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Keyword: decomposition, income inequality, income sources, Gini index

JEL Cassification: D31, D33, D63, H22, H23

Two classical decompositions of the Gini index by income sources: interpretation of contribution terms

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Abstract

Natural Gini decomposition (Rao, V. M., 1969, J R Stat Soc Ser A, 132:418–425) and marginal Gini decomposition (Lerman, R. I., and Yitzhaki, S., 1985, Rev Econ Stat 67:151–156) are the most popular and widely used methods to reveal the contributions of various income sources to total income inequality. Their acceptance and highly spread empirical application have persisted in the face of criticism of the former method. This paper aims to "liberate" the natural Gini decomposition from two major critiques: that the method should be abolished because a uniformly distributed income source obtains zero contribution and that contribution terms lack a meaningful interpretation. Regarding the latter critique, it is shown that the contribution of a certain income source expresses inequality reduction due to the replacement of this source by a marginal uniformly distributed counterfactual income. Concerning the former critique, the argument is as follows: natural Gini decomposition belongs to the absolute inequality view, which commands that equal additions of income leave inequality unchanged. In this sense, it is perfectly normal that the natural Gini decomposition obtains zero contribution of the uniformly distributed income.

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1 Introduction

The total income of an individual or a household is composed of different sources.[1] These sources affect total income distribution differently, i.e., their contributions to total income inequality vary in two aspects: (a) the direction of the contribution, i.e., whether an income source acts to augment or diminish total income inequality, and (b) the relative magnitude of the contribution.

The contributions of income sources to total income inequality were studied within different economic fields. Development economics researchers have been questioning whether nonagricultural income contributes positively or negatively to total income inequality (e.g., Hussain et al., 1994; Adams, 1994) and scrutinizing the role of remittances from abroad (e.g., Stark et al., 1986; Kimhi, 2010). In labour economics, the impact of the head's versus spouse's earnings on total income inequality has been extensively investigated (e.g., Layard & Zabalza, 1979; Cancian & Reed, 1998). Public economics researchers have been assessing the roles of various taxes and benefits on income inequality (e.g., Lerman & Yitzhaki, 1994; Fuest et al., 2010; Avram, Levy and Sutherland, 2014). In addition to these studies focused on specific types of income sources, there is a significant body of research interested in all types of income sources, aiming to explain cross-country and intertemporal differences in economic inequality (e.g., Jenkins, 1995; Milanovic, 1999; Paul, 2004; García-Peñalosa and Orgiazzi, 2013).

Methodologically, the starting point in this research area was the 1969 paper of Indian agricultural economist Vidyanand Madiman Rao, titled "Two Decompositions of

[1] These sources, sometimes called "factor components", typically include employment, self-employment, capital and property incomes, pension and nonpension social benefits, and private transfers. Many studies also include "negative incomes", such as direct taxes and social insurance contributions.

Concentration Ratio" (Rao, 1969).[2] Rao shows that the Gini index of total income equals the sum of the products of the Gini-concentration indices of source incomes and their shares in total income. The decomposition of the Gini income of total income was later "reinvented" and extended by several scholars (Kakwani, 1977; Fei et al., 1978; Pyatt et al., 1980; Lerman and Yitzhaki, 1985). Henceforth, we call this method the "natural Gini decomposition" (NGD).

NGD has become the most popular decomposition of inequality by income sources. However, its interesting history is marked by methodological controversies. There are three main points of critique.

First, according to Shorrocks (1982), the "decomposition rule" implied by NGD is just one of indefinitely many possible rules. Shorrocks (1982) derived "natural" decomposition rules for several most popular inequality indices: For the Gini index, the natural rule obtains NGD. Similarly, the variance of total income is decomposed into the sum of covariances between income sources and the total income; this is the "natural decomposition of the variance" (NVD). Shorrocks (1983) empirically shows that different decomposition rules obtain divergent results for given data. Which rule is the proper one? Shorrocks (1982) demonstrates that the rule implied by NVD is the only rule satisfying his premises and conditions. Hence, Shorrocks (1982) concludes that this "unique rule" should be applied in decompositions of all inequality indices – not only the variance.

Second, according to NGD, a uniformly distributed income source (UDS) has a zero contribution to income inequality.[3] This result is unacceptable for some scholars. Namely, if every person receives a transfer of an identical amount, income inequality should decrease because the gain of the poor is relatively larger than the gain of the rich (Podder, 1993; Podder

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[2] On life and scientific achievements of V. M. Rao, see Deshpande (2015).

[3] This is also true for NVD.

and Chatterjee, 2002; Kimhi, 2011). For these scholars, a “proper” decomposition should obtain a negative (positive) contribution for a positive (negative) UDS.

Third, some scholars have objected that contribution terms obtained by NDG cannot be reasonably interpreted. Podder (1993) notes that these contributions do not satisfy any of the four interpretative conditions proposed by Shorrocks (1988). Suppose that, in a certain empirical case, the Gini index of total income is 0.4, while the contribution of capital income is 0.1. A common interpretation of this result is that “capital income is responsible for 25 per cent of total income inequality”. However, can we interpret this result more intuitively, for instance, “total income inequality would fall by 25 per cent if inequality of capital income were eliminated”?[4] Unfortunately, we can’t.[5]

NGD has survived all these hits, which is evidenced by the fact that it is still widely used in empirical research. An important role in sustaining its popularity was played by Lerman and Yitzhaki (1985), whose contributions are as follows. First, they provided convincing arguments against the critique posed by Shorrocks (1982). Second, the re-established specification of NGD – first proposed by Pyatt, Chen & Fei (1980) – which replaces the income source’s Gini-concentration index with the product of the source’s Gini index and its Gini correlation with total income. According to Lerman and Yitzhaki (1985), this specification “appears more compelling and less arbitrary”. Third, they proposed a new decomposition of the Gini index. The partial derivatives of the Gini index of total income are obtained with respect to proportional changes in each income source. We call this method the “marginal Gini

[4] This question is in line with the Shorrocks (1988) interpretative condition D; see section

[5] However, NGD is not the only decomposition which fails to satisfy these conditions. Shorrocks (1988) shows that they are not fulfilled even by NVD – whenever income sources are correlated.

decomposition" (MGD).[6] MGD obtains elasticities of income sources, which have a very intuitive meaning.

Lerman and Yitzhaki (1985) use NGD and MGD in a single package, with each decomposition providing its own information. NGD provides "contributions to inequality", while MGD obtains "marginal effects" of proportional changes in source incomes. A single-package approach has gained broad support.[7] In fact, it is nowadays more common to see NGD together with MGD than as a standalone method. A desirable feature of MGD is that a positive UDS has a negative contribution (i.e., elasticity). However, MGD is not an "exhaustive" decomposition: its contribution terms do not add up to the value of an inequality index (as is the case for NGD and NVD), but their sum equals zero. Therefore, several scholars have proposed alternative decompositions of standard inequality indices, which also obtain a negative contribution for a positive UDS, but are exhaustive (Morduch and Sicular, 2002; Paul, 2004; Araar, 2006; Nembua, 2012).

