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Capturing the distribution sensitivity among the poor in a multidimensional framework. A new proposal

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Abstract

This paper aims to explore properties that guarantee that multidimensional poverty indices are sensitive to the distribution among the poor, one of the basic features of a poverty index. We introduce a generalization of the *monotonicity sensitivity axiom* which demands that, in the multidimensional framework too, a poverty measure should be more sensitive to a reduction in the income of a poor person, the poorer that person is. It is shown that this axiom ensures that poverty diminishes under a transfer from a poor individual to a poorer one, and therefore it can also be considered a straightforward generalization of the *minimal transfer axiom*. An axiom based on the notion of ALEP substitutability is also introduced. This axiom captures aversion to both dispersion of the distribution, and attribute correlation, and encompasses the *multidimensional monotonicity sensitivity axiom* we propose. Finally, we review the existing multidimensional poverty families and identify which of them fulfil the new principles.

Keywords: Multidimensional poverty, distributional dispersion, transfer principle, attribute dependence.
JEL Classification: D30, D63.

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1 Introduction

Sen (1976) proposes that a poverty measure should be sensitive to the distribution among the poor, and greater weight should be attached to the poorer in society. He introduces the *transfer axiom*, which requires that a poverty index should decrease when a transfer of income is made from one individual to a poor person with a lower income. This axiom admits transfers that can imply a change in the number of poor individuals. Donaldson and Weymark (1986) suggest a weaker proposal, the *minimal transfer axiom*, demanding that the donor should also be poor, that is, it requires that poverty should decrease when a transfer of income is made from a poor person to a poorer one.

These axioms rely on the idea that social states in which poor people are equal are better. A branch of poverty literature has been developed based on the belief that poverty need not rely on egalitarianism to justify a focus on the worse off.[1] According to this view, if poor person i is worse off than poor person j, benefits to the former are more important. In one sense this is because person i is worse off than j, but this has nothing to do with their relative situation. It is rather that both are worse off than they might have been, although at different levels. Poverty should reflect a poor person's deprivations independently of whether others are richer or indeed, poorer.

The *monotonicity sensitivity axiom* introduced by Kakwani (1980) depicts this idea in an uncontroversial way. It specifically requires that a poverty measure should be more sensitive to a drop in a poor person's income; the poorer the person happens to be.[2] Even though this axiom seems not to be directly concerned with transfers, it is interesting to note that this

[1] See Barrientos (2010) for more detail.
[2] Kakwani (1980) originally proposed this axiom in terms of individual's rank, that is, comparing the increase in the poverty levels under decreases of incomes of two individuals with different ranks. He also proposes two other sensitivity axioms for a poverty measure based on income transfers.

axiom and the *minimal transfer axiom* are equivalent (Zheng (1997)). Consequently the justifications for each of these two axioms are valid for both.

The need for a multidimensional approach to the measurement of poverty has already been emphasized, and a number of axioms have been introduced trying to capture multidimensional poverty. Some of them attempt to generalize those existing in the unidimensional framework.[3] In this respect, this paper proposes a generalization of the *monotonicity sensitivity* axiom, henceforth MS, which demands that in a multidimensional context too, a poverty measure should be more sensitive to a decrease in any attribute of a poor individual, the poorer the person is. To our knowledge, up to now, no generalization of this axiom has been introduced.

By contrast, different generalizations of the *minimal transfer axiom* have been proposed, such as the *multidimensional transfer axiom*, henceforth MTP,[4] explored by Tsui (2002) and Bourguignon and Chakravarty (2003), and the *one dimensional transfer principle*, henceforth OTP, introduced by Bourguignon and Chakravarty (2003).[5] However, MTP and OTP deal with transfers between any two poor individuals, one not necessarily poorer than the other in all dimensions. Somehow, the absence of this requirement makes both MTP and OTP too demanding and prevents them from being considered as proper generalizations of the *monotonicity sensitivity axiom*. In the context of multidimensional inequality measurement, Fleurbaey and Trannoy (2003) propose the *Pigou-Dalton bundle dominance criterion* which considers only transfers between two individuals, one unambiguously richer than the other.[6] Taking into account this kind of transfer, this paper proves that it is equivalent for a poverty measure to fulfil MS or to satisfy that the poverty level diminishes under a transfer from a

[3] A thorough survey of the literature on multidimensional poverty is provided by Chakravarty (2009) and Kakwani and Silber (2008).
[4] MTP is the poverty counterpart to the *uniform majorization criterion* of multidimensional inequality indices proposed by Kolm (1977). Tsui (2002) refers to it as the *poverty non –increasing minimal transfer axiom with respect to uniform majorization criterion*.
[5] These two principles are defined in Section 3.1.
[6] Díez et al. (2007) analyze the relationships between this criterion and others commonly used in a multidimensional inequality framework.

