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1. Introduction

In the 1950s and 1960s many development economists put forth the notion of "trickle-down" according to which the poor will ultimately profit from growth. As emphasized by Kakwani and Son (2018), one of the arguments of this view is that the rich are those who can generate economic activities and this will increase the probability for a poor to be employed.

In the 1970s some development economists started to object to such an approach (e.g. Ahluwalia 1976a and 1976b) when they realized that poverty did not significantly decrease and that the income growth rate among the poor was quite smaller than that in the whole population. This explains also the development of an important literature on the concept of pro-poor growth (see, for example, Kakwani and Pernia, 2000; Kakwani and Son, 2000; Dollar and Kraay, 2002; Ravallion and Chen, 2003; Son, 2004; Kraay, 2006; Son and Kakwani, 2008; Foster and Szekely, 2008; Deutsch and Silber, 2011).

Other economists preferred to stress the fact that a sizable middle class is an important factor in economic development (see, for example, Landes, 1998; Easterly, 2001; Birdsall, 2007a and 2007b; Pressman, 2007; Loayza et al., 2012; Bussolo et al., 2014; Boushey and Hersh, 2014). In fact Bhalla and Kharas (2013, page 6) drew our attention to the fact that "the notion of the 'middle class' has roots that go back millennia, originating as a concept in the writings of Aristotle, who defined it as owners of property and thus the people best positioned to rule the state. According to him, they were a moderating force with both the capability and incentive for sober governance, but through its long history, the middle class has been linked to a wide range of concepts from thriftiness to democratic spirit to unchecked consumerism." More recently Birdsall (2013, page 11) argued that "recent growth in India, Africa, China and much of Latin America…will be more likely to be sustained and institutionalized, because an independent middle class has become big enough and politically powerful enough, to be a force for good government and equal opportunity growth". For Lopez-Calva (2013, page 15), it is economic security "that defines a person as middle class. Individuals who are above the poverty line and who have a low risk of falling into poverty may have characteristics in terms of risk-taking capabilities, investment decisions, consumption patterns and the like that differ from the characteristics of those individuals who are just above the poverty line".

Atkinson and Brandolini (2013) in their study of the middle class in selected OECD countries started by adopting definitions based on income but they also examined the role of property and wealth, as well as that of the occupational structure. In fact Ferreira et al. (2013), looking at what happened in Latin America, stressed the fact that "those entering the middle class are more educated, more likely to live in urban areas and work in the formal sector. It appears also that middle class women tend to have fewer children and a higher labor force participation rate than women belonging to the poor or vulnerable groups".

Most of the definitions of the middle class focus however on income and there have been various ways of defining the lower and upper bounds of the middle class: 75% to 125%; 60% to 225%; 50% to 150% of the median household income; $10 a day and the 90^{th} percentile of the income distribution (see, for example, Nissanov et al., 2010, and Nissanov, 2017, for a review of these definitions). An alternative approach to defining the middle class was proposed by Massari et al. (2009) for Italy and Nissanov and Pittau (2016) for Russia, using a non-parametric tool, the so-called "relative distribution". Another interesting approach is the mixture model method (see, McLachlan and Peel, 2000) which was applied by Pittau et al. (2010) to the world income distribution (close to 100 countries) and by Nissanov (2017) to Russian income data.

Foster and Wolfson (2010) took a different approach to the study of the middle class, one that was based on the concept of polarization curves, and derived an index of bi-polarization[1]. Foster and Wolfson took a relative approach to the measurement of bi-polarization, while Chakravarty et al. (2007) focused on an absolute approach to the measurement of bi-polarization.

As stressed by Lasso de la Vega et al (2010), many polarization measures proposed in the literature assume some invariance condition. Choosing a specific measure is based on a value judgment regarding the measurement of polarization. Relative polarization measures are assumed to remain unchanged under equi-proportionate variations in all incomes (i.e. if all incomes change at the same rate). An absolute measure is supposed to remain invariant under equal absolute changes in all incomes. Relative and absolute measure may be viewed as representing two ends of a spectrum of possible measures. Intermediate measures of polarization are a combination of the absolute and relative types of measures and they usually use a parameter to adjust the relative importance of each component (relative or absolute) in the intermediate measure.

[1] This paper was originally written in 1992 and was widely cited over the years until it was finally published in 2010.

Lasso de la Vega et al. (2010) surveyed some of the intermediate measures suggested in the literature. Chakravarty and D'Ambrosio (2010) adopted the notion of intermediateness proposed by Pfingsten (1986), which requires that any combination of an equal proportional increase in all incomes, and an equal amount increase in all incomes, should not change the polarization level. They suggested an intermediate polarization measure based on a combination of the absolute polarization measure of Chakravarty et al. (2007) and of Foster and Wolfson's (2010) relative measure.

For inequality and poverty measurement, Zheng (2007a, 2007b, 2007c) suggested to replace the traditional invariance conditions with the unit consistency axiom. This property demands that the inequality or poverty rankings, remain invariant to the choice of measurement units. Following Zheng's proposal, Lasso de la Vega et al. (2010) introduced a new family of Krtscha (1994) type intermediate bipolarization indices. They show that these indices are the only intermediate polarization indices that satisfy the unit consistency axiom.

These different approaches (relative, absolute, intermediate) to the measurement of the middle class will be the basis for the derivation of pro-middle class growth measures. While, in previous work, Peled and Silber (2019) took a relative approach to pro-middle class growth, the present paper emphasizes more the absolute and intermediate approaches to pro-middle class growth. The paper is organized as follows. Section 2 derives an absolute measure of pro-middle class growth while Section 3 defines a measure of pro-middle class growth that uses the approach of Lasso de la Vega et al. (2010) to intermediate polarization. Section 4 presents then an empirical illustration based on Israeli data or the period 1995-2011 while concluding comments are given in Section 5. The detailed derivation of the measures of pro-middle class we propose is given in the Appendix.

2. Deriving an absolute measure of pro-middle class growth

Following earlier work by Foster and Wolfson (2010), Chakravarty et al. (2007) showed that bi-polarization indices can be relative or absolute. A relative index is supposed to remain invariant if all incomes increase by the same percentage, while an absolute index will remain invariant under equal absolute changes in all incomes. Choosing between these two approaches is clearly a matter of value judgement.

In their study of absolute polarization, Chakravarty et al. (2007) scaled up the Foster-Wolfson (second) bi-polarization curve by the median m in order to derive an "Absolute Polarization Curve (APC)". This curve indicates, for any population proportion, how far the total income enjoyed by that proportion is from the corresponding income that it would receive if everyone were to receive the median income.

Assume that the incomes x_i are ranked in decreasing order $(x_1 \geq \cdots \geq x_i \geq \cdots \geq x_n)$, that n is the size of the population and n_m the rank of the median, the APC ordinate corresponding to the population proportion (k/n) is then

$$AP[x, (\frac{k}{n})] = (1/n) \sum_{n_m < i \leq k} (m - x_i) \qquad \text{when } x_i < m \qquad (1)$$

and as

$$AP[x, (\frac{k}{n})] = (1/n) \sum_{k < i \leq n_m} (x_i - m) \qquad \text{when } x_i > m \qquad (2)$$

As can be seen, for a typical income distribution the *APC* will decrease monotonically until the median income is reached and then increase monotonically. Note also that by dividing $AP(x;p)$ by the median m, one obtains the (second) bi-polarization curve proposed by Foster and Wolfson (2010).

Chakravarty et al. (2007) show also that the area under the Absolute Polarization Curve APC is an absolute index of polarization.

It can then be shown (see, Appendix A, for a proof) that the total area *A* under the APC (on both the R.H.S. and the L.H.S. of the APC) may be expressed as

$$A = \left(\frac{1}{8}\right) (\mu^{ER} - \mu^{FP}) \qquad (3)$$

where

$$\mu^{ER} = \sum_{i=1}^{n/2} [(2i-1)/(n^2/4)]\, x_i \qquad (4)$$

and

$$\mu^{FP} = \sum_{i=\left(\frac{n}{2}\right)+1}^{n} [(2(n-i+1)-1)]/(n^2/4)]\, x_i \qquad (5)$$

We then show in the Appendix that the change ΔA over time in the area *A* may be written as

$$\Delta A = (\frac{1}{8})(\Delta x^{ER} - \Delta x^{FP}) \qquad (6)$$

where

$$\Delta x^{ER} = \sum_{i=1}^{n/2} \left(\frac{2i-1}{(n^2)/4}\right) \Delta x_i \qquad (7)$$

$$\Delta x^{FP} = \sum_{i=1}^{n/2} \left(\frac{2i-1}{(n^2)/4}\right) \Delta x_{n-i+1} = \sum_{i=\left(\frac{n}{2}\right)+1}^{n} \left(\frac{2(n-i+1)-1}{n^2/4}\right) \Delta x_i \quad (8)$$

so that

$$\Delta A > 0 \text{ if } \Delta x^{ER} > \Delta x^{FP} \quad (9)$$

3. Defining pro-middle class growth using the approach of Lasso de la Vega et al. (2010) to intermediate polarization

While a relative polarization curve is homogenous of degree zero in all incomes, an absolute polarization curve, as was stressed in the previous section, is invariant under equal changes (positive or negative) in incomes. Following the approach of Lasso de la Vega et al. (2010) to an intermediate view of polarization, we can define as follows a Krtscha-type (see, Krtscha, 1994, for a definition of intermediate inequality) of intermediate polarization curve. Assume that $\{x\}$ refers to the distribution of incomes at time 0 and $\{y\}$ to the distribution of incomes at time 1. The ordinates of the intermediate polarization curve at time 0 will be (see, Lasso de la Vega et al., 2010)

$$IPC\left(x; \left(\frac{k}{n}\right), \lambda\right) = \frac{1}{n} \sum_{k \le i \le n_m} \frac{m - x_i}{(m_x)^\lambda} \quad , \quad 1 \le k \le n_m \quad (10)$$

$$IPC\left(x; \left(\frac{k}{n}\right), \lambda\right) = \frac{1}{n} \sum_{n_m \le i \le k} \frac{x_{i-m}}{(m_x)^\lambda} \quad , \quad n_m \le k \le n \quad (11)$$

where $0 \le \lambda \le 1$, m_x is the median income at time 0, and (n_m/n) corresponds to the mid-point on the horizontal axis.

Define now π_x , π_y and $\Delta\pi$ as $\pi_x = (m_x)^\lambda$, $\pi_y = (m_y)^\lambda$ and $\Delta\pi = (m_y)^\lambda - (m_x)^\lambda$, (12)

where m_x and m_y are respectively the medians of the income distributions at times 0 and 1.

In what follows we will call π_x and π_y the “transformed intermediate median incomes” at times 0 and 1.