Despite all the benefits provided, Lerman and Yitzhaki (1985) do not explain why NGD obtains a zero contribution of a UDS and whether such results is right or wrong. Moreover, they do not provide alternative interpretations of UDS's contribution terms, in terms of Shorrocks's (1988) conditions. These two points are the topics of the current paper.

We first reconsider the interpretative conditions for NGD. Two propositions are made. First, in a special empirical case, when counterfactual replacement income does not affect the ranking of income units, it can be said that the *k*th source's contribution measures the reduction

[6] Paul (2004) extended the marginal approach to several other inequality indices.

[7] Examples of empirical studies based on the single-package framework of Lerman and Yitzhaki (1985) include Garner (1993), Rozelle (1994), Flückiger & Silber (1995), Reardon & Taylor (1996), Cheng (1996), Garner & Terrell (1998), Zhou (2009), Amarante (2016), Krstić (2021), Kanbur, Wang & Zhang (2021).

of inequality due to the replacement of source *k* income with a uniformly distributed equal-mean counterfactual income. Second, in a general case, it is shown that the *k*th source's contribution equals inequality reduction following a *marginal* replacement of source *k* income with an equal-mean uniformly distributed counterfactual income. We provide analogous interpretations for MGD contributions.

Second, we scrutinize the issue of the zero contribution of a UDS. We provide evidence that the NGD belongs to the absolute inequality view, according to which equal additions of income for all income units leave inequality unchanged. The repercussions are as follows: in contrast to some authors' claims, there is nothing wrong with NGD showing the zero contribution of the UDS; this is in perfect harmony with the absolute inequality view. Analogously, according to MGD, an income source whose distribution is proportional to total income has a zero contribution to total income inequality. This is consistent with the relative inequality view, which proposes that proportional additions of income for all income units leave inequality unchanged.

In real-world examples UDS does not exist. However, the authors of empirical studies are often faced with a problem that is related to the issue of the zero-UDS contribution. Namely, certain income sources behave dually when subjected to NGD versus MGD. In public finance studies, an example is public pensions, which are inequality increasing according to NGD and inequality reducing according to MGD. In the studies of rural income, the same occurs for, e.g., crop income (Davis et al, 2010; Kimhi, 2011), whereas spouse's earnings have the same destiny in the research on labour income composition. Our analysis helps to clarify why such contradictory results occur. Again, the solution to the puzzle lies in different inequality views taken by NGD versus MGD. We scrutinize the issue using an empirical example of 15 EU countries, for which the results are based on EUROMOD.

The paper is organised as follows. Section 2 presents NGD and MGD. Section 3 is devoted to the interpretation of contribution terms. First, we briefly speak about the standard interpretation of contribution terms in the empirical literature (section 3.1). We then analyse the Shorrocks (1988) interpretative conditions and present a case when NGD indeed does fulfil some of these conditions (section 3.2). This is followed by general propositions related to marginal replacements (section 3.3). Section 4 analyses the zero contribution of the UDS issue. Section 5 presents an empirical exercise for 15 EU countries. Section 6 concludes.

2 Decompositions

2.1 Basic concepts

Let y represent total income, which is composed of K income sources, $x_1, \ldots, x_K$, so that $y = \sum_{k=1}^{K} x_k$. The mean total income and the mean source k income are denoted as $\bar{y}$ and $\bar{x}_k$, respectively. The share of source k income in total income is given by $\varphi_k = \bar{x}_k/\bar{y}$. The cumulative distributions of total income and source k income are denoted as F^y and F^{x_k}, respectively.

1^{v}, $\bar{x}_k^{\mathrm{v}}$ and $\bar{y}^{\mathrm{v}}$ denote the vectors with the same dimension as y and all elements equal to 1, $\bar{x}_k$ and $\bar{y}$, respectively. Let $\tilde{x}_k^{\mathrm{v}} = \varphi_k y$ denote a counterfactual income vector, whose values are proportional to total income; note that the mean of $\tilde{x}_k^{\mathrm{v}}$ equals $\bar{x}_k$.

The relative Gini index of total income is obtained as $G(y) = 2\mathrm{cov}(y, F^y)/\bar{y}$. The relative Gini-concentration index of source k income with respect to the cumulative distribution of total income is defined as $D^y(x_k) = 2\mathrm{cov}(x_k, F^y)/\bar{x}_k$. The absolute counterparts of these two indices are obtained as $\Gamma(y) = 2\mathrm{cov}(y, F^y)$ and $\Delta^y(x_k) = 2\mathrm{cov}(x_k, F^y)$, respectively.

Suppose that a new income source is provided to all persons in a society, such that $x_u = \upsilon 1^{\mathrm{v}}$, where υ is a positive constant. This is a uniformly distributed income source (UDS). Since the covariance of x_u and any other vector is zero, $D^y(x_u) = \Delta^y(x_k) = 0$. How is the inequality of total income affected by the introduction of the UDS? We obtain that $G(y + x_u) = \frac{1}{1+\upsilon} G(y)$ and $\Gamma(y + x_u) = \Gamma(y)$. Thus, the relative Gini index is reduced by $\frac{\upsilon G(y)}{1+\upsilon}$, while the absolute Gini index remains unchanged.

In the analysis below, we will rely on the concept of the pseudo-Gini index of a variable z, which always assumes the cumulative distribution of the *actual* total income: $D^y(z) = 2\mathrm{cov}(z, F^y)/\bar{z}$. Thus, regardless of the structure of z, the ranking of income units is always the same.[8]

2.2 *Natural decomposition*

Early scholars hoped that $G(y)$ could be expressed simply as the sum of the Gini indices $G(x_k)$ weighted by the shares φ_k. However, Rao (1969) demonstrated that $G(y) < \sum_k^K \varphi_k G(x_k)$ and revealed that:

$$G(y) = \sum_{k=1}^{K} \varphi_k D^y(x_k) \tag{1}$$

Expression from equation (1) has become the most popular decomposition of total income inequality. We call it the natural Gini decomposition (NGD). Although revealed in the late 1960s, the push for its use and further development is given a decade later by Kakwani (1977), Fei, Ranis and Kuo (1978), Fields (1979), Pyatt, Chen and Fei (1980), whose theoretical and

[8] Note that $D^y(x_k)$ is a special version of $D^y(z)$.

empirical investigations have helped to deepen Rao's findings.[9] Pyatt, Chen & Fei (1980) introduced the following variant of NGD:

$$G(y)=\sum_{k=1}^{K}\varphi_k R^y(x_k)G(x_k) \tag{2}$$

where $R^y(x_k)$ is the "Gini correlation" term:

$$R^y(x_k)=\frac{\operatorname{cov}(x_k,F^y)}{\operatorname{cov}(x_k,F^{x_k})} \tag{3}$$

$R^y(x_k)$ takes the values in the interval $[-1,1]$. If the source *k* income is a monotonically increasing (decreasing) function of total income, $R^y(x_k)=1$ ($R^y(x_k)=-1$). The values closer to -1 or 1 indicate a strong relationship, while the values near 0 represent a weak relationship between x_k and y. By decomposing $D^y(x_k)$ into $R^y(x_k)$ and $G(x_k)$, equation (2) provides valuable additional information about the relationship between source incomes and total income.[10] Nevertheless, the actual results regarding the contributions of income sources to total income inequality are the same as in the original decomposition from equation (1). The nominal and proportionate contributions of the source k income in the case of NGD are:

$$C_k^R(G)=\varphi_k D^y(x_k)=\varphi_k R^y(x_k)G(x_k) \tag{4}$$

[9] Fei, Ranis and Kuo (1978) refer to $D^y(x_k)$ as the "pseudo Gini coefficient", while Pyatt, Chen and Fei (1980) call it the "concentration ratio". Rao (1969) uses the term "concentration ratio" for both $G(y)$ and $D^y(x_k)$.