poor individual to a poorer one. Consequently, MS may be considered not only as a generalization of the *monotonicity sensitivity* axiom, but also as an alternative and straightforward generalization of the *minimal transfer axiom.*

Some other specific issues in the multidimensional context have also been analyzed. For example, Bourguignon and Chakravarty (2003) suggest an interesting principle, the *nondecreasing poverty under correlation increasing switch axiom*, NDCIS, which is concerned with the sensitivity of poverty to the correlation between distributions of attributes. This property establishes conditions over the poverty measure when attributes are regarded as substitutes.[7] In turn, the *poverty nondecreasing rearrangement*, PNR; proposed by Tsui (2002), also involves correlation increasing switches, but for all the attributes. The motivation for this principle is that poverty should not decrease when the rearrangement makes the poorer poor worse off. In order to be sensitive to attribute dependence and attribute dispersion, most of the existing multidimensional poverty measures have been derived assuming MTP along with PNR or NDCIS.

In the context of multidimensional inequality Lasso de la Vega et al. (2010) introduce a principle which captures aversion to both dispersion and attribute correlation. It is based on the notion of ALEP substitutability. None of the distributional axioms introduced in the multidimensional poverty literature are able to capture both issues together. Here, we introduce the ALEP *principle* in the context of multidimensional poverty and show that this axiom encompasses MS, and PNR.

In this paper, the implication of these new axioms, MS and ALEP, are examined. We prove that on separable poverty families, invoking MTP and PNR, or MTP and NDCIS, is more demanding than ALEP, and consequently than MS. As a result, the families derived in Bourguignon and Chakravarty (2003), Chakravarty et al. (1998), Chakravarty and Silber

[7] The corresponding property for attributes considered as complements assures that that poverty should not increase under a correlation increasing switch.

(2008), and Tsui (2002), fulfil these new axioms. We review other families that are non consistent with them.

The paper is structured as follows. Section 2 begins with a brief introduction of the notation and basic definitions. Then, Section 3 reviews some multidimensional axioms concerned with the distribution among the poor, and proposes the new ones; MS and ALEP. In Section 4 the implications of these criteria are examined and some important multidimensional poverty families are reviewed. Finally, Section 5 offers some conclusions.

2. Notation and basic definitions

We consider a population of $n \geq 2$ individuals. The number of attributes relevant to assess poverty is $k \geq 2$, where *k* is given and fixed.

A multidimensional distribution among the population is represented by an $n \times k$ real matrix *X*. The *ij*th entry of *X*, denoted by x_{ij}, represents the *i*th individual's amount of the *j*th attribute. The *i*th row of *X,* denoted by x_i, is individual *i*'s vector of attributes. We denote $M(n,k)$ the class of all $n \times k$ real matrices over the non-negative real elements. Let *D* be the set of all such matrices, $D = \bigcup_{n \in N} M(n,k)$, where *N* is the set of positive integers. When the class of all $n \times k$ real matrices over the positive real elements is considered, $M_+(n,k)$, the corresponding set of all such matrices would be denoted by $D_+ = \bigcup_{n \in N} M_+(n,k)$. Comparisons of vectors of attributes are denoted as follows: $x_q \geq x_p \ if \ x_{qj} \geq x_{pj}$ for all $j = 1,...,k$, $x_q > x_p \ if \ x_{qj} \geq x_{pj}$ and $x_p \neq x_q$.

Regarding the identification of the poor through the specification of a poverty line, let's consider $z_j > 0$ to be the minimum quantity of the *j*th attribute for a subsistence level. An individual i is deprived as regards attribute j if $x_{ij} < z_j$. Let $z = (z_1, z_2, ..., z_k) \in \mathbb{R}^k_{++}$ be the vector of thresholds for all the dimensions, and let x^*_{ij} be the censored value of x_{ij} defined as $x^*_{ij} = \min\{x_{ij}, z_j\}$. A number of different approaches may be used to identify the multidimensional poor in a society. [8] Nevertheless the results in this paper may be established regardless of the identification method selected.

In this paper a multidimensional poverty index is a non-constant real value function $P : D \times \mathbb{R}^k_{++} \to \mathbb{R}$. For any $X \in D$ and $z \in \mathbb{R}^k_{++}$, $P(X, z)$ determines the extent of poverty in a social situation represented by the distribution matrix X and the threshold vector z. The index P may be assumed to satisfy the following properties, which are straightforward generalizations of those suggested for a single dimensional poverty index:

* *Focus:* For any $(X, z) \in D \times \mathbb{R}^k_{++}$ and for any person i and attribute j such that $x_{ij} \geq z_j$, and increase in x_{ij} does not change the poverty level $P(X, z)$.