We now define the shares ω_i and σ_i as

$$\omega_i = \left(\frac{y_i}{n\pi_y}\right) \quad (13)$$

and

$$\sigma_i = (x_i/n\pi_x) \quad (14)$$

In a recent analysis of the measurement of (relative) pro-middle class growth, Peled and Silber (2019) had defined the variation ΔP_{FW} over time in the traditional Foster and Wolfson (2010) bipolarization index as

$$\Delta P_{FW}=\left\{\left[\sum_{i=1}^{\frac{n}{2}}\left(\frac{(2i-1)}{n}\right)\{[w_i]-[s_i]\}\right]\right\}$$
$$-\left\{\left[\sum_{i=\left(\frac{n}{2}\right)+1}^{n}\left(\frac{2(n-i+1)-1}{n}\right)\{[w_i]-[s_i]\}\right]\right\} \quad (15)$$

where the shares w_i and s_i were defined as $w_i=\frac{y_i}{nm_y}$ and $s_i=\frac{x_i}{nm_x}$.

It may be observed that the analysis of Peled and Silber (2019) corresponds in fact to the case where λ in (13) and (14) is equal to 1[2].

We may therefore extend (13) to the more general case where $0\leq\lambda\leq 1$ and define the change ΔP_{FW}^{λ} in the "intermediate bipolarization index" as

$$\Delta P_{FW}^{\lambda}=\left[\sum_{i=1}^{\frac{n}{2}}\left(\frac{(2i-1)}{n}\right)\{[\omega_i]-[\sigma_i]\}\right]$$
$$-\left[\sum_{i=\left(\frac{n}{2}\right)+1}^{n}\left(\frac{2(n-i+1)-1}{n}\right)\{[\omega_i]-[\sigma_i]\}\right] \quad (16)$$

where σ_i and ω_i were defined previously.

Let us also introduce the following definitions:

$$\eta_i=\frac{y_i-x_i}{x_i}=\frac{\Delta x_i}{x_i} \quad (17)$$

$$\overline{\eta}^{\lambda}=\frac{(m_y)^{\lambda}-(m_x)^{\lambda}}{(m_x)^{\lambda}}=\frac{\pi_y-\pi_x}{\pi_x}=\frac{\Delta\pi}{\pi_x} \quad (18)$$

$$\varphi_i=\frac{\left(\frac{2i-1}{n^2/4}\right)x_i}{x^{ER}} \quad (19)$$

with $\sum_{i=1}^{n/2}\varphi_i=1$

$$\Psi_i=\frac{\left(\frac{[2(n-i+1)-1]}{n^2/4}\right)x_i}{x^{FP}} \quad (20)$$

with $\sum_{i=\left(\frac{n}{2}\right)+1}^{n}\Psi_i=1$.

η^{ER} and η^{FP} are the weighted average of growth rates, above and below the median, respectively

[2] Foster and Wolfson's index is defined as twice the area under the second order polarization curve. The absolute polarization (Chakravarty et al. 2007) and the Krtscha-type intermediate polarization indices are defined as the area under the curve, without multiplying it by 2. We will adhere to Foster and Wolfson's index and multiply the absolute and intermediate polarization indices by 2.

$$\eta^{ER} = \sum_{i=1}^{n/2} \varphi_i \eta_i \quad (21)$$

$$\eta^{FP} = \sum_{i=\left(\frac{n}{2}\right)+1}^{n} \Psi_i \eta_i \quad (22)$$

We then show in the Appendix that

$$\Delta P_{FW}^{\lambda} = \left(\frac{1}{4}\right)\left\{\left[\left(\frac{x^{ER}}{(m_x)^{\lambda}}\right)\frac{\left(\eta^{ER}-\overline{\eta}^{\lambda}\right)}{1+\overline{\eta}^{\lambda}}\right] + \left[\left(\frac{x^{FP}}{(m_x)^{\lambda}}\right)\frac{\left(\overline{\eta}^{\lambda}-\eta^{FP}\right)}{1+\overline{\overline{\eta}^{\lambda}}}\right]\right\} \quad (23)$$

We therefore can state that a sufficient condition for growth to be pro-middle class, is that the growth rate of the "transformed intermediate median income" of the whole population is higher than that of the weighted average of the growth rates of the rich, and smaller than the weighted average of the growth rates of the poor, both sets of weights being defined in (19) and (20). It should be noted that since $x^{ER} > x^{FP}$, the gap between η^{ER} and $\overline{\eta}^{\lambda}$ receive a higher weight. Thus, there may be cases in which a relatively small gap between the rich's income growth rate and the (intermediate) median growth rate will outweigh a bigger gap between the poor's income growth and the median income growth.

We also show in the appendix that in the case of λ=0 (absolute polarization)

$$\Delta P_{FW}^{\lambda} = \left(\frac{1}{4}\right)\left(\Delta x^{ER} - \Delta x^{FP}\right) \quad (24)$$

As expected, equation (24) is equivalent to equation (6), the only difference being a factor of 2, as already mentioned in footnote 2.

4. An empirical illustration

4.1. The database

The database is a set of income surveys conducted by Israel's Central Bureau of Statistics for the years 1995-2011[3]. During this period, significant changes in welfare and tax policies took place, as well as major changes in participation and employment patterns.

We have data on several income types, for households and individuals: for households we consider the household's income from salaried work, economic income (including work income, pensions and capital income, and excluding allowances and transfers) household total income and net disposable income. We also consider the household total and net equivalized income (adjusted for family size according to the Israeli/OECD equivalence scale[4]). Other income sources, such as allowances or capital income were either too volatile or were only available for a small part of the sample and hence were not taken into account. For individuals we consider income from salaried work (wage) and wage per hour worked. Individuals who did not work or did not receive a wage were excluded from the database. All the incomes were expressed at 2011 prices.

Each income survey includes data on approximately 32,000 to 35,000 individuals and 13,000 to 15,000 households[5]. Since our methodology requires a fixed number of observations, we divided the sample into 1,000 groups with similar sample weights, according to the relevant income type variable.

4.2 The results

Figure 1 presents intermediate bipolarization indices $IP(\lambda)$ (defined as the area below the relevant *IPC*) for selected income types during the period 1995-2011. As mentioned before,

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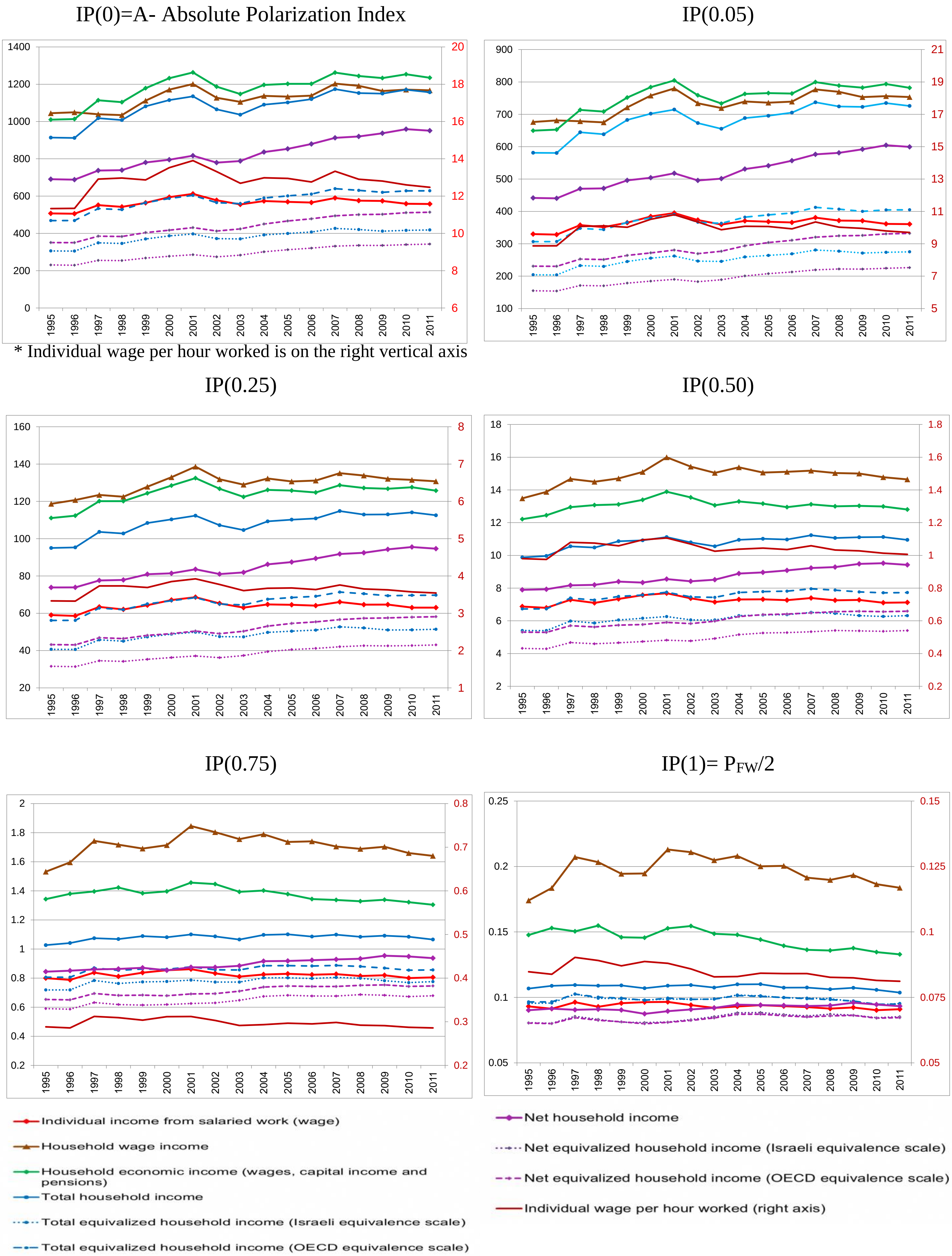
[3] Until 2011 labor force surveys were conducted every quarter. Every individual/household was asked to answer the questions in this survey during two consecutive quarters. There was then an interruption of two quarters and then again the individual/household was asked to fill the labor force survey questionnaire during two additional quarters. At the fourth panel the individual/household was also asked to fill a questionnaire on his/her income (the income survey). But this practice has been interrupted in 2011 because of a major change in the labor force survey.

[4] The Israeli equivalence scale assigns the value 1.25 to the first household member, 0.75 to the second, 0.65 to the third, 0.55 to the fourth and fifth members, 0.50 to the sixth and seventh, 0.45 to the eighth and 0.40 to each additional member. The most recent OECD equivalence scale amounts to dividing the household income by the square root of of the size of the household.