[10] The variant of NGD from equation (2) was reinvented by Lerman and Yitzhaki (1985). In subsequent research, almost all authors cite Lerman and Yitzhaki (1985), and not Pyatt, Chen & Fei (1980), as the originators of equation (2). Lerman and Yitzhaki (1985) have also generalised NGD in realm of the so-called single-parameter Gini index.

$$c_k^R(G) = \frac{C_k^R(G)}{G(y)}$$

We now multiply both sides of equation (1) by $\bar{y}$ to obtain the decomposition of the absolute Gini index, $\Gamma(y)$, as follows:

$$\Gamma(y) = \sum_{k=1}^{K} \Delta^y(x_k) \tag{5}$$

where $\Delta^y(x_k)$ is the absolute Gini-concentration index of source *k* income with respect to the cumulative distribution of total income; $\Delta^y(x_k) = \bar{x}_k D^y(x_k)$. First, notice that the new decomposition is even simpler than the one from equation (1): the absolute Gini index is a pure sum of absolute concentration indices. Second, observe that two decompositions obtain the same proportionate contributions:

$$c_k^R(\Gamma) = \frac{\Delta^y(x_k)}{\Gamma(y)} = \frac{\varphi_k D^y(x_k)}{G(y)} = c_k^R(G) \tag{6}$$

Therefore, it can be said that the decomposition from equation (5) is equivalent to NGD. More precisely, it is NGD in its pure form.

2.3 Marginal decomposition

Lerman and Yitzhaki (1985) proposed a substantially different approach to determine how different income sources affect total income inequality. Assume that income source k is proportionally increased, i.e., that each person's income is multiplied by $1 + e$, where e is a small number (e.g., 0.01 or 0.001). The change of $G(y)$ induced by the mentioned marginal change in x_k, divided by e, equals:

$$\frac{\dot{G}(y)_{|x_k \to (1+e)x_k}}{e} = \varphi_k(R^y(x_k)G(x_k) - G(y)) = \varphi_k(D^y(x_k) - G(y))$$

Assume that the analogous marginal changes are obtained for all sources $k = 1, .., K$. The sum of these changes, $\sum_k^K \dot{G}(y)/e$, equals zero because $G(y)$ is not affected by a proportional increase of y:

$$\sum_{k=1}^{K} \varphi_k(D^y(x_k) - G(y)) = 0 \tag{7}$$

From equation (7), we can deduce two types of "contributions" of the source *k* income:

$$M_k^{LY}(G) = \varphi_k(D^y(x_k) - G(y)) \tag{8}$$

$$\eta_k^{LY}(G) = \varphi_k\left(\frac{D^y(x_k)}{G(y)} - 1\right) \tag{9}$$

where $\eta_k^{LY}(G)$ are obtained simply as $M_k^{LY}(G)/G$ and represent "elasticities".[11] Conveniently, we can express the elasticity as $\eta_k^{LY}(G) = c_k^R(G) - \varphi_k$, where $c_k^R(G)$ is the proportionate contribution according to NGD.

3 Interpretation of contribution terms

3.1 Standard interpretation

Interpretation of NGD contribution terms from equation (4) is straightforward. The term $C_k^R(G)$ represents a part of $G(y)$ that is ascribed to source *k*. In empirical studies, the use of

[11] Derivation of these equations is presented in detail in Stark, Taylor, and Yitzhaki (1986). For alternative derivation, see Podder (1993).

$c_k^R(G)$ is more appealing; this term measures a proportion of total income inequality that can be attributed to income source *k*.

Each source's contribution depends on its share in total income, φ_k. Thus, the two sources may have identical concentrations, measured by $D^y(x_k)$, but the contribution of the source with, say, 10 times higher share, will be 10 times greater. Therefore, to neutralise the impact of the shares, one can divide contribution terms $c_k^R(G)$ by φ_k, i.e., which breaks down to comparing the terms $D^y(x_k)$.

MGD's contribution terms have a substantially different meaning. The sums of contributions $M_k^{LY}(G)$ and $\eta_k^{LY}(G)$ for all *K* income sources equal zero, which means that at least some sources have a negative contribution. In most empirical studies, researchers are using $\eta_k^{LY}(G)$ rather than $M_k^{LY}(G)$. The term $\eta_k^{LY}(G)$ is interpreted as follows: a 1 per cent proportional increase in source *k* income would result in a $100 \cdot \eta_k^{LY}(G)$ per cent increase in the Gini index of total income. The greater the positive (negative) elasticity is, the larger the increase (decrease) in total income inequality.

It is slightly more difficult to explain the terms $M_k^{LY}(G)$. The clear message is given by the sign of the term, but the magnitude *per se* does not tell much. However, it makes sense to compare the values $M_k^{LY}(G)$ for two different sources (of the same sign, preferably) to arrive at the statement as follows: source j has a $M_j^{LY}(G)/M_m^{LY}(G)$ larger marginal contribution than source m.

Notice a few problems in the interpretation of the terms $M_k^{LY}(G)$. Imagine that all *K* income sources have identical relative distributions, i.e., $D^y(x_1) = \cdots = D^y(x_k) = G(y) > 0$, but their shares in total income are different, i.e., $\varphi_1 \neq \cdots \neq \varphi_K$. MGD will indicate that all the terms $M_k^{LY}(G)$ are zero, i.e., that none of the income sources contributes to inequality. However, if this is the case, where does inequality come from?

Take another example, where total income is the sum of two sources: labour income ($j = 1$) and capital income ($j = 2$). Suppose that capital income is a minor source: $\varphi_2 = 0.05$. Additionally, capital income is much more concentrated among the rich than labour income: $D^y(x_1) = 0.3$ and $D^y(x_2) = 0.7$. According to equation (1), $G(y) = 0.32$. MGD says: $M_1^{LY}(G) = -0.019$ and $M_2^{LY}(G) = 0.019$. Thus, all the credit for total income inequality is given to capital income, whereas labour income reduces inequality. This result seems counterintuitive because we would expect that labour income contributes positively to total income inequality, at least to some degree.[12]

3.2 Shorrocks's interpretative conditions

The previous section has explained the basic interpretation of NGD contribution terms. However, can we obtain more knowledge about income sources' distributive impacts from decomposition results? This question was posed by Shorrocks (1988), who proposed a test to check whether contribution terms can be used to arrive at certain more substantive and intuitive interpretations. The "interpretative conditions" or questions are shown in Table 1, where they are applied to the specific case of NGD.

