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Abstract

This paper proposes a graphical tool based on the copula function, namely the multivariate tail concentration function, to represent the dependence structure on the tails of a multivariate joint distribution. We illustrate the use of this function to measure dependence between poverty dimensions. In particular, we analyse how multivariate tail dependence between the dimensions of the AROPE rate evolved in the EU-28 between 2008 and 2018. We find evidence of lower tail dependence in all EU-28 countries, although this dependence is time-varying over the period analysed and the effect of the Great Recession on this dependence is not homogeneous over all countries.

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1 Introduction

There is a broad consensus that poverty is a multidimensional phenomenon. This has led, over the last years, to a large and still growing literature on multidimensional poverty indices, which constitute the dominant approach to study poverty in a multivariate framework. Actually, some international institutions have already adopted this approach. For instance, the United Nations Development Program adopted, in 2010, the Multidimensional Poverty Index, which considers three dimensions: education, health and standard of living. In the European Union (EU) poverty is monitored through the AROPE (At Risk Of Poverty or social Exclusion) rate, which also considers three dimensions: income, work intensity and material deprivation. However, this type of indices are not sufficiently sensitive to an important aspect of multidimensional poverty, namely the degree of dependence between the dimensions; see Duclos and Tiberti (2016) or Seth and Santos (2019). To overcome this drawback, some authors have proposed to complement the information given by those indices with copula-based measures of multivariate dependence between dimensions. Our paper fits into this stream of research, which started with the seminal work of Decancq (2014) and the follow-up papers of Pérez and Prieto (2015) and Pérez and Prieto-Alaiz (2016).

To understand the importance of multivariate dependence when analysing multidimensional poverty, consider the following example. Imagine a society, say Society A, where one individual scores the highest in all dimensions of poverty, another individual scores the second highest in all dimensions and so on, until the last individual, who scores the lowest in all dimensions. On the other hand, consider another society, say Society B, with exactly the same distributional profile in each dimension but where individuals that score low in some dimensions can attain high positions in other dimensions. Then, in Society A there is arguably more concentration of deprivation than in Society B, which could exacerbate overall poverty; see Atkinson and Bourguignon (1982) and Bourguignon and Chakravarty (2003). That is, for two different societies the marginal distributions of the achievements may be the same but their joint distributions may differ, leading to different degrees of multivariate dependence, which, as Chakravarty (2018, ch.1) argues, is an intrinsic characteristic of the notion of multivariate analysis. Hence, in order to appropriately account for the multivariate nature of poverty, the analysis must be sensitive to the association between its dimensions.

To face this goal, in this paper we focus on one aspect of multivariate dependence, namely the dependence in the tails of the joint distribution, and we pay special attention to multivariate lower tail dependence. In a poverty setting, this concept becomes specially important, since it captures how likely it is that an individual who is extremely low-ranked in one poverty dimension, say income, is also extremely low-ranked in the rest of the dimensions considered. The measures we propose are based on transforming the outcomes of one individual in all poverty dimensions into the positions of this individual across dimensions as compared to other individuals. In doing so, the joint distribution of the transformed variables turns out to be a copula, and tail dependence coefficients based on copulas can be applied. The copula approach allows the construction of multivariate generalizations of bivariate scale-free measures of dependence which are appropriate in non-Gaussian and possibly non-linear contexts, such as the ones we face in multidimensional poverty analysis.

Despite its theoretical appeal and its popularity in fields such as finance (Caillault and Guégan, 2005; Reboredo et al., 2015; Matkovskyy, 2019) or environmental sciences (Aghakouchak et al., 2010; Serinaldi et al., 2015), the concept of tail dependence has only recently been applied in welfare economics in D'Agostino et al. (2022), who restricts their analysis to the bivariate setting. Therefore, to the best of our knowledge, our paper provides a pioneering contribution in this field as it faces, for the first time, the measurement of multidimensional tail dependence with an application to poverty analysis. In particular, the following questions will be addressed. Is there multivariate tail dependence between the poverty dimensions in the EU-28? If so, is this dependence symmetric or asymmetric, i.e, is it different in the lower and the upper tails? Is it similar in all the EU-28 countries? And finally, has multivariate tail dependence between poverty dimensions changed since the Great Recession? By answering these questions, we hope to shed light on the complex nature of poverty and its possible implications for policy makers.

Our work builds on previous related work that uses copula-based techniques to measure dependence between poverty dimensions. For instance, García-Gómez et al. (2021) analysed the evolution of multivariate dependence between the dimensions of the AROPE rate in the EU-28 over the period 2008-2014, but using orthant dependence measures. Those measures are based on averaging the departure from independence throughout the whole distribution, whereas in the paper at hand we focus on a particular part of the joint distribution, namely the tails, and the coefficients used are based on conditional probabilities rather than on averages. In this sense, the current paper complements the results in García-Gómez et al. (2021) by dealing with a different aspect of multivariate dependence and covering a broader and more recent period of time. The aforementioned paper by D'Agostino et al. (2022) is closest to the current one in terms of the dependence measures used. These authors provide a novel application of tail dependence concepts in poverty analysis but limited to a bidimensional setting. In particular, the authors use a semi-parametric copula approach to estimate lower tail dependence between pairs of dimensions of the AROPE rate in Europe in 2009 and 2018. Our paper generalises this work by incorporating a multidimensional approach and provides a more comprehensive picture of the evolution of poverty in the EU-28 by covering years 2008, 2014 and 2018. Also, the estimation method we use is different, as we carry out a fully non-parametric estimation.

The contribution of this paper is twofold. From a methodological perspective, we propose the multivariate tail concentration function (TCF), based on the work of Venter (2001) and Durante and Sempi (2015) for the bivariate case, as a graphical tool to analyse the degree of multivariate tail dependence between the dimensions of poverty. As we will see, this function has several advantages. In spite of its multivariate nature, it allows to represent, in a bidimensional unit square, the degree of multivariate dependence in both tails of the joint distribution, regardless of the number of dimensions considered. Moreover, it avoids the cumbersome task of estimating asymptotic tail dependence coefficients. From an empirical perspective, our paper illustrates the use of the multivariate TCF by analysing the evolution of tail dependence between the three dimensions of the AROPE rate in the EU-28 over the period 2008-2018, with special attention to lower tail dependence. We find evidence of lower tail dependence in all EU-28 countries, although this dependence is time-varying over the period analysed and the effect of the Great Recession on this dependence is not homogenous over all countries.

The rest of the paper is structured as follows. Section 2 summarises the basic properties of copulas. It also introduces the concept of multivariate tail dependence and the multivariate TCF and discusses how to estimate this function non-parametrically. Section 3 is devoted to the empirical application on the evolution of multivariate tail dependence between poverty dimensions in Europe over the period 2008-2018. Finally, Section 4 concludes the paper with a summary of the main results.

2 Methodology

As we pointed out above, one key aspect of multidimensional poverty traditionally overlooked in the literature is the interdependence between the different dimensions. In particular, a higher degree of dependence means higher concentration of deprivations and this could make overall poverty worse; see Atkinson and Bourguignon (1982) and Bourguignon and Chakravarty (2003). Hence, to have a complete picture of poverty, it is necessary to incorporate the analysis of multivariate dependence between its dimensions. However, the measurement of dependence in a fully multidimensional setting, that is, when more than two variables are considered, is challenging and requires special care, since some bivariate dependence properties are not preserved in higher dimensions; see Durante et al. (2014).

