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Abstract

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Keyword: Opportunity, Compensation, Domain restriction

JEL Cassification: D63

SEPARABLE UTILITY AND TASTE-INDEPENDENCE: AN ESCAPE ROUTE FROM THE OPPORTUNITY PARADOX

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ABSTRACT. We provide an escape route from the opportunity paradox, which is described as a conflict between the *ex ante* and the *ex post* perspectives of compensation, by restricting the preference domain. Taste-independent utility is introduced as a property of preferences such that individuals' maximized utility levels are the same regardless of their tastes for work. Using the optimal income taxation model, we demonstrate that if parametric utility functions are separable in consumption and labor supply, then they are taste-independent. We obtain a compatibility theorem when utility functions are quasilinear in consumption.

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1. INTRODUCTION

We demonstrate that there is an escape route from one of the eight paradoxes in welfare economics listed in Fleurbaey (2019): the opportunity paradox. This paradox refers to the tension that arises when considering redistribution policies to achieve opportunity equality that accommodates individual responsibility, as explored in a flourishing stream of theoretical and empirical literature (cf. Ferreira and Peragine, 2016). "The general structure of such theories relies on a distinction between responsibility characteristics and circumstance characteristics. Inequalities due to the former are deemed acceptable, unlike inequalities due to the latter" (Fleurbaey, 2019, p. 674).

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Incompatibilities between such principles have been shown since Fleurbaey (1994, 1995); however, according to Fleurbaey and Peragine (2013), "the well documented conflicts between the compensation principles and various reward principles are but an aspect of a broader conflict between *ex ante* and *ex post* perspectives" (p. 126). Thus, it is worthwhile to unravel the logical incompatibility regarding equality of opportunity at this "deeper" level (p. 119). Focusing on utility that is quasilinear in consumption, we show that there is no incompatibility when the preference domain is restricted to separable utility with respect to consumption and labor supply.

Here, we briefly illustrate the opportunity paradox following Fleurbaey (2019, sec. 6), which can be seen in the (ℓ, c) space in Figure 1, where ℓ is labor supply and c is consumption. As is conventional in the fair taxation literature (e.g., Fleurbaey and Maniquet, 2006), we treat labor supply and wage rate as responsibility and circumstance characteristics, respectively. Also, as is often the case with the *ex post* perspective of compensation, we temporarily require the reduction of consumption (or disposable income) inequality, not utility inequality.[1]

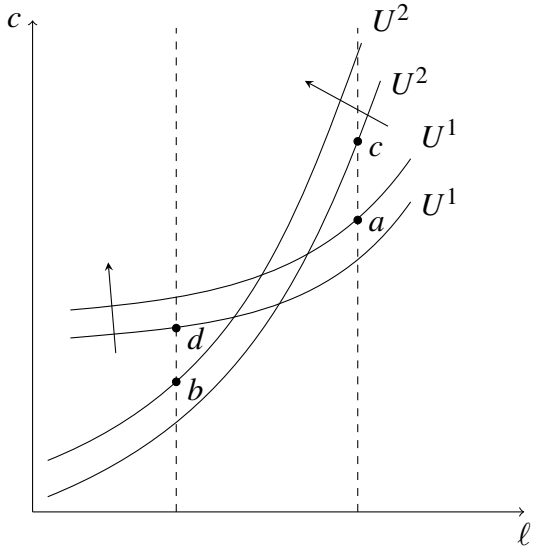

FIGURE 1. Crossing indifference curves of four individuals.

[1]Fleurbaey (2008, Ch. 9, Sec. 9.5) and Fleurbaey (2019, Sec. 6) draw a similar, but not identical, figure showing opportunity sets, setting the vertical axis as the outcome to show opportunity paradoxes. The example given in Fleurbaey (2019) seems to imply that the outcome may be income.

Figure 1 shows the allocations of four individuals. Suppose points a, b, c, and d represent labor supply and consumption at their maximized utility. The two types of preferences are exhibited by the indifference curves U^1 and U^2, and the arrows on them indicate the direction of preferences. Individuals at points a and c share the same responsibility characteristic, labor supply, as do points b and d. As the *ex post* perspective of compensation requires a reduction of consumption inequalities between individuals with the same responsibility characteristic, labor supply, regardless of their preferences, it is desirable to reduce the difference between points a and c as well as between points b and d. However, these changes widen the gaps between the indifference curves of U^1 and of U^2. This is undesirable according to the *ex ante* perspective of compensation, which requires a reduction in the gaps in "the opportunities offered to individuals (as measured by the possible well-being levels achieved with given circumstances for the various values of responsibility characteristics)" (Fleurbaey, 2008, p. 239). Therefore, the *ex post* and the *ex ante* perspectives of compensation are incompatible in general.

To address this issue, we first consider compensation that is aimed at reducing inequalities between utilities, not incomes.[2] We suppose utilities to be interpersonally comparable for evaluating social orderings because "(t)he eschewal of interpersonal comparisons of utilities eliminates the possibility of taking note of inequality of utilities" (Sen, 2017, p. 17).[3] Namely, we focus on interpersonally comparable utility as the informational basis. Nevertheless, the paradox remains. Considering utility equalization, the *ex ante* perspective of compensation also requires a reduction in the gaps between the indifference curves of U^1 and those of U^2 because the *ex ante* perspective of compensation requires reducing the opportunity gaps between individuals.[4]

[2]As Fleurbaey and Maniquet (2018) mention, there is a "possibility, more respectful of individual preferences, ... to take utility as the outcome (assuming there is a comparable measure of utility)" (p. 1045).

[3]For a detailed justification of the construction of individual well-being measures that respect individual preferences and depend on the bundles of goods consumed by the individuals, especially with nonclassical goods such as labor supply, see Fleurbaey and Maniquet (2019).

[4]The graphical movement is the same as the previous argument; that is, for the *ex ante* approach, we require that individuals sharing the same preferences have equal opportunity to enjoy the same utility, but they can choose their labor/leisure time and income. It may seem odd that the outcome, or utility, is constant even if the responsibility characteristic, or labor supply, changes. Although they share the same utility, they may have different incomes, labor supply, and leisure time, and these are opportunities that

Furthermore, although we cannot compare utilities between points a and c, or points b and d, there is a possibility that the *ex post* perspective of compensation may, in general, require reduction between them, just as when we consider income equality. That is, the paradox can still occur in the unrestricted domain;[5] therefore, secondly, we propose a property of preference that we call *taste-independence.* It assumes that the maximized utility levels are the same for individuals with the same wage rate but different tastes for work. It also reflects respect for individual responsibility; hence, the taste for work is also considered a responsibility characteristic. We argue that labor supply and taste for work are *ex post* and *ex ante* responsibility characteristics, respectively. When we consider utility as individual well-being or outcome, this distinction of responsibility characteristics enables us to clearly define the *ex ante* perspective of compensation and to plainly discuss the opportunity paradox using the concept of opportunity sets in a responsibility–outcome space.

We provide a compatibility theorem by introducing a preference domain restriction. We show that one of the preference domain restrictions commonly used in practice, namely separable utility (viz., separable in consumption and labor supply) exhibits the taste-independent property.[6] Focusing on utility that is quasilinear in consumption, we then demonstrate that the opportunity paradox does not occur under the separable—and taste-independent—restrictions on the preference domain.[7]

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individuals can choose as a result of their maximization behavior. Further, by considering taste for work, which is considered to be *ex ante* responsibility characteristic, as argued in the following discussion, is not described by the horizontal axis; thus, the indifference curves only represent correspondences between *ex post* responsibility characteristic and income along the same outcome, or utility. See also footnotes 18 and Section 4.3 for further discussion regarding opportunity sets.

[5]Indeed, Fleurbaey and Peragine (2013) show the general incompatibility.

[6]Note that our separability assumption is different from the outcome separability in terms of responsibility and circumstance characteristics (e.g., Bossert, 1995; Bossert and Fleurbaey, 1996). In our model, responsibility characteristic is labor supply (and taste for work), and circumstance characteristic is wage rate. We do not assume utility to be separable in labor supply and wage rate because consumption is a combination of the two.