[12] For a similar example, see Aaberge and Aslaksen (1996).

Table 1 Shorrocks's interpretative questions

Question	*Does the contribution $C_k^R(G)$ show…*	*Formula*
A	…the inequality due to source *k* incomes alone?	$C_k^R(G) = G(x_k)$?
B	…the reduction in inequality that would result if this source of income was eliminated?	$C_k^R(G) = G(y) - G(y - x_k)$?
C	…the inequality that would be observed if this was the only source of income differences, and all other incomes were allocated evenly?	$C_k^R(G) = G(\bar{y}^{\mathrm{v}} - \bar{x}_k^{\mathrm{v}} + x_k)$?
D	…the reduction in inequality that would follow from eliminating differences in source *k* income?	$C_k^R(G) = G(y) - G(y - x_k + \bar{x}_k^{\mathrm{v}})$?

Generally, none of these questions can be positively answered by NGD contribution terms. However, there exist certain empirical exceptions when some of the questions can receive a positive answer. Imagine "well-behaved" hypothetical data in the sense that exclusion of a certain income source or its replacement by a counterfactual does not change the ordering of income units by total income; such data are presented in Table 2. Let us focus on source 1, whose contribution according to NGD equals $C_k^R(G) = 0.120$. Condition A is not satisfied since $C_k^R(G) \neq G(x_1) = 0.267$. Condition B is not satisfied because $G(y) - G(y - x_1) = 0.352 - 0.422 = -0.07$. However, conditions C and D are fulfilled: $G(\bar{y}^{\mathrm{v}} - \bar{x}_1^{\mathrm{v}} + x_1) = 0.12$ and $G(y) - G(y - x_1 + \bar{x}_1^{\mathrm{v}}) = 0.352 - 0.232 = 0.12$.

Table 2 Hypothetical population

Unit	x_1	x_2	x_3	y	$y-x_1$	$\bar{y}^{\text{v}}-\bar{x}_1^{\text{v}}$ $+x_1$	$y-x_1$ $+\bar{x}_1^{\text{v}}$	$y-x_1$ $+\tilde{x}_1^{\text{v}}$
#1	3	0	4	7	4	14	13	7.15
#2	6	0	4	10	4	17	13	8.50
#3	9	2	4	15	6	20	15	12.75
#4	12	8	4	24	12	23	21	22.80
#5	15	25	4	44	29	26	38	48.80
Total	45	35	20	100	55	100	100	100
Mean	9	7	4	20	11	20	20	20
φ_k	0.450	0.350	0.200					
$G(\cdot)$	0.267	0.663	0.000	0.352	0.422	0.120	0.232	0.390
$D^y(\cdot)$	0.267	0.663	0.000	0.352	0.422	0.120	0.232	0.390
$C_k^R(G)$	0.120	0.18	-0.114	0.157				
$c_k^R(G)$	0.341	0.659	0.000	1.000				
$M_k^{LY}(G)$	-0.038	0.109	-0.070	0.000				

From this analysis, we conclude that NGD "provisionally" satisfies two of the conditions proposed by Shorrocks, namely, C and D. Focusing on condition D, we make the following:

Proposition 1. In special empirical cases, contribution $C_k^R(G)$ measures the reduction in inequality that would follow from the replacement of source *k* income with an equal-mean uniformly distributed counterfactual income. (Proof is given in Appendix 1.)

Shorrocks's (1988) questions only relate to natural decompositions such as NGD and NVD. However, we can extend the analysis to marginal decompositions as well. This leads us to the following:

Proposition 2. In special empirical cases, contribution $M_k^{LY}(G)$ measures the reduction in inequality that would follow from the replacement of source *k* income with an equal-mean counterfactual income proportional to total income. (Proof is given in Appendix 1.)

In our hypothetical case, the MGD contribution of source 1 income is $M_1^{LY}(G) = -0.038$. Observe that $G(y) - G(y - x_1 + \tilde{x}_1^{\text{v}}) = 0.352 - 0.390 = -0.038$, which equals $M_1^{LY}(G)$.

3.3 Marginal replacements

Propositions 1 and 2 are valid in special empirical cases when the replacement of income source does not change the rank of income units. However, can we state similar propositions for a general case? According to Proposition 1, which is based on Shorrocks's condition D, 100 per cent of source income is replaced by a uniformly distributed counterpart. However, what if such replacement occurs *on the margin*?

Proposition 3. Contribution $C_k^R(G)$ measures the reduction in inequality that would follow from the marginal replacement of source *k* income with an equal-mean uniformly distributed counterfactual income. (Proof is given in Appendix 1.)

What about the marginal replacement of source *k* income with an equal-mean counterfactual income proportional to total income?

Proposition 4. Contribution $M_k^{LY}(G)$ measures the reduction in inequality that would follow from the marginal replacement of source *k* income with an equal-mean counterfactual income proportional to total income. (Proof is given in Appendix 1.)

4 The issue of the uniformly distributed source

Inequality views command how additional income should be divided among persons in society so that inequality remains unchanged. Suppose that the total income of country Z will be increased from 2 to 3 billion, but "inequality" should not increase or decrease in the process. How much each person's income should be changed, i.e., what share of an additional 1 billion should be ascribed to each person? The relative inequality view says: in proportion with their current incomes. The absolute inequality view commands that each person should obtain an identical amount.

The relative and absolute inequality views have been discussed in the inequality literature since its beginning.[13] However, the relative view is still prevalent in both theoretical and empirical studies. In contrast, there are only a handful of empirical studies using absolute inequality indicators, and they mainly belong to the area of "trends in global inequality".[14] Thus, the absolute view is widely disregarded. In the field of inequality decompositions by income sources, there is practically no mention of different inequality views. In other words, most researchers assume the relative inequality perspective.

Imagine that the government introduces a new benefit that all individuals receive in the same amount. The introduction of such a uniformly distributed benefit (UDB) will cause a relative inequality index of total income inequality to decrease. Specifically, if UDB is defined as $x_u = \beta 1^{\mathrm{v}}$, where β is a positive constant, the relative Gini index of total income will decrease by $\frac{\beta G(y)}{1+\beta}$ (see section 2.1).

