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Increasing skill premium and education decisions: Higher intra-skilled inequality and lower inter-skill mobility.

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JEL Cassification: I21, J24, J31, J62.

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1. Introduction

This paper shows that an increase in earnings inequality between skilled and unskilled workers (a higher skill premium) typically encourages higher education attending, but it also (i) favours children from skilled families by reducing intergenerational mobility between the skilled and the unskilled (‘*inter-skill mobility*’) and (ii) fosters inequality across skilled workers (‘*intra-skilled inequality*’) when higher education is costly. The rationale is based on education decisions and the impact depends on education financing. If they apply to both advanced and emerging economies, those findings particularly concern the former because the skill premium has increased in all advanced countries in the last decades.

Since the eighties, advanced economies have experienced a substantial increase in earnings inequality. This inequality upsurge has been observed between skilled and unskilled workers but also and significantly across skilled workers. Growing earnings inequality has come with a significant rise in tertiary education enrolment and in the proportion of skilled workers in the population. Those observed facts suggest that the massification of tertiary education has been accompanied by a rising skill divergence across educated workers. In addition, a number of works diagnose either a decrease or at least a stagnation of intergenerational mobility in earnings and education in several advanced countries in the last thirty years.

We investigate the impact of changes in skilled and unskilled wages and in the related education cost upon skill accumulation and intergenerational skill mobility. In this purpose, we develop a model of intergenerational skill investment where the decision to attend the university at the end of compulsory basic education depends on the net return to skill and on the effort necessary to pursue tertiary studies.

Assuming an increase in the real skilled wage, a decrease in the real unskilled wage and thereby a decrease in the skill premium (ratio of the skilled to the unskilled wage), we analyse the impact of those changes on the individuals’ education behaviour depending on their parents’ skill and on the way they finance higher education. Three means of financing are considered: 1) parental bequest conditioned by education; 2) freely provided higher education; 3) loans to the students guaranteed by the parents. We find that 1) in the three cases, the change in wages normally increases university enrolment, but 2) this can come with (i) a greater difference in skill between the individuals born in skilled families and those born in unskilled families and (ii) a higher positive impact on enrolment for the former, except in the case of freely provided higher education. This tends to lessen inter-skill mobility and to widen inequality across skilled workers.

In a nutshell: when higher education is not publicly and freely provided, growing earnings inequality between the skilled and the unskilled fosters upward mobility, but more for individuals from skilled families, and some individuals from unskilled families could undergo a decrease in they tertiary education level. Those findings are in line with a set of facts observed in advanced economies in the last decades.

This contribution is original in several respects. First, the approach sets a linkage between skilled/unskilled earnings inequality in one generation and earning inequality across skilled workers at the next generation. Second, by considering several means of financing, we show that costly higher education enlarges the gap between skilled and unskilled families in university attending by several ways, even if both types of families can benefit from an increase in tertiary education enrolment. Finally, our findings suggest that globalization and technological change have not only generated overall earnings inequality and polarization, but also a growing differentiation among skilled workers.

Section 2 exposes some stylised facts and briefly presents the economic literature related to the subject. Section 3 displays the model general framework and section 4 the education decisions depending on university financing and on the individual’s family type, skilled or unskilled. We introduce changes in skilled and unskilled wages in section 5 and we analyse their impact on education decisions and intergenerational skill mobility. Our major findings are discussed and we conclude in section 6.

2. Facts and Literature

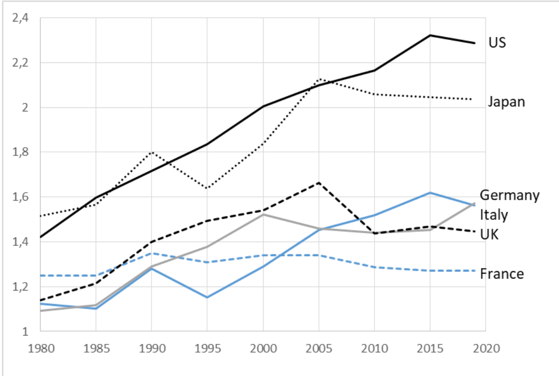
2.1. Facts

The analysis is based on several stylised facts.

First, within advanced economies, the last four decades have been characterised by an increase in inequality. Inequality has risen between skilled and unskilled workers (Fig.1) as well as across skilled workers (Fig.2).

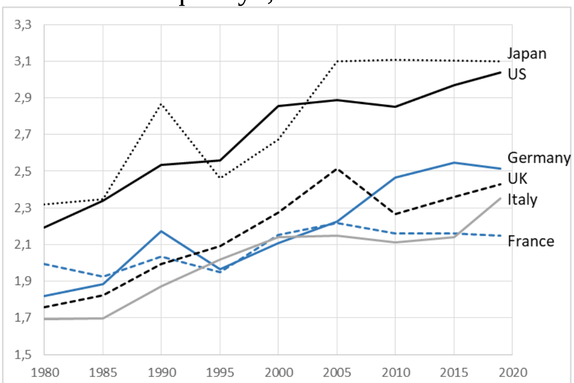
Second, the rise in the skill premium has occurred despite a significant increase in the proportion of skilled workers in the working population (Figs. 3) and a large increase in the proportion of students by generation (Fig. 4).

Fig. 1. Skilled/unskilled earnings inequality*, five advanced economies, 1980-2019



Source: WID. Pre-tax income. Individuals.* Ratio $\frac{\textit{Income share of percentile } \text{p80-p100}}{\textit{Income share of percentile } \text{p30-p70}}$. The 20% lowest incomes are not considered to erase part time jobs and the poorest households, and the interval p70-p80 because the related individuals can have a tertiary degree or not depending on the period and the country.

Fig. 2. Between-skilled inequality*, five advanced economies, 1980-2019



Source: WID. Pre-tax income. Individuals. * Ratio $\frac{\textit{Income share of percentile } \text{p90-p100}}{\textit{Income share of percentile } \text{p80-p90}}$. The highest skills are assumed to be in the p90-p100 decile and the skilled (tertiary education) with a lower level in the decile p80-p90.

Fig. 3. Persons with tertiary education 24-65 y.o. (%)

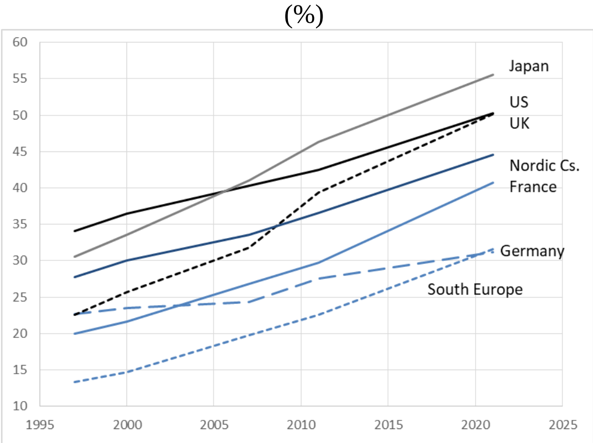
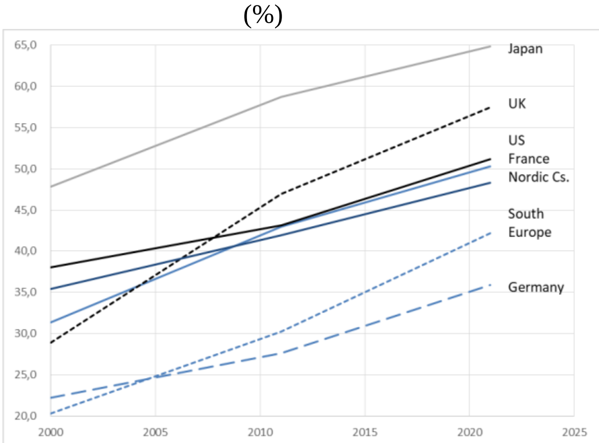


Fig. 4. Persons with tertiary education, 24-35 y.o. (%)



Source: OECD Stat. Dataset: Educational attainment. Note: The data reliably picture the variation for each country, but they are rather inadequate to compare the countries because of differences in tertiary education systems and in the scope of tertiary education across countries. In particular, tertiary education comprises a proportion of the post-secondary non tertiary in Japan and the UK. In Germany, the number of students in post-secondary non tertiary education is very high and not recorded, which tends to reduce the weight of persons with tertiary education.

Finally a number of works indicate that, following a continuous increase in the post – World War II period, intergenerational earnings mobility has either decreased or remained constant for the generations born from the seventies in most advanced countries (see, e.g., Aaronson & Mazumder, 2008, Mazumder, 2012, and Chetty et al., 2014, for the US; Blanden et al., 2004, 2007, and Nicoletti & Ermisch, 2007, for the UK.; Ben-Halima et al., 2014, and Lefranc, 2018, and for France; Harding & Munk, 2019, for Denmark; Connolly et al., 2022, for Canada, etc.).

2.2. Literature

Our approach investigates the impact of inequality between skilled and unskilled workers on education decisions, intergenerational skill mobility and between-skilled inequality. The related literature is large since it concerns skilled/unskilled inequality, education decisions, between skilled inequality and the relation between inequality and intergenerational mobility.

The starting point is the increase in inequality between skilled and unskilled workers. Three major explanations have been given to this rise in inequality, i.e., (i) globalization, particularly trade with emerging countries, (ii) factor-biased technical progress and (iii) institutional changes. Those explanations are typically based on a demand – supply – institutions framework defined by Freeman & Katz (1994) and Katz & Autor (1999).[1] Both skill-biased technical progress and North-South trade tend to increase the demand for skill and enlarge thereby the wage gap between skilled and unskilled workers. Institutions and labour market policies can foster inequality if they reduce redistribution and they can generate unemployment when the increase in inequality is at least partially prevented. We do not present here the different explanations of the increase in inequality and we refer to the large number of reviews on the subject.[2] The general diagnosis on this subject is that the three factors have fostered skilled/unskilled inequality, but (i) their impact differ across countries, sectors and periods, and (ii) they typically interact.

The key mechanisms of our approach are based on the impact of an increase in the return to skill upon education decisions. A large literature has analysed the different effects of a rising skill premium on education decisions. Most of those works relate the rising skill premium either to trade between advanced and emerging countries (North-South trade) or to technological change. In all cases, it is possible to distinguish two opposite effects. First, a higher return to skill logically encourages education because of its *incentive effect*. Second, the rise in the skilled

[1] Se the general presentation of this framework in Hellier (2013a)
[2] See, e.g., Singh & Dhumale (2004), Chusseau et al. (2008), Chusseau & Dumont (2013), Helpman (2017) etc.

wage increases the cost of education because this activity is skill-intensive. This *education cost effect* hampers human capital accumulation, particularly for families with low incomes and when the credit market is imperfect.

In the case of trade, the incentive effect has been put forward in the seminal article of Findlay & Kierzkowski (1983) who insert human capital accumulation into a North-South Heckscher-Ohlinian model with skilled and unskilled labour. A number of works have subsequently extended this analysis by introducing additional assumptions (e.g., Grossman & Helpman, 1991; Janeba, 2003; Falvey et al., 2010; Borissov & Hellier, 2013, etc.). Still based on the impact of trade on the return to skill, the cost of education effect is in contrast primary in the models developed by Cartiglia, (1997) and Eicher (1999).

In the models of Dinopoulos & Segerstrom (1999) and Auer (2015), trade interacts with technological change so as to encourage skill acquisition in advanced economies.

The same two effects are generated when the increase in the skill premium results from technological change. As regards the technology – inequality – education nexus, an interesting dynamics has been highlighted by Goldin & Katz (2007, 2009). The rationale is as follows. A high skill premium fosters education and skill accumulation. The resulting decrease in the skilled labour cost promotes skill-biased technical change, re-increasing the skill premium and encouraging education once again, resetting thereby the same sequence. This generates a fluctuation in inequality and a positive skill dynamics.

The economic analysis of growing inequality across skilled workers has focused on the emergence of superstars.[3] Since Rosen's seminal article (1981) in which the difference in talents is amplified by earnings, several additional factors have been highlighted to explain the dramatic increase in superstars' incomes: the selection of a limited number of winners through stochastic processes (Adler, 1985; Chung & Cox,1994), a growing demand for general managerial skills at the expense of firm-specific skills (Murphy & Zabojnik, 2007; Frydman, 2019), the firms' size (Gabaix & Landier, 2008) and an increase in wage competition between managers (Subramanian, 2013). Our contribution is different because it relies between-skilled growing inequality to the education decisions of individuals.

Finally, following the observed relationship between inequality and intergenerational earnings mobility (the 'Great Gatby Curve') put forward by Corak (2012, 2013), a large literature has attempted to explain and confirm the negative impact of inequality on intergenerational mobility (see the review article by Durlauf et al., 2022).

[3] The large literature on polarization and the shrinking weight of the middle class is not tackled here because a large part of the middle class has no tertiary education degree.

3. The model

3.1. Individuals and Education

Individuals are identified by their dynasty *i* (sequence of successive generations linked by a parent-child relationship) and their generation *t*.

In each dynasty, there is one individual per generation and *individual* (*i*,*t*) depicts the *t*-th generation in dynasty *i*.

Individuals differ 1) in their family backgrounds measured by their parents' human capital and 2) in their personal aptitudes randomly distributed across individuals in the interval $\left[\underline{a},\bar{a}\right]$ at each generation.

Education is composed of two stages, basic education which is compulsory and higher education provided by the university which depends on the individual's choice.

All individuals firstly follow compulsory basic education which provides them for the basic human capital:

$$b_{it} = \delta_B \times a_{it} \times h_{it-1}{}^{\eta}, \quad 0 < \eta < 1 \tag{1}$$

where δ_B depicts the productivity in basic education which depends on both public expenditures and educational technologies, a_{it} is individual (*i*,*t*)'s personal aptitude and h_{it-1} is the parent's human capital.

Human capital is equal to basic human capital b_{it} for individuals who do not pursue higher education and to skilled human capital s_{it} (defined below) for those who enter the university.