[7]According to Fleurbaey and Maniquet (2018), "(t)here are two main views on utilities. According to the first view, ... utilities are empirical objects that only need to be measured and can be used as the inputs of a social welfare function, ... According to the second view, utilities themselves, not just the social welfare function, are normative indexes that need to be constructed" (pp. 1034–1035). Regarding the first view, "(o)ne can distinguish two main approaches that adopt this view. In the first approach, utilities refer to the subjective self-assessments of well-being" (p. 1035). We adopt this approach and consider restricting such utility to be separable in consumption and labor supply.

The remainder of the paper is organized as follows. We present the formal settings of the model in section 2. We define the preference domain restrictions, taste-independence, separability, and quasilinearity in section 3. We present the axioms and the main result in section 4. We provide concluding remarks in section 5.

2. THE MODEL

An *economy* E is composed of a finite set of individuals partitioned into a finite number of *taste types* and *wage rate types*. The set of taste type is $N(E) = \{1, ..., n\}$, with $n \geq 2$, and the set of wage rate type is $M(E) = \{1, ..., m\}$, with $m \geq 2$.

Individuals have conceivably different *tastes* for work, denoted by attribute parameter θ^i. The taste type is indicated by a superscript. Let c be *consumption* and ℓ be *labor supply* ($0 \leq \ell \leq 1$). The *preference* of an individual with taste θ^i is represented by an identical real-valued parametric *utility function* u;[8] that is,

$$U^i \equiv u(c, \ell; \theta^i), \text{ for } i \in N(E). \tag{1}$$

The utility function is increasing in c and decreasing in ℓ, and it is smooth and quasi-concave. We restrict it to be separable and taste-independent, as defined in the next section.

Let w_j be *wage rate*, and let T be *transfers* if positive and *taxes* if negative. The wage rate type is indicated by a subscript. The *disposable income* is determined by

$$w_j \ell + T, \text{ for } j \in M(E). \tag{2}$$

Individuals with taste θ^i and wage rate w_j maximize, or optimize, their utility subject to their disposable income:

$$\max_{0 \leq \ell \leq 1} u(c, \ell; \theta^i) \tag{3}$$

$$\text{s.t. } c = w_j \ell + T. \tag{4}$$

[8]See, for example, Deaton and Muellbauer (1980, Ch. 9), Fleurbaey and Hammond (2004, Sec. 6.2). Note that preference heterogeneity can be considered by using the taste parameter θ^i.

The utility of individuals with taste θ^i and wage rate w_j is denoted by

$$U_j^i \equiv u(w_j \ell_j^i + \tau(w_j, \ell_j^i, \theta^i), \ell_j^i; \theta^i), \text{ for } i \in N(E),\ j \in M(E), \tag{5}$$

where ℓ_j^i is the corresponding labor supply. The transfers/taxes $\tau(w_j, \ell_j^i, \theta^i) = T$ are determined by the government as a *redistribution policy function* τ of w_j, ℓ_j^i, and θ^i.

Let $\bar{\ell}_j^i$ be the labor supply optimally chosen by individuals with taste θ^i and wage rate w_j, and let $\tau(w_j, \bar{\ell}_j^i, \theta^i)$ be the corresponding transfers/taxes.[9] Moreover, let $\bar{\ell}_j$ indicate the labor supply optimally chosen by an individual with wage rate w_j but with an arbitrary taste parameter, and let $\tau(w_j, \bar{\ell}_j)$ indicate transfers, determined by the government as a function of w_j and $\bar{\ell}_j$. We focus on such allocations obtained by individuals' utility maximization behavior when we consider the *ex post* perspective of compensation.

A *social ordering function* defines, for every economy E in domain D, an ordering $\succ_{(E)}$ over all possible maximized individual utilities, where $\tau \succ_{(E)} \tau'$ means that redistribution policy $\tau(\cdot)$ is socially better than policy $\tau'(\cdot)$. The domain D over which these social ordering functions $\succ_{(E)}$ are defined is the set of economies that satisfies the abovementioned conditions. Finally, we evaluate the orderings and do not restrict our attention to the tax and transfer policies that satisfy the government's budget balance.

3. PREFERENCE DOMAIN RESTRICTION

3.1. **Taste-independence.** We introduce a property of preference such that the maximized utility levels of individuals with the same wage rate w_j but different tastes for work θ^i are evaluated as the same.[10] Taste-independence is regarded as respect for the individuals' freedom to choose the labor supply–consumption bundle (ℓ, c) at a given wage rate. Moreover, it respects individual responsibility; that is, individuals

[9]Individual optimal labor supply $\bar{\ell}_j^i$ and the corresponding government transfers/taxes $\tau(w_j, \bar{\ell}_j^i, \theta^i)$ are determined by simultaneously solving the optimization problems of the individuals and the government's redistribution policy function.

[10]The following statements provide some justification for the property on the set of primitive preferences that are in the economy (cf. footnote 7) as well as for separable utility, which actually implies taste-independence according to Theorem 1.

with low income due to high disutility of work would enjoy the same level of utility as individuals with high labor supply and high income because they have more leisure time.[11] In short, we can treat individuals with the same circumstance characteristic, or wage rate.[12] Thus, taste-independent utility is defined as follows.

Definition 1 (Taste-independence)**.** A utility is *taste-independent* if two individuals' utilities are the same whenever they maximize over the same budget set $w_j\ell + T$, where wage rate w_j and transfers/taxes T are given, regardless of their taste parameter θ^i.

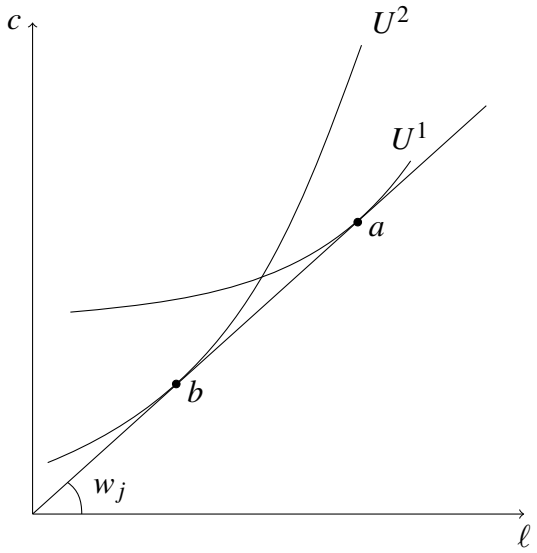

FIGURE 2. Taste-independence between two individuals with the same wage rate but different tastes for work: individuals represented by points a and b enjoy the same utility level.

In the (ℓ, c) space in Figure 2, the indifference curves of two preferences are depicted. Both are tangent to the same line with slope w_j, which means that individuals on points a and b share the same wage rate. Taste-independence then assumes the utilities obtained at points a and b are the same.

[11] We should often consider the high marginal disutility of work, such as that of individuals with (mental) illnesses or disabilities that do not depreciate their wage rates. In such cases, we need utility functions other than separable ones to derive desirable policies for those individuals.

[12] In fact, this is one of the standard requirements of responsibility, or reward, principles.

3.2. **Separability.** We introduce the *separable utility* as a preference domain restriction as follows:[13]

$$U_j^i = u\left[\mu(w_j\ell + T;\theta^i), \nu(\ell;\theta^i)\right], \tag{6}$$

where μ and ν are the corresponding *specific satisfaction functions* of consumption, which is equal to disposable income, and labor supply, respectively. Assuming this preference domain restriction, we have the taste-independent property as shown in Theorem 1.

Theorem 1. *The parametric separable utility functions are taste-independent.*[14]

Proof. We show that the maximum values are constant regardless of taste parameter θ^i, where wage rate w_j and transfers/taxes T are given.

