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On the measurement of population weighted relative indices of mobility and convergence, with an illustration based on Chinese data*

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Abstract

This paper extends previous work on income-weighted measures of distributional change by defining in a unified framework population weighted and relative indices of structural and exchange mobility and measures of σ- and β-convergence. The analysis focuses on both the anonymous (comparison of cross- sections) and non-anonymous case (panel data) and unconditional as well conditional measures of pro-poor growth are defined. The empirical illustration, based on urban China data of non-retired individuals from the China Family Panel Studies, compares incomes in 2010 and 2014 and shows the usefulness of the tools introduced in the present study. It turns out that, during the period examined, there was β-convergence and slight σ-divergence, non-anonymous growth was pro-poor while anonymous growth was not. Income growth favored individuals with low levels of education as well as younger people in the non-anonymous case, but not in the anonymous case.

Keywords: β-convergence, σ-divergence, exchange mobility, structural mobility, pro-poor growth, China.

JEL Classification: D31, I32, O15.

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1. Introduction

In an important paper deriving axiomatically measures of distributional change, Cowell (1985) started by asking what is meant by distributional change. In the context of mobility, distributional change simply aims at estimating the amount of movement taking place when a vector *x* of individual incomes becomes a vector *y*, of the same population size. Such a change generally involves a re-ranking of the individuals. Even without re-ranking there may be some amount of distributional change if there was a change in the inequality of the distribution. Distributional change may however refer to another issue, that of horizontal inequity, when a given distribution (e.g. that of post-tax incomes) is compared to a reference distribution (e.g. that of pre-tax incomes). Here also there may be re-ranking, but even without re-ranking one may be interested in checking how distant the pre-tax are from the post-tax incomes. Cowell's (1985) study was a follow up to a previous paper of his (Cowell, 1980) where he proposed a measure of distributional change related to the concept of generalized entropy. Both papers took a relative approach to distributional change while Berrebi and Silber (1983), borrowing ideas from Kolm (1976), proposed an absolute measure of distributional change. Later on, Silber (1995) introduced a Gini-related index of distributional change. Jenkins and van Kerm (2006) extended this measure, using the concept of generalized Gini. They provided an analytical framework in which changes in income inequality over time are related on one hand to the re-ranking of individuals that took place during the period examined, on the other hand to the pattern of income growth across the income range. The Jenkins and van Kerm approach linked in fact the concept of distributional change to that of pro-poor growth. Starting around the year 2000 there has indeed been a growing literature on the concept of pro-poor growth (see, Deutsch and Silber, 2011, for a short review of this literature). A distinction is generally made between an approach where growth is labeled as pro-poor if the incomes of the poor grow, while another view assumes that growth is pro-poor if the incomes of the poor rise proportionally more than the average income (see, for example, Kakwani, 2004; Ravallion, 2004; Dollar and Kray, 2002). This literature makes also a distinction between an anonymous approach where two or more cross-sections of incomes are compared, and a non-anonymous approach based on panel data (see, for example, Fields et al., 2003; Grimm, 2007; Nissanov and Silber, 2009).

Nissanov and Silber (2009) stressed in fact the similarity between an analysis of pro-poor growth and that of convergence. While the literature on pro-poor growth focused generally on individual or household data, the emphasis in the literature on convergence was clearly on macroeconomic data, since this type of study had a solid theoretical foundation based on growth theory. Following previous work on the concept of convergence (see, for example, Baumol, 1986; Barro and Sala-i-Martin, 1992; Mankiw, Romer and Weil, 1992), Sala-i-Martin (1996) introduced the concepts of σ- and β-convergence[2]. For Sala-i-Martin (1990) "there is absolute β-convergence if poor economies tend to grow faster than rich ones". As far as σ-convergence is concerned, Sala-i-Martin (1990) states that "a group of economies are converging in the sense of σ if the dispersion of their real per capita GDP levels tends to decrease over time". O'Neill and van Kerm (2004) then showed the link that exists between the measurement of β-convergence and that of tax progressivity. They measured σ-convergence as the change over time in the Gini coefficient and decomposed this change in the Gini coefficient into two components, one corresponding to β-convergence, the other to the concept of leapfrogging (see, for example, Brezis et al., 1993, for this notion of leapfrogging). The component reflecting β-convergence measures in fact the extent to which poor economies grow faster than rich ones. As stressed by O'Neill and van Kerm this term is parallel to the notion of vertical equity in the tax literature (see, Reynolds and Smolensky, 1977; or Kakwani, 1977) while leapfrogging corresponds to re-ranking.

Such a unified framework of analysis, as that proposed by O'Neill and van Kerm, appears also in a recent paper by Dhongde and Silber (2016) who defined a set of income weighted measures of mobility, convergence and pro-poor growth, in the non-anonymous as well as in the anonymous case.

This paper contributes however in several ways to the literature. First, like Dhongde and Silber (2016), it proposes a unified framework to analyze mobility, convergence and pro-poor growth, but here the indices derived are population rather than income weighted measures. Second it sheds light on the role played by individual income changes in macro-economic growth and stresses the role of education and age. Third, and most importantly, it shows, using data on non-retired individuals in urban China, how crucial is the distinction between an anonymous and a non-anonymous approach to pro-poor growth.

[2] These concepts were actually introduced in the Ph.D. thesis of Sala-i-Martin (1990).

The paper is organized as follows. Section 2, 3 and 4 describe the methodology. In Section 2 we define the notion of population weighted approach to measuring distributional change and show how this concept allows one to measure mobility, convergence and pro-poor growth. While the measures derived in Section 2 were linked to concepts of Lorenz and relative concentration curves, we generalize this approach in Section3 by applying the notion of generalized Gini index. Whereas in Section 2 and 3 income was both the variable under scrutiny and the criterion for ranking individuals, in Section 4 we show that it is possible to define the income related distributional change by selecting another ranking criterion. Section 5 shows the usefulness of our approach by presenting an empirical illustration based on Chinese data covering urban areas and non-retired individuals. Finally, concluding comments are given in Section 6.

2. A population weighted approach to measuring distributional change

2.1. Notation

As in Dhongde and Silber (2016) let x_i and y_i refer to the absolute income of the i^{th} observation and $\bar{x}$ and $\bar{y}$ to the average incomes at times 0 and 1 in a population of *n* individuals, respectively.[3] Define the absolute changes in incomes, Δx_i and $\Delta\bar{x}$, as $\Delta x_i = (y_i - x_i)$ and $\Delta\bar{x} = \bar{y} - \bar{x}$. Let $s_i = (x_i/n\bar{x})$ and $w_i = (y_i/n\bar{y}) = (x_i + \Delta x_i)/n(\bar{x} + \Delta\bar{x}))$ refer to the income shares at times 0 and 1. Upon simplification, the difference $(w_i - s_i)$ may be expressed as

$$w_i - s_i = \left(\frac{1}{n\bar{x}}\right)\left(\frac{(\bar{x}\Delta x_i - x_i\Delta\bar{x})}{(\bar{x}+\Delta\bar{x})}\right) \tag{1}$$

Now define η_i and $\overline{\eta}$ as $\eta_i = (\Delta x_i)/x_i$ and $\overline{\eta} = (\Delta\bar{x}/\bar{x})$, where η_i denotes the growth in income of observation *i* and $\overline{\eta}$ denotes the growth in average income; then, as shown in Dhongde and Silber (2016), expression (1) implies that

$$w_i - s_i = s_i\frac{(\eta_i - \overline{\eta})}{1+\overline{\eta}} \tag{2}$$

[3] For the ease of exposition, we refer to *i* as an individual here. However, *i* may also represent a population centile, a region, or a country, depending on the application.