At the end of basic education, individual (*i*,*t*) takes her/his decision on university attending by maximising the utility u_{it} which depends positively on her/his net income once working and negatively on the personal effort e_{it} s/he must provide to obtain a higher education degree:

$$u_{it} = u\left(w_{it}, e_{it}\right) = \left(w_{it}\right)^{\beta}\left(\bar{e} - e_{it}\right)^{1-\beta} \tag{2}$$

where $\bar{e}$ is the maximum personal effort an individual can provide, assumed identical for all individuals, and w_{it} is individual (*i*,*t*)'s net income.

The individuals' education behaviour comprises two stages. They firstly calculate the optimal effort and the net income related to university attending. They subsequently compare the utility of higher education (university) with the utility of basic education and select the option which provides the highest utility.

If the individual does not attend the university, s/he possesses one unit of simple (unskilled) labour and s/he is paid w_L (wage per unit of unskilled labour) which is her/his net income.

If the individual has a university degree, her/his net income is equal to her/his wage (wage per unit of skill w_H multiplied by the individual's skill s_{it}) reduced by the cost of university paid by the individual, c_{it}, which depends on her/his selected skill level, her/his personal characteristics and the financing means. The cost is nil ($c_{it} = 0$) and the net income is equal to $w_H s_{it}$ if the university is paid by someone else (the State, the parents etc.) without refunding.

Individuals select their education strategy by considering the wage corresponding to each skill level at the moment of their decision, i.e., the wages at their parents' generation. To simplify, the generation (time) is omitted when denoting wages.

The individual's expected net income is thus:

$$w_{it} = \begin{cases} w_L & \text{if entering the labour market after basic education} \\ w_H s_{it} - c_{it} & \text{if attending the university} \end{cases} \quad (3)$$

Higher education is comprised of a continuum of degrees. The individual's skill (skilled human capital) is equal to the degree s/he obtains.

Obtaining a degree implies to make a personal effort and to pay a fee. The higher the degree, the higher the corresponding effort and fee. Effort and fee are thus complements in the higher education function which can be written:

$$s_{it} = \min\left\{b_{it}\left(1+\delta_H e_{it}\right), \lambda f_{it}\right\} \quad (4)[4]$$

where (i) f_{it} is the quantity of higher education services bought by the individual, (ii) δ_H is a parameter depicting the productivity of effort which partially depends on public subsidies to higher education, and (iii) λ depicts the productivity of educational technology (a higher λ entails a lower cost for one given educational attainment).

[4] We could have defined the relation between effort and skill as $s_{it} = b_{it}\left(1+\delta_H {e_{it}}^{\theta}\right)$, $0 < \theta < 1$. This indicates that the marginal skill produced by the effort is decreasing In this case, the optimal effort is the unique positive root of the equation $(1-(1-\theta)\beta)\delta_H e_{it} + (1-\beta){e_{it}}^{1-\theta} - \beta\theta\delta_H \overline{f} = 0$ (proof available upon request). As there is no direct analytic solution to this equation, this would have made the following reasoning and proofs very cumbersome without changing the analysis outcomes.

The complementarity between f_{it} and e_{it} is logic because a higher f_{it} indicates that the individual buys more knowledge which necessitates more effort to be assimilated.

The expression $b_{it}\left(1+\delta_H e_{it}\right)$ in function (4) indicates that basic education is an input of skill acquisition and that the effort efficiency depends on the human capital level reached at the end of basic education.

The fee corresponding to the utilisation of a quantity f_{it} of education services is $w_H f_{it}$. This indicates that educational services are produced with skilled labour.

Complementarity in the higher education function (4) makes that an efficient choice implies:

$$s_{it} = b_{it}\left(1+\delta_H e_{it}\right) = \lambda \times f_{it} \qquad (5)$$

Consequently, the education cost to achieve the skill level s_{it} is $w_H f_{it} = w_H s_{it} / \lambda$.

3.2. Inequality between skilled and unskilled workers

We do not endogenise the variation in wages which can, as noted in section 2.2, result from several causes (globalization, technological change, institutional changes etc.). We assume an exogenous increase in the real wage per unit of skill w_H, an exogenous decrease in the real wage per unit of unskilled (simple) labour w_L, and thereby an increase in the skill premium $\omega \equiv w_H / w_L$. Making those variation in wages endogenous just consists in adding a simple production framework to the model. Doing this would only encumber the presentation without changing anything in the analysis.

4. Education decision

We distinguish three ways to finance higher education. In the first, university studies are paid by parental bequests depending on parents' incomes and conditional to university attending. This can put a constraint on the individuals' higher education choices. The second assumes a costless access to higher education. This corresponds to universities being fully financed by public expenditures. The third funding pattern is based on individuals' borrowing, with differences in interest rates depending on the individual's family. This renders education more costly for unskilled families. Other financing patterns are discussed in the conclusion and presented in Appendix C and D.

4.1. Education-oriented parental altruism

The parents divide their income between the family consumption and the bequest to their child to finance higher education.

University financing is fully paid by the parents without children's refunding, and parental altruism is conditioned by university attending. Consequently, individuals cannot freely utilise their parents' bequests. If not spent for education, the bequest is lost. In addition, the young have no access to credit. Those assumptions make higher education free for individuals $\left(c_{it}=0\right)$ and its funding limited by the parents' bequest.

The potential bequest q_{it-1} is a proportion ρ of parent's net income w_{it-1}:[5]

$$q_{it-1}=\rho w_{it-1} \tag{6}$$

As the skilled wage is the cost of one unit of higher education service, the individual's financing constraint is:

$$w_H f_{it} \leq \rho w_{it-1} \tag{7}$$

4.1.1. *Education choice*

The individual firstly maximises the utility function (2) subject to the higher education function (5) and to the financial constraint (7). This determines the optimal effort and income corresponding to university attending. Then the individual compares the related utility with the utility deriving from basic education and selects the education strategy which maximises her/his welfare. Being in the working life, the individual distributes her/his income between the family consumption and the bequest for her/his child's education.

Without financial constraint, the optimal choice in higher education yields the following values of the individual's effort, skill and fee (proofs in Appendix A; the hat indicates an optimal value without financial constraint):

$$\hat{e}_{it}=\beta\overline{e}-\frac{1-\beta}{\delta_H} \tag{8}$$[6]

[5] Corresponding to the maximization programme: $\max v_{it-1}=\begin{cases}(1)\ c_{it-1} & ,\ c_{it-1}\leq w_{it-1}-w_H f_{it},\ if\ w_H f_{it}<\hat{q}_{it-1}\\ (2)\ c_{it-1}^{1-\rho}q_{it-1}^{\rho}, & c_{it-1}+q_{it-1}\leq w_{it-1},\ if\ w_H f_{it}=\hat{q}_{it-1}\end{cases}$,

where q is the bequest, $\hat{q}_{it}$ is the solution of (2) and $w_H f_{it}$ the child's university fee.

[6] It is assumed that $\delta_H\overline{e}>(1-\beta)/\beta\Rightarrow\hat{e}_{it}>0$.

$$\hat{s}_{it} = \beta(1+\delta_H \bar{e})b_{it} \tag{9}$$

$$\hat{f}_{it} = \frac{\beta}{\lambda}(1+\delta_H \bar{e})b_{it} \tag{10}$$

The corresponding financing constraint ($w_H f_{it} \le \rho w_{it-1}$) is:

$$b_{it} \le \frac{\lambda\rho}{\beta(1+\delta_H \bar{e})} \frac{w_{it-1}}{w_H} \tag{11}$$

If the financial constraint is ineffective (condition (11) is fulfilled at the optimum values (8)–(10)), individual (i,t) attends the university if the related utility $u\left(w_H \hat{s}_{it}, \hat{e}_{it}\right)$ is higher than the utility of entering the labour market at the end of basic education $u\left(w_L, 0\right)$.

If the financial constraint is effective (condition (11) is rejected at the optimum values (8)–(10)), then individual (i,t)'s buying of higher education services if attending the university, $\tilde{f}_{it}$, and the related effort and skill are (proofs in Appendix A; the tilde indicates an optimum value when the financial constraint is effective):

$$\tilde{f}_{it} = \rho \frac{w_{it-1}}{w_H} \tag{12}$$

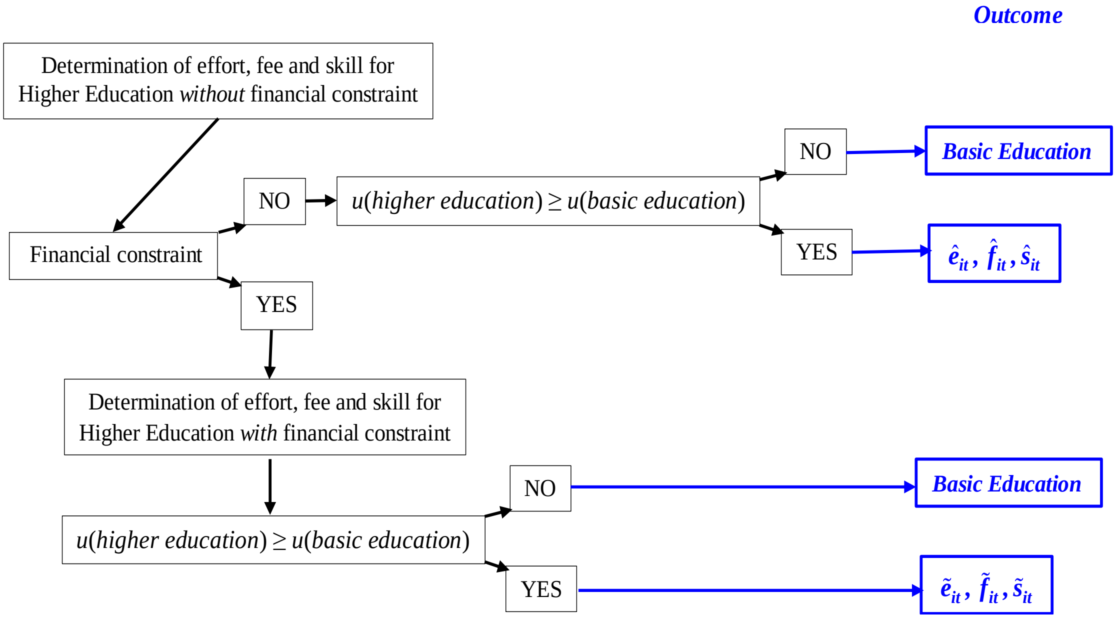
$$\tilde{e}_{it} = \frac{\lambda\rho}{\delta_H b_{it}} \frac{w_{it-1}}{w_H} - \frac{1}{\delta_H} \tag{13}$$

$$\tilde{s}_{it} = \lambda\rho \frac{w_{it-1}}{w_H} \tag{14}$$

The individual then attends the university if the corresponding utility $(w_H \tilde{s}_{it})^{\beta}(\bar{e} - \tilde{e}_{it})^{1-\beta}$ is higher than the utility of entering the labour market at the end of basic education $(w_L)^{\beta}(\bar{e})^{1-\beta}$.

The decision tree corresponding to the education choice is pictured in Schema 1. This decision tree clearly shows that the outcomes, constraints and conditions of the education decision directly depend on the individual's family background, i.e., on the parents' human capital h_{it-1} and their related income w_{it-1}.

Schema 1. Higher education decision tree



4.1.2. *Individuals born in unskilled families*

Individuals born in unskilled families are such that $h^L_{it-1} = b_{it-1}$ and $w^L_{it-1} = w_L$ (superscript L indicates an unskilled family).

The financial constraint (Eqs. 11 with $b_{it} = \delta_B a_{it} s_{it-1}{}^{\eta}$) is ($\omega \equiv w_H / w_L$ is the skill premium):

$$a_{it} \leq \varphi^L(\omega, b_{it-1}) = \frac{\lambda\rho}{\beta\delta_B(1+\delta_H\bar{e})}\omega^{-1} b_{it-1}{}^{-\eta} \tag{15}$$

An increase in the skill premium ω strengthens the financial constraint. Likewise, a high human capital at the end of basic education increase the skill objective and reinforces the probability of being constrained (since the constraint can be written $b_{it} \leq \frac{\lambda\rho}{\beta(1+\delta_H\bar{e})}\omega^{-1}$).

The condition for attending the university (the related utility is higher than the utility brought by basic education) for individuals born in unskilled families depends on whether they are constrained or not. This condition can be written (Appendix A):

$$a_{it} \geq a^L_{\varphi}(\omega, b_{it-1}) = \frac{\beta(1+\delta_H\bar{e})}{1+\delta_H\bar{e}-(\lambda\rho)^{-\beta/(1-\beta)}\delta_H\bar{e}}\varphi^L(\omega, b_{it-1}) \text{ if they are constrained} \tag{16}$$

$$a_{it} \geq a^L_{no\varphi}(\omega, b_{it-1}) = \frac{1}{\lambda\rho}\left(\frac{1}{1-\beta}\frac{\delta_H\bar{e}}{1+\delta_H\bar{e}}\right)^{\frac{1-\beta}{\beta}}\varphi^L(\omega, b_{it-1}) \text{ if they are unconstrained} \tag{17}$$

It can be easily verified that:

$$a_{\varphi}^{L} \underset{<}{\overset{>}{=}} \varphi^{L} \Leftrightarrow a_{no\varphi}^{L} \underset{<}{\overset{>}{=}} \varphi^{L} \tag{18}$$

The equivalence (18) stipulates that the values a_{φ}^{L} and $a_{no\varphi}^{L}$ are always either both higher than φ^{L} or both lower. As a consequence, two situations are possible (Eqs. A11 and A17 in Appendix A):

1) When $\frac{\delta_H \bar{e}}{1+\delta_H \bar{e}} > (1-\beta)(\lambda\rho)^{\frac{\beta}{1-\beta}}$, then all the unconstrained individuals from unskilled families select basic education, and the sole individuals who attend the university are those with an ability higher than $a_{\varphi}^{L}(\omega, b_{it-1})$.

2) When $\frac{\delta_H \bar{e}}{1+\delta_H \bar{e}} < (1-\beta)(\lambda\rho)^{\frac{\beta}{1-\beta}}$, then all the financially constrained individuals from unskilled families select the university as well as the financially unconstrained with the highest basic education level (those with an ability higher than $a_{no\varphi}^{L}(\omega, b_{it-1})$).

