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Lidia Ceriani

Vladimir Hlasny

Paolo Verme

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Lidia Ceriani
Georgetown University, Washington DC, USA.
Vladimir Hlasny
Ewha Womans University, Seoul, South Korea.
Paolo Verme
World Bank, Washington DC, USA.

Abstract

The paper discusses the main issues related to negative and zero incomes that are relevant for the measurement of poverty. It shows the prevalence of non-positive incomes in high- and middle-income countries, provides an analysis of the sources and structure of these incomes, outlines the various approaches proposed by scholars and statistical agencies to treat non-positive incomes, and explains how non-positive incomes and alternative correction methods impact the measurement of standard poverty indexes. It is argued that negative and zero incomes cannot be treated equally in terms of household well-being and that standard methods used by practitioners fail to recognize this fact likely resulting in overestimations of poverty.

Keyword: Welfare measurement; Well-being; Poverty targeting; High- and middle-income countries; Survey non-response; Negative incomes; Zero incomes; Extreme income corrections

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[1] Georgetown University, Washington DC, USA.
[2] Ewha Womans University, Seoul, South Korea.
[3] World Bank, Washington DC, USA. The paper is forthcoming in the "Research Handbook on Measuring Poverty and Deprivation", Elgar Handbooks in Development series, Jacques Silber, Editor.

1. INTRODUCTION

Disposable household income is the variable of choice for distributional analysis in the majority of high- and middle- income countries, notably in Europe and the Americas. Income data from household budget surveys can contain zero or negative values because income can be subject to significant variations across periods of time. Some households may experience periods of no income or negative incomes because of temporary variations in savings and investments, losses from self-employment, or various accounting or measurement practices.

Negative and zero incomes are a non-negligible feature of income data in both numbers and size (Ravallion, 2017; Verma and Betti, 2010). Despite their importance, these values tend to be dropped or replaced with little analysis of their true nature and meaning for poverty and well-being analyses. Negative values are very often excluded from empirical distributional analysis, either by dropping households exhibiting negative incomes, setting negative incomes to zero, or bottom coding extreme values to some arbitrary value. For example, negative incomes are set to zero in 20 out of 35 countries in the definitions of income used for the analysis of poverty and inequality among OECD countries (AUS, CAN, CZE, DNK, FIN, FRA, DEU, HUN, ISR, JPN, KOR, NLD, NZL, NOR, SVN, SWE, CHE, TUR, GBR, USA) while an additional five countries (HUN, JPN, NLD, PRT, TUR) use bottom coding to replace observations below 1% of median disposable income (OECD, 2021).

Understanding the bottom tail of income distributions is important for poverty from measurement and policy perspectives. Negative and zero values are problematic for poverty and inequality measures that require logarithmic or power transformations and may not represent household well-being accurately because of accounting or measurement practices (Stich, 1996). The bottom tail of the distribution includes the income group most in need of assistance and the primary target of social protection programs. Misclassifying households reporting non-positive incomes has an impact on targeting exercises such as

Proxy Means Testing (PMT) resulting in larger inclusion and exclusion errors. This has direct negative consequences on the livelihood of the poor.

This chapter discusses the prevalence and sources of non-positive incomes, the effect on the measurement of poverty, the range of solutions and their impact on poverty measures. The paper concludes that a proper classification of bottom incomes can alter poverty measures non-trivially.

2. PREVALENCE OF NON-POSITIVE INCOMES

The presence of non-positive incomes is quite common in household surveys; negative incomes can be a significant portion of total incomes, and zero incomes can also be highly prevalent in number. According to Ravallion (2017), about 400 of the 700 income surveys included in the World Bank's PovcalNet data repository have non-positive values and, according to Verma and Betti (2010), 3% of total Disposable Household Incomes (DHI) among European Union Surveys on Income and Living Conditions (EU-SILC) data are negative or zero (see also Van Kerm, 2007; Hlanky et al., 2021; Székely et al., 2007). In 12 of the Luxembourg Income Study (LIS) surveys analyzed by Hlasny et al. (2021), negative incomes accounted for over one percent of nonzero incomes, numbered up to 584 observations in a national survey, and could be as large in absolute value as 754% of mean nationwide positive income.