2.2. A population-weighted measure of the distributional change and equivalent growth rate

Let us now plot the cumulative population shares $\left(\frac{1}{n}\right), \left(\frac{2}{n}\right), \dots \left(\frac{i}{n}\right), \dots, \left(\frac{n-1}{n}\right), 1$ on the horizontal axis and the corresponding cumulative values of the shares s_i and w_i on the vertical axis in a one by one square, these shares being ranked according to a given criterion. We therefore obtain two curves, one, C_s, plotting (on the vertical axis) the cumulative income shares at time 0 versus (on the horizontal axis) the corresponding population shares, the other, C_w, doing the same for the cumulative income shares at time 1. These two so-called concentration curves are increasing, starting at point (0, 0) and end at point (1, 1). They may of course cross once or more the diagonal. Let us first compute the area A_s situated below the curve C_s. Assuming n observations, it is identical to the sum of a triangle and of $(n-1)$ trapezoids. It is easy to check that this area may be expressed as

$$A_s = \left(\frac{1}{2}\right)\left(\frac{1}{n}\right) s_1 + \left(\frac{1}{2}\right)\left(\frac{1}{n}\right)\left[\sum_{i=2}^{n} \left(s_i + 2\sum_{j=1}^{i-1} s_j\right)\right] \tag{3}$$

Similarly the area A_w situated below the curve C_w will be expressed as

$$A_w = \left(\frac{1}{2}\right)\left(\frac{1}{n}\right) w_1 + \left(\frac{1}{2}\right)\left(\frac{1}{n}\right)\left[\sum_{i=2}^{n} \left(w_i + 2\sum_{j=1}^{i-1} w_j\right)\right] \tag{4}$$

The difference between these two areas will then be written as

$$A_w - A_s = \left(\frac{1}{2}\right)\left(\frac{1}{n}\right)(w_1 - s_1) + \left(\frac{1}{2}\right)\left(\frac{1}{n}\right)\left[\sum_{i=2}^{n}\left[(w_i - s_i) + 2\sum_{j=1}^{i-1}\left(w_j - s_j\right)\right]\right] \tag{5}$$

Combining (2) and (5) we then obtain

$$A_w - A_s = \left(\frac{1}{2}\right)\left(\frac{1}{n}\right)\left(\frac{1}{1+\overline{\eta}}\right)\left\{\sum_{i=1}^{n}\left[s_i\left(\eta_i - \overline{\eta}\right) + \sum_{j=1}^{i-1}\left[2s_j\left(\eta_j - \overline{\eta}\right)\right]\right]\right\}$$

$$A_w - A_s = \left(\frac{1}{2}\right)\left(\frac{1}{n}\right)\left(\frac{1}{1+\overline{\eta}}\right)\sum_{i=1}^{n}\left(\eta_i - \overline{\eta}\right)s_i(2n+1-2i) \tag{6}$$

Let us call *CHANGE* the value of twice the difference $(A_w - A_s)$. We then derive that

$$CHANGE = \sum_{i=1}^{n} \tau_i \frac{(\eta_i - \overline{\eta})}{(1+\overline{\eta})} = \sum_{i=1}^{n} \tau_i \frac{(1+\eta_i)-(1+\overline{\eta})}{(1+\overline{\eta})} = \sum_{i=1}^{n} \tau_i \left[\frac{(1+\eta_i)}{(1+\overline{\eta})} - 1\right] \tag{7}$$

with

$$\tau_i = \left(\frac{2n+1-2i}{n}\right) s_i \tag{8}$$

But expression (7) may also be written as

$$CHANGE = \frac{\sum_{i=1}^{n} \tau_i(1+\eta_i)}{(1+\bar{\eta})} - \left[\sum_{i=1}^{n} \tau_i\right] = \frac{\sum_{i=1}^{n} \tau_i + \sum_{i=1}^{n} \tau_i \eta_i}{(1+\bar{\eta})} - \left[\sum_{i=1}^{n} \tau_i\right]$$

$$CHANGE = \left[\sum_{i=1}^{n} \tau_i\right] \left\{\frac{\left(1+\sum_{i=1}^{n} \varphi_i \eta_i\right)}{(1+\bar{\eta})} - 1\right\} = \left[\sum_{i=1}^{n} \tau_i\right] \left\{\frac{(1+\eta_E)}{(1+\bar{\eta})} - 1\right\} \tag{9}$$

with

$$\varphi_i = \frac{\tau_i}{\sum_{i=1}^{n} \tau_i} \tag{10}$$

and

$$\eta_E = \sum_{i=1}^{n} \varphi_i \eta_i \tag{11}$$

where η_E is the "equally distributed equivalent growth rate".

Since x_i is the income of individual i at time 0 and $s_i = (x_i/n\bar{x})$ we derive

$$\sum_{i=1}^{n} \tau_i = \sum_{i=1}^{n} \left(\frac{2n+1-2i}{n}\right)\left(\frac{x_i}{n\bar{x}}\right) = \sum_{i=1}^{n} \left(\frac{2n+1-2i}{n^2}\right)\left(\frac{x_i}{\bar{x}}\right) \tag{12}$$

Since $\sum_{i=1}^{n}(2n+1-2i) = n^2$, $\sum_{i=1}^{n} \left(\frac{2n+1-2i}{n^2}\right) x_i = x_E$ in (12) is a weighted average of the different incomes. Note that the weights are higher, the lower the rank of the individuals, given the ranking criterion selected. Moreover the difference between the weights of two individuals who are adjacent in the ranking is always equal to $(2/n^2)$. It is easy to check that when the individuals are ranked by increasing income, x_E turns out to be Atkinson's "equally distributed equivalent level of income" at time 0, assuming we use Gini's social welfare function (see, Donaldson and Weymark[4], 1980). $E_0 = (x_E/\bar{x})$ is then identical to Gini's measure of equality (the complement to 1 of Gini's famous inequality index) at time 0.