In this paper, we propose to measure the degree of multivariate dependence between poverty dimensions using the copula methodology. This methodology allows to study several concepts of multivariate dependence that go beyond the widely known notion of linear correlation, which is only appropriate for measuring bivariate linear relationships in the context of elliptical distributions. In particular, we focus on the concept of tail dependence, which relates to the degree of dependence in the joint (lower or upper) tail of a multivariate distribution, that is, the dependence between extreme events. In a multidimensional poverty setting, this concept, and specifically that of lower tail dependence, becomes specially relevant, since it captures the probability that an individual who is extremely low-ranked in one poverty dimension is also extremely low-ranked in the other dimensions considered. In this section, we summarise some basic concepts on the copula function and discuss the concept of tail dependence, starting with the bivariate case and then moving to the more challenging and scarcely addressed multivariate framework. For a review of other concepts of multivariate dependence, such as orthant dependence or multivariate concordance, see Schmid et al. (2010) and Joe (2014). For applications of these concepts in welfare economics see Decancq (2020), Matkovskyy (2020), Terzi and Moroni (2020) and Tkach and Gigliarano (2020) and García-Gómez et al. (2021) and the references therein.

2.1 Copulas: basic concepts

The copula approach focuses on the positions of the individuals across the variables, rather than on the values that these variables attain for such individuals. In particular, let the random vector $\mathbf{X} = (X_1, \ldots, X_d)$ represent the relevant d dimensions of poverty and let F_i denote the marginal distribution of dimension i, with $i = 1, \ldots, d$. Then, each original variable X_i is transformed by applying the so-called *probability integral transformation*, obtaining a transformed variable $U_i = F_i(X_i)$, with $i = 1, \ldots d$. These transformed variables attach to each individual in the population its relative position in all dimensions. For instance, an individual with position vector $(1, \ldots, 1)$ will be top-ranked in all dimensions, i.e., he/she will be the "richest" one in terms of income, health, education, etc. From probability theory, the transformed variables $U_1, \ldots, U_d$ are standard uniform random variables $U(0,1)$ and the joint distribution of the vector $\mathbf{U} = (U_1, ..., U_d)$ turns out to be the copula function C. Therefore, the copula is a d-dimensional cumulative distribution function, $C : \mathbf{I}^d \rightarrow \mathbf{I}$, with $\mathbf{I} = [0,1]$, whose univariate marginals are $U(0,1)$. So, for a given real vector $\mathbf{u} \in \mathbf{I}^d$, the value $C(\mathbf{u})$ represents the proportion of individuals in the population with positions outranked by $\mathbf{u}$, i.e, $C(\mathbf{u}) = p(\mathbf{U} \leq \mathbf{u}) = p(U_1 \leq u_1, \ldots, U_d \leq u_d)$. For instance, $C(0.2, ..., 0.2)$ will represent the probability that a randomly selected individual is simultaneously in the 1^{st} quintile ("low-ranked") in all dimensions.

From an statistical point of view, the most important result of the theory of copulas is given by Sklar's theorem (Sklar, 1959). This theorem establishes that, given a $d-$dimensional random vector $\mathbf{X} = (X_1, ..., X_d)$ with joint distribution function $F(\mathbf{x}) = F(x_1, ..., x_d) = p(X_1 \leq x_1, ..., X_d \leq x_d)$ and univariate marginal distribution functions $F_i(x_i) = p(X_i \leq x_i)$, for $i = 1, ..., d$, then there exists a copula $C : \mathbf{I}^d \rightarrow \mathbf{I}$ such that, for all $(x_1, \ldots, x_d) \in R^d$,

$$F(x_1, ..., x_d) = C(F_1(x_1), ..., F_d(x_d)). \tag{1}$$

Conversely, if C is a $d-$copula and $F_1, ..., F_d$ are univariate distribution functions, the function F defined in (1) is a joint distribution function with margins $F_1, ..., F_d$. Thus, copulas link joint distribution functions to their univariate marginals. If $F_1, ..., F_d$ are all continuous, the copula C in (1) is unique. Otherwise, C is uniquely determined on Range $F_1 \times ... \times$ Range F_d. Over the rest of this section we will assume that the marginal distributions $F_1, ..., F_d$ are all continuous, although some issues arising when dealing with possibly non-continuous variables will be duly pointed out in subsection 2.4. For a detailed discussion on the pitfalls related to non-continuity of the marginal distributions, see Genest and Nešlehová (2007) and the references therein.

Two particularly important copulas are worth mentioning. First, the independent copula, $\Pi(\mathbf{u}) = u_1 \times \cdots \times u_d$, which accounts for the case where the variables $X_1, ..., X_d$ are independent. Second, the commonotonic copula, $M(\mathbf{u}) = \min(u_1, \ldots, u_d)$, which represents maximal dependence, that is, when the outcomes in all dimensions are ordered in the same way. Another important function, which is not a copula itself but it is related to the copula C, is the survival function, $\bar{C} : \mathbf{I}^d \to \mathbf{I}$, defined as:

$$\bar{C}(\mathbf{u}) = p(\mathbf{U} > \mathbf{u}) = p(U_1 > u_1, \ldots, U_d > u_d),$$

where $\mathbf{U} = (U_1, ..., U_d)$ is a random vector of variables $U(0,1)$ whose joint distribution function is the copula C. In our setting, the survival function accounts for the probability of being simultaneously "rich" in all dimensions. For instance, $\bar{C}(0.8, \ldots, 0.8)$ will represent the probability that a randomly selected individual is simultaneously in the 5^{th} quintile ("high-ranked") in all poverty dimensions.

2.2 Bivariate tail dependence

In the bivariate case, tail dependence measures the degree of dependence in the lower tail or the upper tail of a bivariate distribution. Although several ways of measuring tail dependence have been proposed, the most commonly used measures are the tail dependence coefficients introduced by Sibuya (1960), which are defined as follows. Given a bivariate random vector $\mathbf{X} = (X_1, X_2)$ with joint distribution function F, marginal distribution functions F_1 and F_2 and copula C, the lower tail dependence coefficient of X_1 and X_2 is defined as

$$\lambda_L = \lim_{u \to 0^+} Pr\big(X_2 \leq F_2^{-1}(u) | X_1 \leq F_1^{-1}(u)\big) = \lim_{u \to 0^+} \frac{C(u,u)}{u}, \tag{2}$$

and the upper tail dependence coefficient is given by

$$\lambda_U = \lim_{u \to 1^-} Pr\big(X_2 > F_2^{-1}(u) | X_1 > F_1^{-1}(u)\big) = \lim_{u \to 1^-} \frac{\bar{C}(u,u)}{1-u}; \tag{3}$$

provided that the limits above exist. Thus, the lower tail dependence coefficient in (2) gives the asymptotic probability that a random variable becomes small, given that another random variable is also small. Similarly, the upper tail dependence coefficient in (3) gives the asymptotic probability that a random variable exceeds a high quantile, given that another random variable also exceeds such quantile. In our setting, if $\mathbf{X} = (X_1, X_2)$ represents two relevant dimensions of poverty for a population, say income and health, the lower tail dependence coefficient λ_L is the most relevant one, since it would measure the limit probability that an individual is extremely low-ranked in one dimension (i.e, income) given that he/she is extremely low-ranked in the other dimension (i.e, health).