Consider the implicit form of (6):

$$u\left[\mu(w_j\ell + T;\theta^i), \nu(\ell;\theta^i)\right] - U_j^i = 0. \tag{7}$$

Differentiating with respect to θ^i,

$$\frac{\partial u}{\partial \mu}\cdot\frac{\partial \mu}{\partial \ell}\cdot w_j\frac{\partial \ell}{\partial \theta^i} + \frac{\partial u}{\partial \nu}\cdot\frac{\partial \nu}{\partial \ell}\cdot\frac{\partial \ell}{\partial \theta^i} = 0 \tag{8}$$

$$\frac{\partial u}{\partial \mu}\cdot\frac{\partial \mu}{\partial \ell}\cdot w_j + \frac{\partial u}{\partial \nu}\cdot\frac{\partial \nu}{\partial \ell} = 0. \tag{9}$$

Solving (7) and (9) simultaneously, we have an envelope curve of the parametric separable utility function (6).

Meanwhile, the first-order condition of (6) is

$$\frac{\mathrm{d}U_j^i}{\mathrm{d}\ell} = \frac{\partial u}{\partial \mu}\cdot\frac{\partial \mu}{\partial \ell}\cdot w_j + \frac{\partial u}{\partial \nu}\cdot\frac{\partial \nu}{\partial \ell} = 0. \tag{10}$$

[13]The definition of separable utility follows Leontief (1947a,b), Gorman (1959, 1987), and Atkinson and Stiglitz (1976); that is, since it is a two-good model, $\partial^2 U_j^i/\partial\mu(\cdot)\partial\nu(\cdot) = 0$. Consumption is considered as a composite good, and disposable income is substituted to it to simplify the following analysis.

[14]The converse is not proven; that is, there may be other functional forms of taste-independent utility.

Therefore, the envelope curve is a set of maximum points of the utility. At the same time, the derivative, or the slope, of the envelope curve is always zero. By continuity,[15] the envelope curve of the parametric separable utility function (6), or the set of maximum points of (6), is a line parallel to the horizontal axis ℓ of a labor supply–utility space, (ℓ, U_j^i), which implies our required result.[16] □

Example 1. Parameter θ^i represents taste for work; thus, it is usually attached to variable ℓ. We can consider the following two forms of parametric separable utility functions. When taste parameter θ^i is a multiplier of labor supply ℓ,

$$U_j^i = u\left[\mu(w_j\theta^i\ell + T), \nu(\theta^i\ell)\right], \tag{11}$$

and when taste parameter θ^i is an exponent of labor supply ℓ,

$$U_j^i = u\left[\mu(w_j\ell^{\theta^i} + T), \nu(\ell^{\theta^i})\right]. \tag{12}$$

We can confirm that (the proof of) Theorem 1 holds under these specifications. Consider the implicit form of (11):

$$u\left[\mu(w_j\theta^i\ell + T), \nu(\theta^i\ell)\right] - U_j^i = 0. \tag{13}$$

Differentiating with respect to θ^i,

$$\frac{\partial u}{\partial \mu}\cdot\frac{\partial \mu}{\partial \ell}\cdot w_j\ell + \frac{\partial u}{\partial \nu}\cdot\frac{\partial \nu}{\partial \ell}\cdot\ell = 0 \tag{14}$$

$$\frac{\partial u}{\partial \mu}\cdot\frac{\partial \mu}{\partial \ell}\cdot w_j + \frac{\partial u}{\partial \nu}\cdot\frac{\partial \nu}{\partial \ell} = 0. \tag{15}$$

Solving (13) and (15) simultaneously, we have an envelope curve of the parametric separable utility functions (11).

[15] Continuity follows from the smoothness of the utility function.

[16] See Figure 3 and Figure 4 for examples.

The first-order condition of (11) is

$$\frac{\mathrm{d}U_j^i}{\mathrm{d}\ell} = \frac{\partial u}{\partial \mu}\cdot\frac{\partial \mu}{\partial \ell}\cdot w_j\theta^i + \frac{\partial u}{\partial \nu}\cdot\frac{\partial \nu}{\partial \ell}\cdot\theta^i = 0 \tag{16}$$

$$\frac{\partial u}{\partial \mu}\cdot\frac{\partial \mu}{\partial \ell}\cdot w_j + \frac{\partial u}{\partial \nu}\cdot\frac{\partial \nu}{\partial \ell} = 0. \tag{17}$$

Hence, the envelope curve is a set of maximum points of the utility.

In the same way, consider the implicit form of (12):

$$u\left[\mu(w_j\ell^{\theta^i}+T), \nu(\ell^{\theta^i})\right] - U_j^i = 0. \tag{18}$$

Differentiating with respect to θ^i,

$$\frac{\partial u}{\partial \mu}\cdot\frac{\partial \mu}{\partial \ell}\cdot w_j\ell^{\theta^i}\log\ell + \frac{\partial u}{\partial \nu}\cdot\frac{\partial \nu}{\partial \ell}\cdot\ell^{\theta^i}\log\ell = 0 \tag{19}$$

$$\frac{\partial u}{\partial \mu}\cdot\frac{\partial \mu}{\partial \ell}\cdot w_j + \frac{\partial u}{\partial \nu}\cdot\frac{\partial \nu}{\partial \ell} = 0. \tag{20}$$

Solving (18) and (20) simultaneously, we have an envelope curve of the parametric separable utility functions (12).

The first-order condition of (12) is

$$\frac{\mathrm{d}U_j^i}{\mathrm{d}\ell} = \frac{\partial u}{\partial \mu}\cdot\frac{\partial \mu}{\partial \ell}\cdot w_j\theta^i\ell^{\theta^i-1} + \frac{\partial u}{\partial \nu}\cdot\frac{\partial \nu}{\partial \ell}\cdot\theta^i\ell^{\theta^i-1} = 0 \tag{21}$$

$$\frac{\partial u}{\partial \mu}\cdot\frac{\partial \mu}{\partial \ell}\cdot w_j + \frac{\partial u}{\partial \nu}\cdot\frac{\partial \nu}{\partial \ell} = 0. \tag{22}$$

Therefore, each of their envelope curve is a set of maximum points of the utility, and by the same logic, each of the envelope curves of the parametric separable utility function is a line parallel to the horizontal axis ℓ of a labor supply–utility space, which again implies our required result.

3.3. **Quasilinearity in consumption.** Among the separable utility functions, we focus on utility functions that are *quasilinear in consumption* for analytical simplicity. It is,

however, not an essential assumption to obtain our required result, but it helps us to concentrate on the situations in which the *ex post* perspective of compensation applies.

The quasilinear utility function has the property that the marginal rate of substitution (MRS) between ℓ and c depends only on ℓ, which ensures that indifference curves of the same preference can all be obtained from any one of them by arbitrary translations parallel to the horizontal (labor supply) axis (Mas-Colell et al., 1995, p. 45). This enables us to consider lump-sum transfers/taxes without changing individual labor supply. The quasilinear assumption is often used for the specification in the optimal income tax literature (e.g., Salanié, 2011, pp. 101–107). Roemer (1996, pp. 297–301) uses quasilinear utility functions to examine the redistribution mechanism for equality of opportunity. More recently, Saez and Stantcheva (2016) assumes utility functions to be quasilinear in consumption to "rule out income effects on earnings which greatly simplifies optimal tax formulas" (p. 26).

In general, through lump-sum transfers, individuals' maximization behavior may change their labor supply, and that immediately precludes a comparison between the utilities of individuals with the same labor supply. Without this assumption, we only consider situations when individuals' labor supply are invariant by lump-sum transfers, but these situations can be captured by quasilinearity in consumption.

The following example shows utility functions that are quasilinear in consumption when there are no transfers/taxes.

