for all $u^0, u^1 \in \mathbb{R}_+$.

Now define $y = \ell(t+1)u^0$, $z = \ell(t+2)u^1$, $\overline{\ell}(t+r, y) = \ell(t+r+1)u^0$ and $\widehat{\ell}(t+r, z) = \ell(t+r+2)u^1$. Substituting, (8) implies

$$\overline{\ell}(t+r, y) + \widehat{\ell}(t+r, z) = G^r(y+z)$$

for all $t, r \in \mathbb{N}_0$ and for all $y, z \in \mathbb{R}_+$. This is a Pexider equation defined on the domain $\mathbb{R}_+$ and, by definition, the possible values of the functions $\overline{\ell}$ and $\widehat{\ell}$ (and, thus, the possible values of G^r) are bounded below by zero. Therefore, the solutions of this functional equation are such that there exist functions $d\colon \mathbb{N}_0 \to \mathbb{R}_+$ and $e\colon \mathbb{N}_0 \to \mathbb{R}_+$ such that

$$\overline{\ell}(t+r, y) = d(r)y + e(r)$$

for all $t, r \in \mathbb{N}_0$ and for all $y \in \mathbb{R}_+$; see, for instance, Aczél (1966, p. 46 and p. 142). Also, note that d and e cannot depend on t because G^r does not. Using the definition of $\overline{\ell}$, it follows that

$$\ell(t+r+1)u = d(r)\ell(t+1)u + e(r) \tag{9}$$

for all $t, r \in \mathbb{N}_0$ and for all $u \in \mathbb{R}_+$. Setting $t = 0$ in (9), it follows that

$$\ell(r+1)u = d(r)\ell(1)u + e(r) \tag{10}$$

and, therefore,

$$\ell(t+r+1)u = d(t+r)\ell(1)u + e(t+r) \tag{11}$$

for all $t, r \in \mathbb{N}_0$ and for all $u \in \mathbb{R}_+$. Setting $r = 0$ in (10), we obtain

$$\ell(1)u = d(0)\ell(1)u + e(0)$$

for all $u \in \mathbb{R}_+$. Setting $u = 0$, it follows that $e(0) = 0$. Once this is established, we can choose any $u > 0$ to conclude that $d(0) = 1$. Substituting (10) in (9), it follows that

$$\ell(t+r+1)u = d(r)\left[d(t)\ell(1)u + e(t)\right] + e(r)$$

and, together with (11),

$$d(t+r)\ell(1)u + e(t+r) = d(r)\left[d(t)\ell(1)u + e(t)\right] + e(r)$$

for all $t, r \in \mathbb{N}_0$ and for all $u \in \mathbb{R}_+$. This is equivalent to

$$\ell(1)u\left[d(t+r) - d(t)d(r)\right] = d(r)e(t) + e(r) - e(t+r)$$

for all $t, r \in \mathbb{N}_0$ and for all $u \in \mathbb{R}_+$. Because $\ell(1)$ is positive and the right side of this equation does not depend on u, both sides must be identical to zero and, therefore, it follows that

$$d(t + r) = d(t)d(r) \tag{12}$$

for all $t, r \in \mathbb{N}_0$. Setting $\delta = d(1)$, a simple induction argument together with (12) establishes that $d(t) = \delta^t$ for all $t \in \mathbb{N}$. Setting $u = 0$ in (10), it follows that $e(t) = 0$ for all $t \in \mathbb{N}_0$. Using this observation together with $d(t) = \delta^t$ in (10), we obtain

$$\ell(t + 1)u = \delta^t \ell(1)u$$

and, choosing any $u > 0$, it follows that $\ell(t + 1) = \ell(1)\delta^t$ for all $t \in \mathbb{N}_0$ or, equivalently,

$$\ell(t) = \ell_0 \delta^{t-1} \tag{13}$$

for all $t \in \mathbb{N}$, where $\ell_0 = \ell(1) > 0$. Because ℓ is positive-valued, it follows that $\delta > 0$ and because ℓ is decreasing, it follows that $\delta < 1$ hence $\delta \in (0, 1)$.

To obtain the class of parameter functions g that apply to the gains experienced in each period, we can reproduce the above argument with the hypothesis that $p, q \in \mathbb{R}_-$ are such that $p \leq q$ (instead of the hypothesis that $p, q \in \mathbb{R}_+$ are such that $p \geq q$) to obtain the existence of $g_0 \in \mathbb{R}_{++}$ and $\sigma \in (0, 1)$ such that

$$g(t) = g_0 \sigma^{t-1} \tag{14}$$

for all $t \in \mathbb{N}$.

It remains to be shown that $\delta = \sigma$ and that $\delta \in (0, \min\{\ell_0/g_0, g_0/\ell_0\})$. To accomplish this, we employ resource-variation monotonicity. Using (13) and (14) in (1), it follows that the expression

$$\ell(t) - g(t + 1) = \ell_0 \delta^{t-1} - g_0 \sigma^t$$

is decreasing in t. Treating t as a continuous variable for convenience (which clearly does not involve any loss of generality), we can differentiate to obtain the condition

$$\ell_0 \delta^{t-1} \ln(\delta) - g_0 \sigma^t \ln(\sigma) < 0$$

for all $t \in \mathbb{N}$. Rearranging, we obtain

$$\frac{\delta^{t-1}}{\sigma^t} > \frac{g_0 \ln(\sigma)}{\ell_0 \ln(\delta)} > 0 \tag{15}$$

for all $t \in \mathbb{N}$, where these inequalities follow because both δ and σ are in the open interval $(0, 1)$. Likewise, resource-variation monotonicity implies that

$$\ell(t+1) - g(t) = \ell_0 \delta^t - g_0 \sigma^{t-1}$$

is increasing in t. Differentiating again, it follows that

$$\ell_0 \delta^t \ln(\delta) - g_0 \sigma^{t-1} \ln(\sigma) > 0$$

for all $t \in \mathbb{N}$, which implies that

$$\frac{\delta^t}{\sigma^{t-1}} < \frac{g_0 \ln(\sigma)}{\ell_0 \ln(\delta)} < \infty \tag{16}$$

for all $t \in \mathbb{N}$.

If $\delta < \sigma$, we obtain

$$\lim_{t\to\infty} \frac{\delta^{t-1}}{\sigma^t} = \lim_{t\to\infty} \frac{1}{\sigma} \left(\frac{\delta}{\sigma}\right)^{t-1} = 0,$$

contradicting (15) which requires that this ratio be bounded below by the positive number

$$\frac{g_0 \ln(\sigma)}{\ell_0 \ln(\delta)}.$$

If $\delta > \sigma$, it follows that

$$\lim_{t\to\infty} \frac{\delta^t}{\sigma^{t-1}} = \lim_{t\to\infty} \delta \left(\frac{\delta}{\sigma}\right)^{t-1} = \infty,$$

contradicting (16) which demands that this ratio be bounded above by the finite number

$$\frac{g_0 \ln(\sigma)}{\ell_0 \ln(\delta)}.$$

Finally, note that (7)—and, thus, (1)—is satisfied only if $\delta \in (0, \min\{\ell_0/g_0, g_0/\ell_0\})$. ■

Proof of Theorem 3. Follows immediately from Theorem 2 and the definition of the loss-priority property. ■

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