It is clear that being financially constrained goes with a higher probability to attend the university. This is because unskilled families are unconstrained when their skill objective is low. But then, keeping unskilled could typically bring them a higher utility.

The human capital level, the effort and the education expenditure when attending the university for constrained and unconstrained individuals are determined in Appendix A.

4.1.3. *Individuals born in skilled families*

Individuals born in skilled families are such that $h_{it-1}^{H} = s_{it-1}$ and $w_{it-1}^{H} = w_H s_{it-1}$, where superscript H indicates an unskilled family.

The financial constraint (Eq. 11) is thus $b_{it} \leq \frac{\lambda\rho}{\beta(1+\delta_H \bar{e})} s_{it-1}$, which can be written since $b_{it} = \delta_B a_{it} s_{it-1}{}^{\eta}$:

$$a_{it} \leq \varphi^{H}(s_{it-1}) = \frac{\lambda\rho}{\beta(1+\delta_H \bar{e})\delta_B} s_{it-1}{}^{1-\eta} \tag{19}$$

When individuals come from skilled families, their financial constraint is independent from wages. This is because the wage per unit of skill w_H is also the price of education services. Hence, the increase in the former offsets the negative impact of the increase in the latter.

When financially ***unconstrained,*** the individual's effort, skill, fee and utility if s/he attends the university are determined in Appendix A. The related condition for unconstrained individuals to attend the university is (proof in Appendix A):

$$a_{it} \geq a_{no\varphi}^{H}\left(\omega, s_{it-1}\right) = \frac{1-\beta}{\beta\delta_B\delta_H\overline{e}}\left(\frac{1}{1-\beta}\frac{\delta_H\overline{e}}{1+\delta_H\overline{e}}\right)^{1/\beta}\omega^{-1}{s_{it-1}}^{-\eta} \tag{20}$$

Note that since $\partial a_{no\varphi}^{H} / \partial\omega < 0$ an increase in the skill premium ω relaxes the condition to enter the university.

When financially ***constrained***, the individual's effort, skill, fee and utility if s/he attends the university are depicted in Appendix A. The related condition for constrained individuals to attend the university is (proof in Appendix A):

$$a_{it} \geq a_{\varphi}^{H}\left(\omega, s_{it-1}\right) = \frac{\beta}{1-\frac{\delta_H\overline{e}}{1+\delta_H\overline{e}}\left(\lambda\rho\omega s_{it-1}\right)^{-\beta/(1-\beta)}}\varphi^{H}\left(s_{it-1}\right) \tag{21}$$

The complete analysis of function (21) is made in Appendix A, section A.3.2. It can be noted that $\partial a_{\varphi}^{H} / \partial\omega < 0$ for $a_{\varphi}^{H} > 0$, which indicates that an increase in the skill premium relaxes the condition to attend the university.

4.2. Free higher education

The cost of higher education is paid by the State and higher education is freely provided to individuals. Hence, the cost of higher education is nil for all individuals ($c_{it} = 0,\ \forall(i,t)$).

From the individuals' point of view, this situation is identical to the case of parental altruism without funding constraint as modelled in section 4.1. This leads to the following condition to select the university:[7]

$$a_{it} \geq a^{free}\left(\omega, h_{it-1}\right) = \frac{(1-\beta)^{\frac{\beta-1}{\beta}}}{\beta\delta_B\delta_H\overline{e}}\left(\frac{\delta_H\overline{e}}{1+\delta_H\overline{e}}\right)^{1/\beta}\omega^{-1}{h_{it-1}}^{-\eta} \tag{22}$$

with $h_{it-1} = b_{it-1}$ if the individual is born in an unskilled family and $h_{it-1} = s_{it-1}$ if s/he is born in a skilled family.

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[7] This condition is identical to conditions (17) and (21) determined in section 2.1 for unskilled and skilled unconstrained families respectively.

Since there is no higher education cost, the condition to select the university is the same for all individuals and only depend on the basic education level $b_{it} = \delta_B \times a_{it} \times h_{it-1}{}^{\eta}$ and the skill premium ω .

4.3. University funding through borrowing

The funding of higher education is now ensured by loans to the students. Those loans are guaranteed by the parents. Consequently, the interest rates on student loans include a risk premium which is based on the parents' incomes. The higher this income, the lower the risk premium and the interest rate on the children's borrowing.

The university cost can thus be written:

$$c_{it}(s_{it}) = R_{it} w_H f_{it} \tag{23}$$

with $R_{it} = 1 + r_{it}$ being individual (i,t)'s borrowing cost, r_{it} the interest rate paid by this individual, and f_{it} the amount of higher education services necessary to obtain the skilled human capital s_{it} ($f_{it} = s_{it} / \lambda$).

The borrowing cost R_{it} is defined by the function:

$$R_{it} = R\left(\underline{w}_{it-1}\right), \ \frac{\partial R}{\partial \underline{w}_{it-1}} < 0, \ \frac{\partial^2 R}{\partial \underline{w}_{it-1}{}^2} > 0, \ R\left(\underline{w}_{it-1}\right) \xrightarrow[\omega_{it-1} \to +\infty]{} \bar{R} \geq 1 \tag{24}$$

where $\underline{w}_{it-1}$ is parent (i,t-1)'s *gross* income (i.e., without deduction of the parent's education cost), which indicates that the lender (the bank) does not know the grand parent's skill and bases the interest rate on the apparent income.

Let $\hat{s}_{it}$ be individual (i,t)'s optimal skill if s/he decides to attend the university (this value is determined in Appendix B, Eq. B2). Then, the net income from higher education is $w_{it}^H = w_H \hat{s}_{it} - c_{it}(\hat{s}_{it}) = \left(1 - R_{it} / \lambda\right) w_H \hat{s}_{it}$, and the related condition for choosing to attend the university is (proof in Appendix B):

$$a_{it} \geq \frac{1-\beta}{\beta \delta_B \delta_H \bar{e}} \left(\frac{1}{1-\beta} \frac{\delta_H \bar{e}}{1+\delta_H \bar{e}} \right)^{1/\beta} \frac{\lambda}{\lambda - R_{it}} \omega^{-1} h_{it-1}{}^{-\eta} \tag{25}$$

The above condition depends on the family's type (skilled or unskilled) which determines both h_{it-1} and $R_{it} = R\left(\underline{w}_{it-1}\right)$.

4.3.1. *Unskilled families*

In an unskilled family, the borrowing cost is $R_{it} = R(w_L),\ \forall(i,t)$, and the parent's human capital $h_{it-1} = b_{it-1}$. Consequently, the condition (25) to choose higher education is:

$$a_{it} \geq \alpha^L\left(\omega, w_L, b_{it-1}\right) = \frac{1-\beta}{\beta\delta_B\delta_H\bar{e}}\left(\frac{1}{1-\beta}\frac{\delta_H\bar{e}}{1+\delta_H\bar{e}}\right)^{1/\beta}\frac{\lambda}{\lambda - R(w_L)}\omega^{-1}b_{it-1}{}^{-\eta} \qquad (26)$$

The function $\alpha^L\left(\omega, w_L, b_{it-1}\right)$ is such that $\frac{\partial\alpha^H}{\partial\omega} < 0$ and $\frac{\partial\alpha^H}{\partial w_L} < 0$. An increase in the skill premium firstly releases the condition to select the university. Second, in addition to its pro-education impact through the skill premium, a decrease in the unskilled wage w_L has a negative impact on education by moving upward the interest rate paid by individuals from unskilled families, which raises the cost of university attending.

4.3.2. *Skilled families*

In a skilled family, the borrowing cost is $R_{it} = R(w_H s_{it-1})$, and the parent's human capital $h_{it-1} = s_{it-1}$. The condition (25) to choose higher education is now:

$$a_{it} \geq \alpha^H\left(\omega, w_H, s_{it-1}\right) = \frac{1-\beta}{\beta\delta_B\delta_H\bar{e}}\left(\frac{1}{1-\beta}\frac{\delta_H\bar{e}}{1+\delta_H\bar{e}}\right)^{1/\beta}\frac{\lambda}{\lambda - R\left(w_H s_{it-1}\right)}\omega^{-1}s_{it-1}{}^{-\eta} \qquad (27)$$

The function $\alpha^H\left(\omega, w_H, s_{it-1}\right)$ is such that $\frac{\partial\alpha^H}{\partial\omega} < 0$ and $\frac{\partial\alpha^H}{\partial w_H} < 0$. Hence, both an increase in the skill premium and an increase in the skilled wage release the condition to select the university for children born in skilled families. First, an increase in the skill premium raises the return to higher education relative to the return to basic education, which incites individuals to select higher education. Second an increase in the skilled wage reduces the interest rate paid by skilled families and thereby the cost of attending the university.

5. Growing inequality

We now analyse the impact of an increase in the real skilled wage w_H, a decrease in the real unskilled wage w_L, and hence an increase in the skill premium ω.

So as to determine the effect of an increase in the skill premium on education decisions and intergenerational mobility, we construct the space of higher education (set of individuals who select higher education) in each configuration (university funding through parental bequest; free higher education; university funding through borrowing) and we analyse the impacts of the changes in wages upon the education decision and intergenerational mobility depending on the family type, skilled or unskilled.

As regards intergenerational mobility, we make a double distinction depending on their direction (upward and downward) and their between or within–skill group dimension.

Upward mobility refers to an intergenerational increase in skill, and it can take the form of between–skill group upward mobility (individuals from unskilled families selecting to attend the university and joining thereby the group of skilled workers) and within–skill group upward mobility (individuals from skilled families increasing their skill level relative to their parents).

Similarly, downward mobility reflects an intergenerational decrease in skill, with between–skill group downward mobility referring to the move from the skilled to the unskilled group and within–group downward mobility describing a decrease in skill from one generation to the next inside the group of skilled families. Note that individuals from unskilled families cannot be downward-mobile (between or within) since they have no skill and there is nothing below unskilled labour.

5.1. University funding through parental bequest

When university is financed by parents' bequest, we know that both the number of individuals selecting higher education and their skill level depend on their family type, skilled or unskilled (section 4.1).

5.1.1. *Unskilled families*

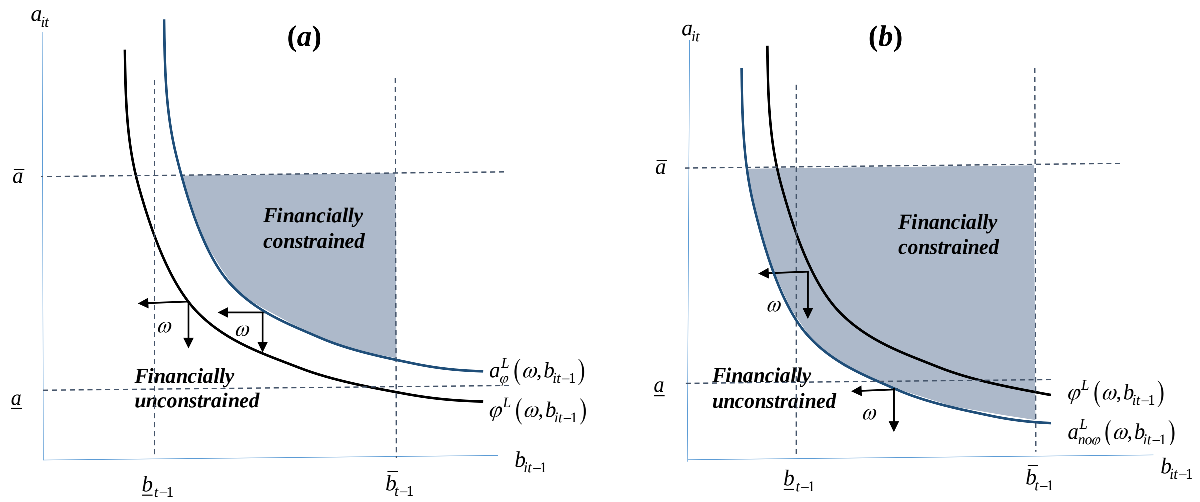
The map $\left[h_{it-1} \times a_{it}\right]$ portrays all individuals defined by their personal characteristics. In this map, Figs. 1 show the two possible configurations of higher education decision in the case of *unskilled families* when university attending is funded by parental bequest.

The *x*-axis depicts the parents' human capital (here basic education b_{it-1}) located in the interval $\left[\underline{b}_{t-1}, \bar{b}_{t-1}\right]$ and the *y*-axis the individual's ability. The curve φ^L draws the financial constraint (all individuals above φ^L are constrained, see relation (15)) and the curves a_φ^L and

$a^L_{no\varphi}$ picture the condition to select higher education when being constrained and unconstrained respectively (individuals above those curves select the university, see conditions (16) and (17)).

Fig. 1a portrays the situation in which only constrained individuals attend the university and Fig. 1b the situation in which all financially constrained individuals as well as the unconstrained individuals with the highest basic education level select higher education.

Fig. 1. Education decision of individuals from unskilled families



The grey surfaces depict the set of individuals who select the university. In each case, the impact of an increase in the skill premium ω on the curves displacement is indicated. It is clear that an increase in the skill premium strengthens the financial constraint (more individuals are constrained) but increase the number of individuals from unskilled families selecting higher education (because the highest return to skill incite them to enter the university).