Combining (9) and (12) we then end up with

$$CHANGE = E_0 \left\{ \frac{(1+\eta_E)}{(1+\bar{\eta})} - 1 \right\} \tag{13}$$

2.3. The non-anonymous case: measuring population-weighted income mobility and $\beta-$convergence

Following earlier work on an approach to the computation of the Gini index based on the use of the so-called *G*-matrix (see, Silber, 1989), Silber (1995) defined a population weighted measure J_{GP} of distributional change as

$$J_{GP} = e'G(s-w) \tag{14}$$

where e' is a 1 by *n* vector of the population shares $\left(\frac{1}{n}\right)$, *s* and *w* are (n by 1) vectors of the income shares s_i and w_i that were defined previously, but both sets of shares are ranked by decreasing values of the incomes x_i at time 0. Finally, G is a *n* by *n* square matrix, whose typical element is equal to 0 when $i = j$, to -1 when $j > i$ and to +1 when $i > j$. Silber (1995) has also shown that this index J_{GP} is actually equal to twice the area lying between the curves C_s and C_w previously defined. Note that in drawing these curves C_s and C_w, both sets of shares, s_i and w_i, are ranked by increasing values of the shares s_i at time 0. Since the area between the diagonal and the curve C_s is equal to $(1/2)(e'Gs)$, while the area between the diagonal and the curve C_w is equal to $(1/2)(e'Gw)$, we derive that

$$CHANGE = 2(A_w - A_s) = 2\left\{\left[\frac{1}{2} - (1/2)(e'Gw)\right] - \left[\frac{1}{2} - (1/2)(e'Gs)\right]\right\} = (e'Gs) - (e'Gw) = J_{GP} \tag{15}$$

[4] Donaldson and Weymark (1980) ranked the incomes by decreasing rather than increasing values, hence the difference in the formulations.

Since the shares s_i are ranked by increasing values, the curve C_s is in fact the Lorenz curve at time 0. The shares w_i, however, are ranked by increasing values of the incomes at time 0 so that the curve C_w may lie below or above the curve C_s and it may even cross once or several times the diagonal. It should thus be clear that if *CHANGE* is positive the curve C_w will lie more above than below the curve C_s so that the area A_w will be greater than the area A_s. This observation implies that the poorer individuals did on average better than the richer ones so that we can conclude that there was what is called $\beta-$convergence. Obviously if *CHANGE* is negative there would be $\beta-$divergence.

<u>A simple illustration</u>:

Assume 3 individuals whose income at time 0 are respectively 1, 2 and 7 while their corresponding incomes at time 1 are 3, 7 and 5.

It is then easy to find out that the measure *CHANGE* is equal to 0.311. The "equivalent growth rate" η_E turns then out to be equal to 1.28 which is much higher than the average growth rate $\overline{\eta}$ (which is equal to 0.5) so that we certainly can conclude that there was income convergence during the period. We should remember that this "equivalent growth rate" η_E which was defined in (10) gives in the present case a higher weight to the individuals who at time 0 had a low income.

The properties of the index J_{GP} were derived and listed in Proposition 1 of Silber (1995). The main properties may be summarized as follows: the index J_{GP} is invariant to homothetic changes of the individual incomes. The effect on J_{GP} of an income swap is greater, the greater the difference between the swapped incomes and that between the ranks of the individuals who swap their incomes. Finally, if a sum Δ is transferred from individual j to individual f (assuming $s_j > s_f$ and no change in the ranking of the individuals), the value of the index J_{GP} will be an increasing function of Δ.

Note also that the gap between the curves C_w and C_s, at a point corresponding to the i first observations, may be expressed as $GAP_i = \sum_{j\le i}(w_j - s_j)$. If this gap is positive, it implies, using (2), that

$$GAP_i = \sum_{j\le i} s_j \left(\frac{\eta_j - \overline{\eta}}{1+\overline{\eta}}\right) > 0 \leftrightarrow \left[\frac{\sum_{j\le i} s_j \eta_j}{\sum_{j\le i} s_j} \frac{\sum_{j\le i} s_j}{1+\overline{\eta}}\right] - \left[\frac{\sum_{j\le i} s_j}{1} \frac{\overline{\eta}}{1+\overline{\eta}}\right] > 0 \qquad (16)$$

Note however that

$$\left(\frac{\sum_{j\le i} s_j \eta_j}{\sum_{j\le i} s_j}\right) = \left(\frac{\sum_{j\le i} x_j \eta_j}{\sum_{j\le i} x_j}\right) = \overline{\eta_J} \qquad (17)$$

where $\overline{\eta_J}$ is a weighted average of the growth rate of the i first observations.

Combining (16) and (17) we derive that

$$GAP_i = \left(\sum_{j\le i} s_j\right) \frac{\overline{\eta_J} - \overline{\eta}}{1+\overline{\eta}} \qquad (18)$$

We therefore conclude that the gap between the curves C_w and C_s, at a point corresponding to the i first observations, will be positive if the (weighted) average growth rate of the i first observations is higher than the average growth rate in the whole population, an intuitive result indeed.

Silber (1995) has also proven that the index J_{GP} could be expressed as the sum of a component F_{GP} measuring the change in inequality (structural mobility) and another one, P_{GP}, representing re-ranking (exchange mobility). Let us call *v* the vector of the income shares w_i at time 1 when these shares are ranked by their increasing values at time 1. In such a case we may express the measure CHANGE defined in (9) as

CHANGE = STRUCTURAL MOBILITY + EXCHANGE MOBILITY (19)

where

$$STRUCTURAL\ MOBILITY = [\sum_{i=1}^{n} \tau_i]\left\{\frac{(1+\eta_E^{s\to v})}{(1+\overline{\eta})} - 1\right\} \qquad (20)$$

In (20) τ_i is defined as previously since the starting vector is also the vector of the shares s_i. $\overline{\eta}$ is also defined as previously since the shares v_i and w_i are the same shares, just classified differently. The "equivalent growth rate" $\eta_E^{s\to v}$ is the one obtained when the change which takes place is the one observed when moving from vector *s* to vector *v*.

We may also define *EXCHANGE MOBILITY* as

$$EXCHANGE\ MOBILITY = [\sum_{i=1}^{n} \theta_i]\left\{\frac{(1+\eta_E^{v\to w})}{(1+\overline{\eta^{v\to w}})} - 1\right\} \qquad (21)$$

where $\eta_E^{v\to w}$ is the one obtained when the change which takes place is the one observed when moving from vector *v* to vector *w*. Note $\overline{\eta^{v\to w}} = 0$ since the shares v_i and w_i are the same shares, just ranked differently. Finally θ_i is defined, using (9), as

$$\theta_i = \left(\frac{2n+1-2i}{n}\right) v_i \qquad (22)$$

where, as mentioned previously, the shares v_i are ranked by increasing values.