When negative incomes are present in a survey, they vary across households suggesting that the values represent some meaningful differences in household incomes' components. In surveys that contain negative incomes, these incomes persist across survey waves indicating that they are not occasional episodes of mismeasurement. When negative incomes do not appear in survey data released by statistical agencies, the reason is often associated with standard adjustments introduced by these agencies such as bottom coding, imputation, or substitution.

Scholars agree that non-positive incomes can be problematic (Schutz, 1951) but the classic solutions proposed to address these problems, such as elimination or bottom coding, do not question whether these incomes represent poor households or not. In some cases, negative incomes can indicate true economic distress. For example, in Myanmar, as many as 40% of farmers have negative incomes and most of these farmers can be classified as poor (Thant and Calkins, 2009). But in other cases, negative incomes can be the result of temporary expenditures, investments, or accounting practices on the part of self-employed individuals and business owners who should otherwise be considered as wealthy. Some of the households reporting negative or zero income may well be non-poor and this should be assessed before non-positive incomes are treated.

3. SOURCES OF NON-POSITIVE INCOMES

In a study of bottom incomes based on LIS data, Hlasny et al. (2021) find that the main source of negative DHI appears to be negative self-employment income. High tax, high social security withholding and high self-paid social security contributions account for negative incomes in some countries. Households with these attributes may not be truly income poor. Whether their negative incomes reflect accurately households' current welfare, or whether they are artifacts of some accounting practices, data-entry errors or statistical agencies' treatment, these questions should be investigated. Verma and Betti (2010) report that: "*It is likely that such values appear in large numbers as a result of deducting social transfers from the household's actual disposable income without adequately considering that outgoings (already deducted from income) may be conditional on the availability to the household of the social transfer income component which is being removed*" (p.64).

Zero incomes differ from negative incomes in some important respects and the two classes of incomes should not be treated equally, for example by bottom coding both classes of incomes at zero. Zero incomes are often caused by post-survey adjustments such as bottom coding or replacing missing observations with zeros, where missing values may be caused by item non-response, data-entry errors or censoring at zero during the survey phase or the post-survey data cleaning. Zero incomes could thus be associated with various issues that the survey documentation typically fails to explain. Understanding who is who among households showing zero incomes is essential for generating a consistent ordering among households, and measuring poverty correctly.

Hlasny et al. (2021) also report the distribution of self-employment income, undue liabilities for taxes and social security contributions, and the burden of social security contributions alone among households with zero or negative incomes. Negative self-employment income is the primary source of negative DHI in three-quarters of all surveys. The remaining cases are due to unduly high self-paid social security contributions and other costs, such as high property taxes, loan repayment, or negative inter-household transfers (e.g., alimonies, remittances, family transfers; Eurostat 2006). The prevalence of negative incomes, and the contribution of individual factors – self-employment income, social security contributions, and others – differ across countries and years.

Interestingly, when country datasets are sorted by the frequency of negative DHI, negative household income from self-employment shows up as the top source of their prevalence. By contrast, when datasets are sorted by the relative magnitude of negative incomes, high inter-household transfers and undue social security and other burdens dominate as sources of the high level of negative incomes. The prevalence of negative incomes is thus primarily due to negative self-employment incomes, while the extreme values of negative incomes are typically due to extremely high social security contributions, non-income taxes, and paid remittances (Hlasny et al., 2021).

Van Kerm (2007) finds that employee income is the most common source of income in the EU-SILC surveys, but self-employment income is the most problematic source of income being often negative and varying widely across individuals and surveys. This variation is partly explained by the nature of self-employment income and partly by statistical artefacts such as different modes of data collection or imputation rules, or differences in non-response. The author also discusses other sources of potential issues such as old age benefits, rents from property, interhousehold transfers and taxes, but concludes that these sources of incomes are rather innocuous for the measurement of poverty or inequality because of their relatively small range and similarities across countries. Instead, interest and dividends vary significantly and have extreme values, thus becoming the second most critical source of potential problems after self-employment.