In contrast, according to NGD, the contribution of UDB to total income inequality equals zero, i.e., $\operatorname{cov}(\beta 1^{\mathrm{v}}, F^y) = 0 \Rightarrow D^y(x_u) = 0 \Rightarrow C_k^R(G) = 0$ (recall section 2.1). For an analyst who applies NGD to decompose the relative Gini index, the UDS's zero-contribution must seem contradictory. On the one hand, according to NGD, the UDS's impact is inequality neutral. On the other hand, according to the relative inequality view, inequality should decrease when UDB is introduced. This puzzle has been tantalising researchers for a long time and sparked many discussions (Podder, 1993; Kimhi, 2011; Cancian and Reed, 1998). The issue of the UDS's zero contribution has also motivated numerous attempts to upgrade the decomposition methodology (Morduch and Sicular, 2002; Paul, 2004).

[13] See seminal contributions of Kolm (1969), Atkinson (1970), Kolm (1976a, 1976b) and Moyes (1987, 1988). For a discussion of the "absolute/relative question", see Atkinson & Brandolini (2004).

[14] For such studies, see Atkinson & Brandolini (2010) and Anand & Segal (2015).

However, the problem should vanish if we accept the following proposal: *NGD is placed within the absolute inequality view*. Consequently, the conclusions based on NGD should be valid in the absolute inequality framework but are not necessarily valid within the relative inequality framework. Let us use the UDB as an example to confirm this. Absolute inequality of total income will remain unchanged upon the introduction of a new UDB. Therefore, the zero contribution of UDB is a perfectly valid result – if observed from a correct perspective.

We have seen that NGD of the relative Gini index obtains identical proportionate contributions as NGD of the absolute Gini index. Scholars have always been analysing the relative Gini index and were expecting that its decomposition should behave according to the relative inequality view. However, NGD is intrinsically an absolute inequality decomposition, whereas its relative inequality form is only a transposition; equation (1) can be obtained simply by multiplying equation (5) by $\bar{y}$.

However, what about the relative inequality view: what is the proper decomposition of the relative Gini index that would fit this framework? It appears that we have presented it above: MGD. The UDB will be ascribed a *negative* contribution; $M_u^{LY}(G) = -\varphi_u G(y)$. This property is consonant with the relative income view and ensures acceptance of MGD among the researchers, especially those disappointed in NGD and other similar decompositions.

We can continue searching for evidence that NGD and MGD belong to the absolute and relative inequality views, respectively. Suppose that all income sources are simultaneously increased by a marginal uniform amount, equal to $e\varphi_k 1$[v]. Such a change does not affect absolute inequality. Alternatively, imagine that instead of a uniform increase, there is a proportional increase equalling $e\varphi_k x_k$; such a change leaves relative inequality unchanged. We now have two propositions, which should establish additional evidence about NGD's and MGD's belonging to different inequality views.

Proposition 5. A uniform marginal increase of all sources, which preserves their shares in total income, does not affect proportionate contributions $c_k^R(G)$. (Proof is given in Appendix 1.)

Proposition 6. The proportional marginal increase of all sources does not affect contributions $M_k^{LY}(G)$. (Proof is given in Appendix 1.)

The first and the second proposition confirm that absolute and relative inequality views lie behind NGD and MGD, respectively.

5 Empirical example

We use data on incomes, taxes and benefits produced by EUROMOD for fifteen EU countries in 2017. EUROMOD, based on EU-SILC data, enables a detailed presentation of various income sources (Sutherland and Figari, 2013). Incomes, taxes and benefits are grouped into eight sources: (1) *labour income*, comprising income from employment and self-employment, (2) *capital income*, capturing income from financial and nonfinancial property, (3) *other private income*, including the income of young persons and students, and private transfers received, (4) *public pensions*, (5) *means-tested benefits* (MTBs), (6) *non-means-tested benefits* (NMTBs), (7) *social insurance contributions* (SICs), and (8) *personal income taxes* (PITs).[15]

The income reference year is 2017. The vectors $x_k = [x_{k,1}, \dots, x_{k,n}]'$, where $x_{k,i}$ represents the source *k* income of person *i*, are obtained from the empirical data as follows: $x_{k,i} = X_{k,i}/e_i$, where $X_{k,i}$ is the monetary amount of source *k* income received by person *i*'s household and e_i is person *i*'s household equivalence scale factor.

[15] Income sources (1) to (4) are in gross terms, i.e., before paying SICs and PITs. Employment income does not include employer SICs. Therefore, employer SICs are not included in source (7).

Table A1 in Appendix 2 presents the shares of income sources in total income. Labour income is the primary income source in all countries, followed by public pensions. Capital income has a tiny share in most countries but is important in several countries (the Netherlands, Denmark, Sweden, Spain and Italy). On average, SICs and PITs represent 12.6 per cent and 20.4 per cent of total income, respectively, but their importance varies significantly among the countries observed.

Table 3 Nominal NGD contributions

	Labour income	Capital income	Other private income	Public pensions	Means-tested benefits	NMTBs	SICs	PITs	Total income
DE	0.418	0.028	-0.003	0.065	-0.014	-0.002	-0.059	-0.133	0.300
DK	0.390	0.104	-0.002	0.039	-0.017	-0.018	-0.004	-0.221	0.271
EE	0.387	0.011	-0.001	-0.016	-0.004	0.009	-0.013	-0.071	0.301
EL	0.307	0.030	0.004	0.134	-0.013	0.001	-0.052	-0.078	0.332
ES	0.337	0.032	0.000	0.096	-0.011	0.005	-0.016	-0.100	0.342
FR	0.247	0.069	0.000	0.135	-0.018	-0.003	-0.037	-0.097	0.297
HR	0.351	0.009	0.001	0.066	-0.009	0.000	-0.068	-0.049	0.301
HU	0.315	0.007	0.001	0.129	-0.003	-0.011	-0.057	-0.044	0.337
IE	0.527	0.020	0.002	0.002	-0.031	-0.005	-0.024	-0.177	0.315
IT	0.342	0.026	0.002	0.152	-0.007	0.024	-0.036	-0.145	0.359
NL	0.357	0.115	-0.001	0.019	-0.021	0.000	-0.066	-0.120	0.284
PL	0.342	0.005	0.001	0.066	-0.016	0.002	-0.039	-0.072	0.288
SE	0.337	0.061	-0.001	0.060	-0.011	-0.019	-0.018	-0.128	0.282
SI	0.289	0.019	0.000	0.068	-0.016	0.003	-0.058	-0.056	0.249
SK	0.259	0.001	-0.001	0.051	-0.008	-0.010	-0.033	-0.031	0.228
AV	0.347	0.036	0.000	0.071	-0.013	-0.001	-0.039	-0.101	0.299

Notes: DE - Germany, DK - Denmark, EE - Estonia, EL - Greece, ES - Spain, FR - France, HR - Croatia, HU - Hungary, IE - Ireland, IT - Italy, NL - The Netherlands, PL - Poland, SE - Sweden, SI - Slovenia, SK - Slovakia, AV - simple average for all countries

Table 3 shows that according to NGD, labour income is the most important contributor to total income inequality, followed by public pensions and capital income. While its contribution is the lowest among the three mentioned sources, capital income is the most potent contributor to inequality per money unit; this can be seen in Table A2 in Appendix, which presents the ratios $c_k^R(G)/\varphi_k$. When this ratio exceeds 1, a source contributes overproportionately to total income inequality. Other examples of potent contributors are means-tested benefits and PITs, but they act to reduce inequality.