We therefore end up with

$$EXCHANGE\ MOBILITY = \sum_{i=1}^{n} \left(\frac{2n+1-2i}{n}\right) v_i\, \eta_E^{v\to w} \qquad (23)$$

<u>An illustration</u>:

Assuming, as before, 3 individuals whose income at time 0 and 1 are respectively 1, 2 and 7 and then 3, 7 and 5, it is easy to check that

CHANGE = 0.311, $\eta_E^{s\to w} = 1.28$ and $\overline{\eta^{s\to w}}$ = 0.5

STRUCTURAL MOBILITY = 0.222, $\eta_E^{s\to v} = 1.06$ and $\overline{\eta^{s\to v}}$ = 0.5

EXCHANGE MOBILITY = 0.089, $\eta_E^{v\to w} = 0.108$ and $\overline{\eta^{v\to w}}$ = 0

It is easy to verify that *CHANGE* = *STRUCTURAL MOBILITY* + *EXCHANGE MOBILITY*.

Note also that $\eta_E^{s\to v}$ is positive because (relative) inequality decreased between times 0 and 1 while $\eta_E^{v\to w}$ is positive because the second poorest individual receives 7 at the end of the move described here rather than 5 as received originally (there is no change in the income of the poorest individual).

2.4. The anonymous case: measuring σ-convergence

Since here we ignore by assumption the identity of the individuals, we have to assume, going back to our numerical illustration, that the vector of the incomes at time 0 is {1, 2 7} while that of the incomes at time 1 is {3, 5, 7}.

The corresponding individual growth rates $(\text{the } \eta_i\text{'s})$ between the two periods are then respectively 2, 1.5 and 0 while the average growth rate is, as before, $\bar{\eta}$ = 0.5. It is then easy to find out that this time the measure *CHANGE*, is equal to 0.222 while the "equivalent growth rate" η_E is equal to 1.06. Since the average growth rate $\bar{\eta}$ is equal to 0.5 which is smaller than the value of η_E, we can conclude that there was structural mobility, as a consequence of the decrease in inequality between times 0 and 1. The indicator *CHANGE* measures therefore also the extent of what is called σ-convergence in the literature. Since we deal now with the anonymous case, there is evidently no exchange mobility

Note also that the absolute value of the indicator *CHANGE* and that of the Gini index are identical. A quick computation shows that the Gini index at time 0 is equal to 0.4 while at time 1 it is equal to 0.178 and the difference between these two Ginis is precisely 0.222: in other words inequality decreased by 0.222 and this decrease (-0,222) is identical in absolute value to the value of the indicator *CHANGE*.

2.5. Measuring pro-poor growth

2.5.1. Anonymous pro-poor growth

The literature on pro-poor growth (see, for example, Kakwani and Pernia, 2000) took originally an anonymous approach to the topic. Such an approach is evidently relevant when one wants to compare two cross sections.

Let us therefore assume that a poverty line *z* has been defined and that the proportion of poor in the population is *(q/n)*. Define now an equivalent growth rate among the centile groups that were poor at time 0 and call it $\eta_{E,pro-poor}^{Anonymous}$. Using (8), (10) and (11) we derive that

$$\eta_{E,pro-poor}^{Anonymous} = \sum_{i=1}^{q} \varphi_i \eta_i \tag{24}$$

with

$$\varphi_i = \frac{\tau_i}{\sum_{i=1}^{q} \tau_i} \tag{25}$$

and

$$\tau_i = \left(\frac{2q+1-2i}{q}\right) s_i' \tag{26}$$

with

$$s_i' = \frac{s_i}{\sum_{i=1}^{q} s_i} \tag{27}$$

where *i* refers to a given centile. If $\eta_{E,pro-poor}^{Anonymous} > \overline{\eta}$, growth has been pro-poor in the anonymous sense, since the originally "poor" centile groups experienced a growth rate that is higher than the average growth rate in the population.

2.5.2. Non anonymous pro-poor growth

We can define in a similar way a measure $\eta_{E,pro-poor}^{Non\ anonymous}$ of pro-poor growth for the non-anonymous case and write that

$$\eta_{E,pro-poor}^{Non\ anonymous} = \sum_{i=1}^{q} \varphi_i \eta_i \tag{28}$$

In (28) however, the subscript *i* does not refer, as in the anonymous case, to a given centile group, but to a given individual whose income is known at times 0 and 1. If $\eta_{E,pro-poor}^{Non\ anonymous} > \overline{\eta}$, non-anonymous growth has been pro-poor. As in the anonymous case, $\eta_{E,pro-poor}^{Non\ anonymous}$

takes into account the inequality in growth rates among the poor since different weights are attached to the various individuals.

3. Generalized population-weighted measures of distributional change, convergence and pro-poor growth

Jenkins and Van Kerm (2006) showed that changes in income inequality over time are related to the pattern of income growth across the income range and the reshuffling of individuals in the income pecking order. Such a breakdown is quite similar to that suggested by Silber (1995) who showed that a Gini-related measure of distributional change may be decomposed into a component measuring the change in inequality and another one reflecting the extent of re-ranking that took place over time. Jenkins and van Kerm (2006) used the so-called generalized Gini index to measure inequality and their approach was extended by Dhongde and Silber (2016) to the analysis of convergence and pro-poor growth but they derived income-weighted measures of distributional changes. In the present paper we propose, like Jenkins and van Kerm (2006), population-weighted measures of such changes but we show how the generalized Gini index may be also used to derive measures of convergence and pro-poor growth.

3.1. The anonymous case

This is the case where we have two cross-sections and we compare anonymous income distributions at time 0 (the set of incomes x_i) and 1 (the set of incomes y_i).

Using Atkinson's (1970) concept of "equally distributed equivalent level of income", Donaldson and Weymark (1980) have defined a generalized Gini index I_{GG} as

$$I_{GG} = 1 - \{\sum_{i=1}^{n}[((i/n)^{\gamma} - ((i-1)/n)^{\gamma})](x_i/\bar{x})\} \tag{29}$$

where x_i is the income of individual i (at time 0) with $x_1 \geq \cdots \geq x_i \geq \cdots \geq x_n$ where n is the number of individuals, γ is a parameter measuring the degree of distribution sensitivity (γ>1 and the higher γ, the stronger this sensitivity) while $\bar{x}$ is the average income in the population at time 0.

The population-weighted change in inequality between times 0 and 1 will hence be expressed as

$$\Delta I_{GG,P} = \{1 - \{\sum_{i=1}^{n}[((i/n)^{\gamma} - ((i-1)/n)^{\gamma})](y_i/\bar{y})\}\} - \{1 - \{\sum_{i=1}^{n}[((i/n)^{\gamma} - ((i-1)/n)^{\gamma})](x_i/\bar{x})\}\}$$

$$\leftrightarrow \Delta I_{GG,P} = \sum_{i=1}^{n}\left[\left(\frac{i}{n}\right)^{\gamma} - \left(\frac{i-1}{n}\right)^{\gamma}\right]\left((x_i/\bar{x}) - (y_i/\bar{y})\right) \qquad (30)$$

where $x_1 \geq \cdots \geq x_i \geq \cdots \geq x_n$ as well as $y_1 \geq \cdots \geq y_i \geq \cdots \geq y_n$.