Verma and Betti (2010) provide a taxonomy of possible errors typically found in the tails of EU-SILC income distributions. They find that self-employment and, in some cases, capital incomes, are the main source of problems among bottom incomes, also explaining the presence of negative incomes. They argue that negative and zero disposable household incomes are not useful measures of well-being while they are also not easily handled by several poverty measures.

Households with negative incomes often appear to be as well off as, or even better off than other households in their material wellbeing. Negative-income households do not appear to have unduly low consumption, and sometimes have higher total consumption, food consumption, and home ownership than households with positive incomes. In relation to households' outflows for mortgage and loan repayment, completion of secondary education, or health, once again no clear concerns arise, and if anything, negative-income households appear to outperform other groups (Brewer, 2017; Hlasny et al., 2021). Zero-income households, by contrast, appear to be materially deprived in their outcomes. They have lower total consumption, lower home ownership rate and lower debt maintenance than the national means, even as their education, health and urban residence status are not clearly behind those among positive-income households (Hlasny et al. 2021). This leads to the question of whether correction methods for negative and zero incomes consider these issues.

4. CORRECTION APPROACHES FOR NON-POSITIVE INCOMES

Adjustment methods for non-positive incomes fall into two categories: 1) Reweighting, whereby original observations are kept intact while weights are recalibrated, and 2) Replacing, whereby weights are kept intact but observations are replaced by other values (Hlasny and Verme, 2017). Trimming and truncation of non-positive incomes are extreme examples of reweighting where weights are set to zero, a practice adopted by Eurostat and many scholars (Bavier, 2008, for example). Reweighting observations using the inverse probability of non-response is sometimes used to correct for self-selected missing observations (Korinek et al., 2007). Censoring, bottom coding or winsorizing are examples of replacing practices (for example, Eurostat, 2006, Gottschalk and Smeeding, 2000, and Van Kerm, 2007) where bottom observations are replaced by some moment of the income distribution, a minimum such as the trimming threshold, or zeroes. Using OECD income data, for instance, Thewissen et al. (2018) advise to set negative incomes to zero, because bottom-coding them is more appropriate than deleting them outright. Replacing can also be implemented by imputing non-positive and missing incomes using other variables present in the survey (Gottschalk and Smeeding, 2000). Sandoval and Urzua (2009) proposed replacing households' negative incomes with their expenditures, noting that some households reporting non-positive incomes show other indicators of well-being that are more consistent with non-poor households. Only a few scholars, such as Raffinetti et al. (2017), retain non-positive incomes as an accurate representation of bottom incomes.

The enduring problem with these classical approaches to non-positive incomes is that they do not use all information available within or out of surveys to assess well-being, they do not replace unreliable zero or negative incomes with more realistic values, and they produce income distributions that are truncated or have discontinuous point-mass at the bottom (Ostasiewicz and Vernizzi, 2017). These classical corrections should thus be complemented by more advanced reweighting and replacing methods.

Correction methods also differ according to the source of inputs used for reweighting or replacing – within-survey or out-of-survey data supplementation (Hlasny and Verme, 2017). For example, among parametric-modeling studies, Székely et al. (2007) used regression methods based on within-survey personal and household characteristics to predict incomes of households reporting missing or zero incomes. Other studies have looked outside of the survey of interest, and attempted to link problematic incomes in a budget survey using another survey with higher availability or reliability of income proxies such as population censuses or tax records (Bourguignon, 2018; Flachaire et al., 2021). Linking of Estonian income survey data with tax records revealed that employment incomes at the bottom of the distribution are particularly affected by tax evasion, resulting in underreporting of true earnings by 17% of the surveyed population (Paulus, 2015). Linking of US income survey to food-stamp administrative data showed that social assistance failed to be reported in the survey by over one-third of housing-assistance recipients, 40% of food-stamp recipients and 60% of general-assistance recipients, resulting in sharply underestimated bottom incomes (Meyer and Mittag, 2019).