* *Monotonicity*: For any $(X, z) \in D \times \mathbb{R}^k_{++}$ and for any person i and attribute j such that $x_{ij} < z_j$, and increase in x_{ij} should decrease the poverty level $P(X, z)$.

* *Continuity*: For any $z \in \mathbb{R}^k_{++}$, P is a continuous function in $X \in D$.

* *Normalization:* For any $(X, z) \in D \times \mathbb{R}^k_{++}$ if $x_{ij} \geq z_j$ for all i and j, then $P(X, \underline{z}) = 0$.

* *Symmetry:* For any $(X, z) \in D \times \mathbb{R}^k_{++}$, $P(X, z) = P(\prod X, z)$ where $\prod$ is any $n \times n$ permutation matrix.

* *Replication invariance:* For any $(X, z) \in D \times \mathbb{R}^k_{++}$, $P(X, z) = P\left(X^{(l)}, z\right)$, where $X^{(l)}$ is a l-fold replication of X, that is, $X^{(l)} = \left(X^1, X^2, ..., X^l\right)$ with each $X^i = X$, and $l \geq 2$ is arbitrary.

[8] See Atkinson (2003), Gordon et al. (2005), Bourguignon and Chakravarty (2003) among others.

Since the focus on poverty measurement is the deprivations of the poor people, the first two postulates are considered as basic axioms for a poverty index. *Focus* demands that the extent of poverty remains unchanged under an increase of any attribute with respect to which a person is not deprived. According to the *monotonicity axiom*, poverty should decrease under an increase of any attribute as regards which a person is deprived. Moreover, *continuity* ensures that the poverty index does not abruptly change under small changes in any individual's attribute. *Normalization* requires P to be equal to 0 when nobody is deprived in any attribute. If *symmetry* is assumed the individuals are not distinguished by anything other than their attributes, and poverty indices which are invariant under replications allow populations of different sizes to be compared.

The following property is related to the partitioning of the population into subgroups, and the interdependencies between the subgroup poverty levels and global poverty. It basically establishes that if the population is split into groups according to social characteristics, such as region, race, gender and so on, then overall poverty is the population weighted average of the subgroup poverty levels. Poverty measures which fulfil this property allow us to calculate the contribution of each subgroup to overall poverty.

In the extreme case where each group is constituted by a single individual, this axiom may be written as follows

* *Separability:* P is separable if there exists a function $p:\mathbb{R}_{+}^{k}\times\mathbb{R}_{++}^{k}\rightarrow\mathbb{R}$ such that:

$$P(X,z)=\frac{1}{n}\sum_{i=1}^{n}p(x_i,z) \text{ for all } X\in D \qquad (1)$$

It may be worth noting that function p is the same for all the individuals and it is usually interpreted as individual i's multidimensional poverty level.

One intuitive implication of this property is that an increase in the poverty of one group increases overall poverty, that is, the multidimensional version of the *subgroup consistency axiom*[9] introduced by Tsui (2002) is satisfied by any decomposable measure.

3. Multidimensional axioms concerned with distribution among the poor. New proposals

In this section we introduce some axioms concerned with distribution among the poor, and analyze the relationships between them and others often invoked in this field.

First we propose a straightforward generalization of the *monotonicity sensitivity axiom* proposed by Kakwani (1980), which requires that the increase in poverty due to a decrease in any attribute of a poor person should be greater, the poorer the person is. Let us consider two poor individuals, *i* and *j*, such that the vector of attributes of individual *i* is smaller than that of individual *j*, i.e., $x_p < x_q$. Let us assume that it is possible to decrease any component of the two vectors by the same absolute amount. *Monotonicity* assumes that the poverty level increases under the two transformations. However, the multidimensional version of the *monotonicity sensitivity axiom* goes beyond *monotonicity,* and requires that the increase in poverty should not be smaller under the former decrease, that of the poorer person's vector of attributes. The following definition captures this idea.

Definition 1: Let $X \in D$. Distribution Y is derived from X by a decrement (increment) δ to individual *i* if:

i) $y_m = x_m$ for all $m \neq i$

ii) $y_i = x_i - \delta$ ($y_i = x_i + \delta$) where $\delta = \left(\delta_1, ..., \delta_k\right) \in \mathbb{R}_+^k$ with at least one $\delta_j > 0$.

[9] This axiom is introduced in the one-dimensional context by Foster and Shorrocks (1991).

* *Multidimensional monotonicity sensitivity axiom,* MS*:* P satisfies MS if $P(Y;z)-P(X,z)\geq P(Y';z)-P(X,z)$ for any $X\in D$, and for all Y, and Y' matrices derived from X by a decrement δ to poor individual *p,* and to poor individual *q*, respectively, such that $x_p < x_q - \delta$.