[5] The coverage of the income surveys increased in 1997 so that the number of observations almost doubled. The survey for 1995 includes data for about 17,000 individuals and 7,000 households.

Figure 1: Intermediate Polarization Indices, 1995-2011



$IP(0)$ is the Absolute bipolarization index and $IP(1)$ is a relative bipolarization index, equal to half of P^{FW}.

Figure 1 emphasizes the significant differences between the absolute polarization index *IP(0)* and the relative polarization index *IP(1),* while the intermediate polarization indices, as there name suggests, are somewhere in between. As λ increases, the intermediate polarization indices give a picture similar to that given by the relative polarizations index, while with smaller values of λ the picture becomes similar to that obtained with the absolute polarization index.

At the beginning of the period (1995-2000) absolute polarization increased for all income types. That is, all incomes moved further apart from the median in real absolute terms. Divergence started in the following periods, as shown in Figure 1. While the absolute polarization of household wage, economic and total household income had the same patterns of change, the absolute polarization of individual income from salaried work decreased and the polarization of net household income continued its upward trend almost unimpededly. These conflicting trends reflect some major developments in labor market participation and changes in policy, as will be explained below.

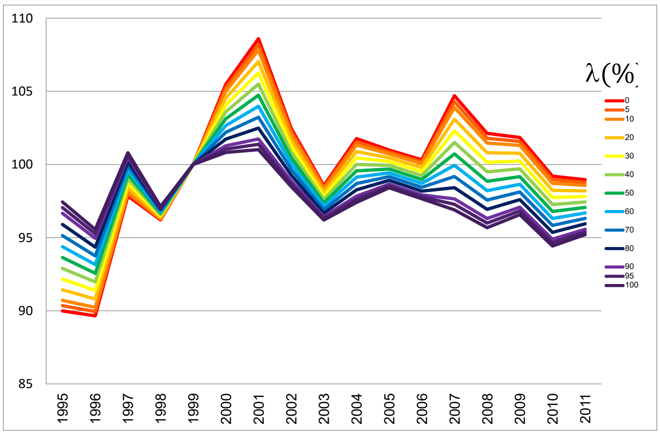
Relative polarization developed quite differently. The increase in polarization at the beginning of the period was shorter and less significant for most of the income types. After a period of stabilization, polarization then decreased for all market incomes (individual and household wage incomes, economic and total household income). Polarization of net household income increased between 2000 and 2004, and remained at this relative high level afterwards.

In order to compare the development of polarization according to different levels of intermediate polarization, we brought all the indices to a common level by setting 1999 as the reference year, where all indices are given a common value of 100. We then calculate each intermediate index relative to this reference year. Figure 2 displays the result of this calculation for different types of income. The red lines are for absolute polarization ($\lambda = 0$) and the purple lines are for relative polarization ($\lambda = 1$). It is clear that different levels of intermediation give significantly different descriptions of polarization, not only regarding the magnitude of the change, but also regarding its direction. For example, the absolute polarization of total and net household income increased between 1999 and 2000, while the relative polarization decreased. This was also true between 2004 and 2006, while the opposite occurred between 2001 and 2002. Over the whole period,

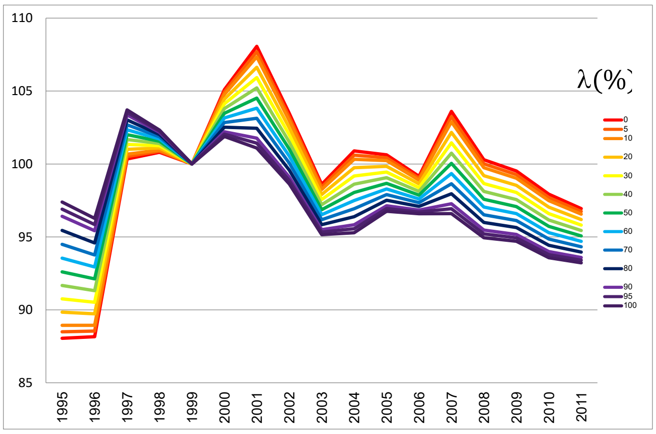
Figure 2: Intermediate Polarization Indices, 1995-2011

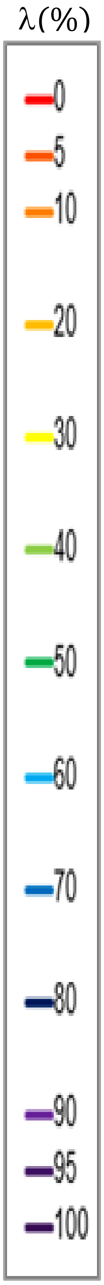
1999=100

Individual income from salaried work (wage)

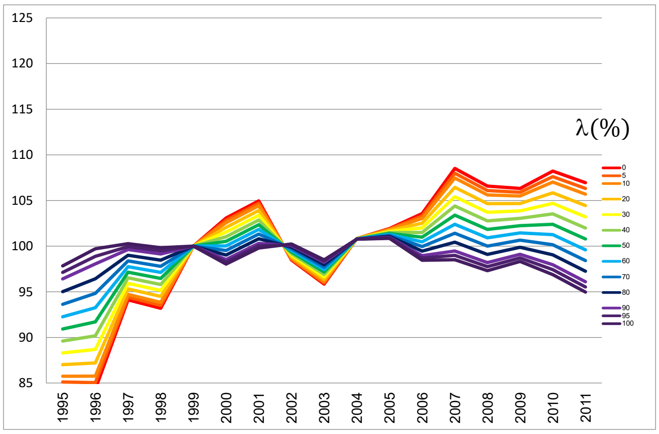


Individual wage per hour worked

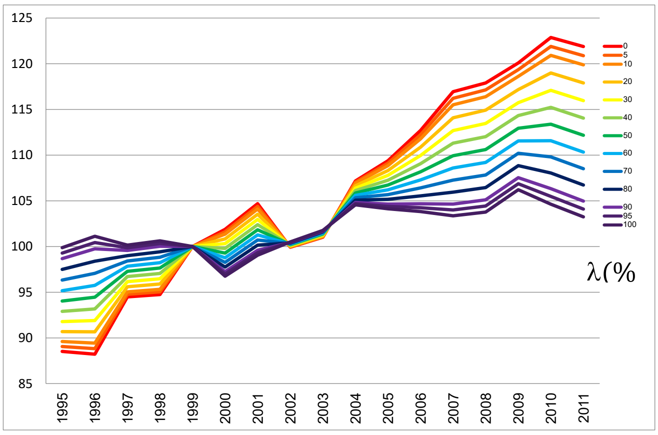




Total Household Income

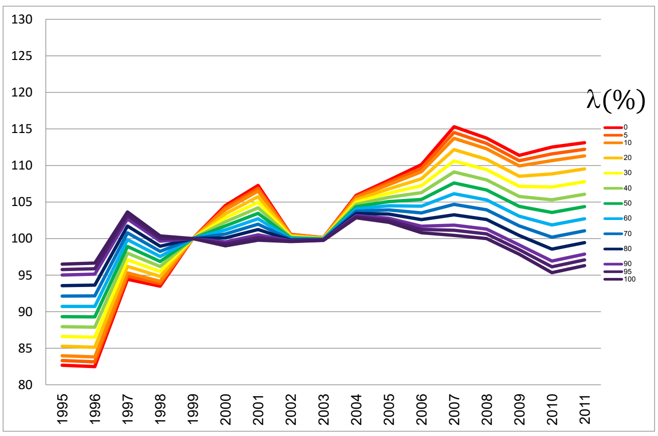


Net household income



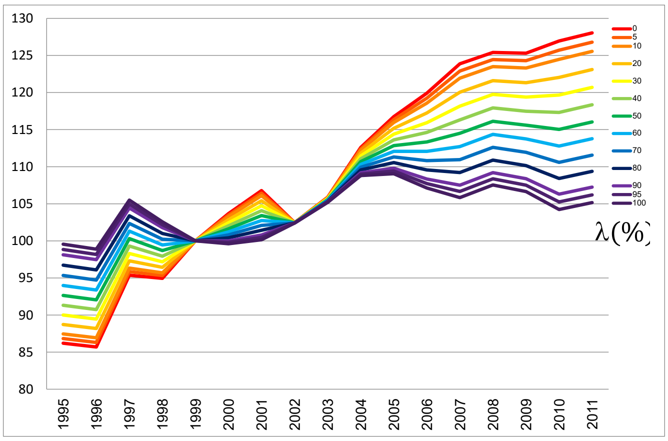
Total Equivalized Household Income

(Israeli equivalence scale)



Net Equivalized Household Income

(Israeli equivalence scale)



relative polarization of total household income decreased, while the absolute polarization sharply increased.

Table 1 gives some descriptive statistics for various income types at five points in time. As expected, due to the effect of assortative mating on inequality and bipolarity, household economic income and household wage income are the most polarized sources of income, regardless of the index used to measure polarization. Net income is less polarized than market incomes, as allowances and progressive taxes work to lower inequality and thus to narrow the spread around the median and reduce bipolarization. Equivalized incomes are less polarized than the original incomes (for both total and net incomes) since assuming economies of scale lowers the distance between the larger households at the bottom of the distribution and the smaller households at the top. OECD assumes higher economies of scale in comparison with the Israeli equivalence scale, and thus the mean and median of equivalized income are higher when computed according to the OECD scale, as is the absolute polarization index. There is, however, no significant difference as far as the intermediate and relative indices are concerned, (see also Figure 1)

The development of polarization indices reflects some significant changes in labor force participation patterns and economic policies that occurred in Israel during the period 1995-2011. Following an increase in social welfare spending, that took place during the last two decades of the twentieth century, a major budgetary cut took place in 2003. Social welfare payments were reduced, and eligibility criteria became stricter. In parallel, there was a decrease in income tax rates for the middle and top income brackets. As a result, the contribution of the government's direct intervention to reduce income inequality and polarization decreased. In addition, the change in policy, together with renewed economic growth, led to a significant increase in the participation and employment rates of population subgroups that previously were under-represented in the labor market, such as the ultra-orthodox Jews and Arab women. The higher labor force participation and employment rates led to an increase in wage and market incomes for the lower social strata of the population, and therefore to a reduction in the degree of inequality and relative bi-polarization of these incomes. However, this impact of higher labor force participation was offset by that of the tax reduction, which was concentrated on the middle and top income brackets, so that the combined effect on net income bi-polarization was much smaller.