Example 2. The quasilinear utility function of taste parameter θ^i as a multiplier of labor supply ℓ, which corresponds to (11), can be represented as

$$U_j^i = w_j \theta^i \ell - (\theta^i \ell)^2. \tag{23}$$

The quasilinear utility function of taste parameter θ^i as an exponent of labor supply ℓ, which corresponds to (11), can be represented as

$$U_j^i = w_j \ell^{\theta^i} - (\ell^{\theta^i})^2. \tag{24}$$

The graphs of the parametric family of utility functions (23) and (24) are shown in Figure 3 and Figure 4, respectively. As is implied by Theorem 1, we can observe that each family of utility functions with the same wage rate w_j but four different tastes for work has the same maximized level.

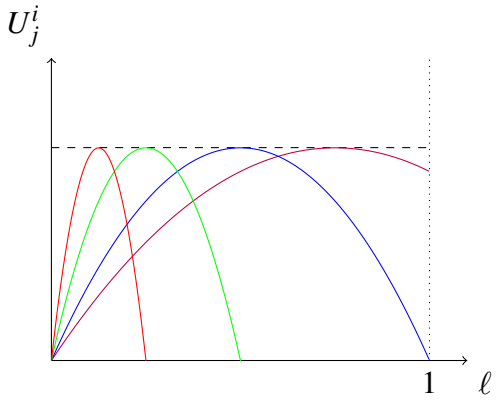

FIGURE 3. $U_j^i = w_j\theta^i\ell - (\theta^i\ell)^2$.

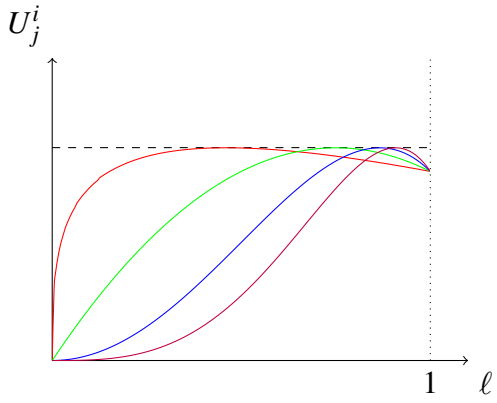

FIGURE 4. $U_j^i = w_j\ell^{\theta^i} - (\ell^{\theta^i})^2$.

4. COMPATIBILITY THEOREM

4.1. **Axioms.** We follow the axioms of Fleurbaey and Peragine (2013), but we modify them in our settings. The *ex post* approach to compensation focuses on inequality of utilities as a consequence of individuals' utility maximization behavior. This approach aims to reduce utility inequality between individuals, regardless of their taste parameter, but with the same *ex post* responsibility characteristic: labor supply.[17]

[17]It corresponds to the *Ex Post Compensation* axiom in Fleurbaey and Peragine (2013), "which says that it is good to reduce inequalities in outcomes between two cells sharing the same effort level but having unequal circumstances" (p.122), where a "cell is a set of individuals with the same characteristics" (p. 121). We just restrict our attention to utility as the outcome in Fleurbaey and Peragine (2013) (cf. footnotes 1 and 2). Also, our axiom is in fact stronger than those of Fleurbaey and Peragine (2013) because they constantly require strict inequality signs. Indeed, stronger axioms are better for our purpose of showing compatibility.

Ex Post Compensation. For all $E \in D$, $\tau \succ_{(E)} \tau'$ if there are $i, j \in M(E)$, such that $\bar{\ell}_i = \bar{\ell}_j$,

$$\begin{aligned} u(w_i\bar{\ell}_i + \tau'(w_i,\bar{\ell}_i),\bar{\ell}_i;\cdot) &> u(w_i\bar{\ell}_i + \tau(w_i,\bar{\ell}_i),\bar{\ell}_i;\cdot) \\ &\geq u(w_j\bar{\ell}_j + \tau(w_j,\bar{\ell}_j),\bar{\ell}_j;\cdot) > u(w_j\bar{\ell}_j + \tau'(w_j,\bar{\ell}_j),\bar{\ell}_j;\cdot), \end{aligned} \tag{25}$$

and $u(w_k\bar{\ell}_k + \tau(w_k,\bar{\ell}_k),\bar{\ell}_k;\cdot) = u(w_k\bar{\ell}_k + \tau'(w_k,\bar{\ell}_k),\bar{\ell}_k;\cdot)$ for all $k \in M(E) \setminus \{i, j\}$.

As defined in section 2, the labor supply $\bar{\ell}_i$ and $\bar{\ell}_j$ are optimally chosen by individuals facing wage rates w_i and w_j, but with an arbitrary taste parameter. Transfers/taxes are determined by the government as a function τ of w_i and $\bar{\ell}_i$ as well as w_j and $\bar{\ell}_j$. Inequality can occur due to a difference in circumstance characteristics, wage rates, and the respective transfers/taxes. This axiom focuses on the different allocations of $\tau(w_i,\bar{\ell}_i)$ and $\tau'(w_i,\bar{\ell}_i)$, and also $\tau(w_j,\bar{\ell}_j)$ and $\tau'(w_j,\bar{\ell}_j)$, and it tries to achieve utility equality between individuals with the same labor supply $\bar{\ell}_i = \bar{\ell}_j$ by changing transfers/taxes.

The *ex ante* approach to compensation aims to reduce utility inequality between individuals, regardless of their labor supply, but with the same *ex ante* responsibility characteristic: taste for work.[18]

[18]It corresponds to the *Strong Ex Ante Compensation* axiom in Fleurbaey and Peragine (2013), which "seeks situations in which two types are clearly unequal in terms of the perspectives offered by their circumstances and the respective transfer policies" (p. 122), where "a type is a set of individuals with the same circumstances" (p. 121); that is, they evaluate the budget sets faced by individuals with different circumstances. Comparing individuals with the same taste for work is an alternative way to achieve the identical goal while considering utility as an outcome and introducing parametric utility functions. This is because both of the axioms aim to equalize "the opportunities offered to individuals (as measured by the possible well-being levels achieved with given circumstances for the various values of responsibility characteristics" (Fleurbaey, 2008, p. 239), where the responsibility characteristic is labor supply. In this paper, the well-being levels, or outcomes, are considered to be utilities, and they can vary between individuals with the same circumstance but with different preferences; thus, we need to compare individuals with the same taste for work. It is also stronger than those of Fleurbaey and Peragine (2013) because they constantly require strict inequality signs.

Ex Ante Compensation. For all $E \in D$, $\tau \succ_{(E)} \tau'$ if there are $i, j \in N(E)$, $i, j \in M(E)$,

$$\begin{aligned} u(w_i\ell_i^i + \tau'(w_i, \ell_i^i, \theta^i), \ell_i^i; \theta^i) &> u(w_i\ell_i^i + \tau(w_i, \ell_i^i, \theta^i), \ell_i^i; \theta^i) \\ &\geq u(w_j\ell_j^i + \tau(w_j, \ell_j^i, \theta^i), \ell_j^i; \theta^i) > u(w_j\ell_j^i + \tau'(w_j, \ell_j^i, \theta^i), \ell_j^i; \theta^i) \end{aligned} \tag{26}$$

and

$$\begin{aligned} u(w_i\ell_i^j + \tau'(w_i, \ell_i^j, \theta^j), \ell_i^j; \theta^j) &> u(w_i\ell_i^j + \tau(w_i, \ell_i^j, \theta^j), \ell_i^j; \theta^j) \\ &\geq u(w_j\ell_j^j + \tau(w_j, \ell_j^j, \theta^i), \ell_j^j; \theta^j) > u(w_j\ell_j^j + \tau'(w_j, \ell_j^j, \theta^j), \ell_j^j; \theta^j), \end{aligned} \tag{27}$$

and $u(w_k\ell_k^k + \tau(w_k, \ell_k^k, \theta^k), \ell_k^k; \theta^k) = u(w_k\ell_k^k + \tau'(w_k, \ell_k^k, \theta^k), \ell_k^k; \theta^k)$ for all $k \in N(E) \setminus \{i, j\}$ and $k \in M(E) \setminus \{i, j\}$.