Notice that this axiom demands that the poorer individual in the initial social situation *X,* should also be the poorer in the final situations *Y* and *Y'*. Consequently the relative position of the individuals involved can not be reversed. A strong version of this axiom may be defined, strong MS, which allows the possibility of changes in their relative positions; it suffices to consider $x_p < x_q$ instead of $x_p < x_q - \delta$ in the definition of MS.

The following proposition shows how MS can also be interpreted as a multidimensional version of the *minimal transfer axiom*, since it also extends the basic idea behind this principle, that is, that a transfer from a poor person to a poorer one, which preserves the order, diminishes the poverty level.[10] Before presenting this result, we need to formalize this type of transfer. This is based on the definition of a PDB transfer in Fleurbaey and Trannoy (2003) as a sequence of progressive transfers of some attributes between two unambiguously ordered individuals.

Definition 2: Let $X, Y\in D$. Distribution *Y* is derived from *X* by a *PDB transfer* if there exist two individuals p and q such that:

i) $y_m = x_m$ for all $m\neq p,q$

ii) $y_q = x_q - \delta$ and $y_p = x_p + \delta$ where $\delta = (\delta_1,...,\delta_k)\in \mathbb{R}^k_+$ with at least one $\delta_j > 0$.

iii) $y_q \geq y_p$

[10] As already mentioned, Zheng (1997) shows that, in the unidimensional setting, the *monotonicity sensitivity axiom* (Kakwani (1980)) and the *minimal transfer axiom* (Donaldson and Weymark (1986)) impose the same restrictions on a poverty measure.

Note that conditions ii) and iii) imply that, in both the initial and the final distributions, individual *q* is richer than individual *p* in all the attributes.

Proposition 1: *A poverty measure* $P: D \times \mathbb{R}^k_{++} \rightarrow \mathbb{R}$ *satisfies MS if and only if* $P(Y,z) \leq P(X;z)$ *for any* $X \in D$ *and for all Y matrices derived from X by a finite sequence of PDB transfers among the poor.*

Proof: We only prove the necessity part of this proposition, since the proof of the sufficiency is straightforward following a similar reasoning.

Let $X \in D$. If *P* satisfies MS then for all Y, and Y' matrices derived from *X* by an increment δ to poor individual *p,* and to poor individual *q*, respectively, such that $x_p < x_q$, and $x_p + \delta < x_q$ we have $P(Y;z) \leq P(Y';z)$ providing that no one is becoming non-poor after the increments. Noting $x_q + \delta$ as t_q, and x_i as t_i for all $i \neq q$, and rewriting matrices *Y* and *Y'* with this notation, it follows that matrix *Y* can be interpreted as derived form *Y'* by a PDB transfer among the poor and we have the result.

Q.E.D.

Other axioms which consider redistributions between poor individuals are traditionally assumed in the multidimensional context. In the rest of this section we revise some of them. Since in this framework many types of redistributions among the poor are imaginable, each of them leading to a different criterion for ordering multidimensional poverty distributions, we present these criteria classified by the type of transformations involved. First we consider axioms concerned with transfers between the poor, and then rearrangements between them. A new axiom, ALEP, considering both types of transformation will be introduced in the last subsection.

3.1 *Taking into account dispersion of attribute distributions*

The two properties considered below are proposed in order to capture dispersion of attribute distributions. The first one, proposed by Bourguignon and Chakravarty (2003), involves a progressive transfer between two poor individuals, involving a fixed amount of some attribute in which both individuals are deprived.[11]

Definition 3: Let $X, Y \in D$. Distribution Y may be derived from distribution X by a *Pigou-Dalton progressive transfer of attribute l* if there exist two individuals p and q such that:

i) $y_{qj} = x_{qj}$, $y_{pj} = x_{pj}$ for all $j \neq l$; and $y_m = x_m$ for all $m \neq p, q$,

ii) $y_{ql} = x_{ql} - \delta$ and $y_{pl} = x_{pl} + \delta$ where $\delta > 0$

iii) $y_{ql} \geq y_{pl}$

* *One dimensional transfer principle,* OTP*: P* satisfies OTP if $P(Y, z) \leq P(X; z)$ for any $X \in D$ and for all Y matrices derived from X by a finite sequence of Pigou Dalton progressive transfers among the poor of some attributes with respect to which they are deprived.

Tsui (2002) and Bourguignon and Chakravarty (2003) analyze the implications of another axiom that involves multidimensional transfers, replacing the original bundles of attributes of any pair of poor individuals by some convex combination of all their attributes.

Definition 4: Let $X, Y \in D$. Distribution Y may be derived from distribution X by a *uniform majorization transfer* if there exists a $n \times n$ bistochastic matrix B that is not a permutation matrix such that $Y = BX$.