The coefficients λ_L and λ_U are bounded between 0 and 1 inclusive. In particular, if $C = \Pi$, that is, if X_1 and X_2 are independent, then $\lambda_L = \lambda_U = 0$ and if $C = M$, that is, if X_1 is a strictly increasing function of X_2 (or vice versa), then $\lambda_L = \lambda_U = 1$. Moreover, according to Joe (2014), we say that C has lower (respectively, upper) bivariate tail dependence if $\lambda_L > 0$ (respectively, $\lambda_U > 0$), whereas if $\lambda_L = 0$ (respectively, $\lambda_U = 0$), we say that C has no lower (respectively, upper) bivariate tail dependence. Theoretical values of λ_L and λ_U have been derived for the most popular families of parametric copulas; see, for example, Joe (1997), Malevergne and Sornette (2006) and Nelsen (2006).

Since the tail dependence coefficients are by definition asymptotic measures, their estimation is not straightforward. In fact, as Joe (2014) argues, the empirical measure of tail dependence for data does not really exist because of the limit and the best that can be done is to apply estimation procedures, either parametric or non-parametric. In a non-parametric framework, we can mention the proposals of Huang (1992), Joe et al. (1992), Capéraà et al. (1997), Frahm et al. (2005), Schmidt and Stadtmüller (2006) and Schmid et al. (2010). For parametric techniques, we refer the interested reader to Frahm et al. (2005) and Supper et al. (2020) and the references therein. Some empirical applications of bivariate tail dependence coefficients can be found in Caillault and Guégan (2005), Reboredo et al. (2015) or Matkovskyy (2019) in finance and in

Aghakouchak et al. (2010) or Serinaldi et al. (2015) in environmental sciences. See also the most recent work of D'Agostino et al (2022) in welfare economics.

2.3 Multivariate tail dependence

While bivariate tail dependence has been relatively well-studied, the multivariate framework has been scarcely addressed. One of the reasons for this is that, when going from the bivariate to the multivariate case, things become more difficult, from both a theoretical and a practical perspective. To start with, the presence of more than two dimensions brings additional difficulties to the definition of tail dependence. As a result, different proposals to measure multivariate tail dependence can be found in the literature; see, for example, Frahm (2006), Schmid and Schmidt (2007) and the discussion in Gijbels et al. (2020).

In this paper, we follow Hua and Joe (2011) and Joe (2014) and limit our attention to the following multivariate lower and upper tail dependence coefficients, respectively:

$$\lambda_L^d = \lim_{u \to 0^+} \frac{C(u, \ldots, u)}{u}, \tag{4}$$

$$\lambda_U^d = \lim_{u \to 1^-} \frac{\bar{C}(u, \ldots, u)}{1 - u}, \tag{5}$$

where C is a d-dimensional copula and $\bar{C}$ is its survival function. That is, we consider the probability that a group of $d-1$ variables take simultaneously very small (large) values, given that the remaining variable takes very small (large) values. In the case of independence, that is, if $C = \Pi$, $\lambda_L^d = \lambda_U^d = 0$, whereas in the case of maximal dependence, that is, if $C = M$, $\lambda_L^d = \lambda_U^d = 1$. Moreover, Fernández-Sánchez et al. (2016) show that, for any $d \geq 3$, the coefficients λ_L^d and λ_U^d are non-increasing in d. As expected, for $d = 2$, the coefficients in (5) and (6) reduce to those in (3) and (4), respectively.

Notice that, in our multidimensional poverty setting, the coefficient λ_L^d in (4) becomes specially relevant, since it measures the probability that an individual is simultaneously extremely low-ranked in $d-1$ relevant dimensions of poverty given that he/she is extremely low-ranked in the remaining dimension. For example, if we consider that income, education and health are the three dimensions of poverty, λ_L^d measures the limit probability that an individual that is extremely low-ranked in one dimension, say income, is also simultaneously extremely low-ranked in both education and health.

As in the bivariate case, the multivariate tail dependence coefficients defined above only give an asymptotic approximation of the behaviour of the copula in the joint tail of the distribution. Some authors argue that this is a rather limited notion of dependence and propose going beyond this asymptotic definition, by considering the tail behaviour at some finite points near the corners of the hypercube, which can be more informative. To that aim, Sweeting and Fotiou (2013) consider the following functions:

$$\lambda_L^d(u) = \frac{C(u,\dots,u)}{u}, \tag{6}$$

$$\lambda_U^d(u) = \frac{\bar{C}(u,\dots,u)}{1-u}, \tag{7}$$

for any $u \in (0,1)$. These functions allow to study multivariate tail dependence at a sub-asymptotic level and are a multivariate generalisation of the functions proposed by Venter (2001) and Manner and Segers (2011) for the bivariate case.

In this paper, we propose to merge the functions defined in (6) and (7) into a unique function that provides information about the degree of multivariate dependence in both the lower and the upper tail of the joint distribution. In particular, we define the multivariate tail concentration function (TCF) as the function $q_C^d : (0,1) \to \mathbf{I}$, given by:

$$q_C^d(u) = \frac{C(u,\dots,u)}{u}\mathbf{1}_{(0,0.5]} + \frac{\bar{C}(u,\dots,u)}{1-u}\mathbf{1}_{(0.5,1)} \tag{8}$$

where $\mathbf{1}_A$ denotes the indicator function on a set A. This is a multivariate generalisation of the function considered by Venter (2001), Patton (2013) and Durante et al. (2015) in a bivariate setting. However, the multivariate TCF is not necessarily continuous, since, with $d > 2$, $C(0.5, \ldots, 0.5)$ is not necessarily equal to $\bar{C}(0.5, \ldots, 0.5)$ and thus $q_C^d(0.5^-)$ does not necessarily coincide with $q_C(0.5^+)$. Moreover, when $C = M$, that is, in the case of maximal dependence, $q_M^d(u) = 1 \ \forall u$, and, in the case of independence, that is, when $C = \Pi$, then $q_\Pi^d(u) = u^{d-1}\mathbf{1}_{(0,0.5]} + (1-u)^{d-1}\mathbf{1}_{(0.5,1)}$.

The multivariate TCF in (9) constitutes a very powerful tool since, despite its multivariate nature, it allows to represent, in a bidimensional unit square, the degree of multivariate dependence in the joint tails of a multidimensional distribution, regardless of the number of dimensions considered.

To better appreciate the usefulness of this function, Figure 1 displays the trivariate TCF (q_C^3) for three different trivariate parametric copulas (the Clayton copula in the left panel; the Frank copula in the central panel; and the Gumbel copula in the right panel) for different degrees of dependence (given by the parameter θ); see Joe (2014) and Durante and Sempi (2015) for a description of these copulas. To ease the interpretation, in all panels, the trivariate TCF of the independent copula Π is represented in blue and the TCF of the commonotonic copula M is represented in red.