Table 1: Descriptive statistics by income types, for selected time period

	Median	Mean	Mean to median ratio	x^{ER}	x^{FP}	A= IP(0)	IP(0.01)	IP(0.05)	IP(0.25)	IP(0.5)	IP(0.75)	P^{FW} = IP(1)*2
Individual income from salaried work (wage)												
1995	5,448	7,247	1.3	8,062	4,004	507	465	330	59	7	0.8	0.19
1999	5,899	8,162	1.4	8,922	4,413	564	517	365	64	7	0.8	0.19
2003	6,043	8,224	1.4	9,013	4,569	556	509	359	63	7	0.8	0.18
2007	6,374	8,773	1.4	9,536	4,815	590	541	381	66	7	0.8	0.19
2011	6,131	8,308	1.4	9,115	4,653	558	511	361	63	7	0.8	0.18
Individual wage per hour worked												
1995	134	173	1.3	193	102	11	11	9	3	1	0.3	0.17
1999	148	197	1.3	217	114	13	12	10	4	1	0.3	0.17
2003	153	216	1.4	223	121	13	12	10	4	1	0.3	0.17
2007	159	214	1.3	232	125	13	13	10	4	1	0.3	0.17
2011	154	205	1.3	222	122	12	12	10	4	1	0.3	0.16
Household wage income												
1995	6,002	8,613	1.4	10,860	2,502	1,045	958	676	119	13	1.5	0.35
1999	5,718	9,099	1.6	11,217	2,321	1,112	1,020	721	128	15	1.7	0.39
2003	5,397	8,732	1.6	10,800	1,957	1,105	1,014	719	129	15	1.8	0.41
2007	6,284	9,966	1.6	12,244	2,619	1,203	1,102	777	135	15	1.7	0.38
2011	6,352	9,793	1.5	12,164	2,826	1,167	1,069	753	131	15	1.6	0.37
Household economic income (wages, capital income and pensions)												
1995	6,841	9,709	1.4	11,789	3,709	1,010	925	650	111	12	1.3	0.30
1999	8,080	11,778	1.5	13,950	4,519	1,179	1,077	752	124	13	1.4	0.29
2003	7,720	11,240	1.5	13,423	4,244	1,147	1,049	733	122	13	1.4	0.30
2007	9,258	13,102	1.4	15,478	5,378	1,262	1,152	800	129	13	1.3	0.27
2011	9,293	12,837	1.4	15,421	5,545	1,235	1,127	782	126	13	1.3	0.27

Table 1: Descriptive statistics by income types, for selected time periods (cont.)

	Median	Mean	Mean to median ratio	x^{ER}	x^{FP}	A= IP(0)	IP(0.01)	IP(0.05)	IP(0.25)	IP(0.5)	IP(0.75)	P^{FW} = IP(1)*2
Total household income												
1995	8,559	11,455	1.3	13,178	5,867	914	835	581	95	10	1.0	0.21
1999	9,910	13,736	1.4	15,488	6,837	1,081	986	683	108	11	1.1	0.22
2003	9,643	13,203	1.4	14,962	6,670	1,037	946	655	105	11	1.1	0.21
2007	10,917	14,953	1.4	16,921	7,533	1,174	1,069	737	115	11	1.1	0.22
2011	11,162	14,728	1.3	16,970	7,716	1,157	1,054	726	113	11	1.1	0.21
Total equivalized household income (Israeli equivalence scale)												
1995	3,203	4,287	1.3	4,779	2,331	306	282	204	41	5	0.7	0.19
1999	3,739	5,153	1.4	5,642	2,682	370	341	245	47	6	0.8	0.20
2003	3,753	5,046	1.3	5,612	2,647	371	341	246	47	6	0.8	0.20
2007	4,292	5,797	1.4	6,463	3,049	427	392	281	53	7	0.8	0.20
2011	4,391	5,711	1.3	6,463	3,114	419	385	275	51	6	0.8	0.19
Net household income												
1995	7,656	9,180	1.2	10,968	5,442	691	632	442	74	8	0.8	0.18
1999	8,637	10,612	1.2	12,426	6,183	780	713	496	81	8	0.9	0.18
2003	8,574	10,433	1.2	12,375	6,069	788	720	501	82	9	0.9	0.18
2007	9,772	12,090	1.2	14,212	6,912	912	832	576	92	9	0.9	0.19
2011	10,197	12,435	1.2	14,781	7,172	951	867	599	95	9	0.9	0.19
Net equivalized household income (Israeli equivalence scale)												
1995	2,859	3,468	1.2	3,996	2,150	231	213	155	32	4	0.6	0.16
1999	3,302	4,020	1.2	4,558	2,416	268	247	179	35	5	0.6	0.16
2003	3,323	4,024	1.2	4,663	2,396	283	261	189	37	5	0.6	0.17
2007	3,864	4,714	1.2	5,446	2,793	332	305	219	42	5	0.7	0.17
2011	4,020	4,835	1.2	5,632	2,890	343	315	226	43	5	0.7	0.17

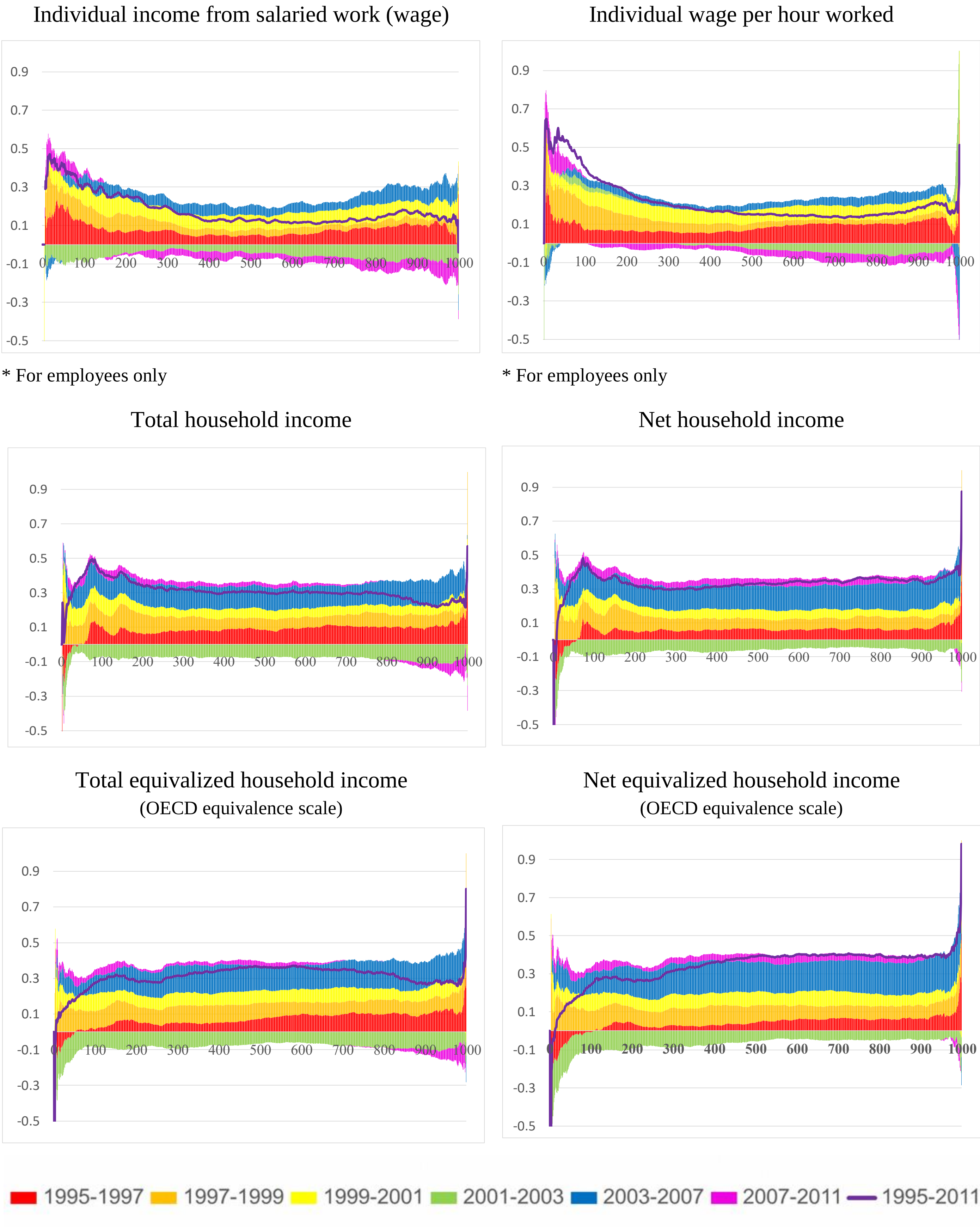
The development of absolute bi-polarization was somewhat different from that of relative bi-polarization, since it also reflects changes in the median income. For example, the economic downturn between 2001 and 2003 was reflected in a decrease in median incomes (in real terms) and thus, in a decline of absolute polarization. The economic recovery in 2004-2007 led to an increase in median incomes, which intensified the increase in polarization. Thus, there was a noticeable increase in absolute polarization during this period while relative polarization increased in a much more moderate manner or even decreased in some cases. In general, over the whole period, both μ^{ER} and μ^{FP} ("equally distributed equivalent level of income among the "rich" and the weighted average of the incomes of the "poor") are highly correlated with the median. Thus, even when the spread around the median widened in absolute terms, it became narrower in relative terms. This is especially true when comparing the absolute polarization of net income, which followed a continuous up trend, and the relative polarization of net income, which remained almost unchanged between 2004 -2011.

A closer look at the impact of the various economic developments that took place during the period is given in Figure 3, which displays the changes in different types of income during selected time periods and in Table 2 which summarizes the distributional changes of various income types during selected time periods.

Overall, in most of the sub-periods, individuals and households at all income levels enjoyed positive real growth in incomes. Two exceptions were the period between 2001 and 2003, in which there was a significant decline in real incomes, for most of the income types and over all income levels, and the period between 2007 and 2011, in which the lower strata saw a small increase in real incomes while for the higher strata there was a decrease in real incomes.

Figure 3 emphasizes the different impact of economic policy on the lower strata and the top strata. Over the whole period- 1995-2011, the growth of wage and total incomes was in favor of the "poor", those with incomes below the median. The opposite is true for net income, which increased more above the median. As expected, the differences between total and net incomes growth are more significant above the median, as the lower strata bear a negligible part of the tax burden. Furthermore, after the major cut in allowances that took place in 2003, there was a significant increase in participation and employment rates of population sub-groups that previously were under-represented in the labor market. In spite of this increase, the total household income of the lower deciles increased by about the same rate as the median and top

Figure 3: Changes in various income types during selected time periods



incomes, which were less affected by the increase in employment. This is mainly a result of the cut in allowances that offset the increase in household wage income.