[11] This progressive transfer means that the less deprived individual in the transferred attribute gives some amount of it to the more deprived one.

* *Multidimensional transfer axiom,* MTP*:* *P* satisfies MTP if $P(Y,z) \leq P(X;z)$, for any $X \in D$ and for all *Y* matrices derived from *X* by a *uniform majorization transfer* among the poor.

According to MTP, poverty should not increase when the poor individuals become closer in the attribute space by a transfer of all the dimensions in the same proportion. It can be seen that whereas OTP involves only transfers of attributes with respect to which both poor individuals are deprived; MTP deals with transfers in all dimensions.

These two axioms can be regarded as multidimensional generalizations of the *minimal transfer axiom*, since OTP and MTP reduce to it when only one attribute is considered. Note that, when more attributes are taken into account, these two axioms deal with transfers between any pair of poor individuals, and neither of them imposes the condition that the transfer has to be carried out from a poor person to a poorer one.

It is obvious that any poverty measure fulfilling OTP also satisfies MS, whereas the converse is not true, as we are going to see in the next section.

3.2 When the attribute dependence matters

The axioms in section 3.1 do not take account of the statistical dependence between the distributions of the attributes, a crucial feature for measuring inequality according to Atkinson and Bourguignon (1982). Bourguignon and Chakravarty (2003) introduce the *nondecreasing poverty under correlation increasing switch axiom, NDCIS* one way in which this dependence is taken into account. This axiom is concerned with a correlation increasing switch between two poor individuals of two attributes as regards which both are deprived. This rearrangement means that a poor person who has a lower amount of an attribute gets a lower amount of the other. If the attributes are substitutes this axiom requires poverty not to decrease under such a switch.

Definition 5: Let $X, Y \in D$. Distribution X may be derived from distribution Y by a *correlation increasing switch* if there exist two individuals p and q, and two attributes, h and l, such that (i) $x_{ph} = \min\{y_{ph}, y_{qh}\}$, $x_{pl} = \min\{y_{pl}, y_{ql}\}$, (ii) $x_{qh} = \max\{y_{ph}, y_{qh}\}$, $x_{ql} = \max\{y_{pl}, y_{ql}\}$ and (iii) $x_m = y_m$ for all $m \neq p, q$, and $x_{qj} = y_{qj}$, $x_{pj} = y_{pj}$ for all $j \neq h, l$.

* *Nondecreasing poverty under correlation increasing switch axiom,* NDCIS*:* A multidimensional poverty measure P satisfies NDCIS if $P(Y,z) \leq P(X;z)$ for any $Y \in D$ and for all X matrices derived from Y by a permutation of rows, and a finite sequence of correlation increasing switches between two poor individuals who are deprived as regards the two attributes concerned.

The authors argue that, if the attributes involved in the transfer are substitutes, poverty decreases less with an increase in one attribute, for instance *h*, for individuals with larger quantities of another attribute, say *l*.

When there are only two attributes considered, a correlation increasing switch between them can be regarded as a regressive rearrangement, in the sense that one poor person is becoming the better-off, and the other the worse off. However, when more attributes are taken into account, this idea is not true in general, since a correlation increasing switch does not necessarily involve comparisons between two poor individuals, one unambiguously poorer than the other. The next axiom introduced by Tsui (2002) is concerned with a transfer under which the bundles of attributes of two poor individuals are rearranged, so that one receives at least as much of every attribute as the other, and more of at least one attribute.

Definition 6: Let $X, Y \in D$. Distribution X may be derived from distribution Y by a *correlation increasing transfer* if there exist two individuals p and q such that

(i) $x_{pj} = \min\{y_{pj}, y_{qj}\}$ for all $j = 1,...,k$, (ii) $x_{qj} = \max\{y_{pj}, y_{qj}\}$ for all $j = 1,...,k$ and (iii) $x_m = y_m$ for all $m \neq p, q$.

* *Poverty non-decreasing rearrangement*, PNR*:* *P* satisfies PNR if $P(Y,z) \leq P(X;z)$ for any $Y \in D$ and for all *X* matrices derived from *Y* by a permutation of rows and a finite sequence of correlation increasing transfers among the poor, with no one becoming non-poor due to the transfers.

PNR ensures that a poverty measure captures aversion to correlation between dimensions, and implicitly assumes that all the attributes are substitutes.

When only two attributes are considered NDCIS and PNR coincide. Moreover, when more are taken into account, they differ since NDCIS also admits rearrangements between two non ordered poor, i.e., one not necessarily better off.