The education level of individuals born in unskilled families if they attend the university is (Appendix A, Eqs. A2 and A14):

$$\tilde{s}^L_{it} = \lambda\rho\omega^{-1}, \qquad \textit{if they are financially constrained} \qquad (28)$$

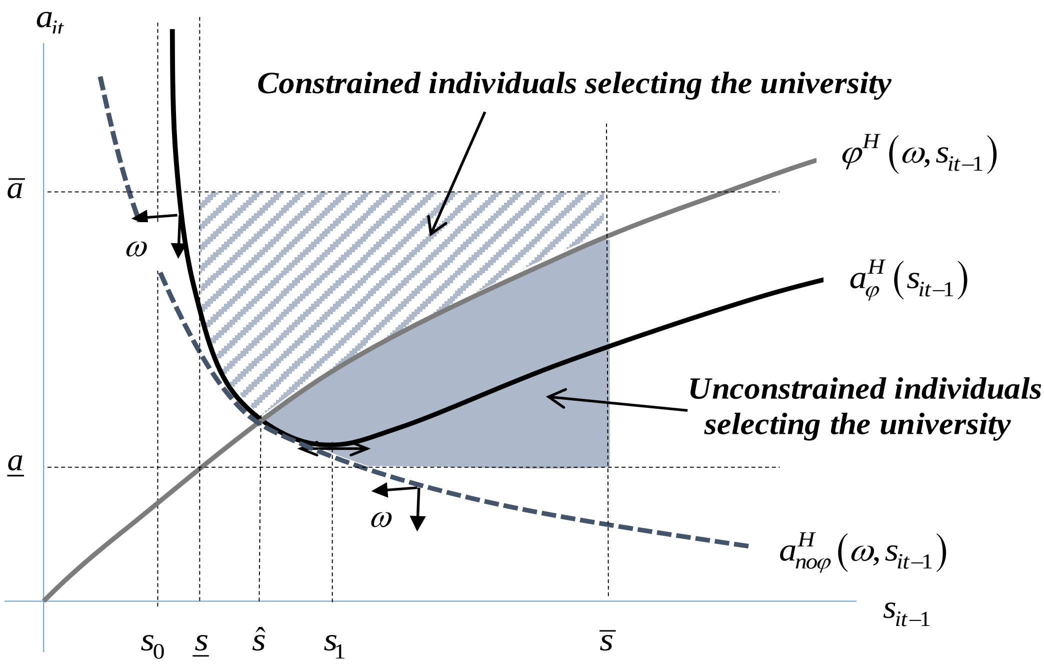
$$\hat{s}^L_{it} = \beta(1+\delta_H\bar{e})\delta_B a_{it} h_{it-1}{}^{\eta}, \qquad \textit{if they are financially unconstrained} \qquad (29)$$

Eqs. (28) and (29) show that the increase in the skill premium reduces the skill level of children from constrained unskilled families, but not for the unconstrained. As the financially constrained account for the majority (or even the entirety) of the individuals who attend the university when they come from unskilled families, it is clear that the rising skill premium significantly decrease the average skill of individuals from unskilled families. As regards unskilled families as a whole, this move can be counteracted by the fact that a higher proportion of their children attend the university.

5.1.2. *Skilled families*

Based on the results determined in section 4.1.2 and 4.13, Fig. 2 pictures university attending in skilled families in the map (s_{it-1}, a_{it}), with parents' skills being distributed in the interval $[\underline{s}, \overline{s}]$. The construction of Fig. 2 is explained in Appendix A, section A.3.2.

Fig. 2. Individuals from skilled families attending the University



Note: The functions $\varphi^H(s_{it-1})$, $a^H_\varphi(\omega, s_{it-1})$ and $a^H_{no\varphi}(\omega, s_{it-1})$ are defined in Appendix A. Individual (*i*,*t*) is constrained if $a_{it} \geq \varphi^H(s_{it-1})$ and selects the university if $a_{it} \geq a^H_\varphi(\omega, s_{it-1})$; s/he is unconstrained if $a_{it} < \varphi^H(s_{it-1})$ selects then the university $a_{it} \geq a^H_{no\varphi}(\omega, s_{it-1})$. Logically, the 3 curves intersect at the same skill $\hat{s} = \frac{1}{\lambda\rho}\left(\frac{1}{1-\beta}\frac{\delta_H \overline{e}}{1+\delta_H \overline{e}}\right)^{\frac{1-\beta}{\beta}} \omega^{-1}$. Finally, $s_0 = (1-\beta)^{(1-\beta)/\beta}\hat{s} < \hat{s}$ and $s_1 = \left(1+\frac{\beta\eta}{1-\eta}\right)^{\frac{1-\beta}{\beta}}\hat{s} > \hat{s}$.

All the individuals above the curve $\varphi^H(s_{it-1})$ are financially constrained and those below are financially unconstrained (constraint (20)). All constrained individuals above the curve $a^H_\varphi(\omega, s_{it-1})$ select to attend the university as well as all the unconstrained individuals above the curve $a^H_{no\varphi}\left(\omega, s_{it-1}\right)$. As in Figs. 1, the moves of the curves in relation to the skill premium ω are indicated.

The moves in wages do not modify the financial constraint whereas an increase in the skill premium displaces downward both curves a^H_{φ} and $a^H_{no\varphi}$, relaxing thereby the condition to attend the university for both constrained and unconstrained families.

The grey surface depicts the set of unconstrained individuals from skilled families who select to attend the university and the dashed surface those who are constrained and select the university.

An increase in the skill premium firstly raises the number of children from skilled families who enter the university by relaxing the attending condition for both constrained and unconstrained individuals.

The skill level of individuals from skilled families is (Appendix A, section ?):

$$\tilde{s}^H_{it} = \lambda \rho s_{it-1}, \qquad \textit{if they are financially constrained} \tag{30}$$

$$\hat{s}^H_{it} = \beta(1+\delta_H \overline{e})\delta_B a_{it} s_{it-1}{}^{\eta}, \quad \textit{if they are financially unconstrained} \tag{31}$$

Eqs. (30) and (31) show that the changes in wages have no impact on the skill level for individuals from skilled families selecting the university.

The above analyses of the education decisions when the individuals' higher education is financed by conditional parental bequests as well as the results established in sections 4.1.2 and 4.1.3 lead to the following proposition:

Proposition 1. *When higher education is financed through conditional parental bequest, the increase in the skill premium leads to:*

1) A strengthening of the financial constraint for individuals born in unskilled families and no change in this constraint for those born in skilled families.

2) An increase in upward mobility of all individuals born in unskilled families but a decrease in the skill attainment of those who are financially constrained.

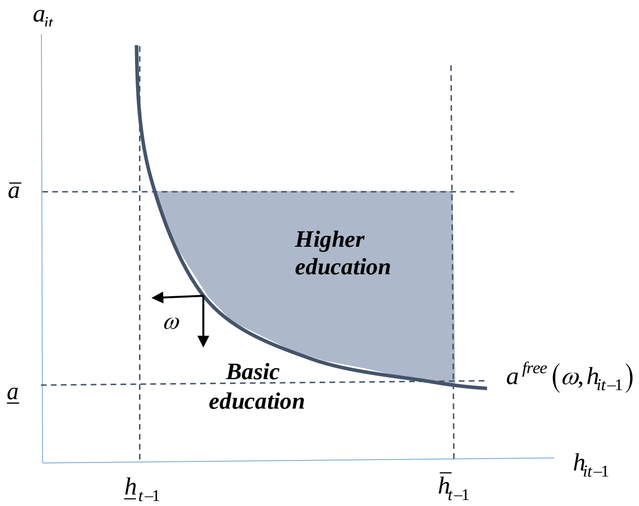
3) A decrease in downward between-skill mobility of all individuals born in skilled families, without change in the skill attainment for those who would have selected higher education without increase in the skill premium.

In summary, when higher education is financed by conditional parental bequests, a growing earnings inequality between skilled and unskilled workers expands the number of individuals enrolled in tertiary education whatever their family type, skilled or unskilled. Nevertheless, concerning individuals from unskilled families, this enrolment expansion comes with a decrease in skill attainment of those who are financially constrained. This decrease is damaging because the constrained individual are the majority (or even the entirety) of the individuals from unskilled family. They are also the most educated and able at the end of basic education. Consequently, inequality tends to increase in the set of skilled workers.

5.2. Free higher education

When higher education is freely provided by the State, the condition for university attending is given by relation[8] $a_{it} \geq a^{free}\left(\omega, h_{it-1}\right)$ as pictured in Fig. 3 where the parents human capital belongs to the interval $\left[\underline{h}_{t-1}, \overline{h}_{t-1}\right]$. The curve a^{free} is valid for both skilled and unskilled families. As an increase in the skill premium moves the curve a^{free} downward ($\partial a^{free} / \partial \omega < 0$), individuals from both skilled and unskilled families benefit from a relaxation of the condition to attend the university.

Fig. 3 Education decision with freely provided higher education



As the displacement to the left of the curve a^{free} concerns the individuals at the bottom of basic education spectrum, the relaxing of the condition to select the university primarily benefit

[8] Eq. 22 in section 4.2.

to children from unskilled families (a large share of children born in skilled families would have selected higher education even without increase in the skilled wage).

As regards higher education attainment, the change in wages has no impact on the optimal level[9] $\hat{s}_{it} = \beta(1+\delta_H \bar{e})\delta_B a_{it} h_{it-1}{}^{\eta}$ for given innate characteristics (a_{it}, h_{it-1}) because this is independent from wages when university attending is costless.

Proposition 2. *When higher education is freely provided by public authorities, the rise in the skill premium entails:*

1) An increase in the enrolment in higher education which primarily benefits to children from unskilled families.

2) No change in the skill attainment of those who would have selected the university without rise in the skill premium.

5.3. University financing through borrowing

When higher education is financed by loans to the students with the interest rate depending on the parents' income, the changes in wages act through two different channels. First, as in both preceding models, the increasing skill premium incites individuals to select higher education. But, in contrast with the preceding models, the negative effect of the increase in education cost is no longer offset by the similar increase in the return to skill because the changes in wages impact the borrowing costs.

We can then make a clear difference between individuals depending on their family skill.

5.3.1. *Individuals from unskilled families*

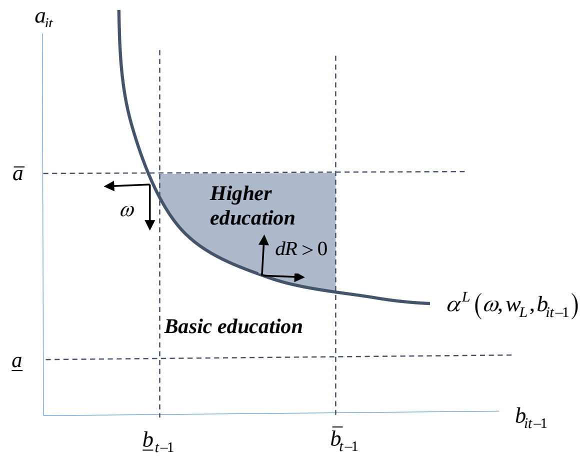
In the case of unskilled families, the decrease in the unskilled wage raises the credit risk, the interest rate, the borrowing cost and finally the higher education cost. This cost effect tends to discourage university attending. Hence, two opposite effects act on the education decision. The incentive effect linked to the increase in the skill premium boosts university attending whereas the cost effect linked to the rise in the personal interest rate dampen university attending.

Those two opposite effects are pictured in Fig. 4 where the curve $\alpha^L(\omega, w_L, b_{it-1})$ defines the condition to select the university. The parents' (basic) human capital is inside the interval

[9] See Appendix A, Eq. (A2).

$\left[\underline{b}_{t-1},\bar{b}_{t-1}\right]$. All the individuals above α^L choose to attend the university. The increasing skill premium ω moves curve α^L to the left and increase thereby the number of individuals selecting higher education. In contrast, the decrease in w_L raises the borrowing cost $R(w_L)$, displacing α^L to the right and reducing thereby the set of individuals choosing higher education. The total impact of the moves in wages on university attending is thus ambiguous for children from unskilled families. This impact depends on the respective strength of the incentive and cost effect. Finally note that if the cost effect dominates the incentive effect for a decrease in w_L , the rejected individuals are those with a high ability as shown by Fig. 4.

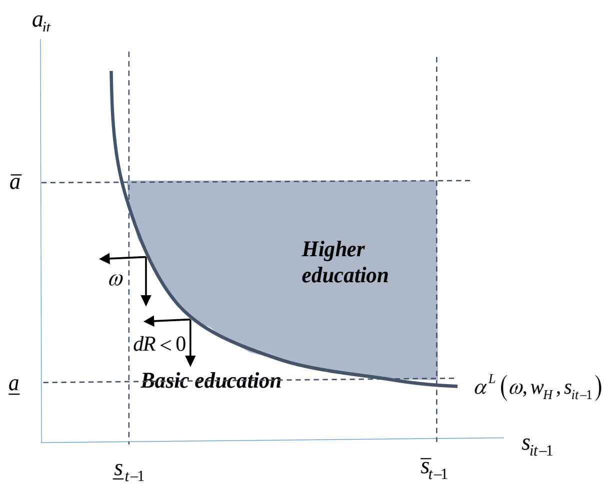
Fig. 4. Funding through borrowing: Education decision in unskilled families



5.3.2. *Individuals from skilled families*

Fig. 5 pictures the education choice of individuals born in skilled families when university is financed by borrowing.

Fig. 5. Funding through borrowing: Education decision in skilled families.



For individuals born in skilled families, both the incentive effect and the education cost effect slacken the condition to select the university. The rise in the real skilled wage w_H reduces the credit risk, the interest rate and the borrowing cost paid by skilled families. This lessens the higher education cost and make more individuals from skilled families to attend the university.

Finally, in both skilled and unskilled families, the change in wages has no impact on the skill level when attending the university, determined by the relation $\hat{s}_{it} = \beta(1+\delta_H \overline{e})\delta_B a_{it} h_{it-1}$ which is independent from wages (Appendix B, Eq. B2).

Proposition 3. *When higher education is funded by loans to the students, rise in the skilled wage, decrease in the unskilled wage and related increase in the skill premium entails:*

1) For children born in unskilled families, an ambiguous impact on university attending, which depends on the respective intensity of the pro-education incentive effect linked to the rise in the skill premium and the education-reducing cost effect linked to the increase in the interest rate.

2) For children born in skilled families, an increase in the number of individuals selecting higher education, both the incentive and the education cost effects encouraging university attending.

3) No change in the skill level of individuals who would have chosen the university without move in wages.

In summary, when university attending is financed through borrowing, the increasing inequality between skilled and unskilled workers benefits to skilled families compared to unskilled families. At the best, both skilled and unskilled families send more children to universities but the impact on the former is significantly higher than for the latter. When the effect of the decrease in unskilled wages on the credit risk premium is sufficiently high, the number of children from unskilled families attending the university decreases whereas this number increases, possibly substantially,[10] for those born in skilled families because they benefit from both the incentive and the education cost effects.

[10] Obviously, the magnitude of this increase depends on the share of children from skilled families who would have entered the university without change in wages. If this share is very high, the impact remains modest.