It is easy to check that if γ=2, $\Delta I_{GG,P}$ in (30) will be expressed as

$$\Delta I_{GG,P}^{\gamma=2} = \sum_{i=1}^{n}\left[\left(\frac{2i-1}{n^2}\right)\right]\left[\left(\frac{x_i}{\bar{x}}\right) - \left(\frac{y_i}{\bar{y}}\right)\right] = \left(\frac{x_E}{\bar{x}}\right) - \left(\frac{y_E}{\bar{y}}\right) \qquad (31)$$

where x_E and y_E are respectively Atkinson's "equally distributed equivalent levels of income" at time 0 and 1, assuming we use Gini's social welfare function. Moreover from (31) we derive that

$$\Delta I_{GG,P}^{\gamma=2} = \left[1 - \left(\frac{x_E}{\bar{x}}\right)\right] - \left[1 - \left(\frac{y_E}{\bar{y}}\right)\right] = I_G^x - I_G^y \qquad (32)$$

which is the difference[5] between the Gini indices at times 0 and 1.

Note that if we rank the incomes at times 0 and 1 by increasing values, expression (30) will be written as

$$\Delta I_{GG,P} = \sum_{i=1}^{n}\left[\left(\frac{(n-i+1)}{n}\right)^{\gamma} - \left(\frac{n-i}{n}\right)^{\gamma}\right]\left((x_i/\bar{x}) - (y_i/\bar{y})\right) \qquad (33)$$

so that, when γ=2, $\Delta I_{GG,P}$ will be expressed as

$$\Delta I_{GG,P} == \sum_{i=1}^{n}\left(\frac{2n-2i+1}{n^2}\right)[(x_i/\bar{x}) - (y_i/\bar{y})] \qquad (34)$$

[5] Donaldson and Weymark (1980) had already shown that if γ =2, the index I_{GG} becomes equal to the traditional Gini index of inequality.

3.2. The non-anonymous case

3.2.1. Decomposing the distributional change index $J_{GG,P}$

Let now the sets of incomes $\{y_i\}$ and $\{x_i\}$ refer to the incomes at time 1 and 0, ranked by increasing values of the incomes at time 0. Let $\{t_i\}$ refer to the incomes at time 1, ranked by increasing incomes (at time 1).

Extending (33) we may then express the distributional change $J_{GG,P}$ between times 0 and 1 as

$$J_{GG,P} = \sum_{i=1}^{n} \left[\left(\frac{(n-i+1)}{n} \right)^{\gamma} - \left(\frac{n-i}{n} \right)^{\gamma} \right] \left((x_i/\bar{x}) - (y_i/\bar{y}) \right) \tag{35}$$

$$\leftrightarrow J_{GG,P} = \sum_{i=1}^{n} \left[\left(\frac{(n-i+1)}{n} \right)^{\gamma} - \left(\frac{n-i}{n} \right)^{\gamma} \right] \{ [(x_i/\bar{x}) - (t_i/\bar{t})] + [(t_i/\bar{t}) - (y_i/\bar{y})] \} \tag{36}$$

The first expression on the R.H.S. of (36) is evidently the difference between the value of the generalized Gini index at times 0 and 1, that is, the change in inequality between times 0 and 1. The second expression on the R.H.S. of (36) measures the amount of re-ranking that took place between times 0 and 1, since the sets of incomes $\{y_i\}$ and $\{t_i\}$ refer to the same incomes, just ranked differently.

3.2.2. Checking for convergence

From (35) we derive that

$$J_{GG,P} = n \sum_{i=1}^{n} \left[\left(\frac{(n-i+1)^{\gamma}}{n^{\gamma}} - \frac{(n-i)^{\gamma}}{n^{\gamma}} \right) \right] \left((x_i/n\bar{x}) - (y_i/n\bar{y}) \right) = n \sum_{i=1}^{n} \left[\left(\frac{(n-i+1)^{\gamma}}{n^{\gamma}} - \frac{(n-i)^{\gamma}}{n^{\gamma}} \right) \right] (s_i - w_i) \tag{37}$$

where, as previously, $s_i = \left(\frac{x_i}{n\bar{x}} \right)$ and $w_i = \left(\frac{y_i}{n\bar{y}} \right)$.

Combining (2) and (37) we obtain

$$J_{GG,P} = n \sum_{i=1}^{n} \left[\left(\frac{(n-i+1)^{\gamma}}{n^{\gamma}} - \frac{(n-i)^{\gamma}}{n^{\gamma}} \right) \right] \left(s_i \frac{(\eta_i - \bar{\eta})}{1+\bar{\eta}} \right) \tag{38}$$

$$\leftrightarrow J_{GG,P} = n \sum_{i=1}^{n} \psi_i \left[\frac{(1+\eta_i)-(1+\bar{\eta})}{1+\bar{\eta}} \right] = n \sum_{i=1}^{n} \psi_i \left[\frac{(1+\eta_i)}{(1+\bar{\eta})} - 1 \right] \tag{39}$$

where

$$\psi_i = \left(\frac{(n-i+1)^\gamma}{n^\gamma} - \frac{(n-i)^\gamma}{n^\gamma}\right) s_i \tag{40}$$

Expression (39) may be also written as

$$J_{GG,P} = n\sum_{i=1}^{n} \psi_i \frac{(1+\eta_i)}{(1+\bar{\eta})} - n\sum_{i=1}^{n} \psi_i = n\left(\sum_{i=1}^{n} \psi_i\right)\left[\sum_{i=1}^{n} \frac{\psi_i}{\sum_{i=1}^{n} \psi_i} \frac{(1+\eta_i)}{(1+\bar{\eta})} - 1\right]$$

$$\leftrightarrow J_{GG,P} = n\left(\sum_{i=1}^{n} \psi_i\right)\left(\frac{(1+\eta_{E,GG,P})}{(1+\bar{\eta})} - 1\right) \tag{41}$$

where $\xi_i = \frac{\psi_i}{\sum_{i=1}^{n} \psi_i}$ (42)

and $\eta_{E,GG,P} = \sum_{i=1}^{n} \xi_i \eta_i$ (43)

$\eta_{E,GG,P}$ is clearly the "equally distributed equivalent growth rate" derived from the generalization of the Gini index.