3.3 Taking into account attribute dispersion and attribute dependence.

None of the axioms above captures aversion towards both attribute correlation and attribute dispersion. For doing so, we also propose the following axiom which is based on the notion of ALEP substitutability. [12]

Definition 7: Let $X, Y \in D$. Distribution *Y* is derived from *X* by a *compensating transfer* if there exist two individuals *p* and *q* such that:

i) $x_q > x_p$

ii) $y_m = x_m$ for all $m \neq p, q$

iii) $y_q = x_q - \delta$ and $y_p = x_p + \delta$ where $\delta = (\delta_1,...,\delta_k) \in \mathbb{R}_+^k$ with at least one $\delta_j > 0$

iv) $y_q \geq x_p$

[12] ALEP stands for Auspitz-Lieben-Edgeworth-Pareto.

This kind of transfer encompasses the definition of a PDB transfer and a correlation increasing transfer. The first is obtained when the ranking between the individuals is not reversed. The second, when the amount of the attribute transferred is equal to the difference between the endowments of the individuals involved in the transfer. The corresponding principle for a poverty measure can be the following:

* *ALEP principle,* ALEP*: P* satisfies ALEP if $P(Y) \le P(X)$ for any $X \in D$ and for all *Y* matrices derived from *X* by a finite sequence of compensating transfers among the poor.

Since a correlation increasing switch is also a compensating transfer, ALEP entails PNR. Furthermore, taking into account proposition 1 and the fact that a PDB transfer is a particular type of compensating transfer, it follows that ALEP also implies MS. Consequently, ALEP entails MS and PNR, which capture dispersion and correlation of the attribute distributions respectively. In addition, the following proposition allows a different interpretation of ALEP in terms of sensitivity of the poverty measure to a drop in a poor person's attribute, the poorer the person is. In fact, proposition 2 shows that ALEP coincides with strong MS when the limiting case is also assumed, that is when the vectors of attributes of both individuals *i* and *j* in definition of strong MS satisfy $x_p \le x_q$.

Proposition 2: *A poverty measure* $P : D \times \mathbb{R}^k_{++} \to \mathbb{R}$ *satisfies ALEP if and only if* $P(Y;z) - P(X,z) \ge P(Y';z) - P(X,z)$ for any $X \in D$, and for all Y, and Y' matrices derived from *X* by a decrease δ to poor individual *p,* and to poor individual *q*, respectively, such that $x_p \le x_q$.

Proof: To prove the sufficiency, it suffices to follow a similar reasoning to that of the necessity part of proposition 1, considering a compensating transfer instead of a PDB transfer. The necessity is straightforward following a similar argument.

Q.E.D.

4. Implications of the new axioms

This section reviews some important multidimensional poverty families and examines the implications of MS and ALEP. As most multidimensional families are separable measures, in the following we consider only poverty measures that can be written according to equation (1).

First, we briefly recall the implications of the existent multidimensional criteria for a separable poverty function, already explored by Tsui (2002) and Bourguignon and Chakravarty (2003).

The following lemma is easily obtained adapting the result in Tsui (2002, proposition1), which in turn is based on a well known result of Foster and Shorrocks (1991). It shows that any separable poverty measure may be constructed using a two-stage procedure. In the first step, a function, denoted by ϕ, is used to aggregate the poor individual's bundle of attribute deprivations into a statistical summary. Then, these statistics are summed. This result is presented in the following lemma. [13]

Lemma 1: *A poverty measure $P : D \times \mathbb{R}^k_{++} \to \mathbb{R}$ satisfies separability if and only if there exists a continuous and non increasing function $\phi : \mathbb{R}^k_{+} \times \mathbb{R}^k_{++} \to \mathbb{R}$ such that, for every $X \in D$,*

$$P(X,z) = \frac{1}{n}\sum\nolimits_{1 \le i \le n} \phi(x_i, z) \text{ for all } X \in D \qquad (2)$$

where $\phi(x_i, z) \equiv \phi\left(\left(\min(x_{i1}, z_1), ..., \min(x_{ik}, z_k), z\right)\right)$; and $\phi(x_i, z) = 0$ when $x_{ij} \ge z_j$ for all $j = 1, ..., k$.

[13] This lemma is implicitly states in Tsui (2002), although he actually refers to subgroup consistent poverty functions.

Tsui (2002) and Bourguignon and Chakravarty (2003) examine the implications of MTP for a separable measure. Specifically, they prove that MTP is equivalent to requiring that the function ϕ in equation (2) be convex. In addition, they show that the set of poverty measures satisfying OTP is more restrictive than those satisfying MTP, since it includes only separable poverty functions that are also additive across attribute components. In particular they prove that if a multidimensional separable poverty measure P fulfilling OTP possesses second partial derivatives then the following relationship holds

$$P(X;z) = \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{k} p_j\left(x_{ij}, z_j\right) \quad (3)$$

where $p_j(\ )$ is the individual poverty function associated with attribute j.