Except in Estonia, public pensions have a positive contribution to total income inequality according to NGD. This result may seem controversial: recipients of public pensions often belong to a lower tier of the income distribution, and the average pension tends to be below the average salary income. However, since pensions typically depend on previous earnings, which are unequally distributed, there may be a lot of *absolute* inequality involved. Since NGD belongs to the absolute inequality view, it is perfectly normal that public pensions have an augmenting effect on (absolute) inequality. In contrast, the critics of NGD will come on their own with the results of MGD for public pensions, which are given in Table 4. Here, public pensions are inequality reducing in most (i.e., 9 of 15) countries.[16]

While SICs is a strong equalising instrument according to NGD, MGD says that their role is almost insignificant in most countries. SICs depend on current earnings, and are typically proportional with earnings. Therefore, SICs reduce absolute income differentials, but do not affect relative ones.

The case of means-tested benefits versus PITs is another comparison, which nicely reflects the differences between NGD and MGD. Table A3 in Appendix shows the ratios $M_k^{LY}(G)/\varphi_k$, which are highest for means-tested benefits; the average for all countries is -0.97 versus only -0.29 for PITs. The situation is much different regarding the ratios $c_k^R(G)/\varphi_k$, which are similar for these two income sources (Table A2). Obviously, taxes lose their relative importance against benefits when we move from the absolute view (reflected in NGD) to the relative inequality view (reflected in MGD).

[16] Recall that, according to MGD, a contribution of public pensions is negative when they are less concentrated than total income, i.e., when $D^y(x_k) < G(y)$.

Table 4 MGD contributions

	Labour income	Capital income	Other private income	Public pension s	Means-tested benefits	NMTBs	SICs	PITs	Total income
DE	0.108	0.014	-0.006	-0.012	-0.020	-0.016	-0.003	-0.065	0.000
DK	0.095	0.061	-0.004	-0.018	-0.035	-0.034	0.001	-0.067	0.000
EE	0.111	0.005	-0.002	-0.067	-0.006	-0.013	-0.003	-0.025	0.000
EL	0.039	0.013	-0.003	0.011	-0.025	-0.003	-0.003	-0.029	0.000
ES	0.047	0.015	-0.002	0.012	-0.023	-0.007	0.005	-0.047	0.000
FR	0.011	0.039	-0.003	0.046	-0.032	-0.015	-0.001	-0.046	0.000
HR	0.075	0.004	-0.003	-0.014	-0.013	-0.007	-0.014	-0.030	0.000
HU	-0.022	0.003	-0.003	0.028	-0.006	-0.023	0.004	0.019	0.000
IE	0.201	0.011	0.000	-0.038	-0.052	-0.018	-0.010	-0.094	0.000
IT	0.037	0.007	-0.001	0.026	-0.015	0.005	-0.002	-0.057	0.000
NL	0.071	0.057	-0.003	-0.012	-0.038	-0.017	0.001	-0.058	0.000
PL	0.072	0.003	-0.001	-0.020	-0.029	-0.004	0.000	-0.020	0.000
SE	0.077	0.037	-0.002	-0.011	-0.017	-0.042	0.000	-0.042	0.000
SI	0.052	0.012	-0.001	0.006	-0.022	-0.013	-0.005	-0.029	0.000
SK	0.043	0.000	-0.002	-0.004	-0.011	-0.019	0.008	-0.015	0.000
AV	0.068	0.019	-0.002	-0.004	-0.023	-0.015	-0.001	-0.040	0.000

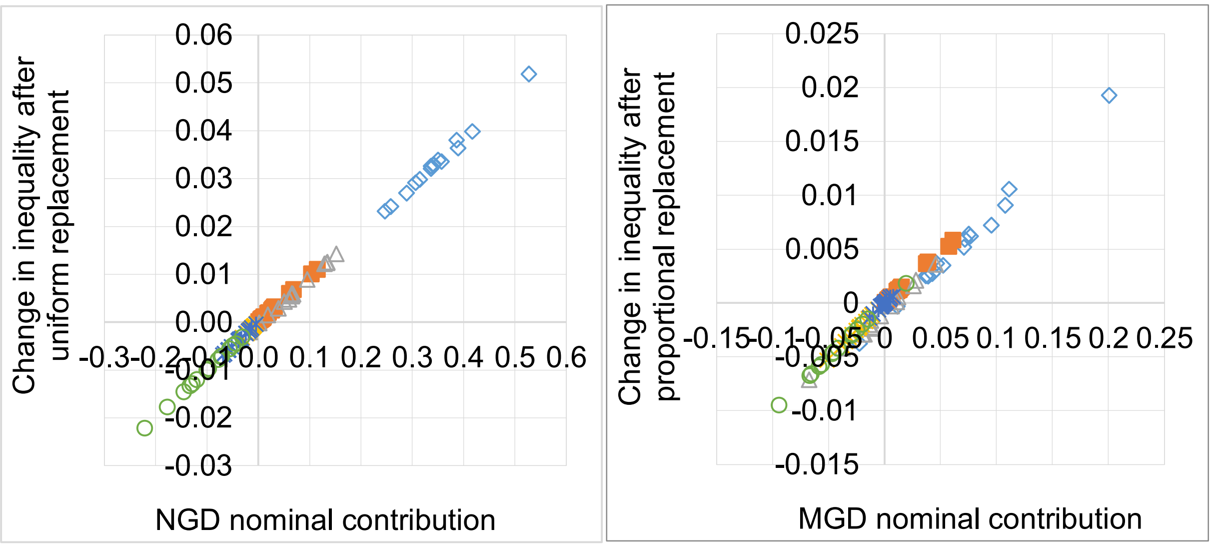
Notes: See notes for Table 3.

Imagine policymakers speculating about the ways to reduce total income inequality in their countries. The following proposal is made: 10 per cent of each person i's public pension ($x_{4,i}$) is going to be replaced by a uniform amount equalling 10 per cent of the mean public pension, $\bar{x}_4$. How much inequality will be wiped out by this reform? The answer to this question is almost readily available for the fifteen countries from our sample; we just have to multiply the figures in Table 3, column "Public pensions", by 10 per cent. Thus, one obtains reductions in the Gini index of up to 0.0152 points in Italy, which is quite significant. In Estonia, the reform would increase inequality by 0.0016 points.

The first proposal is based on Proposition 3. The second proposal is rooted in Proposition 4 and formulated as follows: 10 per cent of each person i's public pension ($x_{4,i}$) will be replaced by 10 per cent of $\varphi_4 y_i$. The results can be read from the column "Public pensions" in Table 4; just multiply the figure by 10 per cent. In many countries, this proposal would increase inequality, while inequality reduction would be highest in France; 0.0046 Gini points. The second proposal is more naturally applied to taxes: what if 10 per cent of each person i's actual PITs ($x_{8,i}$) is replaced by a counterfactual that is proportional to actual total income, i.e., by $0.1\varphi_8 y_i$? Because PITs are typically progressive, this change increases total income inequality, particularly in Ireland, by almost 1 Gini point.