6. Discussion and conclusion

We have analysed the impact of an increase in inequality between skilled and unskilled workers on individuals' education decisions and skill attainments and on the mobility between skilled and unskilled families. The major finding revealed in our three propositions are summarised in Table 1.

Table 1. Education decision according to the financing method and the family type.

Family type / *Financing*	**Unskilled family**	**Skilled family**
Conditional parental bequest	– Increase in the number of individuals attending the university. – Decrease in the skill attainment of the financially constrained.	– Increase in the number of individuals attending the university. – No change in the skill level of those who would have selected the university without change in wages.
Borrowing	– Ambiguous impact on university attending, depending on the respective intensity of the incentive and the cost effect. – No change in the skill level for those who would have attended the university without rising skill premium.	– Increase in the number of individuals attending the university. – No change in the skill level for those who attend the university with or without change in wages.
Free provision	– Increase in the number of individuals attending the university.	– Increase in the number of individuals attending the university

Table 1 clearly shows that, if skilled – unskilled inequality normally expands the number of individuals pursuing tertiary education, children from skilled families are the prime beneficiaries, except in the case of costless (freely provided) higher education. Individuals from skilled families are privileged and those from unskilled families disadvantaged through several channels.

First, when the university is financed by parental bequest, individuals from unskilled families who are financially constrained suffer a decrease in their skill attainment. Those are the majority (or even the entirety) of the university attendants from unskilled families and the most hurt are typically those with the highest ability.

Second, when the university is financed by loans to the students, the increase in university attending primarily improves the position of those from skilled families. This is because both the incentive and the education cost effects enlarge their participation whereas the two effects have opposite impacts on individuals from unskilled families, with even a decrease in their attending when the cost effect is sizeable.

Finally, the only situation in which the increasing between-skill inequality has a similar impact on both types of families is when tertiary education is freely provided to individuals. In this case, the increase in the number of individuals attending the university is even higher for the unskilled-born because their share in the unskilled is typically higher before the rise in the skill premium.

Several additional education financing means could obviously be considered.

First, the case in which the parental bequest is not conditioned by higher education has not been treated here because, even if parental bequests and inheritance are common behaviours, the knowledge of the value of those transfers by the children at the time when they decide for their education strategy and the deduction of university fees from this value are unlikely. Nevertheless, the modelling of this case leads to results which are close to those of conditional bequests (proofs in Appendix C). The moves in wages strengthen the financial constraint for individuals born in unskilled families, but not for individuals born in skilled families. Moreover, those moves normally increase the number of children entering the university, but this impact is stronger for skilled than for unskilled families. Finally, the skill level significantly decreases for the constrained individuals from unskilled families whereas it only slightly decreases for constrained individuals from skilled families.

Second, the government can grant subsidised loans to the students so as to erase the risk premium paid by individuals from unskilled families. In such a situation, the results are rather similar to the case of free higher education but the positive impact for individuals from unskilled families is weaker (see Appendix D).

Third, several financing means can operate simultaneously. For instance, skilled parents with high incomes can fully pay for their children's tertiary education whereas education is at least partially financed by refundable loans in families with lower incomes. In such situations the outcomes are combinations of those revealed by each case presented in Table 1.

Our results provide an explanation to the concomitance observed in a number of advanced countries and highlighted in section 2.1 of (i) higher participation in tertiary education, (ii) higher skill and income inequality across skilled workers, and (iii) lower or stationary intergenerational skill mobility. In addition, the increasing cost of higher education linked to the reforms implemented in several countries, particularly the US and the UK, tend to reinforce the pro-skilled families bias of the rise in inequality by increasing the impacts of parental bequest and/or borrowing in university funding.

Other factors can of course explain several of those observed developments, in particular a two-tier higher education system which discriminate between standard universities open to a large number of applicants and elitist universities which select a limited number of students (see, e.g., Bergh & Fink, 2009, Brezis & Hellier, 2018, Kaganovich & Su, 2019).

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Appendix A. Education financed by conditional parental bequest

A1. Optimum when attending the university

The maximisation programme which determines individual (i,t)'s optimal effort and income when attending the university without financial constraint is:

$$\max_{s_{it},e_{it},f_{it}} u_{it} = \left(w(s_{it})\right)^{\beta}\left(\bar{e}-e_{it}\right)^{1-\beta}, \quad \text{s.t.: } w(s_{it}) = w_H s_{it},\ s_{it} = b_{it}(1+\delta_H e_{it}) = \lambda f_{it},\ b_{it} = \delta_B a_{it} h_{it-1}{}^{\eta}.$$

This can be written: $\max_{e_{it}} u_{it} = \left(w_H b_{it}\right)^{\beta}\left(1+\delta_H e_{it}\right)^{\beta}\left(\bar{e}-e_{it}\right)^{1-\beta}$.

The so-determined optimal values when the individual is *financially* ***unconstrained*** are (the hat indicates the optimum without financial constraint):

$$\hat{e}_{it} = \beta\bar{e} - \frac{1-\beta}{\delta_H} \tag{A1}$$

$$\hat{s}_{it} = \beta(1+\delta_H\bar{e})b_{it} = \beta(1+\delta_H\bar{e})\delta_B a_{it} h_{it-1}{}^{\eta} \tag{A2}$$

$$\hat{f}_{it} = \frac{\beta}{\lambda}(1+\delta_H\bar{e})b_{it} = \frac{\beta}{\lambda}(1+\delta_H\bar{e})\delta_B a_{it} h_{it-1}{}^{\eta} \tag{A3}$$

The *financial constraint* ($w_H f_{it} \le \rho w_{it-1}$) is:

$$a_{it} \le \varphi\left(h_{it-1}\right) = \frac{\rho\lambda}{\beta\delta_B(1+\delta_H\bar{e})}\frac{w_{it-1}}{w_H} h_{it-1}{}^{-\eta} \tag{A4}$$

When *financially* ***constrained***, the individual is limited in her/his education expenditure by the parent's bequest and $w_H f_{it} = \rho w_{it-1}$. As $s_{it} = b_{it}(1+\delta_H e_{it}) = \lambda f_{it}$, we have (the tilde indicates the optimum when the attending the university and being financially constrained):

$$\tilde{f}_{it} = \rho\frac{w_{it-1}}{w_H} \tag{A5}$$

$$\tilde{s}_{it} = \lambda\rho\frac{w_{it-1}}{w_H} \tag{A6}$$

$$\tilde{e}_{it} = \frac{\lambda\rho}{\delta_H}\frac{w_{it-1}}{w_H}\, b_{it}{}^{-1} - \frac{1}{\delta_H} \tag{A7}$$

A.2. Education decision in unskilled families

Superscript L indicates an unskilled family and $\omega \equiv w_H / w_L$ is the skill premium.

The financial constraint (A4) is:

$$a_{it}^L \leq \varphi^L(\omega, b_{it-1}) = \frac{\lambda\rho}{\beta\delta_B(1+\delta_H\bar{e})}\omega^{-1}b_{it-1}{}^{-\eta} \tag{A8}$$

It can be easily verified that: $\frac{\partial \varphi^L}{\partial b_{it-1}} < 0,\ \frac{\partial^2 \varphi^L}{\partial b_{it-1}{}^2} > 0,\ \frac{\partial \varphi^L}{\partial \omega} < 0,\ \frac{\partial^2 \varphi^L}{\partial \omega^2} > 0.$

A.2.1. *Unconstrained unskilled families*

When financially *unconstrained* ($a_{it} \leq \varphi^L$), individual (i,t)'s demand for education services, effort and skill when attending university are determined by Eqs. (A1) – (A3). The related utility ($u\left(w_H\hat{s}_{it}^L, \hat{e}_{it}^L\right) = \left(w_H\hat{s}_{it}^L\right)^\beta \left(\bar{e} - \hat{e}_{it}^L\right)^{1-\beta}$) and the condition to select the university ($u\left(w_H\hat{s}_{it}^L, \hat{e}_{it}^L\right) \geq u\left(w_L, 0\right)$)) are:

$$u\left(w_H\hat{s}_{it}^L, \hat{e}_{it}^L\right) = w_H{}^\beta \frac{\beta^\beta(1-\beta)^{1-\beta}}{\delta_H{}^{1-\beta}}\left(1+\delta_H\bar{e}\right)b_{it}{}^\beta \tag{A9}$$

$$a_{it} \geq a_{no\varphi}^L(\omega, b_{it-1}) = \frac{1}{\lambda\rho}\left(\frac{\delta_H\bar{e}}{(1-\beta)(1+\delta_H\bar{e})}\right)^{\frac{1-\beta}{\beta}} \varphi^L(\omega, b_{it-1}) \tag{A10}$$

with $\frac{\partial a_{no\varphi}^L}{\partial \omega} < 0$ and $\frac{\partial a_{no\varphi}^L}{\partial b_{it-1}} < 0$.

Proof of (A10): $u\left(w_H\hat{s}_{it}^L, \hat{e}_{it}^L\right) \geq u\left(w_L, 0\right) \Leftrightarrow w_H{}^\beta \frac{\beta^\beta(1-\beta)^{1-\beta}\left(1+\delta_H\bar{e}\right)}{\delta_H{}^{1-\beta}}\left(\delta_B a_{it} b_{it-1}{}^\eta\right)^\beta \geq w_L{}^\beta \bar{e}^{1-\beta}$.

After re-arranging: $a_{it} \geq a_{no\varphi}^L(\omega, b_{it-1}) = \frac{1}{\lambda\rho}\left(\frac{\left(\delta_H\bar{e}\right)^{1-\beta}}{(1-\beta)^{1-\beta}\left(1+\delta_H\bar{e}\right)^{1-\beta}}\right)^{1/\beta} \varphi_L\omega, \left(b_{it-1}\right)$.

The condition for $a_{no\varphi}^L(\omega, b_{it-1})$ to be higher (lower) than $\varphi^L(\omega, b_{it-1})$ is that $\frac{1}{\lambda\rho}\left(\frac{\delta_H\bar{e}}{(1-\beta)(1+\delta_H\bar{e})}\right)^{\frac{1-\beta}{\beta}}$ be higher (lower) than 1, which yields after rearranging:

$$a_{no\varphi}^L(\omega, b_{it-1}) \gtreqless \varphi^L(\omega, b_{it-1}) \Leftrightarrow \frac{\delta_H\bar{e}}{1+\delta_H\bar{e}} \gtreqless (1-\beta)\left(\lambda\rho\right)^{\frac{\beta}{1-\beta}} \tag{A11}$$

A.2.2. *Constrained unskilled families*

All individuals born in unskilled families and financially ***constrained*** make the same education expenditure (since unskilled parents have the same income w_L). They consequently have the same skill and their related effort decreases with their basic education:

$$\tilde{f}_{it}^{L} = \rho\omega^{-1} \tag{A12}$$

$$\tilde{e}_{it}^{L} = \frac{1}{\delta_H}\left(\frac{\lambda\rho}{\omega b_{it}} - 1\right) \tag{A13}$$

$$\tilde{s}_{it}^{L} = \lambda\rho\omega^{-1} \tag{A14}$$

For a financially constrained individual, the utility generated by higher education is:

$$u\left(w_H \tilde{s}_{it}^{L}, \tilde{e}_{it}^{L}\right) = \left(\lambda\rho w_L\right)^{\beta}\left(\overline{e} + \frac{1}{\delta_H} - \frac{\lambda\rho}{\delta_H}\omega^{-1} b_{it}^{\ -1}\right)^{1-\beta} \tag{A15}$$

And the constrained individual (i,t) selects to attend the university if and only if $u\left(w_H \tilde{s}_{it}^{L}, \tilde{e}_{it}^{L}\right) \geq u\left(w_L, \overline{e}\right)$, i.e.:

$$a_{it} \geq a_{\varphi}^{L}\left(\omega, b_{it-1}\right) = \frac{\beta(1+\delta_H\overline{e})}{1+\delta_H\overline{e} - (\lambda\rho)^{-\beta/(1-\beta)}\delta_H\overline{e}}\varphi^{L}\left(\omega, b_{it-1}\right) \tag{A16}$$

Proof of (A16): $u\left(w_H \tilde{s}_{it}^{U}, \tilde{e}_{it}^{U}\right) \geq u\left(w_L, \overline{e}\right) \Leftrightarrow a_{it} \geq \frac{\lambda\rho / \delta_B}{1+\delta_H\overline{e} - (\lambda\rho)^{-\beta/(1-\beta)}\delta_H\overline{e}}\frac{w_L}{w_H} b_{it-1}^{\ -\eta}$, and after rearranging: $a_{it} \geq a_{L}^{\varphi}\left(b_{it-1}\right) = \frac{\beta(1+\delta_H\overline{e})}{1+\delta_H\overline{e} - (\lambda\rho)^{-\beta/(1-\beta)}\delta_H\overline{e}}\varphi_L\left(\omega, b_{it-1}\right)$.

Eq. (A16) shows that the condition for $a_{\varphi}^{L}\left(\omega, b_{it-1}\right)$ to be above (below) $\varphi^{L}\left(\omega, b_{it-1}\right)$ is that $\frac{\beta(1+\delta_H\overline{e})}{1+\delta_H\overline{e} - (\lambda\rho)^{-\beta/(1-\beta)}\delta_H\overline{e}} > (<)\ 1$, which can be written:

$$a_{\varphi}^{L}\left(b_{it-1}\right) \begin{array}{c} > \\ = \\ < \end{array} \varphi^{L}\left(\omega, b_{it-1}\right) \ \Leftrightarrow\ \frac{\delta_H\overline{e}}{1+\delta_H\overline{e}} \begin{array}{c} > \\ = \\ < \end{array} (1-\beta)(\lambda\rho)^{\beta/(1-\beta)} \tag{A17}$$

Considering Eqs. (A11) and (A17), it comes that $a_{\varphi}^{L}\left(b_{it-1}\right)$ and $a_{no\varphi}^{L}\left(b_{it-1}\right)$ are either both higher or both lower than $\varphi^{L}\left(b_{it-1}\right)$.