PNR is usually assumed in order to impose that the measure be sensitive to attribute dependence. These authors prove that for a separable function PNR is fulfilled if and only if ϕ in equation (2) is L-superadditive, that is, $\phi(x_p)+\phi(x_q) \geq \phi(y_p)+\phi(y_q)$ for all $y_p, y_q \in \mathbb{R}_{++}^k$ and for all x_p, x_q as specified in definition 6. Bourguignon and Chakravarty (2003) also show that for every separable function fulfilling NDCIS ϕ is L-superadditive. Thus PNR is fulfilled when NDCIS is assumed.

Now we are going to explore the relationships between these axioms and the two new ones proposed in this paper.

A primary consequence of ALEP for a separable poverty function is that function ϕ in equation (2) has non decreasing increments, that is

$$\phi\left(x_p+\delta\right)-\phi\left(x_p\right) \leq \phi\left(x_q+\delta\right)-\phi\left(x_q\right) \quad (4)$$

for all $x_p \leq x_q$ and for all $\delta \in \mathbb{R}_{+}^k$.[14]

[14] Its counterpart for a utility function is defined in terms of non increasing increments. When ϕ is twice continuously differentiable on $\mathbb{R}_{++}^k$, then equation (4) holds if and only if $\phi_{hl}^{''} \geq 0$ for all $h,l=1,...,k$ (Chipman (1977)).

From proposition 1 it is easy to see that any separable measure consistent with MS satisfies equation (4) whenever $x_p + \delta \le x_q$.

Our next proposition shows that requiring validity of MTP and PNR, or MTP and NDCIS, for a separable function is more demanding than invoking ALEP and consequently MS.

Proposition 3: *If a separable poverty measure* $P: D \times \mathbb{R}^k_{++} \to \mathbb{R}$ *satisfies MTP and PNR, or* MTP and NDCIS, *then it fulfils ALEP.*

Proof: Since for a separable function NDCIS implies PNR, it suffices to prove that if P satisfies MTP and PNR, then it fulfils ALEP.

Let x_p and x_q be the two poor individuals' bundles involved in a compensating transfer. Let's assume that they only transfer an amount δ_1 of the first attribute and let $\delta = \left(\delta_1, 0, ..., 0\right)$. Given that P is a separable poverty measure, it suffices to prove that $\phi\left(x_p\right) + \phi\left(x_q\right) \ge \phi\left(x_q - \delta\right) + \phi\left(x_p + \delta\right)$ where $x_q - \delta$ and $x_p + \delta$ are the two individuals' bundles after the transfer, and $x_p \le x_q - \delta$. Let's consider $u = \left(x_{q1}, x_{p2}, \ldots, x_{pk}\right)$ the bundle of an additional poor individual. Then

$\phi\left(x_p\right) + \phi\left(x_q\right) + \phi\left(u\right)$

$\ge \phi\left(x_q\right) + \phi\left(x_p + \delta\right) + \phi\left(u - \delta\right)$ by *separability* and MTP,

$\ge \phi\left(x_q - \delta\right) + \phi\left(x_p + \delta\right) + \phi\left(u\right)$ by PNR.

Again by *separability* we get the result. *Q.E.D.*

This proposition 3 makes a simple but important observation about the relationships of these criteria for separable poverty functions. Most of the poverty classes in the literature are separable functions fulfilling MTP and PNR, or MTP and NDCIS. Then, proposition 3 ensures that all of them satisfy ALEP, and consequently MS. In the following we will review some of these families.

Chakravarty et al. (1998) obtain the general form of relative separable poverty measures which are additive across attributes and fulfil MTP, that is given by

$$P\left(X;z\right)=\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{k}a_j h\left(\frac{x_{ij}}{z_j}\right) \quad (5)$$

where $h:\mathbb{R}_+\to\mathbb{R}$ is continuous, non increasing and convex; and $h\left(x_{ij}/z_j\right)=0$ for all $x_{ij}\geq z_j$.[15] As can be seen, these indices are insensitive to a correlation increasing switch, and then all of them also satisfy NDCIS and PNR. Chakravarty and Silber (2008) review different classes of indices when some functional forms for function *h* are selected in equation (5). The following classes are multidimensional extensions of the Chakravarty (1983), Watts (1968) and Foster et al. (1984) poverty indices, respectively.[16] As all these indices measure poverty restricting attention to the censored values of x_{ij}, x_{ij}^*, they may be expressed in terms of these values as follows

$$P_{CM}\left(X;z\right)=\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{k}a_j\left(1-\left(\frac{x_{ij}^*}{z_j}\right)^{e_j}\right) \text{ where } 0<e_j\leq 1 \quad (6)$$

$$P_{WM}\left(X;z\right)=\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{k}a_j\log\left(\frac{z_j}{x_{ij}^*}\right) \text{ where } a_j\geq 0\text{, and } a_j>0 \text{ for some } j \quad (7)$$

$$P_{FGTM}\left(X,z\right)=\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{k}a_j\left(1-\frac{x_{ij}^*}{z_j}\right)^{\alpha_j} \text{ where } \alpha_j\geq 1 \quad (8)$$