- INSERT FIGURE 1 HERE -

Several conclusions emerge from this figure. First, the three copula models are related to different shapes of the TCF. In particular, Clayton copulas (left panel) are asymmetric, showing more dependence in the lower tail than in the upper tail, since $q_C^3(u) > q_C^3(1-u)$ for $0 < u < 0.5$, that is, $\lambda_L^3(u) > \lambda_U^3(1-u)$. The contrary occurs in the case of Gumbel copulas (right panel), which displays more dependence in the upper tail than in the lower tail, since $q_C^3(u) < q_C^3(1-u)$ for $0 < u < 0.5$, that is, $\lambda_L^3(u) < \lambda_U^3(1-u)$. Finally, Frank copulas (central panel) show a rather symmetric behaviour, since $q_C^3(u) = q_C^3(1-u)$ for $0 < u < 0.5$, that is, $\lambda_L^3(u) = \lambda_U^3(1-u)$. Second, Figure 1 shows that, for the three models, as the degree of dependence increases (as the parameter θ increases), the trivariate TCF moves upwards and closer to the upper bound given by the maximal dependence copula M, whereas as dependence decreases (as θ decreases), the trivariate TCF approaches the lower bound corresponding to the trivariate TCF of the independent copula Π. Therefore, the position of the TCF wih respect to these two bounds gives important information on the degree of multivariate tail dependence.

2.4 Non-parametric estimation of the TCF

Now, the question arises on how to estimate the multivariate TFC in practice, since the copula C is unknown and must be estimated from the data. In order to do that, let $\{(X_{1j}, ..., X_{dj})\}_{j=1,...,n}$ be a sample of n serially independent random vectors from the d-dimensional vector $\mathbf{X} = (X_1, ..., X_d)$ with associated copula C. Then, it is possible to estimate non-parametrically the copula C by the corresponding empirical copula, namely

$$\hat{C}_n(\mathbf{u}) = \frac{1}{n}\sum_{j=1}^{n}\prod_{i=1}^{d}\mathbf{1}_{\{\widetilde{U}_{ij}\leq u_i\}}, \text{ for } \mathbf{u} = (u_1, ..., u_d) \in \mathbf{I}^d, \tag{9}$$

where $\mathbf{1}_A$ denotes the indicator function on a set A and $\widetilde{U}_{ij}$ are the transformed data to $[0,1]$ by scaling ranks, i.e.

$$\widetilde{U}_{ij} = R_{ij}/n,$$

where R_{ij} denotes the *rank* of X_{ij} among $\{X_{i1}, ..., X_{in}\}$, with $i = 1, ..., d$ and $j = 1, ..., n$. Similarly, the survival function $\bar{C}$ can be estimated non-parametrically by its corresponding empirical version, given by

$$\hat{\bar{C}}_n(\mathbf{u}) = \frac{1}{n}\sum_{j=1}^{n}\prod_{i=1}^{d}\mathbf{1}_{\{\widetilde{U}_{ij}> u_i\}}, \text{ for } \mathbf{u} = (u_1, ..., u_d) \in \mathbf{I}^d. \tag{10}$$

Following the work of Patton (2013) and Durante et al. (2015) in the bivariate setting, we propose to estimate the multivariate TCF by replacing in (8) both the copula and the survival function with their empirical counterparts in (9) and (10), respectively. In doing so, the empirical version of the multivariate TCF is given, for any $t \in (0,1)$, by:

$$\hat{q}^d_C(t) = \frac{\hat{C}_n(t,\ldots,t)}{t}\mathbf{1}_{(0,0.5]} + \frac{\hat{\bar{C}}_n(t,\ldots,t)}{1-t}\mathbf{1}_{(0.5,1)}. \tag{11}$$

That is,

$$\hat{q}^d_C(t) = \begin{cases} \frac{\hat{C}_n(t,\ldots,t)}{t}, & \text{if } t \in (0,0.5], \\ \frac{\hat{\bar{C}}_n(t,\ldots,t)}{1-t}, & \text{if } t \in (0.5,1). \end{cases}$$

In our framework, if we consider income, health and education as the three dimensions of poverty, evaluating this function at a given threshold, say $t = 0.2$, we can estimate the probability that an individual that is in the first quintile in income is also simultaneously in the first quintile in both health and education.

So far, we have assumed that the marginal distributions are continuous, which ensures the existence of a unique copula C in Sklar's Theorem in (1). However, as we will see later, in our empirical application we will have to deal with non-continuous marginals. In this setting, there is not a unique copula in (1) and the values of the measures of tail dependence previously discussed can vary widely even based on the same joint distribution. To overcome this issue, we will estimate the multivariate TCF by using the empirical checkerboard copula as proposed by Genest et al. (2017), which allows to consistently estimate the empirical copula process in the presence of non-continuous data.

3 Evolution of multivariate tail dependence between poverty dimensions in the EU-28 (2008-2018)

As we said in the Introduction, multidimensional poverty depends not only on the proportion of individuals deprived in each dimension but also on the degree of interdependence between dimensions, since higher dependence means higher concentration of deprivations and this could make overall poverty worse. Hence, to provide a more comprehensive picture of multidimensional poverty, in this paper we propose to complement the information given by traditional multidimensional poverty indices with measures of multivariate tail dependence between poverty dimensions. In particular, we apply the multivariate TCF introduced in the previous section to analyse the evolution of tail dependence between the dimensions of the AROPE rate in the EU-28 countries over the period 2008-2018.

3.1 Data and variables

The dimensions of poverty we consider are income, material needs and work intensity. These are the dimensions included in the AROPE rate, which is the headline indicator to monitor poverty and implement effective poverty-reduction policies in the EU in the framework of the Sustainable Development Goals.

The three measures characterising the dimensions of the AROPE rate are defined as follows:

- The measure of income is the equivalised disposable income, which is calculated as the total income of the household, after taxes and other deductions, divided by the equivalised household size.[1]
- The work intensity of a household is the ratio of the total number of months that all

[1]The equivalised household size is defined according to the modified OECD scale, which gives a weight of 1 to the first adult, 0.5 to other household members aged 14 or over and 0.3 to household members aged less than 14.

working-age household members have worked during the income reference year and the total number of months they could have theoretically worked during the same period.[2]

- Material deprivation is originally defined as the enforced lack in a number of essential items, namely: 1) the capacity of facing unexpected expenses; 2) one-week annual holiday away from home; 3) a meal involving meat, chicken or fish every second day; 4) an adequately warm dwelling; 5) a washing machine; 6) a colour television; 7) a telephone; 8) a car; 9) the capacity to pay their rent, mortgage or utility bills. For ease of interpretation we transform this variable into a variable that indicates the number of no-deprivations out of the nine possible, so that the new variable takes the following values: 0 (having all the 9 possible deprivations), 1 (having eight out of the nine aforementioned deprivations), ..., 9 (having no deprivations). Thus, high values of the three variables considered (equivalised disposable income, work intensity, and number of no-deprivations) convey lower chance to be poor, while low values of each variable convey higher chance to be poor.

The data we use comes from the EU-SILC survey, which is the key reference for data on income and living conditions in the EU. In particular, we use the cross-sectional surveys of years 2008, 2014 and 2018.

The unit of analysis is the household. We only work with subsamples of households for which we have complete information for all the three variables. In these subsamples, the sample sizes range from 2429 observations in Malta in 2018 to 14773 observations in Italy in 2008.[3]

3.2 Estimation results

In this section, we discuss the results on the evolution of multivariate tail dependence between the three dimensions of the AROPE rate in the countries of the EU-28 over the period 2008-

[2]Eurostat considers that a working-age person is a person aged 18-59 years, excluding also the students aged 18-24 years.