Other out-of-survey replacing studies use statistical distribution functions to identify regularities and replace bottom incomes with values from parametric distributions. Van Kerm (2007), and Cowell and Van Kerm (2015) fitted an inversed Pareto distribution to negative values. Evidence from studies of wealth is also relevant. Dagum (1999), Jenkins and Jãntti (2005), and Jãntti et al. (2015) proposed fitting an exponential distribution to negative data using a point-mass for zero incomes. Clementi et al. (2012) proposed the Weibull model. Gao and Hu (2013) proposed the exponential distribution for negative pretax quarterly business incomes at the onset of economic crises and a heavier power distribution such as the Pareto during recessions. Cowell and Flachaire (2015) proposed the use of influence functions to characterize the sensitivity of well-being measures to extreme values, and showed how this method can be used for inference with missing, censored and truncated data.

Other methods aimed at adjusting problematic extreme observations include matching and machine learning methods such as random forest. Blundell and Costa Dias (2009), and Ceriani et al. (2019) proposed matching estimators to assess the accuracy of components of the welfare aggregate. Hlasny et al. (2021) pursued parametric modeling as well as random forest imputation of incomes using households' characteristics, where non-positive incomes represent the treated group and positive incomes the non-treated group. Machine learning algorithms can improve the accuracy of imputation, and random forest in particular has been very effective in prediction exercises as compared to standard econometric models (Haziza and Beaumont, 2007; Zabala, 2015; Athey and Imbens, 2019). This also holds for poverty predictions as shown by a recent World Bank experiment (Fitzpatrick et al., 2018).

Of course, correction methods carry their own pitfalls, and no method can be considered as the gold standard for adjusting negative incomes. The data and negative incomes at hand play a role in the selection of the correction method. This points to the importance of analyzing negative incomes to understand their sources and understand which observations can be safely assimilated to poor households and which cannot.

5. THE IMPACT OF NON-POSITIVE INCOMES ON POVERTY MEASUREMENT

The choice of retaining non-positive incomes may have non-trivial effects on the poverty measures. Cowell and Victoria-Feser (1996) found that most poverty measures are robust to data contamination provided that the income distribution is bounded at the bottom. When it is not because of negative incomes, measurement issues emerge.

Some poverty indices are not defined on negative and zero incomes. For instance, the poverty index introduced by Watts (1964) is defined on a logarithmic transformation of the censored (to the poverty line) income distribution and is not defined on negative and zero incomes. The Foster, Greer and Thorbecke (FGT) poverty measures (Foster et al., 1984) do not pose computational issues, but may behave aberrantly if negative incomes are retained. Consider the FGT poverty measures' standard formulation:

$$FGY_{\alpha} = \frac{1}{N}\sum_{i=1}^{H}\left(\frac{z-y_i}{z}\right)^{\alpha},$$

where *N* is the population, *z* the poverty line, *H* the population below the poverty line, *y* is income and α is a parameter representing inequality aversion.

The first observation is that the poverty rate in the presence of zero or negative incomes simply classifies these incomes as any other income below the poverty line and the corresponding households are classified as poor. With $\alpha = 1$ or $\alpha = 2$, and some $y_i = 0$, the poverty gap and the severity of poverty indexes increase as compared to distributions with positive incomes only. That is because the sum of the distances from the poverty line becomes 1 for all households with zero income. The normalized poverty gap becomes even larger than 1 for households with negative incomes. This increases the poverty gap and the severity of poverty indexes as compared to distributions with no negative values.

More aberrantly, the one-to-one relationship between poverty orderings and the α-degree stochastic dominance of partial orderings (Foster and Shorrocks, 1988) does not hold anymore. If one income distribution has unambiguously less poverty according to FGT_1, the same may not be true using FGT_2 and FGT_3 when income distributions have negative incomes. Also, it may be that $FGY_\alpha(z_1) > FGY_\alpha(z_2)$ when $z_1 < z_2$. With a higher poverty threshold, poverty might be lower when there are negative incomes, since $\frac{z_1-y}{z_1} > \frac{z_2-y}{z_2}$ if $y < 0$ (Sandoval and Urzua, 2009). The presence of negative or zero incomes can make poverty measurement impossible or alter its ordering.