As defined in section 2, the labor supply ℓ_i^i, ℓ_j^i, ℓ_i^j, and ℓ_j^j are those of individuals facing wage rates w_i, w_j, and taste parameters θ^i, θ^j.[19] Transfers/taxes are determined by the government as a function τ of these variables and types. Again, inequality can occur due to a difference in circumstance characteristics, wage rates, and the respective transfers/taxes. This axiom focuses on the different allocations of the transfers/taxes and tries to achieve utility equality between individuals, regardless of their labor supply ℓ_i^i, ℓ_j^i, ℓ_i^j, or ℓ_j^j, but within the same taste parameter θ^i or θ^j.

4.2. **Statement of the main result.** We provide the compatibility theorem, which is an escape route from the opportunity paradox. The first step is that the *ex post* perspective of compensation, **Ex Post Compensation**, requires a reduction in the inequality of utilities, not consumption or disposable incomes. Thus, we have to compare the utility levels, the "height" of the utility functions, which cannot be observed in the indifference curves. The second step is that using the separable—and taste-independent—domain restriction resolves this difficulty.

[19] Note that superscripts and subscripts, i, j, k, identify the types of individuals.

We provide an intuitive proof of the compatibility theorem using Figure 5. Points a, b, c, and d are all displayed at the same places as in Figure 1. Point a represents the allocation optimally chosen by an individual with wage rate w_j and taste parameter θ^1, point b represents the allocation of an individual with w_j and θ^2, and point f represents the allocation of an individual with w_i and θ^1, where $w_i < w_j$. Points d and e represent the allocations of individuals with w_i and θ^1 who receive the respective lump-sum transfers.[20]

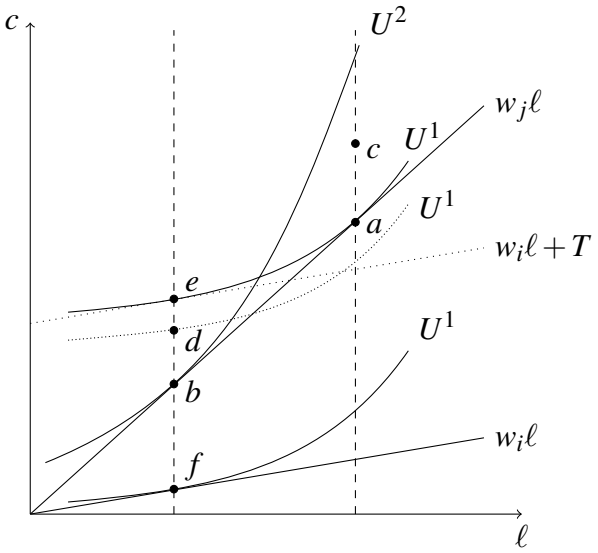

FIGURE 5. No opportunity paradox by introducing separable utility and taste-independence: both **Ex Post Compensation** and **Ex Ante Compensation** require that the individual on point d be compensated to e.

Suppose points a and b are laissez-faire allocations of individuals with wage rate w_j; that is, they are on the same budget line $w_j\ell$. By *taste-independence*, a and b exhibit the same utility levels for individuals with θ^1 and θ^2. Furthermore, points a and e have the same utility levels for individuals with θ^1 because the points are on the same indifference curve. Therefore, **Ex Post Compensation** requires lump-sum transfers to the individual on point d to e, not to b, to realize the same utility level between individuals on points b and d, which does not violate **Ex Ante Compensation**.[21] In

[20]They have the same labor supply due to the assumption of quasilinearity in consumption.

[21]It may seem unreasonable to accept a transfer policy to change point d to e because it expands income inequality. However, consider, for instance, a proportional income tax that changes the wage rate of

this example, point d is actually the point at which an individual on point f, as the laissez-faire allocation, is given a certain amount of lump-sum transfers, which **Ex Post Compensation** deems insufficient.[22] The same argument applies to point c. Theorem 2 shows that this is not an exceptional case.

Theorem 2. ***Ex Post Compensation*** *and* ***Ex Ante Compensation*** *are compatible when the parametric utility functions are quasilinear in consumption.*

Proof. We prove that we can obtain allocations required by both **Ex Post Compensation** and **Ex Ante Compensation** in general.[23] There are two types of individuals (i.e., wage rate type and taste type) in our model; thus, the minimal society we need to consider consists of 2×2 (types of) individuals.

First, we construct all possible transfers/taxes required by **Ex Post Compensation**. Second, we show that arbitrary transfers/taxes required by **Ex Ante Compensation** are compatible with them.

We consider four individuals indicated by 1 to 4. They have wage rate w_i or w_j ($w_i \neq w_j$) and taste parameter θ^i or θ^j ($\theta^i \neq \theta^j$). As a result of their maximization behavior, the four individuals have labor supply $\bar{\ell}^i_i$, $\bar{\ell}^j_i$, $\bar{\ell}^j_j$, or $\bar{\ell}^i_j$, and receive lump-sum transfers/taxes denoted by $T_1, T_2, T_3, T_4 \neq 0$, respectively.

The maximized utility of each individual is indicated by

$$U_1^* \equiv U_i^i = u(w_i\bar{\ell}^i_i + T_1, \bar{\ell}^i_i; \theta^i), \qquad U_2^* \equiv U_i^j = u(w_i\bar{\ell}^j_i + T_2, \bar{\ell}^j_i; \theta^j), \tag{28}$$

$$U_3^* \equiv U_j^j = u(w_j\bar{\ell}^j_j + T_3, \bar{\ell}^j_j; \theta^j), \qquad U_4^* \equiv U_j^i = u(w_j\bar{\ell}^i_j + T_4, \bar{\ell}^i_j; \theta^i). \tag{29}$$

Moreover, assume, without loss of generality,

$$\bar{\ell}^i_i = \bar{\ell}^j_j,\ \bar{\ell}^j_i = \bar{\ell}^i_j. \tag{30}$$

individuals with preference θ^1 at points d, e, or f as the laissez-faire allocation. Their maximization behavior moves those points to a. Also, note that the outcome in Fleurbaey and Peragine (2013) can be utility (cf. footnotes 1, 2, and 17); hence, their *ex post* perspective of compensation would require the same policy if the outcome is utility that is separable in consumption and labor supply.

[22] These lump-sum transfers are implementable due to the assumption of quasilinearity in consumption.

[23] It does not depend on any particular social welfare function.

By the property of *quasilinearity*, the following laissez-faire allocations exist:

$$u(w_i\bar{\ell}_i^i, \bar{\ell}_i^i; \theta^i),\ u(w_i\bar{\ell}_i^j, \bar{\ell}_i^j; \theta^j), \tag{31}$$

$$u(w_j\bar{\ell}_j^j, \bar{\ell}_j^j; \theta^j),\ u(w_j\bar{\ell}_j^i, \bar{\ell}_j^i; \theta^i). \tag{32}$$

By *taste-independence*, which is assured by Theorem 1,

$$u(w_i\bar{\ell}_i^i, \bar{\ell}_i^i; \theta^i) = u(w_i\bar{\ell}_i^j, \bar{\ell}_i^j; \theta^j), \tag{33}$$

$$u(w_j\bar{\ell}_j^j, \bar{\ell}_j^j; \theta^j) = u(w_j\bar{\ell}_j^i, \bar{\ell}_j^i; \theta^i). \tag{34}$$