[3]We do not have data for Croatia in 2008 and for Ireland, Slovakia and the UK in 2018.

2018 based on the estimation of the multivariate TCF described in Section 2. Since we deal with non-continuous variables, we use the empirical checkerboard copula as proposed by Genest et al. (2017). Throughout this section, special emphasis will be given to multivariate lower tail dependence, which refers to the probability that a household that is very low-ranked in one of the dimensions is also very low-ranked in the rest of the dimensions considered.

Figure 2 displays, for the EU-28 countries, the estimated trivariate TCF for years 2008 (blue line), 2014 (red line) and 2018 (green line) together with 95% standard bootstrap confidence intervals using 1000 bootstrap replications. As a benchmark, the theoretical trivariate TCF of independence is displayed in black. The choice of these years allows us to study the change in multivariate tail dependence between the tree dimensions of the AROPE rate in the period of economic crisis (2008-2014) and also in the period of recovery (2014-2018). In all cases, the TCFs are calculated for $t \in [0.05, 0.95]$ over 100 points, so that the left part of the TCF (for $t \in (0, 0.5]$) accounts for lower tail dependence and its right part (for $t \in (0.5, 1)$) measures upper tail dependence.

- INSERT FIGURE 2 HERE -

Several conclusions emerge from Figure 2. First, there is clear evidence of lower tail dependence between poverty dimensions since, for the three years considered, the estimated TCFs are above the theoretical TCF of the independence case for $t < 0.5$. This means that, in the EU-28, there is a positive probability that a household that is low-ranked in one of the dimensions of the AROPE rate is also simultaneously low-ranked in the rest of the two dimensions. From a multidimensional poverty perspective, this result has important policy implications, since low positions in one dimension tend to extend to other dimensions, creating a vicious circle that exacerbates the poverty conditions of individuals. Second, Figure 2 reveals cross-country differences in the shapes of the TCF. On one hand, there are countries, such as Croatia, Cyprus, Hungary or Poland, where the curves are rather symmetric. This means that, in these countries, dependence in the lower tail of the joint distribution seems to be similar to dependence in its upper tail. On the other hand, in most of the EU countries (see, for instance, Austria, Belgium, Finland, France, Germany, Italy or Spain), the TCF is not symmetric, as observations in the lower tail are somewhat more dependent than observations in the upper tail, since $\hat{q}^3_C(t) > \hat{q}^3_C(1-t)$ for $0 < t < 0.5$. To better appreciate this feature, let us focus on Belgium. In this country, in 2008, the probability of being simultaneously in the first quintile in two poverty dimensions, given that you are in the first quintile in the other dimension, was $\hat{q}^3_C(0.2) = 0.4$, which is more than twice the probability of being in the last quintile in two poverty dimensions, given that you were in the last quintile in the other dimension, which was $\hat{q}^3_C(0.8) = 0.18$. Hence, in most of the EU-28 countries, the conditional probability of cumulative deprivation tends to be higher than the mirrored conditional probability of cumulative affluence.

Figure 2 also allows to analyse the temporal evolution of the patterns of multivariate tail dependence between 2008 and 2018. In this respect, it is worth noting that, for each country, the shape of its TCF hardly changed over this period, maintaining either the symmetry or asymmetry previously described. Nevertheless, we do find changes in the degree of multivariate tail dependence between poverty dimensions in the different EU-28 countries over the period analysed. If we focus on the changes in the levels of lower tail dependence, the main remarkable feature is that the Great Recession had a different impact in the EU-28 countries. In particular, we find countries where no significant changes in lower tail dependence can be appreciated over the whole period analysed, since the left part of the TCF of 2008 (blue line), 2014 (red line) and 2018 (green line) seem to be close together. This is the case of Belgium, Bulgaria, Finland, Germany, Latvia, Lithuania and Malta. However, we find many countries where the Great Recession led to an increase on lower tail dependence between 2008 and 2014, but display different patterns over the recovery period. For instance, in Italy and Greece, lower tail dependence between poverty dimensions increased between 2008 and 2014 but this increase was followed by a decrease over the period 2014-2018. As a result, in these countries the level of multivariate lower tail dependence between the AROPE rate dimensions in 2018 was similar to that observed in 2008. By contrast, we find many countries where the increase in lower tail dependence between 2008 and 2014 was not followed by a decrease afterwards. This is the case of Austria, Cyprus, Denmark, France, Hungary, the Netherlands, Poland, Portugal, Slovenia, Spain and Sweden. Actually, in these countries, lower tail dependence between poverty dimensions was still higher in 2018 than in 2008. Finally, we find three countries which do not fit in the profiles discussed so far. One of these countries is Romania, where lower tail dependence between poverty dimensions remained stable between 2008 and 2014 but increased afterwards. The other two countries are Estonia and Luxembourg, where lower tail dependence hardly changed between 2008 and 2014 and decreased between 2014 and 2018.

In order to get a better insight into the evolution of lower tail dependence, Table 1 provides the results of some tests for the significance of the changes in the trivariate TCF using the 1^{st} quintile as threshold. In particular, this table displays, in columns 1, 2 and 3, the estimated values (with bootstrap standard errors in parenthesis) of $q_C^3(0.2)$ for years 2008, 2014 and 2018, respectively. Columns 4, 5 and 6 present the results of a two-independent sample one-side t-test with unequal variances, calculated using bootstrap standard errors, to test the significance of the change in $q_C^3(0.2)$ in the periods 2008-2014, 2014-2018 and 2008-2018, respectively. The corresponding p-value (in parenthesis) is computed assuming asymptotic normality of the t-statistic.

The results in column 6 reveal that, in many EU-28 countries, $q_C^3(0.2)$ significantly increased between 2008 and 2018. That is, over this period, there has been a significant increase in the probability that a household that is in the 1^{st} quintile in one poverty dimension is also in the 1^{st} quintile in the other two poverty dimensions. Moreover, in most of these countries, this overall increase was mainly due to the increase over the period 2008-2014. This is the case of Austria, Denmark, France, Hungary, Malta, Poland, Spain and Slovenia. However, in other countries, like The Netherlands and Sweden, that increase over 2008-2014 was also followed by a significant increase in $q_C^3(0.2)$ over the recovery period of 2014-2018. Unlike the previous countries, in Italy, $q_C^3(0.2)$ significantly increased over the crisis period and significantly decreased afterwards in such a way that dependence in 2018 was similar to that observed in 2008. Finally, it is worth mentioning that only in Estonia and Luxembourg was $q_C^3(0.2)$ significantly lower in 2018 than in 2008.

To sum up, we find evidence of multivariate lower tail dependence in all EU-28 countries, but we also find that this dependence is time-varying over the period analysed and the effect of the Great Recession is not homogeneous over all EU-28 countries.

4 Conclusions

There is a broad consensus that poverty should be regarded as a multidimensional phenomenon involving not only income but also other non-monetary aspects such as health, education or labour status. As a result, over the last years a vast and still growing literature on multidimensional poverty measurement has emerged. In this literature, the main focus has traditionally been on the proposal of multidimensional poverty indices. However, most of these indices, and specially some of the most widely used, are not sufficiently sensitive to the degree of multivariate dependence between poverty dimensions, a crucial aspect that should be taken into account when analysing multidimensional poverty, since a higher degree of multivariate dependence leads to a higher concentration of deprivations, which exacerbates poverty.