6. THE IMPACT OF CORRECTION METHODS ON POVERTY MEASUREMENT

Bottom-coding negative incomes at zero has no effect on the poverty rate because this correction replaces incomes below a poverty threshold with other values below this threshold. However, the poverty gap and severity measures are reduced as compared to the uncorrected distributions. This can be good or bad depending on whether negative values are true representation of household well-being. If they are, these corrections underestimate the poverty gap and severity indexes. If they are not, they overestimate these measures but less than the original situation with negative values. This correction is presumably the least harmful as it is most likely that some negative values are proper representations of household well-being and others are not.

Truncating negative incomes – compared to bottom-coding at zero or the original untouched distribution – reduces the poverty headcount and the other FGT poverty measures. Truncating zero incomes reduces poverty further. Without any knowledge on the capacity of negative and zero incomes to represent true well-being, this practice most likely leads to a significant underestimation of poverty. Unfortunately, this practice is popular among practitioners who use it as a standard data cleaning method.

Fitting parametric functions to bottom incomes has been attempted by a few scholars with mixed results (Dagum, 1999; Jenkins, 2005; Van Kerm, 2007, Jäntti, 2015; Cowell and Flachaire, 2015). The Pareto distribution, for example, does not fit bottom observations well whereas generating observations from parametric distributions remains a theoretical exercise because no counterfactual of true income values is available to researchers. Irrespective of the parametric function of choice, this approach may result in poverty rates below those generated with negative incomes if households with these incomes were all classified as poor and the parametric function replaces their incomes with positive values. If the parametric function can generate negative values, this approach may result in the same or lower poverty rates as the ones generated by non-corrected distribution.

The search for an optimal parametric function at the bottom should not focus on bottom incomes (as the top incomes literature focusing on representing top incomes) but should focus on capturing household well-being accurately. The problem at the bottom is not so much related to missing incomes (as for top incomes) but to the fact that incomes at the bottom are not good proxies of household well-being. Therefore, the search of an optimal distribution function should aim at capturing true household well-being rather than incomes.

This leads to the question of conditioning parametric functions to other variables present in surveys (even when income is not) that proxy household well-being such as education, housing, assets and others. These variables can provide a counterfactual for households reporting negative or zero incomes. This can be done with standard OLS regression by predicting income based on other household characteristics, or methods such as propensity score matching or random forest. These methods may result in the identification of some households reporting negative incomes as “non-poor” and others as “poor.” The result is a set of poverty measures showing lower poverty than the original distributions.

The churning in the identification of poor individuals following the parametrization at the bottom may lead to changes in the poverty line, particularly if the poverty line is defined in relative terms. If some very low incomes end up being replaced by higher values, the poverty line may increase. The simultaneous increase in some very low incomes and the poverty line has an indeterminate effect on the poverty rate.

Finally, any modification of the income distribution at the bottom (including bottom coding, truncation, parametrization) may result in a different poverty profiling, with important consequences for policy, e.g. targeting.

7. CONCLUSION

Income is not a good metric to measure household well-being or rank households when incomes are non-positive. Poverty measures are designed to capture household well-being, and well-being can be measured with a variety of metrics of which income is only one. For at least some categories of households, well-being is not properly measured by income and can be better assessed using alternative metrics such as consumption or expenditure, or by adjusting income using other proxies of well-being. Indeed, in low and middle-income countries, poverty is almost invariably measured with consumption or expenditure, which cannot be negative and can hardly be nil.

To the extent that households with negative incomes and those with zero incomes are estimated to exhibit different socio-economic status, they must be treated using separate correction approaches. Moreover, since missing incomes (unit or item nonresponses, or statistical agency truncations) are also likely to be selected based on households' means, and to comprise poor households, correcting for them may lead to correcting some of the exclusion error in poverty measurement.

A grey area for poverty measurement also involves very low positive incomes. These should again be distinguished from negative or zero incomes because they are less likely to result from adjustments by statistical agencies. Similarly to negative incomes, these incomes could be artificially low because of accounting practices and may represent the non-poor, like rich people reporting low income for tax or alimony avoidance purposes. The question is where to draw the line between incomes that proxy well-being reasonably well and those that do not, a question that in our knowledge has not been addressed. Extending the analysis to a greater range of bottom incomes – the extreme 5-10 percent, or incomes falling short of household consumption – promises to yield more determinate corrections. These corrections may provide a dynamic benefit of reduced volatility of poverty indexes, and may relate better to multidimensional poverty indexes as these indexes typically contain some of the proxies from surveys such as education.