A.3. Education decision in skilled families

Superscript H indicates a skilled family. The financial constraint (A4) is:

$$a_{it} \le \varphi^H\left(s_{it-1}\right) = \frac{\lambda\rho}{\beta(1+\delta_H\overline{e})\delta_B} s_{it-1}{}^{1-\eta} \tag{A18}$$

A.3.1. *Unconstrained skilled families*

From Eqs. (A1) – (A3):

$$\hat{e}_{it}^H = \beta\overline{e} - \frac{1-\beta}{\delta_H} \tag{A19}$$

$$\hat{f}_{it}^H = \frac{\beta}{\lambda}(1+\delta_H\overline{e})\delta_B a_{it} s_{it-1}{}^{\eta} \tag{A20}$$

$$\hat{s}_{it}^H = \beta(1+\delta_H\overline{e})\delta_B a_{it} s_{it-1}{}^{\eta} \tag{A21}$$

$$u\left(w_H\hat{s}_{it}^H, \hat{e}_{it}^H\right) = w_H{}^{\beta}\frac{\beta^{\beta}(1-\beta)^{1-\beta}}{\delta_H{}^{1-\beta}}\left(1+\delta_H\overline{e}\right)\left(\delta_B a_{it} s_{it-1}{}^{\eta}\right)^{\beta} \tag{A22}$$

Thus, the condition for unconstrained individual from unskilled families to attend the university, $u\left(w_H\hat{s}_{it}^S, \hat{e}_{it}^S\right) > u\left(w_L, 0\right)$, is:

$$a_{it} \ge a_{no\varphi}^H\left(s_{it-1}\right) = \frac{\left(\delta_H\overline{e}\right)^{\frac{1-\beta}{\beta}}}{\beta(1-\beta)^{\frac{1-\beta}{\beta}}\delta_B\left(1+\delta_H\overline{e}\right)^{1/\beta}}\frac{w_L}{w_H} s_{it-1}{}^{-\eta} \tag{A23}$$

The curve drawing the financial constraint in skilled families $\varphi^H\left(s_{it-1}\right)$ intersects the curve defining the condition to attend the university for unconstrained families, $a_{no\varphi}^H\left(s_{it-1}\right)$, at the skll:

$$\hat{s} = \frac{1}{\lambda\rho}\left(\frac{\delta_H\overline{e}}{(1-\beta)\left(1+\delta_H\overline{e}\right)}\right)^{\frac{1-\beta}{\beta}}\frac{w_L}{w_H} \tag{A24}$$

A.3.2. *Constrained skilled families*

From Eqs. (A5) – (A7) with $h_{it-1} = s_{it-1}$, we have:

$$\tilde{s}_{it}^H = \lambda\rho s_{it-1} \tag{A25}$$

$$\tilde{e}_{it}^H = \frac{\lambda\rho}{\delta_H\delta_B a_{it}} s_{it-1}{}^{1-\eta} - \frac{1}{\delta_H} \tag{A26}$$

$$\tilde{f}_{it}^{H} = \rho s_{it-1} \tag{A27}$$

And the corresponding utility:

$$u\left(w_H \tilde{s}_{it}^{H}, \tilde{e}_{it}^{H}\right) = \left(\lambda\rho s_{it-1}\right)^{\beta}\left(\bar{e} - \frac{\lambda\rho}{\delta_H \delta_B a_{it}} s_{it-1}{}^{1-\eta} + \frac{1}{\delta_H}\right)^{1-\beta} w_H{}^{\beta} \tag{A28}$$

Individual (*i,t*) selects to attend the university if $u\left(w_H \tilde{s}_{it}^{H}, \tilde{e}_{it}^{H}\right) \geq u\left(w_L, 0\right)$. This yields the following two conditions which must be concurrently fulfilled:

$$a_{it} > a_{\varphi}^{H}\left(\omega, s_{it-1}\right) = \frac{1}{\delta_B} \frac{\left(\lambda\rho\right)^{\frac{1}{1-\beta}} s_{it-1}{}^{\frac{\beta}{1-\beta}+1-\eta}}{(1+\delta_H \bar{e})\left(\lambda\rho\right)^{\frac{\beta}{1-\beta}} s_{it-1}{}^{\frac{\beta}{1-\beta}} - \delta_H \bar{e}\, \omega^{-\frac{\beta}{1-\beta}}} \tag{A29}$$

Proof of Condition (A29): $u\left(w_H \tilde{s}_{it}^{H}, \tilde{e}_{it}^{H}\right) > u\left(w_L, 0\right) \Leftrightarrow \lambda\rho s_{it-1}{}^{1-\eta} < \left(1 + \delta_H \bar{e} - \delta_H \bar{e}\left(\lambda\rho\omega s_{it-1}\right)^{-\frac{\beta}{1-\beta}}\right)\delta_B a_{it}$.

If $1 + \delta_H \bar{e} - \delta_H \bar{e}\left(\lambda\rho\omega s_{it-1}\right)^{-\frac{\beta}{1-\beta}} < 0 \Leftrightarrow s_{it-1} < \frac{1}{\lambda\rho}\left(\frac{\delta_H \bar{e}}{1+\delta_H \bar{e}}\right)^{(1-\beta)/\beta} \omega^{-1} \equiv s_0$, then the condition is not fulfilled and the individual select basic education. If $s_{it-1} > s_0$, inequality $\lambda\rho s_{it-1}{}^{1-\eta} < \left(1 + \delta_H \bar{e} - \delta_H \bar{e}\left(\lambda\rho\omega s_{it-1}\right)^{-\frac{\beta}{1-\beta}}\right)\delta_B a_{it}$ can be written $a_{it} > a_{\varphi}^{H}\left(\omega, s_{it-1}\right)$, with $a_{\varphi}^{H}\left(\omega, s_{it-1}\right) = \frac{1}{\delta_B} \frac{\left(\lambda\rho\right)^{\frac{1}{1-\beta}} s_{it-1}{}^{\frac{\beta}{1-\beta}+1-\eta}}{(1+\delta_H \bar{e})\left(\lambda\rho s_{it-1}\right)^{\frac{\beta}{1-\beta}} - \delta_H \bar{e}\, \omega^{-\frac{\beta}{1-\beta}}}$. As $s_{it-1} < s_0 \Rightarrow a_{\varphi}^{H}\left(\omega, s_{it-1}\right) < 0$ and since $a_{it} > 0$, the condition $a_{it} > a_{\varphi}^{H}\left(\omega, s_{it-1}\right)$ is necessary and sufficient for selecting the university.

Analysis of function $a_{\varphi}^{H}\left(\omega, s_{it-1}\right)$

1) $s_{it-1} = 0 \Rightarrow \underline{a}_{\varphi}^{H}\left(\omega, s_{it-1}\right) = 0$

2) $(1+\delta_H \bar{e})\left(\lambda\rho s_{it-1}\right)^{\frac{\beta}{1-\beta}} - \delta_H \bar{e}\, \omega^{-\frac{\beta}{1-\beta}} \begin{matrix} > \\ = \\ < \end{matrix} 0 \Leftrightarrow s_{it-1} \begin{matrix} > \\ = \\ < \end{matrix} \frac{1}{\lambda\rho}\left(\frac{\delta_H \bar{e}}{1+\delta_H \bar{e}}\right)^{\frac{1-\beta}{\beta}} \omega^{-1} \equiv s_0$

$$\Rightarrow a_{\varphi}^{H} \longrightarrow \begin{cases} -\infty & \text{when } s_{it-1} \to s_0 \text{ by the left side} \\ +\infty & \text{when } s_{it-1} \to s_0 \text{ by the right side} \end{cases}$$

3) $\underline{a}_{\varphi}^{H}\left(\omega, s_{it-1}\right) \xrightarrow[s_{it-1} \to +\infty]{} +\infty$

4) $\dfrac{\partial a_{\varphi}^{H}}{\partial s_{it-1}} = \dfrac{(\lambda\rho)^{\frac{1}{1-\beta}} s_{it-1}^{\frac{\beta}{1-\beta}-\eta}}{\delta_B} \dfrac{(1-\eta)(1+\delta_H \bar{e})(\lambda\rho)^{\frac{\beta}{1-\beta}} s_{it-1}^{\frac{\beta}{1-\beta}} - \left(\dfrac{\beta}{1-\beta}+1-\eta\right)\delta_H \bar{e}\left(\dfrac{w_L}{w_H}\right)^{\beta/(1-\beta)}}{\left((1+\delta_H \bar{e})(\lambda\rho s_{it-1})^{\frac{\beta}{1-\beta}} - \delta_H \bar{e}\left(\dfrac{w_L}{w_H}\right)^{\beta/(1-\beta)}\right)^2}$

$$\frac{\partial a_{\varphi}^{H}}{\partial s_{it-1}} > 0 \Leftrightarrow s_{it-1} > \frac{1}{\lambda\rho}\left(\frac{\beta}{(1-\beta)(1-\eta)}+1\right)^{\frac{1-\beta}{\beta}}\left(\frac{\delta_H \bar{e}}{1+\delta_H \bar{e}}\right)^{\frac{1-\beta}{\beta}} \frac{w_L}{w_H} \equiv s_1$$

Table A1 displays the variation $\underline{a}_{\varphi}^{H}(\omega, s_{it-1})$ as a function of s_{it-1} for $\omega > 0$ and $s_{it-1} \in \mathbb{R}^{+} = [0,+\infty]$.

Table A1. Analysis of $a_{\varphi}^{H}(\omega, s_{it-1})$ as a function of s_{it-1}

s_{it-1}	0		s_0		s_1		$+\infty$
$\partial a_{\varphi}^{H} / \partial s_{it-1}$		$-$		$-$	0	$+$	
$a_{\varphi}^{H}(s_{it-1})$	0	↘	$+\infty$ / $-\infty$	↘	$a_{\varphi}^{H}(s_1)$	↗	$+\infty$

In addition:

1) $\dfrac{\partial a_{\varphi}^{H}}{\partial \omega} = -\dfrac{(\lambda\rho)^{\frac{1}{1-\beta}}}{\delta_B} \dfrac{\dfrac{\beta}{1-\beta}\delta_H \bar{e}\,\omega^{-1/(1-\beta)} s_{it-1}^{\frac{\beta}{1-\beta}+1-\eta}}{\left((1+\delta_H \bar{e})(\lambda\rho s_{it-1})^{\frac{\beta}{1-\beta}} - \delta_H \bar{e}\,\omega^{-\beta/(1-\beta)}\right)^2} < 0$

2) $a_{\varphi}^{H}(s_{it-1})$ intersects $\varphi^{H}(s_{it-1})$ at point $\hat{s} = \dfrac{1}{\lambda\rho}\left(\dfrac{\delta_H \bar{e}}{(1-\beta)(1+\delta_H \bar{e})}\right)^{\frac{1-\beta}{\beta}} \dfrac{w_L}{w_H} = \hat{s}$. Hence, the three curves $a_{\varphi}^{H}(s_{it-1})$, $a_{no\varphi}^{H}(s_{it-1})$ and $\varphi^{H}(s_{it-1})$ intersect at the same skill $\hat{s}$.

3) $s_1 = \left(1+\dfrac{\beta\eta}{1-\eta}\right)^{\frac{1-\beta}{\beta}} \hat{s} \Rightarrow \hat{s} < s_1$ and $s_0 = \dfrac{1}{\lambda\rho}\left(\dfrac{\delta_H \bar{e}}{1+\delta_H \bar{e}}\right)^{\frac{1-\beta}{\beta}} \omega^{-1} < \hat{s} \Rightarrow$ the curve $a_{\varphi}^{H}(s_{it-1})$ intersects $\varphi^{H}(s_{it-1})$ in the decreasing part of $a_{\varphi}^{H}(s_{it-1})$ in the interval $]s_0, s_1[$.

Those features draw Fig. 2 in the text.

Appendix B. University financing through borrowing

B.1. Optimum when attending the university

The maximisation programme which determines individual (*i*,*t*)'s optimal effort, skill and income when attending the university is:

$$\max_{s_{it}, e_{it}, f_{it}} u_{it} = \left(w(s_{it})\right)^{\beta} \left(\overline{e} - e_{it}\right)^{1-\beta}, \quad \text{s.t.: } w(s_{it}) = w_H s_{it} - R_{it} w_H f_{it} \text{ and } s_{it} = b_{it}(1+\delta_H e_{it}) = \lambda f_{it}$$

Expressing u_{it} as a function of e_{it}: $\max_{e_{it}} u_{it} = \left(w_H b_{it}\right)^{\beta} \left(1 - R_{it} / \lambda\right)^{\beta} \left(1+\delta_H e_{it}\right)^{\beta} \left(\overline{e} - e_{it}\right)^{1-\beta}$. This maximisation programme determines the following optimal values):

$$\hat{e}_{it} = \beta\overline{e} - \frac{1-\beta}{\delta_H} \tag{B1}$$

$$\hat{s}_{it} = \beta(1+\delta_H \overline{e}) b_{it} \tag{B2}$$

$$\hat{f}_{it} = \frac{\beta}{\lambda}(1+\delta_H \overline{e}) b_{it} \tag{B3}$$

The higher education cost and the net income when attending the university are:

$$\hat{c}_{it} = R_{it} w_H \frac{\beta}{\lambda}(1+\delta_H \overline{e}) b_{it} \tag{B4}$$

$$w_{it} = w_H \hat{s}_{it} - c_{it} = \delta_B \frac{\beta}{\lambda} w_H (1+\delta_H \overline{e}) \left(\lambda - R_{it}\right) a_{it} h_{it-1}{}^{\eta} \tag{B5}$$

The utility of attending the university:

$$u\left(\hat{w}_{it}, \hat{e}_{it}\right) = w_H{}^{\beta} \beta^{\beta} (1-\beta)^{1-\beta} \delta_H{}^{-(1-\beta)} \left(1+\delta_H \overline{e}\right) \left(1 - \lambda^{-1} R_{it}\right)^{\beta} b_{it}{}^{\beta} \tag{B6}$$

Proof of (B6): $u\left(\hat{w}_{it}, \hat{e}_{it}\right) = w_H{}^{\beta} \left(s_{it} - \frac{\beta}{\lambda}(1+\delta_H \overline{e}) R\left(w_{it-1}\right) b_{it} \right)^{\beta} \left(\overline{e} - e_{it}\right)^{1-\beta}$

$$\Rightarrow u\left(\hat{w}_{it}, \hat{e}_{it}\right) = w_H{}^{\beta} \left(\beta(1+\delta_H \overline{e}) b_{it} - \frac{\beta}{\lambda}(1+\delta_H \overline{e}) R_{it} b_{it} \right)^{\beta} \left((1-\beta)\overline{e} + \frac{1-\beta}{\delta_H} \right)^{1-\beta}$$

$$\Rightarrow u\left(\hat{w}_{it}, \hat{e}_{it}\right) = w_H{}^{\beta} \beta^{\beta} (1-\beta)^{1-\beta} \delta_H{}^{-(1-\beta)} \left(1+\delta_H \overline{e}\right) \left(1 - \lambda^{-1} R_{it}\right)^{\beta} b_{it}{}^{\beta}$$

The condition for individual (*i*,*t*) to select university attending ($u\left(\hat{w}_{it},\hat{e}_{it}\right)\geq u\left(w_L,0\right)$) is:

$$a_{it}\geq\frac{1}{\delta_B}\left(\frac{\left(\delta_H\overline{e}\right)^{1-\beta}}{\beta^{\beta}(1-\beta)^{1-\beta}\left(1+\delta_H\overline{e}\right)}\right)^{1/\beta}\frac{w_L}{w_H}\frac{1}{1-\lambda^{-1}R_{it}}h_{it-1}{}^{-\eta} \quad \text{(B7)}$$

Proof of (B7):

$$u\left(\hat{w}_{it},\hat{e}_{it}\right)\geq u\left(w_L,0\right)\Leftrightarrow w_H{}^{\beta}\beta^{\beta}(1-\beta)^{1-\beta}\delta_H{}^{-(1-\beta)}\left(1+\delta_H\overline{e}\right)\left(1-\lambda^{-1}R_{it}\right)^{\beta}b_{it}{}^{\beta}\geq w_L{}^{\beta}\overline{e}^{1-\beta}\Leftrightarrow\text{(B7)}$$

The outcomes and the condition to select the university depend on the parents' human capital and income, i.e., on whether the family is skilled or unskilled.