As mentioned before, a sufficient condition for growth to be pro-middle class is for the growth rate of the "intermediate median income" of the whole population to be higher than that of the weighted average of the growth rates of the rich and smaller than the weighted average growth rate of the poor. When λ=0, the intermediate median income is equal to 1 and thus its growth rate is always 0. Consequently, a sufficient condition for growth to be pro-middle class when λ=0 is for the weighted average of the growth rates of the poor to be higher than the weighted average of the growth rate of the rich. During most of the sub-periods, the relative increase among the poor in the individual income from salaried work was higher than the increase of the intermediate median income. The only exception was during the sub period 2003-2007. The relatively low income growth during that period may be explained by a compositional change, as more households with lower earning capacity increased their labor supply in response to the cut in allowances. The increased participation of such low-wage workers partially offsets the increase in wage income at the lower strata. However, a higher growth at the lower half is by itself insufficient to guarantee pro-middle class growth. Intermediate median income growth should also be higher than the weighted average of the growth rates of the rich. This condition was not met in most of the sub-periods, except for the sub-periods 2001-2003 and 2007-2011 where the growth rate of the rich was also lower than the growth of the intermediate median income. An interesting case is the period 1997-1999, in which absolute and relative polarization went in opposite directions: since both the poor and the rich enjoyed an income growth which was higher than that of the median, the final result was determined by the weights assigned to each of these components. In absolute terms, the higher growth rate above the median outweighed the impact of the higher growth rate below it, so that absolute polarization increased. In relative terms the result was the opposite. It should be noted, however, that both the positive change in the absolute index and the negative change in the relative index are small. A contrast between the change in absolute and relative polarization occurred also during the whole period, 1995-2011, when absolute polarization increased but intermediate and relative polarization decreased.

Except for the very beginning of the period, 1995-1997 and the economic recovery period 2003-2007, the increase in total income during all sub-periods was higher for the poor than for the rich. The total income growth among the poor was also higher than the median and intermediate.

Table 2: Distributional changes of various income types during selected time periods

			λ=0				λ=0.5					λ=1			
	η^{ER}	η^{FP}	ΔA	$\Delta IP^{0.0}$	$\Delta IP^{0.0}$	$\Delta IP^{0.2}$	$\overline{\eta}^{0.5}$	$Gap_{ER}^{0.5}$	$Gap_{FP}^{0.5}$	$\Delta IP^{0.5}$	$\Delta IP^{0.7}$	$\overline{\eta}^{1}$	Gap_{ER}^{1}	Gap_{FP}^{1}	$\Delta IP^{1} = \Delta P_{FW}$
	Percent change						Percent change					Percent change			Percentage points
Individual income from salaried work (wage)															
1995-1997	7.2	5.7	89	81	56	9	2.5	4.6	-3.1	0.8	0.08	5.1	2.0	-0.6	0.6
1997-1999	3.2	4.3	24	22	15	2	1.5	1.7	-2.7	0.1	0.00	3.0	0.2	-1.2	-0.2
1999-2001	8.1	7.5	97	88	60	9	3.7	4.2	-3.7	0.7	0.05	7.5	0.5	0.0	0.2
2001-2003	-6.5	-3.7	-113	-103	-71	-11	-2.4	-4.2	1.4	-1.1	-0.10	-4.7	-1.9	-1.1	-0.9
2003-2007	5.8	5.4	69	63	43	6	2.7	3.0	-2.6	0.5	0.03	5.5	0.3	0.1	0.1
2007-2011	-4.4	-3.4	-65	-59	-40	-6	-1.9	-2.5	1.5	-0.5	-0.04	-3.8	-0.6	-0.5	-0.3
1995-2011	13.1	16.2	101	92	61	8	6.1	6.6	-9.5	0.5	0.01	12.5	0.5	-3.2	-0.4
Total household income															
1995-1997	10.0	8.4	208	188	127	17	4.3	5.5	-3.9	1.4	0.10	8.7	1.2	0.3	0.5
1997-1999	6.8	7.5	127	115	76	10	3.2	3.5	-4.2	0.6	0.03	6.5	0.3	-0.9	-0.1
1999-2001	5.2	5.5	107	97	64	8	2.6	2.6	-2.8	0.5	0.02	5.2	0.0	-0.3	0.0
2001-2003	-8.2	-7.5	-197	-178	-119	-15	-3.8	-4.5	3.8	-1.1	-0.07	-7.5	-0.7	0.0	-0.3
2003-2007	13.1	12.9	274	247	164	20	6.4	6.3	-6.2	1.4	0.07	13.2	-0.1	0.2	0.0
2007-2011	0.3	2.4	-34	-31	-23	-5	1.1	-0.8	-1.3	-0.6	-0.07	2.2	-1.9	-0.2	-0.8
1995-2011	28.8	31.5	486	438	289	35	14.2	12.8	-15.2	2.1	0.08	30.4	-1.2	-0.9	-0.6
Net household income															
1995-1997	6.3	5.9	93	84	57	8	3.2	3.1	-2.6	0.5	0.03	6.4	-0.1	0.5	0.1
1997-1999	6.6	7.3	86	78	52	7	3.0	3.5	-4.2	0.5	0.02	6.0	0.5	-1.2	0.0
1999-2001	5.0	5.3	73	66	44	5	2.8	2.1	-2.5	0.3	0.01	5.7	-0.6	0.3	-0.2
2001-2003	-5.2	-6.8	-57	-51	-33	-3	-3.1	-2.1	3.9	-0.1	0.02	-6.1	1.0	0.8	0.5
2003-2007	14.8	13.9	248	225	150	20	6.8	7.6	-6.7	1.4	0.09	14.0	0.8	0.1	0.3
2007-2011	4.0	3.8	77	70	46	6	2.2	1.8	-1.6	0.4	0.02	4.4	-0.3	0.6	0.0
1995-2011	34.8	31.8	521	471	316	42	15.4	16.8	-14.2	3.0	0.19	33.2	1.2	1.1	0.6

$Gap_{ER}^{\lambda} = \frac{(\eta^{ER} - \overline{\eta}^{\lambda})}{1+\overline{\eta}^{\lambda}}$ $\quad Gap_{FP}^{\lambda} = \frac{\left(\overline{\eta}^{\lambda} - \eta^{FP}\right)}{1+\overline{\eta}^{\lambda}}$

Table 3: Distributional changes during the period 1995-2011, by selected income types

	1995					λ=0	λ=0.5				λ=1			
	Median	x^{ER}	x^{FP}	η^{ER}	η^{FP}	ΔA	$\bar{\eta}^{0.5}$	$Gap^{0.5}_{ER}$	$Gap^{0.5}_{FP}$	$\Delta IP^{0.5}$	$\bar{\eta}^{1}$	Gap^{1}_{ER}	Gap^{1}_{FP}	$\Delta IP^{1} = \Delta P_{FW}$
				Percent change			Percent change				Percent change			Percentage points
Individual income from salaried work (wage)	5,448	8,062	4,004	13.1	16.2	101	6.1	6.6	-9.5	0.5	12.5	0.5	-3.2	-0.4
Individual wage per hour worked	134	193	102	15.1	19.6	2	7.2	7.3	-11.5	0.1	15.0	0.1	-3.9	-0.7
Household wage income	6,002	10,860	2,502	12.0	12.1	250	2.9	8.9	-9.0	2.4	5.8	5.8	-5.9	2.0
Household economic income (wages, capital income and pensions)	6,841	11,789	3,709	30.8	47.6	467	16.6	12.2	-26.6	1.4	35.9	-3.7	-8.6	-2.8
Total household income	8,559	13,178	5,867	28.8	31.5	486	14.2	12.8	-15.2	2.1	30.4	-1.2	-0.9	-0.6
Total equivalized household income (Israeli equivalence scale)	3,203	4,779	2,331	35.2	33.6	225	17.1	15.5	-14.1	1.8	37.1	-1.4	2.6	0.0
Total equivalized household income (OECD equivalence scale)	4,858	7,265	3,515	33.7	33.2	320	16.8	14.5	-14.1	2.0	36.3	-2.0	2.3	-0.3
Net household income	7,656	10,968	5,442	34.8	31.8	521	15.4	16.8	-14.2	3.0	33.2	1.2	1.1	0.6
Net equivalized household income (Israeli equivalence scale)	2,859	3,996	2,150	40.9	34.4	224	18.6	18.9	-13.4	2.2	40.6	0.2	4.4	0.9
Net equivalized household income (OECD equivalence scale)	4,366	6,063	3,255	39.6	33.7	325	18.0	18.3	-13.4	2.6	39.1	0.3	3.9	0.8

$Gap^{\lambda}_{ER} = \frac{(\eta^{ER} - \bar{\eta}^{\lambda})}{1+\bar{\eta}^{\lambda}}$ $Gap^{\lambda}_{FP} = \frac{(\bar{\eta}^{\lambda} - \eta^{FP})}{1+\bar{\eta}^{\lambda}}$

median income growth in most of these sub-periods. The same is true for the whole period, 1995-2011. As a result, relative polarization decreased significantly during all these sub-periods and over the whole period. Absolute polarization, however, increased during 1997-1999 and over the whole period.

Due to a reform in taxes the distributional change of net income was significantly different from that of total income. Except for one sub-period, 1997-1999, the net income growth among the poor was lower than that of the median. When looking at the intermediate median ($\lambda = 0.5$), the opposite is true: for all sub-periods, except for the period between 2001 and 2003, the net income growth among the poor was higher than that of the intermediate median income. The net income growth of the rich was higher than that of the intermediate median growth rate in most of the sub-periods (for $\lambda = 0.5$) and higher than the median income growth rate in fewer cases. Overall, absolute, intermediate and relative polarization increased in most of the subperiods and during the whole period. There were two sub periods in which the indices did not agree and pointed to an opposite direction of change: in 1999-2001 there was an increase in absolute and intermediate polarization in parallel with a decline in relative polarization. The opposite occurred in the following sub-period, 2001-2003.

Table 3 summarizes the distributional change of different types of income during the whole period 1995-2011.

Over the whole period, the degree of absolute bi-polarization increased for all income types. This happened despite a relatively large income growth below the median in all market incomes (individual wage and wage per hour incomes, household wage, economic and total incomes). Even though η^{FP} was larger than η^{ER} for most of these incomes, it was not sufficient to reduce absolute polarization since η^{ER} gets a significantly higher weight.