Also, by *quasilinearity*, there exist $\tilde{T}_1$, $\tilde{T}_2$, $\tilde{T}_3$, and $\tilde{T}_4$ to compensate between individuals with the same taste (e.g., by suitable lump-sum transfers, the government can compensate the individual represented by point f to e in Figure 5), such that

$$\tilde{U}_1 \equiv u(w_i\bar{\ell}_i^i + \tilde{T}_1, \bar{\ell}_i^i; \theta^i) = u(w_j\bar{\ell}_j^i, \bar{\ell}_j^i; \theta^i), \tag{35}$$

$$\tilde{U}_2 \equiv u(w_i\bar{\ell}_i^j + \tilde{T}_2, \bar{\ell}_i^j; \theta^j) = u(w_j\bar{\ell}_j^j, \bar{\ell}_j^j; \theta^j), \tag{36}$$

$$\tilde{U}_3 \equiv u(w_j\bar{\ell}_j^j + \tilde{T}_3, \bar{\ell}_j^j; \theta^j) = u(w_i\bar{\ell}_i^j, \bar{\ell}_i^j; \theta^j), \tag{37}$$

$$\tilde{U}_4 \equiv u(w_j\bar{\ell}_j^i + \tilde{T}_4, \bar{\ell}_j^i; \theta^i) = u(w_i\bar{\ell}_i^i, \bar{\ell}_i^i; \theta^i). \tag{38}$$

By transitivity, combining (33), (35), and (36),

$$\begin{aligned}\tilde{U}_1 \equiv u(w_i\bar{\ell}_i^i + \tilde{T}_1, \bar{\ell}_i^i; \theta^i) &= u(w_j\bar{\ell}_j^i, \bar{\ell}_j^i; \theta^i) \\ &= u(w_j\bar{\ell}_j^j, \bar{\ell}_j^j; \theta^j) = u(w_i\bar{\ell}_i^j + \tilde{T}_2, \bar{\ell}_i^j; \theta^j) \equiv \tilde{U}_2,\end{aligned} \tag{39}$$

and combining (34), (37), and (38),

$$\begin{aligned}\tilde{U}_3 \equiv u(w_j\bar{\ell}_j^j + \tilde{T}_3, \bar{\ell}_j^j; \theta^j) &= u(w_i\bar{\ell}_i^j, \bar{\ell}_i^j; \theta^j) \\ &= u(w_i\bar{\ell}_i^i, \bar{\ell}_i^i; \theta^i) = u(w_j\bar{\ell}_j^i + \tilde{T}_4, \bar{\ell}_j^i; \theta^i) \equiv \tilde{U}_4.\end{aligned} \tag{40}$$

Now, consider applying axioms between the four individuals with U_1^*, U_2^*, U_3^*, and U_4^* using equations (39) and (40). **Ex Post Compensation** requires that the inequality

of utilities be eliminated if either U_1^* or U_3^* is higher than the other. In such a case, the following transfer or tax policy changes, (α) or (β), can cause individuals 1 and 3 to enjoy the same utility level.

(α): changes T_1 to $\tilde{T}_1$ and T_3 to 0, which result in the same utility level $\tilde{U}_1 = \tilde{U}_2$ for individuals 1 and 3.

(β): changes T_1 to 0 and T_3 to $\tilde{T}_3$, which result in the same utility level $\tilde{U}_3 = \tilde{U}_4$ for individuals 1 and 3.

In the same way, **Ex Post Compensation** requires that the inequality of utilities be eliminated if either U_2^* or U_4^* is higher than the other. In such a case, the following transfer or tax policy changes, (γ) or (δ), can cause individuals 2 and 4 to enjoy the same utility level.

(γ): changes T_2 to $\tilde{T}_2$ and T_4 to 0, which result in the same utility level $\tilde{U}_1 = \tilde{U}_2$ for individuals 2 and 4.

(δ): changes T_2 to 0 and T_4 to $\tilde{T}_4$, which result in the same utility level $\tilde{U}_3 = \tilde{U}_4$ for individuals 2 and 4.

Meanwhile, **Ex Ante Compensation** requires that the inequality of utilities be eliminated if either U_1^* or U_4^* is higher than the other. Either of the following combinations of policy changes, (α) and (γ), or (β) and (δ), can cause individuals 1 and 4 to enjoy the same utility level.

- (α) change T_1 to $\tilde{T}_1$ and (γ) change T_4 to 0, which result in the same utility level $\tilde{U}_1 = \tilde{U}_2$ for individuals 1 and 4.
- (β) change T_1 to 0 and (δ) change T_4 to $\tilde{T}_4$, which result in the same utility level $\tilde{U}_3 = \tilde{U}_4$ for individuals 1 and 4.

Furthermore, **Ex Ante Compensation** requires that the inequality of utilities be eliminated if either U_2^* or U_3^* is higher than the other. Either of the following combinations of policy changes, (α) and (γ), or (β) and (δ), can cause individuals 2 and 3 to enjoy the same utility level.

- (γ) change T_2 to $\tilde{T}_2$ and (α) change T_3 to 0, which result in the same utility level $\tilde{U}_1 = \tilde{U}_2$ for individuals 2 and 3.
- (δ) change T_2 to 0 and (β) change T_3 to $\tilde{T}_3$, which result in the same utility level $\tilde{U}_3 = \tilde{U}_4$ for individuals 2 and 3.

Therefore, combinations of transfer or tax policy changes, (α) and (γ), or (β) and (δ), result in allocations that satisfy the requirements of both **Ex Post Compensation** and **Ex Ante Compensation**. □

Remark 1. **Ex Post Compensation** through lump-sum transfers/taxes only covers situations where individuals have utilities that are quasilinear in consumption (i.e., income effects are zero) or when income and substitution effects are offset. Once the labor supply varies according to transfers/taxes, we can no longer apply **Ex Post Compensation**, which compares individuals with the same labor supply. The assumption of quasilinearity with respect to consumption assures the existence of comparable allocations; otherwise, we cannot discuss the logical relationship between **Ex Ante Compensation** and **Ex Post Compensation**. Therefore, quasilinearity is needed so that we can concentrate solely on resolving the opportunity paradox. On the other hand, separability, which is implied by quasilinearity, is the "crucial" assumption for our result.[24]

Remark 2. In the proof, we demonstrate that we can achieve (perfect) equality of utilities between individuals by showing that **Ex Ante Compensation** and **Ex Post Compensation** requires the same direction of transfers/taxes, but this may require the government to implement a large amount of transfers/taxes. The theorem, however, incorporates some medium level or small amount of reductions in inequality, and transfers/taxes could also be "sufficiently small" (Fleurbaey and Peragine, 2013, p. 126–127). This fact may be useful when we take into account the government's budget constraint, which we do not in the present paper.

[24] "All theory depends on assumptions which are not quite true. That is what makes it theory. The art of successful theorizing is to make the inevitable simplifying assumptions in such a way that the final results are not very sensitive. A 'crucial' assumption is one on which the conclusions do depend sensitively, and it is important that crucial assumptions be reasonably realistic" (Solow, 1956, p. 65).

4.3. **Reinterpretation in a responsibility–outcome space.** We demonstrate how the opportunity paradox can be escaped in a responsibility–outcome space. Since we consider the outcome to be utility, Figure 1 is not a responsibility–outcome space; we need to clarify the opportunity sets of individuals as well as the relationship between the existing literature, such as Fleurbaey (2019).[25] As a result, we provide a graphical alternative proof of Theorem 2. For simplicity, in the following discussion, we assume that opportunity gaps are only due to differences in wage rates, not the respective transfers/taxes.

In our framework, the outcome axis represents utility U^i_j, while the responsibility axis should represent two responsibility characteristics: taste for work θ^i as *ex ante* and labor supply ℓ as *ex post* responsibility characteristics. First, by fixing θ^i, we explore what an *ex post* responsibility–outcome, or labor supply–utility, space looks like. In fact, a labor supply–utility space is the same as the space that we draw for utility functions such as in Figure 3 and Figure 4. Hence, an example of the utility functions of three individuals with the same taste but different wage rates is presented in Figure 6. These curves represent opportunity sets of individuals with wage rates $w_1 > w_2 > w_3$, and the same taste θ^3. Each utility is U^3_1, U^3_2, and U^3_3, respectively. As we assume maximization behavior by individuals, points a, b, and c are supposed be achieved.