B.2. Education decision in unskilled families

In an unskilled family, the borrowing cost is $R(w_{it-1})=R(w_L)$ and the parent's human capital $h_{it-1}=b_{it-1}$. Consequently, the condition (B7) to choose higher education is:

$$a_{it}\geq\alpha^L\left(\omega,w_L,b_{it-1}\right)=\frac{1}{\delta_B}\left(\frac{\left(\delta_H\overline{e}\right)^{1-\beta}}{\beta^{\beta}(1-\beta)^{1-\beta}\left(1+\delta_H\overline{e}\right)}\right)^{1/\beta}\omega^{-1}\frac{\lambda}{\lambda-R(w_L)}b_{it-1}{}^{-\eta} \quad \text{(B8)}$$

with $\frac{\partial\alpha^L}{\partial\omega}<0,\ \frac{\partial\alpha^L}{\partial w_L}<0,\ \frac{\partial\alpha^L}{\partial b_{it-1}}<0,\ \frac{\partial^2\alpha^L}{\partial b_{it-1}{}^2}>0$.

B.3. Education decision in skilled families

In a skilled family, the borrowing cost is $R(w_{it-1})=R(w_H s_{it-1})$ and the parent's human capital $h_{it-1}=s_{it-1}$. Consequently, the condition (B7) to choose higher education is:

$$a_{it}\geq\alpha^H\left(\omega,w_H,s_{it-1}\right)=\frac{1}{\delta_B}\left(\frac{\left(\delta_H\overline{e}\right)^{1-\beta}}{\beta^{\beta}(1-\beta)^{1-\beta}\left(1+\delta_H\overline{e}\right)}\right)^{1/\beta}\omega^{-1}\frac{\lambda}{\lambda-R(w_H s_{it-1})}s_{it-1}{}^{-\eta} \quad \text{(B9)}$$

with $\frac{\partial\alpha^H}{\partial\omega}<0,\ \frac{\partial\alpha^H}{\partial w_H}<0,\ \frac{\partial\alpha^H}{\partial s_{it-1}}<0,\ \frac{\partial^2\alpha^H}{\partial s_{it-1}{}^2}>0$.

Appendix C. Unconditional parental bequests

The parents give to their children the bequest ρw_{it-1}. If the child decides to enter the university, the related fee is deduced from this amount and the child adds to her/his income the net bequest $\rho w_{it-1} - w_H f_{it}$ which value is $R\left(\rho w_{it-1} - w_H f_{it}\right)$ in her/his working life (R = 1+r, with r the interest rate corresponding to the time between the moment when the bequest is received and the moment when it is spent). If the child selects to remain at the basic education level, the value of the bequest when being in her/his working life is $R\rho w_{it-1}$ (we assume to simplify that the R is the identical in both cases).

Assuming that the individual is not limited in her education funding, the maximisation of if the individual attends the university can be written:

$$\max_{s_{it}, e_{it}, f_{it}} u_{it} = \left(w(s_{it})\right)^{\beta} \left(\bar{e} - e_{it}\right)^{1-\beta}, \quad \text{s.t.: } 1)\ w(s_{it}) = w_H s_{it} - R w_H f_{it} + R\rho w_{it-1},$$

$$2)\ s_{it} = b_{it}(1 + \delta_H e_{it}) = \lambda f_{it},$$

$$3)\ b_{it} = \delta_B a_{it} h_{it-1}{}^{\eta}$$

$$\max_{e_{it}} u_{it} = \left((1 - R/\lambda) w_H b_{it} (1 + \delta_H e_{it}) + R\rho w_{it-1}\right)^{\beta} \left(\bar{e} - e_{it}\right)^{1-\beta}$$

At the optimum:

$$\hat{e}_{it} = \frac{\beta(1 + \delta_H \bar{e}) - 1}{\delta_H} - \frac{(1-\beta) R\rho}{\delta_H (1 - R/\lambda) b_{it}} \frac{w_{it-1}}{w_H} \tag{C1}$$

$$\hat{s}_{it} = \beta(1 + \delta_H \bar{e}) b_{it} - (1-\beta) \frac{R\rho w_{it-1} / w_H}{1 - R/\lambda} \tag{C2}$$

$$\hat{f}_{it} = \frac{\beta}{\lambda}(1 + \delta_H \bar{e}) b_{it} - (1-\beta) \frac{R\rho}{\lambda - R} \frac{w_{it-1}}{w_H} \tag{C3}$$

The financial constraint ($\rho w_{it-1} \geq w_H f_{it}$) is:

$$\rho w_{it-1} \geq w_H \beta \lambda^{-1} (1 + \delta_H \bar{e}) b_{it} - (1-\beta) \frac{R\rho}{\lambda - R} w_{it-1} \tag{C4}$$

The utility related to basic education (parental bequests are fully inserted in the income):

$$u\left(w(b_{it}), 0\right) = \left(w_L + R\rho w_{it-1}\right)^{\beta} \bar{e}^{1-\beta} \tag{C5}$$

1) Unconstrained individuals

Net income if selecting university when unconstrained

$$w(\hat{s}_{it}) = (1 - R/\lambda) w_H \hat{s}_{it} + R\rho w_{it-1} \quad \text{(C6)}$$

Utility of university when unconstrained ($u\left(w(\hat{s}_{it}), \hat{e}\right) = \left(w(\hat{s}_{it})\right)^{\beta} \left(\bar{e} - \hat{e}\right)^{1-\beta}$):

$$u\left(w(\hat{s}_{it}), \hat{e}\right) = \left((1 - R/\lambda) w_H (1 + \delta_H \hat{e}_{it}) b_{it} + R\rho w_{it-1}\right)^{\beta} \left(\bar{e} - \hat{e}_{it}\right)^{1-\beta} \quad \text{(C7)}$$

Condition for university attending when unconstrained

$$u\left(w(\hat{s}_{it}), \hat{e}\right) > u\left(w_L, 0\right) \quad \text{(C8)}$$

2) Constrained individuals

Fee if university attending

$$w_H \tilde{f}_{it} = \rho w_{it-1} \Rightarrow \tilde{f}_{it} = \rho \frac{w_{it-1}}{w_H} \quad \text{(C9)}$$

Skill, effort, net income and utility when the constrained individual attends the university:

$$\tilde{s}_{it} = \lambda \tilde{f}_{it} = \lambda\rho \frac{w_{it-1}}{w_H} \quad \text{(C10)}$$

$$\tilde{e}_{it} = \frac{\tilde{s}_{it} / b_{it} - 1}{\delta_H} = \frac{\lambda\rho w_{it-1} / w_H b_{it} - 1}{\delta_H} \quad \text{(C11)}$$

$$w(\tilde{s}_{it}) = w_H \tilde{s}_{it} = \lambda\rho w_{it-1} \quad \text{(C12)}$$

$$u\left(w(\tilde{s}_{it}), \tilde{e}_{it}\right) = \left(\lambda\rho w_{it-1}\right)^{\beta} \left(\bar{e} - \frac{\lambda\rho w_{it-1} / w_H b_{it} - 1}{\delta_H}\right)^{1-\beta} \quad \text{(C13)}$$

C.1. Unskilled families

In the case of unskilled families, we have $w_{it-1} = w_L + R_{-1}\rho w_{it-2}$ and $h_{it-1} = b_{it-1}$. The financial constraint (4) is thus:

$$b_{it} \le \frac{(1-\beta)R\rho + \rho\lambda(1 - R/\lambda)}{\beta(1 + \delta_H \bar{e})(1 - R/\lambda)} \frac{w_L + R_{-1}\rho w_{it-2}}{w_H} \quad \text{(C14)}$$

Both the increase in w_H and the decrease in w_L reinforce the financial constraint.

As the parents are unskilled, all the grandparents' bequest are included in their income (no education spending) and the individual's expected income when selecting basic education is:

$$w_L + R\rho w_{it-1} = w_L + R\rho(w_L + R_{-1}\rho w_{it-2}) \quad \text{(C15)}$$

C.1.1. *Unconstrained individuals*

The unconstrained individual's effort $\hat{e}_{it}$, skill, income ($w(s_{it})=(1-R/\lambda)w_H s_{it}+R\rho w_{it-1}$) and utility $u\left(w(\hat{s}_{it}),\hat{e}_{it}\right)=\left(w(\hat{s}_{it})\right)^{\beta}\left(\bar{e}-\hat{e}_{it}\right)^{1-\beta}$ if s/he select the university are:

$$\hat{e}_{it}=\frac{\beta(1+\delta_H\bar{e})-1}{\delta_H}-\frac{(1-\beta)R\rho}{\delta_H(1-R/\lambda)b_{it}}\frac{w_{it-1}}{w_H} \tag{C16}$$

$$\hat{s}_{it}=\beta(1+\delta_H\bar{e})b_{it}-(1-\beta)\frac{R\rho}{1-R/\lambda}\frac{w_{it-1}}{w_H} \tag{C17}$$

$$w(\hat{s}_{it})=\beta\left((1-R/\lambda)(1+\delta_H\bar{e})w_H b_{it}+R\rho w_{it-1}\right) \tag{C18}$$

$$u\left(w(\hat{s}_{it}),\hat{e}_{it}\right)=\frac{\beta^{\beta}(1-\beta)^{1-\beta}}{\left(\delta_H(1-R/\lambda)w_H b_{it}\right)^{1-\beta}}\left((1-R/\lambda)(1+\delta_H\bar{e})w_H b_{it}+R\rho w_{it-1}\right) \tag{C19}$$

The utility when selecting basic education is:

$$u\left(w(b_{it}),0\right)=\left(w_L+R\rho w_{it-1}\right)^{\beta}\bar{e}^{1-\beta} \tag{C20}$$

The condition to select the university if not constrained ($u\left(w(\hat{s}_{it}),\hat{e}_{it}\right)>u\left(w_L,0\right)$ is:

$$Z\left(w_H,w_L,w_{it-1}\right)\equiv\frac{1+\delta_H\bar{e}}{\delta_H\bar{e}}b_{it}-\frac{1}{\beta^{\beta}(1-\beta)^{1-\beta}}\left(\frac{w_L+R\rho w_{it-1}}{\delta_H\bar{e}(1-R/\lambda)w_H}\right)^{\beta}b_{it}^{1-\beta}+\frac{R\rho w_{it-1}}{\delta_H\bar{e}(1-R/\lambda)w_H}>0,$$

with: $w_{it-1}=w_L+R_{-1}\rho w_{it-2}\Rightarrow R\rho w_{it-1}=R\rho\left(w_L+R_{-1}\rho w_{it-2}\right)=R\rho w_L+\varepsilon$

$$Z\left(w_H,w_L\right)=\frac{1+\delta_H\bar{e}}{\delta_H\bar{e}}b_{it}-\frac{1}{\beta^{\beta}(1-\beta)^{1-\beta}}\left(\frac{(1+R\rho)w_L+\varepsilon}{\delta_H\bar{e}(1-R/\lambda)}\right)^{\beta}w_H^{-\beta}b_{it}^{1-\beta}+\frac{R\rho w_L+\varepsilon}{\delta_H\bar{e}(1-R/\lambda)}w_H^{-1}$$

$$\frac{\partial Z}{\partial w_H}=\left(\frac{\beta}{1-\beta}\right)^{1-\beta}\left(\frac{(1+R\rho)w_L+\varepsilon}{\delta_H\bar{e}(1-R/\lambda)}\right)^{\beta}w_H^{-1-\beta}b_{it}^{1-\beta}-\frac{R\rho w_L+\varepsilon}{\delta_H\bar{e}(1-R/\lambda)}w_H^{-2}$$

$$\frac{\partial Z}{\partial w_H}>0\Leftrightarrow\left(\frac{\beta}{1-\beta}\frac{w_H(1-R/\lambda)\delta_H\bar{e}b_{it}}{(1+R\rho)w_L+\varepsilon}\right)^{1-\beta}>\frac{R\rho+\varepsilon/w_L}{1+R\rho+\varepsilon/w_L}$$

$$\frac{\partial Z}{\partial w_L}=-\left(\frac{\beta}{1-\beta}\right)^{1-\beta}\left(\frac{(1+R\rho)w_L+\varepsilon}{\delta_H\bar{e}(1-R/\lambda)}\right)^{\beta-1}\frac{(1+R\rho)}{\delta_H\bar{e}(1-R/\lambda)}w_H^{-\beta}b_{it}^{1-\beta}+\frac{R\rho}{\delta_H\bar{e}(1-R/\lambda)}w_H^{-1}$$

$$\frac{\partial Z}{\partial w_L}<0\Leftrightarrow\left(\frac{\beta}{1-\beta}\frac{w_H(1-R/\lambda)\delta_H\bar{e}b_{it}}{(1+R\rho)w_L+\varepsilon}\right)^{1-\beta}>\frac{R\rho}{1+R\rho}$$

Assuming $\varepsilon \approx 0$, $\dfrac{\beta(1-R/\lambda)\delta_H\bar{e}\omega}{1-\beta}b_{it} > \dfrac{(R\rho)^{1/(1-\beta)}}{(1+R\rho)^{\beta/(1-\beta)}} \Leftrightarrow b_{it} > \dfrac{(1-\beta)(R\rho)^{1/(1-\beta)}}{\beta(1-R/\lambda)(1+R\rho)^{\beta/(1-\beta)}\delta_H\bar{e}\omega}$

is then the condition for both $\dfrac{\partial Z}{\partial w_H} > 0$ and $\dfrac{\partial Z}{\partial w_L} < 0$, i.e., for a relax of the condition to select the university . The bequest (positively depending on $R \times \rho$) must not be too high to discourage the individual to makes a sufficient effort to pursue higher education.