On the other hand, the level of relative polarization decreased for most of the market incomes. This result reflects the rapid growth of wage income in the lower part of the distribution, due to increased labor force participation, and to some extent due to the effect of an increase in the minimum wage. We observe that $\eta_{FP} > \overline{\eta}^{0.5}$ for all income types. However, this relatively rapid increase in market income was offset by a decrease in allowances so that the growth rate of income at the lower end of the distribution was similar or even lower than the growth rate of the median income. Thus $\eta_{FP} < \overline{\eta}^{1}$ for total equivalized income and for net income. For the 'rich' (households whose income is higher than the median income), we observe that $\eta_{ER} > \overline{\eta}$ for the

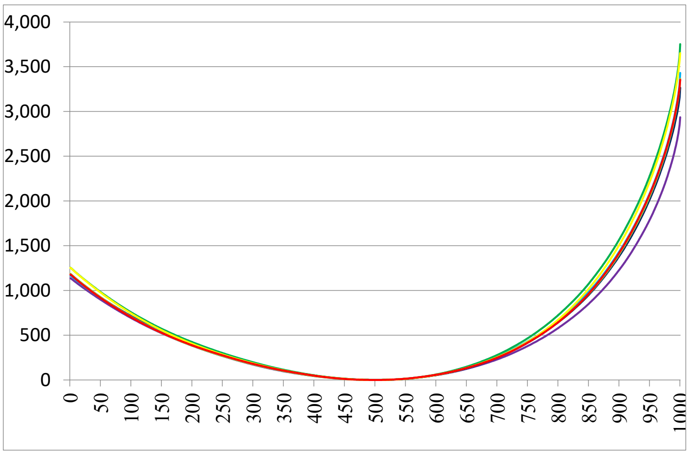
individual income from salaried work, individual wage per hour worked and household wage income, net household income and net equivalized income. This in itself would lead, ceteris paribus, to an increase in the relative bi-polarization of incomes. We also note that $\eta_{ER} < \overline{\eta}$ for household economic income, total household income and total equivalized income. The net result of these changes in the income of the 'rich' and the 'poor' is that the overall degree of relative bi-polarization decreased for individual income from salaried work, individual wage per hour worked, household economic income (the biggest decrease in percentage terms), total household income and total equivalized income, but increased for net income and net equivalized income
As mentioned before, the increase in the relative bi-polarization of net income reflects the combined effect of developments in labor force participation and welfare policy, which affected mainly the lower strata, together with the effect of a major change in income tax that affected mainly the upper-middle incomes.

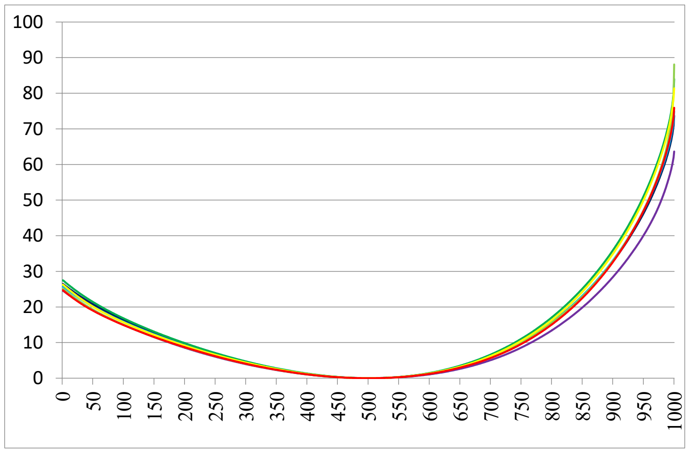
Intermediate bi-polarization curves (*IPC*) are presented in Figure 4. They emphasize the different evolution over time of market and net incomes. The curves also show the differences between absolute, intermediate and relative polarization curves. The *APC (IPC(0))* of wage income decreased slightly below the median. That is, the absolute real change in the wage income of the "poor" was only slightly higher than that of the median. On the right hand side, above the median, there was a more noticeable change in the *APC*, which moved up between 1995 and 2003 and down again afterwards: the rich moved away from the median in real absolute terms at the beginning of the period and got closer to the median afterwards. This was also true for the wage per hour. On the right hand side *IPC(0.5)* and the relative bi- polarization curves, *IPC(1)* changed much less, indicating that the change in the incomes of the rich was proportional to the change in the (intermediate) median. On the left hand side *IPC(1)* slightly decreased. This may reflect the changes in the minimum wage that brought the low- wage workers closer to the median wage. While the *APCs* of wage and wage per hour display the same evolution over time, the *APC* of net household income gives a completely different picture, with a continuous increase in the *APC* on both sides of the median. The *APC* of total household income changed in a similar way at the beginning of the period but by the end of the period there was only a modest change in the right hand side of the *APC*. The changes in total and net household income were proportional to the change in the median, and thus the differences between the curves *IPC(1)* are very small, and the curves are almost completely overlapping.

Figure 4: Intermediate polarization curves

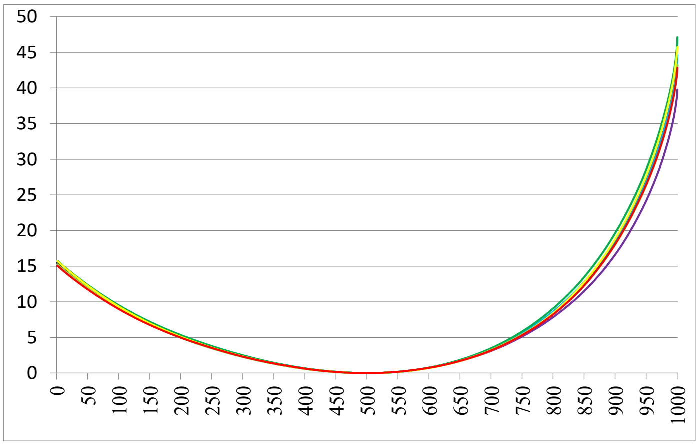
Individual income from salaried work (wage) | Individual wage per hour worked

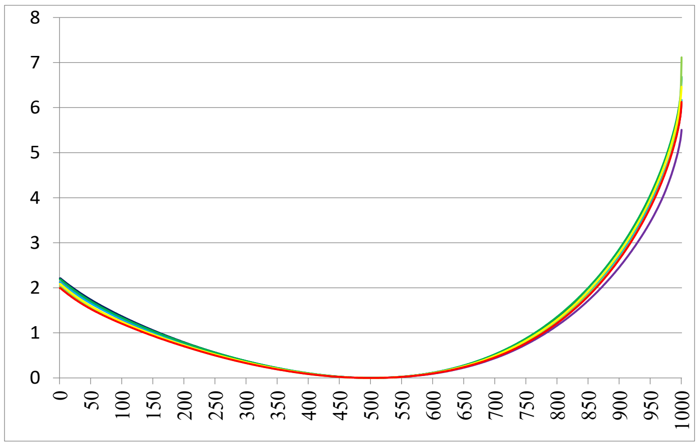
IPC(0)=APC Absolute Polarization Curve



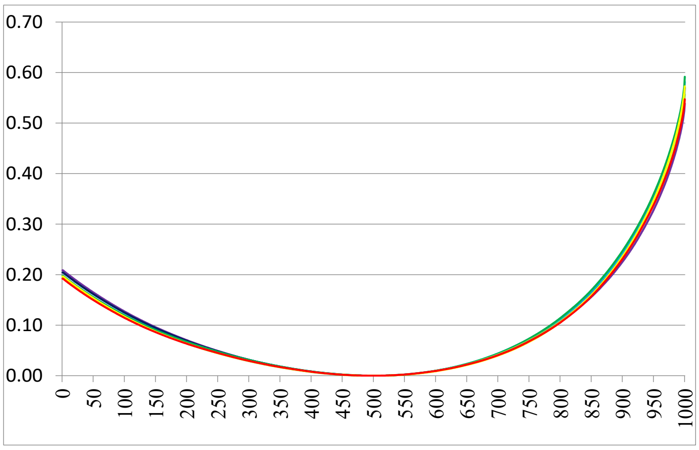


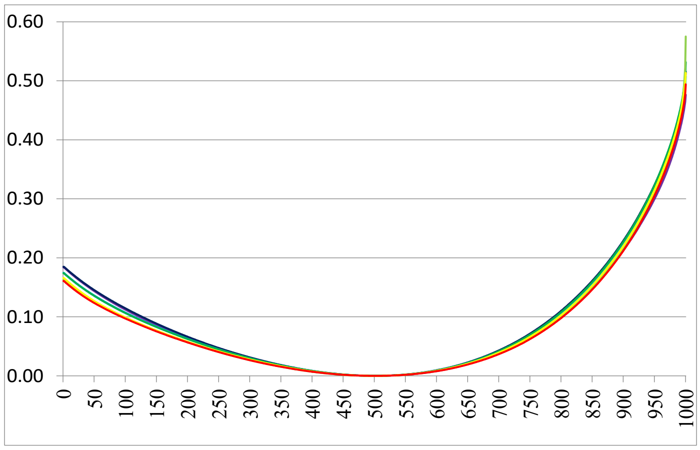
IPC(0.5)





IPC(1)=second order relative bi-polarization curve



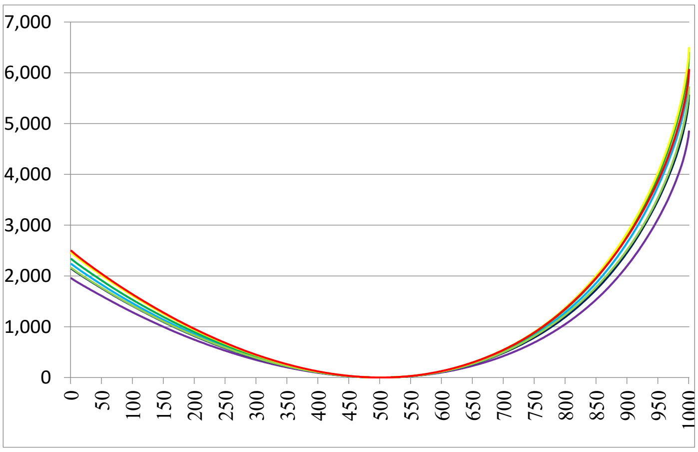


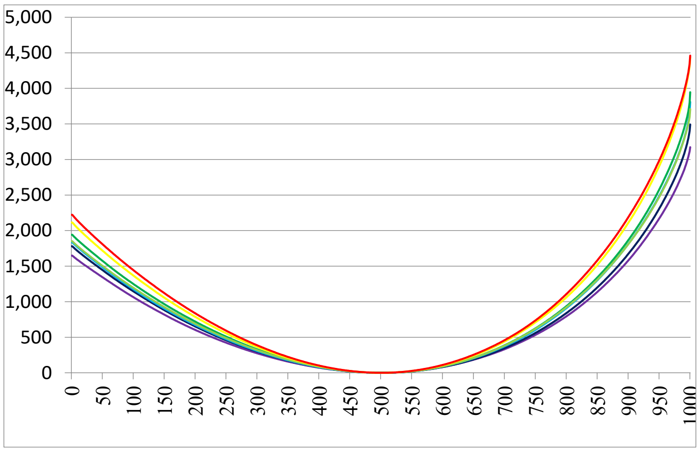
—1995 —1997 —1999 —2001 —2003 —2007 —2011