In this paper, we provide a better picture of multidimensional poverty by incorporating into the analysis the multivariate dependence between its dimensions using the copula methodology, an approach that has recently gained recognition in welfare economics. In particular, we focus on the copula-based concept of multivariate tail dependence. In this framework, we propose the multivariate tail concentration function (TCF) as a powerful graphical tool which allows to represent, in the unit square, the degree of multivariate dependence in both the lower and the upper parts of the joint distribution, regardless of the number of poverty dimensions considered.

We illustrate the use of the the multivariate TCF by analysing the evolution of tail dependence between the dimensions of the AROPE rate (income, work intensity and material deprivation) in the EU-28 countries between 2008 and 2018. Special attention is given to lower tail dependence, which captures somehow the propensity of cumulative deprivations in all poverty dimensions. Our first conclusion is that there is multivariate lower tail dependence between poverty dimensions in the EU-28, that is, low positions in one dimension extend to other dimensions. However, there are cross-country differences in the shape of the TCF: in some countries, this function is symmetric, but in most of them, lower tail dependence tends to be higher than upper tail dependence. Second, we observe that multivariate lower tail dependence is time-varying over the period analysed, but the effect of the Great Recession on this dependence is not homogeneous over all EU-28 countries. In some countries, the level of dependence between poverty dimensions hardly changed over the period analysed. By contrast, in other countries there was a significant increase of the risk of cumulative deprivation over the period 2008-2014, but the post-2014 recovery period allowed to reduce that risk to the levels of 2008. However, there are also countries where the Great Recession strengthened lower tail dependence in such a way that this was still higher in 2018 than in 2008. This means that, in these countries, welfare policies were not able to avoid the vicious cycle of poverty which entails a high probability that a household that is low-ranked in one poverty dimension is also low-ranked in the other dimensions.

To the best of our knowledge, our paper constitutes the first attempt to apply multivariate tail dependence concepts in welfare economics and provides a supportive instrument to have a better understanding of multidimensional poverty. We hope our results to be a wake-up call for a rethinking of the public policy interventions to effectively respond to the consequences of future crises on the more vulnerable households.

References

Aghakouchak, A., Ciach, G., and Habib, E. (2010). Estimation of tail dependence coefficient in rainfall accumulation fields. *Advances in Water Resources*, 33(9):1142 – 1149.

Atkinson, A. B. and Bourguignon, F. (1982). The comparison of multi-dimensioned distributions of economic status. *The Review of Economic Studies*, 49(2):183–201.

Bourguignon, F. and Chakravarty, S. (2003). The measurement of multidimensional poverty. *Journal of Economic Inequality*, 1(1):25–49.

Caillault, C. and Guégan, D. (2005). Empirical estimation of tail dependence using copulas: application to asian markets. *Quantitative Finance*, 5(5):489–501.

Capéraà, P., Fougères, A., and Genest, C. (1997). A nonparametric estimation procedure for bivariate extreme value copulas. *Biometrika*, 84(3):567–577.

Chakravarty, S. R. (2018). *Analyzing multidimensional well-being: A quantitative approach*. Wiley, New York.

DAgostino, A., Deluca, G., and Guégan, D. (2022). Estimating lower tail dependence between pairs of poverty dimensions in europe. *Review of Income and Wealth*, n/a(n/a).

Decancq, K. (2014). Copula-based measurement of dependence between dimensions of well-being. *Oxford Economic Papers*, 66(3):681–701.

Decancq, K. (2020). Measuring cumulative deprivation and affluence based on the diagonal dependence diagram. *Metron*, (78):103–117.

Duclos, J.-Y. and Tiberti, L. (2016). Multidimensional poverty indices: A critical assessment. In *The Oxford Handbook of Well-Being and Public Policy*. Oxford University Press, Oxford.

Durante, F., Fernández-Sánchez, J., and Pappadà, R. (2015). Copulas, diagonals, and tail dependence. *Fuzzy Sets and Systems*, 264:22 – 41. Special issue on Aggregation functions at AGOP2013 and EUSFLAT 2013.

Durante, F., Nelsen, R. B., Quesada-Molina, J. J., and Úbeda-Flores, M. (2014). Pairwise and global dependence in trivariate copula models. In Laurent, A., Strauss, O., Bouchon-Meunier, B., and Yager, R. R., editors, *Information Processing and Management of Uncertainty in Knowledge-Based Systems*, pages 243–251, Cham. Springer.

Durante, F. and Sempi, C. (2015). *Principles of Copula Theory.* Chapman and Hall/CRC, Boca Raton, FL.

Fernández-Sánchez, J., Nelsen, R. B., Quesada-Molina, J. J., and Úbeda-Flores, M. (2016). Independence results for multivariate tail dependence coefficients. *Fuzzy Sets and Systems*, 284:129 – 137.

Frahm, G. (2006). On the extremal dependence coefficient of multivariate distributions. *Statistics & Probability Letters*, 76(14):1470 – 1481.

Frahm, G., Junker, M., and Schmidt, R. (2005). Estimating the tail-dependence coefficient: Properties and pitfalls. *Insurance: Mathematics and Economics*, 37(1):80 – 100.

García-Gómez, C., Pérez, A., and Prieto-Alaiz, M. (2021). Copula-based analysis of multivariate dependence patterns between dimensions of poverty in europe. *Review of Income and Wealth*, 67(1):165–195.

Genest, C. and Nešlehová, J. (2007). A primer on copulas for count data. *ASTIN Bulletin*, 37(2):475–515.

Genest, C., Nešlehová, J., and Rémillard, B. (2017). Asymptotic behavior of the empirical multilinear copula process under broad conditions. *Journal of Multivariate Analysis*, 159:82–110.

Gijbels, I., Kika, V., and Omelka, M. (2020). Multivariate tail coefficients: Properties and estimation. *Entropy*, 22(7):728.

Hua, L. and Joe, H. (2011). Tail order and intermediate tail dependence of multivariate copulas. *Journal of Multivariate Analysis*, 102(10):1454 – 1471.

Huang, X. (1992). *Statistics of bivariate extreme values.* PhD thesis, Erasmus University Rotherdam.

Joe, H. (1997). *Multivariate models and multivariate dependence concepts.* Chapman and Hall/CRC.

Joe, H. (2014). *Dependence Modeling with Copulas.* Chapman and Hall, London, UK.

Joe, H., Smith, R. L., and Weissman, I. (1992). Bivariate threshold methods for extremes. *Journal of the Royal Statistical Society, Series B*, 54(1):171–183.

Malevergne, Y. and Sornette, D. (2006). *Extreme financial risks: From dependence to risk management.* Springer Science & Business Media.

Manner, H. and Segers, J. (2011). Tails of correlation mixtures of elliptical copulas. *Insurance: Mathematics and Economics*, 48(1):153 – 160.

Matkovskyy, R. (2019). Centralized and decentralized bitcoin markets: Euro vs USD vs GBP. *The Quarterly Review of Economics and Finance*, 71:270 – 279.

Matkovskyy, R. (2020). A measurement of affluence and poverty interdependence across countries: Evidence from the application of tail copula. *Bulletin of Economic Research*, 72(4):404–416.