Finally note that, using (40), $\sum_{i=1}^{n} \psi_i$ may be also written as

$$\sum_{i=1}^{n} \psi_i = \sum_{i=1}^{n}\left(\frac{(n-i+1)^\gamma}{n^\gamma} - \frac{(n-i)^\gamma}{n^\gamma}\right) s_i = \sum_{i=1}^{n}\left(\frac{(n-i+1)^\gamma}{n^\gamma} - \frac{(n-i)^\gamma}{n^\gamma}\right)\frac{x_i}{n\bar{x}} = \frac{x_E}{n\bar{x}} \tag{44}$$

where $x_E = \sum_{i=1}^{n}\left(\frac{(n-i+1)^\gamma}{n^\gamma} - \frac{(n-i)^\gamma}{n^\gamma}\right) x_i$ is the "equally distributed equivalent level of income" corresponding to the generalized Gini-related welfare function.

Combining (41) and (43) we end up with

$$J_{GG,P} = \left(\frac{x_E}{\bar{x}}\right)\left(\frac{(1+\eta_{E,GG,P})}{(1+\bar{\eta})} - 1\right) \tag{45}$$

<u>A numerical illustration</u>

<u>First example</u>: convergence

Assume the incomes at time 0 are {1, 2, 7} and at time 1 {5, 6, 9}, so that the lower the income, the higher the growth rate. We hence expect to observe a convergence of the incomes.

It is easy to find out that, when γ=3, the weights ξ_i are respectively: $\xi_1 = (1.9/4)$; $\xi_2 = (1.4/4)$; $\xi_3 = (0.7/4)$

The individual growth rates are 4, 2, (2/7).

The equivalent growth rate $\eta_{E,GG}$ is then 2.65, while the average growth rate is 1, so that there was convergence of the incomes.

Second example:

Assume the incomes at time 0 are {1, 2, 7} and at time {1.1, 2.5, 16.4}. In this illustration we observe that the lower the income, the lower the growth rate, so that we expect that there will be divergence.

The weights ξ_i are, as before, are (1.9/4), (1.4/4), (0.7/4)

The individual growth rates are now 0.1, 0.25 and 1.34.

The equivalent growth rate $\eta_{E,GG}$ is then equal to 0.37 while the average growth rate is 1, so that there was now divergence of the incomes.

3.3. Checking for pro-poor growth

3.3.1. Anonymous pro-poor growth

As in the case of the Gini index, we will say that there is pro-poor growth if

$$\eta_{E,GG,pro-poor}^{Anonymous} = \sum_{i=1}^{q} \delta_i \eta_i > \overline{\eta} \quad (46)$$

with

$$\delta_i = \frac{[(q-i+1)^{\gamma}-(q-i)^{\gamma}]s'_i}{\sum_{j=1}^{q} s'_j[(q-j+1)^{\gamma}-(q-j)^{\gamma}]} \quad (47)$$

with s'_i defined in (27) while the sub-index *i* refers to a given centile.

3.3.2. Non-anonymous pro-poor growth

Here also we use expressions (45) and (46) but the sub-script *i* refers now to a specific individual whose income is known at times 0 and 1.

<u>A simple illustration for the anonymous case.</u>

Assume that the set x_i of incomes at time 0 is {1, 2, 7, 10, 30} and that the corresponding set of incomes t_i at time 1 is {1.1, 2.4, 6.5, 10, 46}. The average growth rate $\overline{\eta}$ will hence be equal to (66-50)/50=0.32

Assume the poverty line z is equal to 6.

The individual growth rates η_i among the poor will then be: $\eta_1 = 0.1$; $\eta_2 = 0.2$

Using (47) we easily derive that the numerator of δ_1 is equal (assuming γ = 3) to $(2^3 - 1^3)(1/3) = 7/3$, while the numerator of δ_2 is equal to $(1^3 - 0^3)(2/3) = 2/3$.

The denominator of δ_1 and δ_2 is hence equal to (9/3) so that $\delta_1 = \left(\frac{7}{9}\right)$ and $\delta_2 = \left(\frac{2}{9}\right)$

and we then conclude that

$\eta^{Anonymous}_{E,GG,pro-poor} = \sum_{i=1}^{q} \delta_i \eta_i$ =(7/9) (0.1) + (2/9) (0.2) = (1.1/9) =0.122$< \overline{\eta} = 0.32$.

Anonymous growth was hence not pro-poor.

<u>A simple illustration for the non-anonymous case</u>

Assume the same set of incomes at time 0, that is *X* ={1, 2, 7, 10, 30} while the corresponding set of incomes w_i at time 1 is {6.5, 46, 1.1, 2.4, 10}. The average growth rate is still

$\overline{\eta}$ = (66-50)/50 = 0.32 but the individual growth rates η_i of the poor (at time 0) are now

(6.5-1)/1 = 5.5; (46-2)/2 =22.

Using again (47) we easily derive that the numerator of δ_1 is equal (assuming γ = 3) to $(2^3 - 1^3)(1/3) = 7/3$, while the numerator of δ_2 is equal to $(1^3 - 0^3)(2/3) = 2/3$.

The denominator of δ_1 and δ_2 is hence equal to (9/3) so that $\delta_1 = \left(\frac{7}{9}\right)$ and $\delta_2 = \left(\frac{2}{9}\right)$

We then conclude that

$\eta_{E,GG,pro-poor}^{Non\ anonymous} = \sum_{i=1}^{q} \delta_i \eta_i$ =(7/9) (5.5) + (2/9) (22) = 9.17> $\overline{\eta} = 0.32$.

Non-anonymous growth was hence pro-poor.

4. Measuring distributional change with another ranking criterion

The analysis has hitherto been conducted by assuming, whether in the anonymous or in the non-anonymous case, that the variable under discussion is also the ranking criterion. Equations (11) and (28) are in fact general formulations that may be used for any ranking criterion. They however show that in computing the "equivalent growth rate" η_E the weight given to an individual depend now on the rank of this individual for the variable that serves as ranking criterion. If the variable under study is, for example, education, it might then be interesting to find out whether progress in education has been more favorable to those who were originally rich or those who were originally poor.

In such a case the observations x_i at time 0 and y_i at time 1 will refer to the educational levels of the individuals at time 0 and 1, but they will be ranked by increasing original incomes (individual incomes at time 0). The same ranking criterion will also be used for the vectors γ_i, φ_i and η_i.

A simple illustration (non-anonymous change):

Assume again 3 individuals whose levels of education (years of education) at time 0 are respectively 5, 9 and 12 while their corresponding levels of education at time 1 (e.g. 10 years later) are 8, 9 and 15.

The corresponding individual growth rates (the $\eta_i{}'s$) between the two periods are then respectively 0.6, 0 and 0.25 while the average growth rate is

$\overline{\eta}$ = (32-26)/26 = (6/26) = 0.231.

Assume now that the original (at time 0) incomes of these same three individuals were 400, 100, 300. It is the easy to find out that in (40) the shares s_i of the educational levels at time 0, ranked by increasing original income at time 0, are respectively equal to (9/26), (12/26), (5/26) while, when γ = 3, the numerators of the weights ξ_i in (42) are (19)(9/26), (7)(12/26), (1)(5/26). The weights ξ_i in (42) are then 0.658, 0.323 and 0.019.