Result: *The move in wages slackens the condition to attend the university if* $b_{it} > \underset{\sim}{b} = \dfrac{(1-\beta)(R\rho)^{1/(1-\beta)}}{\beta(1-R/\lambda)(1+R\rho)^{\beta/(1-\beta)}\delta_H\bar{e}\omega}$ *and it strengthens this condition if* $b_{it} < \underset{\sim}{b}$.

As $\hat{s}_{it} \approx \beta(1+\delta_H\bar{e})b_{it} - (1-\beta)\dfrac{R\rho}{1-R/\lambda}\omega^{-1}$, the increase in ω tends to slightly increase the skill attainment of the individuals who select the university. This is because the rise in the skill premium lessens the relative value of the unskilled parents' bequest and reduces thereby the disincentive to education.

C.1.2. *Constrained individuals from unskilled families*

The skill level, the income and the utility of a constrained individuals from an unskilled family when s/he attends the university are:

$$\tilde{s}_{it} = \lambda\rho\omega^{-1} \qquad \text{(C21)}$$

$$w(\tilde{s}_{it}) = w_H\tilde{s}_{it} = \lambda\rho w_L \qquad \text{(C22)}$$

$$u\left(w(\tilde{s}_{it}), \tilde{e}_{it}\right) = \left(\lambda\rho w_L\right)^{\beta}\left(\bar{e} - \frac{\lambda\rho/\omega b_{it} - 1}{\delta_H}\right)^{1-\beta} \qquad \text{(C23)}$$

The related condition for university attending ($u\left(w(\tilde{s}_{it}), \tilde{e}\right) > u\left(w_L, 0\right)$):

$$b_{it} > \underline{b}_{it} = \frac{\lambda\rho}{\delta_H\bar{e}\left(1 - \left(\dfrac{1+R\rho\left(1+R_{-1}\rho w_{it-2}/w_L\right)}{\lambda\rho}\right)^{\frac{\beta}{1-\beta}}\right)+1}\omega^{-1} \qquad \text{(C24)}$$

We denote $z = 1 - \left(\dfrac{1+R\rho+RR_{-1}\rho^2 w_{it-2}/w_L}{\lambda\rho}\right)^{\frac{\beta}{1-\beta}}$. Then:

$$\frac{\partial z}{\partial w_L} = \left(\frac{1+R\rho+RR_{-1}\rho^2 w_{it-2}/w_L}{\lambda\rho}\right)^{-\frac{1}{1-\beta}} RR_{-1}\frac{\rho}{\lambda}\frac{w_{it-2}}{w_L^{\ 2}} > 0\,;\ \frac{\partial z}{\partial w_L} > 0 \Rightarrow \frac{\partial \underline{b}_{it}}{\partial w_L} < 0$$

The increase in ω expands the number of constrained individuals from unskilled families who enrol the university but the decrease in w_L moderates this impact. Assuming $RR_{-1}\rho^2 w_{it-2} \approx 0$, this moderation effect vanishes.

Result: *An increase in the skill premium ω expands the number of constrained individuals from unskilled families who enrol the university and lessens the skill attainment of those who would have selected the university without rise in ω.*

C.2. Skilled families

In the case of skilled families, we have:

$$w_{it-1} = \begin{cases} w_H s_{it-1} & \text{if parents constrained} \\ w_H s_{it-1} + \sigma_{it-1} & \text{if parents unconstrained} \end{cases} \tag{C25}$$

with $\sigma_{it-1} = R_{-1}\rho w_{it-2} - c(s_{it-1})$, and $h_{it-1} = s_{it-1}$.

The financial constraint is then:

$$\rho w_{it-1} \geq w_H \beta \lambda^{-1}(1+\delta_H \bar{e})b_{it} \tag{C26}$$

C2.1. *Unconstrained individuals from skilled families*

The optimal effort, skill and income when attending the university are:

$$\hat{e}_{it} = \frac{\beta(1+\delta_H \bar{e})-1}{\delta_H} - \frac{(1-\beta)R\rho}{\delta_H(1-R/\lambda)b_{it}} \frac{w_{it-1}}{w_H} \tag{C27}$$

$$\hat{s}_{it} = b_{it}(1+\delta_H \hat{e}_{it}) = \beta(1+\delta_H \bar{e})b_{it} - (1-\beta)\frac{R\rho w_{it-1}/w_H}{1-R/\lambda} \tag{C28}$$

$$w(s_{it}) = \beta(1+\delta_H \bar{e})\left(1-R/\lambda\right) w_H b_{it} + \beta R\rho w_{it-1} \tag{C29}$$

$$w(\hat{s}_{it}) = (1-R/\lambda)(1+\delta_H \bar{e})\beta w_H b_{it} + R\rho w_{it-1} \tag{C30}$$

The utility of university when the individual is unconstrained is:

$$u\left(w(\hat{s}_{it}),\hat{e}_{it}\right) = \beta^\beta (1-\beta)^{1-\beta}\left((1-R/\lambda)w_H(1+\delta_H \bar{e})b_{it} + R\rho w_{it-1}\right)^\beta \left(\frac{\delta_H \bar{e}+1}{\delta_H} + \frac{R\rho w_{it-1}/w_H}{\delta_H(1-R/\lambda)b_{it}}\right)^{1-\beta}$$

The condition to attend the university ($u\left(w(\hat{s}_{it}),\hat{e}\right) > u\left(w(b_{it}),0\right)$) is then:

$$\frac{(1-R/\lambda)(1+\delta_H \bar{e})w_H b_{it} + R\rho w_{it-1}}{\left(w_L + R\rho w_{it-1}\right)^\beta \left(\delta_H \bar{e}(1-R/\lambda)w_H b_{it}\right)^{1-\beta}} > \frac{1}{\beta^\beta(1-\beta)^{1-\beta}} \tag{C31}$$

Assuming that $R\rho\sigma_{it-1} = R\rho\left(R_{-1}\rho w_{it-2} - c(s_{it-1})\right) \approx 0$, we have $R\rho w_{it-1} \approx R\rho w_H$ and the above condition and the optimal skill become:

$$\frac{(1-R/\lambda)(1+\delta_H\bar{e})b_{it} + R\rho}{\left(\omega^{-1} + R\rho\right)^{\beta}\left(\delta_H\bar{e}(1-R/\lambda)b_{it}\right)^{1-\beta}} > \frac{1}{\beta^{\beta}(1-\beta)^{1-\beta}} \tag{C32}$$

$$\hat{s}_{it} = \beta(1+\delta_H\bar{e})b_{it} - \frac{(1-\beta)R\rho}{1-R/\lambda} \tag{33}$$

Result: *The increase in the skill premium relaxes the condition to select higher education. In addition the optimal skill level* $\hat{s}_{it}$ *is unchanged.*

C.2.2. *Constrained individuals from skilled families*

The human capital when selecting the university

$$\tilde{s}_{it} = \lambda\tilde{f}_{it} = \lambda\rho\frac{w_{it-1}}{w_H} = \lambda\rho s_{it-1} + \sigma_{it-1}/w_H \tag{C34}$$

The income when selecting the university (all the bequest is spent for education)

$$w(\tilde{s}_{it}) = w_H\tilde{s}_{it} = \lambda\rho w_{it-1} \tag{C35}$$

The effort when attending the university

$$\tilde{e}_{it} = \frac{\lambda\rho w_{it-1}/w_H b_{it} - 1}{\delta_H}$$

The parents' income

$$w_{it-1} = \begin{cases} w_H s_{it-1} & \text{if the parents are constrained} \\ w_H s_{it-1} + \sigma_{it-1} & \text{if the parents are unconstrained} \end{cases} \tag{C36}$$

with : $\sigma_{it-1} = b_{it-2} - c(s_{it-1})$

The utility of university when constrained is:

$$u\left(w(\tilde{s}_{it}), \tilde{e}_{it}\right) = \left(\lambda\rho(w_H s_{it-1} + \sigma_{it-1})\right)^{\beta}\left(\frac{\delta_H\bar{e} + 1 - \lambda\rho(s_{it-1} + \sigma_{it-1}/w_H)/b_{it}}{\delta_H}\right)^{1-\beta} \tag{C37}$$

And the condition to select the university $u\left(w(\tilde{s}_{it}), \tilde{e}_{it}\right) > u\left(w_L, 0\right)$:

$$b_{it} > \frac{\lambda\rho(s_{it-1} + \sigma_{it-1}/w_H)}{\delta_H\bar{e} + 1 - \delta_H\bar{e}\left(\frac{\omega^{-1}}{\lambda\rho(s_{it-1} + \sigma_{it-1}/w_H)} + \frac{R}{\lambda}\right)^{\frac{\beta}{1-\beta}}} \tag{C38}$$

Result: *Both the increase in the skill premium and the increase in the skilled wage* w_H *relax the condition to select the university. When the grandparents' net bequest (bequest minus the education cost) is positive (the parents were not constrained when they opted for the university), then the increase in the skilled wage* w_H *lessens the skill level. Nevertheless this reduction is very limited and far lower than that experienced by constrained individuals from unskilled families.*

Appendix D. Subsidized loans to students

We assume that the government provides students for subsidised loans. Hence, the borrowing cost is now identical for all students, whatever their family type.

$$\max_{s_{it},e_{it},f_{it}} u_{it}=\left(w(s_{it})\right)^{\beta}\left(\bar{e}-e_{it}\right)^{1-\beta}, \quad \text{s.t.: } w(s_{it})=(1-R/\lambda)w_H s_{it},$$

$$s_{it}=b_{it}(1+\delta_H e_{it})=\lambda f_{it},\ b_{it}=\delta_B a_{it} h_{it-1}{}^{\eta}$$

$$\max_{e_{it}} u_{it}=\left(1+R/\lambda\right)^{\beta}\left(w_H b_{it}\right)^{\beta}(1+\delta_H e_{it})^{\beta}\left(\bar{e}-e_{it}\right)^{1-\beta}$$

$$\left(1-\frac{R}{\lambda}\right)^{\beta}\left(w_H b_{it}\right)^{\beta}\left(\beta\delta_H(1+\delta_H e_{it})^{\beta-1}\left(\bar{e}-e_{it}\right)^{1-\beta}-(1-\beta)(1+\delta_H e_{it})^{\beta}\left(\bar{e}-e_{it}\right)^{-\beta}\right)=0$$

Optimal effort, skill and buying of higher education services when attending the university:

$$\hat{e}=\frac{\beta(1+\delta_H\bar{e})-1}{\delta_H} \tag{D1}$$

We assume $\delta_H>\frac{1-\beta}{\beta}\bar{e}^{-1}\Rightarrow\hat{e}>0$. Then:

$$\hat{s}_{it}=b_{it}(1+\delta_H\hat{e})=\beta(1+\delta_H\bar{e})b_{it} \tag{D2}$$

$$\hat{f}_{it}=\hat{s}_{it}/\lambda=\frac{\beta}{\lambda}(1+\delta_H\bar{e})b_{it} \tag{D3}$$

$$w(\hat{s}_{it})=w_H\beta(1-R/\lambda)(1+\delta_H\bar{e})b_{it} \tag{D4}$$

Utility related to basic education

$$u\left(w_L,0\right)=w_L{}^{\beta}\bar{e}^{1-\beta} \tag{D5}$$

Utility related to university attending ($u\left(w(\hat{s}_{it}),\hat{e}\right)=\left(w(\hat{s}_{it})\right)^{\beta}\left(\bar{e}-\hat{e}\right)^{1-\beta}$)

$$u\left(w(\hat{s}_{it}),\hat{e}\right)=\beta^{\beta}(1-\beta)^{1-\beta}\left((1-R/\lambda)w_H b_{it}\right)^{\beta}\left(\frac{1}{\delta_H}\right)^{1-\beta}(1+\delta_H\bar{e}) \tag{D6}$$

Condition to select tertiary education ($u\left(w(\hat{s}_{it}),\hat{e}\right) > u\left(w_L, 0\right)$):

$$b_{it} > \left(\frac{\left(\delta_H \bar{e}\right)^{1-\beta}}{\beta^{\beta}(1-\beta)^{1-\beta}(1-R/\lambda)^{\beta}(1+\delta_H \bar{e})} \right)^{1/\beta} \omega^{-1} \quad \text{(D7)}$$

Equivalently:

$$a_{it} > \left(\frac{\left(\delta_H \bar{e}\right)^{1-\beta}}{\beta^{\beta}(1-\beta)^{1-\beta}(1-R/\lambda)^{\beta}\delta_B{}^{\beta}(1+\delta_H \bar{e})} \right)^{1/\beta} h_{it-1}{}^{-\eta}\omega^{-1} \quad \text{(D8)}$$