Nelsen, R. B. (2006). *An introduction to copulas.* Springer-Verlag, New York.

Patton, A. (2013). Copula methods for forecasting multivariate time series. In Elliott, G. and Timmermann, A., editors, *Handbook of Economic Forecasting*, volume 2 of *Handbook of Economic Forecasting*, pages 899 – 960. Elsevier.

Pérez, A. and Prieto, M. (2015). Measuring dependence between dimensions of poverty in Spain: An approach based on copulas. In *International Fuzzy Systems Association (IFSA) and European Society for Fuzzy Logic and Technology (EUSFLAT) International Joint Conference*, Gijón, Spain.

Pérez, A. and Prieto-Alaiz, M. (2016). Measuring the dependence among dimensions of welfare: A study based on Spearman's footrule and Gini's gamma. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 24(Suppl. 1):87–105.

Reboredo, J. C., Tiwari, A. K., and Albulescu, C. T. (2015). An analysis of dependence between central and eastern european stock markets. *Economic Systems*, 39(3):474–490.

Schmid, F. and Schmidt, R. (2007). Multivariate conditional versions of Spearman's rho and related measures of tail dependence. *Journal of Multivariate Analysis*, 98(6):1123 – 1140.

Schmid, F., Schmidt, R., Blumentritt, T., Gaißer, S., and Ruppert, M. (2010). Copula-based measures of multivariate association. In Jaworski, P., Durante, F., Härdle, W. K., and Rychlik, T., editors, *Copula Theory and Its Applications*, pages 209–236, Berlin, Heidelberg. Springer.

Schmidt, R. and Stadtmüller, U. (2006). Non-parametric estimation of tail dependence. *Scandinavian Journal of Statistics*, 33(2):307–335.

Serinaldi, F., Bárdossy, A., and Kilsby, C. G. (2015). Upper tail dependence in rainfall extremes: would we know it if we saw it? *Stochastic environmental research and risk assessment*, 29(4):1211–1233.

Seth, S. and Santos, M. (2019). On the interaction between focus and distributional properties in multidimensional poverty measurement. *Social Indicators Research*, 145:503–521.

Sibuya, M. (1960). Bivariate extreme statistics. *Annals of the Institute of Statistical Mathematics*, 11(3):195–210.

Sklar, A. (1959). Fonctions de répartition à n dimensions et leurs marges. *Publications de l'Institut Statistique de l'Université de Paris*, 8:229–231.

Supper, H., Irresberger, F., and Weiß, G. (2020). A comparison of tail dependence estimators. *European Journal of Operational Research*, 284:728–742.

Sweeting, P. and Fotiou, F. (2013). Calculating and communicating tail association and the risk of extreme loss. *British Actuarial Journal*, 18(1):13–83.

Terzi, S. and Moroni, L. (2020). Local concordance and some applications. *Social Indicators Research*.

Tkach, K. and Gigliarano, C. (2020). Multidimensional poverty index with dependence-based weights. *Social Indicators Research*.

Venter, G. (2001). Tails of copulas. In *Proceedings ASTIN*, pages 68–113, Washington, USA.

	2008	2014	2018	t-test Change 08-14	t-test Change 14-18	t-test Change 08-18
AUSTRIA	0.259 (0.013)	0.313 (0.014)	0.298 (0.013)	2.835** (0.002)	-0.753 (0.226)	2.168* (0.015)
BELGIUM	0.396 (0.013)	0.414 (0.013)	0.443 (0.014)	0.959 (0.169)	1.540 (0.062)	2.512** (0.006)
BULGARIA	0.377 (0.016)	0.352 (0.016)	0.365 (0.013)	-1.118 (0.132)	0.626 (0.266)	-0.578 (0.282)
CYPRUS	0.211 (0.015)	0.227 (0.014)	0.256 (0.014)	0.797 (0.213)	1.503 (0.066)	2.246* (0.012)
CZECH REPUBLIC	0.254 (0.009)	0.276 (0.012)	0.275 (0.013)	1.447 (0.074)	-0.012 (0.495)	1.337 (0.091)
GERMANY	0.354 (0.009)	0.371 (0.010)	0.377 (0.011)	1.276 (0.101)	0.377 (0.353)	1.611 (0.054)
DENMARK	0.262 (0.013)	0.302 (0.014)	0.310 (0.015)	2.082* (0.019)	0.397 (0.346)	2.413** (0.008)
ESTONIA	0.306 (0.013)	0.294 (0.014)	0.275 (0.013)	-0.658 (0.255)	-1.014 (0.155)	-1.699* (0.045)
GREECE	0.226 (0.012)	0.210 (0.010)	0.213 (0.007)	-1.006 (0.157)	0.214 (0.415)	-0.970 (0.166)
SPAIN	0.230 (0.008)	0.284 (0.009)	0.301 (0.008)	4.558** (0.000)	1.331 (0.092)	6.352** (0.000)
FINLAND	0.318 (0.010)	0.337 (0.010)	0.342 (0.010)	1.392 (0.082)	0.355 (0.361)	1.730* (0.042)
FRANCE	0.287 (0.009)	0.310 (0.009)	0.321 (0.010)	1.722* (0.043)	0.790 (0.215)	2.477** (0.007)
CROATIA	NA (NA)	0.267 (0.013)	0.333 (0.011)	NA (NA)	3.795** (0.000)	NA (NA)
HUNGARY	0.260 (0.011)	0.308 (0.010)	0.289 (0.013)	3.243** (0.001)	-1.142 (0.127)	1.751* (0.040)
IRELAND	0.351 (0.015)	0.260 (0.013)	NA (NA)	-4.653** (0.000)	NA (NA)	NA (NA)
ITALY	0.247 (0.006)	0.282 (0.008)	0.233 (0.006)	3.481** (0.000)	-4.899** (0.000)	-1.561 (0.059)
LITHUANIA	0.312 (0.015)	0.326 (0.014)	0.344 (0.016)	0.652 (0.257)	0.869 (0.192)	1.450 (0.074)
LUXEMBOURG	0.264 (0.014)	0.262 (0.014)	0.217 (0.014)	-0.087 (0.465)	-2.245* (0.012)	-2.364** (0.009)
LATVIA	0.351 (0.014)	0.334 (0.014)	0.354 (0.014)	-0.854 (0.197)	0.987 (0.162)	0.153 (0.439)
MALTA	0.279 (0.016)	0.320 (0.016)	0.302 (0.019)	1.818* (0.035)	-0.711 (0.239)	0.971 (0.166)
THE NETHERLANDS	0.268 (0.010)	0.333 (0.010)	0.379 (0.010)	4.758** (0.000)	3.234** (0.001)	7.901** (0.000)
POLAND	0.236 (0.008)	0.277 (0.008)	0.283 (0.008)	3.566** (0.000)	0.472 (0.318)	4.247** (0.000)
PORTUGAL	0.240 (0.014)	0.269 (0.012)	0.295 (0.008)	1.609 (0.054)	1.816* (0.035)	3.375** (0.000)
ROMANIA	0.180 (0.011)	0.184 (0.010)	0.231 (0.011)	0.298 (0.383)	3.039** (0.001)	3.288** (0.001)
SWEDEN	0.269 (0.011)	0.326 (0.014)	0.407 (0.015)	3.170** (0.001)	3.897** (0.000)	7.395** (0.000)
SLOVENIA	0.254 (0.009)	0.288 (0.010)	0.294 (0.010)	2.542** (0.006)	0.427 (0.335)	3.008** (0.001)
SLOVAKIA	0.246 (0.012)	0.310 (0.013)	NA (NA)	3.603** (0.000)	NA (NA)	NA (NA)
UK	0.332 (0.011)	0.331 (0.011)	NA (NA)	-0.017 (0.493)	NA (NA)	NA (NA)

Note: Standard errors for the coefficients and p-values for the one-side t-test are displayed in parentheses.
*, ** indicates that the variation is significant at 5%, 1%, respectively.