Since the individual growth rates in education, ranked by increasing original income, are 0, 0.25 and 0.6, the "equally distributed equivalent growth rate" η_E is (0.658)(0)+(0.323)(0.25)+(0.019)(0.6)=0.092 which is less than the average growth rate 0.231. As expected, we conclude that the growth in educational levels was not "pro-poor income", that is it did not favor the lower incomes.

5. An empirical illustration: income mobility and convergence in China 2010-2014

Despite the exceptional income growth experienced by China, there is evidence of a rapid increase in income inequality relative to China's own past as well as to other countries with similar levels of economic development (Xie et al. 2015). Using national accounts, surveys and tax data, Piketty et al. (2019) also corrected the officially under-reported inequality index in China for the period 1978-2015, and found that inequality levels in 2015 were approaching US levels.

We frequently have cross-sectional information on the level of income and the distribution of income in China, but we know little about how growth was distributed in the population. In this section we illustrate how the proposed indices can be used to check whether income growth in China in the period 2010-2014 favored certain categories of individuals. With the change in the economic structure and the new policy tools introduced in recent years, an analysis of the Chinese income growth and its distribution should give us the latest information about the change in income and the income mobility of individuals and the impacts on the income distribution of these economic and policy changes. We therefore try to answer the following questions: Did growth promote convergence? Which type of convergence? Who has benefited more from growth: poor or rich people, more educated or less educated people, older or younger people?

This section uses data of the China Family Panel Studies (CFPS), funded by the 985 Program of Beijing University and carried out by the Institute of Social Science Survey of Beijing University. The CFPS is the first nationally representative survey designed to characterize China's ongoing social transformation by collecting data at the community, family, and individual levels (Xie and Hu, 2014). The CFPS was first launched in 2010 and has since then been conducted every two years. It uses a multi-stage, implicit stratification and a proportion-to-population size sampling method with a rural–urban integrated sampling frame. The sample of CFPS is drawn from 25 provinces/cities/autonomous regions in China excluding Hong Kong, Macao, Taiwan, Xinjiang, Xizang, Qinghai, Inner Mongolia, Ningxia, and Hainan. The population of these 25 provinces/cities/autonomous regions in China (excluding Hong Kong, Macao, and Taiwan) includes 95% of the Chinese total population. Thus, CFPS can be regarded as a nationally representative sample[6].

[6] CPFS data were also used by Kanbur et al. (2017) and Piketty (2019) as nationally representative data, to investigate the evolution of inequality in China.

The CFPS contains a family questionnaire that asked a series of questions pertaining to family income, including labor and non-labor income, expenditures in different categories, and income-generating activities of all family members. The longitudinal design of the CFPS enables the study of trends in income inequality and individual income growth in contemporary China at the micro level.

Regarding the use of sample weights, the CFPS data contain regional subsamples and thus require weighting to be nationally representative (Xie and Hu 2014; Xie and Lu 2015). This is further complicated by sample attrition over time. Following Lu and Xie (2015), we use the restricted sample that includes families that were successfully interviewed in both 2010 and 2014, with panel weights. As stated by Xie et al. (2015), in using weights based on regional and demographic characteristics, we implicitly make use of an unverifiable assumption, often called the 'missing-at-random' assumption (Little and Rubin 2002), that the observations that were lost in the follow-up survey can be approximated by observations with similarly observed regional and demographic characteristics. We restrict our analysis to urban China[7] and males under 60 and females under 55 and drop retired people from the survey and clean the age effect.

The income variable we use in this study is per capita family net income – the total net income from all sources divided by the number of family members. Income is measured in five major categories: (i) wage income (after-tax wages and salaries of individual family members employed in the agricultural or non-agricultural sector, including employer-provided bonuses and in-kind benefits; (ii) income from agricultural production and profits from family-run/owned businesses,

[7] We do not use rural samples in this paper for two reasons. First, the labor market in China is not an integrated one because of the Household Registration System (Hukou), and income sources in rural and urban citizens are quite different. Second, most of the economic growth in China comes from the urban rather than the rural sector. For example, the share of the primary industry GDP in total GDP in 2018 was only 7.2 per cent (data source: website of NBS of China, http://www.stats.gov.cn). So, focusing on the urban sector can help understanding a large part of the economic growth and structural changes in China

(iii) property income (rents of land, housing units and other assets); (iv) transfer income (sum of pensions, various kinds of government aids and allowances, and (v) monetary compensation for government appropriation of land and residential relocation), and other income (private transfers and gifts). [8] Income components are not strictly comparable across the 2010 and 2014 waves of the CFPS. Therefore, as in the analysis we compare 2010 and 2014 using longitudinal data we restrict ourselves to a 'comparable income' measure, the sum of income components that were comparable between the two waves. We adjust the value of income to eliminate the inflation[9]. Then we attach the inflation-adjusted net family income (comparable with year 2010) to each member of the household and then follow the individual across years 2010 and 2014.

The annual samples have been symmetrically truncated with the elimination of 1 percent of the observations at each end of the income distribution. This type of truncation is frequent in intertemporal comparisons due to the possible contamination of the data by anomalies in the extreme values (Cowell et al., 1999). A balanced panel of 4,097 records was generated across these years of data collection containing data on income for both years.

A population-weighted measure of the distributional change and equivalent growth rate.

We start with the non-anonymous measurement of changes in income. Table 1 shows the measure of *CHANGE* (expression 15) that is equal to 0.1856 for the non-anonymous case. The equivalent growth rate, η_E, turns out to be equal to 0.5961 which is much higher than 0.2096, the average growth rate, $\overline{\eta}$, so that we can conclude that there was income convergence during the period, that is, those with lower incomes were, in general, the ones with greater growth rates, fostering

[8] For details of the income component adjustment of CFPS, see Xie, Zhang, Xu and Zhang (2015).
[9] We use the Urban China Consumer Price index, 2010 base year, provided by the National Bureau of Statistics of China

$\beta -$*convergence*. Assuming that the parameter γ is equal to 2, we decompose this change in income into two components: *structural and exchange mobility*. *Structural mobility* is equal to -0.0046. Most of the change comes from *exchange mobility*, that is re-ranking in the distribution, which is equal to 0.1902.

Table 1. Estimates of proposed indices for China 2010-2014.