Then, we consider that the *ex ante* responsibility characteristic, or taste parameter θ^i, also varies. If taste-independence—implied by the separability of utility according to Theorem 1—is satisfied, the utility functions of individuals with two different wage rates and three different tastes for work can be drawn like Figure 7. Individuals with wage rate w_1 have three different tastes, θ^1, θ^2, and θ^3. Their utilities are U^1_1, U^2_1, and U^3_1, respectively. Additionally, individuals with wage rate w_3 have three different tastes, θ^3, θ^4, and θ^5. Their utilities are represented by U^3_3, U^4_3, and U^5_3, respectively. Note that U^3_1 and U^3_3 remain the same as Figure 6, and they have the same taste, but they have different wage rates. Figure 7 shows how, given wage rates w_1 and w_3, the two responsibility characteristics, ℓ and θ^i, determine utility, or outcome. Utility functions

[25] See footnotes 1, 2, and 4.

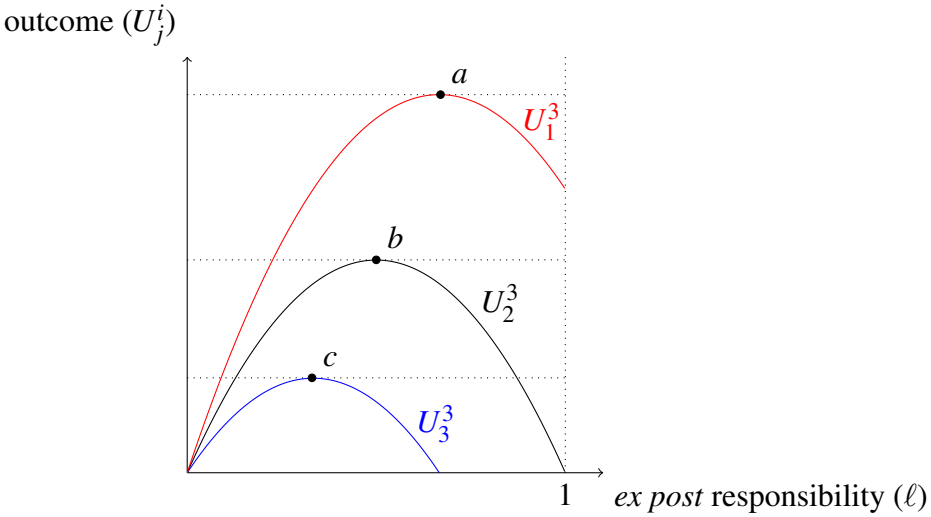

FIGURE 6. Opportunity sets of individuals (fixing the *ex ante* responsibility characteristic, or taste for work).

are continuous with respect to taste parameter θ^i; thus, utilities with the same wage rate but different tastes can densely exist. Therefore, opportunity sets, which describe the correspondence between responsibility characteristics and outcome, can be represented by the envelope curves (lines) of utilities for each wage rate when maximization behavior is supposed. The area below the lines are included if maximization behavior is not supposed.

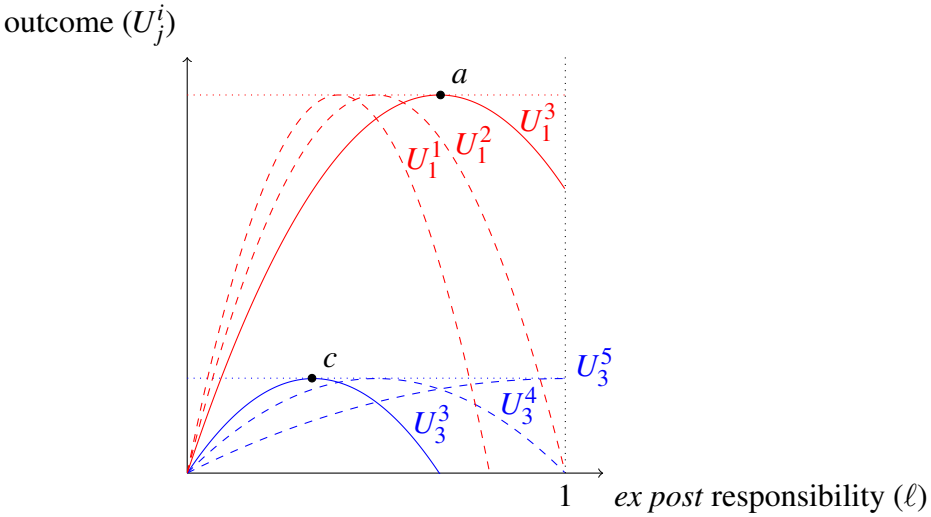

FIGURE 7. Opportunity sets of individuals whose utilities are taste-independent: dashed curves represent the corresponding sets to the change of the *ex ante* responsibility characteristic, or taste for work.

The argument so far can be described simply in Figure 8. We observe that the outcome is constant according to responsibility characteristics. This suggests that individuals with the same wage rate have equal opportunity to enjoy the same utility, even if they choose different labor/leisure time and incomes on their own responsibility. This is exactly what the taste-independence property implies, and the reason why it can be recognized as a responsibility requirement. However, by Theorem 1, it is derived from separability of utility; thus, we do not need to impose this as an axiom.

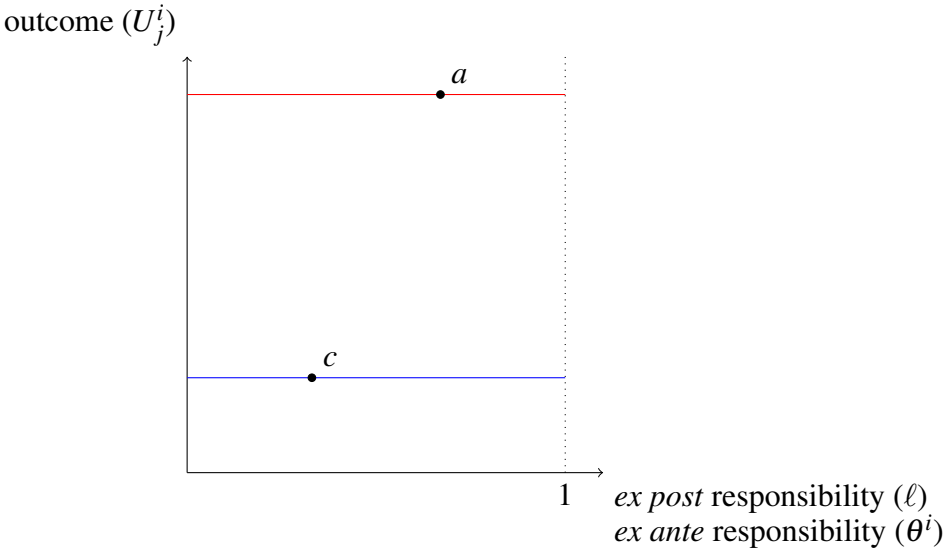

FIGURE 8. Opportunity sets of individuals whose utilities are taste-independent.

Now, we can illustrate how the taste-independence property works for escaping from the opportunity paradox. **Ex Ante Compensation** aims to reduce the utility inequality between individuals with the same taste for work, such as points a and c, which share the same taste for work θ^3. *Anywhere* such comparable individuals (i.e., those with same taste for work but different wage rates) exist, including a and c on the two lines, **Ex Ante Compensation** requires a reduction in the gaps of the two lines, or opportunity sets. In other words, the *ex ante* perspectives of compensation require that individuals with the same taste for work but different circumstances, have equal opportunity to enjoy the same utility, but can choose their labor/leisure time and income.[26]

[26]See also footnote 4.

Meanwhile, **Ex Post Compensation** aims to reduce the utility inequality between individuals with the same labor supply but different circumstances. Again, *anywhere* such comparable individuals (i.e., those with the same labor supply) exist on the two lines, **Ex Post Compensation** always requires a reduction in the gaps of the two lines. Therefore, **Ex Ante Compensation** and **Ex Post Compensation** require the same direction of transfers/taxes; that is, they are compatible.