Propositions 3 and 4 have dealt with marginal changes. However, 10 per cent is far from a tiny change. Are these propositions applicable in the case of such a relatively large change? We can check that by computing the actual changes induced by the above-described 10 per cent replacements. These changes are presented in Figure 1 for uniform replacements (left panel) and proportional replacements (right panel). There is a very strong correlation between NGD nominal contributions and total inequality changes induced by uniform replacements (left panel). In the case of proportional replacements (right panel), the correlation is somewhat weaker but still quite strong. This means that 10-per-cent replacements can be reasonably identified with marginal replacements.

Figure 1. Shares of income sources in total income



Notes: (1) *Left plot*. The horizontal axis shows contributions $C_k^R(G)$ from Table 3 for different countries and sources. The vertical axis shows reductions in the Gini index of total income following a uniform 10-per cent replacement of source incomes. (2) *Right plot*. The horizontal axis shows contributions $M_k^{LY}(G)$ from Table 4 for different countries and sources. The vertical axis shows reductions in the Gini index of total income following a proportional 10-per cent replacement of source incomes.

6 Conclusion

Natural Gini decomposition (NGD; Rao, 1969) and marginal Gini decomposition (MGD; Lerman and Yitzhaki, 1985) are the two most popular and widely used methods to reveal the contributions of various income sources to total income inequality. Their acceptance and highly spread empirical application have persisted in the face of criticism of the former method. The main critique is the one regarding the contribution of a uniformly distributed income source (UDS), which is zero according to NGD, although such a source is deemed as inequality reducing.

This paper has clarified that such an appraisal is unjustified. To understand why, we first have to recognise two different notions of income inequality: relative and absolute. Relative inequality indices are prevalent in the literature, and the Gini index is probably the most popular representative. When thinking about income inequality most people have the relative inequality concept in mind. However, if absolute inequality is taken into account, we realise that NGD belongs to it. Namely, according to the absolute inequality view, uniform income additions to all persons in a society do not change inequality. This is equivalent to the notion that inequality is unchanged upon the inclusion of newly created UDS into total income or exclusion of the existing UDS from total income. In favour of the argument about the absolute nature of NGD stands a proposition that a uniform marginal increase of all sources, which preserves their shares in total income, does not affect proportionate contributions $c_k^R(G)$. A similar proposition is derived for MGD contributions, demonstrating that MGD reflects the relative inequality view, which commands that proportional income additions leave inequality unchanged.

The paper has also dealt with the critique that NGD contribution terms “are not amenable to any sensible interpretation” (Podder, 1993).[17] We have shown that NGD contribution term for source k represents the reduction of inequality due to marginal replacement of source k income with a uniformly distributed equal-mean counterfactual. In an empirical example using 15 EU countries, it was shown that “marginal” does not necessarily have to be very small. Namely, the proposition holds even if the magnitude of replacement is 10 per cent. Thus, for example, the contribution of public pensions in Italy according to NGD is 0.152, which can be interpreted as follows. If 10 per cent of each person i’s public pension is replaced by a uniform amount equal to 10 per cent of the mean public pension, total income inequality would drop by 0.0152 Gini points.

While the theoretical and empirical literature is constantly upgraded with new methods to reveal the contributions of income sources to inequality by income sources (e.g., Kyzyma et al, 2022), NGD and MGD will continue to occupy an important role in the hands of practitioners in different fields of economics in the future. Therefore, it is important to be aware of their normative and interpretative aspects. This is also important when reading the conclusions of past studies, which were based on these methods.

[17] Similarly, Cancian and Reed (1998) say “that the decomposition component [$C_k^R(G)$] is not a meaningful measure of the effect of [source k income] on family income inequality.”

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Conflicts of Interest Statement. The author has no conflict of interest to declare.

Data Availability Statement. The data used in this study are not publicly available, and restrictions apply. The data can be made available from the Joint Research Centre and EUROSTAT. For more information, see https://euromod-web.jrc.ec.europa.eu/access-euromod.

Appendix 1 Proofs

Proof of Proposition 1

The pseudo-Gini index of actual total income equals:

$$D^y(y) = \sum_{k=1}^{K} \varphi_k D^y(x_k) = G(y)$$

whereas for the counterfactual vector $y - x_k + \bar{x}_k^{\mathrm{v}}$, we have that

$$D^y(y - x_k + \bar{x}_k^{\mathrm{v}}) = D^y(y) - \varphi_k D^y(x_k) + 0^{\mathrm{v}}$$

and correspondingly, $D^y(y) - D^y(y - x_k + \bar{x}_k^{\mathrm{v}}) = \varphi_k D^y(x_k) = C_k^R(G)$.□

Proof of Proposition 2

For the counterfactual vector $y - x_k + \tilde{x}_k^{\mathrm{v}}$, we have that

$$D^y(y - x_k + \tilde{x}_k^{\mathrm{v}}) = D^y(y) - \varphi_k D^y(x_k) + \varphi_k D^y(y)$$

and correspondingly, $D^y(y) - D^y(y - x_k + \tilde{x}_k^{\mathrm{v}}) = \varphi_k\big(D^y(x_k) - G(y)\big) = M_k^{LY}(G)$.□

Proof of Proposition 3

Suppose that $100\% \cdot (1-e)$ of source *k* income is replaced by a counterfactual income; how would such a change affect inequality? In this transition, the source *k* income is changed from x_k to $x_k - ex_k + e\bar{x}_k^{\mathrm{v}}$. We obtain the impact of the first change as follows:

$$\frac{\dot{G}(y)_{|x_k \to (1-e)x_k}}{e} = -\varphi_k(D^y(x_k) - G(y))$$

The impact of the second change is calculated as follows:

$$\frac{\dot{G}(y)_{|x_k \to x_k + e\bar{x}_k^{\text{v}}}}{e} = -\varphi_k G(y)$$

By combining two changes, we obtain the total impact of the marginal replacement of source *k* income with a uniformly distributed counterfactual:

$$\frac{\dot{G}(y)_{|x_k \to (1-e)x_k + e\bar{x}_k^{\text{v}}}}{e} = -\varphi_k\big(D^y(x_k) - G(y)\big) - \varphi_k G(y) =$$

$$-\varphi_k D^y(x_k) = -C_k^R(G).\square$$

Proof of Proposition 4

The transition from x_k to $x_k - ex_k + e\tilde{x}_k^{\text{v}}$ is divided into two parts: $x_k \to x_k - ex_k$ and $x_k \to x_k + e\tilde{x}_k^{\text{v}}$, where $\tilde{x}_k^{\text{v}} = \varphi_k y$. We have already seen above the effect of the first part. Regarding the second part, it is easy to see that $\dot{G}(y)_{|x_k \to x_k + e\tilde{x}_k^{\text{v}}} = 0$. Therefore, the combined effect equals

$$\frac{\dot{G}(y)_{|x_k \to (1-e)x_k + e\tilde{x}_k^{\text{v}}}}{e} = -\varphi_k\big(D^y(x_k) - G(y)\big).\square$$