		η_E	$\overline{\eta}$	CHANGE	Structural	Exchange
$\gamma = 2$	Non –anonymous	0.5961	0.2096	0.1856	-0.0046	0.1902
	Anonymous	0.2000		-0.0046		
Sensitivity						
$\gamma = 3$	Non -anonymous	0.8539				
	Anonymous	0.1434				
$\gamma = 4$	Non -anonymous	1.0600				
	Anonymous	0.0813				
$\gamma = 5$	Non -anonymous	1.2356				
	Anonymous	0.0229				
$\gamma = 10$	Non -anonymous	1.8780				
	Anonymous	-0.1944				

Source: CFPS 2010 and 2014

Now, we analyze the anonymous case that allows us to conclude that there was a slight σ-divergence. It then turns out that the equivalent growth rate, when individuals are not identified, is equal to 0.2000 and is smaller than the average growth rate, but the difference is not significant. We can therefore conclude that income growth in urban China in the period 2010-2014 was slightly smaller for lower incomes deciles, resulting in trifling σ-divergence, and at the same time income growth was greater for initially lower income individuals, when they are identified.

Results for the case where the sensitivity parameter takes different values are shown in the lower rows ($\gamma = 3, 4, 5$ and 10) of Table 1. For higher values of γ the equivalent growth rates are higher in the non-anonymous case and smaller in the anonymous case, becoming even negative when $\gamma = 10$. This means that the higher the weight attached to lower income individuals, the greater the non-anonymous convergence and the anonymous divergence. To sum up, during the period examined, on the one hand income growth was higher for those at the lower end of the distribution, and the lower the position, the higher the growth. On the other hand lower income quantiles experienced lower growth rates in general, and when we attach a greater weight to the lowest part of the distribution this fact is even more pronounced.

Checking for pro-poor growth.

Now we focus on poor individuals. We work with the international poverty line of 1.9 US $ (in 2011 PPP $) a day from the World Bank. In Table 2 we estimate equivalent growth rates for the anonymous and non-anonymous cases among the individuals considered as poor.

Table 2. Equivalent growth rates for poor individuals.

		η_E	$\overline{\eta}$
$\gamma = 2$	Non –anonymous	3.2328	0.2096
	Anonymous	-0.5981	
Sensitivity			
$\gamma = 3$	Non -anonymous	3.5202	
	Anonymous	-0.6500	
$\gamma = 4$	Non -anonymous	3.6355	
	Anonymous	-0.6830	
$\gamma = 5$	Non -anonymous	3.6420	
	Anonymous	-0.7054	
$\gamma = 10$	Non -anonymous	3.2617	
	Anonymous	-0.7515	

Source: CFPS 2010 and 2014

We obtain interesting results that are in agreement with the previous observations we made. The non-anonymous equally distributed equivalent growth rate is 3.2328 and is hence much greater than the average growth rate, $\overline{\eta} = 0.2026$. Non-anonymous growth was hence clearly pro-poor. The anonymous equally distributed equivalent growth rate is equal to -0.5981 so that anonymous income growth was clearly non-pro-poor. The different conclusions derived in the anonymous and non-anonymous approaches are due to the fact that there is a reshuffling of the positions of the individuals, which is not taken into account in the anonymous approach. We can even state that poorer quantiles experienced a decline in income. We assess this result under different sensitivity parameter in the lower rows of Table 2 ($\gamma = 3, 4, 5$ and 10). For higher values of γ, the non-anonymous equivalent growth rates are higher than the average growth rate while the anonymous equivalent growth rates are more negative. We can therefore expect to reach similar conclusions (pro-poor non-anonymous growth and non-pro-poor anonymous growth) when adopting lower poverty lines.

Conditional convergence

In this part of the empirical analysis we check whether income growth was more favorable to those individuals with a higher educational level or to older people. We therefore apply the conditional measures of distributional change previously defined. We work with two conditioning variables: the education and age of the individuals.

For education the following categories are distinguished: 1 Illiterate/Semi-literate; 2 Primary school; 3 Junior high school; 4 Senior high school; 5 2- or 3-year college; 6 4-year college/Bachelor's degree; 7 Master's degree; 8 Doctoral degree. We rank individuals according to their level of education and check whether those with a higher educational level experienced a more favorable income growth. Similarly we rank individuals by their age and check whether income growth was higher for younger or older people.

Table 3 gives the equally distributed equivalent growth rate for the anonymous and non-anonymous case regarding the two conditional variables.

Table 3. Equivalent growth rates, conditioning on education and age.

	η_E	η_E
	Cond. to education	Cond. to age
Non -anonymous	0.2240	0.2165
Anonymous	0.2109	0.2101

Source: CFPS 2010 and 2014

We conclude that given that the non-anonymous equally distributed equivalent growth rates for individuals ranked according to their educational level, 0.2240, is greater than the average growth rate, 0.2096, income growth favored individuals with lower levels of education, while no significant conclusion can be drawn in the anonymous case.

Regarding age, we conclude that income growth was more pro-young people in the non-anonymous case (the equally distributed equivalent growth rate was 0.2165) but no conclusion can be drawn in the anonymous case (the equally distributed equivalent growth rate was 0.1997).

We believe that these results confirm that China has reached the Lewis Turning Point (LTP)[10]. Thus at the beginning of the 21st century (around 2004-2005), some enterprises in the coastal cities began to find it was difficult to hire enough workers, hence the use of the expression “migrant workers shortage” in the media. At the same time, the wages of migrant workers in the urban labor market began to rise sharply. Zhang et al. (2011) and Li et al. (2012) argued that China’s economy reached the LTP, or began to face labor shortages around 2005 in China. However, they do not provide convincing evidence. Applying Minami’s (1968) method of identifying the LTP in Japan to a nationally representative rural household sample from NBS of China, Zhang et al. (2018) provided evidence suggesting that the Chinese economy reached the LTP around 2010, rather than in 2005. As a consequence we expect that incomes of those workers with low-skilled or who are young will increase faster than those of the older workers or of the workers having a high education. It is thus not surprising that the findings of our empirical analysis showed that in urban China, during the period 2010-2014, income growth favored individuals with low levels of education and young people.

[10] In economic development the Lewis turning point occurs when there is no more surplus rural labor.

6. Conclusions

Following previous work on the parallelism between the study of income convergence in the growth literature and that of vertical and horizontal equity in the tax literature, as well as on the similarity between the notions of convergence and pro-poor growth, the present paper extended recent work by Dhongde and Silber (2016) on income-weighted measures of distributional change. It introduced population weighted measures of structural and exchange mobility, of conditional and unconditional σ- and β-convergence and of anonymous and non-anonymous pro-poor growth. These measures should help revealing the micro structural changes that took place behind the macro economic growth. For example, the empirical illustration based on urban non-retired Chinese panel data concluded that income growth in urban China, during the period 2010-2014, was slightly smaller for lower incomes deciles, leading to some trifling σ-divergence. At the same time there was β-convergence, that is, income growth was higher for individuals with an initially lower income. Non-anonymous growth was clearly pro-poor, while anonymous income growth was clearly not pro-poor, a result robust to a wide range of values of the sensitivity parameter. Income growth favored individuals with lower levels of education, and younger people individuals in the non-anonymous case, while this claim does not hold for the anonymous case. All these results are actually in line with recent studies that indicate that China seems to have reached the Lewis Turning Point around 2010.

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