Table 1: Estimated value of $q_C^3(0.2)$ in EU-28 countries for years 2008, 2014 and 2018 and t-tests for the significance of its variation over the periods 2008-2014, 2014-2018 and 2008-2018.

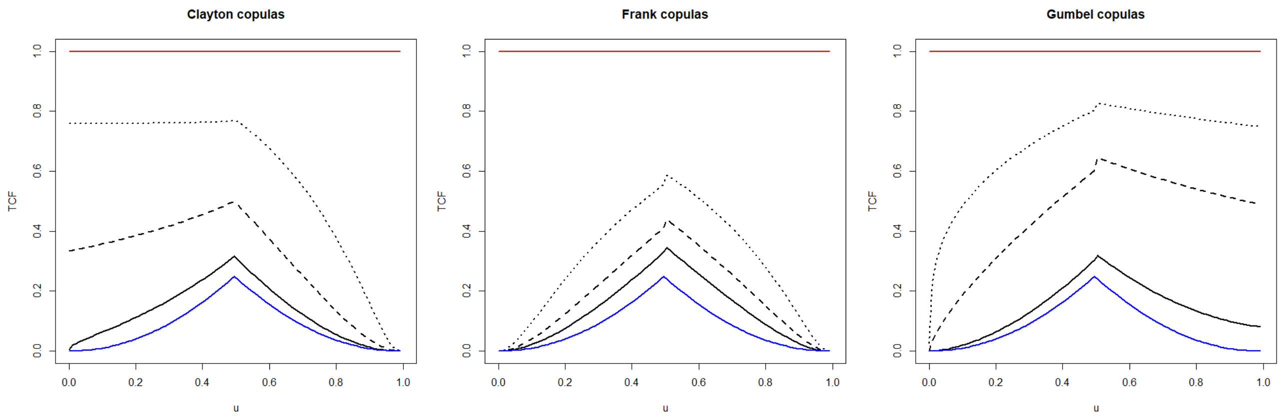

Figure 1: Trivariate TCFs for some trivariate parametric copulas. Left panel represents Clayton copulas with $\theta = 0.1$ (solid), $\theta = 1$ (dashed) and $\theta = 2$ (dotted). Central panel represents Frank copulas with $\theta = 1$ (solid), $\theta = 2$ (dashed) and $\theta = 4$ (dotted). Right panel represents Gumbel copulas with $\theta = 1.1$ (solid), $\theta = 2$ (dashed) and $\theta = 4$ (dotted). In all panels, the trivariate TCF of copulas Π and M are represented in blue and red, respectively.

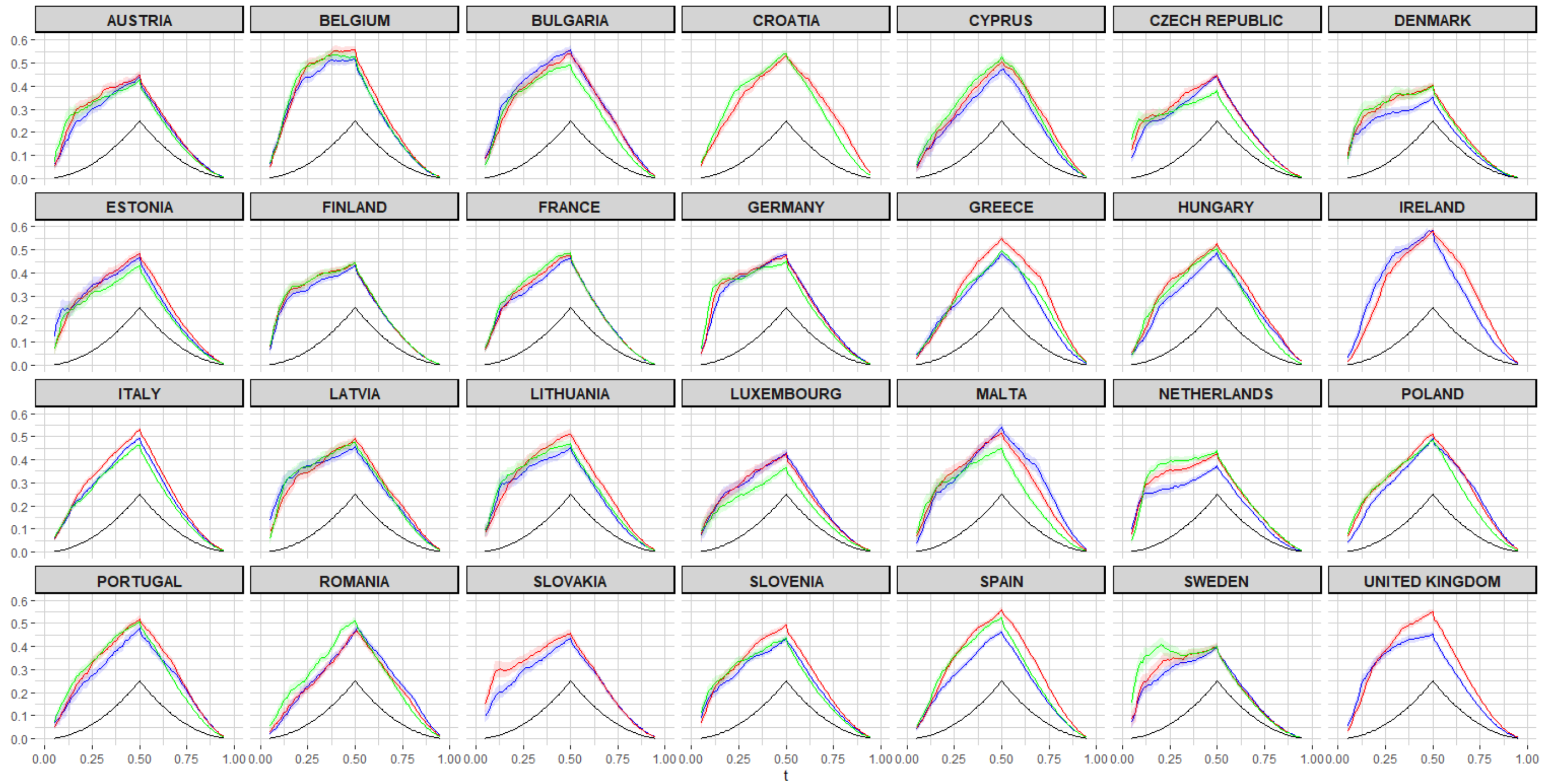

Figure 2: TCF for the EU-28 countries and years 2008 (blue), 2014 (red) and 2018 (green) with bootstrap confidence intervals and TCF of independence (black).