Proof of Proposition 5

A uniform change constitutes of adding $e\varphi_k 1^{\text{v}}$ to each of the *K* sources, i.e., $\sum_{k=1}^{K} e\varphi_k 1^{\text{v}} = e1^{\text{v}}$ in total. The post-change Gini index of total income and post-change concentration index of source *k* income are equal to:

$$G(y^*) = \frac{1}{1+e} G(y)$$

$$D^y(x_k^*) = \frac{1}{1+e} G(y)$$

Taking into account that the change does not affect φ_k, the decomposition of the post-change total income is as follows:

$$G(y^*) = \sum_{k=1}^{K} \varphi_k D^y(x_k^*)$$

$$\frac{1}{1+e} G(y) = \sum_{k=1}^{K} \varphi_k \frac{1}{1+e} D^y(x_k)$$

In comparison with the decomposition of actual total income, the new decomposition has greater contribution terms for each source, but the proportionate contributions are identical.□

Proof of Proposition 6

This is trivial since the proportional changes of all elements in x_k do not affect $G(y)$ and $D^y(x_k)$.□

Appendix 2 Other tables

Table A1 Shares in total income

	Labour income	Capital income	Other private income	Public pension s	Means-tested benefits	NMTBs	SICs	PITs
DE	1.031	0.046	0.010	0.257	0.021	0.046	-0.186	-0.224
DK	1.086	0.158	0.006	0.213	0.067	0.060	-0.021	-0.568
EE	0.913	0.019	0.005	0.169	0.006	0.072	-0.031	-0.154
EL	0.807	0.050	0.020	0.370	0.037	0.010	-0.148	-0.147
ES	0.847	0.049	0.006	0.245	0.034	0.035	-0.062	-0.154
FR	0.794	0.101	0.007	0.301	0.047	0.043	-0.122	-0.171
HR	0.917	0.016	0.012	0.265	0.014	0.023	-0.182	-0.064
HU	1.001	0.010	0.012	0.301	0.008	0.037	-0.182	-0.187
IE	1.038	0.029	0.005	0.127	0.066	0.041	-0.044	-0.262
IT	0.850	0.054	0.008	0.353	0.020	0.054	-0.094	-0.246
NL	1.007	0.204	0.008	0.110	0.059	0.061	-0.235	-0.215
PL	0.938	0.007	0.006	0.299	0.044	0.023	-0.135	-0.183
SE	0.924	0.085	0.002	0.255	0.021	0.084	-0.064	-0.307
SI	0.949	0.026	0.005	0.247	0.026	0.067	-0.211	-0.108
SK	0.945	0.003	0.006	0.240	0.012	0.040	-0.176	-0.069
AV	0.937	0.057	0.008	0.250	0.032	0.046	-0.126	-0.204

Notes: See notes for Table 3.

Table A2 Proportionate NGD contributions divided by the absolute shares of income sources

	Labour income	Capital income	Other private income	Public pension s	Means-tested benefits	NMTBs	SICs	PITs
DE	1.35	2.02	-1.02	0.85	-2.24	-0.15	-1.05	-1.97
DK	1.32	2.43	-1.41	0.68	-0.92	-1.10	-0.76	-1.43
EE	1.40	1.96	-0.51	-0.32	-2.47	0.40	-1.38	-1.53
EL	1.15	1.78	0.54	1.09	-1.05	0.18	-1.05	-1.60
ES	1.16	1.90	0.10	1.14	-0.96	0.43	-0.75	-1.89
FR	1.05	2.30	-0.17	1.51	-1.29	-0.20	-1.02	-1.90
HR	1.27	1.93	0.28	0.83	-2.15	0.03	-1.25	-2.56
HU	0.93	2.02	0.29	1.28	-1.15	-0.90	-0.93	-0.69
IE	1.61	2.24	1.07	0.05	-1.47	-0.43	-1.75	-2.14
IT	1.12	1.35	0.59	1.20	-1.01	1.25	-1.06	-1.64
NL	1.25	1.99	-0.39	0.60	-1.29	0.03	-0.99	-1.96
PL	1.27	2.35	0.28	0.77	-1.27	0.34	-1.00	-1.37
SE	1.29	2.54	-1.80	0.84	-1.83	-0.79	-1.01	-1.48
SI	1.22	2.89	-0.12	1.10	-2.47	0.20	-1.10	-2.07
SK	1.20	1.69	-0.54	0.93	-3.15	-1.06	-0.81	-1.94
AV	1.24	2.09	-0.19	0.84	-1.65	-0.12	-1.06	-1.75

Notes: See notes for Table 3.

Table A3 MGD contributions divided by the absolute shares of income sources

	Labour income	Capital income	Other private income	Public pensions	Means-tested benefits	NMTBs	SICs	PITs	Total income
DE	0.10	0.31	-0.61	-0.05	-0.97	-0.35	-0.02	-0.29	0.10
DK	0.09	0.39	-0.65	-0.09	-0.52	-0.57	0.07	-0.12	0.09
EE	0.12	0.29	-0.45	-0.40	-1.05	-0.18	-0.11	-0.16	0.12
EL	0.05	0.26	-0.15	0.03	-0.68	-0.27	-0.02	-0.20	0.05
ES	0.05	0.31	-0.31	0.05	-0.67	-0.20	0.08	-0.31	0.05
FR	0.01	0.39	-0.35	0.15	-0.68	-0.36	-0.01	-0.27	0.01
HR	0.08	0.28	-0.22	-0.05	-0.95	-0.29	-0.07	-0.47	0.08
HU	-0.02	0.34	-0.24	0.09	-0.72	-0.64	0.02	0.10	-0.02
IE	0.19	0.39	0.02	-0.30	-0.78	-0.45	-0.23	-0.36	0.19
IT	0.04	0.12	-0.15	0.07	-0.72	0.09	-0.02	-0.23	0.04
NL	0.07	0.28	-0.39	-0.11	-0.65	-0.28	0.00	-0.27	0.07
PL	0.08	0.39	-0.21	-0.07	-0.65	-0.19	0.00	-0.11	0.08
SE	0.08	0.43	-0.79	-0.04	-0.80	-0.50	0.00	-0.14	0.08
SI	0.06	0.47	-0.28	0.03	-0.86	-0.20	-0.03	-0.27	0.06
SK	0.05	0.16	-0.35	-0.02	-0.95	-0.47	0.04	-0.22	0.05
AV	0.10	0.31	-0.61	-0.05	-0.97	-0.35	-0.02	-0.29	0.10

Notes: See notes for Table 3.

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