The policy implications of this research are clear. Our results are relevant for the assessments of poverty depth, fiscal redistribution, aid targeting, or tax evasion. Since the problems of poverty and unequal economic opportunities have been linked to civil discontent in some world regions, a better understanding of the scale and character of these problems can help informing policies aimed at social justice.

References

Athey, S., & Imbens, G. (2019). Machine learning methods economists should know about. *arXiv* (1903.10075).

Bavier, R. (2008). Reconciliation of income and consumption data in poverty measurement. *Journal of Policy Analysis and Management*, *27*(1), 40–62.

Blundell, R., & Costa Dias, M. (2009). Alternative approaches to evaluation in empirical microeconomics. *Journal of Human Resources, 44*(3), 565–640.

Bourguignon, F. (2018). Simple adjustments of observed distributions for missing income and missing people. *Journal of Economic Inequality*, *16*(2), 171-188.

Brewer, M., Etheridge, B., & O'Dea, C. (2017). Why are households that report the lowest incomes so well-off?. *Economic Journal, 127*, F24-F49. https://doi.org/10.1111/ecoj.12334

Ceriani, L., Olivieri, S., & Ranzani, M. (2019). Evaluating the accuracy of homeowner self-assessed rents in Peru. Policy Research Working Paper, 8983, World Bank, Washington, DC.

Clementi, F., Gallegati, M., & Kaniadakis, G. (2012). A generalized statistical model for the size distribution of wealth. *Journal of Statistical Mechanics: Theory and Experiment*, P12006.

Cowell, F. A., & Flachaire, E. (2015). Statistical methods for distributional analysis. In A. B. Atkinson and F. Bourguignon (Eds.), *Handbook of Income Distribution*, Vol 2, pp. 359–465. Elsevier.

Cowell, F. A., & Van Kerm, P. (2015). Wealth inequality: A survey. *Journal of Economic Surveys*, *29*(4), 671–710.

Cowell, F. A., & Victoria-Feser, M.-P. (1996). Poverty measurement with contaminated data: A robust approach. *European Economic Review*, *40*(9), 1761–1771.

Dagum, C. (1999). A study on the distributions of income, wealth and human capital. *Revue Europeenne des Sciences Sociales*, *37*(133), 231–268.

Eurostat (2006). Some proposals on the treatment of negative incomes. EU-SILC Documents TFMC-15/06, European Commission.

Fitzpatrick, C. A., Bull, P., & Dupriez, O. (2018). Machine learning for poverty prediction: A comparative assessment of classification algorithms. Technical report, World Bank Knowledge for Change (KCP) Program.https://github.com/worldbank/ML-classification-algorithms-poverty/tree/master/competition_winning_models_malawi/malawi/1st%20Place .

Flachaire, E., Lustig, N., & Vigorito, A. (2021). Underreporting of top incomes and inequality: An assessment of correction methods using linked survey and tax data. CEQ Institute working paper.

Foster, J., Greer, J., & Thorbecke, E. (1984). A class of decomposable poverty measures. *Econometrica, 52*, 761–765.

Foster, J., & Shorrocks, A. (1988). Poverty orderings and welfare dominance. *Social Choice and Welfare, 5*(2/3), 179-198.

Gao, J., & Hu, J. (2014). Financial crisis, Omori's law, and negative entropy flow. *International Review of Financial Analysis*, *33*, 79–86.

Gottschalk, P., & Smeeding, T. M. (2000). Empirical evidence on income inequality in industrialized countries. Ch.5 in A.B. Atkinson and F. Bourguignon (eds.) *Handbook of Income Distribution 1*, pp. 261–307.

Haziza, D., & Beaumont, J.-F. (2007). On the construction of imputation classes in surveys. *International Statistical Review*, *75*, 25–43.