Figure 4 (cont.): Intermediate polarization curves

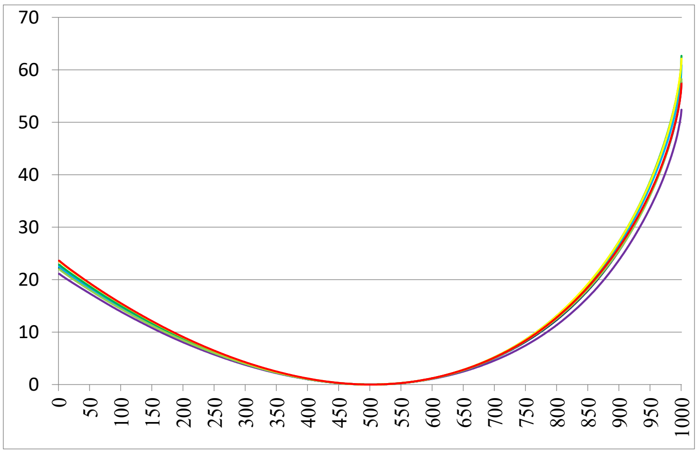
Total household income Net household income

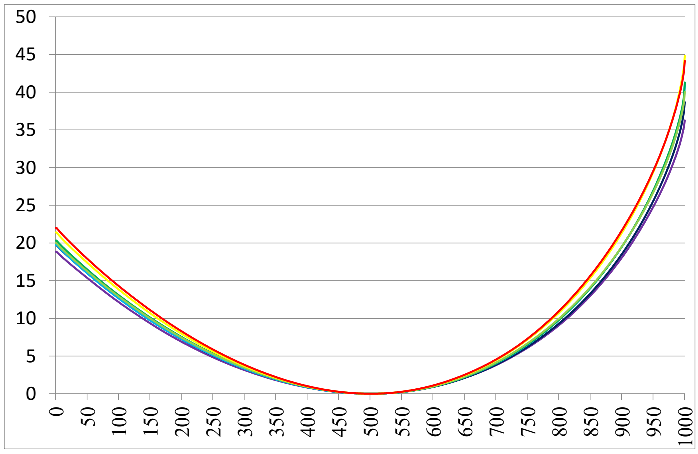
IPC(0)=APC Absolute Polarization Curve



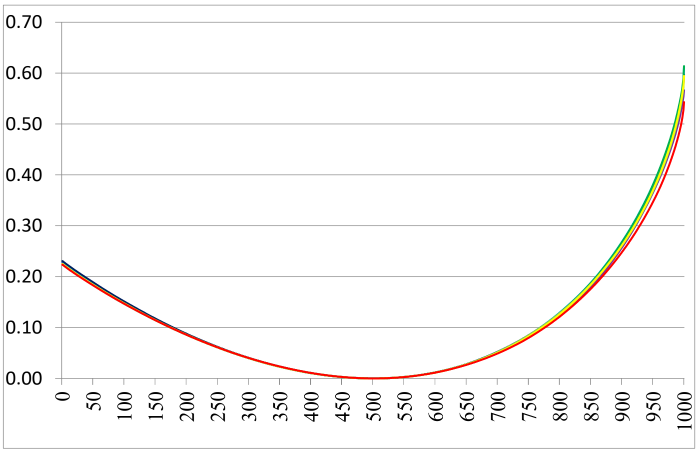


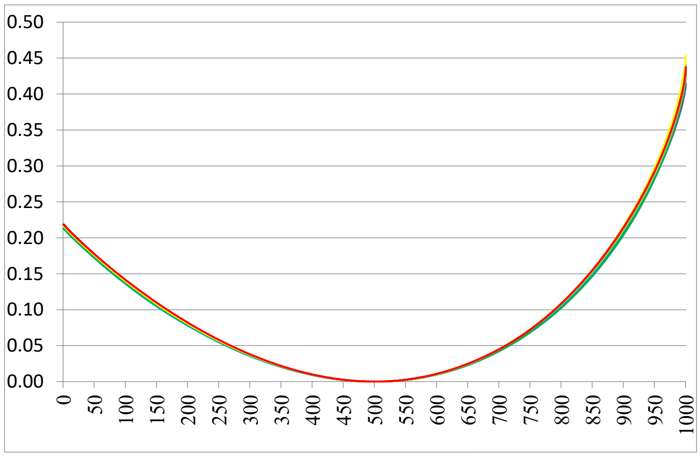
IPC(0.5)





IPC(1)=Second order relative bi-polarization curve





—1995 —1997 —1999 —2001 —2003 —2007 —2011

5. Concluding comments

This paper proposed a broader definition of pro-middle class growth, using the approach of Lasso de la Vega et al. (2010) to intermediate polarization. It appears that a sufficient condition for growth to be pro-middle class is for the growth rate of the “intermediate median income” of the whole population to be higher than that of the weighted average of the growth rates of the rich and smaller than the weighted average growth rate of the poor, the ‘rich’ and the ‘poor' being respectively those with an income higher and lower than the median income. An empirical illustration based on Israeli data for the period 1995-2011 showed that in absolute terms growth was not pro-middle class for any income type. In contrast, growth was pro-middle class in relative terms for all market incomes (individual income from salaried work, individual wage per hour worked, household economic income, total household income and total equivalized income). Growth was not pro-middle class for net income and net equivalized income, even in relative terms. This is a consequence of the combined effect of developments in labor force participation, welfare policy changes and major modifications in income tax rates. The intermediate polarization measures indicate that there was no pro middle class growth for all income types and for most of the values of λ, with an exception in the case of specific market income types and specific (high) values of λ. In these cases we observe a small decrease in polarization**.**

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Appendix A

1) On an absolute measure of pro-middle class growth

Let us first compute the area *R* under the *APC* on the R.H.S. of the median (incomes higher than the median income). We may note that at the rank n_m , which corresponds to the median, the height of the *APC* is zero. To simplify the demonstrations we will assume that *n* is even.

Using (2) and the well-known expressions for the areas of a triangle and of a trapezium, we derive that

$$R = \left(\frac{1}{2}\right)\left(\frac{1}{n}\right)\left\{\left[\left(\frac{1}{n}\right)\left(x_{n_m-1} - m\right)\right] + \left[2\left(\frac{1}{n}\right)\left(x_{n_m-1} - m\right) + \left(\frac{1}{n}\right)\left(x_{n_m-2} - m\right)\right]\right\} + \ldots$$

$$+\left(\frac{1}{2}\right)\left(\frac{1}{n}\right)\left\{2\left(\frac{1}{n}\right)\left[\left(x_{n_m-1} - m\right) + \left(x_{n_m-2} - m\right) + \cdots + (x_2 - m)\right] + \left(\frac{1}{n}\right)(x_1 - m)\right\} \qquad \text{(A-1)}$$

$$\leftrightarrow R = (1/n)^2\left\{\left(\frac{1}{2}\right)x_1 + \left(\frac{3}{2}\right)x_2 + \cdots + \left(\frac{2i-1}{2}\right)x_i + \cdots + \left(\frac{2(n_m-1)-1}{2}\right)x_{n_m-1}\right\}$$

$$-(1/n)^2\left\{\left(\frac{1}{2}\right)m + \left(\frac{3}{2}\right)m + \cdots + \left(\frac{2i-1}{2}\right)m + \cdots + \left(\frac{2(n_m-1)-1}{2}\right)m\right\} \qquad \text{(A-2)}$$

$$\leftrightarrow R = (1/2)\left[\sum_{i=1}^{\frac{n}{2}}\left(\frac{2i-1}{n^2}\right)(x_i - m)\right] = \left(\frac{1}{8}\right)\left[\sum_{i=1}^{\frac{n}{2}}\frac{(2i-1)}{\left(\frac{n^2}{4}\right)}(x_i - m)\right] = \frac{\mu^{ER} - m}{8} \qquad \text{(A-3)}$$

where μ^{ER} may be labeled the "equally distributed equivalent level of income among the "rich" and defined, in the case of the Gini index, (see, Berrebi and Silber, 1989) as

$$\mu^{ER} = \sum_{i=1}^{n/2}[(2i-1)/(n^2/4)]\, x_i \qquad \text{(A-4)}$$

It is easy to verify that $\sum_{i=1}^{n/2}(2i-1)/(n^2/4) = 1$.

If we now compute the area *L* under the APC on the L.H.S. of the median (incomes lower than the median income), we derive, using (1), that

$$L = \left(\frac{1}{2}\right)\left(\frac{1}{n}\right)\left\{\left[\left(\frac{1}{n}\right)\left(m - x_{n_m+1}\right)\right] + \left[2\left(\frac{1}{n}\right)\left(m - x_{n_m+1}\right) + \left(\frac{1}{n}\right)\left(m - x_{n_m+2}\right)\right]\right\} + \ldots +$$

$$\left(\frac{1}{2}\right)\left(\frac{1}{n}\right)\left\{2\left(\frac{1}{n}\right)\left[\left(m - x_{n_m+1}\right) + \left(m - x_{n_m+2}\right) + \cdots + (m - x_{n-1})\right] + \left(\frac{1}{n}\right)(m - x_n)\right\} \qquad \text{(A-5)}$$

$$\leftrightarrow L = \left(\frac{1}{n}\right)^2 \left\{\left[\left(\frac{1}{2}\right) m + \left(\frac{3}{2}\right) m + \cdots + \left(\frac{(2(n-n_m)-1)}{2}\right) m\right]\right\}$$
$$-\left(\frac{1}{n}\right)^2 \left\{\left[\left(\frac{1}{2}\right) x_n + \left(\frac{3}{2}\right) x_{n-1} + \cdots + \left(\frac{(2(n-n_m)-1)}{2}\right) x_{n_m+1}\right]\right\}$$

$$\leftrightarrow L = \left(\frac{1}{2}\right) \sum_{i=\left(\frac{n}{2}\right)+1}^{n} \left\{\left[\frac{[2(n-i+1)-1]}{n^2}\right] (m - x_i)\right\} = \frac{(m-\mu^{FP})}{8} \quad \text{(A-6)}$$

where μ^{FP} is a weighted average of the incomes of the "poor" and is defined as

$$\mu^{FP} = \sum_{i=\left(\frac{n}{2}\right)+1}^{n} [(2(n-i+1)-1)]/(n^2/4)]\, x_i \quad \text{(A-7)}$$

Here also it is easy to verify that

$$\sum_{i=(n/2)+1}^{n} [2(n-i+1)-1] = \left(\frac{n^2}{4}\right)$$

Note that, as in the definition of μ^{ER}, the definition of μ^{FP} indicates that the further away an individual is from the median income, the smaller the weight given to this individual. In other words μ^{FP} gives, for the subpopulation of "poor", a higher weight the less "poor" the "poor" is.