An interesting poverty family of indices which fulfil MTP and NDCIS is suggested by Bourguignon and Chakravarty (2003). It is given by

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[15] They, instead of requiring that poverty be additive across attributes, introduce *factor decomposability* which also requires that the non negative sequence of $\{a_j\}$ satisfies $\sum_{j=1}^{k}a_j=1$

[16] In equation (7), $X\in D_+$.

$$P_{BC}(X,z)=\frac{1}{n}\sum_{i=1}^{n}\left[\sum_{j=1}^{k}a_j\left(1-\frac{x_{ij}^*}{z_j}\right)^{\eta}\right]^{\alpha/\eta} \text{ where } \eta>1, \text{ and } \alpha\geq\eta \qquad (9)$$

Tsui (2002) assumes the *multidimensional subgroup consistency axiom*, and characterizes the family of poverty indices which fulfill MTP and PNR given by

$P_{TM}(X,z)=\frac{1}{n}\sum_{i=1}^{n}\left[\prod_{j=1}^{k}\left(\frac{z_j}{x_{ij}^*}\right)^{\alpha_j}-1\right]$ where the non negative parameters α_j 's have to be chosen such that function $\phi(\theta)=\prod_{j=1}^{k}\theta_j^{-\alpha_j}$, $\theta_j\in(0,1]$ is convex with respect to $(\theta_1,\ldots,\theta_k)$.[17]

Taking into account proposition 3, it is clear that all the indices above satisfy MS and ALEP.

An alternative of interest arises from the approach followed by Maasoumi and Lugo (2008). They derive multidimensional poverty indices based on the information theory. As regards this family examples can be found to prove that MS, and consequently ALEP, are not fulfilled. To see this, let us consider a society with two individuals *p* and *q*, whose bundles of attributes are $(4,10)$ and $(2,10)$, respectively, and suppose that the vector of thresholds for both dimensions is $z=(5,12)$. Notice that both individuals are deprived in both attributes. Let us assume two different transformations leading to two different social situations, A and B. In the first case, only the first attribute of the poorer individual *p* is decreased by 1 unit, and the bundles of attributes become $(4,10)$ and $(1,10)$ (social situation A). In the second case, only the first attribute of the richer individual *q* is decreased by the same amount, and the bundles of attributes come to be $(3,10)$ and $(2,10)$ (social situation B). According to MS the increase in poverty in the first case should not be smaller than in the second. However, we get that the

[17] In this family $X\in D_+$.

poverty level in social situation A is 0.6708 whereas in social situation B it is 0.7035 if the Maasoumi and Lugo poverty index given by

$$P_{ML}\left(X;z\right)=\frac{1}{n}\sum_{i=1}^{n}\left(\max\left(1-\frac{\left(\sum_{j=1}^{k}w_j x_{ij}{}^{\theta}\right)^{1/\theta}}{\left(\sum_{j=1}^{k}w_j z_j{}^{\theta}\right)^{1/\theta}};0\right)\right)^{\alpha},$$

where $\alpha=0.5$, $\theta=-10$, and $w_1=w_2=0.5$ is considered.

5. Conclusions

This paper has explored a new axiom, MS, which demands that a multidimensional poverty measure should be more sensitive to a decrease in any attribute of a poor person, the poorer the person happens to be. It has been shown that it equivalently imposes that a transfer from a poor person to a poorer one should diminish the poverty level. This axiom may be considered as a multidimensional generalization of both the *monotonicity sensitivity axiom* (Kakwani (1980)) and the *minimal transfer axiom* (Donaldson and Weymark (1986)). These two axioms, although based on different requirements, have the same consequences for a poverty measure. There already exist other properties proposed in the literature, but none of them encompasses the ideas behind these two principles.

Moreover, an extension of this new principle, referred to as ALEP, which encompasses sensitivity to the dispersion of attribute distributions and to the correlation between them, has also been considered.

Finally, this paper has revealed that there exist some multidimensional poverty indices that violate the new axioms. Nevertheless, we have shown that the existing multidimensional poverty families which fulfill MTP and PNR, or MTP and NDCIS, satisfy ALEP and MS, and consequently preserve the underlying motivations behind the *monotonicity sensitivity axiom* and the *minimal transfer axiom*.

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