Hlasny, V., Ceriani, L., & Verme, P. (2021). Bottom incomes and the measurement of poverty and inequality. *Review of Income and Wealth*, forthcoming.

Hlasny, V., & Verme, P. (2017). The impact of top incomes biases on the measurement of inequality in the United States. Working Papers 452, ECINEQ, Society for the Study of Economic Inequality.

Jãntti, M., Sierminska, E. M., & Van Kerm, P. (2015). Modeling the joint distribution of income and wealth. Measurement of Poverty, Deprivation, and Economic Mobility. *Research on Economic Inequality*, *23*, 301–327.

Jenkins, S. P., & Jãntti, M. (2005). Methods for summarizing and comparing wealth distributions. Working Paper 2005-05, ISER.

Korinek, A., Mistiaen, J. A., & Ravallion, M. (2007). An econometric method of correcting for unit nonresponse bias in surveys. *Journal of Econometrics*, *136* (1), 213–235.

Meyer, B. D., & Mittag, N. (2019). Using linked survey and administrative data to better measure income: Implications for poverty, program effectiveness and holes in the safety net. *American Economic Journal: Applied Economics*, *11*(2), 176-204.

OECD (2021). OECD Income Distribution Database (IDD). Country-specific detailed information for latest year and Wave 7, https://www.oecd.org/social/income-distribution-database.htm

Ostasiewicz, K., & Vernizzi, A. (2017). Decomposition and normalization of absolute differences, when positive and negative values are considered: applications to the Gini coefficient. *Quantitative Methods in Economics*, *18*(3), 472–491.

Paulus, A. (2015). Tax evasion and measurement error: an econometric analysis of survey data linked with tax records. Institute for Social & Economic Research Working Paper 2015-10.

Raffinetti, E., Siletti, E. & Vernizzi, A. (2017). Analyzing the effects of negative and non-negative values on income inequality: Evidence from the Survey of Household Income and Wealth of the Bank of Italy (2012). *Social Indicators Research, 133*, 185–207.

Ravallion, M. (2017). A concave log-like transformation allowing non-positive values. *Economics Letters, 161*, 130–132.

Rawal, V., Swaminathan, M., & Dhar, N. S. (2008). On diversification of rural incomes: A view from three villages of Andhra Pradesh. *Indian Journal of Labour Economics*, *51*(2), 229–248.

Sandoval, H. H., & Urzua, C. M. (2009). Negative net incomes and the measurement of poverty: A note. *Revista de Administración, Finanzas y Economía*, *3*(1), 29–36.

Schutz, R. R. (1951). On the measurement of income inequality. *American Economic Review*, *41*, 107–122.

Székely, M., Lustig, N., Cumpa, M., & Mejía-Guerra, J. A. (2007). Do we know how much poverty there is?. *Oxford Development Studies*, *32*(4), 523–558.

Stich, A. (1996). Inequality and negative income. *Journal of the Italian Statistical Society*, *5*, 297–305.

Thant, P. P., & Calkins, P. (2009). Measuring poverty and income inequality under negative incomes: A case study from Myanmar. Chiang Mai University Journal of Economics, 13(1), 86–119.

Thewissen, S., Kenworthy, L., Nolan, B., Roser, M., & Smeeding, T. (2018). Rising income inequality and living standards in OECD countries: How does the middle fare?. *Journal of Income Distribution*, *27*(2), 1–23.

Van Kerm, P. (2007). Extreme incomes and the estimation of poverty and inequality indicators from EU-SILC. Working Paper 2007-01, CEPS-Instead IRISS.

Verma, V., & Betti, G. (2010) Data accuracy in EU-SILC (2010). Chapter 3 in Atkinson and Marlier (EDS.) Income and Living Conditions in Europe, Eurostat Statistical Book, Luxembourg.

Watts, H. W. (1964). An economic definition of poverty. Institute for research on poverty. Madison, WI: University of Wisconsin.

Zabala, F. (2015). Let the data speak: Machine learning methods for data editing and imputation. Working Paper 31, Conference of European Statisticians, United Nations Economic Commission for Europe.