Combining (A-3) and (A-6) we derive that the total area A under the APC (on both the R.H.S. and the L.H.S. of the APC) is

$$A = R + L = \left(\frac{1}{8}\right)(\mu^{ER} - \mu^{FP}) \quad \text{(A-8)}$$

Assume now that $\{x\}$ refers to the distribution of incomes at time 0 and $\{y\}$ to the distribution of incomes at time 1. Call respectively $A_0, A_1, x^{ER}, y^{ER}, x^{FP}$ and y^{FP} the total areas under the APC curves and the weighted average of incomes μ^{ER} and μ^{FP} at times 0 and 1. Using (A-8) the change ΔA over time in the area A will then be

$$\Delta A = \left(\frac{1}{8}\right)\{[y^{ER} - y^{FP}] - [x^{ER} - x^{FP}]\} = \left(\frac{1}{8}\right)\{[y^{ER} - x^{ER}] - [y^{EP} - x^{FP}]\} \quad \text{(A-9)}$$

If y_j and x_j refer to the income of some individual *j* at times 1 and 0 while $\Delta x_j = (y_j - x_j)$, expression (A-9), using (A-4) and (A-7), will become equal to

$$\Delta A = \left(\frac{1}{8}\right)\left[\sum_{i=1}^{n/2} \left(\frac{2i-1}{(n^2)/4}\right) \Delta x_i - \sum_{i=(n/2)+1}^{n} \left(\frac{2n-2i+1}{(n^2)/4}\right) \Delta x_i\right] \quad \text{(A-10)}$$

$$\leftrightarrow \Delta A = \left(\frac{1}{8}\right)\left[\sum_{i=1}^{n/2} \left(\frac{2i-1}{n^2/4}\right)(\Delta x_i - \Delta x_{n-i+1})\right] = \left(\frac{1}{8}\right)(\Delta x^{ER} - \Delta x^{FP}) \quad \text{(A-11)}$$

where

$$\Delta x^{ER} = \sum_{i=1}^{n/2} \left(\frac{2i-1}{(n^2)/4}\right) \Delta x_i \quad \text{(A-12)}$$

$$\Delta x^{FP} = \sum_{i=1}^{n/2}\left(\frac{2i-1}{(n^2)/4}\right)\Delta x_{n-i+1} = \sum_{i=\left(\frac{n}{2}\right)+1}^{n}\left(\frac{2(n-i+1)-1}{n^2/4}\right)\Delta x_i \tag{A-13}$$

2. Intermediate polarization and pro-middle class growth

Equation (13) and (14) defined the income shares at times 0 and 1

$$\omega_i = \left(\frac{y_i}{n\pi_y}\right) \tag{A-14}$$

$$\sigma_i = (x_i/n\pi_x) \tag{A-15}$$

Using (A-14) and (A-15) we derive that

$$\omega_i - \sigma_i = \frac{y_i}{n\pi_y} - \frac{x_i}{n\pi_x} = \left(\frac{y_i n\pi_x - x_i n\pi_y}{n^2\pi_y\pi_x}\right)$$

$$\leftrightarrow \omega_i - \sigma_i = \left[\frac{(x_i+\Delta x_i)n\pi_x - x_i n(\pi_x+\Delta\pi)}{n^2\pi_x(\pi_x+\Delta\pi)}\right] = \left[\frac{n\Delta x_i\pi_x - n x_i\Delta\pi}{n^2\pi_x(\pi_x+\Delta\pi)}\right]$$

$$\leftrightarrow \omega_i - \sigma_i = \frac{\Delta x_i}{x_i}\frac{x_i}{n\pi_x}\frac{\pi_x}{(\pi_x+\Delta\pi)} - \frac{x_i}{n\pi_x}\frac{\Delta\pi}{(\pi_x+\Delta\pi)}$$

$$\leftrightarrow \omega_i - \sigma_i = \sigma_i\,\eta_i\frac{1}{1+\overline{\eta}^{\lambda}} - \sigma_i\frac{\overline{\eta}^{\lambda}}{1+\overline{\eta}^{\lambda}} = \sigma_i\frac{\eta_i - \overline{\eta}^{\lambda}}{1+\overline{\eta}^{\lambda}} \tag{A-16}$$

Using (A-16) we then obtain

$$\Delta P_{FW}^{\lambda} = \left[\sum_{i=1}^{\frac{n}{2}}\left(\frac{(2i-1)}{n}\right)\left\{\sigma_i\frac{\eta_i-\overline{\eta}^{\lambda}}{1+\overline{\eta}^{\lambda}}\right\}\right] - \left[\sum_{i=\left(\frac{n}{2}\right)+1}^{n}\left(\frac{2(n-i+1)-1}{n}\right)\left\{\sigma_i\frac{\eta_i-\overline{\eta}^{\lambda}}{1+\overline{\eta}^{\lambda}}\right\}\right] \tag{A-17}$$

Note however that

$$\sum_{i=1}^{n/2}\left(\frac{2i-1}{n}\right)\sigma_i = \sum_{i=1}^{n/2}\left(\frac{2i-1}{n}\right)\left(\frac{x_i}{n(m_x)^{\lambda}}\right) = \sum_{i=1}^{n/2}\left(\frac{2i-1}{n^2}\right)\left(\frac{x_i}{(m_x)^{\lambda}}\right)$$

$$\leftrightarrow \sum_{i=1}^{n/2}\left(\frac{2i-1}{n}\right)\sigma_i = \left(\frac{1}{4}\right)\sum_{i=1}^{n/2}\left(\frac{2i-1}{n^2/4}\right)x_i\left(\frac{1}{(m_x)^{\lambda}}\right) = \left(\frac{1}{4}\right)\left(\frac{x^{ER}}{(m_x)^{\lambda}}\right) \tag{A-18}$$

Combining (A-17) and (A-18) we derive that

$$\Delta P_{FW}^{\lambda} = \left\{\left[\left(\frac{1}{4}\right)\left(\frac{x^{ER}}{(m_x)^{\lambda}}\right)\sum_{i=1}^{n/2}(\varphi_i)\left(\frac{\eta_i-\overline{\eta}^{\lambda}}{1+\overline{\eta}^{\lambda}}\right)\right] - \left[\left(\frac{1}{4}\right)\left(\frac{x^{FP}}{(m_x)^{\lambda}}\right)\sum_{i=\left(\frac{n}{2}\right)+1}^{n}(\Psi_i)\left(\frac{\eta_i-\overline{\eta}^{\lambda}}{1+\overline{\eta}^{\lambda}}\right)\right]\right\} \tag{A-19}$$

where

$$\varphi_i = \frac{\left[\left(\frac{2i-1}{n}\right)\sigma_i\right]}{\left[\sum_{i=1}^{\frac{n}{2}}\left(\frac{2i-1}{n}\right)\sigma_i\right]} = \frac{\left[(1/4)\left(\frac{2i-1}{n^2/4}\right)x_i\right]}{\left[\sum_{i=1}^{n/2}(1/4)\left(\frac{2i-1}{n^2/4}\right)x_i\right]} = \frac{\left(\frac{2i-1}{n^2/4}\right)x_i}{x^{ER}} \tag{A-20}$$

with $\sum_{i=1}^{n/2}\varphi_i = 1$

and

$$\Psi_i = \frac{\frac{[2(n-i+1)-1]}{n}\sigma_i}{\sum_{i=\left(\frac{n}{2}\right)+1}^{n}\frac{[2(n-i+1)-1]}{n}\sigma_i} = \frac{\left[(1/4)\left(\frac{[2(n-i+1)-1]}{n^2/4}\right)x_i\right]}{\sum_{i=\left(\frac{n}{2}\right)+1}^{n}\left[(1/4)\left(\frac{[2(n-i+1)-1]}{n^2/4}\right)x_i\right]} = \frac{\left(\frac{[2(n-i+1)-1]}{n^2/4}\right)x_i}{x^{FP}} \quad \text{(A-21)}$$

with $\sum_{i=\left(\frac{n}{2}\right)+1}^{n} \Psi_i = 1$.

Let us now define η^{ER} and η^{FP} as

$$\eta^{ER} = \sum_{i=1}^{n/2} \varphi_i \eta_i \quad \text{(A-22)}$$

$$\eta^{FP} = \sum_{i=\left(\frac{n}{2}\right)+1}^{n} \Psi_i \eta_i \quad \text{(A-23)}$$

Combining (A-19) to (A-23) we then end up with

$$\Delta P_{FW}^{\lambda} = \left(\frac{1}{4}\right)\left\{\left[\left(\frac{x^{ER}}{(m_x)^{\lambda}}\right)\frac{(\eta^{ER}-\overline{\eta}^{\lambda})}{1+\overline{\eta}^{\lambda}}\right] + \left[\left(\frac{x^{FP}}{(m_x)^{\lambda}}\right)\frac{(\overline{\eta}^{\lambda}-\eta^{FP})}{1+\overline{\overline{\eta}^{\lambda}}}\right]\right\} \quad \text{(A-24)}$$

Note that when λ=0, $(m_x)^{\lambda} = \pi_x = 1$ and $\left(m_y\right)^{\lambda} = \pi_y = 1$ so that $\Delta\pi = \pi_y - \pi_x = 0$ and therefore $\overline{\eta}^{\lambda} = \left(\frac{\Delta\pi}{\pi_x}\right) = 0$.

We can then, in such a case, rewrite (A-17) as

$$\Delta P_{FW}^{\lambda} = \left[\sum_{i=1}^{\frac{n}{2}}\left(\frac{(2i-1)}{n}\right)\{\sigma_i \eta_i\}\right] - \left[\sum_{i=\left(\frac{n}{2}\right)+1}^{n}\left(\frac{2(n-i+1)-1}{n}\right)\{\sigma_i \eta_i\}\right]$$

$$\leftrightarrow \Delta P_{FW}^{\lambda} = \left[\sum_{i=1}^{\frac{n}{2}}\left(\frac{(2i-1)}{n}\right)\left\{\left(\frac{x_i}{n\pi_x}\right)\left(\frac{\Delta x_i}{x_i}\right)\right\}\right] - \left[\sum_{i=\left(\frac{n}{2}\right)+1}^{n}\left(\frac{2(n-i+1)-1}{n}\right)\left\{\left(\frac{x_i}{n\pi_x}\right)\left(\frac{\Delta x_i}{x_i}\right)\right\}\right]$$

$$\leftrightarrow \Delta P_{FW}^{\lambda} = \left(\frac{1}{4}\right)\left(\Delta x^{ER} - \Delta x^{FP}\right) \quad \text{(A-25)}$$

As expected, equation (A-25) becomes equivalent to equation (6), the only difference being a factor of 2, as already mentioned in footnote 2.