If taste-independence is not satisfied, the opportunity sets are, in general, no longer horizontal lines but arbitrary nonlinear curves. Hence, responsibility–outcome sets, or opportunity sets, can be described as in Fleurbaey (2008, ch.9, sec. 9.5), Fleurbaey (2019, sec. 6), and Figure 1 as well as Figure 9 in the present paper.[27] In such cases, the *ex ante* and *ex post* perspectives of compensation can conflict. For reference, we restate the same proof as Fleurbaey (2008) in Figure 9 using our axioms **Ex Ante Compensation** and **Ex Post Compensation**.[28]

We now consider that opportunity gaps are due to both different wage rates and the respective transfers/taxes. Solid curves A, B, C, and D represent four opportunity sets. Individuals on A and B share the same wage rate but different transfers/taxes, so do those on C and D. The nonlinearity of the curves indicates that the utilities are not taste-independent; that is, each of the curves represents the maximized utility of individuals with the same wage rate but different tastes for work. **Ex Ante Compensation** seeks individuals with the same taste, and we can discuss this by arbitrarily picking some allocations to compare. However, since the difference between A and B is due the respective transfers/taxes, anywhere individuals with the same taste exist on curves A and B, **Ex Ante Compensation** requires a reduction in the gaps between curves A and B. The same is true for curves C and D. For example, modifications of curves, described by the dashed curves, can be required by **Ex Ante Compensation**, but they

[27]That is, outcome, or utility, varies according to the two responsibility characteristics, or labor supply and taste for work, without taste-independence. See also footnote 4.

[28]Fleurbaey (2008) uses the axiom *Opportunity Dominance*, which implies our **Ex Ante Compensation**, and is setting the vertical axis as well-being. Figure 9.4 in Fleurbaey (2008, p. 238) corresponds to Figure 9 in this paper, and Figure 5 in Fleurbaey (2019, p. 675) corresponds to Figure 1 in this paper. Since, in Figure 1, we consider an labor supply–income, or *ex post* responsibility–income, space, Figure 9 is more suited for describing the opportunity sets. See also footnote 4.

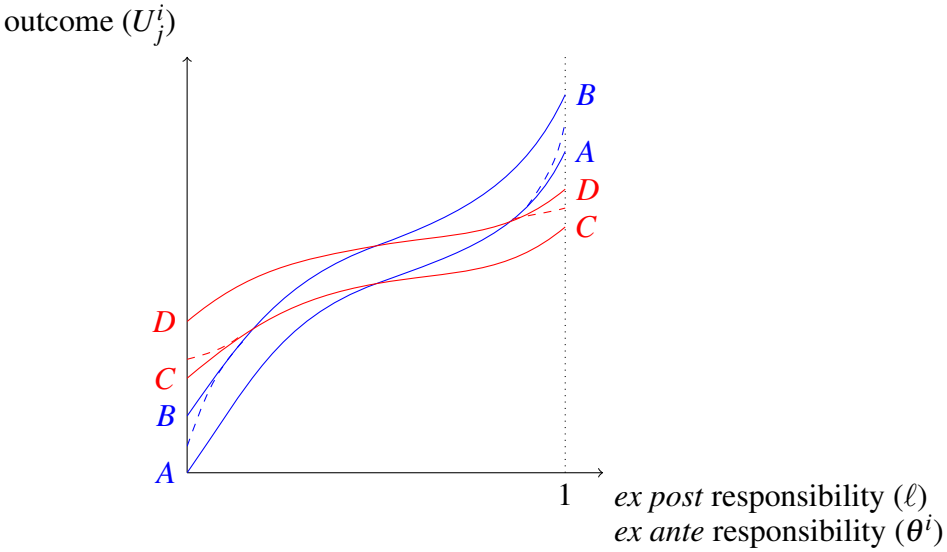

FIGURE 9. Crossing opportunity sets of individuals whose utilities are not taste-independent (reproduced from Fleurbaey, 2008, p. 238, Figure 9.4).

are generally incompatible with **Ex Post Compensation** because the utility inequalities between individuals on B and C with low labor supply as well as individuals on A and D with high labor supply are widened.

Graphically, it may seem that taste-independence is a strong assumption because it requires curves such as A, B, C, D in Figure 9 to be horizontal straight lines, but by Theorem 1, it is a property derived from separability of utility that is not an unusual or extraordinary preference domain restriction.[29]

5. CONCLUDING REMARKS

"(I)n the process of finding some meaningful escape routes from these logical impasses, we are brought to much richer understanding on what makes several social values mutually compatible than otherwise" (Suzumura, 2002, p. 25).

[29]However, this also implies that there may be other preference domain restrictions where the opportunity paradox does not occur.

In this paper, we introduced taste-independence as a property of preferences while taking individual responsibility into account. We then showed that the preference domain restriction, utility separability in consumption and labor supply, ensure the taste-independent property. Consequently, we demonstrated that *ex post* and *ex ante* perspectives of compensation, which focus on utilities, are compatible when the utility functions are quasilinear in consumption. In fact, quasilinearity is not an essential assumption, in the sense that it only makes the *ex post* perspective of compensation apply when there are lump-sum transfers/taxes. Without quasilinearity, we need to consider situations where the labor supply optimally chosen by individuals are invariant to lum-sum transfers/taxes, and these situations can be captured by quasilinear assumption. Therefore, the opportunity paradox does not occur under the separable utility restrictions on the preference domain.

In practice, for instance, when deriving the optimal income taxation formula, restrictions of the preference domain, such as separability, are commonly required. That is, we often solve the maximization problem of the Bergson-Samuelson social welfare function (SWF), whose inputs are separable utilities, subject to incentive compatibility constraints and the government's budget constraint. We face ethical conflicts such as the opportunity paradox when constructing the SWF itself; thus, maximizing it derives a solution (e.g., income taxation formula) that is, at any rate, a compromise of either the *ex ante* or *ex post* perspectives of compensation. However, since we solve the maximization problem with the preference restriction after all, we can derive the SWF on a restricted domain where there is no compatibility (i.e., separable utility). Therefore, we opened up the possibility obtaining a solution that is not a compromise of both the *ex ante* and *ex post* perspectives of compensation.

We conclude by suggesting directions for future research. As declared in footnote 7, we adopted an approach in which utility is a subjective measure of well-being, but such a welfarist approach has "serious weaknesses" (Fleurbaey and Maniquet, 2018,

p. 1035). Further explorations are needed to accommodate critiques such as the "expensive tastes" (e.g., Dworkin, 1981a,b) and the "tamed housewife" arguments (Sen, 1985a,b).[30]

Moreover, as mentioned in footnotes 11 and 21, it is worthwhile to extend our theory to address issues regarding the heterogeneous preferences of individuals with disabilities, such as Cuff (2000) and Boadway et al. (2002), and curbing inequality of incomes, such as (Roemer et al., 2003). Furthermore, applications to income taxation and transfer policies, relating to the existing fair taxation literature (e.g., Schokkaert et al., 2004; Fleurbaey and Maniquet, 2006, 2007, 2011a,b; Jacquet and Van de gaer, 2011; Lockwood and Weinzierl, 2015; Saez and Stantcheva, 2016; Fleurbaey and Maniquet, 2018), are expected.

Finally, as mentioned in footnote 14, there may be other functional forms of taste-independent utility apart from separable ones, and also other preference domain restrictions may resolve the opportunity paradox, as mentioned in footnote 29. Further, other related axioms of the *ex ante* perspective of compensation, or various reward principles, may be compatible with the *ex post* perspective of compensation when taste-independence is satisfied. In other words, we should explore the necessary and sufficient conditions for escaping the opportunity paradox, and such investigations will lead us to a more profound comprehension of the theoretical possibilities beyond the difficulties regarding equality of opportunity.

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[30] *Welfarism* is defined as "requiring that the goodness of a state of affairs be a function only of the utility information regarding that state" (Sen, 1987, p. 39). Our results may imply that there are limitations to using only utility information because just assuming separability leads to expansion of inequality of income through lump-sum transfers/taxes (cf. footnote 21).

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